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PROPAGATION AND DISPERSION OF HYDROMAGNETIC AND ION CYCLOTRON WAVES IN PLASMAS IMMERSED IN MAGNETIC FIELDS

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ABSTRACT

Experimental and theoretical investigations of ion cyclotron waves and related wave phenomena in magnetoplasma are discussed. Experimental facilities for the generation and detection of harmonic ion cyclotron waves and the fast wave associated with the ion cyclotron wave are described. Results are presented from the initial tests of a new preionization device of the reflex P.I.G. type. An analysis is presented of the Stix coil as a means of exciting quasistatic ion cyclotron waves. In the first approximation, coupling to the wave is shown to be a secondary effect, as the primary fields of the Stix coil do not couple directly to the wave. An outline is presented of the formalism for calculating the coupling coefficients of the Stix coil for quasistatic ion cyclotron waves assuming cold plasma boundary conditions. Experimental investigations of propagation of the fast hydromagnetic wave are presented. It is shown that the observed propagation in "bands" can be explained in terms of higher order transverse and azimuthal modes and that, with this assumption, the predicted waveguide cut-off is in agreement with the experimentally observed cut-offs.

FACULTY AND STAFF

The following individuals contributed to one or more areas of the research in progress at various times during the period July 15, 1967 to January 14, 1968.

1. Dr. Arwin A. Dougal; Professor of Electrical Engineering, and Principal Investigator (July 15, 1967 through September 14, 1967; currently "on approved leave" of absence from the University).
2. Dr. Otto M. Friedrich, Jr.; Assistant Professor of Electrical Engineering, and Principal Investigator (September 15, 1967 through January 14, 1968).
3. Dr. Hans Schlüter; Professor of Physics and of Electrical Engineering (September 15, 1967 through January 14, 1968; at "no cost" to the NASA Grant NsG-353).
4. Jimmy G. Melton; Doctoral Candidate, and Research Engineer Assistant III.
5. Nathan B. Dodge; Doctoral Candidate, and Research Engineer Assistant III.
6. Glenn C. Andrew; Undergraduate, and Laboratory Research Assistant I.
7. Mrs. Barbara Himes; Administrative Secretary.

COMMUNICATIONS AND COORDINATION OF RESEARCH WITH NATIONAL AERONAUTICS AND SPACE ADMINISTRATION RESEARCH CENTERS

1. September 11-13, 1967, AIAA Electric Propulsion and Plasmadynamics Conference, Colorado Springs, Colorado: Dr. Otto M. Friedrich, Jr. met with Dr. Robert V. Hess, P. Brockman, D. R. Brooks, and J. Burlock, NASA-Langley Research Center, and discussed recent experimental and analytical research on gaseous plasmas.

2. September 19-22, 1967, Pulsed High-Density Plasma Conference, Los Alamos, New Mexico: Dr. Otto M. Friedrich, Jr. met with Charles J. Michels of NASA-Lewis Research Center, and discussed recent experimental and analytical research on dynamic gaseous plasmas.

3. November 2, 1967, NASA Headquarters, Washington, D. C.: Dr. Otto M. Friedrich, Jr. met with Dr. K. Thom and I. R. Schwartz of NASA Headquarters to discuss research in progress, recent new results, and plans on ion cyclotron and harmonic wave generation and propagation.

4. November 7, 1967, The University of Texas at Austin, Austin, Texas: Dr. K. Thom of NASA Headquarters and Jack Reinmann of NASA-Lewis Research Center visited with Jimmy G. Melton, Nathan B. Dodge, Dr. Otto M. Friedrich, Jr., Dr. Hans Schlüter, and others at the University. Research discussions on new research results on harmonic wave generation and propagation, and nonlinear wave phenomena were held. Future plans and current research on NASA Grant NsG-353 were described.

PAPERS PRESENTED, LECTURES, AND PUBLICATIONS BY FACULTY AND STAFF*

A. Papers Presented:

1. M. Kristiansen and A. A. Dougal, "Plasma Heating and Wave Propagation at Harmonics of the Ion Cyclotron Frequency," Second European Conference on Controlled Fusion and Plasma Physics, Stockholm, Sweden, August 14-17, 1967.

2. Otto M. Friedrich, Jr., "Electrical Breakdown Mechanisms in High-Pressure, Laser-Induced Plasmas, and Strong Blast Waves," Gordon Research Conference on Laser Interaction with Matter, Crystal Inn, Washington, August 14-18, 1967.

3. Arwin A. Dougal and Dennis H. Gill, "Breakdown Mechanisms for Laser-Induced Discharges in Super-High Pressure Gases," Eighth International Conference on Phenomena in Ionized Gases, Vienna, Austria, August 27-September 2, 1967.

4. Hans Schlüter, "Noise Emission of Electrodeless Ring Discharges with Static Magnetic Field," Eighth International Conference on Phenomena in Ionized Gases, Vienna, Austria, August 27-September 2, 1967.

5. Otto M. Friedrich, Jr., Frederic Weigl, and Arwin A. Dougal, "Investigation of Strong Blast Waves and the Dynamics of Laser Induced Plasmas in High Pressure Gases," AIAA Electric Propulsion and Plasmadynamics Conference, Colorado Springs, Colorado, September 11-13, 1967.

6. Howard N. Roberts, Frederic Weigl, and Arwin A. Dougal, "Space-Time Resolved Mach-Zehnder Interferometer Measurements of Cross-Sectional Electron Distribution in Theta Pinch Plasmas," Pulsed High-Density Plasma Conference, Los Alamos, New Mexico, September 19-22, 1967.

7. Guy Walter Haynes and Arwin A. Dougal, "Laser-Induced Breakdown in Super-High Pressure Neon," Twentieth Annual Gaseous Electronics Conference, San Francisco, California, October 18-20, 1967.

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*Consolidates all papers presented, lectures, and publications by faculty and staff.

8. J. G. Melton, N. B. Dodge, and Arwin A. Dougal, "Observation of Frequency Mixing and Generation of Harmonics in a Cylindrical Radio Frequency-Excited Plasma," Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, November 8-11, 1967.

9. Frederic Weigl, Otto M. Friedrich, Jr., and Arwin A. Dougal, "An Adaptation of Holographic Interferometry to Obtain Effective Infinite-Fringe Mach-Zehnder Interferograms with a Non-Critical System," Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, November 8-11, 1967.

10. Howard N. Roberts, Frederic Weigl, and Arwin A. Dougal, "Effects of Pressure, Bias B-Field, and Peak B-Field in Theta-Pinch Plasma on End-Loss, Bias B-Field Trapping, and Neutron Production," Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, November 8-11, 1967.

11. Hans Schlüter, "Laboratory Studies of the Lower Hybrid Resonance," Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, November 8-11, 1967 (invited talk).

B. Lectures Presented:

1. Otto M. Friedrich, Jr., "Report on Gordon Research Conference on Laser Interaction with Matter," Plasma Dynamics and Quantum Electronics Seminar, The University of Texas, Austin, August 21, 1967.

2. Arwin A. Dougal, "Laser Induced Discharges in Super High Pressure Gases, and Mach-Zehnder Interferometry of Dense Plasma Pinches," Interdepartmental Physics Colloquium, Westinghouse Research and Development Center, Pittsburgh, Pennsylvania, September 28, 1967.

C. Publications:

1. M. Kristiansen and A. A. Dougal, "Plasma Heating and Wave Propagation at Harmonics of the Ion Cyclotron Frequency," Proceedings of the Second European Conference on Controlled Fusion and Plasma Physics, Stockholm, Sweden, August 14-17, 1967 (ABSTRACT).

2. Arwin A. Dougal and Dennis H. Gill, "Breakdown Mechanisms for Laser-Induced Discharges in Super-High Pressure Gases," Proceedings of the Eighth International Conference on Phenomena in Ionized Gases, Vienna, Austria, p. 262, August 27-September 2, 1967.

3. Hans Schlüter, "Noise Emission of Electrodeless Ring Discharges with Static Magnetic Field," Proceedings of the Eighth International Conference on Phenomena in Ionized Gases, Vienna, Austria, August 27-September 2, 1967.

4. Otto M. Friedrich, Jr., Frederic Weigl, and Arwin A. Dougal, "Investigation of Strong Blast Waves and the Dynamics of Laser Induced Plasmas in High Pressure Gases," AIAA Bulletin 4, p. 444, September, 1967 (ABSTRACT).

5. Otto M. Friedrich, Jr., Frederic Weigl, and Arwin A. Dougal, "Investigation of Strong Blast Waves and the Dynamics of Laser Induced Plasmas in High Pressure Gases," AIAA Preprint No. 67-696, Colorado Springs, Colorado, September 11-13, 1967.

6. Howard N. Roberts, Frederic Weigl, and Arwin A. Dougal, "Space-Time Resolved Mach-Zehnder Interferometer Measurements of Cross-Sectional Electron Distribution in Theta-Pinch Plasma," Proceedings of the Pulsed High-Density Plasma Conference, Los Alamos, New Mexico, September 19-22, 1967.

7. M. E. Oakes, R. Freeman, and H. Schlüter, "Electron-Ion Hybrid Resonance in Finite Plasma," Bulletin of the American Physical Society 12, p. 192, 1967 (ABSTRACT).

8. M. E. Oakes, H. Schlüter, and B. Wheatley, "Ion-Electron Hybrid Resonance with Partial Propagation along the Magnetic Field," Annals of Physics 41, p. 339, 1967.

9. M. E. Oakes, H. Schlüter, and R. Skipping, "Power Transfer to Finite Plasmas near the Lower Hybrid Resonance," Physics Letters 25A, 1967.

10. Guy Walter Haynes and Arwin A. Dougal, "Laser-Induced Breakdown in Super-High Pressure Neon," Proceedings of the Twentieth Annual Gaseous Electronics Conference, San Francisco, California, October 18-20, 1967 (ABSTRACT).

11. J. G. Melton, N. B. Dodge, and Arwin A. Dougal, "Observation of Frequency Mixing and Generation of Harmonics in a Cylindrical Radiofrequency-Excited Plasma," Abstracts of Papers, Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, pp. 83-84, November 8-11, 1967 (ABSTRACT).

12. Frederic Weigl, Otto M. Friedrich, Jr., and Arwin A. Dougal, "An Adaptation of Holographic Interferometry to Obtain Effective Infinite-Fringe Mach-Zehnder Interferograms with a Non-Critical System," Abstracts of Papers, Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, pp. 10-11, November 8-11, 1967 (ABSTRACT).

13. Howard N. Roberts, Frederic Weigl, and Arwin A. Dougal, "Effects of Pressure, Bias B-Field, and Peak B-Field in Theta-Pinch Plasma on End-Loss, Bias B-Field Trapping, and Neutron Production," Abstracts of Papers, Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, pp. 109-110, November 8-11, 1967 (ABSTRACT).

14. Hans Schlüter, "Laboratory Studies of the Lower Hybrid Resonance," Abstracts of Papers, Ninth Annual Meeting of the Division of Plasma Physics, American Physical Society, Austin, Texas, p. 22, November 8-11, 1967 (invited talk).

ILLUSTRATIONS

II. EXPERIMENTAL FACILITIES

IV. EXPERIMENTAL INVESTIGATION OF HYDROMAGNETIC WAVE PROPAGATION AT LOW MAGNETIC FIELDS

PROPAGATION AND DISPERSION OF HYDROMAGNETIC AND ION CYCLOTRON WAVES IN PLASMAS IMMERSED IN MAGNETIC FIELDS

Jimmy G. Melton and Nathan B. Dodge (Prof. Otto M. Friedrich, Jr.)

I. INTRODUCTION

During the research period covered by this report investigations have been concerned with the excitation and propagation of the quasistatic ion cyclotron wave and of the Alfven compressional wave. The waves are being studied experimentally in an apparatus in which the waves are generated by rf excitation of a Stix coil and studied as they propagate along the magnetic field into a magnetic beach. Theoretical investigations are concerned with deducing how the waves will behave in the experimental geometry.

The observation[1] of waves which propagate near the ion cyclotron frequency and its harmonics has uncovered a new class of quasistatic waves which exist near the ion cyclotron frequency. A recent theoretical treatment[2] has shown that waves which behave qualitatively as observed can be deduced from theory when finite temperature effects are taken into account. Another class of waves has been observed[3] to propagate for very low values of static magnetic field, under conditions favorable for the Alfven compressional mode.

Further experimental investigations[3,4] into these two classes of waves have revealed that they do not behave in all respects as was expected. Measurements of the axial wavelength of harmonic ion cyclotron waves revealed wavelengths much longer than that imposed by the Stix coil, whereas measurements of the cut-off conditions for the low-field waves were not in agreement with theoretical values.

