NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-326

Shock-Spectrum Analysis Program

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FOREWORD

This Report and the program described herein were prepared at the request of Section 294, Environmental Requirements; however, the programming and much of the report writing were done by Section 314, Scientific Programming.

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FIGURES

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FIGURES (contd)

ABSTRACT

The Shock-Spectrum Analysis Program for the 7094 computer computes the primary and residual shock spectra of a digital signal by the recursive filtering method. The program also automatically plots selected output on the SC-4020 plotter and punches the data on cards for additional analysis. The program is written in FORTRAN IV and MAP language.

SHOCK-SPECTRUM ANALYSIS PROGRAM

I. GENERAL PROBLEM DESCRIPTION

The program described in this Memorandum computes the shock spectrum of a signal that has been digitized and formatted on digital magnetic tape. The output data formats are oriented toward applications to acceleration signals, i. e., mechanical shocks as measured with accelerometers.

The shock-spectrum method of analyzing a transient signal is often employed by dynamicists to the analysis of mechanical shocks. The spectrum values represent the peak acceleration response of a simple mechanical oscillator to an acceleration transient $\ddot{x}(t)$ applied as a ground or base acceleration (Fig. 1).

The peak acceleration response [maximum $\ddot{y}(t)$] is presented as a function of oscillator natural frequency $(1/2\pi)(k/m)^{1/2}$. Any value of the damping coefficient may be selected.

In this program two different shock spectra are derived: the "primary" and the "residual." The primary shock spectrum consists of those values of peak acceleration response selected from the $\ddot{y}(t)$ time history over the portion of time for which $\ddot{x}(t)$ is defined. The residual spectrum consists of those values of peak acceleration response occurring in time <u>after</u> that t for which $\ddot{x}(t)$ is defined [i. e., the forcing function $\ddot{x}(t)$ is assumed to be zero outside the range for which it is defined]. Even though the forcing function has ceased, the response may continue, leading to nonzero residual spectra. Of academic interest, and sometimes of practical value, is the fact that the magnitude of the Fourier transform of $\ddot{x}(t)$ is expressed as

$$\left|F(f)\right| = \frac{\ddot{y}(f)}{2\pi f}$$

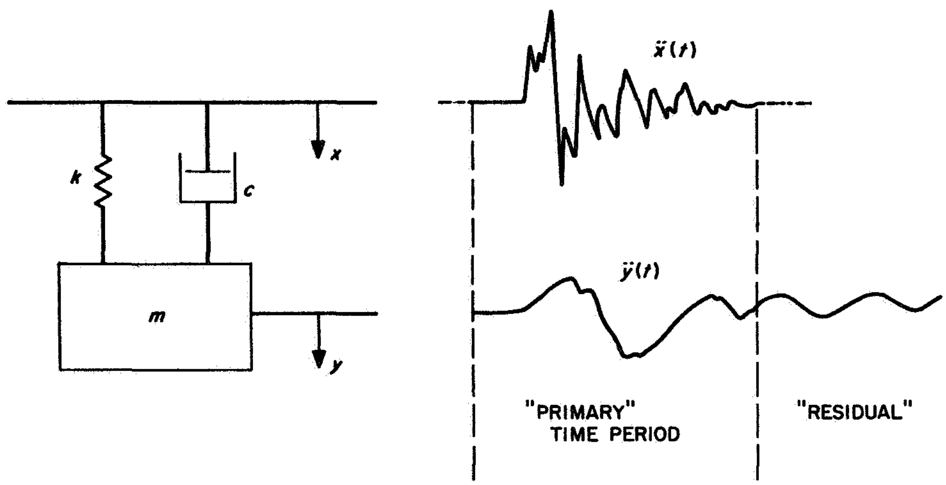

Fig. 1. Simple oscillator model for shock spectrum

where

$F(f)$ = Fourier transform of $\ddot{x}(t)$

$\ddot{y}(f)$ = zero-damping residual shock spectrum of $\ddot{x}(t)$

f = frequency in cps

The computation technique used in this program is a recursive filtering operation, which has proved very efficient in solving the differential equation of motion. A complete description of the technique and computational accuracies is given in Ref. 1.

II. DESCRIPTION OF INPUT

A. Data Tapes

Data tapes are prepared by the Data Analysis Lab with the following restrictions:

(1) One file of data represents the total data sample for one channel.

