DDI-WA; JPL P. O. AA-390672 JPL Task No. DDI-T-002
NAS7-100

ON THE APPLICATION OF A METHOD OF DYNAMIC FILTRATION TO PROBLEMS OF DETERMINING TRAJECTORIES FOR SPACECRAFT

By B. A. Reznikov

Translated from "Kosmicheskiye Issledovaniya" v. IV, No. 4, 1966, pp. 535-544, by Joseph L. Zygielbaum, Data Dynamics, Inc., Los Angeles, California

A comparative characteristic of various methods for the construction of algorithms for the processing of measurement information during the determination of trajectories of spacecraft, is given in this paper. The possibility of constructing new types of algorithms by means of a method of dynamic filtration, are considered. The properties of these algorithms are investigated.

Normally, in course of processing measurement information, data on certain reference trajectories, sufficiently close to the actual orbit of the given spacecraft, are assumed. In such a case, the problem of determining the parameters of the actual trajectory, is reduced to the task of finding corrections for the parameters of the reference trajectory, which in turn permits the broad application of linearization during the un-iterated information processing, as well as during processing with the application of iteration methods. (Ref. 1-5).

Algorithms of processing measurement information, in which linearized estimates are used, are investigated in this paper. These algorithms are divided into two groups; 1. (algorithms which are calculated for the obtaining of estimates after the accumulation of a complete selection of measurements for a given stage of flight); 2. (algorithms which make it possible to obtain consecutively improving estimates in accordance with the flow of measurement information (algorithms of dynamic filtration). Principle attention is paid to the second, comparatively little investigated group of algorithms.

Single problems of the general theory of consecutive filtration were expounded in other scientific papers (Ref. 6-8). Problems connecting with the use of earlier obtained estimates during orbit determination, were considered in a number of papers (Ref. 2, 4, 5). The uniqueness of the paper which we present now is contained in the conduction of investigations on the basis of two matrix forms of generalized estimates (Ref. 9). The latter makes it possible to establish a direct contact between the estimates of dynamic filtration and estimates of a common type. In our work we have obtained the following new results:

1. The characteristics and conditions of equivalence of estimates of various methods of practicing selections of measurements of a complete and increasing volume have been determined.

2. The rules for selection of initial data for the introduction of computer devices into the mode of dynamic filtration have been formulated.

3. Basic versions of the application of a method of dynamic filtration to problems of orbit determination have been investigated, and a corresponding system of algorithmes has been constructed.

4. The possibility of computing several methods of information processing by way of utilization of universal properties of algorithmes of dynamic filtrations has been substantiated.

The application of the method of dynamic filtration in a number of practical cases might improve considerably such important indices, as for instance the speed of obtaining evaluations, the simplicity of algorithme realization, the systematic character of trajectory control, etc.

GENERALIZED ESTIMATES OF FLIGHT PARAMETERS

A generalized, linearalized estimate of a maximum probability for the n-vector of deviations of estimated parameters from the parameters of the reference trajectory Δq is constructed on the basis of data to which the following pertain: Δq_{Π} - preliminary estimate, Δh^* - normal l - dimensional vector of deviations of measurement results with a mathematical anticipation Δh, $K_{q\Pi}$ - correlating matrix of errata of the preliminary evaluation, K_h - correlating matrix of measurement errata. This estimate is determined by the correlation:

$$\Delta\hat{q} = \Delta\hat{q}_{\Pi} + N(\Delta h^{*} - W\Delta\hat{q}_{\Pi}), \tag{1}$$

where W is the matrix of the derivities of the l x m type, each i - line of which represents a transposed vector of partial derivities of the measured function according to the components of vector q:

$$W = \begin{pmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_l^T \end{pmatrix}, \quad w_i^T = \left(\frac{\partial h_i}{\partial q_1}\frac{\partial h_i}{\partial q_2}\cdots\frac{\partial h_i}{\partial q_m}\right), \tag{2}$$

and N is the matrix of l x m type, which has 2 forms of presentation (Ref. 9).

In the absence of a correlation between the errors in determining the vectors Δq_{Π} and Δh the presentation of the matrix N and the corresponding correlating matrix of errata and the estimate K_q in the first form is as follows:

$$N = K_q W^T K_h^{-1}, \quad K_q = (K_{q\Pi}^{-1} + W^T K_h^{-1} W)^{-1}. \tag{3}$$

The equivalent presentation of these matrices in the second form is,

$$N = K_{q\Pi} W^T (K_h + W K_{q\Pi} W^T)^{-1}, \quad K_q = K_{q\Pi} - N W K_{q\Pi}. \tag{4}$$

The sum-matrices, which are subjected to a transformation process in equation (3) and equation (4), have orders which are correspondingly equal to the number of estimated paramaters of m and the number of measurements of l. The rank of the matrices $W^T K_h^{-1} W$ and $W K_{q\Pi} W^T$ when $m > l$ does not exceed l, and when $m < l$ does not exceed m. From this follows that the first form might have an advantage when $m < l$ and the second form when $m > l$. The first form cannot be applied when $K_{q\Pi} = 0$ and $m > l$, and the second form cannot be applied when $K_h = 0$ and $m < l$, since in these cases inverting matrices in equations (3) and (4) become singular. A direct application of the first form is impossible when $K_h = 0$ and in the case of the second form - when $K^{-1}_{q\Pi} = 0$.

