DYNAMICS OF GAS BUBBLES IN AN OSCILLATING PRESSURE FIELD

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ABSTRACT

The behavior of oscillated liquid columns was investigated theoretically and experimentally.

The theory was derived from basic force-momentum relationships for non-viscous and viscous fluids. In the derivations as much generality was retained as possible to permit future application of the theory to a wide range of the parameters involved. In particular, such a general formulation of the theory permits inclusion of resonance effects, the effects of relatively large bubbles compared with the tank, and the influences of the tank structure and the vibrating mechanism. Equations are given for the instantaneous and mean motion of a bubble and it is shown that, for the restrictions applied in previous theories, these equations identically reduce to equations of the previous literature.

The present analysis was extended also to viscous liquids and the effect of viscosity on the stabilization of single bubbles was shown. The results are compared with inviscid theories and experimental results from previous literature.

The experimental investigations were concerned with identifying the mechanisms by which bubbles are produced and clusters developed in the liquid, and establishing the dependence of these mechanisms on the frequency and amplitude of the forced oscillations; viscosity, density, temperature and pressure of the liquid; the concentration of dissolved gases in the liquid; the dynamic deformations and structural properties of the container; and the responses of the vibration exciter system to the frequency characteristics of the liquid-filled container.

From the results, it is deduced that the behavior of the liquid is controlled by a strong feedback mechanism involving liquid, cluster and container; and on the basis of this hypothesis a model is constructed for the generation, development and stabilization of clusters.

Measurements of pressure distributions and the location of clusters were compared with theoretical predictions of previous publications.

Because of the important interference of the vibration exciter system with the dynamics of the liquid motion, the frequency characteristics of that system as a function of the load impedance was studied in some detail.

Recommendations are made for the extension of the present investigations.

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1.0 INTRODUCTION

The behavior of liquids under forced oscillations has been investigated since about the turn of the century, but it was not until relatively recently that a need for intensive research in this field arose. The new importance attributed to this type of fluid motion is largely due to the possibility that bubbles forming in vibrating fuel tanks and lines of large missiles hamper their proper functioning. This supposition resulted from observations of intensely pulsating combustion in the engines of some missiles, accompanied by other periodic phenomena. A tentative explanation given to some of these phenomena is that pulsating bubbles disturb the steady flow of fuel. In extreme cases, the condition caused by bubbles can destroy the vehicle. Besides the efforts to eliminate such detrimental effects, it is also being attempted to exploit the dynamics of vibrating liquids for useful purposes (see e.g. Baird, 1963). To contribute further to the understanding of bubble dynamics in vibrating liquids, basic experimental and theoretical research has been conducted at The University of Tennessee, Department of Engineering Mechanics, since July 1, 1965. This research is sponsored by the George C. Marshall Space Flight Center, Huntsville, Alabama, under Contract No. NAS8-20152. The immediate specific aim of the research project is to establish the relationship between physical properties of liquids and their behavior under vibration.

2.0 THEORETICAL INVESTIGATION

2.1 Previous Research On Bubble Motion in Oscillating Liquids

Some of the earliest theoretical results concerning the dynamics of pulsating bubbles were published by V. Bjerknes (1909) in a book on fields of force. The purpose of this book was to describe and examine the extensive analogy between the force fields of hydrodynamic systems and the fields of electromagnetic forces. To place this work into the proper perspective, one has to remember that the definition and theory of an electromagnetic field were published just a few decades earlier by Maxwell.

Consistently with the general objective of his book, Bjerknes gives the derivation of the instantaneous and the mean force moving the fluid around a pulsating and oscillating bubble, but he does not calculate the resulting motion of the bubble.

More recently, several authors extended Bjerknes' theory to the calculation of the motion of bubbles (Blake, 1949; Buchanan, Jameson and Oedjoe, 1962).

Buchanan, Jameson and Oedjoe (1962) calculated the conditions for stabilization of a bubble at a mean location in an inviscid liquid and with the assumption that the volume changes of the bubble are small.

Another approach to the problem was taken by Bleich (1956). He derived, from Lagrange's equations, two equations which determine the motion of an isolated bubble in an inviscid liquid. He than restricted the analysis to the case that the radial pulsations of the bubble are small (to permit linearization) and that the bubble oscillates about a fixed mean location. Then the equations were solved for the corresponding mean locations of the bubble. Furthermore, the stability of these mean locations was examined and the conclusion was made that stable mean location of an oscillating bubble exists only in elastic vessels.

Kana,Dodge (1964) further developed and refined Bleich's theory. First, Bleich's simplification that the bubble moves in an infinite medium was examined. Kana and Dodge calculated the approximate effect of finite tank size (finite tank radius and finite distance between bubble and liquid surface), but the correction term was not included in the calculations, because it was shown to be small in all cases considered. Further, Kana and Dodge presented a simplified method to account for the finite rigidity of the tank. Their analysis also included a calculation of the dynamic pressure distribution ($\bar{p}$) in the tank. In a later part of this report, measured results will be compared with their theoretical pressure distribution.

During the past three decades or so, most theoretical studies in this field were prompted by observations either in the liquid contents of oscillating tanks or in liquids permeated by ultrasonic sound fields. Earlier, since the first half of the nineteenth century, it was cavitation that gave incentive to theoretical investigations of the liquid flow about bubbles. It was in response to such publications that Lord Raleigh (1917) analyzed the hypothetical flow which would develop if, in an infinite body of incompressible, non-viscous liquid at rest, suddenly a spherical hole would be created. One remarkable result of his calculations is that the location of the maximum pressure is at infinity only during the early development of the flow. After that initial phase, the pressure becomes much higher just a short distance away from the surface of the bubble.

Rayleigh's result was deduced from an equation for the motion of the bubble (cavity) surface which derived from energy considerations. The equation of surface motion was modified by Houghton (1963) to include effects of viscosity and surface tension.

The motion of a bubble in a vertically oscillating non-viscous liquid is the topic of a paper by Jameson and Davidson (1966). The calculations deal with bubbles which execute periodic motion around a fixed point. Their theory is further developed by Jameson (1966) to account for viscosity. In the two studies, the radial motion of the bubble surface was derived from the above mentioned modified Rayleigh equation due to Houghton. The effects of viscosity on the oscillations of the bubble and the pressure field about the bubble was obtained by a method similar to the one developed by Stokes (1851).

Recently Fritz, Ponder and Blount (1964) investigated the effect of important parameters which arise in liquid fuel missile systems on bubble cluster formation. Their experiments produced data on the acceleration levels required for bubble cluster formation as a function of liquid column height, vapor space pressure above the liquid column, and tank wall thickness. Analysis of the data was made in terms of Bleich's theory for an inviscid liquid column undergoing longitudinal oscillation.

Finally, several authors analyzed resonance characteristics of the system composed of the elastic bubble and the mass of the surrounding liquid. The first formula for the calculation of the resonant pulsation frequency has been derived by Minnaert (1933). Refinements of expression were made by Smith (1935) and Baird (1963) to include the effects of surface tension and finite tank dimensions.

2.2 Theory

The preceding review of the literature on the behavior of bubbles in vibrating liquids exemplifies the variety of basically different approaches taken by the authors to develop their theories. In spite of the differences, however, the physical problem is essentially the same in all of these cases, and so the results should inherently be related to each other. In view of this situation, it seems indicated to present a qualitative discussion of the process and of the anticipated characteristics of the theoretical solutions. Such a general discussion may also serve as a guide in the further development of a theory, the beginning of which is outlined in the next section.

Bubbles in vibrating liquids can be observed over a very wide range of conditions. Within that range, the effects of certain factors vary more than an order of magnitude and so their relative importances change. It follows then that the permissible simplifications in theories vary depending on the range of intended applications of these theories.

One of the purposes of the present theory is to predict conditions under which bubbles formed in or entrained into the liquid become stationary.

Experiments performed under conditions of bubble stabilization gave results which permitted comparison of the various forces acting in the liquid. Calculations based on recordings of the tank acceleration indicate that the forces vibrating the liquid are of the same order of magnitude as the gravitational forces. Since it is the driving forces which have to balance the buoyancy and they are of the same order of magnitude, the mechanism preventing the bubbles from rising must be a first order effect.

One also concludes that the phenomenon of bubble stabilization is non-linearly linked to the forced motion. The argument for this conclusion can be the observation that the up and down phases of the periodic tank motion can be symmetrical and still the net motion of the bubble is in one direction, downward.

Further, it is unlikely that a state of resonance in itself can explain this type of bubble motion. Besides theoretical reasoning, evidence against that possibility is given by the experiments in which stationary or downward moving bubbles were observed over wide ranges of the variables, especially the frequency. A state of resonance is unlikely to exist over such wide ranges.

One may note here that gravity, i.e. hydrostatic forces, cannot be linked to the net downward motion of bubbles. Since buoyancy forces the bubbles to move up, a downward motion as a result would require that gravity forces interact with the alternating physical quantities like velocity, acceleration, pressure, etc. Hydrostatic forces, however, are independent of these.

Finally, one expects that viscous forces strongly modify the motion of bubbles, but that they are not the cause of the net downward drift of bubbles. This is suggested by the observation that variation of the viscosity, by using liquids of as widely different viscosities as water, alcohol, and glycerine, (Buchanan (1963), Jameson and Davidson (1966), did not change the conditions for balancing the buoyant force much.

One is led to this conclusion also by a qualitative analysis of the factors controlling viscous forces. First, since these attenuate existing relative motion, they cannot amount but to a fraction of the total force causing the motion of bubbles relative to the fluid. But that force, as we have seen, is of the same order of magnitude as the buoyant force. Secondly, the primary factors determining the viscous forces in a given fluid are the velocity and the surface of the boundary, in this case, the bubble. If, without the interference of viscosity, the velocity variation would be symmetric, it would be left to the difference of the surface area during the upward and the downward part of the bubble motion to generate the required force. This, however, would not suffice to effectively oppose buoyancy because the buoyant force varies with the volume of the bubble and the volume changes are relatively larger than the surface changes, unless some rapid and large distortions would occur in the shape and surface area of the bubble with little volume change. But such fast and extreme distortions have never been recorded.

Even if it is not assumed in the previous paragraph that the velocity oscillations are symmetric, their effect still can be discounted by observation. It is sufficient to demonstrate this on a single case, e.g. when at a moderate frequency a larger bubble becomes stationary or slowly moves downward. The amplitude of the center's periodic displacement is not more than a relatively small fraction of the diameter and, therefore, even the amplitude of the velocity oscillations is small. The differences between the up and down velocities are then even smaller. Thus, the effect of viscosity must be of an order higher than buoyancy.

One must add, however, that in special cases, like the resonance of bubbles, viscous forces probably grow to the point where their magnitude equals a large fraction of the buoyant force.

Assuming the validity of the above discussion, only non-linear interactions of the inertia forces and the pressure can be responsible for the primary control of the motion.

2.2.1 Inviscid Theory

The present analysis is the first step in a plan for the development of a theory of vibrating liquid columns. It is much less specific than most of the existing theories, because it was attempted to exclude as many of the usual simplifying assumptions concerning the liquid motion as possible. In this way, it remains possible to incorporate new informations as they become available from theoretical or experimental investigations.

One such frequently encountered assumption is that the time dependent variables are pure sinusoidal functions of the time. Such an assumption may be adequate to obtain a clue, how certain physical quantities interact, but it is probably not sufficiently accurate for engineering applications. In fact, measurements of tank acceleration, pressure fluctuation and bubble pulsation clearly indicate that--at least under certain conditions--the wave forms of all of these quantities strongly differ from a pure sine wave.

With respect to the phase relationships of the alternating quantities, the most frequently made approximation has been, that they are in phase. Simultaneous measurements of the instantaneous tank acceleration and the instantaneous pressure inside the bubble showed, however, that this is not always the case. In a plexiglas tank of seven inches outside diameter and one-fourth inch wall thickness this phase difference was observed to be as much as 90° at a frequency of 179 cycles per second in methyl alcohol and when a stable bubble cluster was present.

Assuming then that these variations are in phase eliminates some existing effects from the theory.

The above simplifications are probably always justified, when the bubble is small compared with the container diameter, when it pulsates with small amplitudes, maintains spherical shape, and the frequency of the forced oscillation is much lower than the resonant frequency of the bubble. For non-viscous liquids this has been shown to be true by Bleich and Kana Dodge. In practice, however, these conditions are frequently not satisfied. It is hoped, therefore, that a theory can be developed in which these restrictions are relaxed.

A further important factor is the elasticity of the tank. The effects of this, too, have been included in theories only in very simplified forms, if at all. It is easily verified by experiment, however, that the dynamic effects on the tank deformation are strongly fed back to the liquid, altering its motion and the location of the bubble. It would be important, therefore, to take these effects into consideration, but very little information about them is available at this time. This is the reason why they have been mostly ignored or greatly simplified in the past. It is the aim of a new project to shed more light on this aspect of the physical problem.

In the theory as presented below, effects of viscosity are omitted. They are to be restored in the next section. It is expected that inclusion of viscosity will, in some cases, considerably change the present results, and so the latter serve mainly the purpose of showing the relative magnitudes of the primary forces controlling the behavior of bubbles and to show how these forces depend on other physical quantities.

It was pointed out earlier that gravity has no part in the dynamic response of bubbles to the forced vibration. Consequently, it is immaterial for the theoretical investigation of this response in which direction the oscillations of the liquid column occur, and so we shall assume that they take place along a fixed straight line of arbitrary direction and zero gravity will be assumed for the time being. This assumption permits us to write the vector equation for the driving force and the resulting acceleration as an algebraic equation. The pressure gradient, created by this acceleration, is also parallel to the same fixed direction, and varies in time as the acceleration, but it is constant at any one instant throughout the liquid if this is homogeneous.

This part of the motion, the primary motion, is that of a rigid body and it requires that the constraints of the liquid are rigid. Additional motions are superposed on this one when the constraints of the liquid are not rigid. The assumption of rigid body motion also presupposes that a sound or pressure wave having the frequency of the forced vibration is much longer in the liquid than the size of the liquid body in the direction of the oscillations. Under such conditions, the pressure field set up by the acceleration is similar to the hydrostatic pressure field and the resultant pressure force, F, acting over a closed surface, stationary with respect to the liquid, is

$$F = a_\ell \rho_\ell V_b \quad , \qquad (2.2.1\text{-}1)$$

which is analogous to buoyancy. In Equation (2.2.1-1) a_ℓ is the acceleration of the liquid, ρ_ℓ its density, and V_b the volume enclosed by the surface. Such a closed surface can be the boundary of a bubble.

When the closed surface is free to move with respect to the liquid and it contains a mass M_b, the resultant force of F and the inertia force of M_b,

$$F - a_\ell M_b \quad , \qquad (2.2.1\text{-}2)$$

will move the surface with respect to the body of liquid at infinity, or the rigid outer constraints of the liquid. Such relative motion requires relative acceleration of the mass M_b and the liquid around the surface.

The inertia of the surrounding liquid can be expressed formally by an apparent mass, M_a, possessed by the closed surface,

$$M_a = k_1 \rho_\ell V_b \quad , \qquad (2.2.1\text{-}3)$$

where k_1 is a coefficient appropriate to the shape of the closed surface.

When this closed surface is a bubble entirely within the liquid, M_b represents the mass of the gas filling the bubble, and if the bubble is spherical and its diameter small compared with the distance to the nearest wall or free liquid surface, $k_1 \approx 1/2$.

In terms of the above quantities and the velocity v_b of the bubble center relative to the liquid at infinity, the total momentum I_{tr} of the relative motion is

$$I_{tr} = (M_b + M_a)v_b = (M_b + k_1 \rho_\ell V_b)v_b \quad . \qquad (2.2.1\text{-}4)$$

Application of Newton's law gives--neglecting viscous effects--

$$F - a_\ell M_b = a_\ell(\rho_\ell V_b - M_b) = \frac{d}{dt}(M_b + k_1 \rho_\ell V_b)v_b \quad ,$$

$$= (M_b + k_1 \rho_\ell V_b)a_b + v_b \frac{d}{dt}(k_1 \rho_\ell V_b) \quad , \qquad (2.2.1\text{-}5)$$

where $a_b \equiv \frac{dv_b}{dt}$ is the acceleration of the bubble center relative to the liquid far from the bubble.

In many cases, the bubble maintains a nearly spherical shape at all times. In many other cases, the bubble is not spherical but at least one can assume that the shapes taken by the bubble at different moments of the oscillation are geometrically similar. When such an assumption is permissible, that component of the flow which is due to the relative motion of the bubble center remains dynamically similar during the oscillation and k_1 becomes independent of time.

The assumption of geometric similarity of bubble shape is not always accurate enough. When the bubble has the form of a cluster and it resonates to the frequency of the forced motion, its shape becomes strongly distorted during a cycle of the oscillation. The high degree of distortion can clearly be observed on high-speed photographs. For the sake of simplicity, however, here we take k_1 as a constant.

The liquid density ρ_ℓ is also approximately constant by virtue of the above restriction on the relation between the length of pressure waves present and the size of the liquid-filled space. Accordingly, Equation (2.2.1-5) may be simplified to

$$a_\ell(\rho_\ell V_b - M_b) = (M_b + k_1\rho_\ell V_b)a_b + k_1\rho_\ell v_b \frac{dV_b}{dt} \quad . \quad (2.2.1\text{-}6)$$

In the present calculations we assume that the liquid is not everywhere bounded by the container walls, and we shall call the boundary of the liquid not in contact with the container the free surface. We consider such cases because they are more general and more important, as regards the dynamics of bubbles, than a closed, entirely liquid-filled container. For the same reasons, we also assume that the container oscillates in a direction normal to the free surface rather than parallel to it.

Without loss of generality, we may further assume that the container is in such a position that the direction of oscillation is horizontal, since gravity does not enter into the dynamics of the motion analyzed. In this position, the liquid may be prevented from pouring out of the container by a thin elastic membrane as shown in Figure 1.

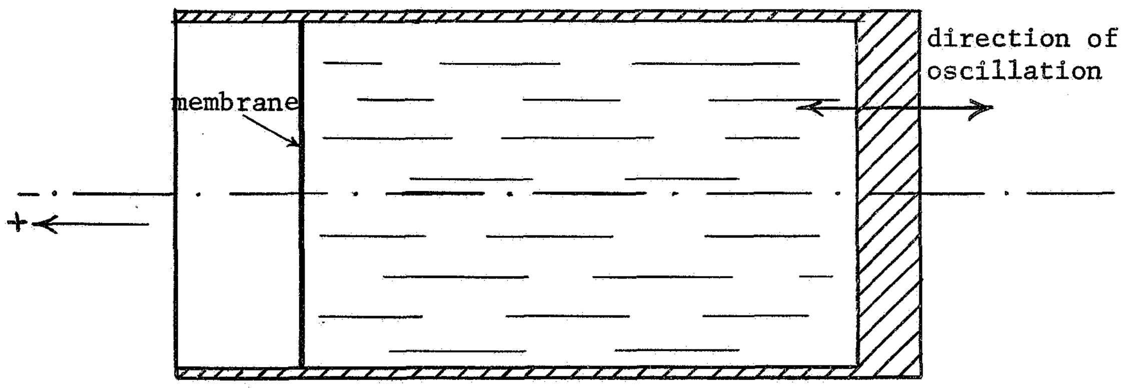

FIGURE 1. ARRANGEMENT OF OSCILLATING CONTAINER

Because of the above configuration, the boundary conditions of the liquid motion are not symmetrical with respect to the alternating directions of the container motion. In other words, the free surface, along which the pressure is constant, is upstream when the container in Figure 1 moves towards the left, and it is downstream when it moves to the right.

Similarly, the bottom of the container, where the fluid has to follow the motion of the wall, is downstream in the first case and upstream in the latter. As a consequence, when the container accelerates to the left, the pressure anywhere inside the liquid is higher than at any arbitrary point during acceleration to the right.

