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# AN INVESTIGATION OF A GRAY, OPTICALLY THICK PLANETARY ATMOSPHERE IN CONVECTIVE - RADIATIVE EQUILIBRIUM

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# AN INVESTIGATION OF A GRAY, OPTICALLY THICK PLANETARY ATMOSPHERE IN CONVECTIVE - RADIATIVE EQUILIBRIUM

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ABSTRACT

Optically thick planetary atmospheres in radiative-convective equilibrium have been investigated by the use of models which have an adiabatic temperature gradient in the troposphere. The gray stratospheric solution employs the discrete ordinate method to match correctly the tropopause boundary condition in which the tropopause location is found by temperature continuity. The models show that convection may increase the surface gray infrared opacity requirements by as much as an order of magnitude in greenhouse type theories as applied, for example, to Venus.

A more approximate, analytical solution is also presented which generally agrees with the more elaborate treatment quite well. One of the major differences between the two types of solutions is the temperature gradient discontinuity at the tropopause predicted by the more exact theory.

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# I. INTRODUCTION

The study of the greenhouse effect on planets such as Venus and Jupiter requires the knowledge of optically thick planetary atmospheres. Almost all of the calculations performed for optically thick planetary atmospheres have assumed radiative equilibrium (for a purely convective calculation, see Sagan and Pollack, 1964). This assumption allows a simple and theoretically concise problem and the solution affords much insight into the structure of the atmosphere. The Earth, however, has an atmosphere that is optically thin and generally in convective equilibrium. Other planets may possess optically thick convective type atmospheres. Hence it seems desirable to see under what conditions an optically thick atmosphere is convective and what are the effects of such convection on atmospheric behavior.

For the Venus problem, Teiger (1965) has constructed radiative-convective model atmospheres which are optically thick. Teiger's gray opacity dependence on pressure followed the dependence empirically observed in the Earth's atmosphere; this dependence is based almost entirely on the variable concentration of water vapor with height. Such opacity variations may not be appropriate for a planet such as Venus. More general opacity dependences are thus needed.

As a step towards understanding convective atmospheres, the present paper will investigate the behavior of gray, optically thick atmospheric models in radiative-convective equilibrium when changes in a reasonable infrared opacity dependence on pressure are made. The use of gray opacities allows a minimum of parameters to be used in the calculations. Hence an opportunity is afforded for greater physical insight into convective planetary atmospheres and comparisons with analytic solutions are possible. This is a logical step to take before the non-gray transfer problem is solved.

## II. PHYSICAL CONCEPTS AND ASSUMPTIONS

In the absence of convection an atmosphere transporting heat flux upwards into space would be in radiative equilibrium since all gases are very poor conductors of heat. Thus the atmospheric structure would be determined by the distribution of opacity (i.e. by the opacity pressure dependence) and by the infrared radiative flux, which, for example, would be constant in a conservative atmosphere. As emphasized by Goody (1964), a convective layer will always form above the planetary surface. The height of this convective layer will depend on the stability of the bulk of the atmosphere to convection. A radiative-convective atmosphere thus occurs. The radiative layer which occupies the upper part of the atmosphere, the convective layer above the surface, and the transition region between the two layers are usually referred to as the stratosphere, troposphere and tropopause, respectively.

The boundary conditions that prevail at a radiative-convective interface are 1.) continuity of the net radiative flux or, equivalently, of the radiative intensity (from the radiative transfer equation) and 2.) temperature continuity (from the convective stability requirement). These two conditions are necessary and sufficient conditions for the solution of the stratosphere and for the location of the tropopause in the atmosphere. Thus the tropospheric temperature profile can also be determined uniquely if an adiabatic troposphere temperature gradient is assumed. These boundary conditions have proved effective for the Earth's atmosphere (Goody, 1964).

In a real atmosphere, solar flux will be absorbed throughout the entire atmosphere as well as at the surface. In order to study convective atmospheres using a minimum number of parameters, the atmosphere is assumed to be conservative. Implications about non-conservative solutions will be presented in the discussion.

Several general assumptions should be stated. Local thermodynamic equilibrium, pure absorption, and time independent solutions are assumed. Diurnal variations of solar flux, latitude effects, and the consequences of global convection are considered averaged. These rather restrictive assumptions still allow local atmospheric structure to be determined although quantitative results must be interpreted carefully due to lack of inclusion of global effects. The atmosphere is treated as plane parallel.

## III. MODEL CONSTRUCTION

The basic strategy employed in the calculations will first be outlined and then discussed in greater detail. The model construction employs the discrete ordinate method to achieve, in the stratospheric solution, constancy of the net radiative flux. In the tropospheric solution, an adiabatic temperature profile is assumed and this profile is used to compute the specific intensities which emerge from the troposphere. These intensities are the boundary conditions that produce an unique stratosphere. The opacity dependence that has been chosen allows a complete solution without prior specification of the surface temperature. The tropopause location is determined by the requirement of temperature continuity.

The stratospheric solution employs the discrete ordinate approximation to the gray radiative transfer equation for a plane parallel, purely absorbing atmosphere. The method and solution have been given by Chandrasekhar. Eight ordinates are used in each hemisphere. The solution to the equation of radiative transfer has been obtained from equation (3-14) of Chandrasekhar (1960), and can be written as:

$$I(\tau,\mu_i) = \frac{\sigma}{\pi} T_s^4 \left[ \sum_{j=1}^{N-1} \left( \frac{L'_j e^{-k_j \tau}}{1 + \mu_i k_j} + \frac{L'_{-j} e^{+k_j \tau}}{1 - \mu_i k_j} \right) + Q' + (\tau + \mu_i) b' \right]$$

