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# CONSIDERATIONS
# ON THE RETURN TRAJECTORIES FROM THE MOON TO THE EARTH

by

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## SUMMARY

Considered in this paper are trajectories beginning near the Moon, emerging from its sphere of action on the first revolution, and approaching the Earth during the first orbit around it. The axial trajectories of the considered trajectory beams pass through the center of the Earth. They begin either on the lunar surface or on orbits of artificial satellites of the Moon (AMS).

Possible types of return trajectories are indicated and their evolution is ascertained with the change of initial velocity modulus. The influence of initial data scattering is approximately analyzed.

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The problem of return voyage from the Moon to the Earth was examined in [1 - 3]. In this work the problem is considered for the case of return from the surface of the Moon, as well as for that from the orbit of Moon's artificial satellite (AMS).

This problem has two aspects: one refers to the possibility of return of automatic devices to Earth as a whole, the other – to the possibility of return of piloted crafts with flat entry into the dense atmosphere layers for example, into a preassigned corridor according to the height of a conditional (conventional)perigee.The width of this corridor is small by comparison with the radius of the Earth [4, 5], and the requirements of precise realization of entry trajectory are incomparably higher than for return trajectory to "Earth in the whole".

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(*) O TRAYEKTORIAKH VOZVRASHCHENIYA OT LUNY K ZEMLE

In the problem of return from the surface of the Moon unquestionable interest is offered by trajectories starting from the region of possible points of vertical landing. Interesting also are trajectories starting vertically, or nearly so, relative to the lunar surface, since for them the control system by takeoff is simplest.

When considering variants of return to Earth from AMS orbit of the type "LUNA-10" and "LUNA-11", it was assumed that the planes of the considered AMS orbits pass approximately through one and the same straight line. This line is the axis of the beam of flight trajectories to the Moon, passing through its center, with flight time of about 3.5 days.

The return trajectories were analyzed with the aid of an approximate method based upon the consideration of selenocentrical and geocentric escape velocities from Moon's sphere of action. This method of study of a multitude of return trajectories allowed us to obtain a good qualitative and an approximate quantitative representation on the influence of various factors and on the basic characteristics of return trajectories.

It is ascertained that there exist only two types of return trajectories, and that there is a minimum initial velocity assuring the return into a preassigned region of Earth's surface. At lower velocities there appears on the Earth's surface a forbidden region into which the return is impossible.

A return with least initial velocities (about 2.58 km/sec) takes place at start from AMS orbit. For a return journey from the Moon's surface with vertical start lower initial velocities are required than for an inclined start, but these velocities are found to be only by a few tens of meters per second greater than the minimum, and this on condition of sufficiently good visibility of the point of start from Earth. When starting from the region of vertical landing, the horizontal direction of the initial velocity is found to be the most advantageous from the standpoint of energy, whereupon its magnitude constitutes $\sim$ 2.65 km/sec.

The approximate method allows us to obtain a representation on the influence of initial data scattering on the return trajectory. The fundamental factors influencing the deviation of the return trajectory from the nominal are the initial velocity and the angular flight range within the sphere of action of the Moon.

## 1. GENERAL CHARACTERISTIC OF THE MULTITUDE OF RETURN TRAJECTORIES

Let us call return trajectories those which begin near the Moon, escape its sphere of action over the first orbit around the Moon and then approach the Earth after having complete around it no more than one revolution.

All trajectories "Moon-Earth" satisfy this definition on the condition that for them the value of Jacobi integration constant in a cicrular restricted three-body problem "Earth-Moon-device" exceed sufficiently its first critical value $h_1$ (see [6], pp.160-173). Trajectories "Moon-Earth" with values of $\underline{h}$ only little exceeding $h_1$, perform numerous revolutions about the Moon and then around the Earth. Thet are extremely sensitive to the scattering of initial data, the corresponding flight times are quite great [6], and in the following these trajectories will not be considered. We shall limit ourselves to the study of return trajectories with flight times of the order of a few days.

An approximate analysis of return trajectories may be conducted within the framework of the two-body problem: in the sphere of action of the Moon we neglect the perturbations due to Earth, and outisde the sphere of action of the Moon we disregard the perturbations due to the latter. Then the return trajectory may be approximated by two arcs of conical cross-sections: the selenocentrical arc with focus at the center of the Moon and the geocentric one with focus at the center of the Earth. The initial geocentric velocity which we shall call escape velocity, is equal in the sphere of action of the' Moon to the sum of selenocentric escape velocity and of the geocentric velocity of Moon's motion. The Moon's orbit may be approximately considered as circular, and this is why the velocity of the Moon ($V_M$) will have a constant value (about 1 km/sec).