PROCESSING OF SAMPLES OF MEASUREMENTS OF A TOTAL VOLUME

The number of measurements in a sample of measurements of a total volume exceeds normally the number of estimated parameters. Therefore, in this case it is expedient to apply the first form of estimate (Ref. 1), (Ref. 3). The correlations of equation (1) and equation (3) can be substituted by an equivalent system of normal equations,

$$(K_{q\pi}^{-1} + W^{T}K_{h}^{-1}W)\,\Delta\hat{q} = K_{q\pi}^{-1}\Delta\hat{q}_{\pi} + W^{T}K_{h}^{-1}\Delta h^{*}, \tag{5}$$

which in the absence of a priori information and independent of measurement errors ($\Delta\hat{q}_{\pi} = 0$, $K^{-1} = 0$, the matrix K_h is diagonal) transfers into a common system of normal equations of the method of least squares.

Application of the algorithm as shown in equation (5) during an orbit determination using the Newtonian method of iteration, was explained in detail in the Reference papers (Ref. 1, 2). In this paper we examine only separate characteristics of this algorithm, necessary for a comparison of various types of algorithms.

1. During the realization of the algorithm in equation (5), a considerable number of operations has to be conducted on the formation of the matrix $W^{T}K_{h}^{-1}W$ and the vector $W^{T}K_{h}^{-1}\Delta h^{*}$. From equation (2) follows that

$$W^{T}K_{h}^{-1}W = \sum_{i,\,j=1}^{l} c_{ij}\mathbf{w}_{i}\mathbf{w}_{j}^{T}, \quad W^{T}K_{h}^{-1}\Delta \mathbf{h}^{*} = \sum_{i,\,j=1}^{l} c_{ij}\Delta h_{j}^{*}\mathbf{w}_{i}, \tag{6}$$

where c_{ij} are the elements of the matrix K_{h}^{-1}. Doing calculations according to the formulas in equation (6), it is necessary to develop and summarize a l^{2} matrix in the form of $c_{ij}\, w_{i}\, w_{j}^{P}$ and l square vectors $c_{ij}\Delta h_{j}^{*}\, w_{i}$, in addition to the determination of the vectors of partial derivatives of w_i and the elements c_{ij}. We must thereby keep in mind all elements of the matrix W until the processing is completed.

Due to the fact that the correlating characteristics of measurement information is still insufficiently studied, and the consideration of these characteristics is generally connected with considerable complications in the process of calculations, the method of least squares is widely used. Estimations conducted with this method prove often to be close to estimations of maximum probability (Ref. 1-3), as far as effectiveness is concerned. Doing the application of the method of least squares, the matrix K_h^{-1} is considered to be diagonal ($c_{ij} = c_{ii}$ when $i = j$, $c_{ij} = 0$ when $i \neq j$), and it follows from equation (6) that

$$W^T K_h^{-1} W = \sum_{i=1}^{l} c_{ii} \mathbf{w}_i \mathbf{w}_i^T, \quad W^T K_h^{-1} \Delta \mathbf{h}^* = \sum_{i=1}^{l} c_{ii} \Delta h_i^* \mathbf{w}_i. \tag{7}$$

The number of developed and summarized matrices and vectors in equation (7) decreases in comparison with equation (6) l x. Correspondingly, the number of opperations decreases. The number of multiplication operations per one measurement comprises * $m^2 + 2m + 1$. We will also note that in the given case it is not necessary to memorize the matrix W, it is sufficient only to compute successfully the vectors w_i in process of developing the correlation of equation (7).

