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MEASUREMENT OF THE DIAMETER OF MERCURY BY THE HERTZSPRUNG METHOD ON NOVEMBER 7, 1960.

H. Camichel, M. Hugon, J. Rösch

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MEASUREMENT OF THE DIAMETER OF MERCURY BY THE HERTZSPRUNG METHOD ON NOVEMBER 7, 1960.

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ABSTRACT. Observations of the transit of Mercury on November 7, 1960, by Hertzsprung's photometric method and their reduction are described. A general interpretation is given of the discrepancy between double-image micrometer and photometric measurements of small discs. Laboratory experiments have been conducted, affording a complete agreement with this explanation. The effect of the limited diameter of the hole used is also discussed and a correction computed. Finally, the proposed values are 6"84 ± 0"03 for the angular diameter at unit distance, and 5.09 ± 0.07 for the density. /410*

I. INTRODUCTION

As A. Dollfus discussed in one of the last issues of this publication (Dollfus, 1963), we took advantage of the passage of Mercury in front of the Sun on November 7, 1960, in order to measure its apparent diameter and in order to confirm the recommendations of commission No. 16 of the I.A.U. Two of us (Camichel and Rösch, 1962), when we published the measurements obtained by means of a double image micrometer, as well as by the photometric method suggested by Hertzsprung, indicated the reasons by which the observed discrepancy between the results obtained with the two methods could be explained. We also stated why the second method should have greater weight. Since then, we have employed laboratory methods (cited by A. Dollfus) which confirm this point of view and lead us to believe with more certainty that preference should be given to the value obtained by the Hertzsprung method, and that it is not advisable to adopt the average of the latter and the one obtained from measurements with the double image micrometer.

We felt it was necessary to justify this choice by publishing the details of our observations of the sky and our experiments in the laboratory.

II. USE OF THE HERTZSPRUNG METHOD

We wanted to use an arrangement designed to eliminate the brightness differences from one point to another of the photosphere and we described it in a note distributed by commission 16 of the I.A.U. before the phenomenon took place, so that the observers could take advantage of it. This apparatus was built, but a period of several weeks of bad weather hindered these tests up to the morning of November 7. Because an anomaly was observed and could not be

* Numbers in the margin indicate pagination in the original foreign text.

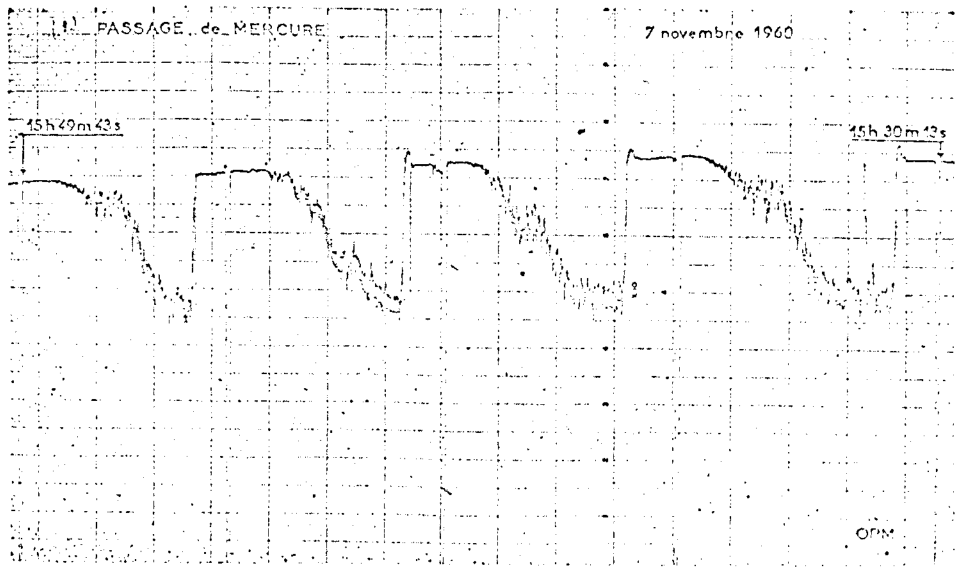

Figure 1. Examples of Recordings (Nos. 5 to 8 in Table I) During the Passage of Mercury in Front of the Sun

(1) - Passage of Mercury.

corrected on location, we used the apparatus in a slightly different way than had been planned. It was considered sufficient to center the image of Mercury in the hole and then, letting the equatorial follow the Sun, the image of the planet was allowed to move out of the hole by its proper motion. The photoelectric current was recorded which shows important fluctuations (see below) and which increases and stabilizes itself when the image is completely outside of the hole. Then the image is brought in again and a new recording is made.

III. ANALYSIS OF THE RECORDINGS /4[illegible]

In this way we obtain 16 recordings, between 14 hours 54 and 16 hours 20 Universal Time. Figure 1 shows four consecutive ones and the following remarks can be applied to them as well as to all others.

(a) In the lower part of the recording, the flux shows rapid variations whose relative importance decreases in proportion to the increase of the mean value, i.e., as the image leaves the hole. The fluctuations become minimal when the hole is illuminated by the solar photosphere not occulted by Mercury.