(2) A record of data contains 250 IBM words.

(3) Each IBM word contains two data points.

(4) The first record of each file is a 60-word header record.

(5) Data tapes are written with a density of 556 bits/in.

1. Header record

Figure 2 shows the header record in detail.

2. Word format

bits	0	1	2	3	4	5	6	7	8	16	17
	S	S	S	S	S	X	X	X	X	X	X
bits	18	19	20	21	22	23	24			34	35
	S	S	S	S	S	X	X			X	X

X bits represent the integer magnitude (counts).

S bits are the sign bits.

0 0 0 0 0 = plus data

1 1 1 1 1 = negative data

If the number is negative the magnitude must be complemented.

HEADER RECORD

TEST.I	DENTIT	Y.....	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX
DATE..	XXXXXX	XXXXXX	TAPE..	bXXXXX	SAMPLE	bRATE.	XXXXXX	SAMPLE	bPERIO
D.....	XXXXXX	SKEWbb	±XXXXX	ENGR..	XXXXXX	OPER..	XXXXXX	LOC.bb	XXXXXX
XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	ID.bNO	bbbbbb	XXXXXX	FILE..	XXXXXX
CALIBR	ATION.	XXXXXX	XXXXXX	XXXXXX	RMKS..	XXXXXX	XXXXXX	XXXXXX	XXXXXX
XXXXXX	XXXXXX	XXXXXX	STARTb	TIMEbb	bbXXX:	XX:XX:	XX:XXX	OOOAAA	BBBCCC

X = INPUT BCD CHARACTER

b = BLANK

AAA = SAMPLE PERIOD (In binary with leading zeros)

BBB = SAMPLE RATE (In binary)

CCC = NUMBER OF WORDS IN PARTIAL RECORD (binary)

All fields are left adjusted with trailing blanks except where noted.

Fig. 2. Sample of header record

B. Control Cards

The shock-spectrum analysis program uses the NAMELIST feature of FORTRAN IV. Data may be punched in columns 2 through 80. All data must be separated by commas. A sample input card is shown in Fig. 3.

The following input symbols must be used:

Symbol	Type	Explanation
$INPUT		Start of data for one case.
NFILE	Fixed point	The largest file number on data tape to be used in run.
IFL	Fixed point	Program options 0 = no, 1 = yes IFL(1) = Rewind data tape between cases. IFL(2) = Read card data. IFL(3) = Plot data. IFL(4) = Punch input data. IFL(5) = Integrate and plot data. IFL(6) = Compute residual shock spectra. IFL(7) = Compute and plot shock spectra. IFL(8) = Printout shock spectra.
CID		Tape ID number (right adjusted). Input format is as follows: CID = 6H _ _ 7 1 2 5, where 7125 is the ID number on tape.
TS		Starting time in seconds.
TT		Total number of seconds to analyze.
NPS	Fixed point	Number of points to skip.

SHOCK SPECTRA PROG. - 5628

ENGINEER: ______ WORK ORDER NO.: ______ TAPE NO.: ______
RUN NO.: ______ TIME ESTIMATE: ______ NO. OF FILES: ______

1	2	
b	$INPUT	
b	CID=6H__6243 ,	IFL = Program Options (Fixed Point) 0 = No, 1 = Yes
b	NFILE= 3 ,	IFL(1) = Rewind data tape between cases
b	TS= 0. ,	IFL(2) = Read card data
b	TT= .09 ,	IFL(3) = Plot data
b	NPS= 0 ,	IFL(4) = Punch input data
b	SR= 0 ,	IFL(5) = Integrate and plot data
b	CALIB= 0. ,	IFL(6) = Compute residual shock spectra
b	FMIN= 10. ,	IFL(7) = Compute and plot shock spectra
b	ND= 3 ,	IFL(8) = Printout shock spectra
b	N= 20 ,	
b	NQ= 3 ,	
b	QQ= 0., 10., 20. ,	
b	IFL= 0, 0, 1, 0, 1, 1, 1, 0 ,	
b	$	

b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
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b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $
b	$INPUTbCID=6H______ ,	______ $

Fig. 3. Sample of input control data

Symbol	Type	Explanation
SR		Sample rate of data on tape. If SR = 0, program will use header record. SR is set = 0 between cases, so user must input SR for each case of data unless he wants to use the header record. (Program uses SR/NPS + 1.)
CALIB		Calibration value. If CALIB = 0, program uses header record. CALIB is set = 0 between cases, so user must input CALIB for each case unless the header record is used.
FMIN		Starting frequency.
ND	Fixed point	Number of decades (MAXIMUM = 5).
N	Fixed point	Number of frequencies per decade.
NQ	Fixed point	Number of Q values (MAXIMUM = 10).
QQ		Q values. If QQ = 0, program uses 1. E6.
$		END OF CASE

NAMELIST input permits the user to input only changes to the preceding case. The exception to this rule is CALIB and SR.