2. A tendency was observed in recent times toward a more complete consideration of statistical properties of measurement information in process of planning the composition and program of measurement, and in specific cases, also during the processing (Ref. 2, 3). It is thereby desirable to simplify to a maximum, the calculation of correlating connections. The first step in this direction is the break-up of the complete sample of measurements into a certain number of groups, the correlation between which can be neglected. Then the matrix K_h becomes quasi-diagonal (Ref. 10):

$$\Delta \mathbf{h}^* = \begin{pmatrix} \Delta \mathbf{h}_1^* \\ \Delta \mathbf{h}_2^* \\ \vdots \\ \Delta \mathbf{h}_s^* \end{pmatrix}, \quad K_h \Rightarrow \begin{pmatrix} K_{1h} & 0 & \dots & 0 \\ 0 & K_{2h} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & K_{sh} \end{pmatrix}, \tag{8}$$

*The operation of calculating w_i is not taken into consideration in this case. A number of methods for the calculation of isochronic derivatives is proposed in the papers of Ref. 1, Ref. 2. Further on, during the comparison of various algorithms, it is proposed that the vectors w_i are computed with an identical method .

where $\Delta h_1^*, \Delta h_2^*, \ldots, \Delta h_s^*$ are the subvectors $K_{1h}, K_{2h}, \ldots, K_{sh}$ are the sub matrices of the groups.

It follows from the correlation in equation (8) that

$$W^T K_h^{-1} W = \sum_{p=1}^{s} W_p^T K_{ph}^{-1} W_p, \quad W^T K_h^{-1} \Delta \mathbf{h}^* = \sum_{p=1}^{s} W_p^T K_{ph}^{-1} \Delta \mathbf{h}_p^*. \tag{9}$$

Further simplification is possible through a specific presentation of the matrix K_{ph}. For instance, in a number of cases the correlating dependents is well described by the exponential law. If the intervals between the measurements of each p - group are thereby equal to $t_{i+1} - t_{i+1} - t_i = T_p$ = const and the number of measurements within the group equals l_p, we have the following presentation of the matrix K_{ph}:

$$K_{ph} = \sigma_p^2 (r^{|i-j|}), \quad r = e^{-\alpha_p T_p} \quad (i, j = 1, 2, \ldots, l_p). \tag{10}$$

The matrix K^{-1}_{ph} in the given case is tri-diagonal (Ref. 10). Consequently,

$$\begin{aligned} W_p^T K_{ph}^{-1} W_p &= \sum_{i=1}^{l_p} c_{pii} \mathbf{w}_{pi} \mathbf{w}_{pi}^T + \sum_{i=1}^{l_p - 1} c_{pi,\, i+1} \mathbf{w}_{pi} \mathbf{w}_{p,\, i+1}^T + \\ &+ \sum_{i=2}^{l_p} c_{pi,\, i-1} \mathbf{w}_{pi} \mathbf{w}_{p,\, i-1}, \\ W_p^T K_{ph}^{-1} \Delta \mathbf{h}_p^* &= \sum_{i=1}^{l_p} c_{pii} \Delta h_{pi}^* \mathbf{w}_{pi} + \sum_{i=1}^{l_p - 1} c_{pi,\, i+1} \Delta h_{p,\, i+1}^* \mathbf{w}_{pi} + \\ &+ \sum_{i=2}^{l_p} c_{pi,\, i-1} \Delta h_{p,\, i-1}^* \mathbf{w}_{pi}, \end{aligned} \tag{11}$$

where

$$c_{pii} = \begin{cases} \dfrac{1}{\sigma_p^2 (1 - r^2)} & \text{when} \quad i = 1, i = l_p \\ \\ \dfrac{1 + r^2}{\sigma_p^2 (1 - r^2)} & \text{when} \quad i = 2, 3, \ldots, l_p - 1 \end{cases}, \quad c_{pi,\, i+1} = c_{pi,\, i-1} = \frac{-r}{\sigma_p^2 (1 - r^2)}. \tag{12}$$

Calculation according to the formulas (11) requires approximately twice the number of operations than calculations according to the formulas (7). For the consideration of each new i - measurement, it is necessary to have only the last w_{pi} and the penultimate w_p, i-1 value of the vector of partial derivatives.

Undoubtedly, the distinction of processing a sample of total volume measurements consists of the fact that we have thereby the best potential possibilities for analyzing the measurement information and obtaining effective estimates. At the same time the processing of measurement samples of an increasing volume has considerable advantages during the solution of a number of problems.

PROCESSING OF SAMPLES OF MEASUREMENTS OF AN INCREASING VOLUME DYNAMIC FILTRATION

In many cases there might arise the necessity for trajectory control and for making decisions in process of receiving measurement information, as well as during the information processing. First of all this pertains to cases related to lengthy and systematic flows of measurement information. A rational processing method in such conditions is the method of dynamic filtration which is characterized by the following: 1) the estimates are conducted in succession in line with the growth of volume of measurement samples, 2) the preceding estimate is used in determining each consecutive estimate. The method of dynamic filtration is directly related to the theory of dynamic programming (Ref. 8) and the theory of Markov processes (Ref. 7, 11). On the other hand, during specific conditions it is possible to establish the relation of the methods to the theory of consecutive analysis by Vald (Ref. 12).