Let the multitude of return trajectories from Moon to Earth by bounded by the combination of trajectories passing at a preassigned limit distance $r_\gamma$ from the center of the Earth. For the case of return to Earth's surface the quantity $r_\gamma$ is equal to the radius of the upper boundary of the terrestrial atmosphere; for the case of return to AES orbit $r_\gamma$ is equal to apogee distance of the satellite. Obviously, $r_\gamma$ $r_M$, where $r_M = 384{,}400$ km is the major semiaxis of lunar orbit. For example, in the first case we have

$$r_\gamma / r_M = 1/60.$$

For limit return trajectories radius $r_\gamma$ is the perigee distance, so that

$$r_2 V_{2\tau} = r_\gamma V_\gamma, \tag{1}$$

where $V_\gamma$ is the velocity in perigee, $r_2$, $V_{2\tau}$ are respectively the geocentric radius and the transverse velocity at the time $t_2$ of device's escape from the sphere of action of the Moon.

From the energy integral we have

$$V_2^2 - \frac{2\mu}{r_2} = V_\gamma^2 - \frac{2\mu}{r_\gamma},$$

whence

$$V_\gamma = V_\text{п}\sqrt{1 + \beta_2 - \nu}, \quad \beta_2 = \left(\frac{V_2}{V_\text{п}(r_\gamma)}\right)^2, \quad \nu = \frac{r_\gamma}{r_2}. \tag{2}$$

Here $V_\Pi = V_\Pi(r_\gamma) - \sqrt{2\mu / r_\gamma}$ is the parabolic velocity at the distance $r_\gamma$. From (1) and (2) we have

$$V_{2\tau} = \nu V_\Pi(r_\gamma)\ \sqrt{1 + \beta_2 - \nu}, \tag{3}$$

whereupon $\nu = r_\gamma / r_M$. For the case of return to Earth's surface $\nu$ 1/60. By order of magnitude quantity $\beta_2$ cannot notably exceed $(V_M / V_\Pi)^2$, for on account of the rise of energy expenditures it is not advantageous to emerge from a sphere of action with selenocentrical velocity U, compensating with great excess the velocity $V_M$ of Moon's motion. For the problem of return to Earth we have $\beta_2 \sim 10^{-2}$.

Conseuuently, by virtue of (3) for the trajectories considered the transverse escape velocity $V_{2\tau}$ $V_\tau^*$, where $V_\tau^* = \nu V_\Pi(r_\gamma)$, i.e. it constitutes only about 0.2 km/sec. This takes place for any limit return trajectories. For the remaining ones

$$V_{2\tau} < V_\tau^* \tag{4}$$

independently of the initial data (the equality here may obviously take place also for all trajectories with $\beta_2 = \nu$ (see (3)). For the return trajectory passing throughvthe center of the Earth, $V_{2\tau} = 0$. It is not difficult to establish that the value of the selenocentrical escape velocity U is not less than $U_* = V_M - V_\tau^*$ 0.8 km/sec, for otherwise the projection $(\vec{U} + \vec{V}_M)_\tau > V_\tau^*$ contrary to (4). The quantity $U_*$ is more than twice the selenocentrical parabolic velocity at the boundary of Moon's sphere of action (constituting no less than 0.4 km/sec). This is why the arc of return trajectory in the sphere of action of the Moon is inescapably a hyperbola.

As for any hyperbolic trajectories at distance from the focus, the directions of escape selenocentrical velocities $\vec{U}$ and radius $\vec{\rho}_*$ for the return trajectories are quite close. Let us estimate the angle between these directions. From selenocentrical energy and area integrals we have

$$V_1^2 - \frac{2\mu'}{\rho_1} = U^2 - \frac{2\mu'}{\rho_*}, \qquad \rho_1 V_1 \sin\alpha_1 = \rho_* U \sin\alpha \tag{5}$$

where $\rho_1$, $V_1$, $\alpha_1$ are the initial senenocentrical radius, velocity and angle between them, $\mu'$ is the product of the gravitational constant by the mass of the Moon, $\rho_* = 66{,}000$ km is the radius of Moon's sphere of action, $\alpha_*$ is the angle between the escape velocity $\vec{U}$ and the escape radius-vector $\vec{\rho}_*$. From (5) we have

(6)

where

Since in (6) $\alpha_*$ decreases monotonically with the rise of $\beta_1$ , it results that the greatest $\alpha_*$ will occur for the smallest $\beta_1$, i. e. for the smallest $U = U_*$. If we then take $\rho_1$ of the order of the Moon's radius, we shall obtain $\lambda_* < 1/30$ and $\alpha_*$ of the order of a few degrees.

