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[redacted]

THE APPLICATION OF FEEDBACK IN MEASUREMENT

by

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October 1967

Supplement to CAPACITIVE PRESSURE TRANSDUCER SYSTEM by

Gerard J. Born
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July 1967

TABLE OF CONTENTS

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LIST OF SYMBOLS

A	gain
C	comparator (with subscripts)
E	error (with subscripts)
e	environment
f	function
G	gain
i	input
R	reference
S	sensitivity
T	transducer or convertor
X	variable with subscripts
Δ	comparison error
ϵ	environmental error
Γ	characteristic error
ζ	characteristic zero error

INTRODUCTION

In recent years, advanced research and technology in science and engineering have created a need for more reliable and more accurate measurements. Developments in the field of instrumentation have resulted in improvements of the existing measuring techniques and have demonstrated that under certain conditions, the application of feedback methods can result in instrumentation performance which is not obtainable by other methods.

Feedback in an instrument results in the original open loop measuring characteristics being replaced by the characteristics of a feedback element. By this principle, undesirable properties can be effectively suppressed. Under certain conditions, the use of feedback in a non-linear instrument can result in a linearized measuring characteristic. A reduction in the time constant of the measurement can often be achieved.

In this report, the basic problems connected with feedback instrumentation are discussed, and are compared with the conventional instrumentation, thereby clearly pointing out the necessary conditions for achieving measurement improvement by the use of feedback in instrumentation design.

CHAPTER 1 ELEMENTS OF MEASUREMENT

In measuring instruments, the physical quantity to be measured and the reference are compared to each other. The elements of a measurement system can be a comparator, a reference, a gain element, and a transducer or converter. In Appendix A the nomenclature associated with this measurement process is described.

The goal of a measurement is obtaining the TRUE VALUE AND THE DIMENSION of the quantity to be measured. The measurement as described above yields only a READING or MEASURED VALUE. Successively, the following steps must be made to obtain a true value and dimension.

1. From the instrument, the READING or MEASURED VALUE is obtained by reading the divisions OF THE SCALE and using an interpolation between the divisions.

2. A correction for the systematic errors is applied by which one obtains the CORRECTED READING or CORRECTED VALUE.

3. After having made several measurements of the same physical quantity (assumed to be constant), one can obtain a measure of the random errors and one obtains the PROBABLE VALUE.

4. From the measurement of the random errors and probable value, one can estimate the TRUE VALUE with a certain amount of confidence.

The true value is the value of the quantity to be measured obtained by an instrument with no systematic or random errors.

It is the purpose of the measurement techniques to minimize the errors. The measure in evaluation of a measurement technique is the possible error reduction.

In Appendix B a classification of instrument errors is developed. For instrumentation system design, the instrument errors are separated into

1. reading error
2. environmental error
3. characteristic or non environmental error
4. dynamic error.

1.1 Comparator and Reference Elements

Consider a simple measurement consisting of only a comparator and reference element without gain or transducer element as shown in Figure 1.

The static comparator error is the sum of the environment and characteristic error of the comparator at the working point where the comparisons are made. There are two comparison errors. The first is the comparison error of the zero; the second is the comparison error of the actual value of the input variable. The comparison error is inherently coupled with each comparison as this error is defined as the error in determining the equality of two quantities with the comparator error equal to zero. Note that is the comparator errors are zero, then the output of a comparison is zero because the comparator determines the point of equality.

In a simple measurement, the comparator and the variable reference are the scales. When the reference is a scale, the comparison error is synonymous with the reading error which consists of a reading error of the zero and of the actual value of the quantity to be measured. The essential feature of a comparator is the additive comparison. A comparison of two quantities is said to be purely additive if:

a. both quantities are of the same dimensions;
b. both quantities do not influence each other.

If this condition is not met, an additional "coupling" error in the comparator can exist. In further discussion, it is assumed that the comparator is purely additive. To sum up, the steps in a simple measurement are:

a. A quantity to be measured is applied to a measuring system which consists of a reference and a comparator.

b. The quantity to be measured is compared to the reference.

c. Two comparisons are made, one to compare the zero of the quantity to be measured with the reference zero, and one to compare the actual value of the quantity to be measured with the reference.

A block diagram of a simple direct measurement is given in Figure 1.