$$i = \pm 1, \ldots, N \quad , \qquad (1)$$

where $\sigma$ is the Stefan-Boltzman constant, $T_s$ is the temperature at the surface, ( $\mu_i > 0$ ) are the discrete ordinates (zeroes of the appropriate Legendre polynomials), N is the number of ordinates per hemisphere, and the $k_j$ are given by equation (3-7), Chandrasekhar (1960). The constants $L'_j$ , $L'_{-j}$ , $b'$ , and $Q'$ are the (2N) constants of integration to be determined by the boundary conditions. These are the specific intensities that enter the stratosphere from above and from below: $I(0, -\mu_i) = 0$, representing the lack of thermal radiation from space; and $I(\tau_T , \mu_i)$ representing the specific intensity at the tropopause ( $\tau_T$ ) which originates from the convective troposphere. In solving for the radiative solution, the coefficients of the 2N constants of integration are considered as a matrix. Inversion of this matrix and multiplication by the intensities which come from the convective layer results in a correctly matched radiative solution above the troposphere. Calculation has shown that eight ordinates per hemisphere produce a net radiative flux constancy in the stratosphere of 0.1% or better for all models. Thus the specific intensities, temperature and other atmospheric parameters can be found in the stratosphere as functions of optical depth. For example, the radiative temperature is given as:

$$T_r(\tau) = T_s \left[ Q' + b'\tau + \sum_{j=1}^{N-1} \left( L'_j e^{-k_j \tau} + L'_{-j} e^{k_j \tau} \right) \right]^{1/4} , \qquad (2)$$

where equation (2) has been obtained from equations (11-30, 31) of Chandrasekhar (1960).

The tropospheric temperature profile is determined by the behavior of the mass absorption coefficient and by thermodynamical considerations. The definition and pressure dependence of the mass absorption coefficient K used in the calculations, is:

$$\frac{d\tau}{dz} = - K(p)\,\rho\,(p) = - K_0 \left(\frac{p}{p_0}\right)^{\alpha} \rho\,(p) \quad , \qquad (3)$$

where $p_0$ is a reference parameter, $\alpha$ is the opacity power dependence parameter, and $K_0$ is defined as $K\,(P_0)$. The particular pressure dependence of the mass absorption coefficient was chosen to approximate the Rosseland mean absorption behavior of many gases. The small temperature dependence of the mass absorption coefficient normally observed for gases is further reduced by the slow variation of temperature with pressure in planetary atmospheres. Hence temperature dependence will be ignored in this paper.

In most planetary atmospheres, heat transport by convection is so efficient that the true temperature gradient in a convective region never becomes superadiabatic by any significant amount (for example, see Sagan 1960). Thus the temperature dependence on optical depth will be determined by the adiabatic condition, equation (3), the equation of state (ideal gas), and the equation of hydrostatic equilibrium, and is given by

$$T(\tau) = T_s \, (\tau/\tau_s)^{\iota}; \quad \iota \equiv C_p \, (\alpha + 1) \quad , \tag{4}$$

where $C_p$ is the molar gaseous specific heat at constant pressure in units of R, $\tau_s$ is the surface infrared optical depth and $\iota$ can conveniently be called the instability parameter. The corresponding temperature gradient is equal to:

$$\nabla T \Big|_{\text{adiabatic}} = - \frac{mg}{C_p R} \quad , \tag{5}$$

where m is the mean molecular weight of the gases, g is the local acceleration of gravity, and R is the gas constant. These formulas are only valid in the troposphere.

The equation of transfer in integral form gives the intensities at the top of the troposphere:

$$I(\tau_T, \mu_i) = \frac{\sigma}{\pi} T_s^4 \, e^{-(\tau_s - \tau_T)/\mu_i} + \int_{\tau_T}^{\tau_s} \frac{\sigma}{\pi} T^4(\tau') e^{-(\tau' - \tau_T)/\mu_i} \frac{d\tau'}{\mu_i} \quad , \tag{6}$$

where the first term on the right hand side represents the contribution to the intensity from the surface, assumed here to emit isotropically. The surface reflection term has been neglected. Equations (4) and (6) indicate that $\frac{\sigma}{\pi} T_s^4$ can be factored out of the convective intensity. Thus, in this case, the radiative constants can be obtained without the prior specification of $T_s$. The surface temperature is then determined by the relation

$T_{eff}^4 = 4/3\ b'\ T_s^4$ , obtained from Chandrasekhar (1960), equation (3-39) where $T_{eff}$ is the effective temperature which characterizes the net radiative flux of the stratosphere. The Simpson quadrature used for the above computations was compared to the algebraic equivalent (at $\iota = 4$); error was negligible.

The stratospheric and tropospheric solutions have been determined; however the two solutions must be made consistent with each other by choosing the correct tropopause optical depth. This is accomplished by the convective stability requirement that the convective tropopause temperature as determined by equation (4) be equal to the radiative tropopause temperature as determined by equation (2). An interpolative routine is used for this procedure. Convergence is so rapid that a negligible tropopause temperature discontinuity (0.001 $^{\circ}$K) is obtained without any appreciable sacrifice of computation time.

The calculation of the net radiative flux in the atmosphere is straightforward. The required formula is (Chandrasekhar, 1960, equation (1-98)),

$$F_r(\tau) = 2\int_{\tau}^{\tau_s} \frac{\sigma}{\pi} T^4(\tau')\, E_2(\tau' - \tau)\, d\tau' - 2\int_0^{\tau} \frac{\sigma}{\pi} T^4(\tau')\, E_2(\tau - \tau')\, d\tau' + \frac{\sigma}{\pi} T_s^4\, E_3(\tau_s - \tau) \quad , \tag{7}$$

where $E_N(\tau)$ is the exponential integral function of order N. The term on the far right represents the surface contribution to the net radiative flux. The quadrature used for the above integration is of the gaussian type with divisions and Christoffel numbers (weights which correspond to the divisions) determined for $E_2$ as the weighting function. (For the calculation of these numbers, see Chandrasekhar (1960); the numbers and method used here come from

Mihalas 1966). The quadratures are accurate to one part in $10^4$ (Mihalas 1966). All of the above calculations were performed on the Princeton IBM 7094 computer.

It is interesting to compare the theory outlined above with a simple analytical solution such as is presented in Appendix A. The less complicated solution may often be useful for qualitative results.

Boundary optical depths, temperatures, temperature gradients and net radiative fluxes are the most fundamental of the physical parameters which describe an atmosphere, from the radiative transfer point of view. These quantities depend only on $\iota$ when the surface temperature and effective temperature of the planet are specified. This is seen explicitly in the equations of this section and those of Appendix A. Values of $T_S = 700^\circ K$ and $T_{eff} = 235^\circ K$ have been assumed. These values are consistent with those used by other authors (e.g. see Sagan 1962).