This report presents the recent progress which has been made in explaining the observed wave phenomena. It has been found that a Stix coil theoretically should not be an efficient exciter of quasistatic ion cyclotron waves, and that the observed low-field waves can be explained in terms of Alfven compressional waves possessing higher order azimuthal modes.

Section II of this report describes the experimental facilities which were used in the investigations. Also described is the construction and testing of a reflex P.I.G. discharge arrangement to be used in future investigations as a preionizer.

In Sec. III the Stix coil is evaluated as a means of exciting quasistatic ion cyclotron waves. A model for the plasma is adopted which assumes cold plasma boundary conditions. It is shown that the Stix coil couples to the waves only as a secondary effect, such that one cannot assume a priori that the waves in the plasma will be of the same wavelength as the Stix coil. A method is outlined whereby the excitation coefficients of the Stix coil for each mode can be computed.

Section IV considers the effect that higher order azimuthal modes will have on the propagation and cut-off conditions for the compressional wave. The problem is analyzed assuming a cold plasma cylinder. It was found that certain azimuthal modes can bring the theoretical cut-off conditions into agreement with the experimental conditions, and, in addition, that the observed propagation "bands" can be explained on the basis of the excitation of different azimuthal modes in each band.

Reference for Section 1

1. M. Kristiansen and A. A. Dougal, "Experimental Investigation of Harmonic Ion Cyclotron Wave Propagation and Attenuation," Phys. Fluids 10, 3, 596 (1967).

2. D. G. Swanson, "Quasi-static Ion Cyclotron Waves," Phys. Fluids 10, 7, 1531 (1967).

3. N. B. Dodge, J. G. Melton, and A. A. Dougal, "Experimental Investigation of Hydromagnetic Wave Propagation at Low Magnetic Fields," Semiannual Status Report No. 9/NASA Grant NsG-353, The University of Texas at Austin, pp. 23-53, July 15, 1967.

4. J. G. Melton, N. B. Dodge, and A. A. Dougal, "Determination of Power Flow in Ion Cyclotron Resonance Plasma Heating," Semiannual Status Report No. 9/NASA Grant NsG-353, The University of Texas at Austin, pp. 4-22, July 15, 1967.

II. EXPERIMENTAL FACILITIES, Jimmy G. Melton and Nathan B. Dodge

(Professor Otto M. Friedrich, Jr.)

The experimental arrangement used during the past report period remains essentially unchanged. It is described in part A below. A proposed addition to the experiment in the form of a new preionization source is discussed in part B.

A. Present Experimental Arrangement

The experimental facilities are arranged as shown in Fig. II-1. Hydrogen or deuterium gas is confined inside a pyrex tube located along the axis of a solenoidal magnetic system. A 300 kilowatt dc power supply provides current for energizing the water-cooled magnet coils. The magnet coils are arranged to provide small (10%) magnetic mirrors at each end of the plasma tube, as shown in Fig. II-2. Rf power is coupled through a Stix-type induction coil into a uniform (±1/3%) magnetic field region adjacent to an 18% magnetic "beach" or region of decreasing magnetic field. Ion-cyclotron and harmonic ion-cyclotron waves produced by the Stix coil may be observed and studied as they propagate into the magnetic "beach," and transfer energy to the plasma through the mechanism of ion cyclotron damping.

Rf power is supplied by a 10 kW (20-30 kW pulsed) transmitter coupled through a directional coupler and LC tuning circuit to the Stix coil. The directional coupler provides a means of measuring incident and reflected power to the plasma. Typical measurements of power are ~20 kW incident and ~10 kW reflected, for a net of 10 kW into the plasma and tuning circuitry. Normally, about 5+ kW is absorbed in the tuning circuitry so that the main rf transmitter is 20-25% efficient in coupling power into the plasma column.

Present preionization equipment is limited to rf sources. A 1.0 kW transmitter provides preionization power coupled between a button electrode at one end of the plasma tube and a steel vacuum flange at the other. Typical ionization percentage is about 1% or less with this device. The preionization pulse may, however, be extended for fairly long times (up to 3 msec), and the

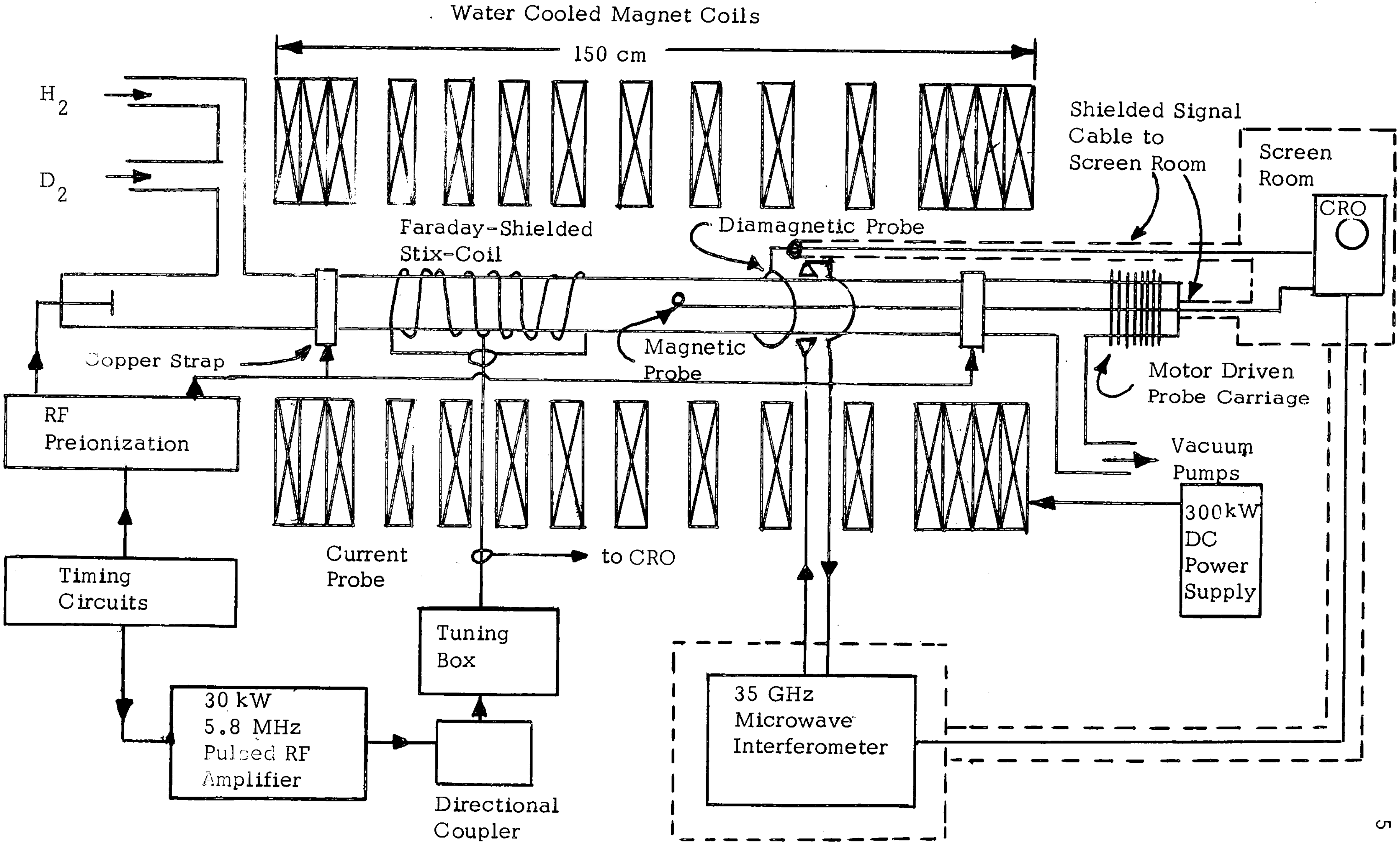


Fig. II-1. Block Diagram of ICHR Experiment.

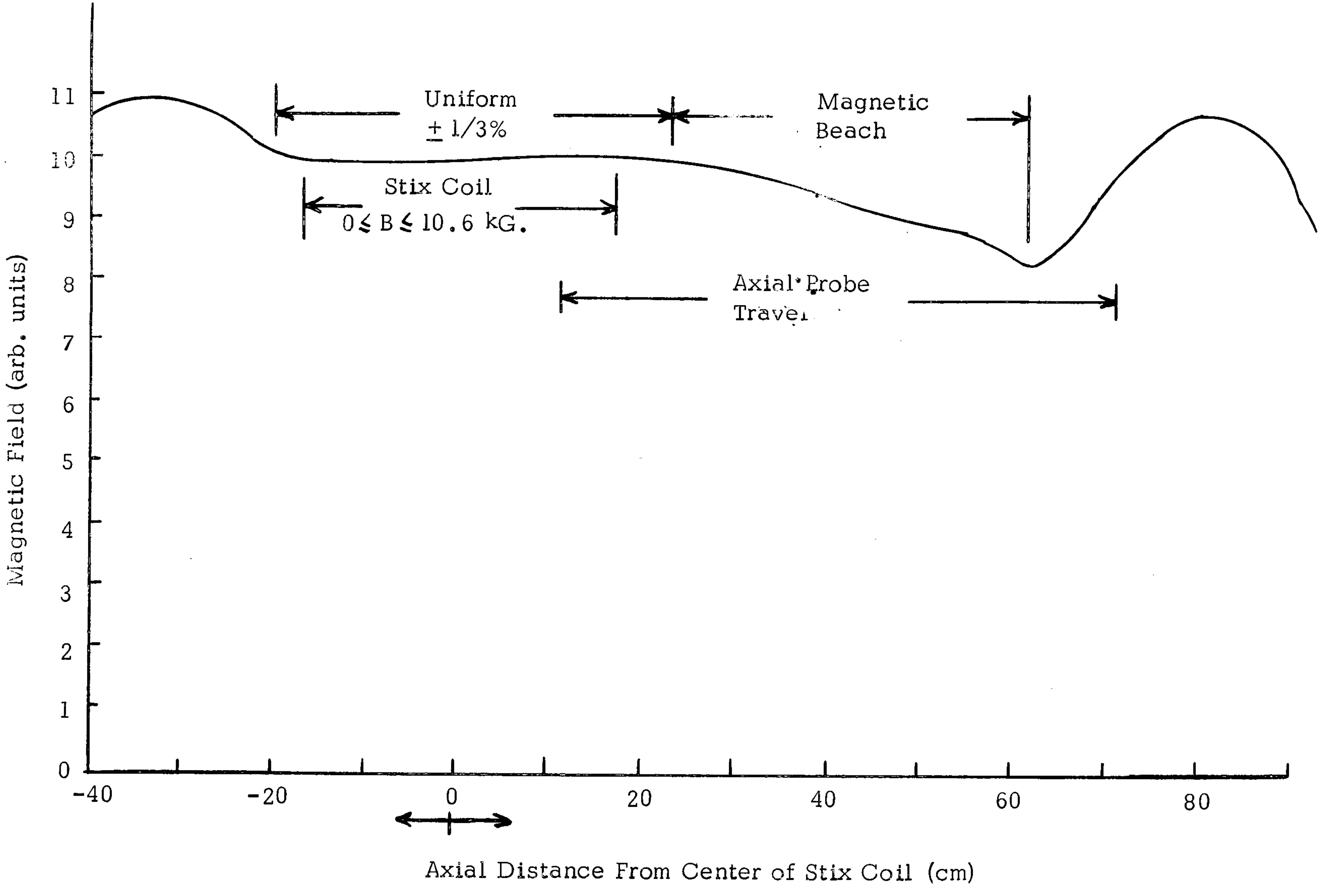


Fig. II-2. Magnetic Field Configuration.

frequency range is adjustable in steps so that operation is possible at 3.5, 7, 14, 21 and 28 MHz.

A short (200-500 μsec) high-power input for preionization is provided by a 15 MHz, 40 kW pulsed rf transmitter. Power is capacitively coupled through two copper braid straps wrapped aroung the pyrex tube at the mirror peaks. Ionization rates up to 3-4% are possible with this preionizer, although it is not as flexible nor as easily tunable as the lower-power rf preionizer.

Diagnostic transducers located at the experimental apparatus include magnetic probes and microwave horns from an 8 mm interferometer. The diamagnetic probe consists of a loop wound outside the plasma tube and placed in the magnetic "beach" region. The loop is movable outside the pyrex tube along the entire "beach" section of the magnetic field.