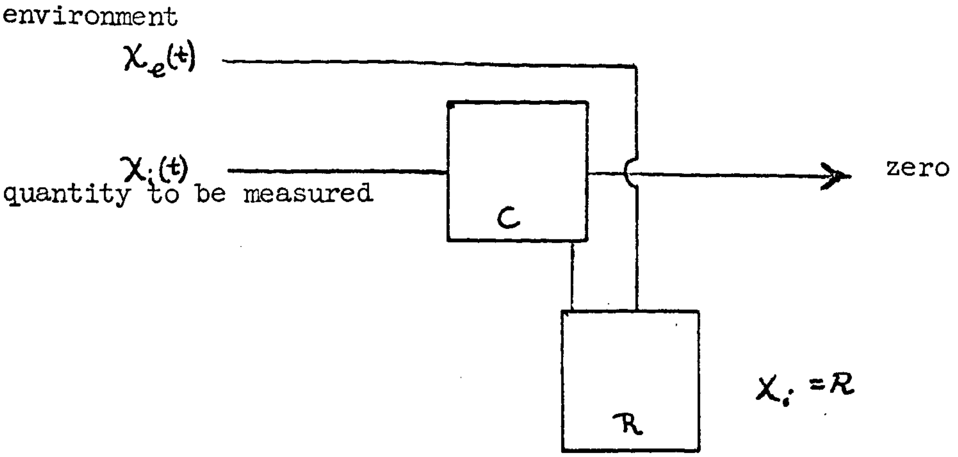

Figure 1

The static errors in each element are expressed as a function of full scale and are assumed to be random errors. In the reference element we have the reference characteristic error Γ_R and the reference environmental error ϵ_R. The relative errors are F_R/R and ϵ_R/R where R is the reference full scale. In the comparator only the zero errors are important. Hence in the comparator we have the comparator characteristic error ζ_C, the environmental error ϵ_C, and the comparison (reading) error Δ. When the full scale magnitude C equals M, the relative errors become Γ_C/C, ϵ_C/C, Δ_C/C where C is the comparator full scale.

The equations of the system are in absolute errors.

$$X_i(t) - (R \pm E_R) \pm E_c \pm \Delta = 0 \qquad (1)$$

$$X_i(t) = R \pm E_R \pm E_c \pm \Delta \qquad (2)$$

Hence, the absolute RMS error E

$$E = \left[E_R^2 + E_c^2 + \Delta^2 \right]^{1/2} \qquad (3)$$

Optimum results are obtained when each of the error terms are as small as possible and all terms are equal. No reduction in error over the simple measurement as described in Figure 3 and Equation ( ) can be made unless better elements are used.

1.2 Gain Elements

Consider a gain element with a gain G.

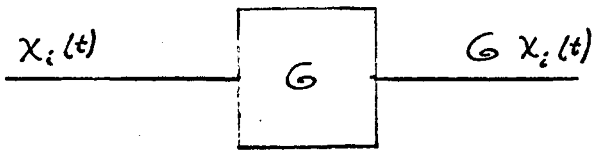

Figure 2.

The errors are measured at the output of the gain element. The quantity to be measured is also amplified with a gain G.

A block diagram of simple measurement with gain elements is given in Figure 3.

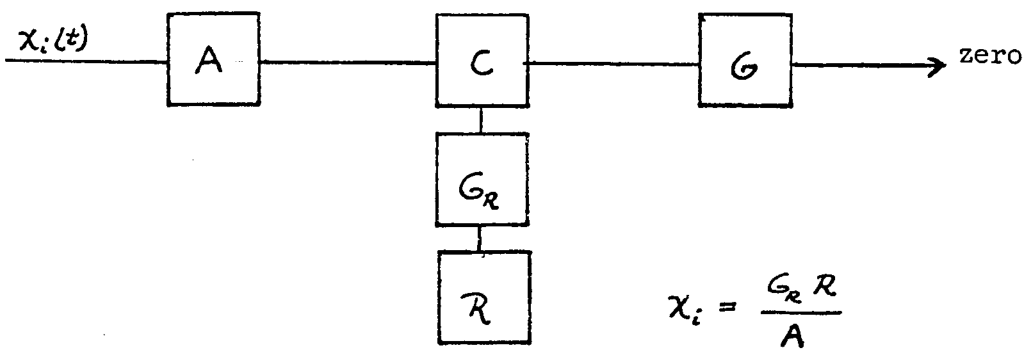

Figure 3.

The system equation is

$$\left\{\left(X_i A \pm E_A\right) - \left[\left(R \pm E_R\right) G_R \pm E_{G_R}\right]\right\} G \pm E_C G \pm E_G \pm \Delta = 0 \quad (4)$$

$$X_i = \frac{G_R R}{A} \pm \frac{E_A}{A} \pm \frac{E_R G_R}{A} \pm \frac{E_{G_R}}{A} \pm \frac{E_C}{A} \pm \frac{E_G}{AG} \pm \frac{\Delta}{AG} \quad (5)$$

The RMS error becomes:

$$E = \left[\left(\frac{E_A}{A}\right)^2 + \left(\frac{E_R G_R}{A}\right)^2 + \left(\frac{E_{G_R}}{A}\right)^2 + \left(\frac{E_C}{A}\right)^2 + \left(\frac{E_G}{AG}\right)^2 + \left(\frac{\Delta}{AG}\right)^2\right]^{1/2} \quad (6)$$

Comparison with equation (3) shows that the comparator error is increased by a factor G_R. In general the gain element should have a gain equal or smaller than one. Assuming a reference gain of one, then further comparison with equation (3) shows that the error of the comparator (C) and reference (R) are reduced by a factor 1/A; the comparison error is reduced by a factor 1/AG. The improvement by using the gain elements has been made at the expense of additional error terms

$$\left(\frac{E_A}{A}\right) , \left(\frac{E_G}{AG}\right) \tag{7}$$

In the gain element G, only errors are significant. In the first amplifier, zero and gain errors are significant and these are reduced by gain A. The ability to keep characteristics and environmental error small depends considerably on the physical form of the gain element A. The use of a gain element A is valuable only when these errors contribute to the total error an amount less than the reduction in the reference, comparator and comparison errors. This shows that, in general, for an element with its associated errors before the comparator the total complete characteristic and environmental error are propagated in the reading. For an element after the comparator only the errors in the zero of the elements appear in the measurement and the Gain changes are of secondary importance. Hence when a gain element is necessary, then it is desirable to place this after the comparator as then only its characteristic zero error and environmental error are propagated.