(b) The stable portions of the curve (without the image of Mercury) are all located on a regular decreasing curve which results from the increase of atmospheric absorption with zenith distance (which was already large at the beginning of the passage). The same curve is found again on a recording made independently (through another objective supported by the same equatorial mounting) in order to control the possible variations of transparency during the duration of a recording. The atmosphere was very regular and no significant variation was observed. The "hooks" which appear before each return to the minimum are due to re-centering motions which bring a region of the photosphere which is closer to the center of the disk, into the hole.

(c) In almost all passages we observed one, and sometimes several, "humps" which had variable amplitude and whose distance corresponded to the centering of the image of the planet in the hole, which heightens the recording for several tenths of seconds.

These particular features can immediately be explained if we assume that the atmospheric turbulence spreads out the image of Mercury until it temporarily touches the edges of the hole. The true curve to be used for measurements would not be an S-curve, passing at equal distances from the band limits which makes up the recording of the recorder, but the <u>lower envelope</u> of this band, excluding the "humps" which correspond to a period where the spreading is such that the image is <u>never</u> entirely in the interior of the hole (this is the phenomenon of the image explosion in a very short time period which is well known.) Explaining the fluctuations when the hole contains the image of Mercury by "swarming" of the granules, which would have a relatively larger effect for a less extended zone, seems rather unlikely, because the difference in the recording from the minimum to the maximum is rather large. Experiments carried /412
out during calibration of the apparatus, which will be discussed later on, confirm the first interpretation and led us to discard the second. We therefore assume that the value of the flux ratio with Mercury/without Mercury was the ratio of the ordinate of the lower envelope of the recording to that of the upper curve. However, as we have already said, this latter decreases progressively due to the increasing atmospheric absorption, so that it is not permissible to pick up the former again at the beginning of each passage and to pick up the second at the end of the passage. Also, it would not be correct to take as the upper ordinate of the recording the one recorded at the end of the preceding passage, because the recentering process introduces a region closer to the center of the disk into the field, and the center-edge effect is not negligible. We then use the auxiliary recording already mentioned to determine the ratio between the ordinates at the beginning and the end of each passage. The ordinate at the end of the passage on the recording "with Mercury", multiplied by this ratio, results in the ordinate at the beginning, to which one refers the ordinate of the lower envelope picked up again at the beginning of the passage.

TABLE 1

Passage No.	1	2	3	4	5	6	7	8
Time, Universal Time	15 h 8 m	15 m	20 m	25 m	30 m	36 m	41 m	46 m
Zenith Distance	75.5	76.7	77.4	78.2	79.0	79.9	80.7	[illegible]
Mercury Area/Hole Area	0.3553	3732	3630	3594	3826	3667	3776	[illegible]
Passage No.	9	10	11	12	13	14	15	16
Time, Universal Time	15 h 51 m	55 m	59 m	16 h 4 m	7 m	11 m	15 m	19 m
Zenith Distance	82.1	82.7	83.2	84.0	84.6	85.3	85.9	[illegible]
Mercury Area/Hole Area	0.3761	3648	3789	3661	3320	3207	3307	[illegible]

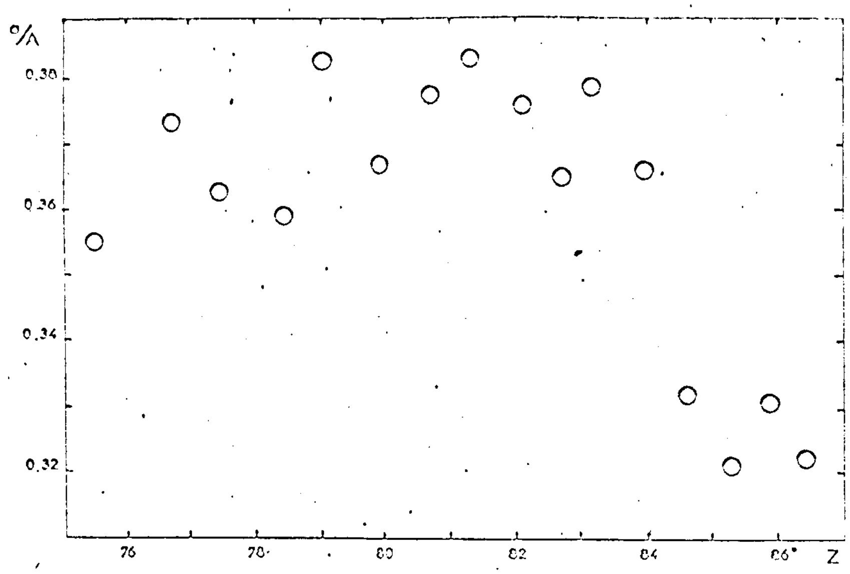

Figure 2. Values Obtained for the Ratio a/A of the area of the Image of Mercury to the Area of the Hole, as a Function of the Zenith Distance z.

The change of the ratio obtained with time is significant. Table 1 shows the complement of this ratio to one (i.e., the ratio of the area a of the image of Mercury to the area A of the hole). Figure 2 shows the variation of this number as a function of the zenith distance.