A magnetic probe is located in a pyrex re-entry tube inside the plasma. It may be manually or automatically moved axially along the plasma column by means of a probe carriage driven through a stainless steel bellows assembly. Radial adjustments made manually allow for measurement across the plasma, and the axial adjustments allow positioning of the probe from under the Stix coil to the far mirror at the lower end of the magnetic beach. $\dot{B}_r$, $\dot{b}_\theta$, and $\dot{b}_z$ may be measured, depending upon the orientation of the coil in the re-entry tube. Both probes are connected through shielded leads to one of the headers which terminate the copper conduits extending from the screen room, as shown in Fig. II-1. All oscilloscopes and the control circuitry for adjusting the magnetic probe position and magnetic field intensity are located in the screen room.

An 8 mm microwave interferometer is located as shown in the schematic adjacent to the experimental apparatus. Because considerable contamination of the crystal detector signals from the interferometer was experienced, due mostly to the rf noise from the preionizers, the entire microwave circuit except for the horn antennas was enclosed in a small screen room. The interferometer is connected by doubly-shielded leads to the copper conduit system of the large screen room.

B. New Modified P.I.G.-Type Preionizer

The techniques of plasma production have long been considered of the utmost importance, both at this facility and elsewhere. Although the 40 kW preionizer described in part A provides sufficient ionization for coupling of ion cyclotron waves, rf preionization is far from ideal in several respects. Since the coupling of rf energy to the gas depends upon the conductivity of the gas, which changes rapidly as energy is added to the gas and the ionization begins to increase, tuning conditions for the preionizer are quite variable. Usually some type of LC tuning circuit is used. It is normally true that one setting of the tuning circuit will provide for maximum power transfer just at the point where "breakdown" (initial production of ions) occurs, but that tuning condition immediately begins to change as the plasma loads the Stix coil. As a result, power transfer rapidly decreases so that after a short time an equilibrium is reached between ionization and recombination, the level of ionization depending on the rf power available. In the present case, because of the relatively low rf power levels available for preionization, only 1-4% ionization is possible at neutral gas densities of 10^{12} to 10^{14}cm^{-3}.

Further, tuning conditions are variable with a number of other parameters, such as the aging of the active elements of the rf transmitters involved. Although rf preionization provides sufficient ionization to allow the observation of ion cyclotron wave phenomena, a better source of ionization is still desirable. Both repeatability and the magnitudes of effects observed are markedly improved by even small increases in preionization.[1] A method of preionization capable of 50+% preionization has been the goal.

The design of a modified P.I.G.-type discharge preionizer has been previously discussed.[2] A somewhat modified version of this preionizer was constructed and assembled separate from the main experiment at the beginning of the last report period. Diagrams of the apparatus, as originally assembled, are shown in Fig. II-3 and II-4.

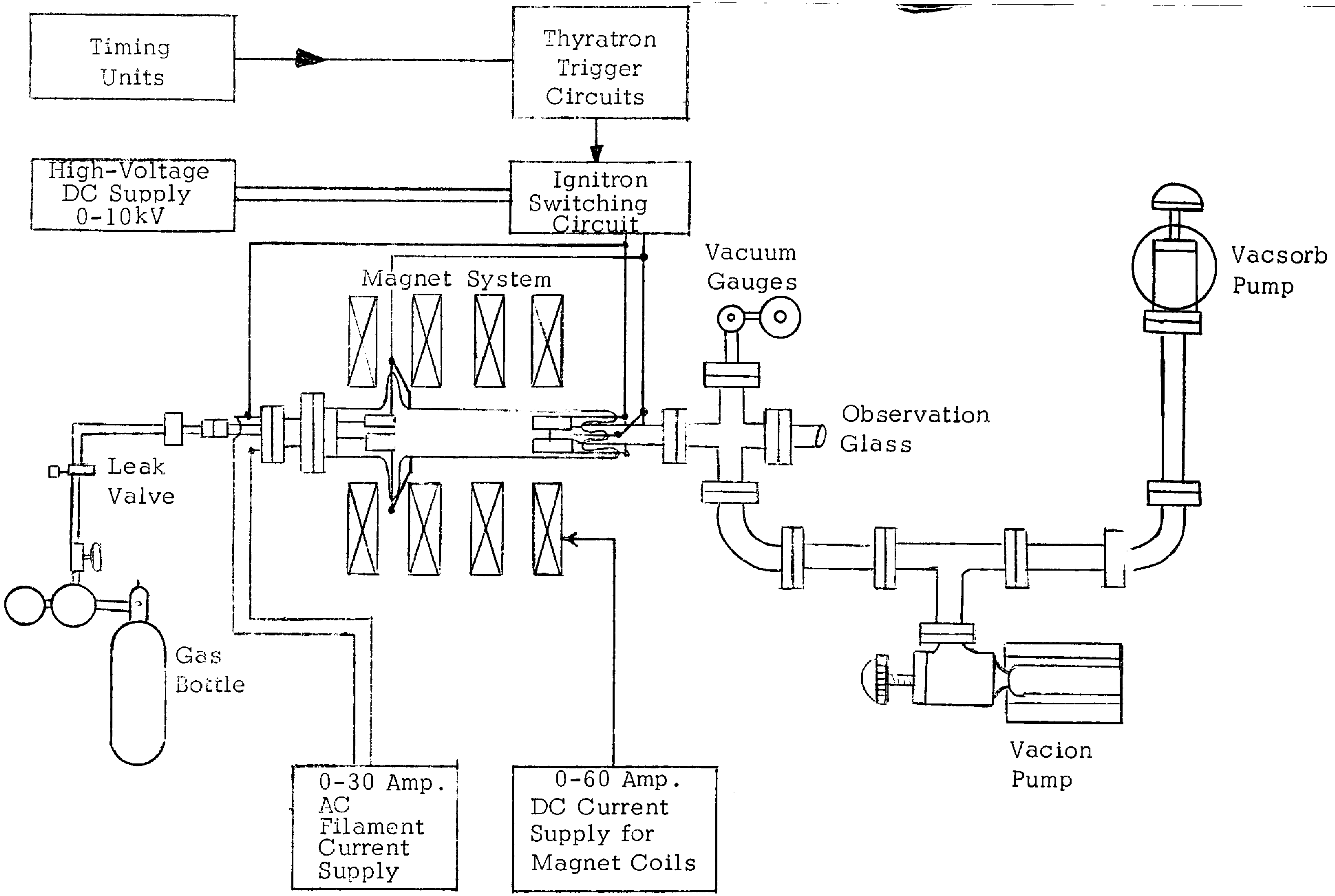
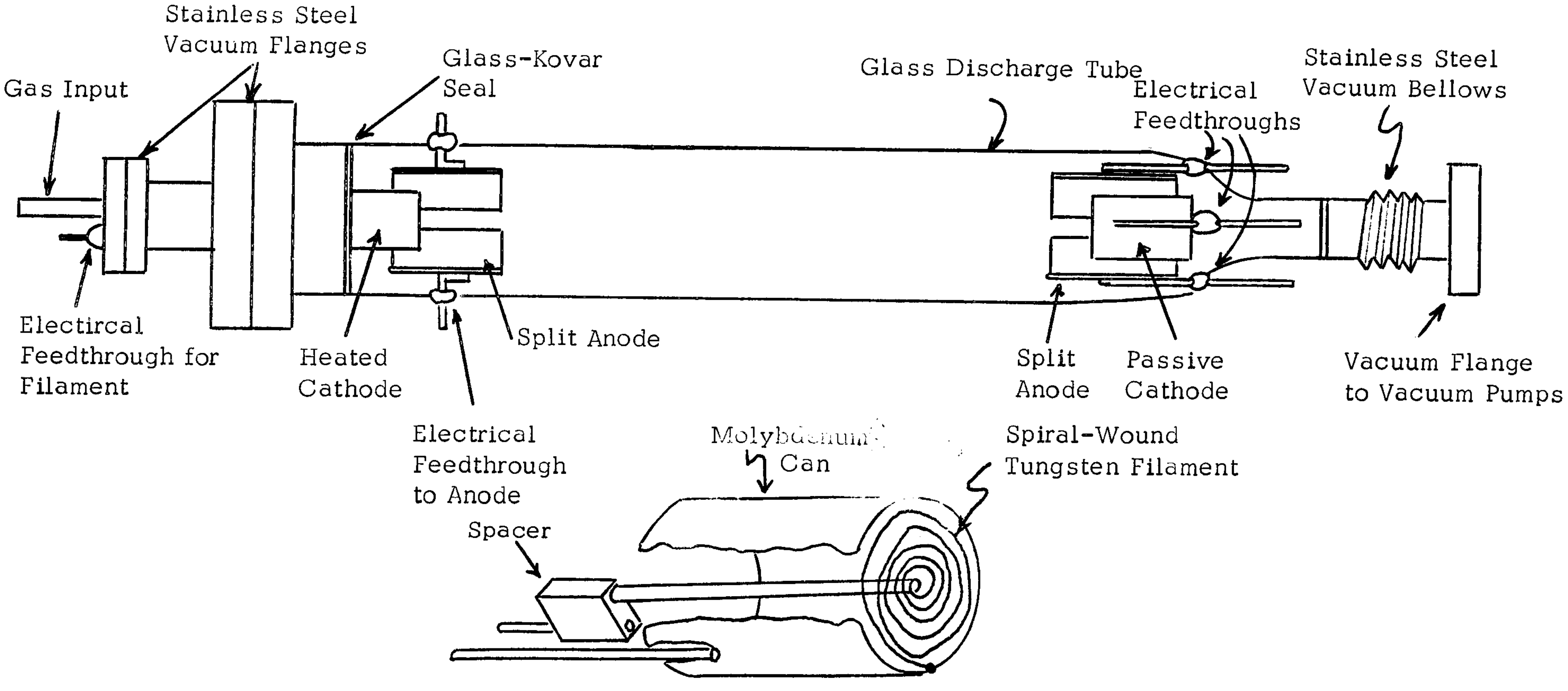


Fig. II-3. Test Apparatus for P.I.G. Discharge Preionizer.



<u>Cut-Away View of Heated Cathode</u>

Fig. II-4. Drawing of P.I.G. Discharge Assembly.

The device consists of a pyrex tube with copper, half-cylinder anode pairs at both ends. At the right end is a pair of half-cylindrical cathodes; these are unheated cathodes. The left cathode assembly is a heated tungsten filament which provides free electrons initially. The cathodes are pulsed negatively, accelerating the electrons axially and producing high ionization through collisions between the electrons and neutral gas particles.

As originally assembled, the modified P.I.G. preionizer operated satisfactorily above about 4 mT. However, higher voltages were necessary to cause breakdown than had been anticipated. Potentials approaching 10 kV resulted in a glow-type discharge down to pressures near 0.5 mT, but ionization was once again marginal. Pulsed potentials of about 5 kV produced 50+% preionization at and above 4 mT. The electron densities produced were verified by the 8 mm microwave interferometer used on the main experiment. The plasma typically reached a maximum in 10-22 μsec, depending on the peak negative pulse voltage and the magnetic field applied to the plasma column, and decayed in 20-40μsec subsequent to reaching this peak. Filament current was about 30 amperes, and magnetic fields ranged from about 400 to 800 gauss.

Since typical neutral gas pressures on the main experiment are in the range 0.1-1.0 mT, modifications were undertaken to improve the performance of the discharge and permit operation of this lower pressure range. A cylindrical oxide-coated cathode was built to provide enhanced electron production. This cathode was completed and installed near the end of the present report period, together with a heavy-duty ac filament current supply capable of providing 100 amperes of current. In preliminary testing, the preionizer has ionized hydrogen gas at 1.2 mT, but performance has been sporadic. Significantly, however, cathode potentials have been limited to -500 to -2000 v.

At the present time a new electrical feedthrough has been designed and is being constructed to meet the higher current requirements of the oxide-coated cathode. In order to further improve the performance of the preionizer around and below 1.0 mT, new anode assemblies are being designed. It is anticipated that during the next report period, these modifications will be completed.

Upon satisfactory completion of tests, the preionizer will be disassembled and installed on the main experiment. The use of the preionizer on the main experiment will provide a highly ionized plasma for ion-cyclotron wave studies in the 0.1 - 1.0 mT pressure range.

References for Section II

1. M. Kristiansen, J. G. Melton, F. C. Harris, N. B. Dodge, and A. A. Dougal, S. S. R. No. 6 on NASA Research Grant NsG-353, Plasma Dynamics Research Laboratory, The University of Texas at Austin, January 15, 1966.