In practical instrumentation, the errors are often proportionally related to the full scale. For minimum error, it is necessary to match the full scale of reference (R) and comparator (C) with the measured (M).

<u>In conclusion:</u>

The reduction of the comparison error can be done by using a gain element after the comparison. The full scale value of the comparator and reference should be matched to the full scale value of the quantity to be measured. When the full scale value of the quantity to be measured is much smaller than the available full scale values of reference and comparator, amplification before the comparison can reduce the absolute error, as the terms

$$(E_R)^2 + (E_C)^2 + (\Delta)^2 \tag{8}$$

are replaced by

$$\left(\frac{E_R}{A}\right)^2 + \left(\frac{E_C}{A}\right)^2 + \left(\frac{\Delta}{AG}\right)^2 + \left(\frac{E_A}{A}\right)^2 + \left(\frac{E_G}{AG}\right)^2 \tag{9}$$

1.3 Convertors or Transducers

In many measurements, the value of the measured quantity cannot be compared directly with a reference; but the measured quantity is first converted by one or more conversions into another physical quantity which can be compared directly with the reference. A conversion is the change of a physical quantity to a quantity of different kind (different dimension) and is done in a convertor.

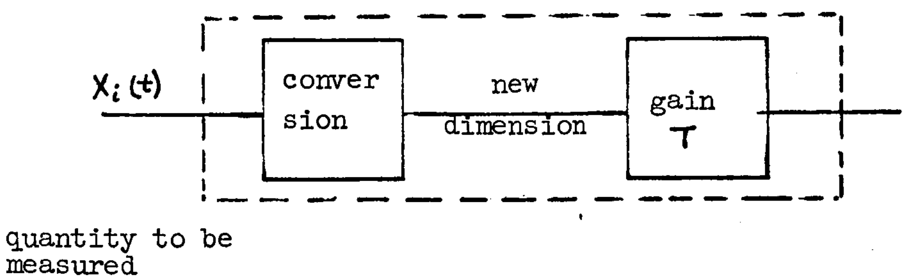

Figure 4

A conversion always consists of a conversion process and a gain process. However, these two functions should always be distinguished. The element containing the conversion process and its gain process is often called the transducer. The conversion process changes the dimensional quantity and has no error. The gain process of the convertor has a characteristic and environmental error. The actual measurement may take place after one or more conversions. Hence, the conversion errors are added into the error equation.

The possible places of a convertor in a measuring system are:

1. Conversion before comparison
2. Conversion between reference and comparison
3. Conversion after the comparison.

The first two positions are the same with respect to the error propagation. They correspond to the position of amplifier A in Figure 5. The total characteristic conversion error is added to the measurement error. In the third position the convertor corresponds to the position of amplifier G in Figure 5, and only the characteristic zero error of the conversion element is propagated into the measurement error.

In conclusion:

In many practical cases, the transducer errors are among the largest errors in a measuring system. An attempt should be made to minimize these errors. The best position for a transducer element is after the comparator as in that case only, the characteristic zero error is propagated into the total measurement error. In this case, the error term of a convertor

$$E_T = \left[\Gamma_T^2 + \epsilon_T^2 + \zeta_T^2 \right]^{1/2} \tag{10}$$

is reduced to

$$\varepsilon_T = \left[\zeta_T^2 + \epsilon_T^2 \right]^{1/2} \tag{11}$$

CHAPTER 2 MEASURING SYSTEMS

In this chapter, the basic measuring systems will be analyzed and the precision (E_p = "sum" of the instrumentation errors) will be determined.

2.1 Direct Measurement

In the direct measurement, the quantity to be measured and for reference is converted so that the physical dimension of the converted quantity to be measured and reference are the same. All conversions take place before the comparator, for example, voltage measurement by a galvanometer. The voltage is converted into current, the current is converted into a force, the force is converted (by a spring) in a displacement and the displacement is compared to a scale.

$$\left[(X_i T \pm E_T) - (R \pm E_R)\right] \pm E_c \pm \Delta = 0 \qquad (12)$$

$$X_i = \frac{R}{T} \pm \frac{E_T}{T} \pm \frac{E_R}{T} \pm \frac{E_c}{T} \pm \frac{\Delta}{T}$$

where T is the transducer full scale.

$$E_p = \left[\left(\frac{E_T}{T}\right)^2 + \left(\frac{E_R}{T}\right)^2 + \left(\frac{E_c}{T}\right)^2 + \left(\frac{\Delta}{T}\right)^2\right]^{1/2} \qquad (13)$$