If now we place a bubble inside the liquid, it will contract during acceleration to the left and expand during acceleration to the right. Of course, change of the bubble volume changes the forces accelerating the bubble. The effect may be shown on Equation (2.2.1-5) after division by $(M_b + k_1\rho_\ell V_b)$, and with some rearrangement, remembering that M_b is constant. Thus,

$$a_b = \frac{\rho_\ell V_b - M_b}{k_1\rho_\ell V_b + M_b} a_\ell - v_b \frac{d}{dt}\left[\ln(k_1\rho_\ell V_b + M_b)\right] , \qquad (2.2.1\text{-}7)$$

or

$$a_b = \left(\frac{1}{k_1}\frac{1}{1+M_b/k_1\rho_\ell V_b}\right)\left(1-\frac{k_1 M_b}{k_1\rho_\ell V_b}\right) a_\ell - v_b \frac{d}{dt}\left[\ln(k_1\rho_\ell V_b + M_b)\right] . \qquad (2.2.1\text{-}8)$$

With the definition

$$\varepsilon \equiv \frac{M_b}{k_1\rho_\ell V_b} , \qquad (2.2.1\text{-}9)$$

we can reduce the above form to

$$a_b = \left(\frac{1-k_1\varepsilon}{1+\varepsilon}\right)\frac{a_\ell}{k_1} - \frac{v_b}{1+\varepsilon}\left[\frac{d}{dt}(\ln V_b) + \frac{d}{dt}(\ln k_1\rho_\ell)\right] . \qquad (2.2.1\text{-}10)$$

The derivative

$$\frac{d}{dt}(\ln V_b) \equiv \frac{1}{V_b}\frac{d\,V_b}{dt} \qquad (2.2.1\text{-}11)$$

in the last term is controlled by the pressure inside the bubble (p_b) and it can be expressed as a function of

$$\frac{1}{p_b}\frac{d\,p_b}{dt} = \frac{d}{dt}(\ln p_b) \quad . \qquad (2.2.1\text{-}12)$$

The pressure p_b, in turn, is determined by the acceleration a_ℓ of the container. If Δp_b is the difference between the pressure during acceleration and the pressure when the container is at rest,

$$\Delta p_b \sim a_\ell \qquad (2.2.1\text{-}13)$$

and so $\frac{d}{dt}(\ln V_b)$ is determined by the acceleration of the container and it is independent of the magnitude of V_b, i.e. the size of the bubble.

The only way the magnitude of V_b directly affects the relative acceleration of the bubble is through ε. Since $(1-k_1\varepsilon)$ and $\frac{1}{1+\varepsilon}$ increase when V_b increases, in a configuration as in Figure 1, an acceleration of the container towards the left produces a smaller relative acceleration of a bubble than an acceleration of equal magnitude towards the right. As a result, oscillations with no net displacement of the container produce a migration of bubbles toward the bottom of the container (here to the right.) This effect is, however, very small. If $\Delta\varepsilon$ is the deviation of ε from its mean value, the net displacement of the bubble from this source is the resulting effect of the variation of $\Delta\varepsilon$ over each cycle. Typically, as in the case of an air bubble in water, ε is about 2.10^{-3} and $\Delta\varepsilon$ a fraction of this.

Clearly, these effects are of high order compared with hydrostatic buoyancy and so negligible when they compete with the latter. Such is the case in an arrangement where the free surface is horizontal and the oscillations are vertical. Then the hydrostatic buoyant force acts in the same direction as the dynamic forces and, therefore, the effects of $\Delta\varepsilon$ become negligible.

The term which is then the most important in Equation (2.2.1-10) for the net displacement of the bubble is

$$\frac{-v_b}{1+\varepsilon}\frac{d}{dt}(\ln V_b) \quad . \qquad (2.2.1\text{-}14)$$

This term represents the inertia force of the increase in apparent mass per unit of time. After substitution of the pressure inside the bubble and neglecting ε:

$$-v_b\frac{d}{dt}(\ln V_b) = +v_b\frac{n}{p_b}\frac{d(p_b)}{dt} \quad . \qquad (2.2.1\text{-}15)$$

The constant n is determined by the thermodynamic process undergone by the gas in the bubble. In general, that is a polytropic process having as limiting cases the isothermal and the adiabatic processes. Since n and p_b are always positive, the sign of this term is determined by

$$v_b \frac{d(p_b)}{dt} \quad . \qquad (2.2.1\text{-}16)$$

An equation for the relative velocity v_b of the bubble is obtained by integration of (2.2.1-10) with substitution of (2.2.1-15)

$$v_b = \int_{t_1}^{t} \left(\frac{1-k_1\varepsilon}{1+\varepsilon}\right)\frac{a_\ell}{k_1}dt + \int_{t_1}^{t} \frac{v_b}{1+\varepsilon}\,\frac{n}{p_b}\,\frac{d\ p_b}{dt}\,dt - \int_{t_1}^{t} \frac{v_b}{1+\varepsilon}\,\frac{d}{dt}(\ln\ k_1\rho_\ell)dt + v_{b_{t_1}} \qquad (2.2.1\text{-}17)$$

where $v_{b_{t_1}}$ represents the relative bubble velocity at time t_1.

Without loss of generality, especially in the case of periodic motion, one can select for t_1 an instant at which $v_{b_{t_1}}$ is zero.

If the container oscillates with period T, the net bubble displacement over a full cycle Δs_b may be expressed formally by a second integration as

$$\Delta s_b \equiv \int_{t_1}^{t_1+T} v_b dt = \int_{t_1}^{t_1+T}\left[\int_{t_1}^{t} \frac{1-k_1\varepsilon}{1+\varepsilon}\,\frac{a_\ell}{k_1}\,dt\right]dt + \int_{t_1}^{t_1+T}\left[\int_{t_1}^{t} \frac{v_b}{1+\varepsilon}\,\frac{n}{p_b}\,\frac{d\ p_b}{dt}dt\right]dt\ (-)$$

$$- \int_{t_1}^{t_1+T}\left[\int_{t_1}^{t} \frac{v_b}{1+\varepsilon}\,\frac{d}{dt}(\ln\ k_1\rho_\ell)dt\right]dt \qquad (2.2.1\text{-}18)$$

Based on the above equations, one can discuss at this point some characteristics of the dynamic behavior of bubbles in oscillating liquids, but because of the large variety of possible modes of the liquid motion and the sizable differences between them, little can be said in full generality. For this reason, certain characteristic classes are separated and discussed by themselves.

1. When the natural frequency of the bubble is much higher than the frequency of a forced harmonic or nearly harmonic oscillation, the bubble size responds almost instantaneously to the pressure in the surrounding liquid and the pressure inside the bubble varies almost in phase with the tank acceleration (see also Bleich 1956, Jameson & Davidson 1966). If in addition the accelerations are not extreme in order to avoid strong distortions, one can write approximately

$$v_b \frac{d(p_b)}{dt} \approx c_1 v_b \frac{d(a_\ell)}{dt} \quad , \qquad (2.2.1\text{-}19)$$

where c_1 is a positive constant if, in reference to Figure 1, the direction to the left is defined as positive.

Since the tank oscillations are nearly harmonic,

$$\frac{d\ a_\ell}{dt} \equiv \frac{d^2 v_\ell}{dt^2} \approx - v_\ell \omega^2 \qquad (2.2.1\text{-}20)$$

and we may write by combining (2.2.1-15), (2.2.1-19) and (2.2.1-20)

$$-v_b \frac{d}{dt} (\ln V_b) \approx - \frac{nc_1}{p_b} v_b v_\ell \omega^2 \quad , \qquad (2.2.1\text{-}21)$$

where ω is the circular frequency.

Substituting this into Equation (2.2.1-10), the relative acceleration of the bubble becomes in approximation

$$a_b \approx \frac{1-k_1\varepsilon}{1+\varepsilon}\frac{a_\ell}{k_1} - \frac{nc_1\omega^2}{1+\varepsilon}\frac{v_\ell v_b}{p_b} - \frac{v_b}{1+\varepsilon}\frac{d}{dt}(\ln k_1\rho_\ell) \qquad (2.2.1\text{-}22)$$

The corresponding substitution into (2.2.1-18) yields the net bubble displacement during a period. The resulting expression, however, can be simplified.

The last term of Equation (2.2.1-22) is negligible because $k_1\rho_\ell$ is practically constant.

In the first term on the right hand side k_1 is a constant, the variations of ε are very small as we have seen earlier, and a_ℓ is a nearly harmonic variable. Consequently, when this term is integrated over a period of the motion, the result is negligible compared with the other integrals.

Finally, if we take into consideration that $\varepsilon<<1$,

$$\Delta s_b \approx -nc_1\omega^2 \int_{t_1}^{t_1+T}\left[\int_{t_1}^{t}\frac{v_\ell v_b}{p_b}\,dt\right]dt \qquad (2.2.1\text{-}23)$$

The value of the integral of Equation (2.2.1-23) is determined by the triple correlation of v_ℓ, v_b and $\frac{1}{p_b}$. It is noteworthy, however, that in the cases under discussion, and also more generally, the sign of this double integral is determined by the correlation of v_ℓ and v_b alone, and p_b only modifies its magnitude. This can be verified by examining the relationship of p_b to $\frac{d\,v_\ell}{dt}$, and the phase relation between the latter and v_ℓ.

In the class of motions being discussed, v_b and v_ℓ are very nearly in phase and so the integrand is positive during almost the whole cycle of the oscillation. As a consequence Δs_b is negative which means that the bubble migrates toward the bottom of the container.

2. When the difference between the frequency of the container oscillation and the resonant frequency of the bubble decreases, the wave forms differ increasingly from a pure harmonic and the phases of the periodically changing variables shift more and more with respect to each other. These changes tend to reduce the magnitude of Δs_b.

Presence of such conditions were observed after large bubble clusters have developed.

All of the above results were derived for configurations in which the container axis and the direction of the oscillation were horizontal, and, therefore, no effects of gravitation appeared in the equations. Gravitational effects, however, do appear when the container is turned in the upright position and it is oscillated in the vertical direction. The necessary change to account for gravity involves addition of a term, the hydrostatic buoyancy, in Equation (2.2.1-1), which becomes

$$F = a_\ell \rho_\ell V_b + g \rho_\ell V_b \qquad (2.2.1\text{-}24)$$

if the positive direction is upward and g is the gravitational constant.

Equation (2.2.1-5) becomes

$$a_\ell(\rho_\ell V_b - M_b) + g\rho_\ell V_b = (M_b + k_1\rho_\ell V_b)a_b + v_b \frac{d}{dt}(k_1\rho_\ell V_b) \tag{2.2.1-25}$$

and the new form of (2.2.1-10) is

$$a_b = \frac{1-k_1\varepsilon}{1+\varepsilon}\frac{a_\ell}{k_1} + \frac{1}{1+\varepsilon}\frac{g}{k_1} - \frac{v_b}{1+\varepsilon}\left[\frac{d}{dt}(\ln V_b) + \frac{d}{dt}(\ln k_1\rho_\ell)\right] \tag{2.2.1-26}$$

Finally, Equation (2.2.1-18) acquires an additive term

$$+ \int_{t_1}^{t_1+T}\int_{t_1}^{t} \frac{1}{1+\varepsilon}\frac{g}{k_1}\, dt\, dt \tag{2.2.1-27}$$

These results can be further improved by properly accounting for the finite dimensions of the container. Until now the derivations corresponded to a model consisting of a finite sized bubble pulsating in an unbounded body of liquid. In such a case, the flow field generated by the pulsations is symmetrical around the center of the bubble. This case is, however, unrealistic because the walls of the container are always at finite distances from the bubble and the flow field is distorted compared with the above one. The consequences can easily be seen qualitatively in the extreme situation in which the bubble occupies the whole cross section of the container. The liquid is then separated into two unconnected regions, one between the bubble and the bottom of the container, which we shall designate with A, and another between the free surface and the bubble, designated with B. Clearly, in the absence of viscosity and when the container is cylindrical, axial oscillations of the tank can be transmitted to region B only through the bubble. Now, if the bubble is soft because of low mean pressure, the liquid in region B will hardly move.

On the other hand, if the bubble is made hard by sufficiently high mean pressure, the natural frequency of the pulsations is much higher than the frequency of the forced oscillations and, as a result, the liquid in region B will oscillate together with region A and the tank, almost like one rigid body.

At a certain intermediate pressure, the bubble will resonate to the oscillation frequency of the tank, and region B will oscillate with large amplitudes.

When the bubble is smaller than the container cross section, the liquid is, of course, a single connected region and the two regions of liquid motion are not as clearly defined as above. Basically, however, the description of the liquid motion will be the same beyond some axial distance from the bubble. This supposition is supported by phase angle measurements of the pressure fluctuations. The pertinent characteristic of this motion is that the axial component of the fluid momentum is not the same on the two sides of the bubble. Consequently, if the bubble begins to move in an axial direction relative to the container, the domains of A and B change at a certain rate and this will be accompanied by a rate of momentum change $\frac{dl_w}{dt}$.

In a sense, it is equivalent to say that the bubble has also an apparent mass dependent on its location.

For further analysis, we can define a velocity change Δv_w and a mass flow rate $\frac{dM_w}{dt}$ in such a way that

$$\frac{dl_w}{dt} \equiv \Delta v_w \frac{dM_w}{dt} \quad . \tag{2.2.1-28}$$

If we arbitrarily adopt the definition

$$\frac{dM_w}{dt} \equiv v_b A_c \rho_\ell \quad , \tag{2.2.1-29}$$

i.e. the rate at which mass changes from region A to region B, when the bubble moves with a velocity v_b in a container of cross sectional area A_c, one expects that the corresponding velocity increment Δv_w will be a function of the rate of volume change of the bubble $\frac{dV_b}{dt}$ divided by A_c and the ratio of the bubble surface area S_b to A_c.

When the bubble moves with a positive velocity v_b and at the same time expands so that $\frac{dV_b}{dt}$ is also positive, the momentum change is negative. Symbolically, we may write

$$\frac{dl_w}{dt} = - \frac{k}{A_c} \frac{dV_b}{dt} \frac{S_b}{A_c} v_b A_c \rho_\ell \quad , \tag{2.2.1-30}$$

where k is a positive valued function varying probably with the relative size of the tank to that of the bubble with their shapes and with the depth of the bubble below the surface.

The volume change in (2.2.1-30) is again expressed by the pressure change through the polytropic relation,

$$\frac{-dV_b}{dt} = \frac{nV_b}{p_b}\frac{dp_b}{dt} \qquad (2.2.1\text{-}31)$$

and the final expression for the rate of momentum change becomes

$$\frac{dI_w}{dt} = k\ n\ v_b\ \frac{V_b \rho_\ell}{p_b}\frac{S_b}{A_c}\frac{dp_b}{dt} \quad . \qquad (2.2.1\text{-}32)$$

This term has to be added to the right hand side of Equation (2.2.1-25) in order to account for the effects of finite container size.

The resulting change in the relative bubble acceleration a_b is a fourth term at the right in Equation (2.2.1-26):

$$a_b = \frac{1-k_1\varepsilon}{1+\varepsilon}\frac{a_\ell}{k_1} - \frac{v_b}{1+\varepsilon}\left[\frac{d}{dt}(\ln V_b) + \frac{d}{dt}(\ln k_1\rho_\ell)\right] - \frac{kn}{k_1(1+\varepsilon)}\frac{S_b v_b}{p_b A_c}\frac{dp_b}{dt} + \frac{1}{1+\varepsilon}\frac{g}{k_1}$$

(2.2.1-33)

As before, we take for a more detailed discussion first those cases in which the resonant frequency of the bubble is much higher than the frequency of the container vibration. This restriction permits application of Equations (2.2.1-19) and (2.2.1-20), and one obtains, after neglecting ε in relation with unity:

$$a_b \approx \frac{a_\ell + g}{k_1} - v_b \left[\frac{d}{dt}(\ln V_b) + \frac{d}{dt}(\ln k_1 \rho_\ell)\right] + \frac{kn}{k_1} \frac{S_b}{p_b A_c} c_1 v_b v_\ell \omega^2 \quad . \tag{2.2.1-34}$$

This approximation is further simplified by disregarding changes of $k_1 \rho_\ell$ and by substituting for $\frac{d}{dt}(\ln V_b)$ as we did earlier. Then we get

$$a_b \approx \frac{g + a_\ell}{k_1} - nc_1 \omega^2 \frac{v_\ell v_b}{p_b}\left(1 - \frac{k}{k_1} \frac{S_b}{A_c}\right) \tag{2.2.1-35}$$

The integrals for the relative bubble velocity and displacement become

$$v_b \approx \int_{t_1}^{t} \frac{g + a_\ell}{k_1} \, dt - nc_1 \omega^2 \int_{t_1}^{t} \left(1 - \frac{k}{k_1} \frac{S_b}{A_c}\right) \frac{v_\ell v_b}{p_b} \, dt \tag{2.2.1-36}$$

and

$$\Delta s_b \approx \int_{t_1}^{t_1+T} \int_{t_1}^{t} \frac{g + a_\ell}{k_1} \, dt \, dt - nc_1 \omega^2 \int_{t_1}^{t_1+T} \int_{t_1}^{t} \left(1 - \frac{k}{k_1} \frac{S_b}{A_c}\right) \frac{v_\ell v_b}{p_b} \, dt \, dt \tag{2.2.1-37}$$

The interpretation of the last equation is that the effect of buoyancy is offset if the value of the second integral is sufficiently large. But the latter depends, among others, on the size of the bubble, in that the value of the integral decreases as the bubble grows. This might be the mathematical formulation of the observation that, under certain conditions, small bubbles move down in an oscillated tank while larger ones move up simultaneously. These last equations are easily related to the equations derived by Bleich which are also the basis for the theoretical calculations of Kana and Dodge.

If we re-substitute Equations (2.2.1-30, 31 and 32) into (2.2.1-33) and neglect ε (i.e. the mass of the gas in the bubble) and $\frac{d}{dt}(\ln k_1\rho_\ell)$, (i.e. liquid compressibility), we get

$$a_b \approx \frac{a_\ell+g}{k_1} - \frac{v_b}{V_b}\frac{dV_b}{dt} + \frac{k}{k_1}\frac{S_b}{A_c}\frac{v_b}{V_b}\frac{dV_b}{dt} \quad , \qquad (2.2.1\text{-}38)$$

or

$$\left(1 - \frac{k}{k_1}\frac{S_b}{A_c}\right)\frac{dV_b}{dt}v_b + V_b\frac{dv_b}{dt} \approx \frac{V_b}{k_1}(a_\ell + g) \quad . \qquad (2.2.1\text{-}39)$$

If we make the restriction that the bubble is small compared to the radius of the tank, i.e. $S_b \ll A_c$, and that the bubble is spherical, as was assumed by Bleich, then $\frac{k}{k_1}\frac{S_b}{A_c}$ becomes negligible next to unity and $k_1 = 1/2$ (factor for apparent mass). Then we can write

$$\frac{d}{dt}(V_b v_b) \approx 2V_b(a_\ell + g) \quad . \qquad (2.2.1\text{-}40)$$

Using Bleich's notations, assumptions for the oscillatory tank motion and result for the radial pulsation of the bubble (Δ), namely

$$v_b \equiv -\dot{\xi}$$

$$V_b = \frac{4}{3}\pi\ (a+\Delta)^3 \approx \frac{4}{3}\pi\ (a^3 + 3\Delta a^2)$$

$$a_\ell = -Ng \cos \omega t$$

and $$\Delta = \frac{\alpha}{3} a \cos \omega t \quad ,$$

Equation (2.2.1-40) becomes, after substitutions

$$\frac{d}{dt}\left[(1 + \alpha \cos \omega t)\dot{\xi}\right] \approx 2(1 + \alpha \cos \omega t)(Ng \cos \omega t - g) \tag{2.2.1-41}$$

which is identical to Bleich's Equation [18].

2.2.2 Viscous Theory

The general problem of predicting the effect of liquid properties on the dynamics of bubble clusters has been discussed in the previous sections. On the basis of experimental evidence and theoretical analysis, it has been established that the cluster dynamics involves the complex interaction of accelerative liquid forces, as influenced by the deformation of the confining vessel, and the inertia force of the bubble cluster and its associated added liquid mass. Interpretation of the experimental data on cluster dynamics is restricted somewhat by the fact that the effect of liquid properties on the motion of single bubbles has not been fully elucidated.