Let us consider now the geometrically escape selenocentrical velocities $\vec{U}$ of a single value U and all possible directions at the moment of time $t_2$ of missile emergence from the sphere of action in a nonrotating system of coordinates $\underline{u}$, $\underline{v}$, $\underline{w}$, of which the axis $\underline{u}$ at the moment of time $t_2$ is directed from Moon to Earth, the axis $\underline{v}$ is directed against the Moon's velocity $\vec{V}_M(t_2)$, and the axis $\underline{w}$ complements axes $\underline{u}$, $\underline{v}$ to the right-hand set of three. The conjunction of the ends of the considered selenocentrical velocities forms a sphere of radius U (see the dotted line of Fig.1). The corresponding conjunction of escape geocentric velocities $\vec{V}_2$ forms by their ends a sphere of radius U (solid line in Fig.1).

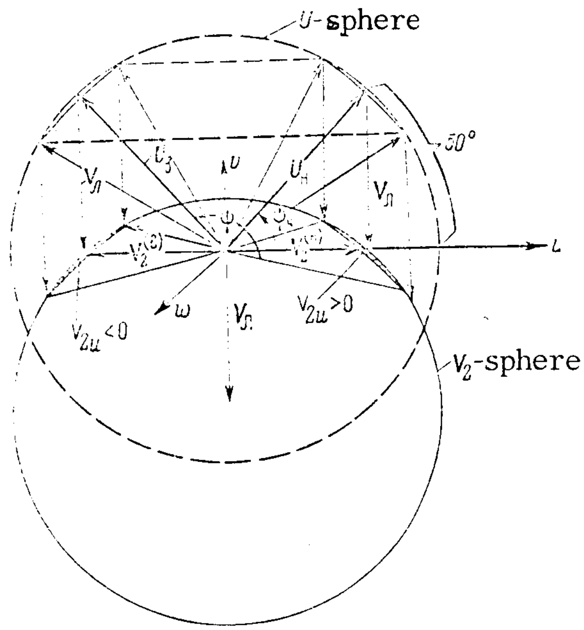
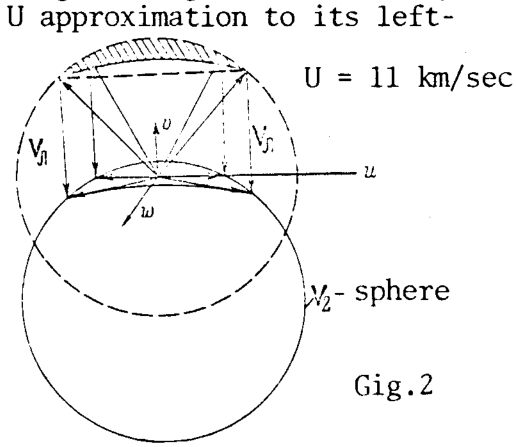



Fig.1 *

Let us separate on this last sphere the regions of directed V satisfying congition (4). It is obvious that these regions are cut out from the sphere by a straight circular cylinder of radius $V_\tau$*, of which the axis coincides with axis $\underline{u}$. As may be seen from Fig.1, for U > U* = $V_M + V_\tau$* these parcels do not merge and have a slightly oval shape. As U U* (U > U*), they stretch and approach one another. At U = U* they are tangent at the point (0, $V_\tau$*, 0). If

$$U_* < U < U^*, \qquad (7)$$

region (4) on the sphere already is singly connected (Fig.2). It is quite stretched for values of U approaching the right-hand boundary of the interval (7) and constricts at a point with U approximation to its left-hand boundary. If $U < U_*$, the return trajectories are absent.




Gig.2

It is obvious that in the case U > U* the limit trajectories of return encompass the geocentric sphere $r = r_\gamma$ from all sides. As U decreases from its value U* on the geocentric sphere $r = r_\gamma$ a forbidden zone appears (from the side approximately opposite to Moon's velocity direction), symmetrical reaative to lunar orbit plane. It no longer is encompassed by return trajectories. As U decreases to $U_*$, this zone spreads over the entire sphere $r = r_\gamma$. Note that the points of region (4) on the $V_2$-sphere

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* In all figures V stands for $V_M$

for which $V_{2u} < 0$ correspond to the drifting away of the device from Earth. Indeed, the corresponding points on the U-sphere are located in its upper left-hand part (Figs. 1 and 2). Since it was shown above that the emergent selenocentrical radius constitutes a small angle with the velocity U, the points of emergence are also located in the upper left-hand half of the sphere of action. If a geocentric radius r is drawn into such a point, its angle with the respective vector $\vec{V}_2$, satisfying condition (4), will be sharp (Figs.1 and 2). Thus, for $V_{2u} < 0$ we have $V_{2r} > 0$.

We shall call the motions with $V_{2u} < 0$ ascending, and those with $V_{2u} > 0$ descending.