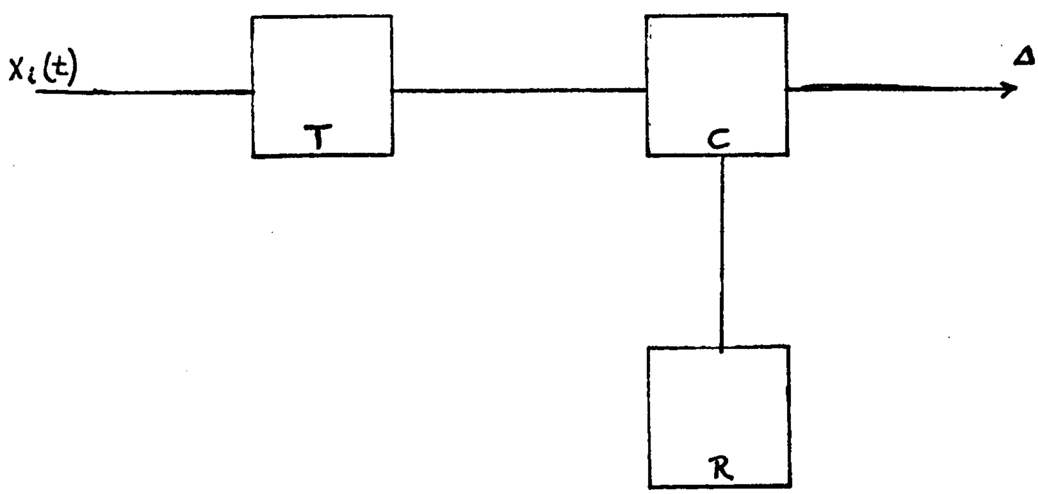

Figure 5

The quantities Γ_T/T and ϵ_T/T are the relative conversion errors. Normally these conversion errors are dominating, in which case the equation becomes

$$E_p \cong \frac{E_T}{T} = \frac{\left[(\Gamma_T)^2 + (\epsilon_T)^2\right]^{1/2}}{T}$$

The advantages of a direct measurement are

1. a relatively simple construction
2. good dynamic response (determined by the measuring elements)

2.2 The Effect of Gain Elements

The influence of gain elements in direct measurement can be evaluated from Figure 6.

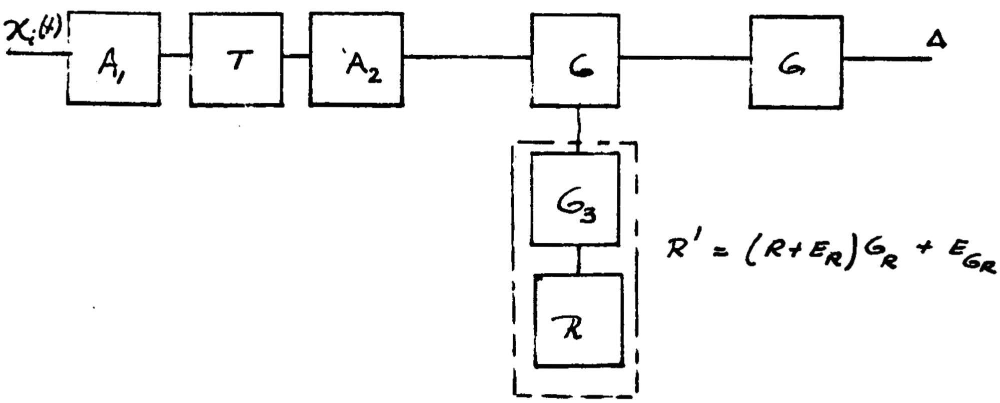

Figure 6

$$\left[\left[\left\{(X_i A_1 \pm E_{A_1}) T \pm E_T\right\} A_2 \pm E_{A_2}\right] - R' \pm E_c\right] G + E_G + \Delta = 0$$

$$X_i = \frac{R\,G_R}{A_1 T A_2} \pm \left(\frac{E_{A_1}}{A_1}\right) \pm \left(\frac{E_T}{A_1 T}\right) \pm \left(\frac{E_{A_2}}{A_1 T A_2}\right) \pm \left(\frac{E_c}{A_1 T A_2}\right) \pm \left(\frac{E_R G_R}{A_1 T A_2}\right) \pm \left(\frac{E_G}{A_1 T A_2 G}\right) \pm \left(\frac{\Delta}{A_1 T A_2 G}\right) \qquad (14)$$

The RMS error is:

$$E_p = \left[\left(\frac{E_{A_1}}{A_1}\right)^2 + \left(\frac{E_T}{A_1 T}\right)^2 + \left(\frac{E_{A_2}}{A_1 T A_2}\right)^2 + \left(\frac{E_R G_R}{A_1 T A_2}\right)^2 + \left(\frac{E_G}{A_1 T A_2}\right)^2 + \left(\frac{E_G}{A_1 T A_2 G}\right)^2 + \left(\frac{\Delta}{A_1 T A_2 G}\right)^2\right]^{1/2} \qquad (15)$$

This equation shows that besides the introduction of an additional error in the additional element, the following effects occur:

a. A gain element after the comparator reduces the comparison error.

b. A gain element between reference and comparator enlarges the reference errors.

c. A gain element between the transducer and comparator reduces the comparator error.

d. A gain element before the transducer reduces the transducer errors.

Hence, by use of gain elements, it is often possible to obtain an error reduction especially when gain elements are used to match the full scale of the elements, as discussed under comparison error reduction (page).