It is seen that the four latter passages result in values which are much smaller than all the others. Since we are dealing with a black spot on a light background, a larger calibration would result in more light in the hole and thus in an exaggerated value of the measured ratio. Figure 2 shows that the four latter passages should be eliminated. The "humps" have therefore overrun the entire recording, which no longer touches the lower envelope which one would obtain for a smaller degree of turbulence. The 12 other values are distributed in such a way that it is difficult to directly determine the position of the maximum of frequency as a function of the value within the interval that they cover. In fact, one rather finds two maxima, one around 0.366, the other around 0.378. We can assume that the latter is closer to the true value, which is affected by a certain inevitable dispersion. The first is due to the overflowing effect of the image of Mercury outside of the hole, which results in systematically lower values. With this hypothesis, one would be led to also eliminate the six smallest values, which are separated from the six larger ones by a considerable gap. The average would therefore be 0.3785. However, since we do not have a large number of measurements, this is only an estimate, and in all strictness, we should take the ensemble average, which is 0.3705. The appearance of the curves on the majority of the recordings results in small values, which makes the explanation given above very likely. We also assume that better measurements (larger hole) which could be made during a later passage /A12 would result in higher values. Later on you will see that a systematic correction must be applied to these numbers and we will examine the numerical repercussions upon the value of the diameter of Mercury.

IV. CALIBRATION OF THE APPARATUS

We attempted to reproduce conditions in the laboratory which were as close as possible to those under which the observations must be made. Without touching any of the adjustments of the apparatus, we projected the image of a uniformly illuminated region in the focal plane with a known enlargement. The image was obtained by means of an objective which had the same relative aperture as the one used on the sky. Its focal length was large enough (315 cm) so that the pupils, located inside the apparatus, occupied essentially the same positions. In front of this uniform region, we first placed opaque discs of known diameter which were increased in such a way as to reproduce the passage of the planet. . In order to have an independent control method, we also used the straight edge of an opaque screen, in order to determine the diameter of the hole from the curve which gives the flux transmitted during the course of the progressing obturation.

a. Passage of Calibrated Discs

They were made from steel balls. The ball is cemented with colophane to a glass plaque and then rubbed with emery on a flat plane, until the point beyond the diametrically opposite plane is obtained, so that the edge profile makes an acute angle. Its plane side is then cemented to a glass lamina which can be moved in front of the illuminated region , after microscopic measurements of several section diameters have been carried out. The dimensions used extended from 3.0 to 4.2 mm. They can be considered to be known within a probable error of $\pm$ 3μ.

The calibration method consists of measuring a ratio a/A, just as in the sky measurements, for various discs, and to obtain, by graphic interpolation, the diameter of the one which would have resulted in the same ratio as Mercury. If the enlargement of the projection of the discs is known, the linear diameter of the image of Mercury can be derived. Its angular diameter can be determined using the focal length of the objective used in the sky measurements.

The essential difference between these measurements carried out in the laboratory and those made in the sky is the fact that they do not show the fluctuations observed when Mercury was centered in the hole. We took advantage of /414 this arrangement in order to find an explanation of the phenomenon, which we were effectively able to reproduce by causing a strong thermal perturbation of the trajectory traveled by the beam between the source and the objective. The fluctuations already appeared when an electric radiator was used. They become very substantial if mixing of the air masses is introduced by agitating a paddle in the vicinity of the apparatus. Figure 3 shows very clearly that the trace obtained without turbulence is a good one, as we have assumed. The lower envelope of the trace is perturbed. If the perturbation is strong, the trace leaves this envelope and shows "humps" of the same type as were observed in the sky.

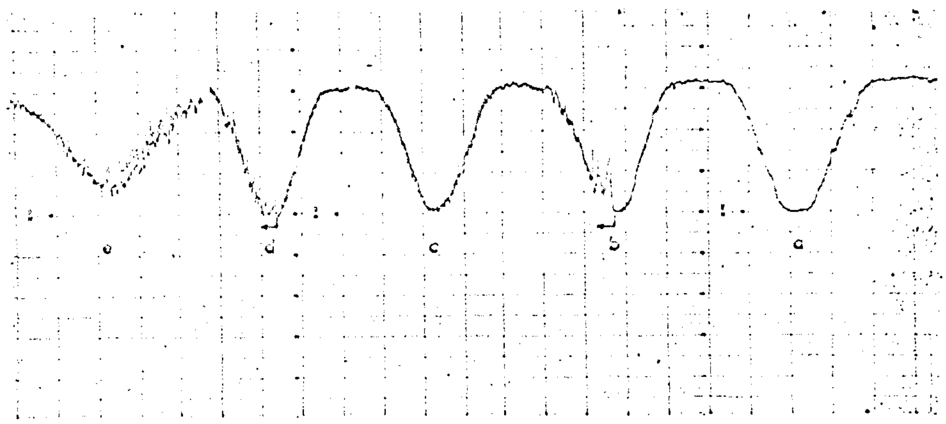

Figure 3. Passage of a Disc in Front of a Uniform Background; (a) Without Perturbation; (b) and (d) 500 W Radiator Under the Beam, Air Mixing Beginning with the Arrow; (c) 500 W Radiator, Without Mixing; (e) 2 kw Radiator, Without Mixing.

It was easy to replace the uniform background by a photospheric granulation device having the desired scale. Experience has shown that this overstimulation of true conditions changes nothing in the phenomena.

The interpolation mentioned above directly results in the angular diameter of Mercury. However, the calculation of the probable error is not easy. It is better to use ratios measured with different discs for calculating many values of the hole diameter and for evaluating the probable error for measurements of this type, and to compare the value obtained after obturation of the hole by a screen having straight edges. A definite value of the diameter of the hole is then decided upon and the diameter of Mercury is derived. The results are as follows:

Diameter of the disc (mm)	2.951	3.189	2.592	4.008	4.198
Diameter of the hole virtually reduced into the plane of the disc (mm)	5.797	5.830	5.819	5.834	5.853

The average is 5.827 with a probable error of ± 0.010 for one measurement.