1. We will show that when the matrix K_h is in a condition of diagonality or quasi-diagonality and the presence of data $\Delta\hat{q}_\pi$ and $K_{q\pi}$ the algorithm (5) might be substituted by an equivalent (according to the estimate results) algorithm of dynamic filtration in two forms.

We will define the estimate obtained according to the measurements of the s groups by $\Delta q^{(s)}$ and the correlating matrix of errors of this estimate by $K_q^{(s)}$. In accordance with equation (5) and equation (9) we obtain

$$\left(K_{q\pi}^{-1} + \sum_{p=1}^{s} W_p^{T} K_{ph}^{-1} W_p\right)\Delta\hat{q}^{(s)} = K_{q\pi}^{-1}\Delta\hat{q}_\pi + \sum_{p=1}^{s} W_p^{T} K_{ph}\Delta h_p^{*}. \qquad (13)$$

By putting down the analogeous expressions for the estimate of $\Delta\hat{q}^{(s+1)}$ according to s +1 groups and taking into consideration equation (3) and equation (13), we obtain the recurrent correlation

$$\Delta\hat{\mathbf{q}}^{(s+1)} = (K_q^{(s)^{-1}} + W_{s+1}^{T} K_{s+1,h}^{-1} W_{s+1})^{-1} (K_q^{(s)^{-1}} \Delta\hat{\mathbf{q}}^{(s)} + W_{s+1}^{T} K_{s+1,h}^{-1} \Delta\mathbf{h}_{s+1}^{*}). \tag{14}$$

We will assume that $\Delta\hat{q}_{\pi} = \Delta q^{(0)}$, $K_{q\pi} = K_q^{(0)}$ and we will substitute equation 14 with correlations which are analogeous to equation (1) equation (3) and equation (4):

$$\hat{\Delta}\mathbf{q}^{(s+1)} = \Delta\hat{\mathbf{q}}^{(s)} + N^{(s+1)} (\Delta\mathbf{h}_{s+1}^{*} - W_{s+1}\Delta\hat{\mathbf{q}}^{(s)}); \tag{15}$$

$$N^{(s+1)} = K_q^{(s+1)} W_{s+1}^{T} K_{s+1,h}^{-1}, \quad K_q^{(s+1)} = (K_q^{(s)-1} + W_{s+1}^{T} K_{s+1,h}^{-1} W_{s+1})^{-1}, \tag{16}$$

$$N^{(s+1} = K_q^{(s)} W_{s+1}^{T} (K_{s+1,h} + W_{s+1} K_q^{(s)} W_{s+1}^{T})^{-1},$$

$$K_q^{(s+1)} = K_q^{(s)} - N^{(s+1)} W^{(s+1)} K_q^{(s)}. \tag{17}$$

The recurrent formulas (15), (16), (first form) or (15), (17) (second form) might be considered as algorithms of dynamic filtration which make it possible to transfer consecutively from the estimate for s groups of measurements to the estimate for s+1 groups betinning with s = 0. We will name each such transition as a step of dynamic filtration. The magnitude of each step is determined by the number of measurements taken into consideration during the given step. In the case when this number = 1 the vector Δh_{s+1} becomes a scalar quantity ($\Delta h_{s+1} = \Delta h_{s+1}$), the matrix W_{s+1} becomes equal to the vector w_{s+1}, and the matrix K_{s+1}, h becomes equal to the dispersion of the s+1 measurement of σ^2_{s+1}. Then we obtain from equation (17)

$$N^{(s+1)} = \frac{1}{\sigma^2_{s+1} + \mathbf{w}^{T}_{s+1} K_q^{(s)} \mathbf{w}_{s+1}} K_q^{(s)} \mathbf{w}_{s+1}, \quad K_q^{(s+1)} = K_q^{(s)} - N^{(s+1)} \mathbf{w}_{s+1}^{T} K_q^{(s)}. \tag{18}$$

From the above described follows that for measurement information which satisfies the indicated conditions, after any number of s steps of dynamic filtarion with the application of recurrent correlations (15), (16) or (15), (17) or (15), (18), the same estimates are obtained as in the case of processing of a complete sample of measurements of a given volume with the utilization of the algorithm (5).

Of greatest interest in the examined case is the algorithm (15), (18). This algorithm might be considered as a method for solving the system of equations (5) of an arbitrary l-order, which makes it possible to find the estimates for the cases 1, 2, ..., l measurements during the solving process. Thereby, during each step which includes the formation of the corresponding matrices as well as the obtaining of estimates, $2m^2 + 4m$ operations are carried out, i. e. twice as much than doing one step of forming a system of normal equations.