## IV. RESULTS

The results of a number of models are shown in Figures 1-6. Figures 1-3 indicate the effects of changes in the instability parameter and the effects of the transition from a radiative to a convective type atmosphere. Figures 4-6 display atmospheric profiles of models for several different values of the instability parameter.

Surface optical depth and tropopause optical depth are plotted as functions of the instability parameter in Figure 1. The solid lines indicate the results of this paper; the dotted lines indicate the Eddington-Schwarzschild approximation (hereinafter called the E-S approximation). The Eddington solution and the more exact theory both give almost equal surface optical depths for purely radiative atmospheres. This value of surface optical depth is seen to be the minimum surface optical depth. For convective atmospheres, the surface opacity increases and the tropopause opacity decreases as the instability parameter increases. The two solutions behave in a similar fashion, although the E-S approximation has larger surface and tropopause optical depths than the theory of this paper.

Figure 2 displays the surface temperature, tropopause temperature, and boundary temperature for the same models and under the same conditions as in Figure 1. The E-S approximation predicts a fixed boundary temperature that is higher than the almost constant boundary temperature associated with the more exact theory. The rapid change in the tropopause temperature with $\iota$, observed near $\iota = 4$, is a consequence of the rapid change in $\tau_T$ observed in Figure 1 near $\iota = 4$. The more exact theory shows that $T_T$ deviates from $T_s$ before a convective type atmosphere occurs (see insert, Figure 2). This separation is caused by the convective layer just above the surface, which exists even in stable atmospheres because of the surface boundary (see Goody 1964, chapter 8).

In Figure 3 the tropopause radiative temperature gradient, normalized to the adiabatic gradient, and the surface net radiative flux, normalized to the stratospheric flux, are plotted as functions of $\iota$. For radiative type atmospheres, the radiative temperature gradient at the tropopause is sub-adiabatic and decreases linearly to zero as $\iota$ approaches zero. The magnitude of the tropopause temperature gradient discontinuity in this paper's theory increases with an increase in $\iota$.

In radiative type atmospheres, the small convective layer adjacent to the ground produces, in the context of this paper, a decrease in the surface flux at the ground. At this point, the contribution to the net flux that would occur because of hotter layers below is absent. As expected, larger values of $\iota$ are associated with smaller values of surface net radiative flux.

Figure 4 shows temperature atmospheric profiles plotted for three values of the instability parameter: $\iota = 3.5$ which corresponds to a radiative type atmosphere, $\iota = 6$ which corresponds to a moderately convective atmosphere, and $\iota = 8$ which corresponds to a highly convective atmosphere. The difference between the temperature profiles for different values of the instability parameter is apparent. The results of this paper and the E-S approximation agree quite well.

In Figure 5, temperature gradient profiles are shown for the same models as above. The progression from isothermality at the upper boundary to an adiabatic gradient in the convection zone is smooth except for the discontinuity in the temperature gradient at the tropopause associated with the more exact theory. The differences between the two solutions are more apparent in this Figure than in Figure 4.

Net radiative flux profiles for the three models listed above are displayed in Figure 6. The E-S approximation flux curves agree very well with the more exact flux curves in this Figure. The discontinuity seen in the slope of the E-S flux curves at the tropopause is not present in the more exact theory, however. It can be shown that continuity of temperature implies continuity of the first derivative of the net radiative flux across a boundary. Hence the discontinuity in the E-S flux slope at the tropopause is due to the approximation used.

The theory presented in this paper predicts that the surface radiative flux will be smaller than the radiative flux a few optical depths away from the surface; this is consistent with Figure 6. The change in flux near the surface appears discontinuous on the Figure due to the logarithmetic scale. The two solutions match very well in the interior of the atmospheres. This is to be expected since Eddington type approximations are most valid away from outside boundaries.

## DISCUSSION

The most important result derived from the radiative-convective atmospheric models presented in this paper is the large surface infrared opacity requirements in atmospheres whose gaseous constituents produce convective instabilities. In optically thick planetary atmospheres, these instabilities lower the atmospheric temperature gradient and thus cause energy transport from the surface to the top of the atmosphere to occur by convection as well as by radiation. The larger opacity requirement in such atmospheres is a direct result of the more efficient transport of energy by the atmosphere. Only that part of the atmosphere which is transporting energy mostly by convection is able to increase the surface opacity over the radiative minimum surface opacity. Hence it is reasonable to assert that the stability of the upper parts of the atmosphere to convection, or the presence of clouds in only the upper parts of the atmosphere does not alter the surface opacity requirements appreciably. These remarks are borne out by calculations performed with more complex atmospheric models.

The effect of solar energy deposition in the atmosphere can best be understood by referring to Figure 6. For time independent solutions, conservation of energy in the atmosphere demands that the upward net thermal flux just equal the downward solar flux at each atmospheric level. In a purely radiative atmosphere the non-conservative solution that would result when energy deposition is included would increase surface opacity requirements. In a radiative-convective atmosphere, the surface opacity requirements will be increased only if the (non-constant) upward net flux is less than the net infrared radiative flux at that level. In such a case the atmosphere would become radiative at that point and the sub-adiabatic temperature gradients that result would decrease the temperature below. According to Figure 6, however, smaller amounts of atmospheric energy deposition would still leave the convective zone convective; hence no change in the temperature profile or in the surface opacity requirement would occur.

According to Figure 1, gray infrared opacities of over 1000 are needed for an atmosphere whose opacity depends on the square of the pressure. Most optically deep planetary atmospheres will have such an opacity dependence on pressure. In these cases, the opacity dependence will result from the absorption line wings; their dependence follows a square law (cf. Goody 1964, chapter 3). Deeper in the atmosphere, collision-induced transitions may occur; these also have a square law dependence associated with them (cf. Welsh, Crawford and Locke, 1949). This high opacity requirement would mean that, for example, only one part in $10^{400}$ of the radiant energy at the planetary surface can escape to space in order for a Greenhouse effect to work on Venus! It must be emphasized that the above statement applies only to the gray case. Can such high opacities occur on Venus? Although there still is debate about the amount of water in the Cytherean atmosphere, observations (cf. Spinrad, 1962) generally do not allow a sufficient amount of water to achieve the required opacity (Venus 4, however, has reported 1 percent $H_2O$). In addition, Ho, Kaufman and Thaddeus (1966) limit the total surface pressure from consideration of the microwave spectrum. It is probable that, with the above limitation on pressure, an opacity of 1000 cannot be reached by the inclusion of $N_2$ and $CO_2$ alone (Danielson and Solomon, 1966). While dust may play a role in atmospheric absorption, Samuelson (1967) has found dust alone to be insufficient to cause the high surface temperatures observed on Venus (if the correct planetary albedo is used). Non-gray models will probably be required before a satisfactory greenhouse explanation of the Cytherean surface temperature will evolve.