2.3 Balancing Measurement

In a pure balancing type of measurement, the comparison is made directly between the quantity to be measured and a reference having the same physical quantity. The conversions take place after the comparator. Hence the only characteristic zero errors of the convertor appear in the error equation. The reference of the same physical quantity as the quantity to be measured is varied; hence the reference must be measured. The value of the reference is compared to or converted with a scale. Only when there is a better convertor available for the reference measurement can an improvement be made. Take, for example, voltage measurement. The voltage of a reference cell is controlled by a precision decade resistor. The voltage to be measured and the reference voltage are subtracted; the voltage difference is converted into current, the current is converted into force, the force is converted into displacement. Equation (10) applies where Γ_A and ϵ_A are zero. By the use of large gains after the convertor the reading error can be made negligibly small. The error equation becomes (Figure 7)

$$E_p = \left[E_R^2 + E_c^2 + \left(\frac{E_T}{T}\right)^2 + \left(\frac{\Delta}{T}\right)^2 \right]^{1/2} \tag{16}$$

$$\cong \frac{E_T}{T} \quad \text{(when transducer errors are dominaling)}$$

Note that $_T$ and ϵ_T are the errors at the output of the gain element. In a practical measuring system, the reading error Δ is small compared to the other errors, and if necessary can be reduced by a gain element G. Practical amplifiers can be made without practically any zero nonlinearity such as dead zone, etc., so that $_G$ ϵ_G 0. The errors Γ_R, ϵ_R and $_C$, ϵ_C are the errors of the reference and comparator. These are usually small compared to the transducer errors.

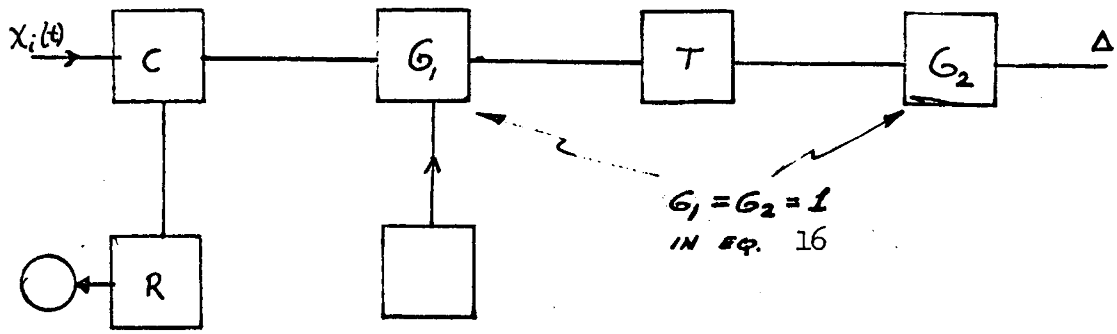

Figure 7

With the gain elements G_1 and G_2, the equations become:

$$\left[\left[\left\{\left(X_i - R \pm \varepsilon_R \pm \varepsilon_c\right) G_1 + \varepsilon_{G_1}\right\} T + \varepsilon_T\right] G_2 + \varepsilon_{G_2} + \Delta = 0 \qquad (17)$$

The RMS error (E) equals:

$$E_p = \left[\left(\varepsilon_R^2 + \varepsilon_c^2 + \left(\frac{\varepsilon_{G_1}}{G_1}\right)^2 + \left(\frac{\varepsilon_T}{G_1 T}\right)^2 + \left(\frac{\varepsilon_{G_2}}{G_1 T G_2}\right)^2 + \left(\frac{\Delta}{G_1 T G_2}\right)^2\right)\right]^{1/2} \qquad (18)$$

This equation shows that under certain conditions (negligible gain errors) a gain element before the transducer can reduce the error part due to the transducer.

To sum up, a balancing measurement (nulling measurement) is made by a nulling procedure and a direct measurement of the balancing quantity (e.g., reference). Such a measuring system consists of a comparator where the quantity to be measured and a variable reference of the same dimension are compared. The comparator output is measured by a direct measurement. First a comparison is made between the zeros of the quantity to be measured and the zero of the reference. The magnitude of the quantity to be measured is obtained by a nulling procedure where the reference is varied until the output of the comparator is equal to zero. Consequently, the variable reference magnitude and the magnitude of the quantity to be measured are equal. The magnitude of the variable reference is read and therefore the magnitude of the quantity to be measured is known. By the measurement of the null, only the characteristic zero error of the elements after the comparator appear. This is the advantage of a balancing measurement. Poor dynamic response and a more complicated system are the disadvantages of this system.

2.4 Feedback Measurement

When feedback is applied to a balancing measurement, the measurement becomes an automatic nulling measurement. An additional amplifier element G is added which uses the output of the balancing measurement to null the measurement by varying the reference. The reference element is read and gives the magnitude of the input quantity to be measured.

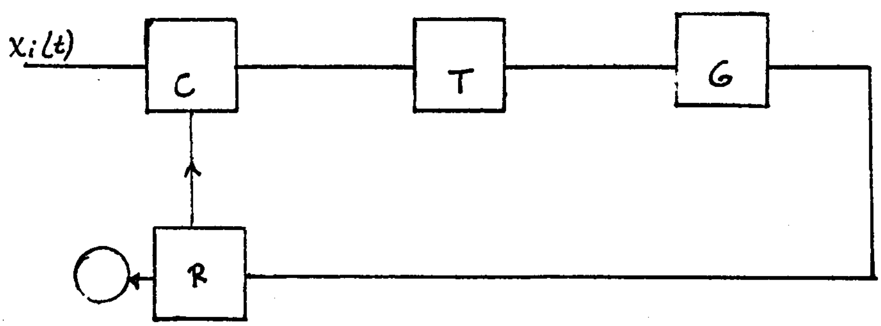

Figure 8

When the loop gain is TG, then the RMS error becomes

$$E_p = \left[E_R^2 + E_C^2 + \left(\frac{G E_T}{TG+1}\right)^2 + \left(\frac{E_G}{TG+1}\right)^2 + \left(\frac{\Delta}{TG+1}\right)^2 \right]^{1/2} \qquad -19-$$

If one compares the automatic balancing measurement with the non-automatic balancing measurement, then the following effects occur:

1. When the added gain G is large and $TG \gg 1$, the term $G/TG + 1 \approx 1/T$ and the static errors of both nulling measurements are (Equations 18, 19) for practical purposes identical, as the additional error due to the term $G/TG + 1$ should be made negligibly small.