Thus, during the processing of a sample of measurements of a large volume, when the operation of a common solution to a system of normal equations has a low specific weight in the overall number of operations, the application of the algorithm (15), (18) leads to a certain increase of this number. In process of finding estimates after each measurement or after each sample of measurements of a small volume, the given algorithm naturally yields a gain in comparison with the numerous solutions to equations (5). Application of the method of dynamic filtration leads always to a simplification of the program.

In a specific case, the given version of processing might be applied during the obtaining of an estimate at a certain starting point of the trajectory but each cycle of calculations according to the Newtonian method of iteration. Additional possibilities are thereby created in respect to the control of the process by way of a comparison of estimates obtained during each step of dynamic filtration.

2. We will assume that a priory information is absent or little trustworthy. Then, assuming that $\Delta\hat{q}_{\pi} = 0$ and selecting in a determined manner the matrix $K_{q\pi}$, it is possible by utilizing the algorithm (15), (18) already after a minimum number of measurements ($l = m$) to obtain an estimate which will be sufficiently close to the estimate obtained according to a complete selection of measurements with a volume of $l = m$.

We will define the estimate obtained when $l = m$, $\Delta\hat{q}_{\pi} = 0$, $K^{-1}_{q\pi} = 0$ by $\Delta\hat{q}_{min}$, and when $l = m$, $\Delta\hat{q}_{\pi} = 0$, $K^{-1}_{q\pi} \neq 0$ by $\Delta\hat{q}'_{min}$. In these cases, the matrix $W = W_m$ is ^{2} and from (5) follows that

$$\Delta\hat{\mathbf{q}}_{\min} = W_m^{-1}\Delta\mathbf{h}_m^*, \quad \Delta\hat{\mathbf{q}}'_{\min} = (K_{q\pi}^{-1} + W_m^{\mathrm{T}} K_{hm}^{-1} W_m)^{-1} W_m^{\mathrm{T}} K_{hm}^{-1} \Delta\mathbf{h}_m^*. \tag{19}$$

We will request that the relation of the standards of difference of vectors $\Delta\hat{q}'_{min}$ and $\Delta\hat{q}_{min}$ to the standard of the vector $\Delta\hat{q}_{min}$ should be less than a certain low number of ϵ:

$$\frac{\| \Delta\hat{\mathbf{q}}'_{\min} - \Delta\hat{\mathbf{q}}_{\min} \|}{\| \Delta\hat{\mathbf{q}}_{\min} \|} < \varepsilon. \tag{20}$$

On the basis of known properties of the standards of vectors and matrices (Ref. 10), we obtained from equation (19)

$$\| \Delta\hat{\mathbf{q}}'_{\min} - \Delta\hat{\mathbf{q}}_{\min} \| \leqslant \| (E + B)^{-1} - E \| \| \Delta\hat{\mathbf{q}}_{\min} \|, \tag{21}$$

where $B = (W_m^P K^{-1}_{hm} W_m)^{-1} K^{-1}_{q\Pi}$, E is a unit matrix.

Assuming that $\| B \| < 1$, after dividing (21) by $\| \Delta q_{min} \|$ and by expending $(E = B)^{-1}$ in series we obtained a condition which is sufficient to carry out the inequality (20):

$$\|B\| < \frac{\varepsilon}{1+\varepsilon} \text{ or } \|K_{q\pi}^{-1}\| < \frac{\varepsilon}{1+\varepsilon} \frac{1}{\| (W_m^{\mathrm{T}} K_{hm}^{-1} W_m)^{-1} \|}. \tag{22}$$

Since $\varepsilon < 1$, then under a condition for executing (22) the inequality $\| B \| < 1$ is also satisfied.

We will note that the expression $(W_m^P K_{hm} W_m)^{-1}$ represents a correlating matrix of estimate errors according to a sample of minimum volume of K_{qm} in the absence of a priory data. We should orient ourselves on a certain maximum possibility for the proposed conditions for processing the value of the standard of this matrix $\| K_{qm} \|_{max}$. Then it follows from (22) that

$$\|K_{q\pi}^{-1}\| < \frac{\varepsilon}{1+\varepsilon} \frac{1}{\|K_{qm}\|_{\max}}. \tag{23}$$

In practical application, it is expedient to select the matrix $K_{q\pi}$ in a diagonal condition with sufficiently large magnitudes of diagonal elements at which the condition (23) is carried out.