A remark is appropriate concerning the two methods of solving the radiative-convective atmospheres described in this paper. The excellent agreement between the E-S approximation and the theory presented in this paper on most of the atmospheric parameters shows that the use of temperature and temperature gradient continuity at the tropopause and the use of the Eddington approximation for the stratosphere appear justified. This agreement was not fully anticipated before detailed comparison was made as evidenced by the results. The more exact theory, however, produces a temperature gradient discontinuity at the tropopause (see Figures 3 and 5). Observations from airplanes in the Earth's atmosphere indicate a sharp change in the temperature gradient at the Earth's tropopause. Hence, it is believed that non-gray models or allowance for a reasonable mixing-length theory of convection would not result in tropopause temperature gradient continuity.

If the tropopause is optically deep in the atmosphere the temperature gradient discontinuity should become negligible since then the Eddington-type diffusivity relation (see Appendix A) should be valid. This relation requires temperature gradient continuity at a boundary by virtue of net radiant flux continuity at the boundary. Figure 3 indicates that atmospheres with deep tropopauses do have negligible tropopause temperature gradient discontinuities in the more exact theory.

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## APPENDIX A

The Eddington approximation, which is for a gray, plane parallel atmosphere in radiative equilibrium and the Schwarzschild criterion for convective instability can be used to obtain an approximate solution to the radiative-convective atmosphere. The radiative temperature gradient is fundamental to this approximation; it is calculated from equation (3), the derivative of the Eddington approximation and the ideal gas law:

$$\left.\frac{dT}{dz}\right|_{\text{radiative}} = -\frac{3}{16}\,\frac{(\alpha + 1)\,g\,\tau}{R(1/2 + 3/4\,\tau)} \quad . \tag{8}$$

This is to be compared with formula (5) for the adiabatic temperature gradient. Equality of the two gradients allows determination of $\tau_T$ , the tropopause optical depth in this approximation,

$$\left.\begin{aligned} \tau_T &= \frac{8}{3}\cdot\frac{1}{\iota - 4} \qquad (\iota > 4) \\ \tau_T &= \infty \qquad (\iota < 4) \\ \iota &= C_p\,(\alpha + 1) \end{aligned}\right\} \quad . \tag{9}$$

The above equations indicate that the point $\iota = 4$ is the transition point from a radiative type atmosphere to a convective type atmosphere (i.e. from an infinitely deep tropopause to a finitely deep tropopause). Thus the designation of $\iota$ as the instability parameter in Section III. The atmosphere is generally considered to be finite in extent and a definite, finite surface temperature is usually desired. In this case $\tau_T$ may not exceed the radiative $\tau_s$ which is determined by the Eddington solution, and the transition value of $\iota$ is given by:

$$\iota_{\text{transition}} = \frac{8}{3(\tau_s)_{\text{radiative}}} + 4 \quad . \tag{10}$$

The surface optical depth is found from the Eddington solution or from equation (4) as:

$$\left.\begin{aligned} \tau_s &= \frac{4}{3}\left[\left(\frac{T_s}{T_{eff}}\right)^4 - \frac{1}{2}\right] && (\iota \leq \iota_{tr.}) \\ \tau_s &= \tau_T\left(\frac{T_s}{T_T}\right)^{\iota} = \frac{8}{3}\cdot\frac{1}{\iota - 4}\left(\frac{1}{2} + \frac{2}{\iota - 4}\right)^{-\iota/4}\cdot\left(\frac{T_s}{T_{eff}}\right)^{\iota} && (\iota > \iota_{tr.}) \end{aligned}\right\} . \qquad (11)$$

Relations similar to some of the above have been derived previously by Sagan and Pollack (1964).

The calculation of the net radiative flux in this approximation uses the ratio of the flux at a point in the convective region with the flux at the tropopause. The net radiative flux is calculated from the diffusivity relation which exists between the net radiative flux and the temperature gradient, obtained if the approximations of Eddington apply. The relation is:

$$F_r = -\frac{16\,\sigma\,\pi\,T^3\,\nabla T}{3\,k(p)\,\rho} \qquad (12)$$

(compare with equation (5-158), Chandrasekhar (1957)). The temperature gradients at the two points are equal due to adiabaticity; ~~the Eddington~~ equation approximation is used for $T(\tau)$ . The final result for the net radiative flux is:

$$\left.\begin{aligned} F_r(\tau) &= \sigma\,T_{eff}^4 && (\iota \leq \iota_{tr} \text{ or } \tau \leq \tau_T) \\ F_r(\tau) &= \sigma\,T_{eff}^4\cdot\left[\frac{\tau}{8}\cdot 3(\iota - 4)\right]^{4/6-1} && (\iota > \iota_{tr} \text{ if } \tau \geq \tau_T) \end{aligned}\right\} . \qquad (13)$$

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## FIGURE CAPTIONS

Figure 1. Surface infrared optical depth and tropopause infrared optical depth for radiative-convective atmospheric models. The visible surface optical depth is zero in these models. The independent variable $\iota$ is the instability parameter, defined in the text. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.