If the velocity V does not exceed the geocentric parabolic velocity $V_{\Pi}(r_2) = \sqrt{2\mu_{\gamma}/r_2}$, the device will turn toward the Earth after a certain time past its escape from the sphere of action of the Moon. In the opposite case it will drift to infinity and the trajectory will not be a return trajectory. This is only possible when $U > V_{\Pi}(r_2)$, whereupon we have

$$1.56 \text{ km/sec} = V_{\Pi}(r_M - \rho_*) > V_{\Pi}(r_2) > V_{\Pi}(r_M + \rho_*) = 1.32 \text{ km/sec}, \qquad (8)$$

where $r_M$ is the radius of the lunar orbit.

Since for the singly connected region (4) $U < U^* \cong 1.2$ km/sec $< V_{\Pi}(r_2 + \rho_*)$, all points of this region do indeed correspond to return trajectories. However, greater flight times and a greater scattering of geographic coordinates of the landing point will correspond to ascending return trajectories than to descending ones. That is why the decsending return trajectories offer greater interest.

## 2. NOMINAL RETURN TRAJECTORIES OF VARIOUS FORMS

1. We shall classify the nominal return trajectories by initial data. We shall refer to the first type the return trajectories from the lunar surface and to the second type those from the orbit of an AMS.

It is appropriate to subdivide the trajectories of return from lunar surface into two forms: trajectory with vertical start and those with inclined start (

For the determination of initial nominal trajectory data with vertical start we shall consider the spheres of escape velocities, selenocentrical $\vec{U}$ and geocentric $\vec{V}_2$ (Fig.1) at a fixed initial velocity $V_1$, for which the escape velocity $U > U^*$. Then there will be on the $\vec{V}_2$-sphere two vectors $\vec{V}_2^{(B)}$ and $\vec{V}_2^{(H)}$ respectively for the ascending and the descending motion by trajectories hitting the center of the Earth or a preassigned point of the Earth's surface. We shall denote the respective vectors of the escape selenocentrical velocity by symbols $\vec{U}_H$ and $\vec{U}_B$. The angle $\psi$ of these vectors' projections on the Moon' orbit plane (Figs 1 and 3) with direction $\xi(t_2)$ from Moon to Earth will be respectively denoted by $\psi_H$ and $\psi_B$. When hitting the center of the Earth, vectors $\vec{V}_2$ and $\vec{U}$ lie in the lunar orbit plane; when hitting the point of the ground surface located under the lunar orbit plane, vectors $\vec{V}_2$ and $\vec{U}$ also rise above that plane.

The angle $\phi$ of vector $\vec{U}$'s rise above the lunar orbit plane at vertical start evidently is the selenocentrical latitude $\phi_1$ of the starting point. The selenocentrical longitude $\lambda_1$ of the point of start (counted from meridian of the direction Moon-Earth) on the angle $\phi_M = \omega T_{1.2}$exceeds (Fig.3)

$$\lambda_1 = \psi + \omega T_{1.2}, \tag{9}$$

where $\omega$ is the mean motion of the Moon, and $T_{1.2}$ is the flight time from the Moon's surface to its sphere of action. For the problem considered $T_{1.2} < 17h$ (see [6], Fig.12 ), so that $T_{1.2} < 9°$.

In the following we shall take for the nominal trajectory the descending one. It is interesting in that for it the point of start is visible from the Earth, while the flight times and the influence of errors in the initial data are less than for the ascending trajectory. The point of start for the descending trajectory will be so much the nearer the center of the visible Moon's disk as the initial velocity is greater. For infinitely great initial velocitieis it coincides with the indicated center ($\lambda_1 = 0$), and as the velocity decreases to the value $V_{1m}$, corresponding to the selenocentrical velocity on the sphere of action $U = V_M$ (so as to hit the center of the Earth), the point of start reaches the limb of the visible disk of the Moon ($\lambda \simeq 90°$). For initial velocities of the order of 1 km/sec angles $\lambda \approx 55° - 65°$ are obtained. This follows also from properties of motion reversibility [7] by symmetrical trajectories relative to the plane $\xi\zeta$ (axis $\zeta$ being directed toward the northern hemisphere orthogonally to lunar orbit plane and axis $\xi$ being constantly directed from Moon to Earth), if we take into account that for the point of missile's fall from Earth to Moon with selenocentrical entry velocity into the sphere of action of the Moon of the order of 1 km/sec we have $\lambda_1 = -55 - -65°$. Note that the points of verical start for ascending motions are about symmetrical to the nominal ones relative to the plane $\eta\zeta$.