We will assume that the dynamic filtration is realized with the utilization of the correlation (15), (18), $\Delta \hat{q}_{\pi}$ is accepted to be equal to 0, and $K_{q\pi}$ is selected in accordance with (23). Then on the basis of (19), (20) and the material described in section 1, it is possible to ascertain that after m steps the estimate coincides with $\Delta \hat{q}_{min}$ and will coincide with the estimate $\Delta \hat{q}_{min}$ with any given accuracy which is determined by the magnitude of ϵ. By further increasing the number of measurements the degree of coincidence with the results of a normal solution to a system of normal equations is either preserved or improved if we do not take into consideration the calculation errors. Thus the algorithm which is determined by the correlation (15), (18), (23), might be considered as a matter for solving a system of normal equations with a method of least squares of an arbitrary order $l \geq m$, which differs by the fact that with its help the solution is realized in correspondence with the flow of measurement information. We obtain thereby estimates for the cases m, m + 1 . . . l measurements.

We will note that the correlation (23) which characterizes the lower limits for selection of elements of the matrix $K_{q\pi}$, is constructed without consideration of rounding off errors. In actuality, an increase in the elements of the matrix $K_{q\pi}$ leads to a determined road of rounding off errors during the first step of dynamic filtration. In cases when the magnitude $\| K_{qm} \|_{max}$, which is used in determining $K_{q\pi}$, proves to be very high, this circumstance might reflect substantially on the accuracy of the obtained estimates. In these conditions and also in conditions when the matrix W_m is singular, it is expedient to concentrate on the obtaining of a good coincidence with the results of common estimates according to a complete sample, not after m, but after a certain number of $l_i > m$ steps. In the correlation (23) we substitute thereby instead of $\| K_{qm} \|_{max}$ a lesser magnitude $\| K_{q l_1} \|_{max}$ which leads to a decrease in the required values of the diagonal elements of matrix $K_{q\pi}$.

3. We will examine the possibilities of calculating the correlations between measurement errors during the realization of the dynamic filtration, with a step which

equals 1. If we should conduct within each group measurements at equally spaced time intervals and the correlating matrix K_{ph} is in the form of (10), then, by utilizing (5), (9), (11) and (12), it is possible to find the recurrent correlations which combine the estimates according to measurement samples from the p-group when this p-group of measurements contains j-measurements as well as j + 1 measurements ($\Delta q^{(p,j)}$ and $\Delta q^{(p,j+1)}$):

$$\Delta\hat{\mathbf{q}}^{(p,j+1)} = K_q^{(p,j+1)}\left(K_q^{(p,j)-1}\Delta\hat{\mathbf{q}}^{(p,j)} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j}\Delta\dot{h}_{p,j} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j+1}\Delta\dot{h}_{p,j+1}\right), \tag{24}$$

$$K_q^{(p,j+1)} = \left(K_q^{(p,j)-1} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j}\mathbf{w}^T_{p,j} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j+1}\mathbf{w}^T_{p,j+1}\right)^{-1},$$

where

$$\mathbf{w}'_{p,j} = -\frac{r}{1-r^2}(\mathbf{w}_{p,j+1} + r\mathbf{w}_{p,j}), \quad \mathbf{w}'_{p,j+1} = \frac{1}{1-r^2}(\mathbf{w}_{p,j+1} - r\mathbf{w}_{p,j}). \tag{25}$$

We will introduce a new vector $\Delta q'^{(p,j)}$,

$$\Delta\hat{\mathbf{q}}'^{(p,j)} = \left(K_q^{(p,j)-1} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j}\mathbf{w}^T_{p,j}\right)^{-1}\left(K_q^{(p,j)-1}\Delta\hat{\mathbf{q}}^{(p,j)} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j}\Delta\dot{h}_{p,j}\right). \tag{26}$$

By defining the inverted sum-matrix in (26) by $K'^{(p,j)-1}_q$ we obtain from (24) and (26)

$$\Delta\hat{\mathbf{q}}^{(p,j+1)} = \left(K_q'^{(p,j)-1} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j+1}\mathbf{w}^T_{p,j+1}\right)^{-1}\left(K_q'^{(p,j)-1}\Delta\hat{\mathbf{q}}'^{(p,j)} + \frac{1}{\sigma_p^2}\mathbf{w}'_{p,j+1}\Delta\dot{h}_{p,j+1}\right). \tag{27}$$

The correlations (26), (27) are of the same structure as the correlation of (14) in the case of one measurement within the group ($W_{s+1} = W, K_{s+1,h} = \sigma^2_{s+1}$). Consequently these correlations can be analogously converted into the form of (15), (18).