If IFL(2) = 1, the program expects raw data to be on cards. The following data must be added to input deck:

(1) The first card following the $ on the previous input data is a comment card. Any information desired by the user may be punched in card columns 1 through 72.

(2) Following the comment card, data must be punched with a 6E12. 5 format. The last data point should be a large number, i. e., 1. E25, to indicate end of data. A maximum of 6000 data points may be read in.

(3) Calibration value is set = 1., and mean is set = 0 for card data. User must supply all other required input.

III. EQUATIONS

A. Preliminary Calculations Used for Program Control

The effective sample rate is defined as

$$S_R = \frac{S_{RT}}{S_{RF}}$$

where

S_{RT} = true sample rate of data (May be input on control card or set = 0. If S_{RT} = 0, the sample rate on header record is used.)

S_{RF} = NPS + 1, where NPS is the number of points to skip (input)

The first and last data points used for the analysis are defined as

$$N_i = t_i S_R$$

$$N_t = N_i + \Delta t S_R$$

where

t_i = start time (input)

Δt = number of seconds to analyze (input)

The frequencies at which the spectra are computed are defined as

$$f_1 = f_{min}$$

$$f_i = f_{i-1} 10^{1/n}, \qquad i = 2, 3, \cdots, n$$

where

f_{min} = initial frequency (input)

n = number of frequencies per decade (input)

B. Data Conversion

The sample mean is computed for the data from the record of data preceding the record that contains t_i. If $t_i = 0$, the first data record is used. The sample mean is defined as

$$\bar{u} = \frac{1}{500} \sum_{i=1}^{500} u_i$$

where

u_i = data values in one record of data

The raw data are then transformed to have a zero mean value by the following relation:

$$\ddot{x}_i = (u_i - \bar{u})K, \qquad i = 1, 2, \cdots, N$$

where

N = total number of data points

K = calibration constant for data used

C. Data Integration (if requested by the user)

Velocity is defined by

$$\dot{x}_i = \Delta t \sum_{j=1}^{i-1} \ddot{x}_j + \frac{\Delta t \ddot{x}_i}{2}, \qquad i = 1, 2, \cdots, N$$

where

$$\Delta t = \frac{1}{S_R}$$

$\ddot{x}$ = acceleration data

N = total number of data points

Displacement is defined by

$$x_i = \Delta t \sum_{j=1}^{i-1} \dot{x}_j + \frac{\Delta t \dot{x}_i}{2}, \qquad i = 1, 2, \cdots, N$$

D. <u>Filter Coefficients</u>

$$A = \frac{2\pi f_i \Delta t}{Q_j}$$

$$B = Ae^{-A/2}\left[\frac{2Q_j^2 - 1}{\sqrt{4Q_j^2 - 1}}\right]\sin\left[\frac{A}{2}\sqrt{4Q_j^2 - 1}\right] - \cos\left[\frac{A}{2}\sqrt{4Q_j^2 - 1}\right]$$

$$C = Ae^{-A/2}\cos\left[\frac{A}{2}\sqrt{4Q_j^2 - 1}\right]$$

$$D = e^{-A}$$

where

$j = 1, 2, \cdots, NQ$

$i = 1, 2, \cdots, NF$

NQ = number of Q values (input)