2. Let us now examine the return journey to Earth from a preassigned point of the lunar surface, whereupon we shall limit ourselves only to descending motions. Note that if the given point does not coincide with that of verical start, the minimum initial velocity exceeds $V_{1m}$. Thus, for instance, for a start with velocity $V_{1m}$ from the landing region of the station "LUNA-9" $\phi_1 \approx 10°$, $\lambda_1 \approx -60°$ the angular remoteness of the flight in the sphere of action $\Phi_p$ (*) does not exceed 135° (**), whereas in order to hit the center of the Earth the angular velocity $\Phi_\Pi = 90° + 60° = 150° > \Phi_p$. is prerequisite. For th e initial velocity responding to the value $U = 1.4$ km/sec, we shall obtain (at horizontal start) $\Phi_p \simeq 120°$, while from Fig.1 we find $\Phi_\Pi = 50° + 60° = 110°$. Now we see that $\Phi_p > \Phi_\Pi$. Consequently, there exists such an initial velocity, for which for a horizontal start we have

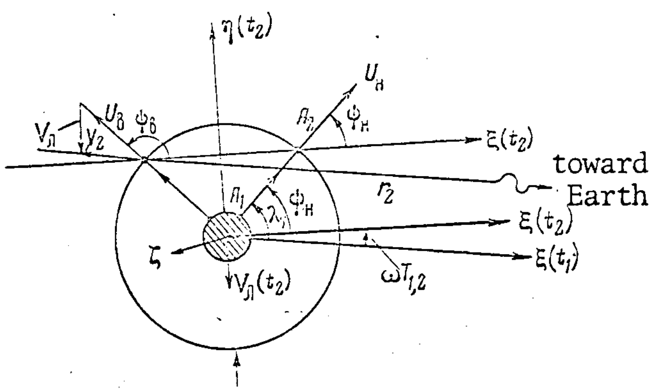
$$\Phi_p = \Phi_\Pi \tag{10}$$

(*) even when the elevation angle of the initial velocity vector $\theta_1 = 0$.
(**) The numerical data used here may be obtained with the aid of the graph (Fig.1,4), of the work ref. [6].

This velocity constitutes for $\lambda_1 = 60°$, $\phi_1 < 10°$ about 2.65 km/sec, corresponding to $U \cong 1.2$ km/sec. Note that for it we have $\Phi_p \cong 125°$, while from Fig.1,4 of the work [6], $\Phi_\Pi = 65° + 60° = 125°$).

No solution exists for lower initial velocities, while for greater velocities the solution exists only for a certain inclined start ($0 < \theta_1 < 90°$). The greater the initial elevation angle, the greater the required initial velocity for a fixed initial point. Thus, the horizontal start is the most advantageous from the standpoint of energy.

Note that as the initial point approaches the point $\phi = 0°$, $\lambda = 90°$, the minimum required initial velocity at vertical start decreases monotonically to $V_{1m}$ value.




Sphere of action
$\rho = \rho*$
Fig. 3

3. Let us finally consider the start from AMS orbit of the "LUNA-10" - "LUNA-11" type. Assume that the nominal selenocentrical trajectory "Earth-Moon", passing through the center of the Moon, with a flight time of about 3.5 days, is the axis of a beam of selenocentrical trajectories that may be used for the creation of an AMS. At the point of encoun er with the Moon this axis constitutes with the direction Moon-Earth an angle of about 60°. Fron the energetic standpoint, most advantageous is the transition to satellite orbit from the line of apsides of selenocentrical trajectory lying in the orbit plane of the AMS. Below we shall consider only the transitions (transfers) close to the most advantageous.

For a photographic AMS the satellite's orbit inclination to the orbit plane of the Moon of 90° ($i_p \approx 90°$) may be appropriate. This means that the orbit plane of the AMS will constitute at the time $t_u$ of its escape an angle of about 60° with the direction $\xi(t_u)$ Moon-Earth (Fig.4). Consequently, yhe longitude of the ascending node of satellite's orbit for a hyperbola passing to the north of the Moon, will constitute about 300°, and for a hyperbola passing to the south of the Moon, the indicated longitude will constitute about 120°. Since the angle between hyperbola's asymptotes, with which transfer to satellite's orbit is materialized is about 90°, the transfer point will not be visible from Earth for a close photographic AMS.

For the emergence (escape) from the sphere of action with the velocity $U = 1.2$ km/sec along the return trajectory it is necessary (Fig.4) that the asymptote of this trajectory at time of escape constitute, as the axis of a beam of possible escape trajectories, an angle of about 60° with the direction Moon - Earth. For the escape with energy close to minimum the asymptote must constitute a small angle with the AMS's orbit plane, which takes place twice a month.