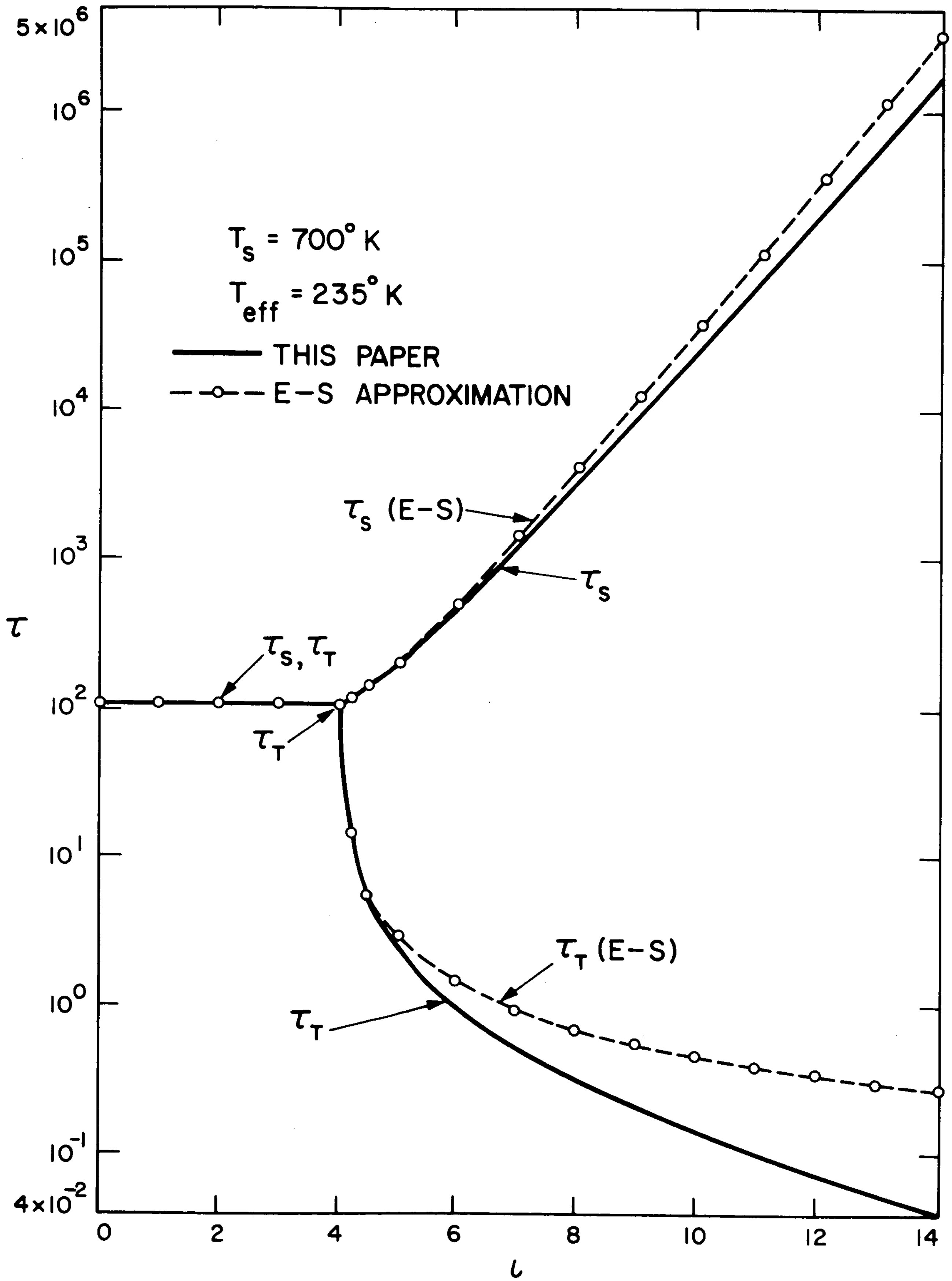
Figure 2. Surface temperature, tropopause temperature, and upper boundary temperature for radiative-convective atmospheric models. The independent variable $\iota$ is the instability parameter, defined in the text. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.

Figure 3. Tropopause radiative temperature gradient, normalized to the adiabatic gradient, and surface net radiative flux, normalized to the stratospheric flux, for radiative-convective atmospheric models. The independent variable $\iota$ is the instability parameter, defined in the text. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.

Figure 4. Temperature profiles for radiative-convective atmospheric models with three values of the instability parameter. The independent variable is the infrared optical depth in the atmosphere. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.

Figure 5. Temperature gradient profiles for radiative-convective atmospheric models with three values of the instability parameter. The temperature gradients are normalized to the adiabatic temperature gradient. The independent variable is the infrared optical depth in the atmosphere. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.

Figure 6. Net radiative flux profiles for radiative-convective models with three values of the instability parameter. The net radiative fluxes are normalized to the stratospheric flux. The independent variable is the infrared optical depth in the atmosphere. The solid curves refer to the theory outlined in Section III; the dotted curves refer to the Eddington-Schwarzschild approximation.