2. J. G. Melton, N. B. Dodge, and A. A. Dougal, S. S. R. No. 8 on NASA Research Grant NsG-353, Plasma Dynamics Research Laboratory, The University of Texas at Austin, January 15, 1967.

III. INVESTIGATIONS OF HARMONIC ION CYCLOTRON WAVES, Jimmy G. Melton and Nathan B. Dodge (Professor Otto M. Friedrich, Jr.)

A. Generation of Harmonic Ion Cyclotron Waves

This section of the report describes the status of harmonic ion cyclotron wave investigations and reports the recent progress which has been made. Current investigations are being conducted to obtain a more complete verification that the observed harmonic ion cyclotron waves are quasistatic ion cyclotron waves. The investigations are directed toward achieving experimental determination of the plasma wavelengths and toward achieving the additions to the quasistatic theory which will allow study in a finite geometry.

The experimental observations of harmonic ion cyclotron waves have been described in a previous journal publication,[1] in a full length technical report,[2] and in previous semiannual status reports.[3,4] The waves are generated in a magnetic beach geometry by rf excitation of a Stix coil. The waves are observed to propagate below the ion cyclotron frequency harmonics. When the waves damp at the cyclotron harmonics, the plasma diamagnetism increases, indicating that heating is occurring. Measurements of the wave fields across the plasma cross-section, by means of movable magnetic probes, show no variation, indicating a higher order transverse mode structure. Measurements of the power input to the plasma during wave propagation show broad coupling resonances, unlike the sharper resonances which occur during good coupling to the ion cyclotron wave.

The theory of quasistatic ion cyclotron waves[5] explains very well qualitatively each of the observations above on the experimental waves, except for the power coupling measurements for which no predictions have been made. However, quantitative agreement can be achieved only by measurements of the wave numers parallel and perpendicular to the magnetic field, $k_{\parallel}$ and $k_{\perp}$, and by determining the density dependence of the waves. Modifications to the experiment to achieve a variable density plasma are described elsewhere in this report. Measurements of $k_{\parallel}$ and $k_{\perp}$ depend on the ability to excite only one wave mode, and measurements so far indicate that more than one mode is present.[4]

Unfortunately, the quasistatic theory presented in Ref. 5 does not specify how the excitation of a given mode occurs. Quasistatic ion cyclotron waves have been studied only in an infinite geometry. Yet, the question of how these waves can be excited, by some mechanism such as a Stix coil, can be answered only by taking into account the boundary effects. The question of exciting these waves also has a bearing on the proposed use of these waves to heat a plasma by ion cyclotron damping without the density limitation encountered in heating by the ion cyclotron wave.[5] Any density limitations which exist should appear when one considers exciting these waves at a plasma boundary.

In the following section, results are presented for a model which assumes a hot, homogeneous plasma with cold plasma boundary effects. Only m=0 azimuthal symmetry is assumed for the circular cylindrical plasma with the quasistatic approximation. The question to be answered is how a Stix coil couples to a quasistatic ion cyclotron wave.

B. Generation of Quasistatic Ion Cyclotron Waves

The problem to be considered is that of a hot, homogeneous, infinitely long plasma cyclinder to which waves are coupled from some external source. We will consider only waves of a quasistatic nature, that is, for which

$$|k^2| >> |K_{ij}| \qquad \text{for all } i, j \qquad \text{(III-1)}$$

at frequencies near the ion cyclotron frequency and its harmonics, but much below the electron plasma and cyclotron frequencies. In Eq. (III-1), k is the wavenumber and K_{ij} are the elements of the equivalent dielectric tensor for the hot plasma.

The dispersion relation[6] greatly simplifies in the quasistatic approximation to

$$\vec{k} \cdot \overleftrightarrow{K} \cdot \vec{k} = 0 , \tag{III-2a}$$

which can be written in more detail for $k_y = 0$ as

$$k_x^2 K_{xx} + 2 k_x k_z K_{xz} + k_z^2 K_{zz} = 0 \tag{III-2b}$$

or $$k_\perp^2 + k_\parallel^2 + \sum_j \sum_{n=-\infty}^{\infty} \frac{\omega_{pj}^2 m_j e^{-\lambda_j} I_n(\lambda_j)}{\kappa T_j} \left[1 + \frac{\omega}{k_\parallel v_j} Z\left(\frac{\omega + n\Omega_{cj}}{k_\parallel v_j} \right) \right] \tag{III-3}$$

$k_\perp$ and $k_\parallel$ are the wave numbers perpendicular and parallel to the static magnetic field, which is in the Z-direction. ω_{pj} and Ω_{cj} are the plasma and cyclotron frequencies, respectively, for the j-th plasma species, V_j is the thermal velocity defined by $V_j = (2KT_j/m_j)^{1/2}$, $I_n(\lambda)$ is a modified Bessel function of the first kind of order n, and $Z(\zeta_n)$ is the plasma dispersion function of Fried and Conte;[7] λ is the hot plasma expansion parameter, $\lambda = \frac{1}{2} k^2 \rho_\perp^2$, where $\rho_\perp$ is the Larmor radius.

If we are not too near one of the $n\Omega_{cj}$, i.e. if

$$|\zeta_{nj}| = \left| \frac{\omega + n\Omega_{cj}}{k_\parallel v_j} \right| >> 1 \qquad \text{for all } n, j \tag{III-4}$$

then, following Swanson,[5] the dispersion relation can be written approximately

$$k_\parallel = \omega \left(\frac{2 m_e}{\kappa T_i} \right)^{1/2} Q\left(\lambda_i , \omega/\Omega_{ci} \right) \tag{III-5}$$

where $$Q(\lambda, \omega/\Omega_{ci}) = \left[\sum_{n=1}^{\infty} \frac{e^{-\lambda} I_n(\lambda)}{1 - (\omega/n\Omega_{ci})^2} \right]^{1/2} \tag{III-6}$$

We note that for plasmas with thermal energy 100eV or less, that Eq. (III.5) is valid except very near $n\Omega_c$.

Equations (III-5) and (III-6) are the dispersion relation for quasistatic ion cyclotron waves in an infinite geometry. We will consider this equation in a bounded geometry, which will impose the additional restriction that the k's be members of a discrete set $(k_{\parallel m}, k_{\perp m})$. In order to be consistent with the use of the hot plasma dispersion relation, we should use boundary conditions appropriate for a hot plasma. However, this problem has been solved only in the limits λ very small and $\lambda \to \infty$. Inclusion of effects of n-th order in λ near $\lambda = 0$ requires the specification of 2(n+1) boundary conditions.[8] Consequently, we choose to treat the simplest case first, in that we assume that the boundary conditions are those for a cold plasma, i.e. in the $\lambda \to 0$ limit.

It is likely to be the case that plasma inhomogeneities near the boundary, such as the plasma sheath, will be as important as finite λ effects in determining the wave properties in the plasma interior. So we can expect this model to yield good answers unless finite λ effects are very drastic near the boundary.

The boundary conditions for the electromagnetic fields across the plasma-vacuum interface can be obtained by integrating Maxwell's equations across an infinitesimal volume element including the interface. If $\vec{n}$ is the inward directed normal, and $[\vec{A}]$ indicates the interface discontinuity $\vec{A}_p - \vec{A}_{vac}$, the jump conditions are,

$$\begin{aligned} \vec{n} \times [\vec{B}] &= \mu_c \vec{J}^* \\ \vec{n} \times [\vec{E}] &= 0 \\ \vec{n} \cdot [\vec{B}] &= 0 \\ \vec{n} \cdot [\vec{E}] &= \sigma^*/\epsilon_0 \end{aligned} \qquad \text{(III-7)}$$

where σ^* is surface charge density and $\vec{j}^*$ is surface charge density. We have assumed that normal velocities are zero at the boundary, $\vec{n} \cdot \langle \vec{V} \rangle = 0$.

From these equations, it follows directly that

$$\text{at } r = r_p, \quad \begin{array}{ll} [B_r] = 0 & [E_r] = \sigma^*/\epsilon_0 \\ [B_\theta] = -\mu_0 \dot{j}_z^* & [E_\theta] = 0 \\ [B_z] = \mu_0 \dot{j}_\theta^* & [E_z] = 0 \end{array} \qquad \text{(III-8)}$$

We use these boundary conditions to examine wave coupling from a Stix coil. The ideal Stix coil can be represented as a current sheet $\vec{j}(z) = j_c(z)\,\hat{\theta}$ at radius $r_o = r_c > r_p$. The Stix coil has phase-reversed windings which have axial periodicity $\lambda_o = 2\pi/k_o$, such that

$$\begin{array}{ll} \vec{j}(z) = j_c \sin k_0 z\, \hat{\theta} & \text{for } -a \leq z \leq a \\ j(z) = 0 & \text{for } |z| \geq a \end{array} \qquad \text{(III-9)}$$

In this idealization, the coil in the absence of plasma induces fields E_θ, B_r, and B_z. If plasma is present, but without any surface charge or surface current, the fields in the plasma are (E_θ, B_r, B_z), together with higher order fields in (E_r, E_z, B_θ) which are induced by the plasma currents flowing in response to the first order fields (E_θ, B_r, B_z). The higher order fields will in turn induce surface charge and surface currents which allow the boundary conditions to be met.

The fields of the wave we wish to excite are those pertinent to the dispersion relation, Eqs. (III-2) and (III-3), which are E_r, E_z, and B_θ. Eqs. (III-2) and (III-3) are applicable to cylindrical geometry only for the case of m = 0 azimuthal symmetry. For this case, however, we see that an ideal Stix coil can excite quasistatic ion cyclotron waves only through the process of mode conversion, whereby the TE mode pattern of the Stix coil couples through the plasma currents to the TM mode of the waves.

Since the physical Stix coil is non-ideal, (specifically it possesses stray E_r and E_z fields) the possibility also exists that excitation is occurring through the higher order fields of the coil. Since these fields would couple directly to E_r and E_z in the plasma, their effect may be greater than that of the mode conversion within the plasma.

There is the third possibility of the excitation of modes with higher order azimuthal symmetry. This possibility has not been studied before in the quasistatic theory since the inclusion of $m \neq 0$ modes replaces the very simple form, Eq. (III-2a), with a more complicated form which includes the additional dielectric tensor element K_{yy}, that is

$$k_x^2 K_{xx} + 2k_x k_z K_{xz} + k_z^2 K_{zz} + k_y^2 K_{yy} = 0 \qquad \text{(III-10)}$$

Physically, excitation of such waves by a Stix coil could occur only through the non-ideal nature of the coil. Azimuthal asymmetry is present in the pitch of the Stix coil turns.

We have determined that excitation of quasistatic ion cyclotron waves by a Stix coil can occur through three possible mechanisms: (1) mode conversion from TE to TM mode inside plasma, (2) coupling from secondary fields of Stix coils (E_r, E_z, B_θ), (3) coupling to higher order azimuthal mode, also from secondary field of Stix coil. All three of these are secondary effects. Thus, one should be able to devise more efficient coupling devices for exciting quasistatic ion cyclotron waves.

To continue the formulation of how the Stix coil excites quasistatic ion cyclotron modes, we consider the fields present in the hot plasma in order to calculate the TE to TM mode coupling. We again exclude higher order azimuthal modes in the interest of simplicity, although they are being included in the overall analysis. The components of the fields parallel to the magnetic field can be expressed in the form[8]

$$B_z(r) = \sum_{\ell=1}^{2} \sum_{n=1}^{\infty} A_{\ell n} J_0(k_{\perp \ell n} r) \qquad \text{(III-11)}$$

$$E_z(r) = \sum_{\ell=1}^{2} \sum_{n=1}^{\infty} -\frac{\omega A_{\ell n}}{k^2_{\perp \ell n}} \beta_{\ell n} J_0(k_{\perp \ell n} r), \qquad \text{(III-12)}$$

where
$$\beta_{\ell n} = \frac{[\gamma_n(\gamma_n - 2K'_{0\ell} + k^2_{\perp \ell n}) + K'_{2\ell}]}{[\gamma_n K'_{z2\ell} + K'_{2\ell}(k_{\parallel n} + K'_{z1\ell})]},$$

$$\gamma_n = k^2_{\parallel n} - K'_1$$

and the K'_1, K'_2, K'_{z2}, K'_0 are elements of the equivalent dielectric tensor. We only indicate where the elements appear in the formulas since their functional form will not have a bearing on the following analysis. The K's are in general functions of frequency and propagation vector $K(\omega, \vec{k})$. The field formulas are correct to zeroth order in λ, consistent with the assumption of cold plasma boundary conditions. The sum on ℓ indicates that 2(n+1) boundary conditions, with n=0, are to be specified, meaning two values of $k_\perp$ are to be determined. The transverse field components, $\vec{E}_\perp$ and $\vec{B}_\perp$, can be expressed in terms of E_z and B_z.