2. The dynamic response of the feedback measuring system is enhanced by the feedback. When the dynamic response of the elements is determined by parameters outside the feedback loop, then the dynamic response is identical to that of the direct measurement. When the parameters which determine the dynamic response are in the feedback loop, then the dynamic response of the feedback systems is enhanced by a factor $G/TG + 1$ as compared to the direct measurement.

APPENDIX A NOMENCLATURE AND DEFINITIONS

A measurement is a comparison of two physical quantities of the same dimensions and the characterization of the physical quantity to be measured in terms of a number and a dimensional expression. A physical quantity is determined by a dimensional expression and a number comparing the quantity with the unit quantity. Hence, a physical quantity is a dimensional quantity.

In a simple measurement, the steps in making a measurement are:

1. Compare the reference zero with the "zero of the input variable."

2. Apply the input variable.

3. Vary the reference until the reference value is equal to the input variable. This is the second comparison.

4. Determine (read) the reference value. Often the simple measurements function (3) and (4) can be combined and done with the human eye, e.g., on a scale. The eye goes over the scale to the point where the reference (number diversion) equals a line or pointer. The reference is determined by reading the number diversion.

The necessary elements to make the simplest measurements are thus a comparator and a variable reference.

The comparator is an element in which the difference between the magnitudes of two quantities of the same kind (dimension) is determined. Hence, the comparator is the element necessary to perform the comparisons of the zero and of the actual value of the quantity to be measured against the zero and the value of the reference.

The reference is a physical quantity which is (presumably) known. In a practical measurement, the reference and the comparator are measuring instruments.

In a measurement, the following properties are of fundamental importance:

1. MEASUREMENT RANGE OR RANGE OF THE MEASURING INSTRUMENT

This is the range (of the values of quantity to be measured) over which the instrument is intended to measure. The measurement range is specified by lower and upper limits.

2. SENSITIVITY OF THE MEASURING INSTRUMENT

This is the ratio of the change in instrument indication to a change in the value of the quantity to be measured or:

Sensitivity = $G = \frac{\Delta x}{\Delta y}$, often called the gage factor or gain,

where Δx = change in instrument indication
Δy = change in quantity to be measured

The sensitivity is often expressed as the variation of the instrument indication to a unit change of the value of the quantity to be measured.

The logarithmic sensitivity is defined as the ratio of the relative change in the instrument indication to the relative change in the quantity to be measured.

$$\text{Logarithmic Sensitivity} \quad S = \frac{\Delta x / x}{\Delta y / y} = \frac{\Delta x}{\Delta y} \cdot \frac{x}{y} = G \cdot \frac{x}{y}$$

where x = instrument indication
Δx = change in instrument indication
y = value of the quantity to be measured
Δy = change in value of the quantity to be measured

3. INSTRUMENT DYNAMIC CHARACTERISTICS

The ability of the instrument to follow the variations of the measured quantity with time is determined by the instrument dynamic characteristics and is often expressed as a frequency response curve.

4. MEASUREMENT ERRORS

The measurement error is the algebraic difference between the indicated value and the true value of the quantity to be measured.

The error can be expressed as an absolute or relative error.

The absolute error is the error in an absolute dimensional quantity.

The relative error is given as a percentage of some quantity, usually in percent of the upper limit of the measurement range (full scale); sometimes the errors are expressed in percent of the indicated value.

An error source is a (variable) physical quantity causing a measurement error. The error is related to the error source by some physical law which can or cannot be described analytically. A variable is called deterministic when it possesses a definite functional dependence on time, while the random parameters appear parametrically. A variable is called random when such a definite functional structure in time cannot be given.

A. SYSTEMATIC ERROR

An error is systematic if both the following conditions are satisfied:

1. The error source is a deterministic variable and its magnitude is known at the time of the measurement.

2. A functional relationship is known and used to describe the relation of the error to its source.

B. RANDOM ERROR

An error is called random if one or more of the conditions for a systematic error are not satisfied. Therefore, the error is random if any one of the following conditions is satisfied:

1. The error source is a random variable or the magnitude of the error at the time of the measurement can be only predicted with some probability.

2. A functional relationship to describe the relation of the error to its source is not known or is not used.

Hence, when the deterministic character of the source or the functional relationship of a systematic error is not used, the error is classified as a random error.

The measurement errors can be divided into DYNAMIC ERRORS and STATIC ERRORS.

The dynamic error occurs due to the time variation of the quantity to be measured. All errors which are not related to the time variation of the quantity to be measured are static errors.

For the measuring performance, the measurement errors can be divided into:

A. LOADING ERROR

The loading error is the difference of the value of the quantity to be measured before and after the measuring device is brought into its measurement position. The loading error can consist of a static and/or dynamic error. The loading error is often one of the most important errors in the measurement. This error can only be determined for a specific measuring system; hence, it will not be further discussed.

B. INSTRUMENT PRECISION

This is the sum total static error of the instrument.