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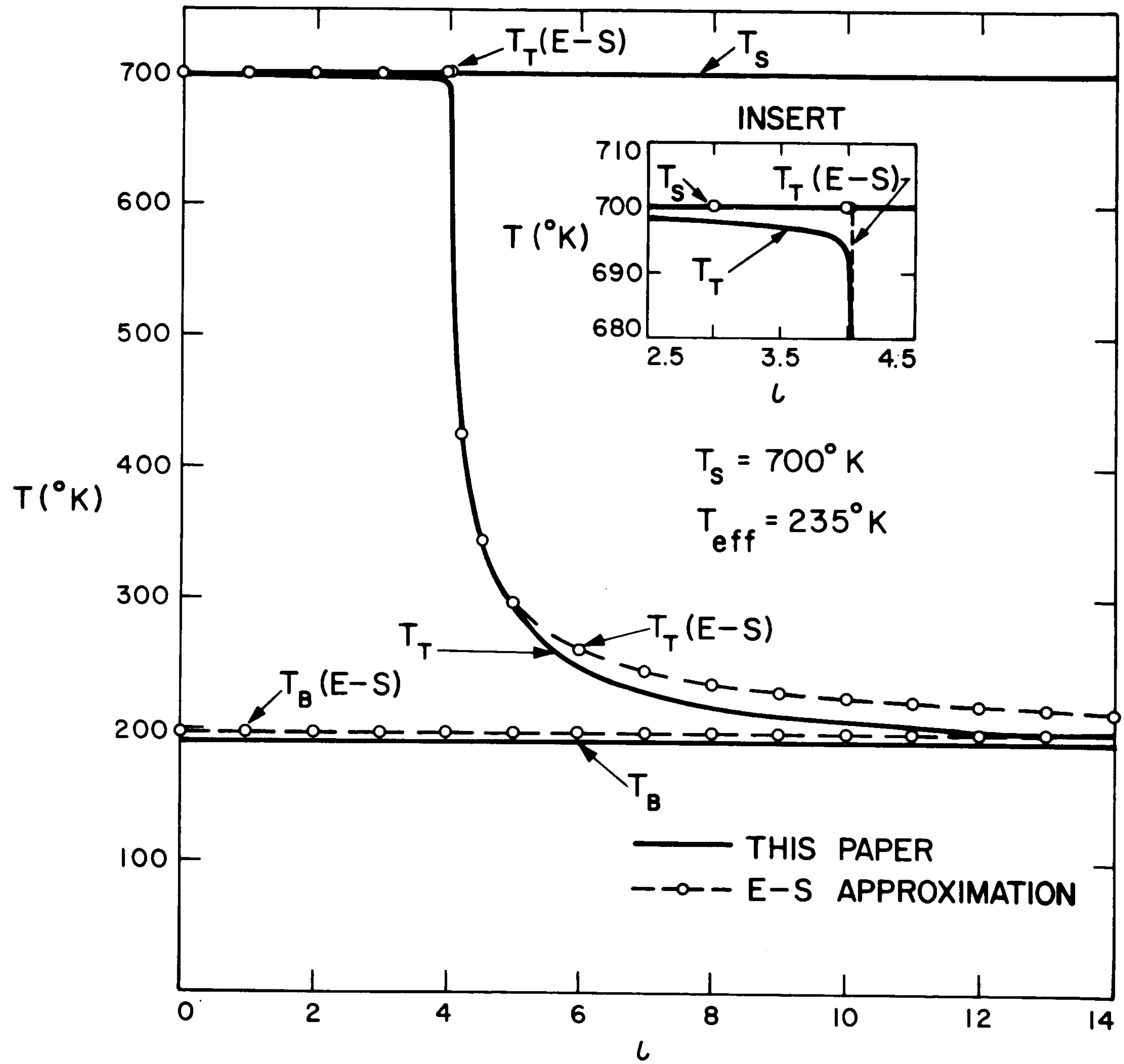

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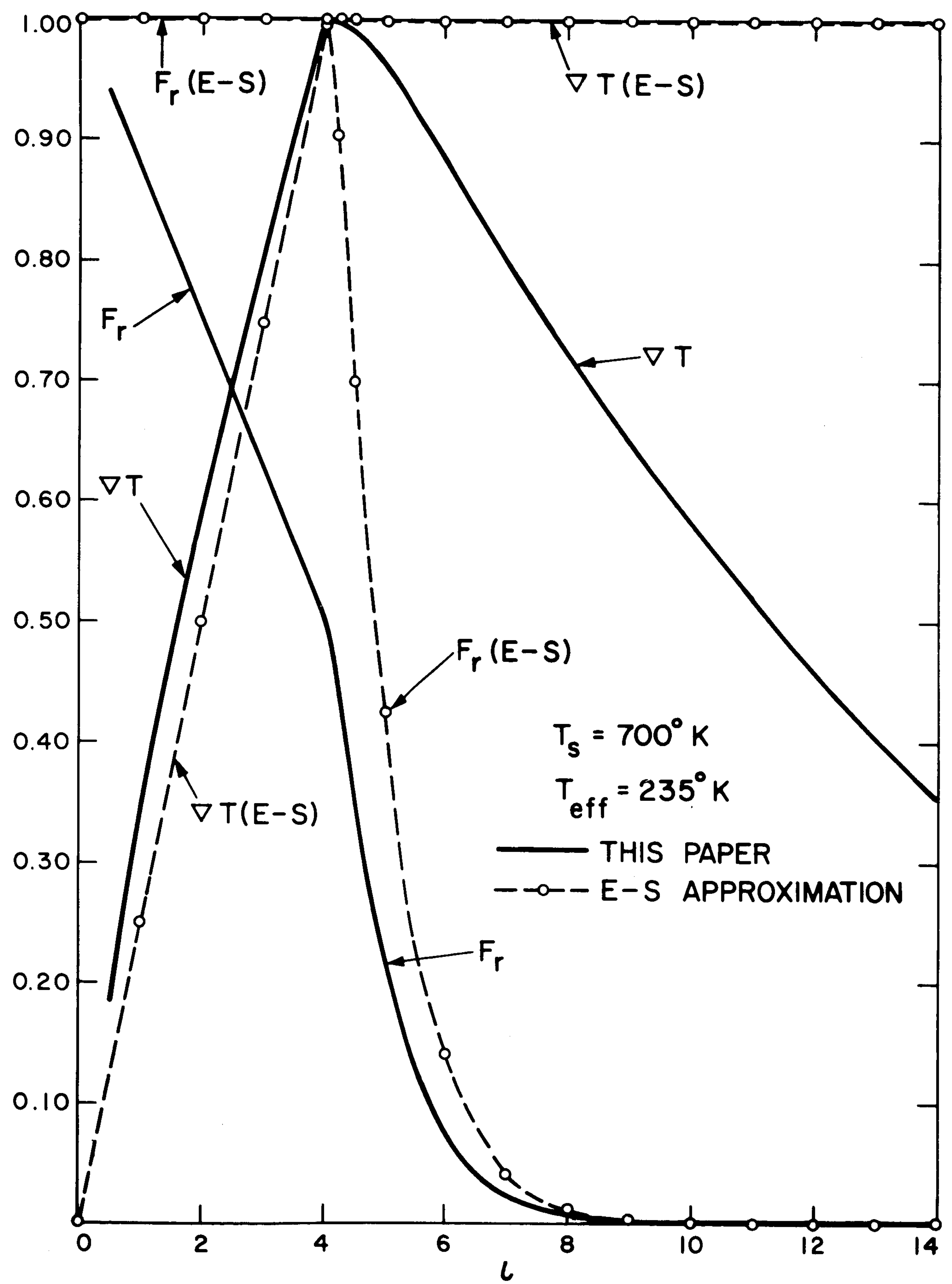