At this point, all the necessary information is present to complete the formal solution of the problem, and it remains only to outline the program of the solution. By matching the field formulas, Eqs. (III-11) and (III-12) and expressions for the transverse components, the coefficients $A_{\ell n}$ are specified. The

Stix coil (or any other exciting structure), being of finite length, excites not just a single axial wavelength, but a finite spectrum of axial wavelengths. This effect can be included by Fourier analysis of the Stix coil current,[6]

$$j^c(k) = j^c \frac{\sin (k-k_0)a}{\pi (k-k_0)} \tag{III-13}$$

Solutions of the dispersion relation, Eqs. (III-5) and (III-6), specify $k_\parallel$ for a given $k_\perp$ determined from (III-11) and (III-12).

The excitation of a particular mode can then be determined relative to the other modes by evaluating the integral

$$\int_{-\infty}^{\infty} \vec{E}_\theta(k) \cdot \vec{j}^{\,c}(k)\, dk , \tag{III-14}$$

for a given mode. This, in effect, determines the power in a mode by weighting that mode by the degree to which the coil excites that mode.

The analysis presented here in outline is presently being extended to include the possibility of higher order azimuthal modes. The extended formulation will then be used to numerically evaluate excitation of quasistatic ion cyclotron waves by a Stix coil. If a reasonable determination of the stray fields of a Stix coil can be arrived at, this effect also will be included in the analysis.

References for Section III

1. M. Kristiansen and A. A. Dougal, "Experimental Investigation of Harmonic Ion Cyclotron Wave Propagation and Attenuation," Phys. Fluids 10, 3, 596 (1967).

2. M. Kristiansen and A. A. Dougal, "High Power RF Plasma Heating and Wave Propagation Near the Fundamental and Harmonics of the Ion Cyclotron Frequency," Technical Report No. 4/NASA Grant NsG-353, The University of Texas at Austin, September 20, 1966.

3. M. Kristiansen, J. G. Melton, N. B. Dodge, and A. A. Dougal, "Propagation and Dispersion of Hydromagnetic and Ion Cyclotron Waves in Plasmas Immersed in Magnetic Fields," Semiannual Status Report No. 8/NASA Grant NsG-353, University of Texas at Austin, January 15, 1967.

4. J. G. Melton, N. B. Dodge, and A. A. Dougal, "Propagation and Dispersion of Hydromagnetic and Ion Cyclotron Waves in Plasmas Immersed in Magnetic Fields," Semiannual Status Report No. 9/NASA Grant NsG-353, University of Texas at Austin, July 15, 1967.

5. D. G. Swanson, "Quasi-Static Ion Cyclotron Waves," Phys. Fluids 10, 7, 1531 (1967).

6. T. H. Stix, Theory of Plasma Waves, (McGraw-Hill Book Company, Inc., New York, 1962).

7. B. D. Fried and S. D. Conte, The Plasma Dispersion Function, (Academic Press, Inc., New York, 1961).

8. D. G. Swanson, "Hot Plasma in a Cyclindrical Waveguide," Phys. Fluids 10, 2, 428 (1967).

IV. EXPERIMENTAL INVESTIGATION OF HYDROMAGNETIC WAVE PROPAGATION AT LOW MAGNETIC FIELDS, N. B. Dodge and J. G. Melton (Professor O. M. Friedrich, Jr.)

A report of detailed investigations of wave propagation at low magnetic fields ($B \leqslant 1000\,G$) was given in the previous Semiannual Report.[1] It was noted that propagation was strongly dependent on magnetic field B and neutral gas pressure p. There were values of B for which propagation would not occur which were interspersed between ranges of B in which propagation of waves of various amplitudes was observed. In all cases except the waves propagating at the very lowest magnetic fields, enhanced ionization occurred during wave propagation. However, the diamagnetic probe information seemed to indicate that the $nT_{\perp}$ product was relatively unchanged, or actually decreased, during wave propagation. Such behavior is not characteristic[2,3] of propagation of the so-called "fast" or right hand circularly polarized hydromagnetic wave, which left the identity of the majority of the observable wave phenomena in doubt.

The completion of new notch-type filters and the improvement of filtering techniques applied to the diamagnetic signal B_D have resulted in much improved diamagnetic signal measurement. These measurements have confirmed that plasma heating does occur in the majority of the low-field wave propagation, so that the identity of the low-field wave phenomena may be made. A low-current dc power supply was also obtained so that much more accurate control was possible over the value of the low magnetic fields that was not possible with the larger 300 kW dc power supply.

Part A of Section IV describes briefly some of the data recently taken with the modifications to the experimental arrangement described above, and discusses some of the general characteristics of wave propagation in the magnetic field range 0 to 1000 G. Part B discusses the waves which propagate in the 800 to 1000 G range in detail, Part C discusses wave propagation in the range 200 - 700 G, and Part D describes wave phenomena below 100 G. Part E summarizes the discussion in Parts A-D and the work which has been accomplished in this study over the past two report periods.

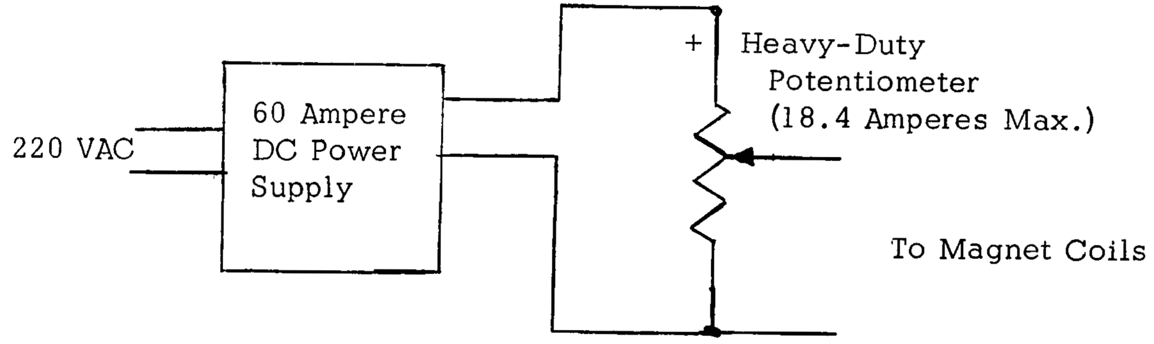
A. Hydromagnetic Wave Propagation at Low Magnetic Fields ($B \leqslant 1000$ G)

Two problems which appeared in the experiments reported in the previous report[1] were the accurate control of magnetic field and the filtering of extraneous noise from the B_D probe. The 300 kW dc power supply was tedious to control at low currents for fields less than about 1 kilogauss, and the ac ripple caused a further uncertainty in the values of magnetic fields.

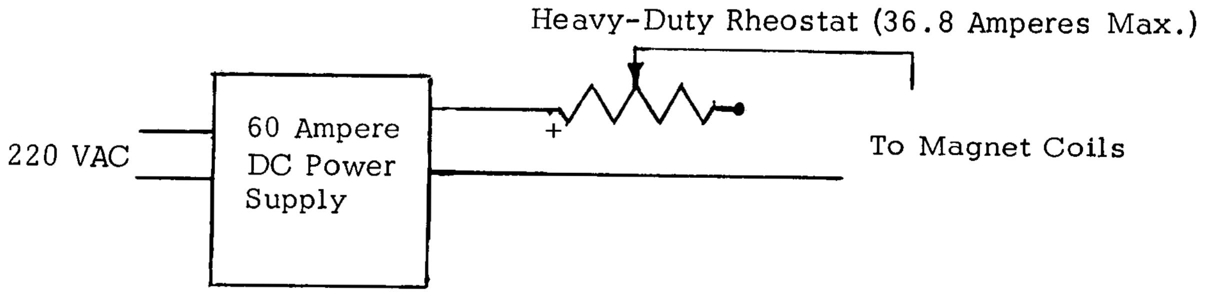
A low current dc power supply was obtained and placed in operation in conjunction with a heavy-duty rheostat and a high-current potentiometer. The resulting current supply permitted very accurate control (± 5 G) of magnetic field in the range 0 - 925 G. The three power supply configurations are shown in Fig. IV-1. Note that the magnet coils were in the normal configuration, including the various electrical connections, used with the 300 kW dc power supply. That is, there were three parallel sets of five coils in series, so that the current in all coils was the same. The physical arrangement of the coils then produced the axial magnetic field contour shown in Fig. II-2. Figure IV-2 shows the calibration curves for the three power supply configurations. These were obtained by the axial placement of a dc magnetic probe, and the magnetic field recorded is that in the flat (Stix-coil) protion of the magnetic contour shown in Fig. II-2. Note that the current shown is at the power supply, not in the coils.

The current-field relation in Fig. IV-2a is of course nonlinear because of the use of the potentiometer. However the readings were repeatable within ± 5 gauss as stated previously. Although this made readings below 100 G more inaccurate, it was only below 40 gauss that control of the magnitude of the field was really difficult. Actually, below 100 G, it was usually possible to set the field on even readings of magnetic field (20, 40, 60, 80 G) within ±1 or 2 gauss, or to the limit of the gaussmeter.

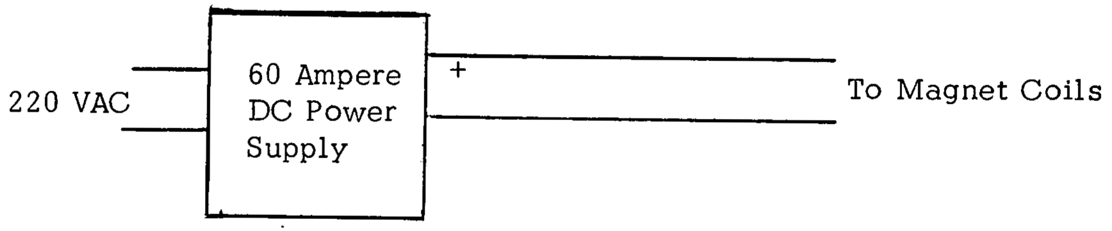
The diamagnetic probe circuit and the notch filter schematic is shown in Fig. IV-3. The 360 Hz pickup from the three-phase transformer in the 300 kW



a. 0-240 Gauss Configuration (See Fig. IV-2a)

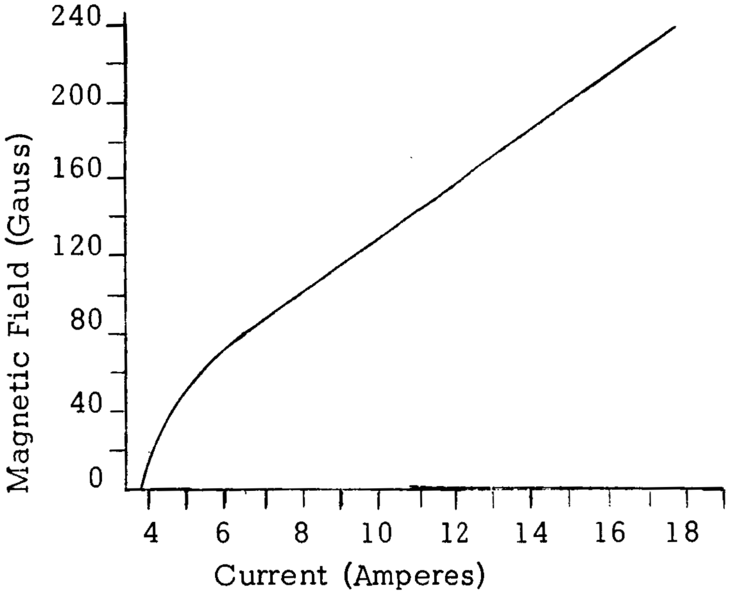


b. 100-525 Gauss Configuration (See Fig. IV-2b)

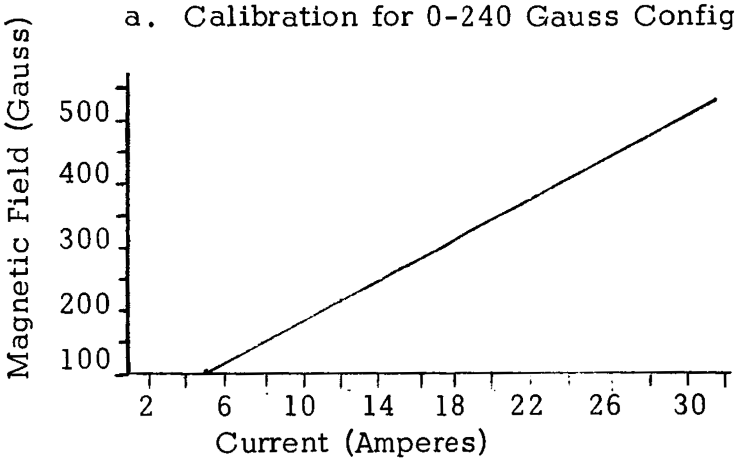


c. 550-925 Gauss Configuration (See Fig. IV-2c)