NF = total number of frequencies

Q_j = Q values (input)

f_i = frequency at which shock spectra are computed

E. Filter Equation

The acceleration data are then filtered using the following transformation:

$$Y_1 = A\ddot{X}_1$$

$$Y_2 = A\ddot{X}_2 + B\ddot{X}_1 + CY_1$$

$$Y_k = A\ddot{X}_k + B\ddot{X}_{k-1} + CY_{k-1} + DY_{k-2}$$

where

$$k = 3, 4, 5, \cdots, NW$$

NW = total number of data points used

F. Peak Value of Response

The peak value of the response is determined as follows:

$$Y_m = [\Delta t\ 0.00764(Y_i + Y_{i+5}) - 0.06458(Y_{i+1} + Y_{i+4}) + 0.55694(Y_{i+2} + Y_{i+3})]$$

where

$$i = 1, 2, 3, \cdots, N$$

N = number of acceleration data points

Then, using the value of m where Y_m occurs, compute the following coefficients:

$$d = Y_{m-3} - Y_{m-4}$$

$$d_2 = (Y_{m-2} - 2Y_{m-3} + Y_{m-4})/2$$

$$d_3 = (Y_{m-1} - 3Y_{m-2} + 3Y_{m-3} - Y_{m-4})/6$$

$$d_4 = (Y_m - 4Y_{m-1} + 6Y_{m-2} - 4Y_{m-3} + Y_{m-4})/24$$

$$d_5 = (Y_{m+1} - 5Y_m + 10Y_{m-1} - 10_{m-2} + 5Y_{m-3} - Y_{m-4})/120$$

$$d_6 = (Y_{m+2} - 6Y_{m+1} + 15Y_m - 20Y_{m-1} + 15Y_{m-2} - 6Y_{m-3} + Y_{m-4})/720$$

$$d_7 = (Y_{m+3} - 7Y_{m+2} + 21Y_{m+1} - 35Y_m + 35Y_{m-1} - 21Y_{m-2} + 7Y_{m-3} - Y_{m-4})/5040$$

$$c_1 = (720d_7 - 120d_6 + 24d_5 - 6d_4 + 2d_3 - d_2 + d)/\Delta t$$

$$c_2 = (-1764d_7 + 274d_6 - 50d_5 + 11d_4 - 3d_3 + d_2)/\Delta t^2$$

$$c_3 = (1624d_7 - 225d_6 + 35d_5 - 6d_4 + d_3)/\Delta t^3$$

$$c_4 = (-735d_7 + 85d_6 - 10d_5 + d_4)/\Delta t^4$$

$$c_5 = (175d_7 - 15d_6 + d_5)/\Delta t^5$$

$$c_6 = (-21d_7 + d_6)/\Delta t^6$$

$$c_7 = d_7/\Delta t^7$$

where

$$\Delta t = \frac{1}{S_R}$$

Using the coefficients, compute the maximum area:

$$Y_A = Y_{m-4} + A(C_1 + A(C_2 + A(C_3 + A(C_4 + A(C_5 + A(C_6 + AC_7))))))$$

where

$$A = 3\Delta t$$

Then recompute Y_A using $A = A + 0.01\Delta t$ and continue until $Y_{A_{MAX}}$ is found; $Y_{A_{MAX}}$ is the value plotted for shock spectra using $\ddot{X}_i$ data from $i = N$, $N + 1, \ldots, NW$, and is computed for each value of Q_i, f_j where

$$i = 1, 2, \cdots, NQ \text{ (MAXIMUM} = 10)$$

$$j = 1, 2, \cdots, NF \text{ (MAXIMUM} = 500)$$

G. Residual Shock

The residual shock is computed as follows:

$$bt = \frac{1}{\sqrt{4Q_j^2 - 1}} \tan^{-1}\left[\frac{\sqrt{4Q_j^2 - 1}}{1 + 2Q_j\omega_0 \dfrac{Y_0}{\dot{Y}_0}}\right]$$

$$R = e^{-bt}\left[Y_0^2 + \left(\frac{\dot{Y}_0 + \dfrac{\omega_0 Y_0}{2Q_j}}{\dfrac{\omega_0}{2Q_j}\sqrt{4Q_j^2 - 1}}\right)^2\right]^{1/2}$$

where

$\omega_0 = 2\pi f_i$

Y_0 = last value of primary data

$\dot{Y}_0$ = 2-point slope using the last two filtered values of the primary data

$j = 1, 2, \cdots, NQ$

$i = 1, 2, \ldots, NF$

IV. DESCRIPTION OF OUTPUT

The following quantities are available to the user. Some quantities are optional, and may be requested on the input form.

A. Printout

1. Input data.
2. Header record.
3. ID, mean, number of data points, maximum data point, mean.
4. Primary/residual shock spectra (optional).