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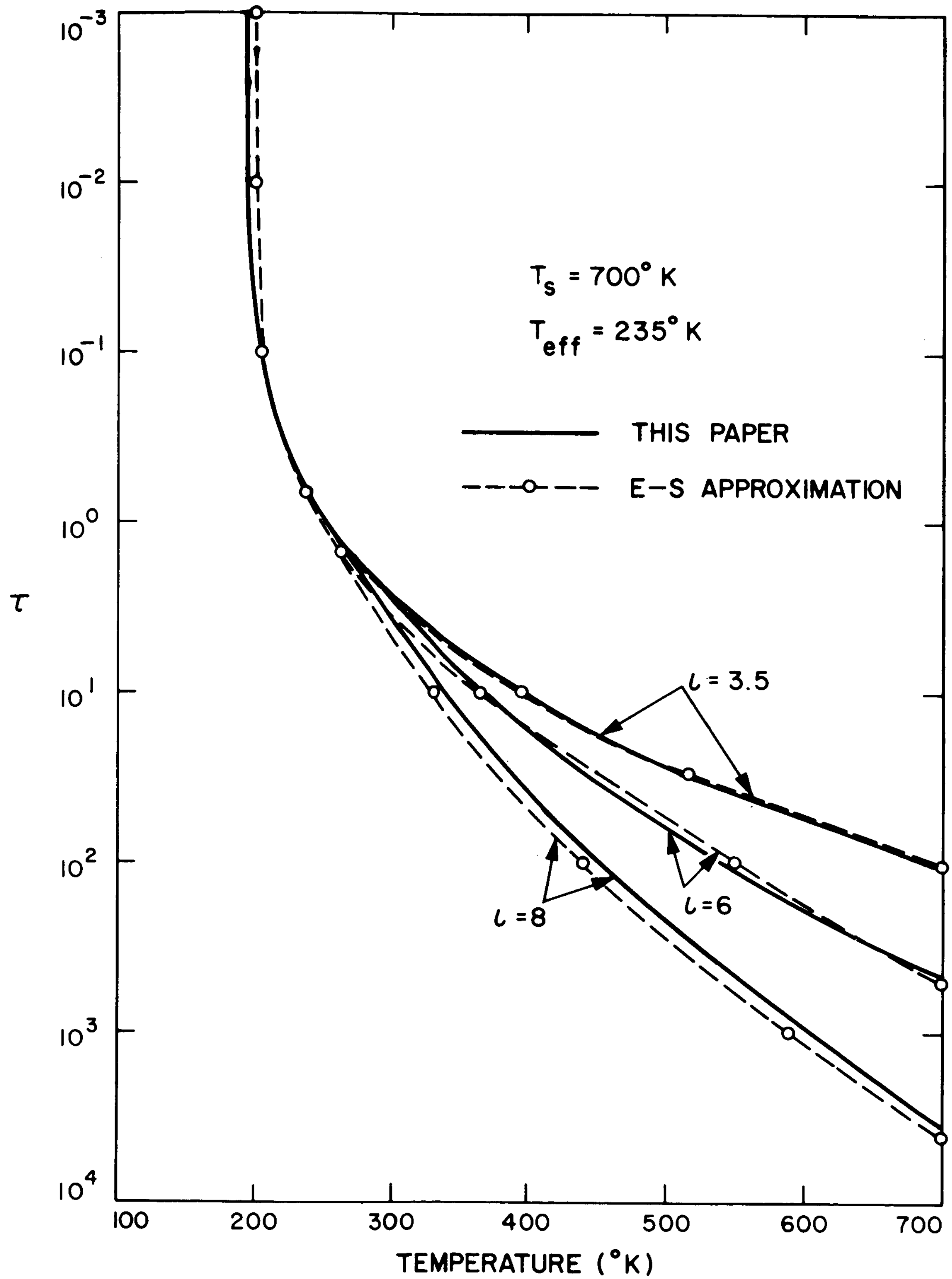