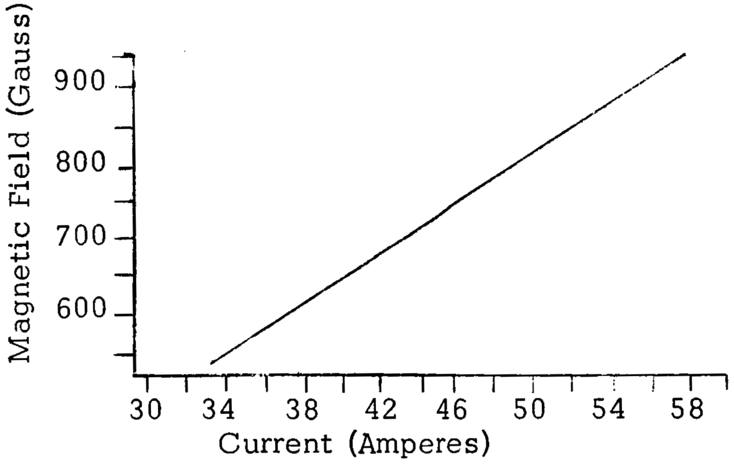
Fig. IV-1. Various Configurations of Low-Current Power Supply To Provide Fields in the Range 0-925 Gauss.



a. Calibration for 0-240 Gauss Configuration (See Fig. IV-1a)



b. Calibration for 100-525 Gauss Configuration (See Fig. IV-1b)



c. Calibration for 550-925 Gauss Configuration (See Fig. IV-1c)

Fig. IV-2. Calibration Curves for Power Supply Configurations Shown in Fig. IV-1.

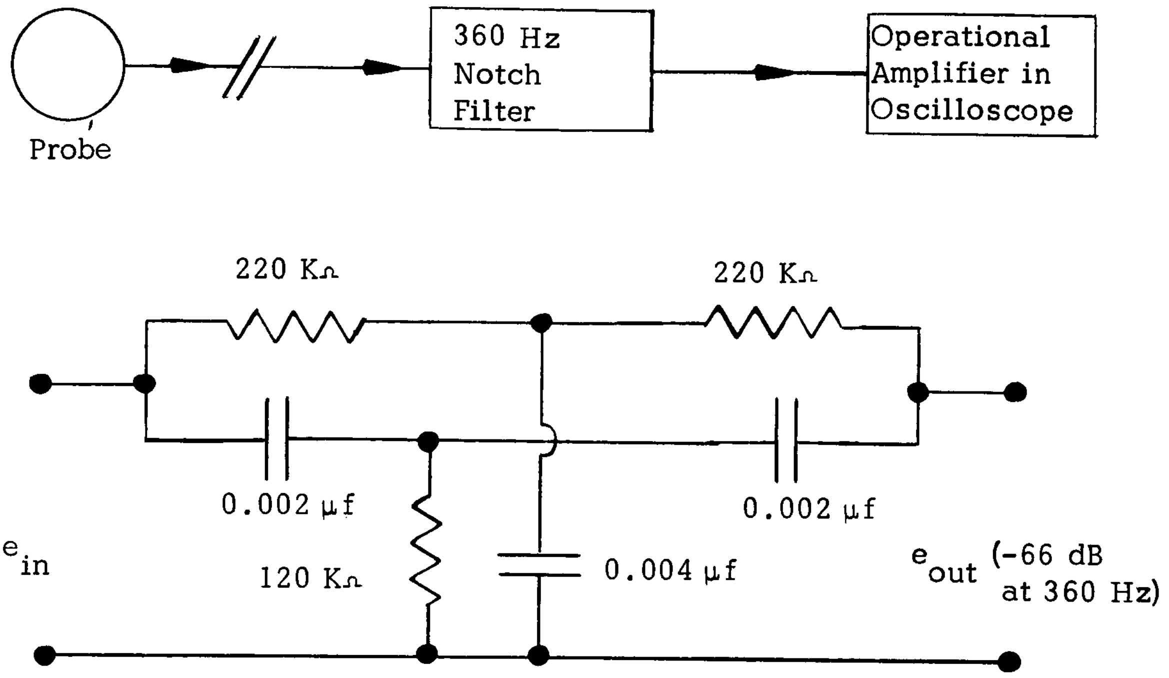


Fig. IV-3. Diamagnetic Probe Circuit and Associated Notch Filter Circuitry.

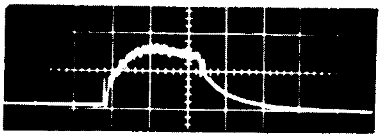


Fig. IV-4. Diamagnetic Signal Trace with Improved Filter and Circuit power supply is substantial at appreciable fields. Sixty Hz pickup is also a problem, with both sources of "noise" normally being large compared to the B_D signal (360 Hz noise picked up by the B_D probe is normally much greater than the signal caused by plasma diamagnetism).

The RC notch filter shown produced -66 db signal levels at 360 Hz with a bandwidth of only a few Hz. Use of a Tektronix type O operational amplifier for amplification and signal integration greatly improves performance over the use of the passive RC integrator. A typical diamagnetic signal trace with the improved circuitry is shown in Fig. IV-4.

Except for the diamagnetic signal data, results with the new low current dc power supply tended to confirm data gathered previously. Propagation is not normally seen below about 0.3 mT, except near 1 kilogauss, although there is an uncertainty in pressure readings of greater than 2 below about 0.3 mT. The ranges of magnetic field for propagation (referred to rather inaccurately as "pass bands" or "propagation bands") and attenuation ("stop bands" or "attenuation bands") are shown in Fig. IV-5.

Above 800 gauss, there is normally propagation, the lower cut-off point depending on the neutral gas pressure but normally decreasing with an increase in pressure. A "stop band" exists centered in the range 700-800 gauss, and a broad region of propagation occurs between 200 and 700 gauss. Below this "band" is another "stop band" in the region centered at 100-200 gauss, and a "propagation band" exists centered normally below about 100 gauss. Once again, the width and actual location of the regions of propagation and attenuation is variable with neutral pressure, although the approximate locations are the same.

Except for the upper band, no proper damping occurs in the magnetic beach. Plasma diamagnetism increases markedly in the central region of propagation as well as in the upper region (this increase in B_D in the upper region was previously noted), but decreases in the lowest propagation band. A typical plot of B_D is shown in Fig. IV-6 with the corresponding plot of $\dot{b}_z$. As noted in the previous

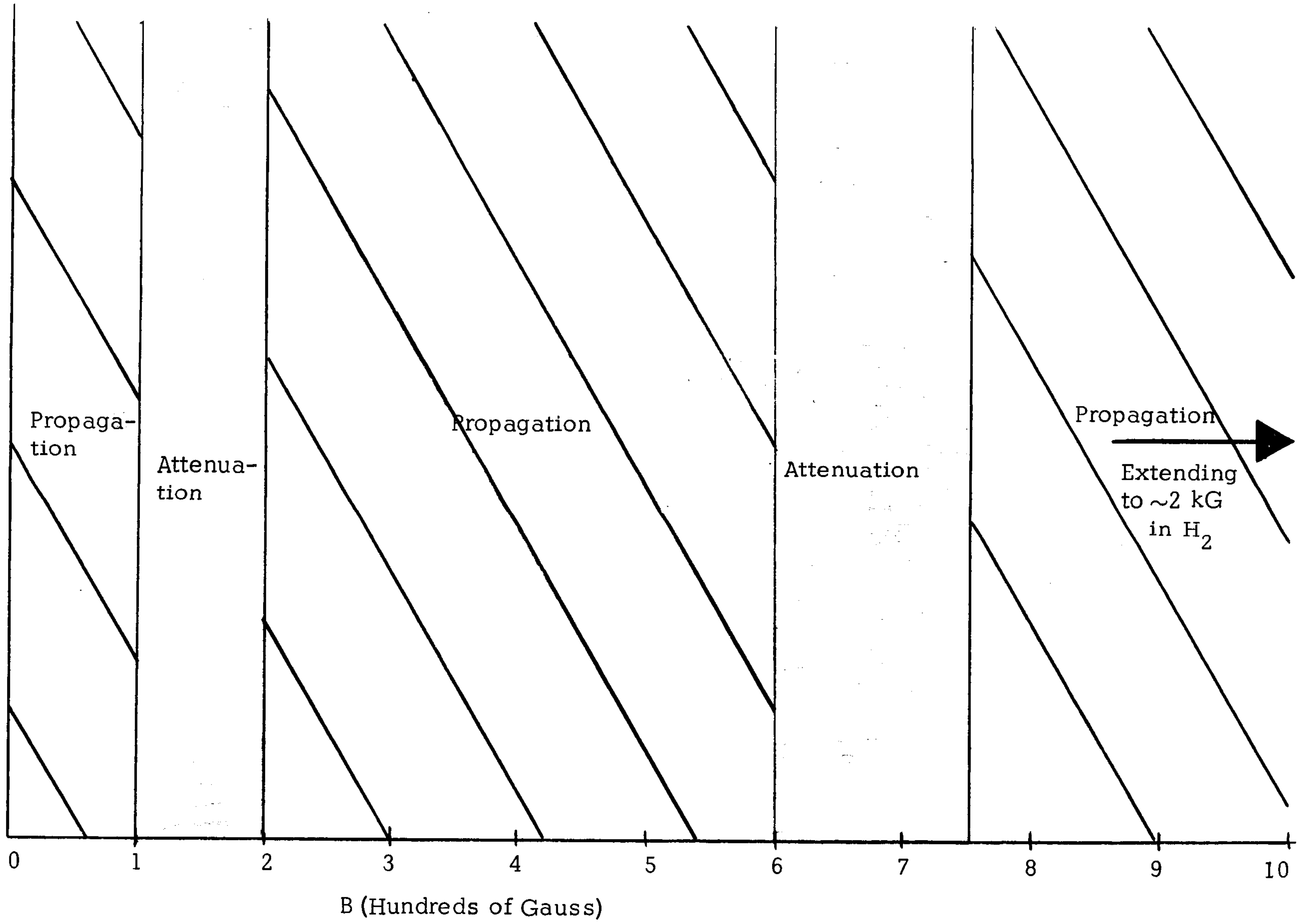
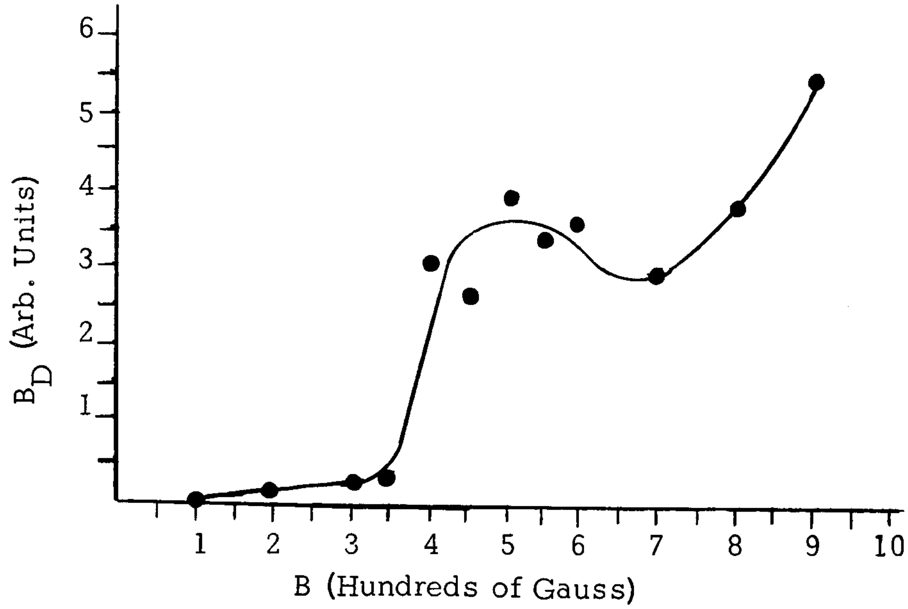
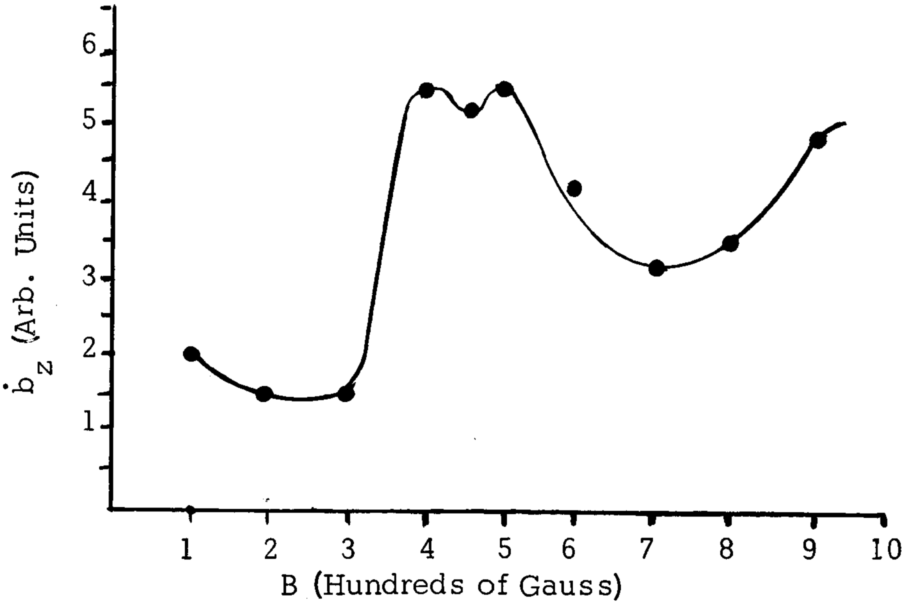