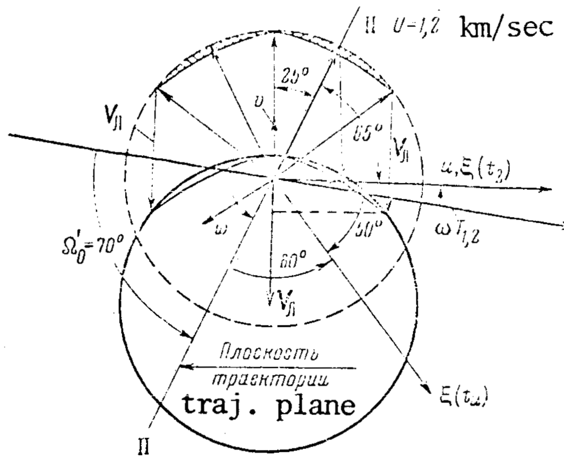



Fig.4

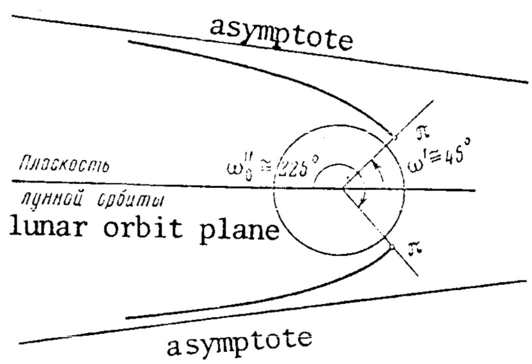



Fig.5

If we take into account the rotation of the direction Moon-Earth during flight time from AMS orbit to the boundary of the sphere of action (about 12 hours) and if we neglect the AMS orbit's precession under the action of perturbing forces, the directions Moon-Earth at the time $t_u$ of transfer to the AMS orbit and at the time $t_1$ of convergence from that same orbit will be differing by about 50°. Consequently, the minimum waiting time in orbit constitutes about 4 days.

For energy expenditures close to minimum it is possible to return to the Earth from AMS orbit only at time intervals multiple of a fortnight (half month). At the same time, for the return trajectory — a hyperbola passing to the south or to the north of the Moon, we shall obtain respectively a longitude $\Omega_0'' = 250°$ or $\Omega_0' = 70°$. (here the angle $\Omega_0$ is counted from the direction Earth-Moon). We shall correspondingly obtain the longitude of the apside line of the return trajectory $\omega_0'' = 225°$ or $\omega_0' = 45°$, provided we take into account that the branch of the hyperbola leading toward the Earth is nearly parallel to the lunar orbit plane and constitutes with the other branch an angle of about 90° (here $\omega_0$ is counted from the Moon's orbit plane, Fig.5). The orbit eccentricity of the photographic AMS was visibly appropriately being taken as zero, so that the height of photographic remain constant. Then, the moment of time of AMS passage through the apside of the return hyperbola obviously is also the initial moment of motion.

## 3. ESTIMATE OF THE REQUIRED PRECISION OF INITIAL DATA

Let us pass to the estimate of the accuracy of initial data required for the return to Earth in the whole, at start from the surface of the Moon or from the AMS orbit. It is obvious that for return to Earth along a nonnominal trajectory the transverse component $V_{2\tau}$ of the escape geocentric velocity must satisfy condition (4), which brings to light the admissible regions on the $V_2$-sphere and consequently also on the U-sphere. The latter regions are precisely those allowing us to judge about the required precisions of the initial data. Indeed, because of errors of initial data the factual vector U, deflecting in direction from the nominal, must not emerge from the admissible region.

The case of vertical start differs essentially from that of the horizontal start by the influence of initial data scattering. Having considered the extreme cases, we may obtain a representation on the intermediate cases also.

As a consequence of the error in the direction of the velocity vector there will appear during the flight from surface to the sphere of action an angular remoteness $\phi$ for a nearly vertical start. We shall determine it from the hyperbola equation [8]

$$p\nu = 1 + (p - 1)\cos\phi - 2\beta_1 \sin\alpha_1 \cos\alpha_1 \sin\phi \qquad (11)$$

where $\nu = \rho_1/\vec{\rho}_*$, $\bar{p} = p/\rho_1$ is the parameter ratio to the initial radius $\rho_1$, $\beta_1 = [V_1/V_{\Pi}(\rho_1)]^2$ $\alpha_1 = 90° - \theta_1$; $\theta_1$ is the elevation angle of the initial velocity vector above horizon. Substituting

$$\bar{p} = 2\beta_1 \sin^2\alpha_1 \text{ and } \lambda = \frac{\sin\phi/2}{\sin\alpha_1}, \qquad (12)$$

we shall obtain

$$\beta_1(\nu - \cos\phi) = \lambda\,(\lambda - 2\beta_1 \cos\frac{\phi}{2}\cos\alpha_1). \qquad (13)$$