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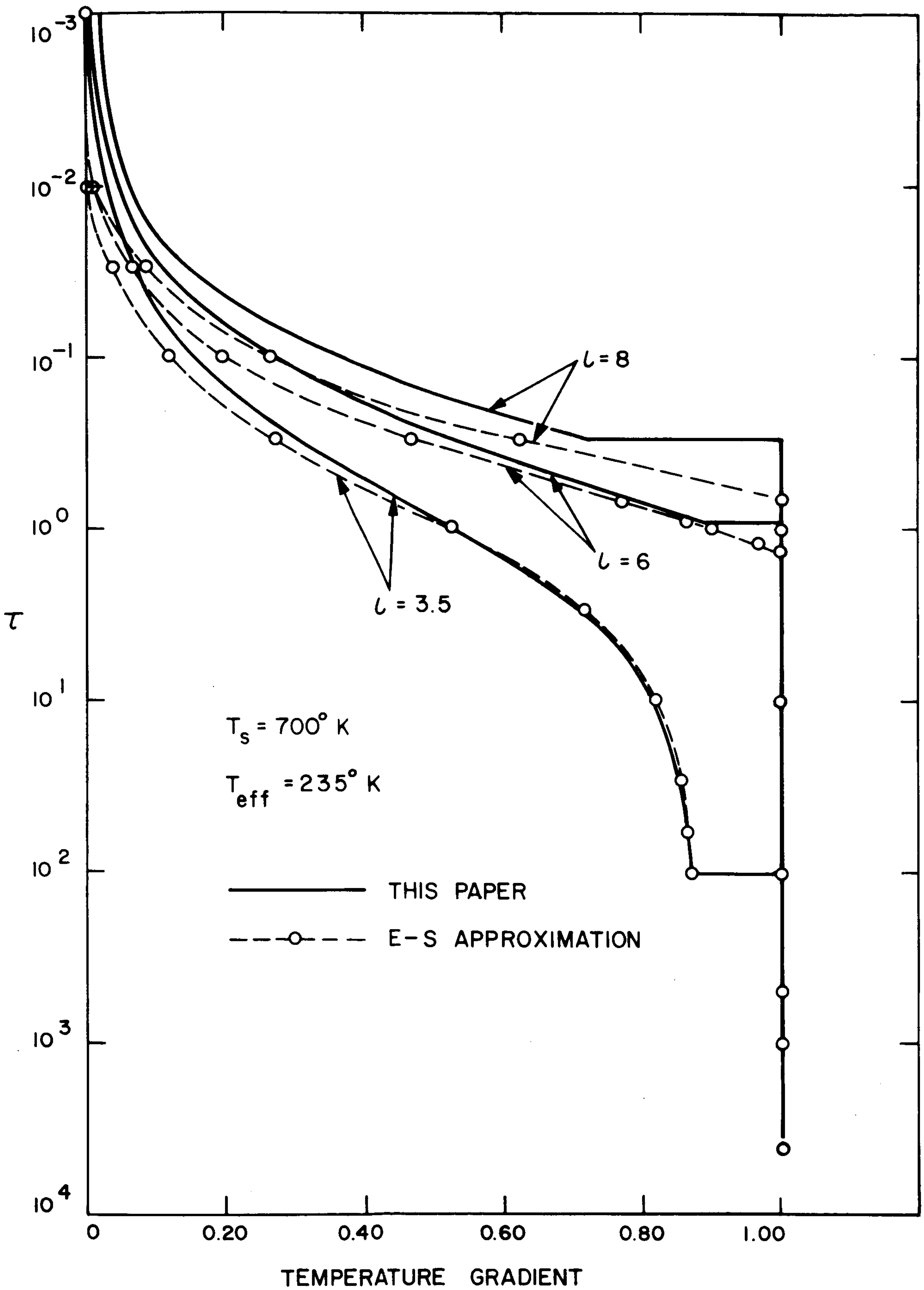
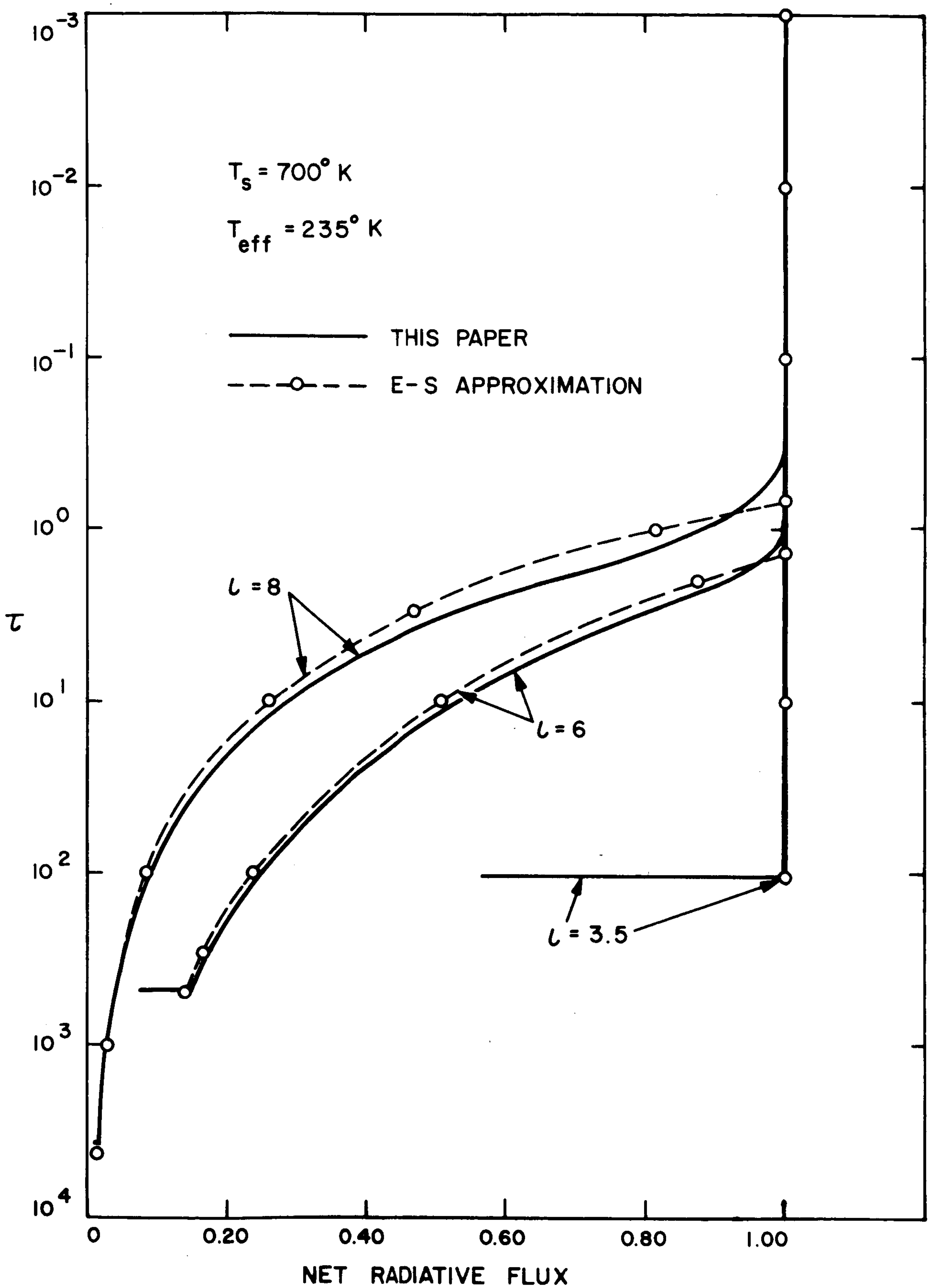
700
TEMPERATURE (°K)



$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
$10^{4}$
τ
0
0.20
0.40
0.60
0.80
1.00
TEMPERATURE GRADIENT
ι = 8
ι = 6
ι = 3.5
$T_s$ = 700° K
$T_{eff}$ = 235° K
THIS PAPER
E-S APPROXIMATION



$T_s = 700° K$
$T_{eff} = 235° K$
THIS PAPER
E-S APPROXIMATION
$\iota = 8$
$\iota = 6$
$\iota = 3.5$
$\tau$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$
0
0.20
0.40
0.60
0.80
1.00
NET RADIATIVE FLUX