This section is concerned with an analysis of the stabilization phenomenon of single gas bubbles in an oscillating liquid to account quantitatively for the effect of liquid viscosity. In essence, the recent theory of Jameson (1966) is modified and used as a basis for developing a criterion for the single bubble stabilization. Discussion of the new viscous theory in comparison to the literature results and suggestions for further study are given at the end of the section. The analysis supports the point made in the previous section that viscosity does not influence in a first order way the accelerative forces required for bubble stabilization.

Extension of the concepts presented in the previous section of this report indicates that the momentum equation for a spherical bubble in a viscous liquid is:

$$\frac{d}{dt}\left[\frac{v_b(\rho_b + k\rho_\ell)u}{g_c}\right] = \frac{v_b\rho_\ell(g - A\omega^2 \sin \omega t)}{g_c} - \frac{6\pi\mu R(1+\beta R)U}{g_c} \quad ; \qquad (2.2.2\text{-}1)$$

where v_b = bubble volume,

ρ_b, ρ_ℓ = bubbles density and liquid density,

u = bubble velocity,

U = relative bubble velocity referred to the oscillating liquid,

R = bubble radius,

A = maximum amplitude of the liquid motion undergoing a displacement $x = A \sin \omega t$,

ω = circular frequency of the liquid oscillation,

t = time,

$\beta = \sqrt{\omega\rho/2\mu}$,

K = apparent mass coefficient for a sphere accelerating in a viscous liquid,

g_c = force-mass conversion factor.

The term on the left side of equation (2.2.2-1) represents the rate of momentum change of the bubble and its apparent mass, and the right side of the equation gives the forces due to gravity, oscillating acceleration, and viscous drag, respectively. The viscous drag term includes the steady state drag and the additional drag for oscillatory relative motion between the bubble and liquid, as developed by Stokes (1851). Further, use is made of Bleich's (February 1956) suggestion that a criterion for bubble stabilization can be obtained by requiring that the sum of the non-periodic terms on the right side of equation (2.2.2-1) be equal to zero. Such a condition will insure that the solution for the bubble velocity from equation (2.2.2-1) will be periodic, and hence, the bubble will be stabilized about a mean position.

Upon using Jameson's theoretical result for the relative bubble velocity U in equation (2.2.2-1), the bubble stabilization condition is found to be the solution of the equation,

$$\frac{g}{\omega^2 A} - \frac{\varepsilon}{2} + \frac{\pi\mu\varepsilon R_o(1 + 2\beta R_o)b}{\rho\ v_o A\omega^2} = 0 \qquad ; \qquad (2.2.2\text{-}2)$$

here,

ε = amplitude factor in the bubble pulsation equations, $V = V_o (1 + \varepsilon \sin \omega t)$ and $R = R_o(1 + \varepsilon \sin \omega t)$ with a theoretical value obtained by Jameson and Davidson (1966), $\varepsilon = \rho h\, \omega^2 A/(g_c P_e + h\rho g)$,

b = theoretical coefficient in Jameson's (1966) equation for relative bubble velocity,

h = depth below liquid surface at which bubble is stabilized,

P_e = gas pressure above the liquid column.

In the development leading to equation (2.2.2-2), it was assumed that the bubble pulsation and liquid oscillation are in phase, and that the bubble pulsation process is isothermal.

A form of equation (2.2.2-2) convenient for comparison to previous theoretical results can be obtained by use of the definitions $N_{AC} = \omega^2 A/g$, $P = 1 + \frac{P_e g_c}{\rho g h}$, and $N_s = fR_o^2/\nu$. The last ratio is the Stokes number and involves the liquid oscillation frequency (f), the mean bubble radius (R_o), and the liquid kinematic viscosity (ν). When these quantities are used in equation (2.2.2-2), the final form is:

$$\frac{N_{AC}}{P^{1/2}} = \left[\frac{1}{2} - \frac{3K_2(1 + \sqrt{\pi N_s})}{8\pi(K_1^2 + K_2^2 N\ )} \right]^{1/2} \quad ; \qquad (2.2.2\text{-}3)$$

where K_1 and K_2 are the theoretical constants which appear in Jameson's equation for the relative bubble velocity and are defined in terms of the Stokes number as

$$K_1 = 1/2 + 9/4\sqrt{\pi N_s}\ ,$$

$$K_2 = 9/4\sqrt{\pi N_s}\ (1 + 9/4\sqrt{\pi N_s})\ .$$

Equation (2.2.2-3) provides a working criterion for predicting the oscillating acceleration necessary to stabilize a single gas bubble in a viscous liquid.

Figure 2 is a plot of Equation (2.2.2-3), along with the bubble stabilization criterion as predicted by Bleich (February 1956) for an inviscid liquid in a tank with rigid wall, and as predicted by Jameson and Davidson (1966) for an inviscid fluid. It should be pointed out that Bleich's equation is based on adiabatic bubble pulsation, while the Jameson and Davidson equation is based on the assumption of isothermal bubble pulsation. These two equations are given in terms of the variables used in Equation (2.2.2-3) as follows:

$$\text{Bleich,}\quad \frac{N}{P^{1/2}} = \sqrt{2\gamma} \qquad (2.2.2\text{-}4)$$

where γ is the specific heat ratio for the bubble;

$$\text{Jameson and Davidson,}\quad \frac{N}{P^{1/2}} = \sqrt{2}\ . \qquad (2.2.2\text{-}5)$$

It is noted that the theoretical result for a viscous liquid is within the bounds given by Equations (2.2.2-4) and (2.2.2-5).

The plot of Equation (2.2.2-3) reveals that at Stokes numbers greater than 100, the criterion for bubble stabilization based on viscous theory approaches the theoretical criterion from the isothermal, inviscid theory. This means that although there exists substantial

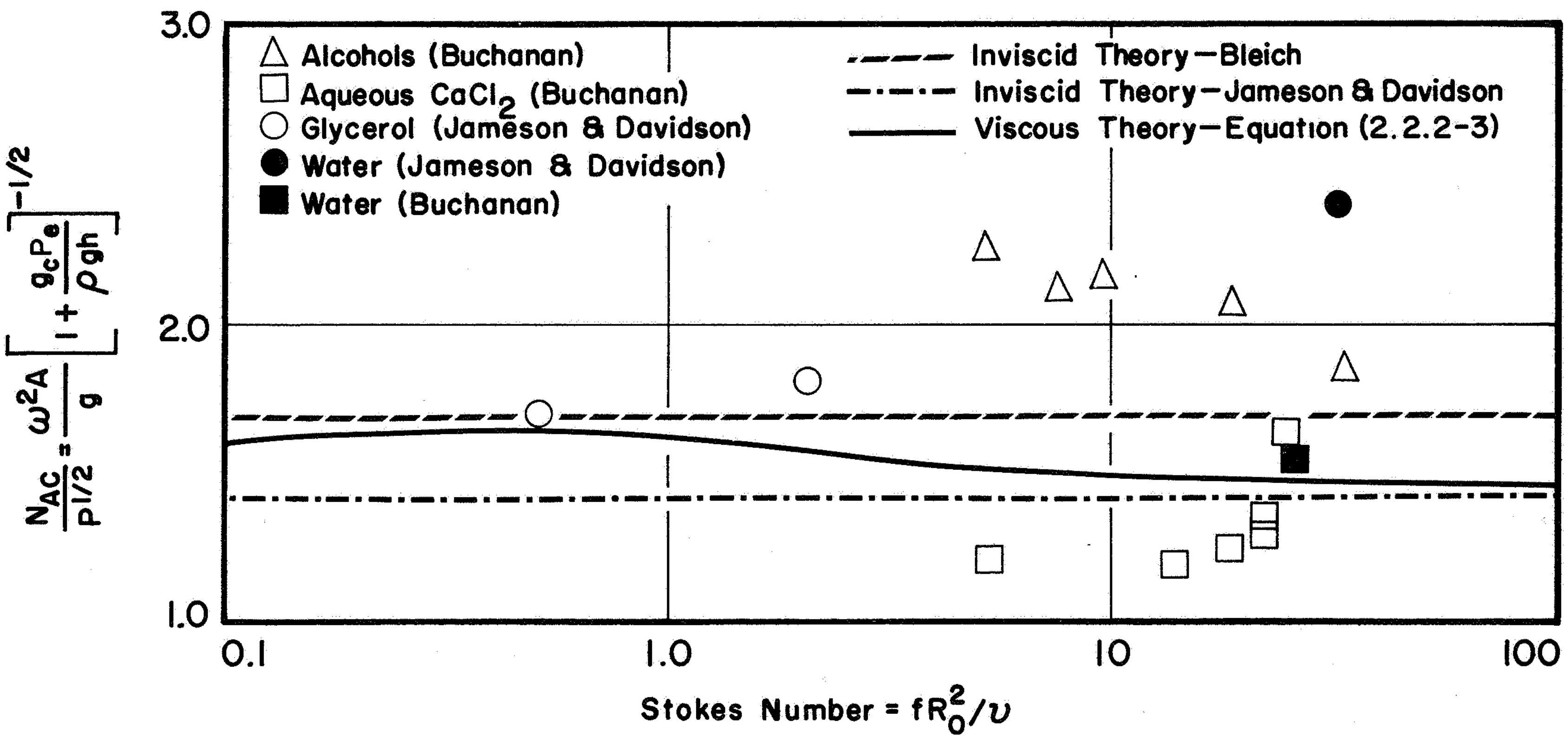

Figure 2. Critical Oscillation Parameter For Single Bubble Stabilization

relative motion between the gas bubble and the oscillating liquid, the viscous effect is small because of the low liquid viscosity. In contrast, the viscous theory criterion at very low Stokes numbers approaches the Jameson-Davidson result because the relative motion between the bubble and the oscillating liquid becomes negligibly small. The maximum viscous effect is shown at a Stokes number near 0.5.*

Experimental data on bubble stabilization for various liquids are also shown in Figure 2. Data points are shown for glycerol solutions and water from the work of Jameson and Davidson (1966), and for aqueous calcium chloride solutions and pure alcohols from the work of Buchanan (1962). None of these investigators reported the bubble sizes for their experimental observations of bubble stabilization acceleration. In Buchanan's paper there is a suggestion that bubbles about 2 mm in diameter were involved in their experiments. In absence of data, a bubble diameter of 2 mm was taken for all the experimental points shown in Figure 2.

The experimental data agree in general with the theoretical predictions, as revealed in Figure 2. The largest differences between the theoretical and experimental values are for the low surface tension alcohols and the high surface tension calcium chloride solutions. The differences are large enough to indicate that surface tension effects need to be incorporated into the theories. In order to accomplish this, it will be necessary to consider quantitatively the surface tension in

*For reference, a 2 mm diameter bubble stabilized at 100 cycles/sec in a typical cryogenic liquid gives a Stokes number of $\approx 10^3$.

the bubble pulsation and in the drag relationship for the relative motion between the bubble and the oscillating liquid. It is known that at Reynolds numbers greater than about 200 the surface tension influences sharply the drag of gas bubbles in liquids (Peebles and Garber, 1953).

The importance of the information on single bubble stabilization in relation to the more complex problem of bubble cluster dynamics has been cited earlier. Although the single bubble stabilization theory appears adequate, a number of details have yet to be established by comprehensive experiments. A list of further studies needed is as follows:

1. Measurement of relative bubble velocity in oscillating liquids under bubble stabilization conditions for liquids to cover the Stokes number range of 10^{-1} to $10^3$and to include the surface tension range of about 20 to 100 dynes/cm.--It is expected that analysis of the surface tension effect and correlation of the experimental data will involve an oscillatory "Weber" number of the form $R_o{}^3f^2\rho/\sigma g_c$, where σ is the liquid surface tension. These experiments should be carried out in rigid vessels with pure liquids and injected gas bubbles of known sizes. The high speed photographic methods employed by Jameson and Davidson (1966) should be suitable for the bubble velocity measurements.

2. Measurement of the mean velocity of bubbles which rise or move downward in oscillating liquid columns.--These experiments will be more easily executed than those involving measurement of the local bubble oscillating velocities described above, and the results will

yield data to test quite critically the existing theories on oscillatory bubble motion.

3. Further analytical investigation of the surface tension effects on oscillatory bubble motion by including the surface tension term in the equation for bubble pulsation and also by accounting for bubble shape changes in the drag relations for bubble motion.

3.0 EXPERIMENTAL INVESTIGATION

3.1 Equipment

The experiments for the study of bubble behavior in oscillating liquids were performed with the aid of an MB Model C 25 H vibration exciter system (Figure 3), which was furnished by the George Marshall Space Flight Center on a loan basis. The containers for the liquids were cylindrical Plexiglas tanks (6-1/2 inches inside diameter) of various wall thicknesses (1/4 inch and 1/2 inch)(Figure 4).

For the measurements of the forced tank oscillations, two instruments were used simultaneously. One was the exciter system's own built-in velocity sensor, the other was an accelerometer mounted on the vibration table at the base of the tanks.

The pressure field inside the liquid was explored with a quartz pressure transducer and a Kistler Universal Dial-gain Charge Amplifier Model 504. Quantitative evaluations of varying transducer signals were based on the recordings of a Sanborn "150" Series Recording System.

Finally, the instantaneous values of periodically varying quantities were measured on the screen of a Tektronix Type 502A Oscilloscope.

3.2 Experimental Techniques

The experiments performed to date can be grouped essentially into four categories as regards the techniques used.

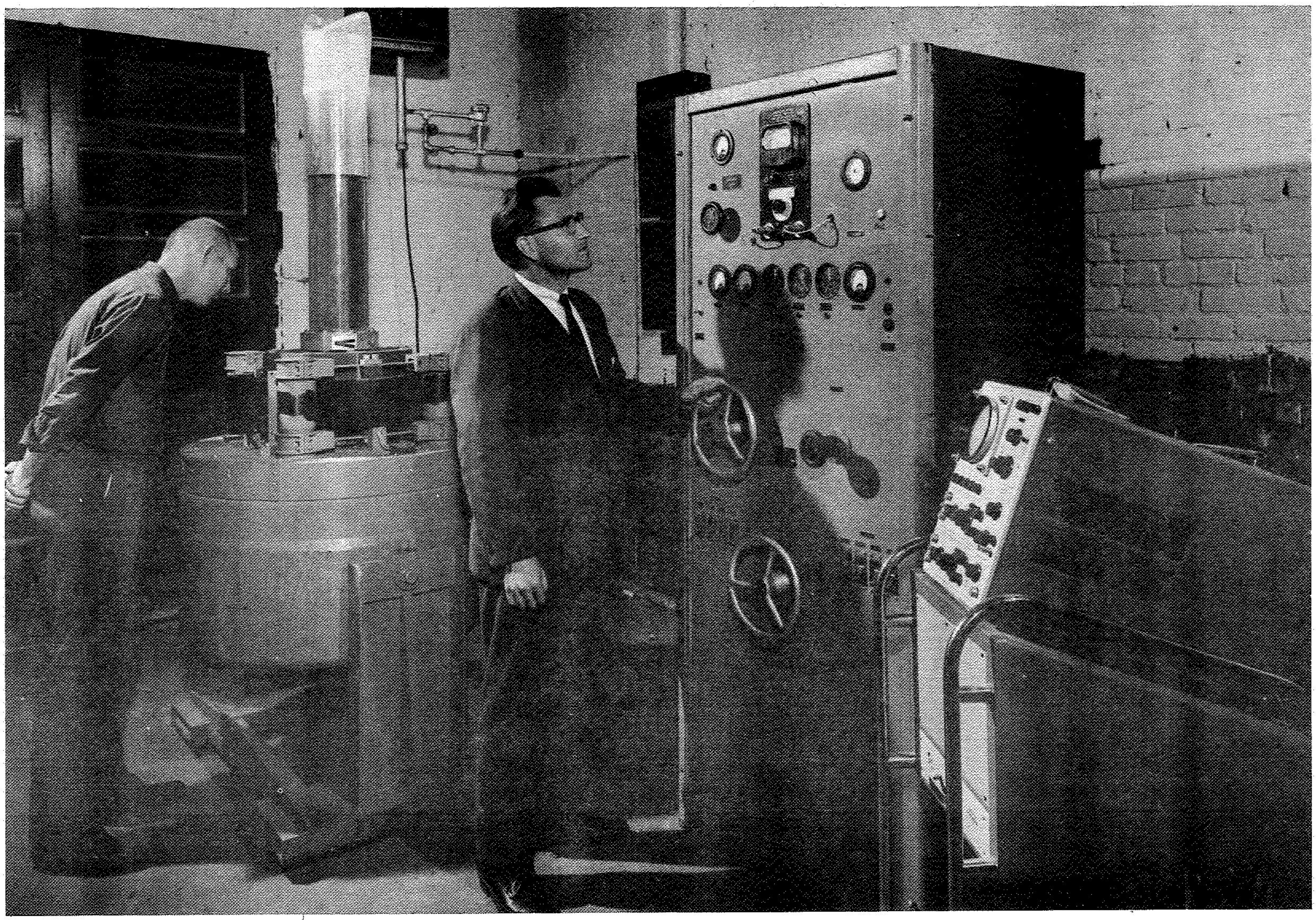

FIGURE 3. VIBRATION EXCITER AND CONTROL CABINET

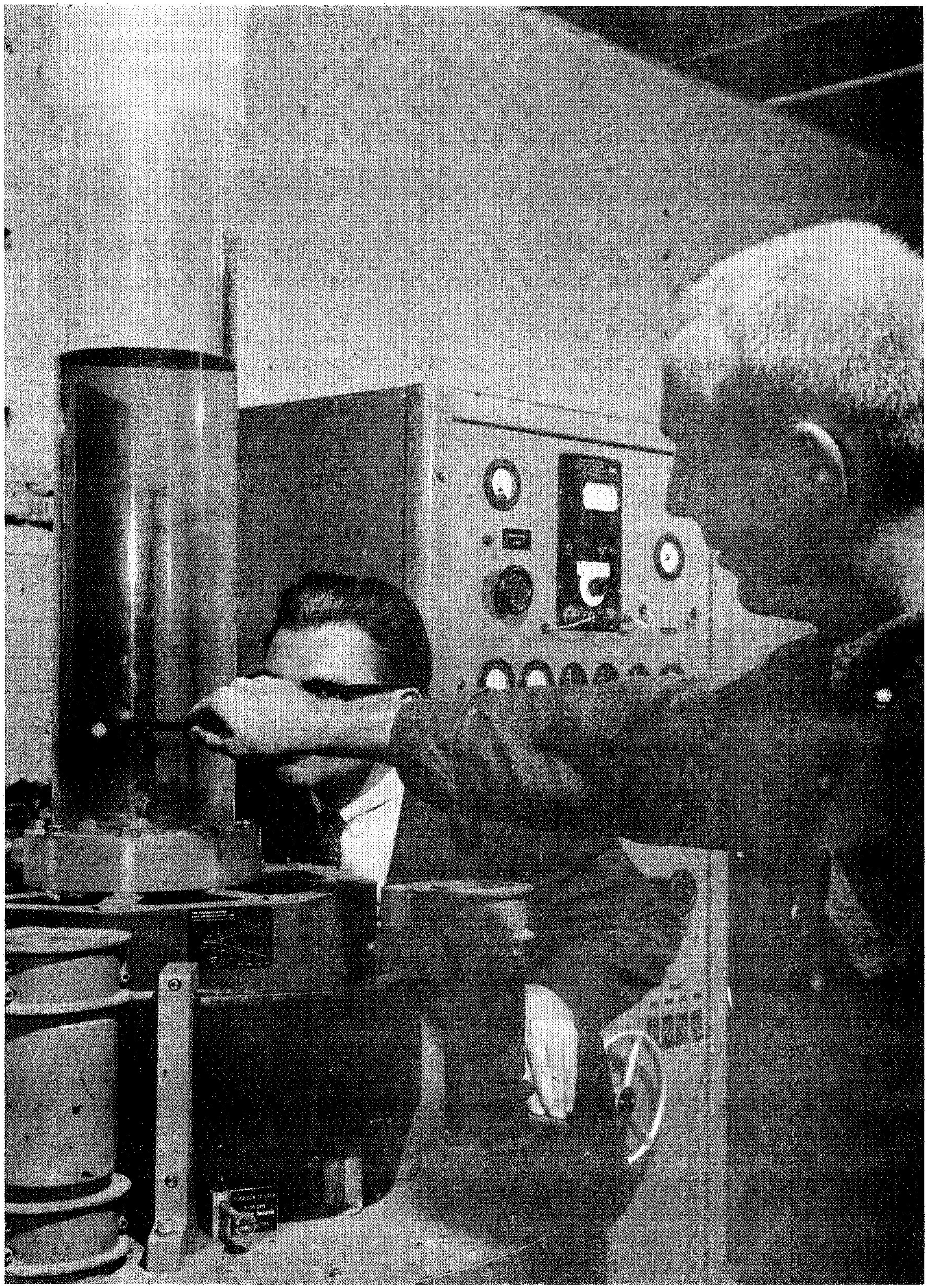

FIGURE 4. OSCILLATING TANK WITH FULLY-DEVELOPED CLUSTER

Experiments of the first category consist of measurements of the length of time which elapsed from the moment the vibration exciter started vibrating until a bubble cluster started forming, or until the cluster developed to its final size, the limit cluster. During these experiments, the frequency and amplitude settings of the vibration table were not changed.