C. INSTRUMENT DYNAMIC ERROR

This is the instrument error due to a time variation of the quantity to be measured (with exclusion of the static error).

APPENDIX B INSTRUMENT ERRORS

The instrument error is the total error contributed by the instrument and consists of static and dynamic errors.

I. STATIC ERRORS

The total static error precision of an instrument can be split into errors due to the major error sources. These are:

A. Reading Error
B. Characteristic Error (gain and zero)
C. Environmental Error

A. READING ERROR

The reading error is the error that an average reader will make in reading the pointer on a scale. This error can be divided into several sub-errors such as

a. Optical Resolution - The average optical resolution of the eye is in the order of 10^{-3} times the distance from the reading.

b. Paralax - This is a function of the distance of the pointer above the scale.

c. Interpolation Error - This error is in the order of 10% of the scale markings.

d. Background Noise - This can cause a vibration of pointer and hence the average position must be estimated. This results in an error in the order of 10% of total vibration deflection.

As is discussed in Chapter 2, the size of the reading error can be largely influenced by the instrumentation system

B. CHARACTERISTIC AND ENVIRONMENTAL ERROR

Consider the output of a measuring element as a function of its input variables. We distinguish between the signal input (X_i) and the environmental input (X_e). The latter can be actually a number of inputs (X_{e1}, X_{e2}, ...). The enviornmental input variables are considered to be the variations from some nominal environment. For a single environmental variable, the output is $X_o = f(X_i, X_e)$. This function can be illustrated by a surface in three dimensional space (O, A', B", C" in Figure B-1).

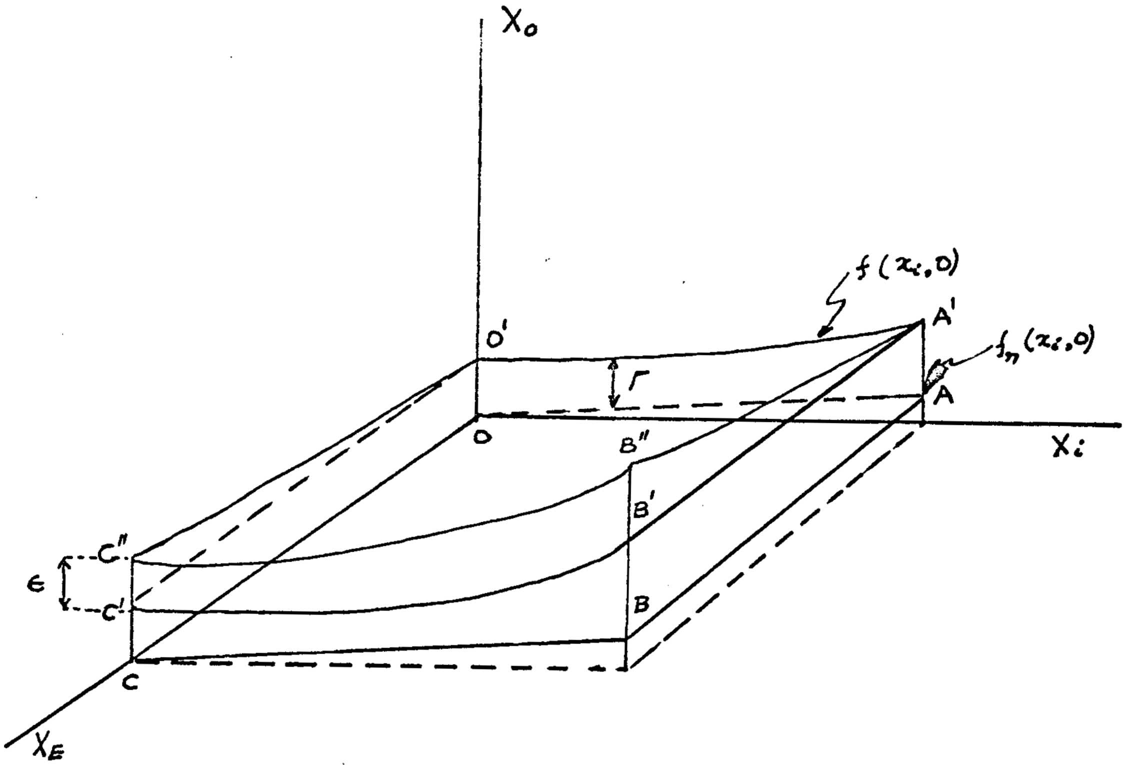

Figure B-1

A perfect signal transmission would be characterized by the function $X_{on} = f_n(X_i, X_e) = f_n(X_i, 0)$. This equality expresses a nominal function as well as the independence from the environment. The function $f_n(X_i, 0)$ is illustrated by the line OA. The function $f_n(X_i, X_e)$ is the OABC plane if $f_n(X_i, 0)$ is a straight line. Even with no environmental error, there will be a deviation from the ideal characteristic OA. This deviation which will be called characteristic error, results in the O'A' characteristic.

Note that this error depends only upon the nature of the output quantity to be measured and not upon the environment. Hence, it might be called "non-environmental error."

The total error with a variation of the nominal environment, or $X_e \neq 0$, is given by BB". It can be considered as the sum of two errors; the characteristic error BB" = AA', and the environmental error, B'B". Note that the O'A'B'C' surface is perpendicular to the X_iX_o plane and that the O'A'B"C" surface intersects the X_iX_o plane in the line O'A'.