The process of dynamic filtration in the examined case is realized as follows: The recurrent formulas of (15), (18) are utilized twice for each measurement. The first time we substitute in them the values $\Delta\hat{q}^{(p,j)}$, $K_q^{(p,j)}$, $\Delta h^*_{p,j}$, $w'_{p,j}$ and $w^T_{p,j}$ and we determine $\Delta\hat{q}'^{(p,j)}$ and $K_q'^{(p,j)}$. The second time, in result of substituting $\Delta\hat{q}'^{(p,j)}$ $K_q'^{(p,j)}$, $\Delta h^*_{p,j+1}$, $w_{p,j+1}$, we determine the values of $\Delta\hat{q}^{(p,j+1)}$ and $K_q^{(p,j+1)}$. Thus we have established a multi-step process during which the basic steps, which take into consideration current measurements, are alternated by auxiliary steps which take into consideration preceding measurements. The properties of such processes approach the Markov processes.* Just as in the case of processing independant information the obtained results are equivalent to the solution of the equation of (5) for a sample of a corresponding volume

The considerable distinction of the examined algorithm is contained in the fact that its application is possible at any number of measurements within a group.

Another way for calculating correlations, in the case when the correlating function of errors is exponential, consists of the prognostication of the magnitude of errors during each step, i.e. it is included in the number of estimated parameters(ref .5) This result, as well as the preceding one, is found in connection with the Markov properties of the Gaussian process, which contains an exponential correlating function(Ref. 11).

Further generalization might be obtained by presenting the correlating matrix K_{ph} in the form of the sum $M_p + 1$ matrices having the following view,

$$K_{ph} = (\sigma_{pii}^2 \delta_{ji}) + \sum_{m=1}^{M_p} \sigma_{pm}^2 (e^{-\alpha_{pm}|t_i - t_j|}) \quad (i, j = 1, 2, \ldots, l_p), \tag{28}$$

where $\delta_{i,j}$ are the Kroneker factors. The presentation of (28) approximates well any correlating dependents which can be found in practice. By including M of additional values in the number of estimated parameters, it is possible to construct a process of dynamic filtration with the use of the correlations (15), (18).

4. One of the applications of the method of dynamic filtration to problems of orbit determination is related to the obtaining of estimates at each point of measurement information flow(ref. 4,5) or at the starting points of the consecutive orbital sectors, at each of which a group of measurements is conducted. In these cases, by utilizing the recurrent correlations (15), (16), (17) and (18), it is necessary

---

* The process of consecutive handling of measurement information contains Markov properties in the case when the estimate of parameters during each step is completely determined by the estimate of the preceding step, as well as the new flow of information.

to take into consideration the transition from an estimate at point t_p to an estimate at point t_{p+1}, which is carried out with the help of the transition matrix $\Phi(t_p, t_{p+1})$ (the matrix of isochronic derivatives of trajectory parameters). For this purpose it is necessary to introduce into the indicated correlation $\Phi(t_p, t_{p+1})\,\Delta\hat{q}^{(p)}$ instead of $\Delta\hat{q}^{(p)}$ and $\Phi(t_p, t_{p+1})\,K_q^{(p)}\,\Phi^T(t_p, t_{p+1})$ instead of $K_q^{(p)}$. The flight between the points of consecutive estimates t_p and t_{p+1} can normally be considered as being very little disturbed, which makes it possible to calculate the matrix of $\Phi(t_p, t_{p+1})$ according to the formulas(ref. 2).

By applying the same methods as in sections 1 and 2 it is not difficult to show that the obtained results in the examined case are equivalent to the determination of the estimate according to a total volume sample at a starting point with a following calculation at the terminal point of measurement.

5. Of particular interest is dynamic filtration with the introduction of corrections to the reference trajectory. In the given case, after finding the estimate $\Delta\hat{q}^{(p)}$ at the point t_p, we conduct a correction of the initial conditions of integration of the reference trajectory at that point as follows:

$$\hat{q}^{(p)} = \hat{q}_{\pi}^{(p)} + \Delta\hat{q}^{(p)}, \tag{29}$$

where $\hat{q}_{\pi}^{(p)}$ is the estimate determined as a result of integration of the sector $[t_{p-1}, t_p]$, $\hat{q}^{(p)}$ are the initial conditions of integration at the sector $[t_p, t_{p+1}]$.

Depending on the number of measurements during estimation, we utilize the recurring correlations (15), (16) or (15), (17), or (15), (18), in which it is necessary to assume that $\Delta\hat{q}^{(p)} = 0$ and to consider a re-calculation of the correlating matrix, i.e. instead of $K_q^{(p)}$ we substitute $\Phi(t_p, t_{p+1})\,K_q^{(p)}\,\Phi^T(t_p, t_{p+1})$.