In the experiments of a second category, pressure fluctuations were measured in the liquid and inside bubble clusters.

For investigations of a third category, a high speed motion picture camera was used to obtain information on the shape and size of pulsating clusters, on the pulsation amplitude and on the turbulence pattern inside clusters.

In the last category, all preliminary experiments can be collected which served the purpose of providing information on the dynamic deformation of the tank; interactions between the tank, vibrating mechanism and the fluid; and the behavior of compressible and incompressible objects placed inside the liquid.

Most of these experimental techniques served more than one purpose. When the technique of time measurements (category 1) was used to measure effects of frequency and oscillation amplitude on the length of formation and development time of clusters, the procedure was as follows: First, the frequency control of the vibration exciter was set to the desired frequency. Then the amplitude control was turned with one quick motion so that the oscillation amplitude rose almost instantly from zero to the desired magnitude. It was attempted to do this in such a way that little or no correction was necessary to obtain the exact amplitude after the first move was made. This insured a well-defined beginning of the experiment. From here on, the vibration system controls were not changed.

Meanwhile, the recording instrument recorded the amplitude of the table acceleration on a moving paper strip. In a typical experiment after the oscillations started, the recorded curve was at first a straight line with only small irregularities. At the moment, however, when the cluster appeared in the liquid, the curve rapidly or even suddenly changed towards larger amplitudes. This happens because the bubble cluster alters the motion of the liquid and with it the load of the inertia forces on the table. That, in turn, alters the amplitude of the table's oscillation.

After the rapid change of slope, the curve describes an arch while the cluster develops. When the cluster reaches its final state, the oscillation amplitude reaches a maximum and settles at a somewhat smaller magnitude. The lengths of time which elapsed during these phenomena were measured off the recordings.

The above description of events fits essentially all of the measurements of this type but there were many variations in the details, e.g. at higher frequencies the amplitude rise becomes greatly reduced or the change can even occur towards smaller amplitudes.

Essentially, the same method was used to measure the changes in cluster formation and development times when the concentration of dissolved gases in the liquid was changed. In this series of experiments, the tank was covered with a thin plastic membrane and filled with liquid until all the air was forced out from under the membrane. This prevented entrainment of bubbles at the surface during oscillation, and the bubbles had to form out of the dissolved gases (air) in the liquid and probably out of vapor.

The experiment consisted of the following sequence of steps. After the table was turned on and a cluster had formed, the oscillation amplitude was turned down again to permit the cluster to rise to the plastic cover. The cluster was removed and replaced by liquid. Then the amplitude was turned up again until the next cluster formed and the procedure was repeated. With the removal of each bubble, the concentration of the dissolved gases in the liquid was reduced.

One modification of this experiment was achieved by replacing the plastic membrane with an inch thick rigid Plexiglas plate.

Other experimental methods will be described later together with discussions of results.

3.3 Experimental Results

One characteristic feature of the present problem is that the phenomena depend on a very large number of factors and the mechanisms seem to be strongly non-linear. As a consequence, it is possible that the liquid column behaves in one way in a certain range of the various variables, and very differently in another range. This is probably the reason why the descriptions of phenomena by different authors frequently differ considerably from each other.

The many variable factors offer a large variety of possible experiments and raise many questions concerning their effects. In the present experimental investigation, it was attempted to collect information on the effects of some of these factors. In the following discussion, the experiments are arranged approximately according to the order of the techniques used as outlined in the preceding paragraphs as long as logical connections are not interrupted.

3.3.1 Bubble Cluster Formation Time

For practical reasons, one of the first questions one may ask concerning the present problem involves the length of time that passes after vibration of the tank has started until a bubble cluster begins to form. This formation time was measured over as wide a range of frequencies and oscillation amplitudes as was possible, with the method of measuring time as described in the previous section. Results obtained with methyl alcohol are collected in Figures 5, 6 and 7, for water in Figures 8 and 9.

In these diagrams, the abscissa indicates the amplitude of the acceleration oscillations which the table executed before clusters appeared. The ordinate is the length of time between the beginning of oscillations and the appearance of a cluster. The curved lines have been fitted to join points belonging to the same frequency for easier viewing. A characteristic that these curves reveal is that they all approach monotonicly a minimum initial acceleration amplitude which depends on the frequency. If the amplitude of the table acceleration