As $\alpha_1 \to 0$, $\phi \to 0$, and for $\lambda$ we obtain the quadratic equation

$$\lambda^2 - \beta_1\lambda + \beta_1(1-\nu) = 0, \qquad (14)$$

whence

$$\lambda = (1-\nu) \Big/ 1 + \sqrt{1 - \frac{1-\nu}{\beta_1}} \qquad (15)$$

From the two solutions we took the one, which , as $V_1 \to \infty$, satisfies the evident relation

$$(\rho_* - \rho_1)\alpha = \rho\,\phi, \text{ whereupon } \lambda_\infty = (1-\nu)\,/\,2.$$

From the determination (12) we have

$$\frac{\partial\phi}{\partial\alpha_1} = 2\lambda = 2(1-\nu) \Big/ 1 + \sqrt{1 - \frac{1-\nu}{\beta_1}} \qquad (16)$$

Introducing instead of $\alpha_1$ the angle $\theta = 90° - \alpha_1$, we shall obtain

$$\partial\phi/\partial\theta_1 = -2\lambda.$$

Besides, for the vertical start we have

$$\frac{\partial\phi}{\partial\rho_1} = 0, \quad \frac{\partial\phi}{\partial V_1} = 0. \qquad (17)$$

For a value $U \approx 1$ km/sec we have $\partial\phi \quad \partial\theta_1 \approx -0.86$. Region (4) on the U-sphere have dimensions close to relatively greatest for values $U_\Pi$ little differing from $V_M$. At the same time the region considered is found to be simply connected. Its anglar dimensions constitute several tens of degrees in length, and about 20° in width.

Since $\partial\phi \,/\, \partial\theta_1 < 1$, the limit errors by angle $\theta_1$ will be of the order of ±10° in the direction of region (4)'s width, that is along the normal to Moon's orbit plane. For the nominal value $U = V$ the errors by velocity U (in the absence of errors by $\theta$) must not exceed ±0.2 km/sec so that region (4) still contain the trajectory considered (for $\delta U < -0.2$ km/sec, region (4) is empty, for $\delta U > +0.2$ km/sec it is doubly connected and the factual vector $\vec{U}$ will be situated between parts of region (4), outside it). The corresponding limit errors $\delta V_1$ of the initial velocity $V_1$, according to energy integral, will satisfy the condition

$$V_1 \delta V_1 = U \delta U \tag{18}$$

i.e. the constitute about 0.06 km/sec. Obviously, most harmful here are the mixed errors, so that the real ones must not exceed values of the order 2°- 3° by $\theta$ and 15 – 20 m/sec by $V_1$.

In case of inclined start, be varying equation (1) we shall obtain

$$\frac{\delta\rho_1}{\rho_1} = 2\frac{1-\cos\varphi}{-\bar{p}^2}(\delta\beta_1\cos^2\theta_1 - 2\beta_1\cos\theta_1\sin\theta_1\delta\theta_1) - \\ -\frac{\sin\varphi}{\cos^2\theta_1}\delta\theta_1 - \left[\left(1-\frac{1}{\bar{p}}\right)\sin\varphi + \mathrm{tg}\,\theta_1\cos\varphi\right]\delta\varphi. \tag{19}$$

Introducing the longitude of the pericenter $\omega$ and the eccentricity $\underline{e}$ we obtain from (19) with the aid of solutions $\bar{p} - 1 = e\cos\omega$; $\bar{p}\,\mathrm{tg}\,\theta_1 = -e\sin\omega$ :

$$\frac{\partial\varphi}{\partial\rho_1} = -\frac{\bar{p}}{\rho_1 e\sin(\varphi-\omega)}, \quad \frac{\partial\varphi}{\partial\beta_1} = \frac{1-\cos\varphi}{\beta_1 e\sin(\varphi-\omega)}$$

$$\frac{\partial\varphi}{\partial\theta} = 2\frac{(1-\cos\varphi)\sin\theta_1 - \beta_1\cos\theta_1\sin\varphi}{e\cos\theta_1\sin(\varphi-\omega)}. \quad (20$$

Taking into account of the dependence of $\beta_1$ on $\rho_1$ we obtain

$$\frac{\partial\varphi}{\partial\rho_1} = -\frac{(\bar{p}\nu+1)-\cos\varphi}{e\sin(\varphi-\omega)}, \tag{21}$$

$$\frac{\partial\varphi}{\partial V_1} = -\frac{2}{V_1}\frac{1-\cos\varphi}{e\sin(\varphi-\omega)}. \tag{22}$$

For a horizontal start ($\theta_1 = 0$) we have

$$\delta_{\rho_1}\varphi = -1{,}46\frac{\delta\rho_1}{\rho_1}, \quad \delta_{V_1}\varphi = -3{,}02\frac{\delta V_1}{V_1}, \tag{23}$$

$$\delta_{\theta_1}\varphi = -1{,}68\,\delta\theta_1. \tag{24}$$

We see that the modulus of the derivative with respect to $\theta_1$ is almost twice as great as for the vertical variant.