In view of the introduced definitions the output can be expressed as a Taylor series of the environmental variable.

$$X_o = f(X_i, X_e) - f(X_i, 0) + \left.\frac{\partial f(X_i, X_e)}{\partial X_e}\right|_{X_i = C} \cdot X_e + \left.\frac{\partial^2 f(X_i, X_e)}{\partial X_e}\right|_{X_i = C} \cdot X_e^2 +$$

When substituting the characteristic error, (Γ)

$$\Gamma = f(X_i, 0) - f_n(X_i, 0)$$

the first order approximation becomes

$$X_o = f_n(x_i, 0) + \Gamma + \epsilon \quad = X_{on} + \Gamma + \epsilon$$

where ϵ denotes the environmental error, or

$$\epsilon = \left.\frac{\partial f(x_i, x_e)}{\partial x_e}\right|_{X_i = C} \cdot X_e$$

With the concepts of characteristic and environmental error, an element can be represented by an ideal element, $f_n(X_i, 0)$, to which the errors are added. Hence, a division of the static instrument error is:

1. The Environmental Error (ϵ) - This error due to a changing environment with a constant transducer input. All physical quantities influencing the performance of a measurement with the exception of the quantity to be measured are considered under environment. In the first approximation, this error is independent of the quantity to be measured. This may be, for example, temperature, acceleration, velocity, vibration, etc.

2. The Characteristic or "Non-Environmental Error" (Γ) - This is the deviation of the instrument output from the physical law predicted output with a constant nominal environment. The characteristic error can be divided with respect to the error source into:

   a. Mobility Error - This error is associated with the smallest level of the quantity to be measured that will produce a variation in the measurement value. This error can be caused by friction and discontinuities such as potentiometer wires.

b. Hysteresis Error - This is the maximum uncertainty in the hysteresis cycle.

c. Calibration Error (zero and gain) - This error consists of two parts:
1. the error in reading the calibration curve
2. the error in making the calibration curve

From the systems point of view, these errors can be grouped together and then separated into

1. The Characteristic Zero Error (ζ) - This is the characteristic error when the input variable is equal to zero. This error is independent of the quantity to be measured.

2. The Characteristic Gain Error (γ) - This is the characteristic error with the characteristic zero error subtracted. Hence, the characteristic gain error is a function of the quantity to be measured.

II. DYNAMIC INSTRUMENT ERRORS

This is the error due to the time variation of the quantity to be measured. The dynamic error is normally expressed as:

A. FREQUENCY RESPONSE CURVES (amplitude and phase) for the measurement or a variable quantity to be measured.

B. RESPONSE TIME (the time to reach 63% of the final value) for the measurement of constant levels.

C. DELAY TIME when the quantity to be measured varies slowly.

III. TOTAL ROOT MEAN SQUARE ERROR OF A MEASURING SYSTEM

A measure of the total error in a measurement system can be obtained by the root mean square error. The total RMS error in a measuring system is the "sum" of the loading error, precision and dynamic error or:

$$E = \left[E_L^2 + E_p^2 + E_D^2\right]^{1/2}$$

The conditions for the use of the RMS error summation are:

1. The errors are independent.
2. The probability distribution of the errors tends to be gaussian (normal probability distribution).
3. The errors are of the same magnitude.

A. INDEPENDENT ERRORS

In practical measuring systems, the condition for independence or errors is the most important one. For example, the environmental error of each of the elements in a measurement chain may not be independent. In the case that errors are dependent or correlated, the RMS error becomes:

$$E = \left[E_1^2 + E_2^2 + E_3^2 + (E_4 + E_5 + E_6)^2\right]^{1/2}$$

where the errors E_4, E_5 and E_6 are dependent or in other words, function of the same parameter. (The errors E_1, E_2, and E_3 are independent)

When the total RMS error is known, then an estimation can be made (assuming the normal distribution) of the most probable value of the measurement. For this estimation, the total RMS error is set equal to the standard deviation (σ) and the confidence bands of 1σ (68%), 2σ (95%), or 3σ (99%) are known.

B. NON-GAUSSIAN PROBABILITY DISTRIBUTIONS

In general, the probability distribution of a measurement tends to be gaussian or normal. However, this should be verified (by analysis or experiment) as otherwise one will assign a most probable value that does not correspond to the true value. The classical example is a sampled voltage measurement. The measure value varies between 0 and the applied voltage. In conclusion, application of the gaussian or normal distribution to measured values should not be done without reservations.

IV. ERROR CLASSIFICATION FOR INSTRUMENT PERFORMANCE

For the performance of an instrument, the errors can be divided into:

A. RESOLUTION - This is a measure of the smallest increment of the quantity to be measured which gives a detectable reading increment. The resolution is determined by the reading error and mobility error. The resolution can be given by:

$$\text{resolution} = \frac{\sqrt{\text{reading error}^2 + \text{mobility error}^2}}{\text{measurement range}}$$

B. PRECISION - This is a measure of the repeatability of the measurement. The precision is determined by the reading error, characteristic error, and environmental error. The precision can be given by:

$$\text{precision} = \frac{\sqrt{\text{reading error}^2 + \text{charac. error}^2 + \text{environ. error}^2}}{\text{measurement range}}$$

C. ACCURACY - This is a measure of how well the reading corresponds to the true value. The accuracy is determined by the reading error and the instrument errors and loading error.