The tendency toward higher values in the case of the largest discs may indicate that the passage of the disc in front of the hole took place slightly too rapidly and that the deviation of the recorder did not reach its minimum entirely. The average value shown above will therefore usually be too high.

b. Passage of the Straight Edge of a Screen

This screen consisted of a razor blade carried by the carrier of a micrometer ocular (reticle and optics removed). The enlargement was the same as in the preceding operation. The flux entering the apparatus was measured for a series of positions of the screen, located on a millimeter scale. /415

Let us draw the curve showing the flux as a function of the position of the screen and let us determine the position which results in a flux which is exactly one-half of the flux found when there is no obturation (which we will assume as the unit). The edge of the screen then passes through the center of the hole. Let us now call x the abscissa of the screen measured from this position of the origin. Let 2ω be the arc of the circle delimited by the non-covered edge of the hole by the edge of the screen, and ρ the ratio of the non-obturated area to the area of the hole. R is the radius of the hole. We have:

$$\rho = \frac{1}{\pi}(\omega - \sin\omega\cos\omega) \text{ and } x = R\cos\omega.$$

The operation consists of selecting a few values of ω, of picking off the values of ρ calculated by the above formula from the experimental curves, and of picking off the experimental values of x. If x and ω are known, R can be derived. We used four values of ω in the region of the curve which results in the best accuracy, as well as the four supplementary values. The average among the eight values obtained for R eliminates the possible error of the origin of the abscissa. The numbers are as follows:

ω (degrees) =	45	60	66	75	(90)	105	114	120	135
ρ =	0.0908	0.1955	0.2482	0.3370	(0.5000)	0.6630	0.7518	0.8045	0.9092
R/x =	1.414	2.000	2.458	3.864	—	3.864	2.458	2.000	1.414

The last line gives the quantities by which the values of x picked off the curves must be multiplied in order to obtain R.

We carried out measurements for four different diameters of the hole. Following are the values obtained for the diameter of the hole (always reduced in the plane of the screen)

Direction:	a	a	a	b	c	d
Diameter (mm)	5.818	5.780	5.770	5.778	5.826	5.862

The average is 5.797 with a probable error of ± 0.025 in one measurement.

The margins of error of the results obtained by the two methods of calibration overlap, but the method of the screen seems to be less accurate than the method of discs. If we take a general average by giving double weights to the latter, we finally obtain 5.815 mm for the fictitious diameter of the hole, with a probable relative error of 1/1000. If we multiply by the enlargement of the calibration system (0.0826 ± 0.0008), and divide by the focal length of the objective used for the sky experiment (6040 ± 5 mm), and then also multiply by the number of seconds in a radian, we obtain the following for the angular diameter of the region isolated on the Sun during the passage of Mercury: 16"40 with a probable error on the order of 0"05.

Let us again consider the values previously found for the ratio of the area of Mercury to the area of the hole, or 0.3705 for the rough average of the measurements and 0.3785 if we only retain the higher values. By multiplying the square roots of these ratios by the diameter of the hole, we obtain the following for the diameter of Mercury during the observation (distance 0.6754)

	9"98	10"09
and at unit distance	6"74	6"81.

The probable error on the hole diameter is translated into an error of 0"02 in the diameter of Mercury at the unit distance. The systematic correction which we will apply in the following can only increase the above values.

V. COMPARISON WITH THE MEASUREMENTS CARRIED OUT WITH THE DOUBLE IMAGE MICROMETER.

Dollfus (1963), in the article mentioned above, gave results of various observations carried out during the same passage. The rough average of the re- /416 sults gives the diameter 6"67, but the average of the observations with the double image micrometer and by photometry results in 6"63 and 6"72, respectively. In addition, the measurement of H. Camichel with the double image micrometer was arbitrarily increased by 1%, and our measurement carried out with the Hertzsprung method is the first value which we have published and is slightly smaller than the definite value to which we have been led (see above). The discrepancy between the two averages could reach at least 0"15, which is considerably larger than the dispersion belonging to each of these two groups.

Experimenters (Dollfus, 1954) were aware of the possibility of a systematic effect in the measurement of the diameter of a small disc whose edges are not well-defined using a double image micrometer. Two possible causes for this exist:

(a) Let us observe a band with rectilinear edges (uniformly bright on a dark background, or vice versa) with a small resolution limit with respect to its size, which we measure by means of a double image micrometer. The photometric profiles of the images are symmetric with respect to the geometric edges, and the illumination at this point is equal to one-half of the uniform illumination. The superposition of the two images results in a uniform field when the geometric edges coincide, and there is no systematic error at all. This is not the case when we have a disc whose diameter is not very large with respect to the limit of resolution, because the diameter of the circle along which the illumination is one-half of the uniform illumination is smaller than the geometric diameter. This is true whether we have a bright disc on a dark background or a dark disc on a bright background. Thus, if the separation of the two images is precisely equal to this diameter, the illumination at the center of the line of centers is increased in the case of dark discs and diminished in the case of bright discs. If the observer tends to obtain a uniform illumination along the