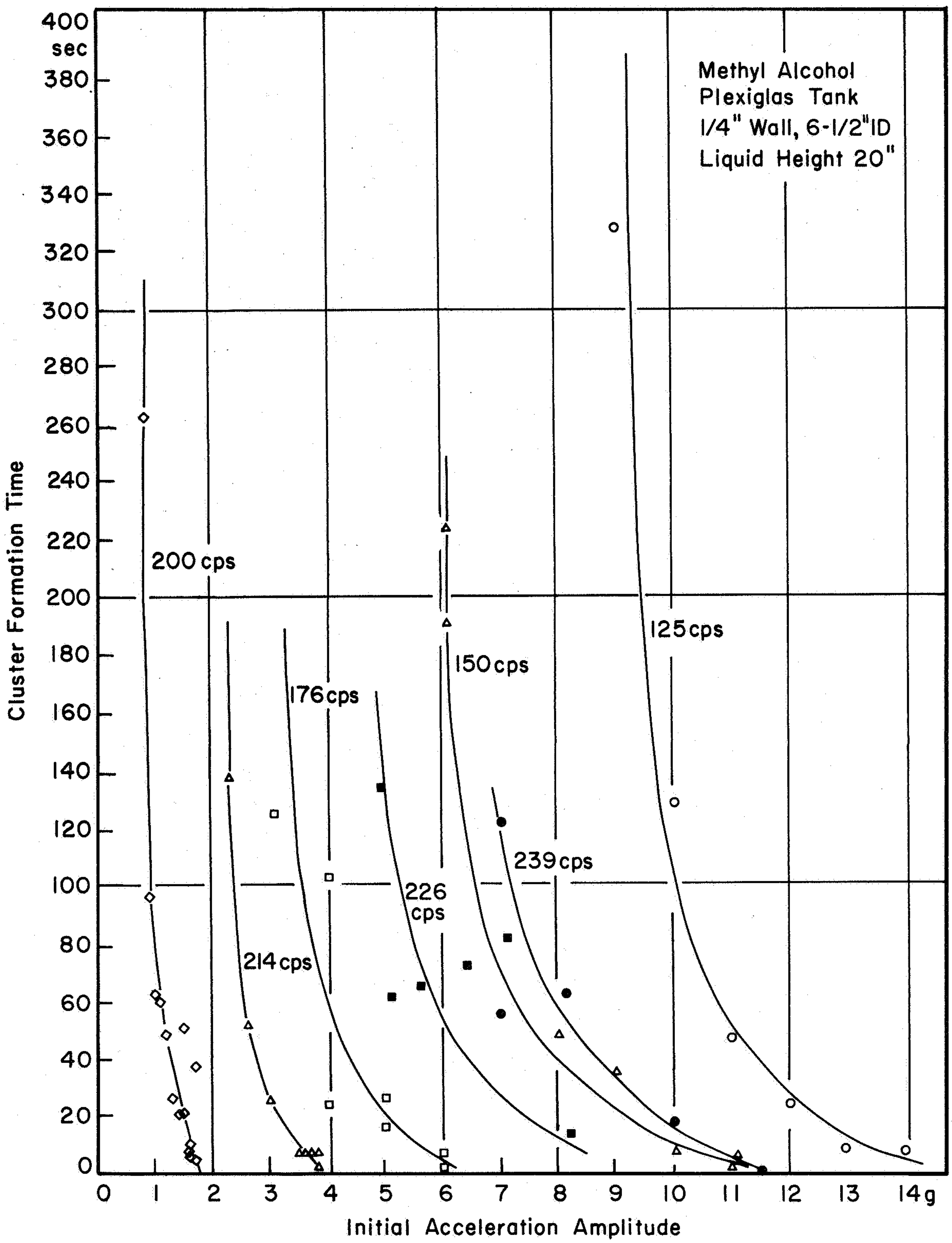

Figure 5. Effect of Frequency and Amplitude on Cluster Formation

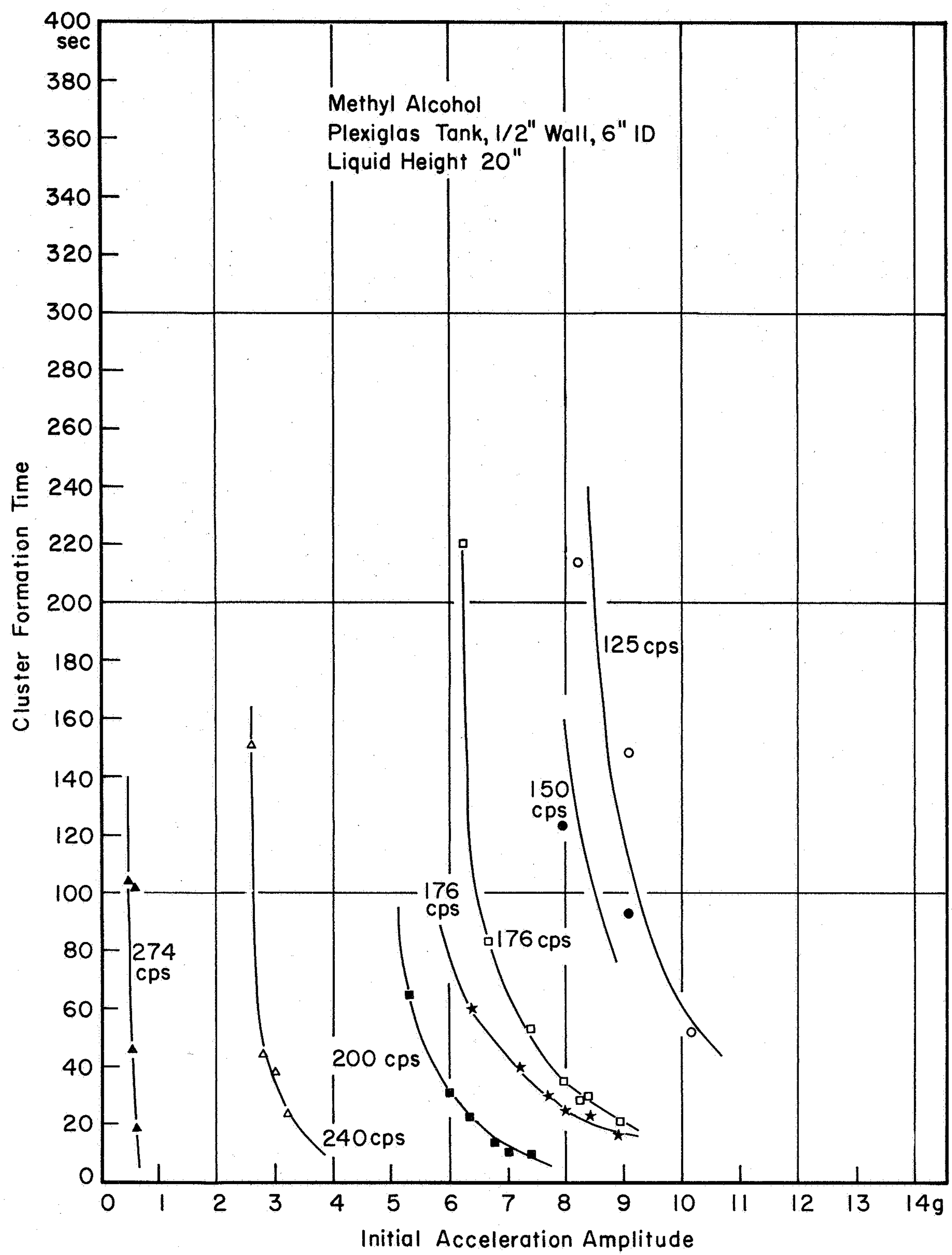

Figure 6. Effect of Frequency and Amplitude on Cluster Formation

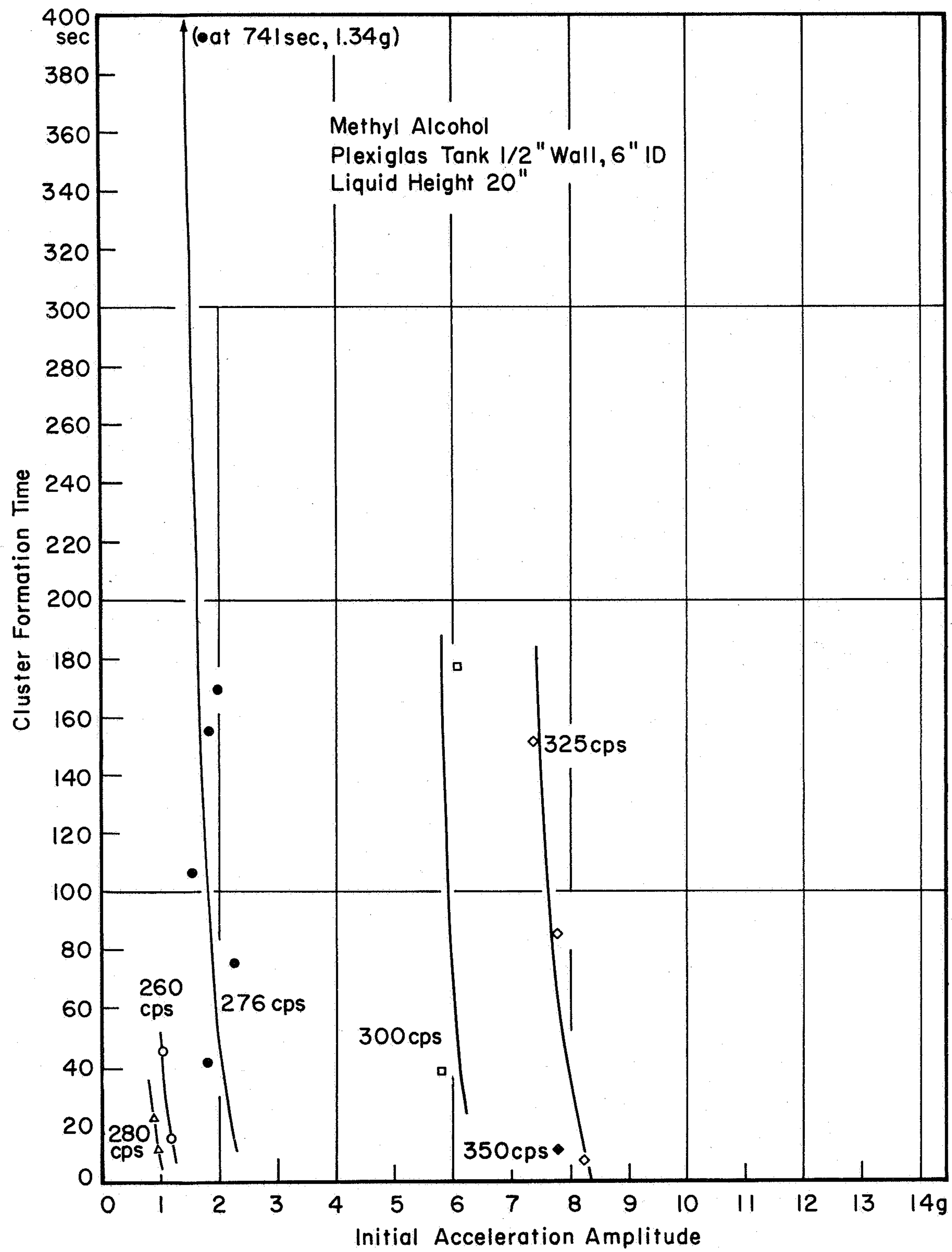

Figure 7. Effect of Frequency and Amplitude on Cluster Formation

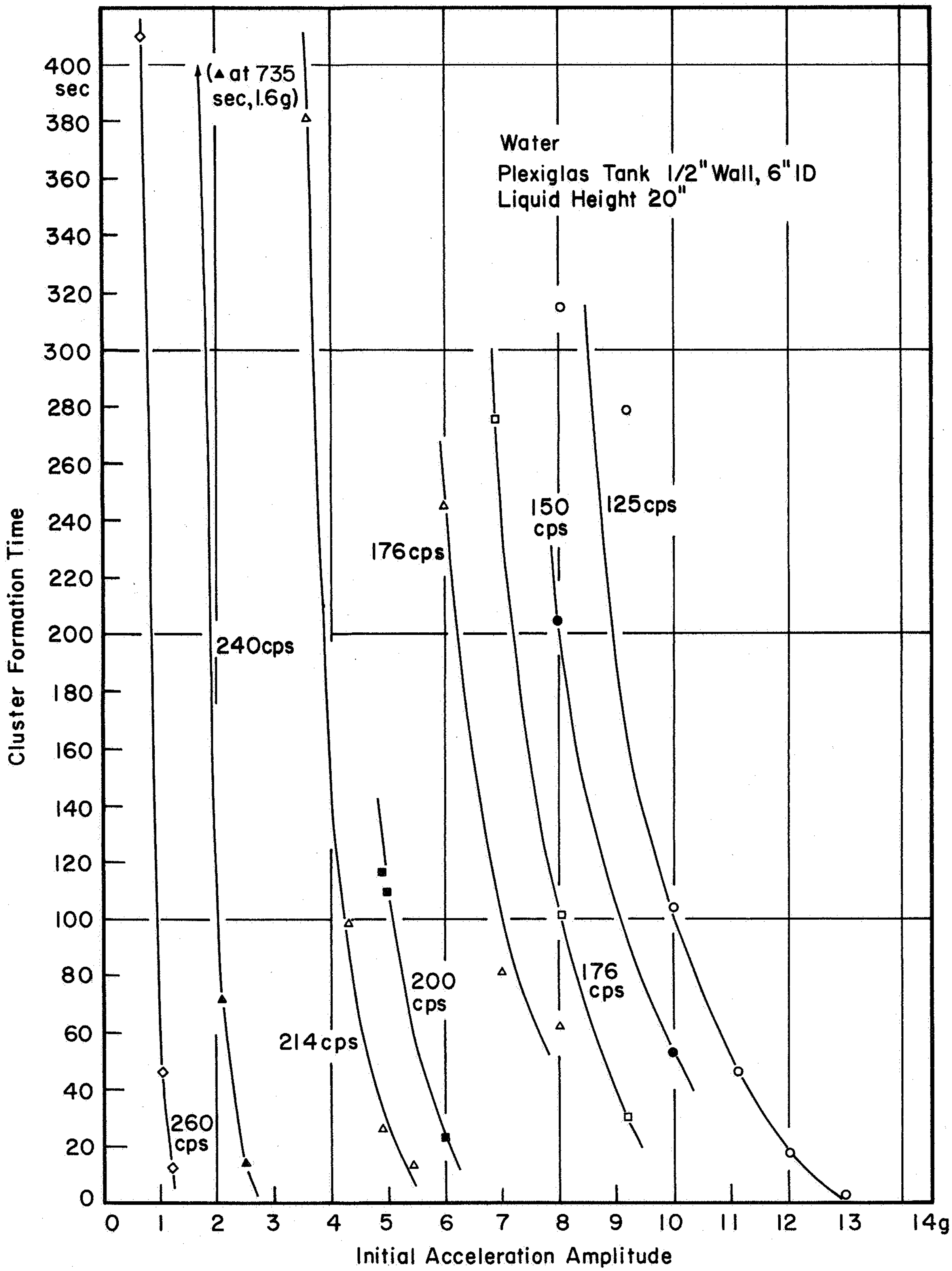

Figure 8. Effect of Frequency and Amplitude on Cluster Formation

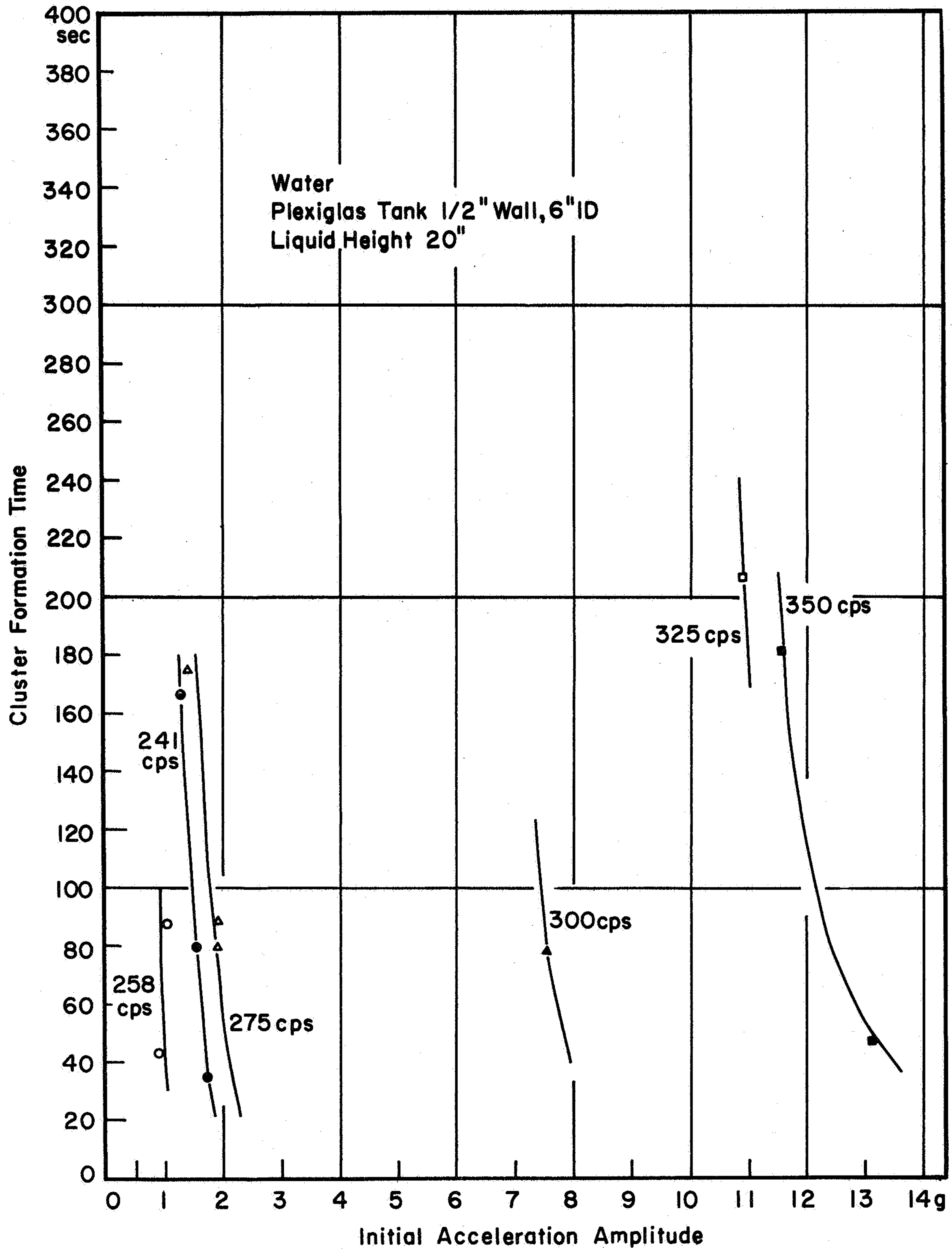

Figure 9. Effect of Frequency and Amplitude on Cluster Formation

was set below that minimum, no clusters formed even though small stationary bubbles may have been present.

It is noteworthy that in Figures 5, 6 and 7, for methyl alcohol, the curves for lower frequencies shift to smaller accelerations as the frequency increases, but for frequencies above about 200 cycles per second and 275 cps respectively, this trend reverses. There is some unexplained scatter between the day to day results, especially at higher frequencies. These results are in agreement with those of Ponder, Blount and Fritz (1964). The effect is almost certainly due to reciprocal interaction between the fluid and the container, possibly affected by the frequency characteristics of bubble nucleation. As Figures 8 and 9 demonstrate, these trends are qualitatively the same for water also.

Another observed effect is also attributed in part to interaction between tank and fluid. This effect is the decrease, as the frequency increases, of the relative difference between the final table oscillation amplitude g_f after a cluster has developed and the initial amplitude g_i. At lower frequencies (for methyl alcohol below about 125 cycles per second) the amplitude jump as the cluster forms is violent and it is accompanied by a disintegration of the fluid into a foamy mixture after which the amplitude returns to its original value. These events occur so fast that only the order of magnitude of the amplitude jump could be observed. As the frequency increases, the magnitude of this amplitude jump rapidly decreases, then it may peak again, but ultimately the change becomes zero and even negative, i.e. the table amplitude is smaller after

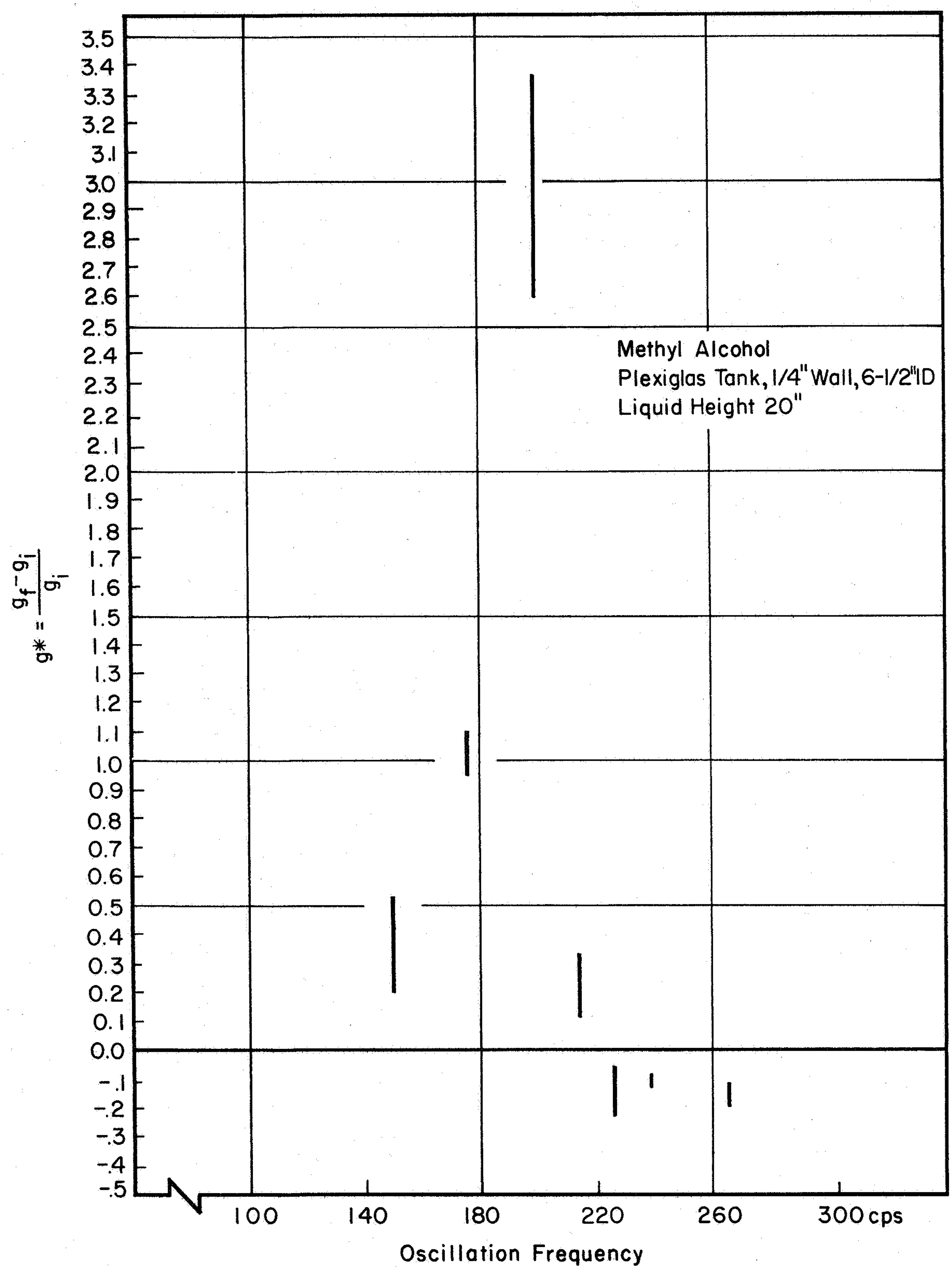

Figure 10. Effect of Cluster on Acceleration Amplitude

the cluster has formed than before. Also, simultaneously with this development, the cluster formation becomes increasingly gradual.

This behavior exhibits resonance characteristics and it raises the question of how the various components, namely bubble, liquid, container and vibration table participate in the process.

To a limited extent, this problem was explored by various means.

First, it was noted that (at least in the frequency range explored; i.e. above 125 cps) the explosive character with which clusters grow in the lower frequency range is not an inherent property of the liquid motion but a feedback phenomenon through the vibration exciter. This conclusion was reached because the clusters developed gradually when the amplitude was kept constant by manually operating the amplitude control.

If, as it appears, the dynamic deformation of the tank has such a large effect on the behavior of the liquid, it is necessary to know the responses of the tank in order to understand how the interaction takes place. Since the present investigations were not designed to cover problems related to the structural properties of the container, only a few, mostly qualitative, experiments could be performed.

In theoretical calculations of the literature it is usually assumed that the tank deforms according to the first mode, i.e. axisymmetrically, with a radial breathing motion. The following simple experiment attempted to verify the validity of this assumption.

Since pressure measurements in the liquid (to be discussed at another place) indicated that the largest pressure fluctuations occur at about the level of the cluster, the tank deformation was checked there.

First a steel band was placed around the tank and it was tightened right at the level of the cluster after a cluster had fully developed. This restricted the symmetric pulsations of the tank and it was expected that the cluster should show some sign of this, but it did not.

Because of the flexibility of the steel band, this experiment did not affect any deformation of the tank which causes no change in the circumference. Therefore, a heavy clamp was used to press on the tank at two points only, over the cluster and at a diagonally opposite point. To this, the cluster responded by turning around along the tank wall to a location between clamped points. The deflection could be as much as about 140°.

The deformation forces required to produce deflection of a bubble cluster are relatively small. With a Plexiglas tank of 1/4" wall thickness, it can be done even with the fingers, provided frequency and amplitude are within a favorable range.

These observations are in agreement with instantaneous pressure measurements along a tank diagonal intercepting the cluster (see Section 3.3.3 and Figure 13). These, too, indicate that the motion is not axially symmetric. Further findings relating to pressure measurements and tank properties will be presented in a later section of this report.

The above observations lead to the question of how the magnitude of the forced oscillations, the tank deformation, the cluster pulsations and the liquid motion relate to each other.

The relationship between volumetric displacements of the tank bottom and the volume changes of the cluster during pulsations was established approximately with a high speed movie camera. From the measured amplitude of the table oscillations and the diameters of the cluster, measured on successive frames of the photograph, it was calculated that the cluster pulsated with volume changes about ten times as large as the volume displacement of the tank bottom.

The explanation for such large pulsations could possibly be resonance of the cluster and liquid column system or large deformations of the tank wall.

Some information to decide which process took place was obtained on the following principle. When two rigid spheres are placed in an incompressible fluid in which the pressure pulsates but there is no relative motion between liquid and spheres, there is no force acting between the spheres. However, if the liquid moves relative to the spheres and the velocity component perpendicular to the line connecting the centers of the spheres is finite, the spheres attract each other (Bernoulli force). This relationship was exploited in the experiment. It was reasoned that if the bubble cluster pulsates just to absorb the in and out bulging motion of the tank wall, then there should be no noticeable velocity increase in the liquid far above the cluster and near the liquid surface when the cluster forms. Consequently, if two rigid spheres of different density than the liquid are placed near each other at this location, they should be unaffected. On the other hand, however, if the cluster and the liquid column above it are in resonance and that is the cause for the large pulsation amplitude, the liquid will oscillate with large velocity amplitudes even at the surface and the spheres should be attracted to each other.

Following this reasoning, two hard plastic spheres of 1/2 inch diameter were suspended along the centerline of the tank beneath the liquid surface with about 1-mm space between them. It was believed that the spheres were heavy enough so that velocities of the order of the tank motion did not move them noticeably, but velocities of ten times that magnitude would. (Conditions were about 170 cps and 4 g acceleration.) Indeed, the spheres hung motionlessly while there was no cluster in the liquid, but the instant the cluster formed, they swung toward each other. It is believed that this supports the assumption that the cluster and the liquid above it are in a state of resonance. Further confirmation of this conclusion is derived from the fact that the size of the cluster was nearly that predicted by Minnaert (1933) and Smith (1935) for resonance at the prevailing frequency. Also, measurements of phase shifts between tank acceleration and the pressure indicated the same condition. These results will be presented later.

Finally, one may mention in this connection the following experiment. At a time when a fully developed stationary cluster occupied a stable location, a tube was lowered into the liquid and a puff of air was blown into it. Instantly, the cluster detached from its site and vented to the surface.

If the end of the tube was below the cluster, the air blown into the liquid moved to the previous site of the cluster and it remained there. This process could be repeated periodically.

Similar was the reaction of the cluster when a small balloon, approximately the size of the cluster, was forced into the liquid. It was, however, noted that the cluster dissolved only at selected combinations of frequency, amplitude and balloon size.

Since this experiment may have important implications for the avoidance of cluster formation in fuel lines and tanks of rockets, it may be recommended that this matter be further explored. For such a program, one can suggest experiments with containers, the inside surface of which has been lined with a compressible elastic material containing trapped bubbles, like wet suits of divers. Other experiments could be performed with oscillated tubes containing regularly spaced small balloons or compressible spheres.

3.3.2 Effect of Dissolved Gas on Cluster Formation Time

Another parameter which seems to influence significantly both the inception of bubbles and the length of time required to fully develop a cluster is the concentration of dissolved gases in the liquid.

A series of runs was made to discover the role of this parameter. The method used for this purpose was that described in Section 3.2, Experimental Techniques. Results of the experiment are shown in Figure 11 where they have been arranged along the abscissa in the chronological order of the individual experimental runs. In this arrangement the abscissa, when measured from left to right, becomes an arbitrary scale of decreasing concentration of gases, probably air. The explanation is given below.

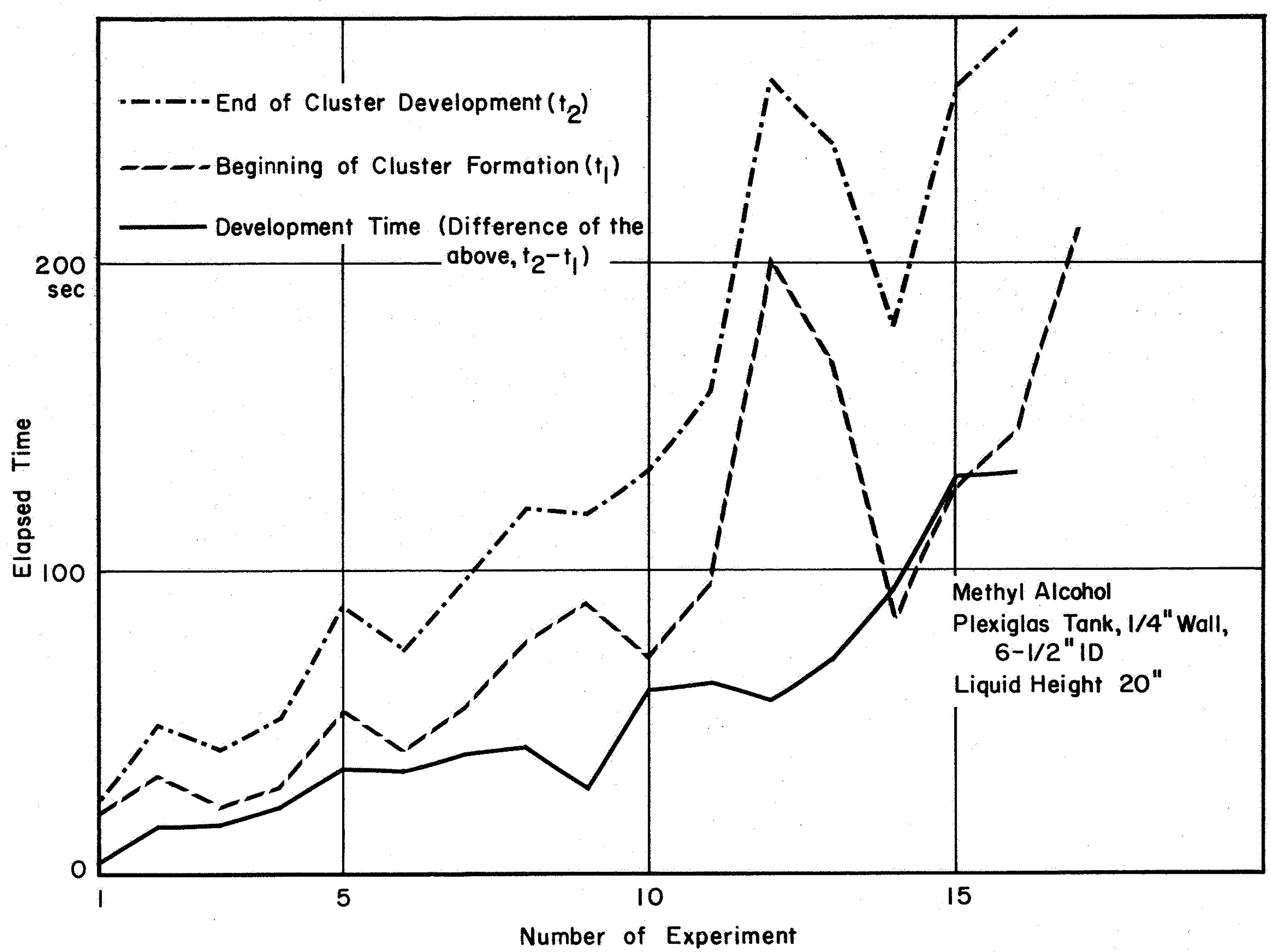

Figure 11. Effect of Gas Concentration on the Formation of Clusters

In Figure 11, the variable t_1 is the length of time measured from the onset of vibration until appearance of a cluster, t_1 is called the formation time. The value of t_2 gives the time from the onset of vibration until the cluster reached its final state, that of the limit cluster. The difference $t_2 - t_1$ is the development time.