At a horizontal start from the surface of the Moon from the region of vertical landings we have $U \cong 1.2$ km/sec (see section 2), $V_1 = 2.65$ km /sec. The limit errors are determined by the part of region (4) for which $V_{2\tau} < 0$. Its dimensions are of the order of 50° in length and 20° in width (Fig.4). The limit (single) errors are obtained by $\theta_1$ near ±10°, by velocity near ±150 m/sec (taking into account (23), (24)). Errors by azimuth must of same order as by $\theta_1$.

At start from satellite orbit the limit errors will be somewhat greater, for the region on the U-sphere can be increased at the expense of transition to lower nominal velocity $V_M = 1.1$ km/sec. This is more advantageous also from the standpoint of energy. It is not difficult to obtain with the aid of computers the motions inside and outside the sphere of action with more precise ranges of errors, assuring the return to Earth as a whole, and also to a preassigned region of the ground surface.

## CONCLUSION

1. The spatial problem of return has the same characteristic singularity as the other problems of flight into the Earth's gravitational field, or that of the Moon: the return trajectories that have extreme properties, belong to lunar orbit plane. Note also that the last of the return trajectories (with minimum selenocentrical velocity) by-passes the Earth in the direction of motion of the Moon, lies in the lunar orbit plane and is tangent to the geocentric sphere $r = r_\gamma$ at a point opposite to the direction toward the Moon at time of device's emergence from its sphere of action.

2. The approximate method expounded is asymptotic, i.e. it is so much the more precise as the mass ratio of attracting bodies is lesser. The Moon to Earth mass ratio ($\sim 1/80$) is already found to be sufficiently small to obtain a good qualitative and approximate quantitative representation on the different properties of return trajectories with the help of the analysis of velocity conjunctions (this is corroborated by computers). The approximate method allows to rapidly find with the aid of a computer those solutions which have the required properties and belong to the regions offering the greatest interest. It allows us to resolve the questions of existence and uniqueness of solutions.

3. The equality condition of the deployed angular range of the flight, which is geometrically indispensable (formula (10) from section 2) at a given initial velocity, has the same theoretical significance as the analogous condition in the problem of hitting the Moon from a preassigned point of the terrestrial surface (see [6], pp.16-47). For initial velocities insufficient for the fulfillment of this equality there is no solutions for the problem.

However, the practical significance of of this effect in the problem of flight from the lunar surface to Earth is notably lesser than in the problem of hitting the Moon from Earth, for the indicated condition is always satisfied when the initial velocity is raised above minimum if only by a few meters per second.

4. From the analysis conducted on the influence of initial data scattering in the problem of return from the orbit of an AMS it may be concluded that the optimum value of the escape velocity from the sphere of action of the Moon constitutes about 1.1 km/sec. Indeed, no escape velocity of 1.2 km/s is required for the return from AMS orbit as in the case of return from the landing area of the station "LUNA-9", and the escape selenocentrical velociry may be decreased to the geocentric velocity of the Moon. However, as is shown by computations, the influence of initial data scattering then increases noticeably. But at velocity of about 1.1 km/sec the region on the sphere of escape velocities (Fig.4) already is simply connected, but still maintains large angular dimensions (i. e. according to the admissible direction deviations of the velocity vector from nominal, the ranges are still sufficiently great).

*** THE END ***

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REFERENCES

[1]. M. I. KOOJ, T. BERGHUIS. Astronaut. Acta, 6, 2-3, 1960.
[2]. I. ALMAR, B. BALAZS. Magyar tud. akad.Mat.Kutato int Kozl, 4, No.2, 1959.
[3]. A. NIMAN. J. British Interpl.Soc. 19, 1, 1963.][ see . Bekessy, K. Toth
[4]. N. I. ZOLOTUKHINA, D. E. OKHOTSIMSKIY. Kosm.Issl. 3, 4, 523, 1965.
[5]. J. CHAPMAN. (Approximate method of investigation of body entry into planet atmospheres) Transl. For.Lit. 1962.
[6]. V. A. YEGOROV. Prostranstvennaya zadacha dostizheniya Luny (Spatial Problem of Raeching the Moon). pp.16-47 "Nauka", 1965.
[7]. A. MIL'. Astronavtika i raketodinamika (Astronautics and Rocket Dynamics) No.23, 1961.
[8]. D. E. OKHTSIMSKIY. Lektsii po dinamike kosmicheskogo poleta (Lectures on the Dynamics of Space Flight). Mekh-Mat, f-t. MGU, 1962.

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