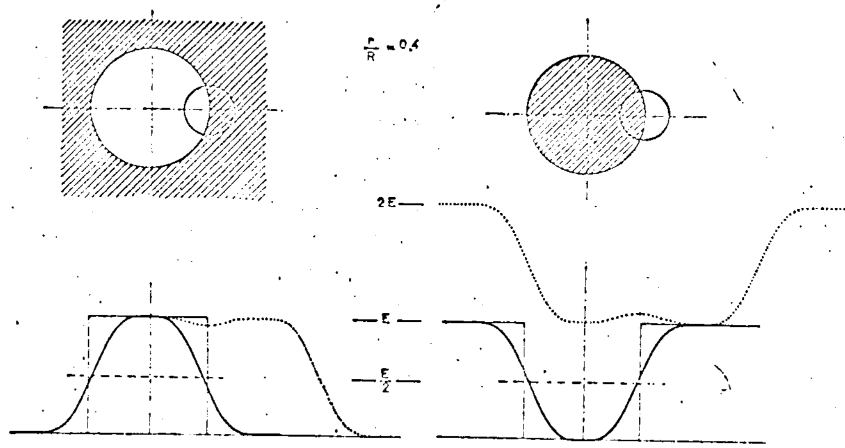

Figure 4. Effect of a Faulty Adjustment Spot of Radius r on the Increase of the Diameter of a Disc with Radius R. On the Left, Disc with Illumination E on a Black Background. On the Right, Black Disc on an Illuminated Background E. Dotted Lines Show the Addition of the Illumination of the Two Images.

line of centers, he will measure a diameter which is too small in the two cases (Figure 4).

(b) In addition to this strictly physical effect, psycho-physiological effects can be superimposed, such as the appearance of "ligaments" at the moment of contact of the two images, and it is difficult to predict the effects on the measurements a priori.

We built a laboratory apparatus which produces an extrafocal image of an opaque disc on a bright, uniform background under conditions suited for showing these deviations (Figure 5).

In front of the diffusor D_1 (opal glass) one places a circular, opaque calibrated disc R (diameter 6 mm), at the focal point of an objective O_1 (Tessar) with a focal length of 21 cm which gives an image at infinity. A photographic objective O_2 with a focal length of 90 mm and with a helicoidal mount produces an image on the second diffusor D_2, which can be distinct or not. This image, which has edges which are more or less spread out, is reduced by an objective with a focal length of 25 mm (Plossl ocular) to a dimension which the image of Mercury had on October 7, 1960 in the focal plane of the 21 cm objective which was used for the measurements with the double image micrometer. It is exactly this same micrometer of the Lyot type (inclinable calcite blade) which is used after a transformation of the image to the magnitude -1, in order to accomodate the increase. /417

In order that the measurements are directly significant, it is important

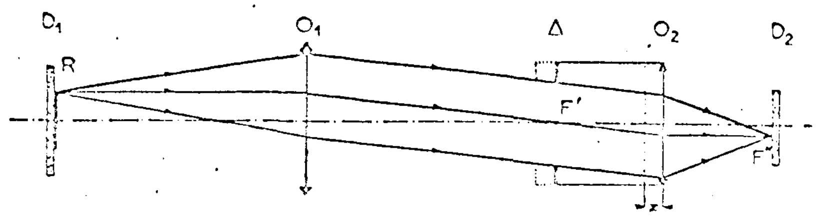

Figure 5. Diagram of the Laboratory Device (see the text for explanations).

that the object observed on the diffusor D_2 be defined as the transformation of an opaque disc with a fixed diameter transformed by a certain dissipation function applied to each of its points. The diameter of the resulting object will be measured for different values of the parameter which characterizes the scale of this dissipation function with respect to the geometric diameter (diameter for zero dissipation). Any dissipation function can be obtained by inserting a screen in the bundle between O_1 and O_2 (parallel light) which has a transparency which varies from one point to another which is precisely represented by the desired function, and by placing D_2 at a certain distance z of the focal plane of O_2. Thus, one obtains the desired dissipation function on D_2, with a size parameter proportional to z. In order that this function can be applied to a disc (or to an arbitrary object) with fixed geometrical dimensions, it is sufficient that the screen be located in the object focal plane of O_2. This is because the principal rays emanating from the different points of the contour of the object at infinity all pass through the focal point, which define a projection with fixed dimensions of this object on O_2. These rays consequently result in a projection on the diffusor D_2 which is identical to itself no matter what the defocalization is.

In this plane one can place a screen whose transparency reproduces the distribution of the illuminations in an arbitrary dissipation figure. In fact, the dissipation functions encountered in practice all give the illumination function near the edge an S-shape whose entire central part will have a slope which is very slightly different from the slope at the point of inflection. Let us draw this curve, and take as the unit on the abscissa scale the radius of the geometric disc. The unit of the ordinate scale will be the ordinate of the origin. If the ordinate perpendicular to the geometric edge is equal to $1/2 \pm y$ $(y > 0)$, and if the slope at the same point is $1/m$, the amount by which the centers of the two discs must be moved closer to each other so that the illumination along the line of centers appears uniform will be obviously equal to 2 my. Even if two dissipation functions result in different relationships between m and y when their size parameter is varied, we may consider that the correction to be made to the measurements will be the same for the two values of the geometric parameters belonging to each function, which results in the same value of the product my.