In these experiments, the tank was covered with a plastic membrane and the contents of the bubbles were forced out of the tank each time after a cluster has formed. In this way the separated gases and vapors were removed and the gas concentration reduced with each successive run. The curves of the figure show how rapidly the time required to form a cluster increases as the amount of dissolved gases diminishes.

The curves for t_1 and t_2 become irregular at lower gas concentrations (after Experiment No. 12), possibly because of the probabilistic nature of nucleation of gas bubbles. Therefore, the difference $t_2 - t_1$ is a better measure of concentration effects.

The series was discontinued with Experiment No. 17 because the cluster which formed at one point could never fully develop. Apparently the concentration became too low to make possible the growth of the cluster beyond a certain size at the applied frequency and amplitude (150cps,3g, methyl alcohol, 1/4 inch tank wall). After Experiment 17 the cover membrane was removed to permit entrainment of air into the liquid. This mixing restored the initial concentration and when the next run was made, the results matched the results of Experiment No. 1 within measuring accuracy.

3.3.3 Effect of Liquid Density on Cluster Formation Time

An unsuccessful attempt was also made to correlate liquid density and the minimum initial acceleration amplitude needed to form clusters. Results are shown in Figure 12. All experiments of Figure 12 were performed at a frequency of 210 cps and with the tank uncovered. Four liquids were used and they were selected to cover a wide range of density. In the diagram, the ordinate of the points gives the smallest initial acceleration amplitude at which clusters developed. These results scatter too much, suggesting that density was not the only effective factor that changed from one liquid to another. Explanation of the apparent chaos requires further investigation.

3.3.4 Pressure Distribution in Vibrating Liquid Column

The second major set of experiments had the purpose of mapping the pressure distribution inside the liquid. The tests were performed in tanks of 1/4-inch and 1/2-inch wall thicknesses with the Kistler Amplifier and a quartz pressure transducer. The liquid was methyl alcohol.

For the systematic exploration of the pressure field, traverses were made with the pressure probe either along a diagonal or along the centerline of the tank. The measured quantity was the instantaneous pressure and it was displayed on the screen of a calibrated oscilloscope. The peak-to-peak differences of the signals could be obtained with the aid of the graticule ruling of the oscilloscope screen.

The evaluated data are presented in Figures 13 and 14 where the ordinate gives the peak-to-peak value of the pressure fluctuations.

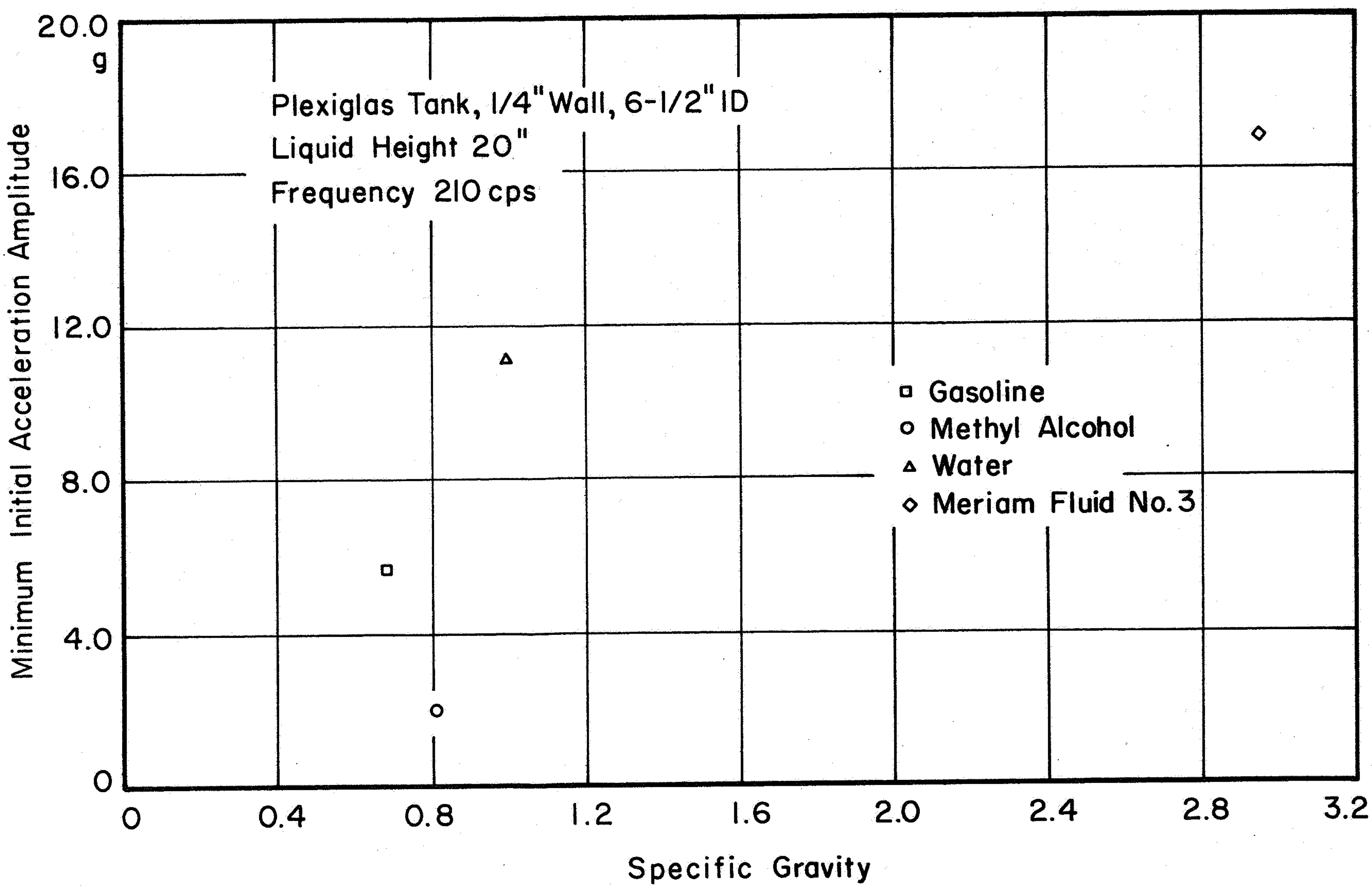

Figure 12. Minimum Initial Acceleration for Cluster Formation

In Figure 13 the traverse starts at the inside (about the center) of the cluster which was at 11 inches from the tank bottom, and continues diagonally to the opposite wall. The frequency was 179 cycles/sec. It is clear from this figure that the maximum of the pressure fluctuations occurs at the cluster and that the distribution of the fluctuations is not symmetric with respect to the tank centerline.

During these measurements, phase shifts with respect to the tank acceleration were also measured but those results will be discussed further below. Here it be just mentioned that the maximum lag of the pressure behind the acceleration, about 90°, occurred also at the cluster. This further demonstrates the resonance character of the process. All these results agree with the observations of Kana and Dodge in whose experiments the cluster developed at the bottom of the tank.

Figure 14 shows how the pressure fluctuations varied along the centerline, but this presentation is not well suited for interpretation and, therefore, the data were further processed.

For Figures 15, 16 and 17, first pressure oscillations were calculated which would occur in the liquid if it oscillated as a rigid body. These results were subtracted from the actually measured values and the differences were plotted. Thus, the pressure distributions in these diagrams represent the effect of the relative motion of the liquid with respect to the vibration table, the dynamic pressure fluctuations. These distributions give also an indication of the dynamic tank deformations.

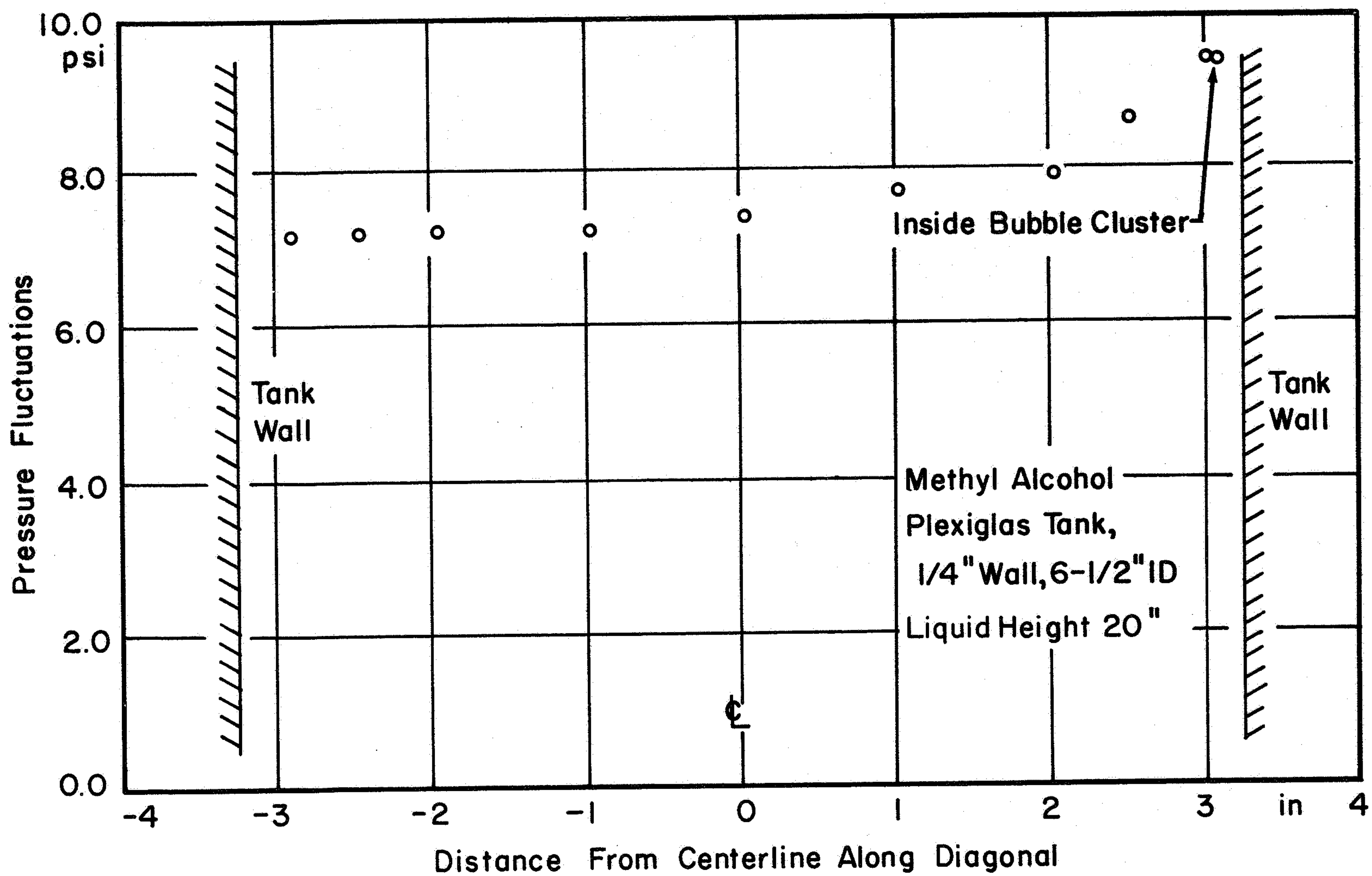

Figure 13. Distribution of Peak-to-Peak Pressure Fluctuations Along a Tank Diagonal Traversing a Bubble Cluster

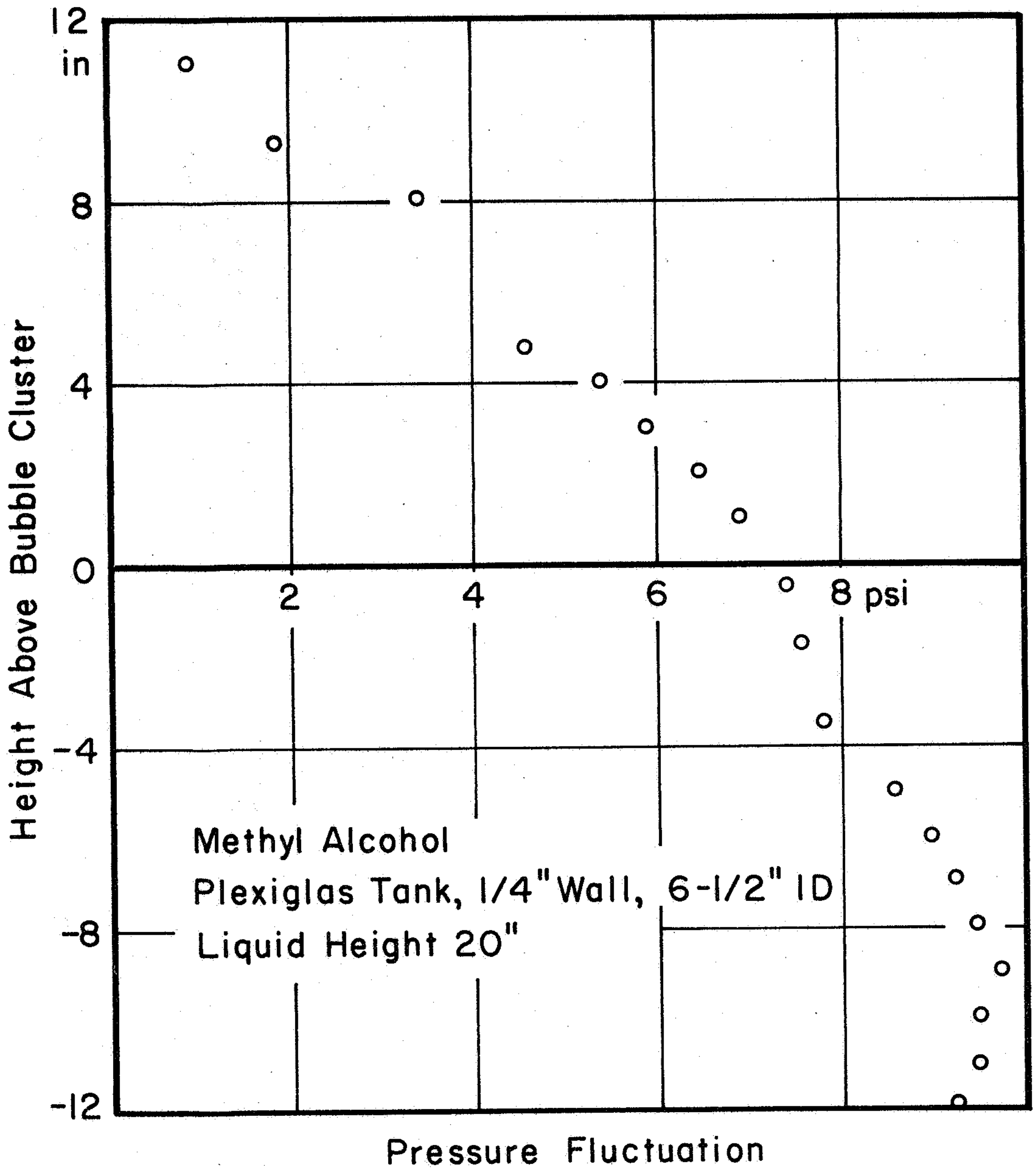

Figure 14. Distribution of Peak-to-Peak Pressure Fluctuations Along Tank Axis

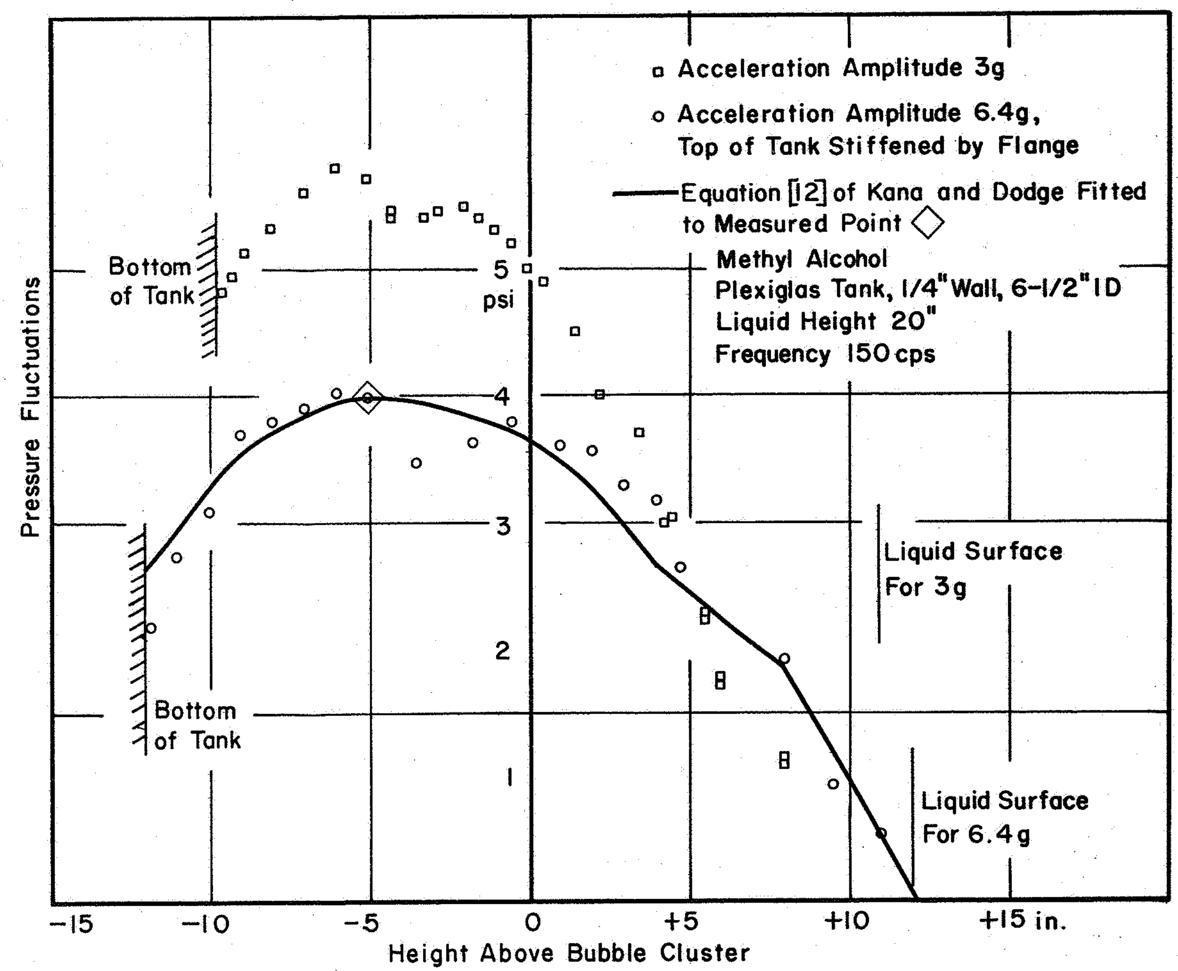

Figure 15. Distribution of Peak-to-Peak Pressure Fluctuations Along Tank Centerline Caused by Relative Motion, and Effect of Tank Construction