Fig. IV-5. Regions of Wave Propagation and Attenuation Below 1000 Gauss in H_2.



a. Diamagnetic Signal (B_D)



b. Magnetic Signal ($\dot{b}_z$)

Fig. IV-6. Magnetic and Diamagnetic Signal During "Fast" Wave Propagation in Hydrogen Gas, 0.8 mT Neutral Pressure.

report,[1] the magnitude of B_D which was plotted was corrected for the magnetic field reduction which occurred.

As was recorded previously, increased ionization occurred except in the wave propagation below 100 gauss. Normally the maximum ionization was 2-4% depending on the neutral gas pressure. Also the density of ion and electrons, magnitude of magnetic signal, and plasma diamagnetism were at a maximum in the neutral pressure range of 0.5-1.0 mT, the exact maxima depending on the magnitude of magnetic field. Normally the largest wave effects and highest electron densities occurred at 500-700 gauss at about 0.6-0.8 mT, although these quantities were increasing around 900-1000 gauss, the upper limit to the survey.

It should be noted that the predominance of information was recorded for hydrogen gas. However, near the end of the experimental measurements, some data were gathered for deuterium gas. This final amount of information is discussed in the summary in Part E. It should be pointed out that such information as was gathered is not complete, though it does support the data gathered for hydrogen gas. Further information on the low field wave propagation is desirable to complete the picture given in Parts B, C, and D. No further studies of these low field phenomena are planned at the present time, however.

B. Wave Propagation in the Range 800-1000 Gauss

Wave propagation in this region is characterized by enhanced plasma density, the highest plasma diamagnetism recorded in the data taken, and moderately high values of wave magnetic signal $\dot{b}_z$. This "band" of propagation was noted in the previous report[1] to extend above the region studied in both hydrogen and deuterium, up to near 2.0 KG in H_2.

As mentioned in the last report, this band of wave propagation appeared to differ from the harmonic ion cyclotron waves, in that it is in the range such that $B < \frac{B_C}{3}$ i.e., the driving frequency is greater than third cyclotron harmonic in hydrogen. Wave attenuation had previously been observed by Kristiansen and Dougal[4] in hydrogen only up to the third cyclotron harmonic. It appears from the data obtained using the low-current power supply, however, that attenuation might also occur at the fourth harmonic. Kirstiansen and Dougal reported harmonic effects at very low neutral gas pressures,[4] and the harmonic effects were strongly pressure dependent. However, at these low neutral pressures, the percent ionization may be a good deal higher than at some higher pressures where the actual density of ions is greatly increased.

Swanson has also shown[5] that the propagation of quasistatic ion cyclotron waves is not density dependent. Such waves should propagate at higher neutral gas pressures, where the plasma density is higher, although the percent ionization is actually lower. At these higher pressures, however, where the ratio of neutrals to ions is higher, ion-neutral collisions would be expected to cause a "smearing out" of the wave effects so that wave propagation and damping would not be sharp. A broad band of wave effects might appear over the range where the cyclotron harmonics propagate, with no sharp damping appearing to occur.

The wave propagation in the 800-1000 gauss range may be regarded, then as the attenuating "tail" of cyclotron harmonic wave propagation, the sharp damping phenomena much reduced by ion-neutral interactions. These waves are not in actuallity "low magnetic field" waves, but represent the harmonic wave band associated with the "hot plasma" effects which stem from the higher order radial mode structure present in the cyclotron waves.

C. Wave Propagation in the Range 200-700 Gauss

The waves which were observed to propagate in this region were characterized by the highest magnitudes of magnetic signal $\dot{b}_z$, increased plasma diamagnetism (as shown in Fig. IV-6), and appreciable increase in electron densities. In all the above respects this behavior is characteristic of the "fast" hydromagnetic wave, as reported previously from other sources.[2,3] In the previous report, however, because of the lack of indication of increased plasma diamagnetism, and because the wave did not appear to match the cut-off conditions imposed by the dispersion relation and boundary conditions, it was tentatively concluded that the wave phenomena in this "pass band" were not attributable to fast wave propagation.

With the improved diamagnetic probe, plasma diamagnetism is observable so that there appears to be an anomaly in the plasma behavior. As discussed in Semiannual Status Report No. 9, possible wave phenomena, other than the fast wave, which might exist are ruled out by order-of magnitude differences in the parameters involved in the various dispersion relations.

The anomaly which then appears to exist is that the only possibility for wave propagation is the fast wave. The wave which <u>does</u> propagate has all the characteristic effects which might be attributable to the fast wave, but it does not appear to obey the cut-off relation.

The cut-off relation is normally written:

$$\frac{\omega}{\Omega_c} > \frac{5.45(10)^7}{r_o}\left(\frac{1}{n_e}\right)^{1/2} \qquad \text{(IV-1)}$$

for a plasma bounded by a vacuum. The driving frequency is $\omega = 2\pi f$, Ω_c is the ion cyclotron frequency, $= \frac{eB}{m_i}$, r_o is the plasma radius, e the electron charge, B the static magnetic field, m_i the ion mass and n_e the electron density.

It was shown previously[1] that for the fast wave to propagate at 500 G., the plasma density would have to be greater than any value actually recorded in the experimental data. However, there are several factors which could either lower the required density, or provide a higher density. First, the radial density of the plasma contains gradients as has been previously noted. The measured plasma density is somewhat averaged, even though a cosinusoidal radial density profile is assumed. A brief photographic study showed evidence of a hollow core of plasma of radius smaller than the plasma tube, so that a more accurate value of plasma density for the cut-off relation might then be 1 x 10^{12}cm^{-3} rather than 5 x 10^{11}. The value of plasma radius to be taken is also open to interpretation, as will be discussed further on in this section. For the present, however, the diameter of the plasma tube (3.0 cm) will be used. In this case, B_c (the cut-off field) would be:

$$B_c = 6.99(10)^{-5} r_0 \sqrt{n} = 210 \text{ gauss} . \qquad \text{(IV-2)}$$

The cut-off field obtained in (IV-2) is higher than that previously derived, but still less than the 600-700 gauss figure observed experimentally. However, the consideration of azimuthal symmetry, in at least one case, may also increase the critical or "cut-off" magnetic field, B_c. This may be seen by considering the equations of the field components of the fast wave in cylindrical coordinates. The cold plasma equations are considered rather than the more complicated hot plasma dispersion relation, for the following reasons.

First, the plasma may be regarded essentially as "cold;" the wave results in only moderate temperature increases so that $T_e \simeq T_i < 10$ eV in all probability. Secondly, the cut-off of the observed wave is very sharp, which rules out (at least in the field region 200-700 gauss) high-order radial mode structure. Thus radial and axial wavelengths are much larger than characteristic dimensions of the plasma (such as the Debye and Larmor radii), so that finite Larmor radius effects may be neglected. Notice, however, that the same is not true in this same experiment for ion cyclotron wave propagation. Also note that all modes above zero order are not ruled out; in particular the first-order azimuthal mode will be seen to be important in the following development.

Although the cold plasma wave equations and dispersion relations are well known,[6] a brief review is necessary in order to provide background for relating the modified fast-wave dispersion relation to the actual plasma case discussed here.

The basic field relations are derived from Maxwell's equations:

$$\nabla \times \vec{E} = i\omega \vec{B} \tag{IV-3}$$

$$\nabla \times \vec{B} = -\frac{i\omega}{c^2} \overleftrightarrow{K} \cdot \vec{E} = -\frac{i}{\omega} \overleftrightarrow{K}' \cdot \vec{E} \tag{IV-4}$$

The various vectors and vector operators are in cylindrical coordinates in this case. The dielectric tensor, $\overleftrightarrow{K}'$, has components:

$$\overleftrightarrow{K}' = \left(\frac{\omega^2}{c^2}\right) \begin{pmatrix} K_1 & K_2 & 0 \\ -K_2 & K_1 & 0 \\ 0 & 0 & K_3 \end{pmatrix} \tag{IV-5}$$

where

$$K_1' = \left(\frac{\omega^2}{c^2}\right) K_1 = \left(\frac{\omega^2}{c^2}\right) \left(1 + \frac{\Omega_p^2}{(\Omega_c^2 - \omega^2)} + \frac{\omega_p^2}{(\omega_c^2 - \omega^2)}\right) \tag{IV-6a}$$

$$K_2' = \left(\frac{\omega^2}{c^2}\right) K_2 = \left(\frac{\omega^2}{c^2}\right) \left(\frac{i\Omega_c \Omega_p^2}{\omega(\Omega_c^2 - \omega^2)} - \frac{i\omega_c \omega_p^2}{\omega(\omega_c^2 - \omega^2)}\right) \tag{IV-6b}$$

$$K_3' = \left(\frac{\omega^2}{c^2}\right) K_3 = \left(\frac{\omega^2}{c^2}\right) \left(1 - \frac{\Omega_p^2 + \omega_p^2}{\omega^2}\right) \tag{IV-6c}$$

The following definitions apply:

$\Omega_p \equiv$ ion plasma frequency $= \left(\frac{ne^2}{m_i \epsilon_o}\right)^{1/2}$

$\omega_p \equiv$ electron plasma frequency $= \left(\frac{ne^2}{m_e \epsilon_o}\right)^{1/2}$

$\Omega_c \equiv$ ion cyclotron frequency $= \frac{eB}{m_i}$

$\omega_c \equiv$ electron cyclotron frequency $= \frac{eB}{m_e}$

$n \equiv$ plasma density (ions or electrons) $(= n_i = n_e)$

$e \equiv$ electron charge $= 1.6(10)^{-19}$ coulomb (IV-7)

$m_e \equiv$ electron mass $= 9.1(10)^{-31}$ kg.

$m_i \equiv$ ion mass $= 1.67(10)^{-27}$ kg.

$\epsilon_o \equiv$ free space permittivity $= 8.85(10)^{-12}$ farad/m

$B \equiv$ static magnetic confining field

$\omega \equiv$ driving frequency or frequency of wave in rad/sec $(= 2\pi f)$

$c \equiv$ velocity of light $= 3(10)^8$ m/sec

$\nabla_x \rightarrow i\bar{k}x$ in the present case; $\frac{\partial}{\partial t} \rightarrow (-i\omega)$

$k \equiv$ the wave number, or absolute value, $|\bar{k}|$, of the wave-vector $\bar{k}$.