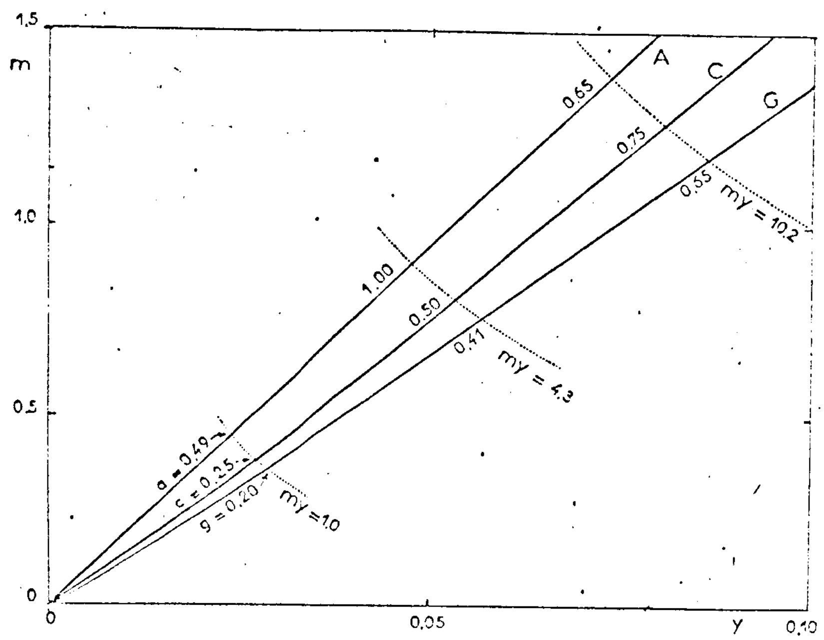

Figure 6. Slope of the profile at the Geometric Edge, as a Function of the Complement y of the Ordinate at the Value 1/2, for Dissipation Functions of Airy (A), Gauss (G), and Defocalization (C).

We carried out the calculation of m and y for the following:

an Airy function whose first black ring has the radius a times the radius of the geometric disc;

a Gauss function whose ordinate is equal to 1/e at a distance from the center equal to g times the radius of the geometric disc;

a "circle" (defocalization) function whose radius is equal to c times that /418 of the geometric disc.

Figure 6 shows m as a function of y for the three functions, where the values of a, g and c are parted along each of the curves. According to what we have assumed to be true, the curves corresponding to equal values of the correction (labeled in percent of the real diameter of the disc) are equilateral hyperbolas and are shown in dotted lines. The intersections of a hyperbola with the three curves result in the values of a, g and c which must have the same effect on the measurements.

We will thus limit ourselves to determining whether, for one type of function, the correction found experimentally is in agreement with the one predicted by the calculation. For convenience, we have chosen the function by simply using a circular diaphragm Δ connected with O_2 and contained in its object focal plane. O_2 and Δ are displaced together while D_2 remains fixed.

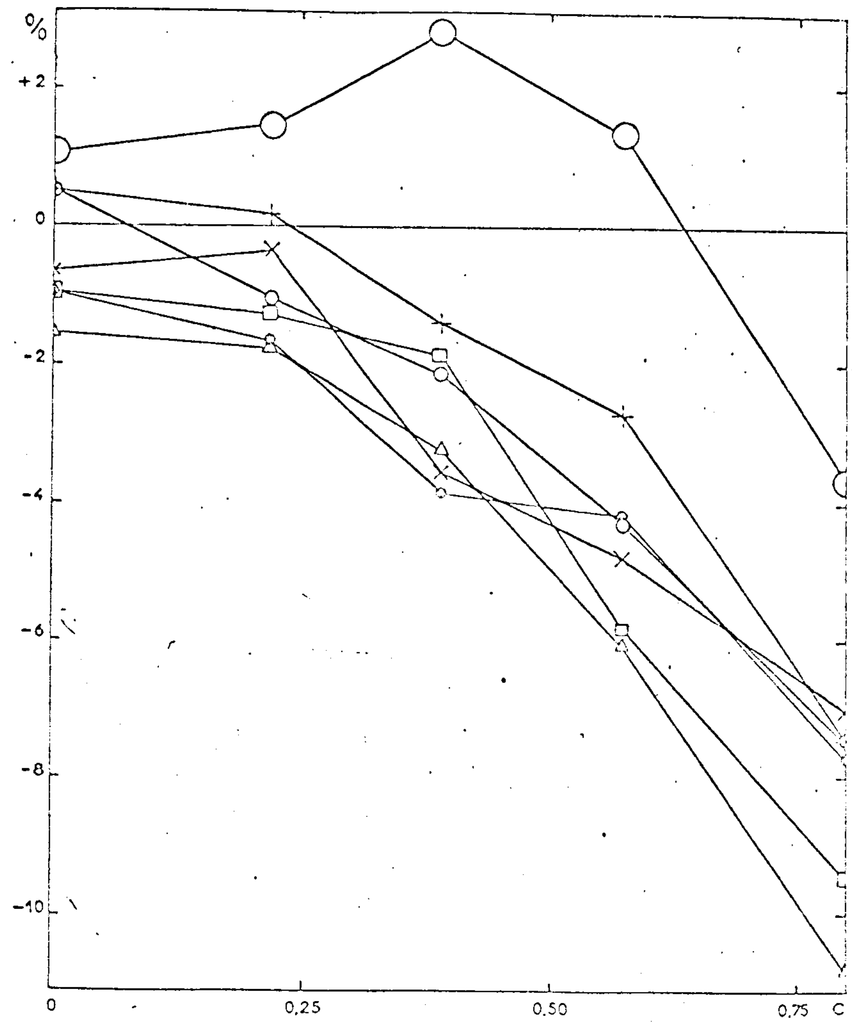

Figure 7. Double Image Micrometer Measurements of the Diameter of a Small, Black Disc on a Bright Background, with a Defocussing Spot with a Radius Equal to c Times the Radius of the Disc. Observers:

△, Ce. Boyer; ○, H. Camichel; +, F. Chauveau; ◯, A. Dollfus; o, M. Hugon; □, ×, J. Rösch.