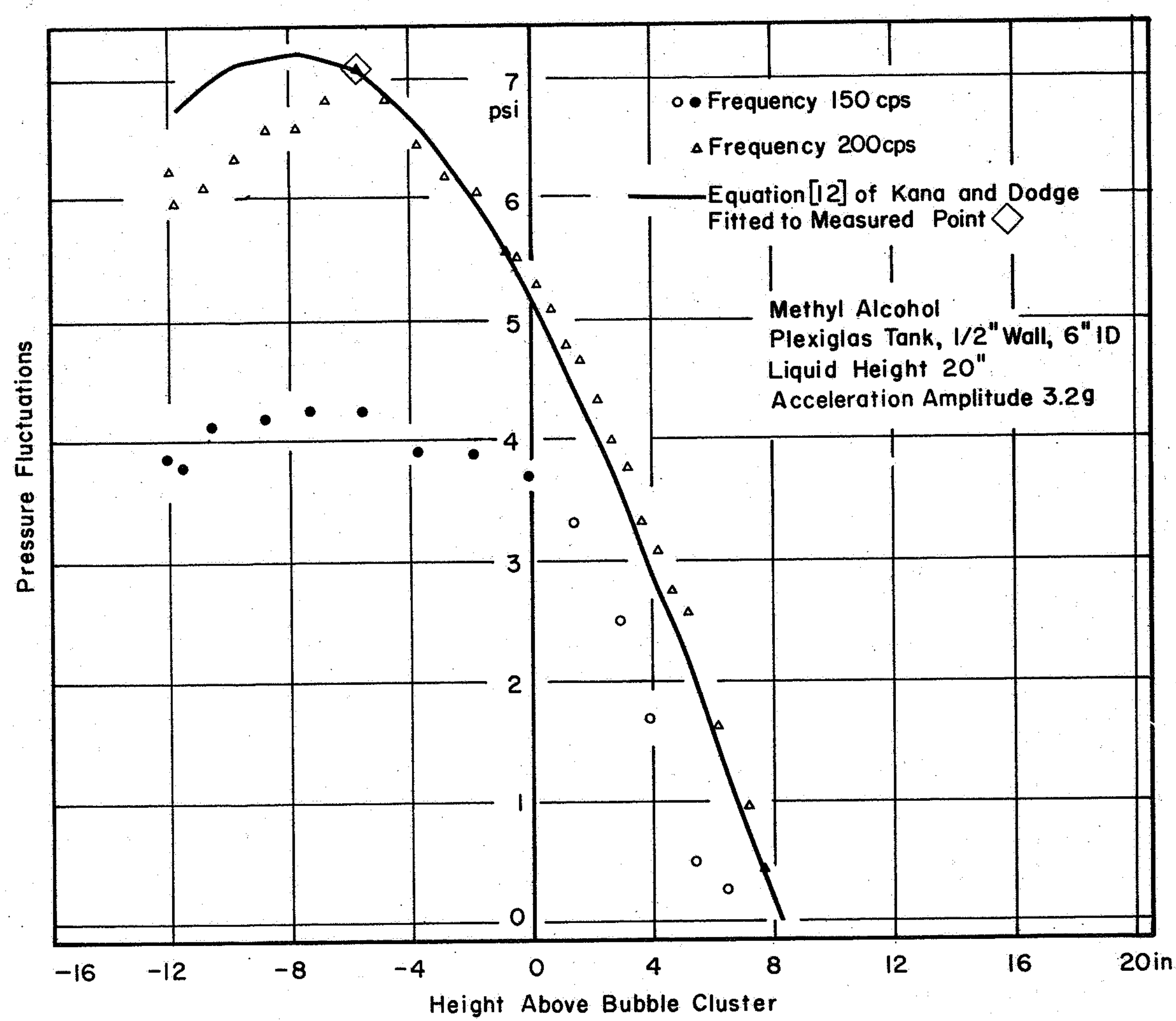

Figure 16. Distribution of Peak-to-Peak Pressure Fluctuations Along Tank Centerline Caused by Relative Motion

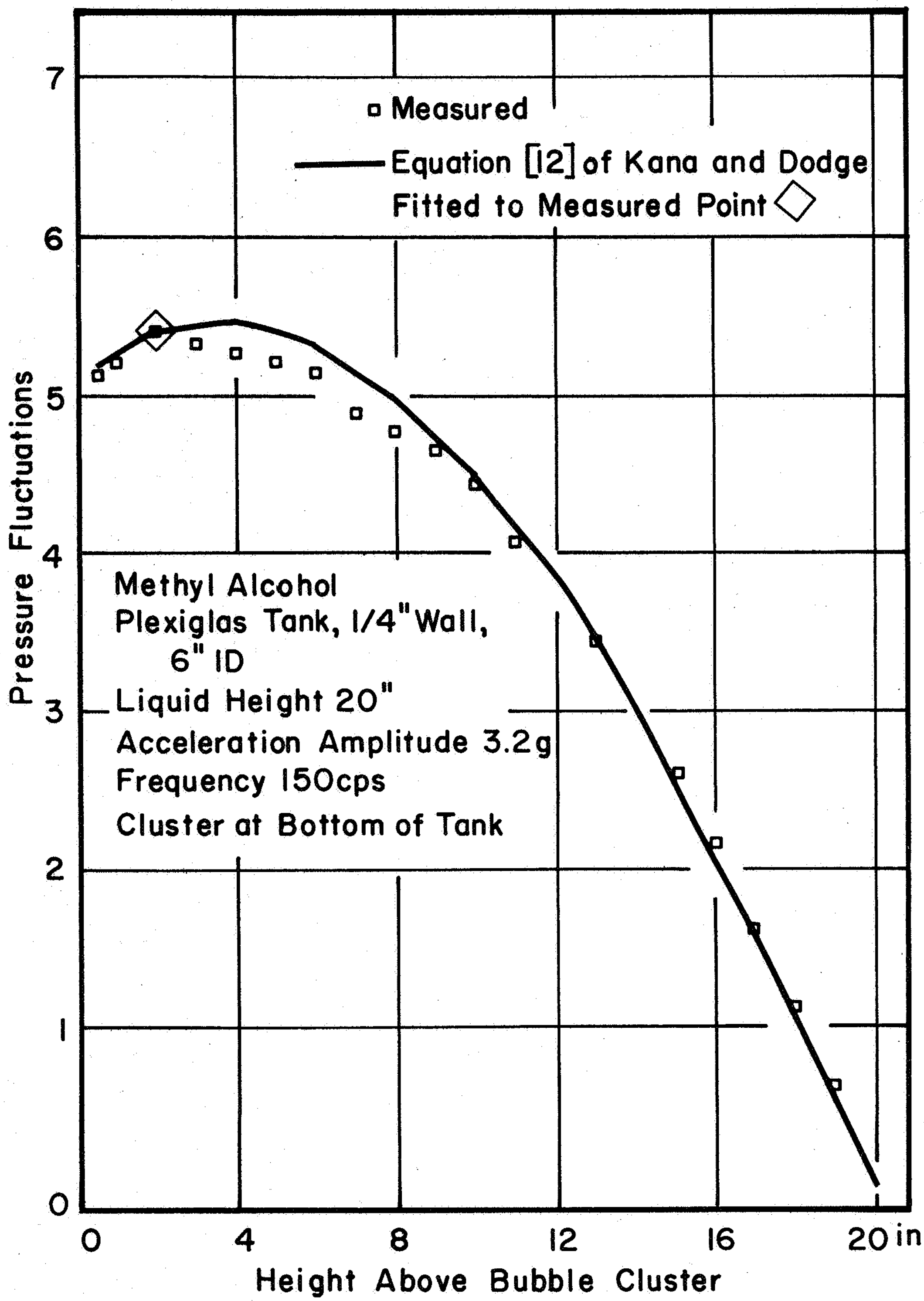

Figure 17. Distribution of Peak-to-Peak Pressure Fluctuations Along Tank Centerline Caused by Relative Motion

The abscissa in Figures 15, 16 and 17 gives the position of the probe measured from the level of the cluster. For reference, the relative positions of the tank bottom and the liquid surface were also marked in Figure 15. The lines for the surface give the latter's positions measured from the cluster at the time when the probe was at the highest measured location, and the lines for the bottom give its positions relative to the cluster when the probe was moved to its lowest measured location. Although in all experiments the liquid column height was 20 inches, the two lines representing the surface do not coincide with each other because the cluster drifted somewhat during the runs and from run to run. The same applies for the lines indicating the tank bottom.

It was possible to compare some of the experimental results with theoretical calculations. The theory on which these calculations were based was developed by Kana and Dodge (1964) and the results are included in Figures 15, 16 and 17 in the form of solid lines. For the calculation of these curves, the time independent part of Equation [12]* of Kana and Dodge was used. Equation [12] is

$$\bar{p}(z,t) = -\rho c \omega x_o \frac{\sin \frac{\omega}{c} z}{\cos \frac{\omega}{c} \ell} \cos \omega t$$

in which $\bar{p}$ = dynamic pressure,

z = distance below surface,

*Equation numbers shown in brackets refer to equations in the literature reference cited.

t = time,

x_o = amplitude of tank displacement,

c = wave velocity,

ℓ = length of liquid column.

The unknown constant c in this equation was determined by substituting a measured value for the pressure $\bar{p}$ and the depth z where the pressure was measured. In each experiment the maximum dynamic pressure and the corresponding depth were selected for substitution in order to make the calculated curve through the highest experimental point plotted in the diagrams. In this way good agreement between experiment and theory could be achieved as far as the pressure distribution was concerned.

The values of c were also calculated with physical dimensions and properties of the tank, liquid and air (Equation [7] of Kana and Dodge). Results of the two methods are compared in Table I.

It was attempted to calculate the pressure distribution also with the values of c derived from Equation [7], i.e. using c = 917 ft/sec instead of c = 652 ft/sec for Tank 1. This, however, gave pressures which were too large by an order of magnitude. The values for c were also used to calculate the location of the cluster by means of Equation [18] of Kana and Dodge and the results were compared with the measured locations. As an example, during the experiment with Tank 1 at 150 cps referred to above in this paragraph and also represented in Figure 15, the cluster was 10 inches below the surface. Equation [18], however, had no realistic solution with c = 652 ft/sec and gave the result 17.5 inches below the surface with the calculated velocity c = 917 ft/sec.

TABLE I

Comparison of Experimental and Calculated Values of Wave Velocity in Vibrating Liquid Columns

Specifications: Tank 1: Plexiglas;

Modulus of elasticity 4.5×10^5 psi

I.D. 6-1/2 inch

O.D. 7 inch

Tank 2: Same as Tank 1 except I.D. 6 inch

Liquid: Methyl Alcohol;

Density 1.57 slugs/ft^3

Compressibility at 20°C 5.825×10^{-8} ft^2/lb.

Estimated volume ratio (cluster to total) for

Tank 1: 7.88×10^{-4}

Tank 2: 9.26×10^{-4}

	c from Equ. [7]	c from measured data and Equ. [12]	frequency
Tank 1	917 ft/sec.	652 ft/sec. cluster 10-12" above bottom.	150 cps
Tank 2	1042 ft/sec.	1085 ft/sec. cluster 11" above bottom	200 cps
Tank 2		813 ft/sec. cluster at bottom	150 cps

When Equation [18] was solved for c using the measured cluster location of 10 inches, the result was between the two earlier values, namely, c = 814 ft/sec. This c resulted with Equation [12] in a calculated pressure distribution which lay between the distributions corresponding to c = 652 ft/sec and c = 917 ft/sec and was about three times higher than the measured results.

In summary, Equation [12] of Kana and Dodge gave good approximation for the pressure distribution if the constant c was adjusted in such a way that the equation was satisfied when the highest measured value for the pressure and the corresponding depth below surface were substituted. Then, however, the calculated location of the bubble from Equation [18] became unrealistic (above surface level). If c was adjusted to give the right bubble location (with Equation [18]), then the pressure distribution, according to Equation [12], was off by a factor of 3.

Independent determination of c from physical properties of tank and liquid resulted with Equations [12] and [18] in pressures which were much too large, by an order of magnitude, and a bubble location off by a factor of 175 per cent.

These discrepancies are not too surprising, since in the derivation of these equations Kana and Dodge assumed that the gas is uniformly distributed in the liquid and that is very different from the situation of the present experiments in which almost all the bubbles were concentrated in one cluster.

For a final comparison, the equilibrium bubble location was also calculated with Bleich's Equation [25] (same as Kana and Dodge: Equation [3]). It gave 41 inches below surface which means 21 inches below tank bottom.

In the curves outlined by the measured points, in Figures 15 and 16, a dip can be observed slightly below the cluster. The cause of this characteristic is not clear, one could only determine that it varies with tank wall thickness and that it can be correlated with some other experimental observations.

Upon examination of the pressure distributions in Figure 15 and 16, one finds that the local maximum and minimum below the cluster are more pronounced in the thin-walled tank than in the tank with the thicker wall. In correlation with this, it was observed that the cluster was much more stable at its location half way between top and bottom of the tank in the thin-walled tank than in the tank with the thicker wall. In the latter tank, the cluster drifted a lot during the experiments and it also easily dropped to the bottom of the tank.

A similar change in behavior was noticed in the same tank when the frequency was changed. In Figure 16 one set of points was obtained at 150 cycles/sec., the other at 200 cycles/sec. Although the row of points for 200 cps does not exhibit a local minimum, it still has a pronounced inversion or maximum near below the cluster. Correspondingly, the cluster was very stable in its location at this frequency. At the lower frequency, the trend of the points has a very weak maximum and minimum below the cluster and the latter's stability was also weak.

One may than conjecture that the bubble is stabilized by a maximum of the pressure distribution.

These observations are consistent with the theory developed in Section 2.2.1.

A natural question in this connection is whether the structure of the tank uniquely determines the dynamic pressure distribution. It seems that it does not. For evidence, one can take the points in Figure 17 and compare them with the points for 150 cycles/sec. in Figure 16. In these two experiments, all conditions were the same except the location of the cluster which was at the bottom of the tank in the experiment of Figure 17, while it was 12 to 16 inches above the bottom during the experiment of Figure 16. In these two cases, the pressure distributions are definitely different and the maxima of the curves vary according to the location of the cluster.

It seems that the dynamic pressure distribution is generated by a strong feedback effect of the cluster and the tank properties constitute only certain dynamic constraints. If this is true, then small bubble analyses, like those of Bleich, Kana and Dodge, Jameson and Davidson, etc., which assume that the bubble does not change the pressure field in the liquid, cannot be applied to fully-developed bubble clusters.

Interpreted in the light of the above discussions and more observations on bubbles and clusters, the birth of a cluster occurs as follows. When the tank vibrates with such frequency and amplitude that bubbles form and the smallest ones are stationary or move downward, but as soon as they grow a little they rise to the surface, bubbles are distributed along the wall in a statistically random manner but with somewhat higher density around certain preferred sites. The locations of these sites seem to be functions of the tank structure and the table motion, including frequency and amplitude.

Each bubble has a small feedback effect on the wall but singly they cause negligible changes. Occasionally, however, a number of small bubbles happen to be so close to each other that their ranges of influence overlap. In such a case, their combined effects on the pressure field directly and by feedback through the wall may add up to an intensity which is sufficient to affect larger bubbles, which normally would rise to the surface, in such a way that they stop or even move towards the group of small bubbles and merge with it.

At this stage, the agglomeration of bubbles exhibits some fatures of clusters. Each individual bubble vibrates vigorously and they churn around each other. Together they could be called a primitive or quasi cluster.

With the arrival of each additional bubble, the agitation becomes more vigorous and with it also the effects on the pressure field and the wall. Soon bubbles stream in continuously from larger and larger distances and the quasi cluster rapidly develops into a full cluster.

Thus, the birth of a cluster from existing bubbles is a chance event triggered by the accumulation of a critical amount of bubbles.

As the cluster approaches its final stage, the pressure and acceleration amplitudes grow towards their resonance values.

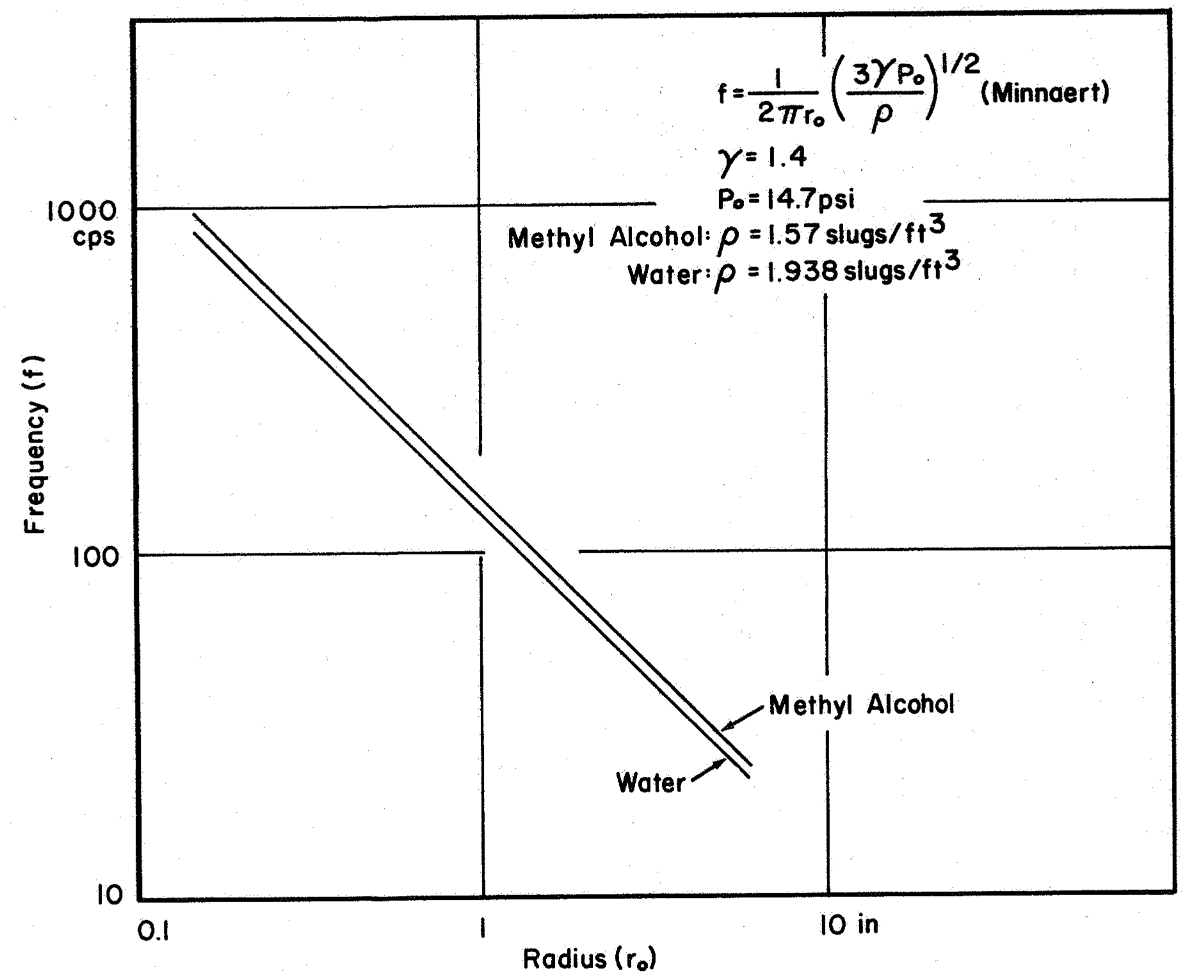

Figure 18. Resonant Frequency of Small Bubbles

Simultaneously their phases shift with respect to each other (see later in this section). Such a phase shift, however, means, according to Equation 2.2.1-37, that the force holding the cluster down decreases, and thus the cluster slowly begins to rise. With decreasing depth, however, the resonant frequency increases, and the phase shift together with the amplitude of the pressure fluctuations decrease. This has two consequences. On the one hand, the cluster begins to lose bubbles and stops growing. On the other hand, reduction of the phase angle increases the downward force. If this opposing trend is strong, it either fixes the cluster at a stable equilibrium location or causes it to move periodically up and down. If it is not sufficiently strong to achieve this, the cluster will slowly continue to grow and rise, and eventually will vent to the surface.

As it was pointed out, growth of a cluster is associated with an increase of pressure fluctuations. One would then expect that such intensified oscillations increase the probability of bubble and cluster formation everywhere in the tank. Indeed, this seems to be the case, especially in water, where one can see several growing satellite clusters, most of which, however, fall into the parent cluster before they could reach full development. In liquids where such additional small clusters do not appear, the reason apparently is that the small individual bubbles stream to the cluster too fast to permit formation of a new primitive cluster. Still it is very frequent that two or three clusters manage to develop. If they are powerful enough to effectively distort the local wall oscillations and pressure field and thus establish the conditions for their own stabilization, they can remain and even successfully compete with the original cluster by attracting and absorbing it.