If the radius of the zone of probable deviations of a spacecraft from the reference trajectory is small, then the results of dynamic filtration without corrections to the reference trajectory, coincide in fact with the results which include such corrections. However when the zone of deviations increases, the advantage of the latter method over the first becomes apparent. The advantage consists of the fact that this method permits in principle to decrease the linearization errors by bringing closer the eference trajectory to the actual trajectory, as we introduce consecutive corrections. Furthermore it is possible in a number of cases to disregard the application of a common process of iteration, i.e. of repeated integrated differential equations of motion, even then when the initial deviations are quite considerable. In the case of determined measurement compositions (for instance when three position functions are measured) and in cases when measurement information is systematically fed to a sufficiently large orbital sector, particularly favorable results might be achieved by the application of this method.

The indicated properties of the method contribute to an essential increase in the speed of obtaining estimates (precisely defined trajectory data might be obtained with an insignificant delay after each new reception of measurements results).

The statistical characteristics of measurement errors and the characteristics of perturbing forces which affect a spacecraft might be considered in course of constructing an algorithm for the processing of measurement information only to a known degree of approximation. For this reason, and also due to the accumulation of calculation errors, we have observed in process of dynamic filtration a determined divergence of values in the elements of the correlating matrix K_q, which is calculated during each step, and the actual correlating matrix of total errors K_{qa}. This divergence might become quite considerable in a case when the dynamic filtration continues over a long period of time. Consequently, during a systematically lengthy flow of information, it is necessary to correct periodically the matrix K_q bringing it closer to the matrix K_{qa}. A possible organization of control of the process of dynamic filtration, by way of calculating the mean square of deviation of the weight unit σ_o^2 on the consecutive sectors. The obtained values of σ_o^2 might be used during the correction of K_q.

During the process of dynamic filtration controlling influences might be formed on the basis of current results of information processing and might be controlled by means of measurements (ref. 4,5). Measurements make it possible to introduce reference trajectory corrections which would correspond to the controlling influences. During the introduction of each such correction to $\Delta\hat{q}^{(i)}$, simultaneously with the correction of the initial estimate of $\hat{q}_{\pi}^{(i)}$ at the i-point, the correlating matrix of errors becomes also corrected:

$$\hat{q}^{(i)} = \hat{q}_{\pi}^{(i)} + \Delta\hat{q}^{(i)}, \quad K_q^{(i)} = K_{q\pi}^{(i)} + K_{qy}^{(i)}, \tag{30}$$

where $K_q^{(i)}$ is the initial correlating matrix of errors, and $K^{(i)}_{qy}$ is the correlating matrix guidance errors of the i - control.

The determination of $\hat{q}^{(i)}$, K and the processing of information obtained from other means of measurement are conducted according to the normal schematic of the method of dynamic filtration with the introduction of corrections to the reference trajectory.

6. In spite of a number of essential distinctions of the method of dynamic filtration with the introduction of corrections to the reference trajectory, this method might prove to be inferior to the Newtonian method of iteration in the process of solving specific problems according to individual readings. This forces a conclusion that it is expedient to consider both methods and to use them when necessary during the various flight stages (ref. 9). An important circumstance in this case is the possibility of carrying out calculations during each cycle of the Newtonian method of iteration according to the recurrent formulas of the method of dynamic filtration (see section 1). As was noted, this increases somewhat the number of operations, however it improves the control of the process and gives the possibility for the development of a universal program. A universal program might also be constructed if, along with the indicated methods, we would also use a method of dynamic filtration without the introduction of corrections to the reference trajectory.

Submitted for publication June 23, 1965.

LITERATURE

1. P.E. Elyasberg, V.D. Yastrebov. "Iskusstv. Sputniki Semli", No.4 publ. AN SSSR, 1960, p. 18.

2. E.A. Akim, T.M.Eneyev. Kosmich. Issled., 1, No. 1,5, 1963.

3. A.V.Brykov. Kosmich,Issled. 2, No.4, 539, 1964.

4. G.L.Smith,S.F.Schmidt,L.A.McGee. Techn.Rept. NASA, No. R-135, 1962.

5. R.H.Battin, ARS Journal, 32, No.11, 1681, 1962.

6. R.E. Kalman. Trans. ASME, ser.D, 82, No.1, 35, 1960.

7. R.L.Stratonovich. Dokl. AN SSSR, 140, No.4, 769, 1961.

8. R.Bellman, S.Dreyfus. Applied Problems of Dynamic Programming. Publ. "Nauka", 1965.

9. B.A. Reznikov, Kosmich. Issled., 4, No.1, 40, 1966.

10. D.K.Fadeyev, V.N.Fadeyeva. Vychislitel 'nyye metody lineynoy algebry (Calculation methods of linear algebra), Fizmatgiz, 1960.

11. D.L. Dub. Vyeroyatnostnyye processy (Probability processes), Izd. Inostr. Lit. 1956.

12. A. Val'd . Posledovatel'nyy analiz (Sequential analysis), Fizmatgiz, 1960.