Figure 7 shows the results obtained by seven different observers. Only one obtained values which differ considerably from the average of the others (and who carried out a smaller number of measurements). Figure 8 shows the comparison of this average and the calculated values of my for the circle function corresponding to the measurements. The difference between the two curves is less than is found between observers. More precisely, let us say that the measurements were carried out in two directions of double decomposition of the micrometer, which are parallel and perpendicular to the line of sight, respectively. The difference between the two series was not significant in any case and changed sign from one measurement to the other so that for each observer

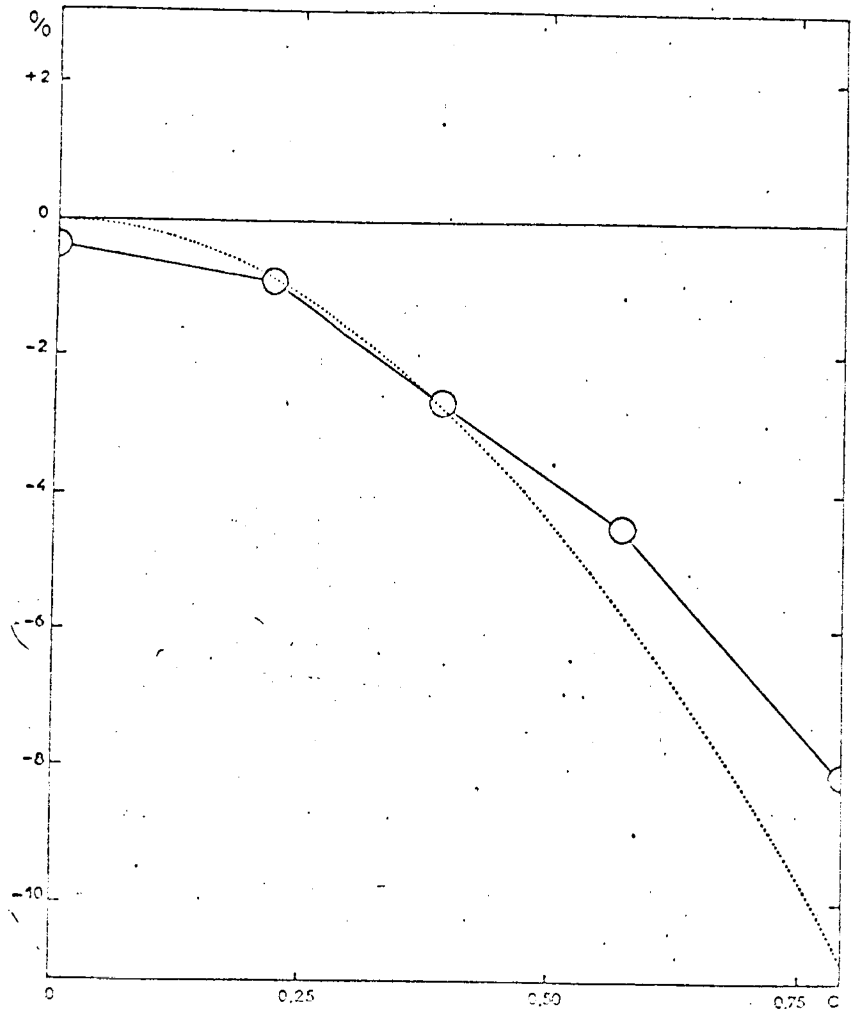

Figure 8. Solid Lines: Average of the Six Observers Grouped in the Best Way. Dotted Lines: Curve Derived from Curve C of Figure 6.

we took the average of the two.

One also notes an excellent agreement between the measurement carried out with the perfectly adjusted micrometer and the value calculated from the diameter of the object and the characteristics of the optical increase.

This agreement makes it possible to exclude physiological effects mentioned /419 above.

Having interpreted the effect of the apparent reduction of the diameter in this way, we may look for the dissipation function capable of explaining the difference between the averages of the diameter of Mercury on November 7, 1960, by the two methods, which is .5 to 3% approximately (it would be illusory to take a more accurate number). Depending on the type of function, one finds either a defocussing circle with a radius of 2", or a Gaussian distribution in which the illumination is divided by e at 1"6 from the center, or an Airy spot whose first dark ring will have a radius of 4".

It is this latter function which is the most plausible one (possibly augmented by an adjustment deficiency which varies with the deformations of the wave surface). The Gauss function, considered in principle, decreases in an exponential manner which does not represent the nature of diffraction phenomena encountered in practice.