If the probability of bubble formation increases too much, the liquid transforms into a foamy mixture and the process may collapse.

In regard to Figure 15 a further remark is in order. Here two sequences of measured points have been plotted which deviate from each other considerably although the experimental conditions were almost identical, the only difference, as far as known, being that the rim of the Plexiglas tank was stiffened by a flange in one of the runs and detached from the flange in the other. This further demonstrates the importance of the mechanical characteristics of the container.

The effect of detachment of the flange was so great on the fluid behavior that the acceleration amplitude had to be reduced from 6.4g, the value it had when the flange was attached, to 3g in order to be able to retain the cluster at the same approximate level in the second experiment as in the first one.

As indicated earlier, the phase shift of the pressure oscillations in relation to the oscillations of the tank accelerations was also measured along with the magnitudes. This was possible because the oscilloscope displayed both signals simultaneously. The phase shift was obtained by measuring the distance between maxima or other corresponding points. This is not a very accurate method and, therefore, only a summary of the results and some conclusions will be listed at this place. More accurate measurements with the proper instruments are planned for future experiments.

In a typical experiment at frequencies between 150 and 179 cps a cluster stabilized at about 11 inches above the bottom of the tank, and it had a core of about 1 inch in diameter. When the pressure probe was inserted into the center of the cluster, the pressure wave shown on the screen was strongly distorted from sinusoidal and so its phase angle could not be determined accurately. The results obtained indicated a lag behind the tank acceleration of either slightly more or somewhat less than 90°. The pressure lagged behind the acceleration at every point in the tank where it was measured and at the boundary of the cluster the lag was 80.5°. By the time the transducer reached the center of the tank along a diagonal, the phase shift was reduced to between 54° and 58°, and at the opposite end of the diagonal the phase shift was 51°.

Along the centerline of the tank, the main changes of the phase angle were restricted to a few cluster radii above and below the level of the cluster. Above that region, the relative phase angle was 37.8° and below it the phase lag stayed constant at 10.8° to within 1/8 inch from the tank bottom.

Two tentative conclusions can be made even on the basis of the presently available results. One is based on the fact that most of the liquid above the cluster is only moderately out of phase with the acceleration while the phase lag of the cluster itself is 90°. This circumstance suggests that the resonating system comprises not only the pulsating cluster and a swinging mass of the liquid but also at least some portion of the tank wall. Otherwise continuity of the liquid would be violated. The second conclusion is that these results support the assumption of the model for the liquid motion which was adopted in Section 2.2.1. There, when the effect of finite tank dimensions on the net displacement of the bubble was calculated, it was assumed that the liquid moves essentially as if it consisted of two parts, one on one side of the bubble moving with one velocity and another one on the other side moving with another velocity, the cluster being in the middle compensating for the difference. Although the above results do not prove that this is necessarily so, they are consistent with this hypothesis. The experimentally established pattern, namely that the phase angle is constant below the cluster and almost constant but different above it, is what one would expect for such a model. Of course, velocity measurements would be necessary to verify these conjectures.

3.3.5 Observations Relating to Origin of Bubbles in Vibrating Liquid Columns

Little has been said until now about how bubbles get into the liquid. In the discussion of this, one has to distinguish between two basically different origins of bubbles. One mechanism by which bubbles are introduced into the liquid is surface turbulence. If the amplitude of the tank oscillations is sufficiently large, the liquid surface becomes unstable and it breaks up in an irregular motion similar to turbulence. In this state, local peaks of the surface may break or separate into droplets by surface tension and be thrown into the air. When they fall back into the liquid, they carry minute amounts of air with them. These small quantities of air form tiny, even non-visible, bubbles and, if the oscillation amplitude is sufficiently large, they remain in the liquid and move downward. Once such a small bubble is present in the vibrating liquid, it apparently encourages gas separation, since it grows visibly on its way down. Growth is also caused by the attraction and coalescence of the synchronously pulsating small bubbles. This description of bubble entrainment suggests that it depends on all the parameters which control the behavior of surfaces.

As expected, the surface of Meriam fluid No. 3, with viscosity of about 23.6 centipoise, remained smooth even at high acceleration amplitudes and also no bubbles could be observed in the liquid. Finally, bubbles were generated by another mechanism from the inside of the liquid at 16.8 g's and 210 cycles/sec, but still no entrainment could be observed at the surface.

The above described mechanism of entrainment accounts also for the observation that bubbles begin to appear at smaller tank oscillations in a thin-walled tank than in a tank with a thicker wall. Because of its lesser rigidity, the thinner wall vibrates with larger amplitudes and shakes the surface more. This observation was made with two tanks having 1/4 inch and 1/2 inch thick walls, respectively.

The other mechanism by which bubbles are produced in the oscillating liquid is nucleation. This phenomenon seems to be very similar to nucleation in boiling. It appears that their dependence on temperature and pressure is similar and also some of their other characteristics. One such characteristic is that small local irregularities of solid boundaries are preferred sites for bubble formation. One can see, at times, a meandering row of fine bubbles, which seem to rise persistently out of a fixed point on the bottom surface of the tank. If the liquid is disturbed, the row of bubbles is carried away but the new bubbles again appear above the original spot. This is similar to what happens in boiling.

The first, hardly visible, bubbles appear about 4-5 mm above the bottom and they grow slowly as they rise. Their spacing is very regular and the manner in which they move and grow strongly suggests that the row starts all the way from the bottom surface but the bubbles are too small to be seen there. This phenomenon is probably related to the following observation.

One of the Plexiglas tanks was constructed in such a way that there was a circular groove (0.025 inch wide and about 1/4 inch deep) in the bottom plate along the inside surface of the wall. This groove was filled intermittently with a plastic solution so that small parts of the groove remained. During tank vibration these pockets very soon contained a small pulsating bubble. These bubbles could be seen there most of the time of vibration and when a cluster moved near one of these pockets, bubbles started streaming out of it in great numbers. It is known that bubbles are generated by cracks in boiling.

Bubbles may appear also anywhere else along the wall and they may or may not adhere to it. As was noted in an earlier paragraph, the inception of bubbles is affected also by the gas concentration in the liquid. As the concentration decreases, the inception of bubbles is more and more delayed. This dependence suggests, among others, that the incipient bubbles consist mainly of the dissolved gases rather than of vapor.

As it could be predicted, bubbles formed more readily when the liquid was at a higher temperature than when it was cooled.

With the equipment presently available, the pressure above the liquid could not be varied, still a way was found to probe dependence of bubble formation inside the liquid on pressure. For this purpose, two experiments were performed. In one the entirely liquid-filled tank was covered with a thin plastic membrane. In the second, it was closed up with a one-inch thick rigid plate, taking care that no air bubbles were left inside.

In the first case, the top of the liquid was always at atmospheric pressure, but the pressure dropped below this at the bottom of the tank every time the tank accelerated downward. In this particular experiment, bubbles formed at 3g acceleration amplitude.

When the top of the liquid was in contact with the rigid lid firmly attached to the tank wall, the pressure distribution became fundamentally different. During the downward acceleration, the top of the liquid was no more at atmospheric pressure, but almost at the pressure which prevailed at the bottom during the upward acceleration. This was caused by the rigid cover which now pushed the liquid downward. Thus the pressure never dropped below atmospheric, which existed when the tank was at rest.

With this configuration, the acceleration amplitude had to be increased to approximately 25g before bubbles appeared. At this point, probably compressibility of the liquid and elasticity of the tank had already a sizeable effect on the pressure variations. One is led to conclude that increased pressure impedes or even inhibits nucleation.

The experiment with the rigid tank top provided also supporting evidence for the validity of the theoretical model applied in Section 2.2.1. There it was pointed out that the net motion of bubbles toward the bottom is to be attributed to the difference in the boundary condition at the top of the liquid and at the bottom. Now, with the thick lid fastened, the boundary conditions became identical at the two ends of the tank and from a dynamical point of view only the action of gravity remained asymmetrical with respect to the oscillatory motion.

Under such conditions, all effects but that of gravity are canceled and bubbles are expected to behave like in containers at rest. The supposition was verified by the fact that bubbles and clusters went immediately to the top whenever they formed even at 25g acceleration.

3.3.6 Effect of Vibration Exciter System Characteristics on Bubble Behavior

Several times in this section, it was mentioned that the vibration exciter system had an important role in influencing the behavior of the liquid and the bubbles. For instance, it was noted that the sudden explosive development of clusters under certain circumstances was due to the response of the exciter table. From the point of view of the exciter system, the tank with its liquid content is an impedance which changes its characteristics as the bubble develops. Since the mass of the exciter table (89 lb) was comparable to that of the full tank (35 lb), frequency characteristics of the tank and liquid had a strong effect on the frequency characteristics of the combined exciter-tank system and thus changes inside the tank, like formation of a cluster, strongly altered the motion of the whole system.

In order to get acquainted with some of the dynamic properties of the whole vibration system, a few experiments were performed. The results are shown in Figure 19. The experiment consisted of varying the frequency and observing the changes in amplitude. In these experiments the table had various types of loads but each had the same weight, 35 lb. The amplitudes of the table oscillations were set very small, about 0.6g or less, much smaller than necessary for bubble formation or even to cause visible agitation of the liquid surface in order to eliminate those phenomena from interfering with the table motion.

One of the loads was a steel block. Because of its rigidity, it did not change the frequency characteristics of the table to any notable extent, as can be seen from Figure 19. The other extreme type of load was mercury filled into pliable plastic bottles. These bottles readily deformed and absorbed the vibrations of the table by extending in the region near their bottoms hardly forcing the bulk of the mercury

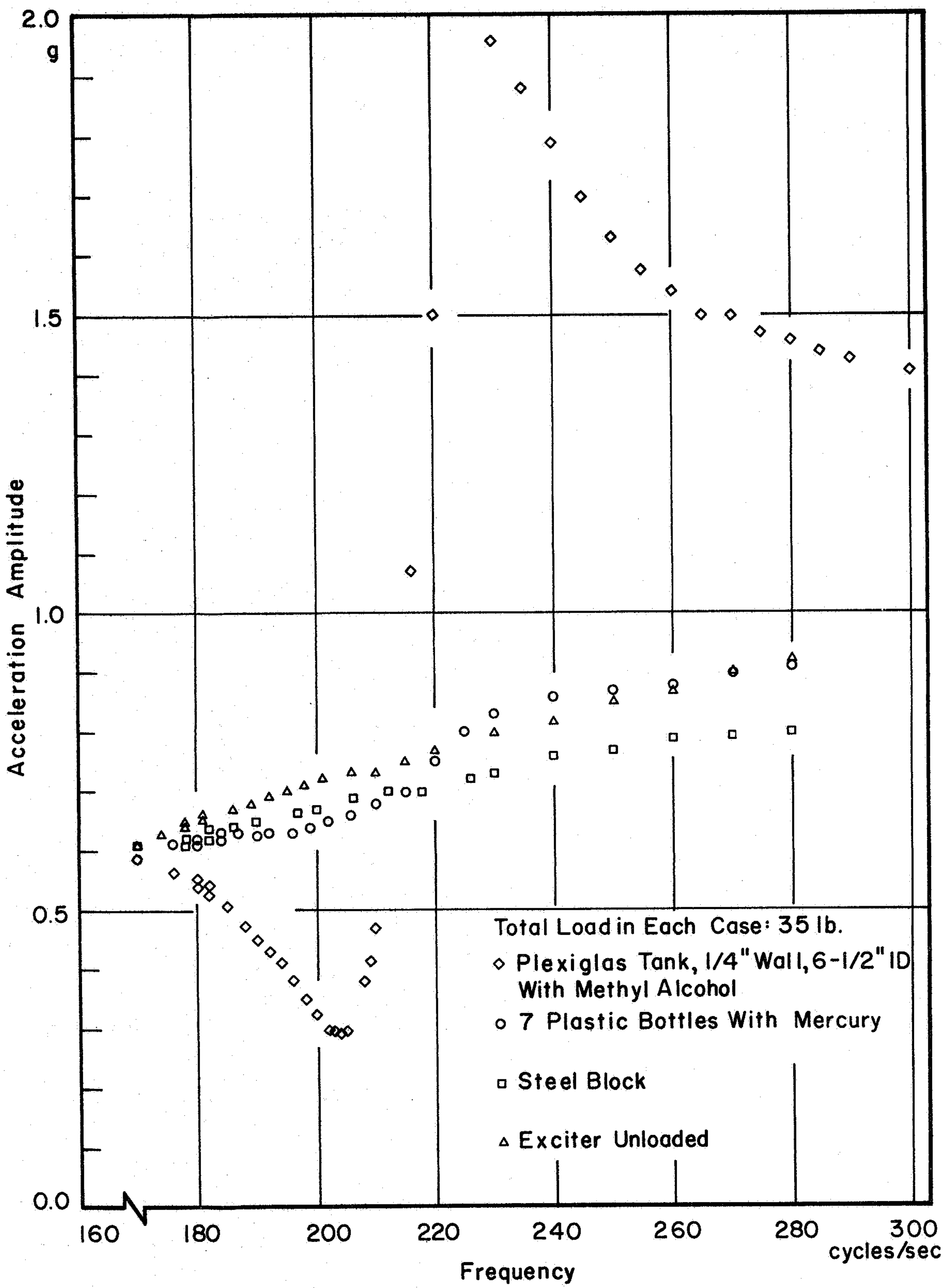

Figure 19. Frequency Characteristic of Vibration Exciter

to move. Consequently, the dynamic effect on the table was almost as if there had been no load at all. For comparison, the table was also tested without load. The results of both of these experiments are also plotted in Figure 19.

Finally, the frequency response was examined with the liquid-filled tank. The plotted points show how strongly and abruptly the oscillation amplitude changed with changing frequency. Several similar but smaller jumps occurred also at higher frequencies (beyond the range of the diagram).

The exact relationships determining this behavior are unknown and, therefore, the effects could not be accounted for properly in the experimental results. It was, however, noticed that the critical frequency at which the big jump occurred changed from day to day and sometimes even during an experimental run and with it changed noticeably some results. For illustration of such effects, in Figures 6 to 8 some of the repeated results were also plotted. For instance, in Figure 6, there are two curves for 176 cycles/sec. These results were obtained on two successive days without any change of the apparatus. The divergence of these two curves and also some inconsistencies in the succession of the other curves in these diagrams are suspected to be connected to changes of the frequency characteristics of the system. It is, however, not clear whether the changes occur in the vibration exciter or in the tank and whether the differences in the results are caused by the changed frequency characteristics or both are caused by variations of some other factors. It is also not known what parameters are effective. Based on records, it seems that the temperature has a certain influence.

Finally, it may be mentioned that the response of the vibration table was so sensitive even at an acceleration amplitude as small as 0.3g that slight amplitude differences could be measured when the 2-inch thick base of the tank was fastened to the table with 2, 4 or 8 bolts, respectively.

4.0 CONCLUSIONS AND RECOMMENDATIONS

Theoretical investigations of the behavior of oscillated viscous and non-viscous liquid columns led to the following conclusions:

1. The present equations for non-viscous liquids explain qualitatively observed behavior of bubbles and clusters even for clusters the sizes of which are comparable to the container cross section. The motion of bubbles and clusters depends on the amplitudes and relative phase angles of the fluctuations of tank and bubble velocities and the pressure.

2. In order to be able to predict liquid behavior correctly when large clusters are present, the theory had to be developed with fewer restrictions than it was done in previous literature.

3. Because of the required generality of the theory, the response of the solid structure has to be solved simultaneously with the liquid motion. Present knowledge of the role of container deformations is not sufficient to furnish the necessary information.

4. The viscous theory predicts the deviations of the bubble stabilization criterion from inviscid flow theories and agrees in general with experimental results. The theory also establishes the need to consider the effects of surface tension.

5. Further extension of the theory is required to predict certain observed characteristics of the cluster development and motion, and of the pressure distribution in the liquid not covered by existing theories.

Experimental results permit the following conclusions:

1. The liquid motion depends on a large number of parameters, and within the range of variations of these parameters several domains with largely different liquid behavior can be separated.

2. Generation of bubbles and the development of clusters strongly depend on oscillations frequency and amplitude, structure of wall surfaces, density and viscosity of the liquid, and concentration of dissolved gases.

At a constant frequency the time required to develop a cluster increases as the oscillation amplitude decreases. The curve approaches asymptotically a minimum required amplitude for formation of a cluster. This minimum amplitude varies with frequency and it decreases at first as the frequency increases, but it increases again beyond a critical frequency.

Reduction of gas concentration in the liquid increases the time required to form bubbles inside the liquid, indicating that the decisive component in the bubble content is gas separated from solution.

3. The deformations of the container have a crucial effect on the development and stabilization of clusters, and the clusters in their turn affect the dynamic deformations of the tank. Because of this feedback effect, the usual "small bubble" assumptions are not adequate as demonstrated by measurements of dynamic pressure distributions.

As a consequence, also, the usual assumption that the bubble pulsates in phase with the tank oscillations does not agree with the measurements of phase angle between pressure and tank acceleration when a cluster is present.

4. Clusters and the surrounding liquid are in a state of resonance and the cluster size is approximately that of the resonant bubble size predicted by theory.

5. Theories assuming uniform distribution of bubbles in the liquid predict pressure distribution and stable bubble location only with limited agreement with experimental results when applied to clusters.

6. Motion of the vibration exciter table is affected by the development of clusters in the tank with the result that the liquid motion is considerably altered to a yet not fully known degree.

Based on the above conclusions, the following recommendations can be made.

For calculations of the liquid and cluster motions, further development of the theory is required.

1. In particular, theories are needed for the detailed calculation of liquid velocity, pressure and tank deformation fluctuations in order to solve the equations for the motion of bubbles and clusters and to determine their stability.

2. The development of a theory of viscous fluids has to be further pursued.

3. There is no theory available for the inception and growth of bubbles in oscillating liquids.

For the experimental investigations of oscillating liquid columns, the following may be recommended:

1. Experiments of the present investigations should be continued over extended ranges of parameters in order to complete the understanding of the mechanisms involved. But in the arrangement of these experiments, it should be attempted to separate the effects of certain factors in order to render the effects of other factors clearer. Ways to achieve this could be the following:

1.1. By performing some experiments with a heavier vibration exciter or with one electronically controlled and possibly by using smaller tanks, distortions of the table motion due to forming clusters and feedback effects of such distortions on the fluid behavior could be eliminated.

1.2. Construction of very rigid tanks would permit investigation of the mechanisms of the fluid alone.

2. Since all experiments were conducted at the same liquid height, experimental information on the stabilization of clusters and response of tank deformation in function of liquid depth is very limited and requires fuller exploration by varying the tank length.

3. It was noted that the structural properties of the container have crucial effects on the liquid motion. For this reason, a major effort is needed to investigate the dynamic responses of oscillating containers filled with a liquid-gas mixture.

4. It will probably require a major experimental effort to establish the mechanism of bubble nucleation in oscillating liquids. The only available information associated with this problem refers to nucleation in boiling and even this information is not complete.

5. Development of new techniques and equipment.

5.1. For the detailed and sufficiently accurate mapping of the instantaneous pressure field inside the entire tank analysis of the spectrum of the pressure with a wave analyzer is required in the absence of bubbles and clusters and under conditions when bubbles and clusters are present.

5.2. Equally needed is the analysis of the liquid velocity both near and far from bubbles and clusters. For these measurements new methods have to be developed and results would yield information on resonant and turbulent conditions.

5.3. It is also recommended to explore the flow field with the technique of birefringent liquids in narrow tanks with essentially two-dimensional liquid motion.

6. Based on previous experimental evidence and theoretical results, viscosity and surface tension play an important role in the behavior of liquids and bubbles. It is suggested that special attention be paid to the effects of these properties on the entrainment, motion and stabilization of bubbles.

7. Experiments with controlled bubble sizes would be required to understand the motion and effects of individual bubbles and to

verify theoretical results. For similar purposes experiments with small balloons, singly or in larger numbers and various distributions, should be performed.

8. Finally, the suppression of clusters might be investigated by inserting compressible elements, e.g. lining, into the oscillating tank.

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