In the experiment under consideration, $\omega = 2\pi(5.8)(10)^6$ rad/sec. Thus, the following relations hold:

$$\begin{array}{ll} \omega \ll \omega_c & \Omega_p \ll \omega_p \\ \omega \ll \omega_p & \Omega_p \ll \omega_c \\ \omega \ll \Omega_p & \Omega_c \ll \omega_c \\ \multicolumn{2}{c}{\Omega_c \ll \omega_p} \end{array} \qquad \text{(IV-8)}$$

The plasma parameters of interest will include $n \simeq 10^{12}\,\mathrm{cm}^{-3}$ and $B \simeq 500$ gauss. It is then possible to include the following relations:

$$\omega_p \simeq \omega_c \; ; \; \omega \simeq 7\Omega_c \; ; \; \omega^2 \gg \Omega_c^2 \qquad \text{(IV-9)}$$

Now, using relations (IV-8) and (IV-9), the components of $\overleftrightarrow{K}'$ in (IV-6) may be reduced as follows:

$$K_1' \simeq -\left(\frac{\Omega_p^2}{c^2}\right) \qquad \text{(IV-10a)}$$

$$K_2' \simeq \frac{-i\omega}{\Omega_c}\left(\frac{\Omega_p^2}{c^2}\right) = \frac{i\omega}{\Omega_c} K_1' \qquad \text{(IV-10b)}$$

$$K_3' \simeq -\left(\frac{\omega_p^2}{c^2}\right) \qquad \text{(IV-10c)}$$

The results in (IV-10b) were obtained by recalling that:

$$\frac{\Omega_p^2}{\Omega_c} \equiv \frac{\omega_p^2}{\omega_c} \qquad \text{(IV-11)}$$

Now, an approximation often made at this point is $K_3' \to \infty$, which somewhat reduces the complexity of the problem. It is not, in the present case, a valid approximation. However, with the other pertinent approximations which have been introduced, the ($K_3' \to \infty$) approximation eliminates the consideration of a mode of interest only for frequencies near the electron cyclotron frequency. Thus in the present case, no loss of generality results in setting:

$$K_3' \longrightarrow \infty \tag{IV-12}$$

which implies:

$$E_z \longrightarrow 0 \tag{IV-13}$$

and considerably simplifies the anticipated algebraic manipulations.

Then, using (IV-3) and (IV-4), substituting for $\overleftrightarrow{K}'$ the values in (IV-10a), (IV-10b), and (IV-12) the Z-component of magnetic flux density as a function of r may be obtained in the following relation:

$$\frac{1}{r}\left[\frac{d}{dr}\left(r\,\frac{dB_z(r)}{dr}\right)\right] - \frac{n^2 B_z(r)}{r^2} + T^2 B_z(r) = 0 \tag{IV-14}$$

where T^2 is defined by the relation:

$$T^2 = \frac{\left[(k^2 - K_1')^2 + K_2'^2\right]}{k^2 - K_1'} \quad . \tag{IV-15}$$

Equation (IV-14) is Bessel's equation, with the general solutions being Bessel's functions of order n of the first and second kind. However, the Bessel function of the second kind is not considered here since it results in values for the fields which grow without bound as $r \to 0$. Since the axis of the plasma column is in the region of interest, this solution is discarded. The resulting solution for B_z is:

$$B_z(r,\theta,z) = \sum_{n=-\infty}^{\infty} \sum_{m} A_n J_n(T_m r)\, e^{i(kz + n\theta - \omega t)} \tag{IV-16}$$

where T_m is T as written previously. The subscript indicates the fact that there are many roots for the equations $J_n(Tr) = 0$, or $J_n'(Tr) = 0$, which will result from the boundary conditions.

The relation which was first used to obtain the cut-off field B_c, Equation (IV-1), above which the fast wave is evanescent, assumed boundary conditions involving a plasma-vacuum interface. In the present case, the plasma is surrounded not only by a glass container and air (which would normally make the above assumption accurate) but a series of copper magnet coils of bore only a few centimeters larger in diameter than the plasma. There are actually a triple set of boundary conditions to be considered (plasma-glass, glass-air, air-coils). However the field magnitudes are in general much larger in the plasma, and they diminish quickly approaching the "waveguide" surface provided by the magnet coils. To a good approximation, then, the tangential E-fields may be assumed to go to zero at some radius between that of the plasma and that of the magnet coils. This radius, r_w, may be considered the radius of a "virtual waveguide" due to the effects of the larger diameter coils and the smaller diameter plasma.

Since $E_z \simeq 0$, the only boundary condition is on E_θ:

$$E_\theta = 0 \Big|_{r=r_w} \qquad \text{(IV-17)}$$

where r_w is the "waveguide" radius, defined above.

The dispersion relation for cylindrical waves may be obtained from the definition in (IV-15):

$$T^2(k^2-K_1') + (k^2-K_1')^2 + K_2'^2 = 0 \qquad \text{(IV-18)}$$

At cut-off, the condition is that $k^2 = 0$, so that (IV-18) becomes

$$-K_1'T^2 + K_1'^2 + K_2'^2 = 0 \qquad \text{(IV-19)}$$

Referring to (IV-10b) the relation $K_2' = \frac{i\omega}{\Omega_c}(K_1')$ is obtained. Substituting into (IV-19):

$$-K_1'T^2 + K_1'^2 - \frac{\omega^2}{\Omega_c^2}K_1'^2 = 0 \quad , \tag{IV-20}$$

$$T^2 = K_1'\left(1 - \frac{\omega^2}{\Omega_c^2}\right) = 0 \quad . \tag{IV-21}$$

Substituting for K_1':

$$T^2 + \frac{\Omega_p^2}{c^2}\left(1 - \frac{\omega^2}{\Omega_c^2}\right) = 0 \quad . \tag{IV-22}$$

or since $\Omega_i^2 << \omega^2$,

$$T^2 - \frac{\Omega_p^2\omega^2}{\Omega_c^2 c^2} = 0 \quad , \tag{IV-23}$$

$$\Omega_c = \frac{\omega\Omega_p}{cT} \quad , \tag{IV-24}$$

or

$$B_o = 1.67(10)^{-4}\left(\frac{r_o\sqrt{n}}{\rho_m}\right) \quad . \tag{IV-25}$$

In (IV-25), B is the confining field in gauss, r_o the plasma radius in cm, n the density in cm^{-3}, and ρ_m the root of the appropriate Bessel function. Now the plasma density is on the order of $10^{12} cm^{-3}$, as stated above, so that;

$$B_o = \frac{167 r_o}{\rho_m} \quad . \tag{IV-26}$$

From equations (IV-3) and (IV-4), it is seen that $E_\theta \propto \frac{\partial B_z}{\partial r}$, since in the range of parameters of interest, the compressional and torsional waves are essentially uncoupled, and $E_z \simeq 0$. Then E_θ may be written:

$$E_\theta = C J_n'(T_m r) \quad . \tag{IV-27}$$

From (IV-17), the values of ρ_m are then obtained:

$$E_\theta\Big|_{r=r_w} = 0 = J_n'(T_m r_w) \tag{IV-28}$$

with r_w as defined above. The roots of Equation (IV-28) will be the ρ_m such that $J'_n(\rho_m) = 0$, with $\rho_m = T_m r_w$. Normally, only the zero-order Bessel functions are considered, since azimuthal modes are ignored. In the present case, however, the smallest value of ρ_m (which leads to the largest value of B_o) is obtained for $n = 1$; that is, by allowing the first order azimuthal mode. This is quite reasonable, in that the exciting or driving mechanism in the actual plasma experiment, the Stix coil, has one azimuthal discontinuity at the point of current input and reversal of coil direction (there are two coil reversal points and three input points but they all lie in the same azimuthal coordinate).

For $n = 1$, (IV-28) becomes:

$$J_1'(\rho_m) = 0 \tag{IV-29}$$

with first root $\rho_m = 1.84$.

The value of r_w, as defined in conjunction with (IV-17), lies between the plasma radius r_o and the magnet coil radius r_c;

$$r_o \lesssim r_w \lesssim r_c \tag{IV-30}$$

If B is set at 700 gauss, normally the field near which "cut-off" of the wave is observed, then r_w may be solved for:

$$r_w \simeq 7.7 \text{ cm}. \tag{IV-31}$$

which lies between the plasma radius of 3.0 cm and the magnet coil radius of 8.9 cm. If the actual coil radius is also used, a range of plasma density quite close to the actual range of values recorded will be obtained corresponding to the range of radii $7.7 \text{ cm} \le r_w \le 8.9 \text{ cm}$.

The correspondence between the calculated and observed values for "fast" wave propagation is very good with the assumptions made above, so that it may be concluded that "fast" wave does occur in the experimental case. The second "stop band" at 100-200 gauss may be attributed to the change in coupling conditions for the rf preionization which reduces the value of plasma density and thus produce a lower "cut-off" value of B as well as an upper value.

D. Wave Propagation in the Region 0-100 Gauss

Wave propagation in this lowest range of B is characterized by increased magnetic signal, $\dot{b}_z$, but with little or no increase in plasma density or diamagnetic signal. As neutral pressure increases, this band extends nearly to the limit $B = 0$.

It has been shown[7] that the coupling coefficients for higher-order azimuthal modes of the fast wave ($n \geqslant 8$) may actually be larger than coefficients for some of the lower order modes. Attenuation for the higher modes is also large, however, so that although their excitation would be somewhat easier than the lower order modes, they would tend to attenuate rapidly. They would not penetrate well into the plasma column and would exist essentially like surface waves, thereby having little influence on plasma density or heating. Furthermore, from the boundary condition on E_θ, it is seen that the values of ρ_m for these waves would be large, corresponding to cut-off at low values of B. The experimental plasma under consideration is driven by a Stix coil, the individual loops of which are wound helically. That is, there is a slight inclination of the plane of each loop from planes exactly perpendicular to the plasma tube axis and the direction of B. Such an assymmetry is sufficient to couple energy to non-symmetrical azimuthal modes, although such slight coil assymmetry is involved that only those modes having the largest coupling coefficients would be detectable.

The lowest band of low-field wave propagation may be attributed to fast wave propagation also. High attenuation of the larger order modes and weak penetration into the plasma account for weak interaction of this band with the plasma column.

E. Summary of Findings on Low-Field Wave Propagation

It is concluded that cyclotron harmonic waves have been seen to propagate at higher neutral gas pressures, although ion-neutral interactions "smear out" sharp damping effects. Since attenuation of these waves occurs between 800 and 1000 gauss, the fourth cyclotron harmonic in hydrogen (attenuating at $B = \frac{Bc}{4}$) may be present in some instances.

Wave propagation in the lower two "pass bands" is attributable to the fast wave when waveguide conditions are assumed, the lower band consisting of higher-order azimuthal modes and interacting only very weakly with the plasma column.

The data taken in deuterium confirm what is seen for hydrogen, although there is some difficulty in interpreting all the data clearly. Since there is a factor of (m_i) in the dispersion relation, one would expect that B_{cutoff} (deuterium) $= 2\,B_{cutoff}$ (hydrogen), and this does occur. The central propagation band which occurs in the 200–700 gauss range in hydrogen extends above 1.0 k gauss in deuterium.

For deuterium, however, the cyclotron harmonics may be seen to propagate up to the eighth[4] ($B = \frac{Bc}{G} = 925$ gauss), which means that there is a range of magnetic field for which cyclotron harmonic waves and the fast wave both propagate. The resultant signals from the plasma ($\dot{b}_z$ and B_D) are large and the influence of each wave on these signals is impossible to determine with the present diagnostic equipment, in all but a few cases. Of most importance, however, is the fact that the data taken confirm the findings reported in preceding sections, and agree with the explanation presented.

F. Plans for the Next Period

No plans are at present being made for further studies of the lower magnetic field propagation bands. Some data will be taken for $B \leqslant 1000$ gauss in connection with the harmonic frequency phenomena investigation which is continuing at present. A brief study is projected after completion of the installation of the new preionizer in order to study its characteristics. Primary emphasis in the experimental activities will be shifted to the other topics discussed in Section I.

References for Section IV

1. J. G. Melton, N. B. Dodge, and A. A. Dougal, S.S.R. No. 9 on NASA Grant NsG-353, The University of Texas at Austin, July 15, 1967.

2. M. A. Rothman and K. Chung, "Heating of Plasma by the Fast Hydromagnetic Wave," MATT-Q-23, pp. 32-37, April, 1966.

3. M. A. Rothman, "Coupling of Waves to a Cylindrical Plasma," Technical Memorandum TM-217, Plasma Physics Laboratory, Princeton University, September, 1965.

4. M. Kristiansen and A. A. Dougal, "Experimental Investigation of Harmonic Ion Cyclotron Wave Propagation and Attenuation," Phys. Fluids, 10, pp. 596-604, March, 1967.

5. D. G. Swanson, "Quasi-Static Ion Cyclotron Waves," Phys. Fluids, 10, pp. 1531-1534, July, 1967.

6. T. H. Stix, The Theory of Plasma Waves, (McGraw-Hill Book Company, Inc., New York, 1962).

7. D. G. Swanson, Private communication.