VI. EFFECT PRODUCED BY BROADENING THE IMAGE ON PHOTOMETRIC MEASUREMENTS. /420

The fluctuations which we found in the recording of the photoelectric current can be explained either by the variation in the broadening of the image between a minimum value and a maximum value, which is more or less stable, or by the random motion of the center of the image, or by the combination of the two. The motion is much less effective than the broadening change, as to a great extent, the flux obtained from the side where the image is extended from the edge of the hole compensates for the flux loss on the other side (the closer the edge of the hole is to the inflection of the profile of the image, the more accurately this will be the case). Therefore, we have only considered the case in which the variation of the broadening alone is present. It is understood that a small fraction of the total effect, which is difficult to estimate, can be due to motion.

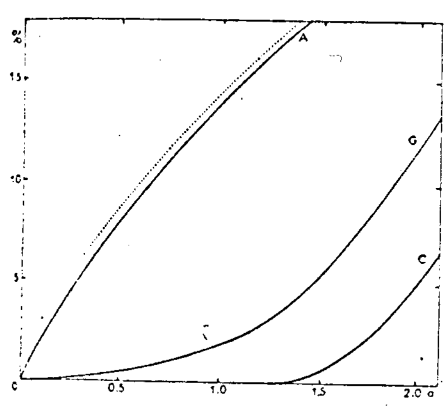

Figure 9. Fraction of the Energy Outside of a Hole With a Diameter of 1.64 Times That of a Disc, for an Airy Function (A) Whose First Black Ring Has a Radius Equal to a Times That of the Disc. The Dotted Line Shows the Real Fraction which is Assumed. G and C are Curves Corresponding to Gauss Functions and Defocalization Functions which Result in the Same Value of my as the Airy Function A.

Figure 9 shows, for the case of a bright disc on a black background as a function of the radius of the Airy spot referred to the radius of the disc, the fraction of light falling outside of a circle whose diameter is 1.64 times the diameter of the observed disc. Therefore, we will have the same effect (by permutating "black" and "light") as for a 10"0 image of Mercury on a 16"4 /421
hole. We show the curves (by making the abscissas correspond which cause the discs to approach each other in the same amount) for the circle function (obviously out of the question for this phenomenon) and the Gauss function which does not fit either, because it is precisely the elongated regions which come into play here. On the other hand, it is well known that the forms of instrument profiles found in practice in general result in higher ordinates at a certain distance from the center than the Airy function, which is the

most representative at the center. The result is that on the graph the real curve must undoubtedly be placed above (dotted curve) the calculated curve, by an amount which is difficult to calculate.

This curve and the recordings lead us to make the following remarks:

(a) The amplitude type of the fluctuations observed (see, for example, in Figure 1 the passage which begins around 15 h 36) corresponds to a difference between the average current and the low envelope on the order of 11% of the difference between the currents with and without Mercury. This quantity measured along the ordinate scales must be entered in Figure 9. The corresponding abscissa is on the order of 3"5, which is in very good agreement with the radius obtained in order to explain the difference between the measurements made by the two methods.

(b) For the four last passages (see Figure 2), the low envelope of the trace already results in an area ratio less than 15% at the maximum value given by the preceding passages. Stated in another way, the broadening will always be greater than 5" at this time, which is completely probable (sec z > 10!).

It can be seen that the fraction outside of the diameter of the hole is not negligible even for very weak broadening: it could be equal to 1.5 to 2% (depending on the value of the excess above the Airy curve) in the case where the theoretical resolution power of the objective of 38 cm, which we use, is reached. Here we are dealing with an unavoidable, systematic effect which of necessity makes it necessary to increase the diameter of Mercury found[1].

Considering that the lower envelope represents the flux received under optimum conditions, the correction to be made to our measurements will increase from 0"05 to 0"07. From this we obtain the limits 6"79 and 6"88 according to which we take the average of all our values or of the six largest ones. We would like to note that the measurements of J. L. Leroy (Dollfus, 1963), carried /422 out with an objective 95 mm in diameter and with a 18"33 hole, must also be corrected between 2 and 2.5% in the diameter, or 0"13 to 0"17. This quantity added to the published value will result in 6"83 to 6"87, which is in very good agreement with the above numbers.

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(1) At the time we finished editing this article, M. Zwaan, from the Utrecht Observatory, was kind enough to send us a note in which he suggested, based on our first publication(Camichel and Rösch, 1962), that our method of analysis cannot separate the measurements of the broadening effect outside of the hole, which he assumed to be important and approximately constant. It seems to us that this property of being constant does not conform with reality. We believe that the lower envelope of the traces corresponds to a minimum broadening near the theoretical value (without which the small dispersion of the measured values would be surprising). It would then suffice to make a small correction to this value which would remain small, and we estimate that by then the density would not be reduced to values on the order of 4.5 as M. Zwann has suggested.

VII. CONCLUSIONS. DIAMETER AND DENSITY OF MERCURY

We believe we have given a quantitative interpretation of the deviation between the measurements carried out with the double image micrometer and the measurements using the Hertzsprung method. We therefore recommend that the value obtained by this method be adopted for the diameter of Mercury at unit distance. Taking into account the systematic correction found in the preceding paragraph and the probable errors determined above, we finally propose the value obtained by giving double weight to the higher values, i.e.,

$$6''84 \pm 0''03 \qquad \text{or} \qquad 4960 \pm 20 \text{ km}$$

which, if associated with the mass found by Rabe, results in the following for the density:

$$5.09 \pm 0.07.$$

This value of the diameter is slightly higher than the one which we gave previously. After all the revisions have been carried out, we consider this result as the final (ne varietur) result of our observations, without discarding the possibility that later observations, which will take into account our experience, will result in a number which is slightly higher.

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