B. SC 4020 Plots

Plot label	Units	Independent variable
ACCELERATION	g	time
VELOCITY	g. sec	time
DISPLACEMENT	g sec^2	time
SHOCK SPECTRUM		
(Acceleration response peak)	g	frequency

Samples of output plots are shown in Fig. 4-9.

C. Punched Cards

One set of primary and residual shock spectra for each Q value is punched with appropriate identification. Figure 10 is a sample of the punched card output.

D. Punched Data Cards

If requested by the user, the raw data may be punched on cards. The first punched card is a header card, followed by the data punched with a 7E10. 3 format.

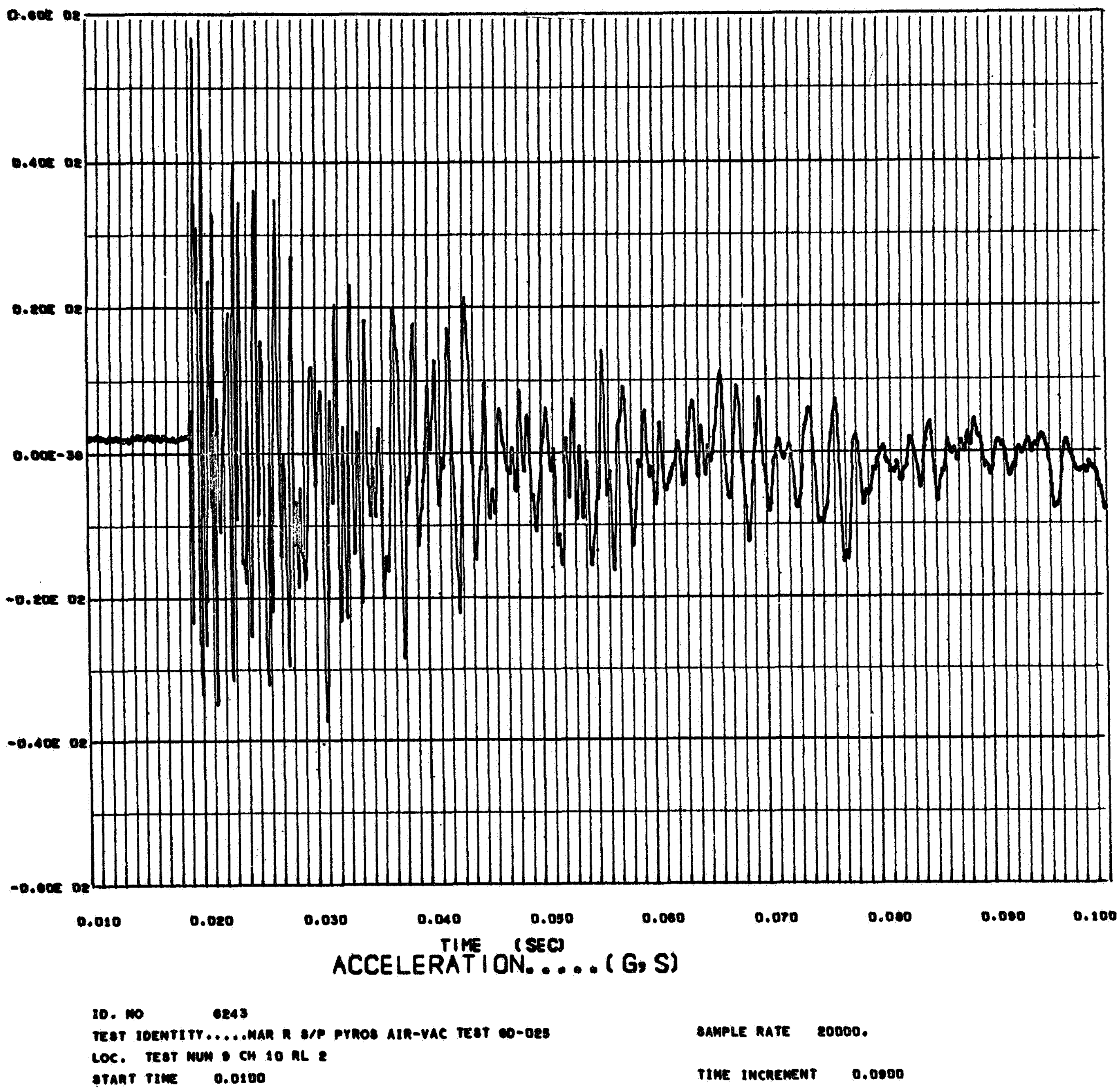
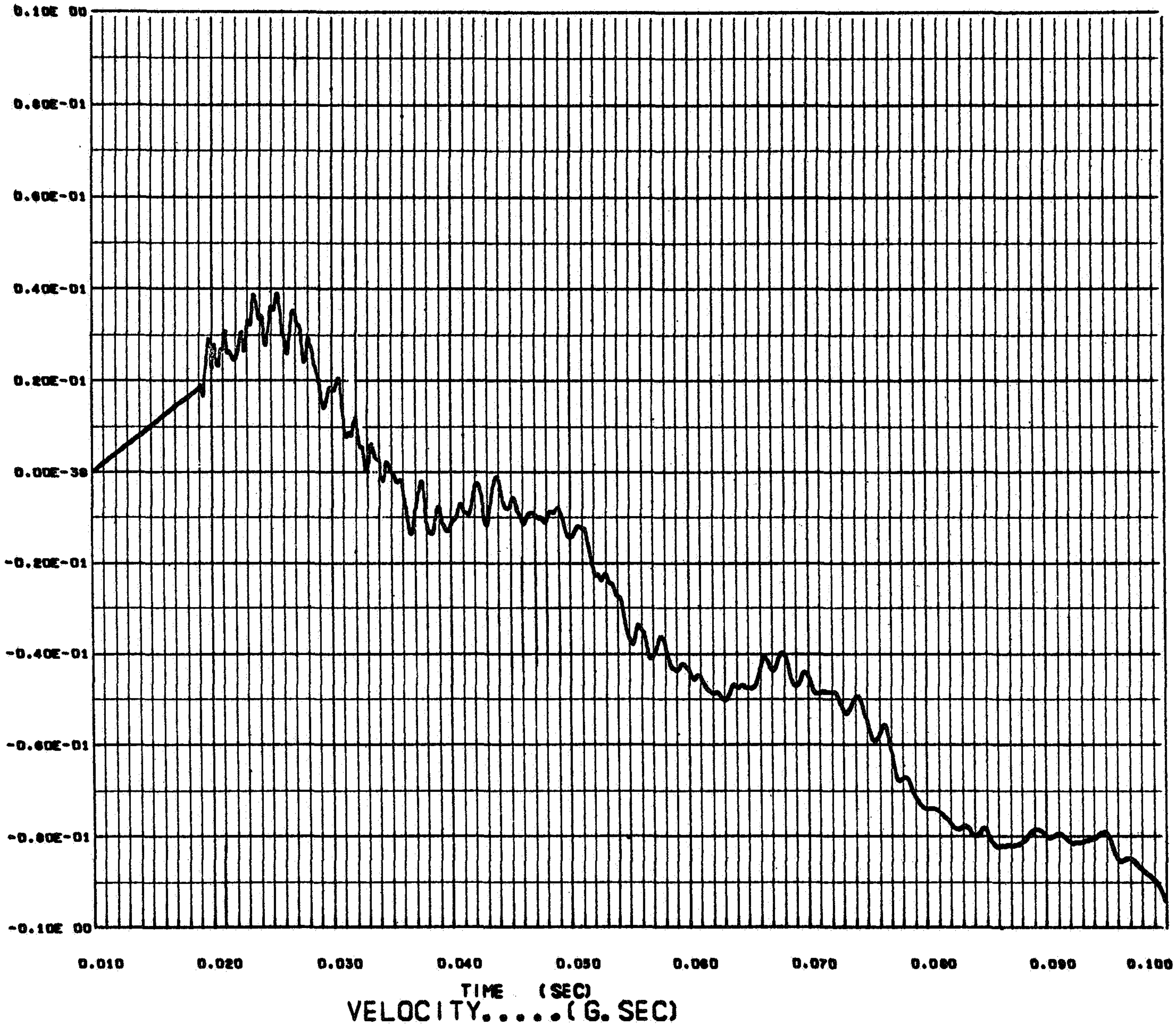


Fig. 4. Sample of acceleration plot



ID. NO 6243
TEST IDENTITY.....MAR R S/P PYROS AIR-VAC TEST 6D-025 SAMPLE RATE 20000.
LOC. TEST NUM 9 CH 10 RL 2
START TIME 0.0100 TIME INCREMENT 0.0900

Fig. 5. Sample of velocity plot

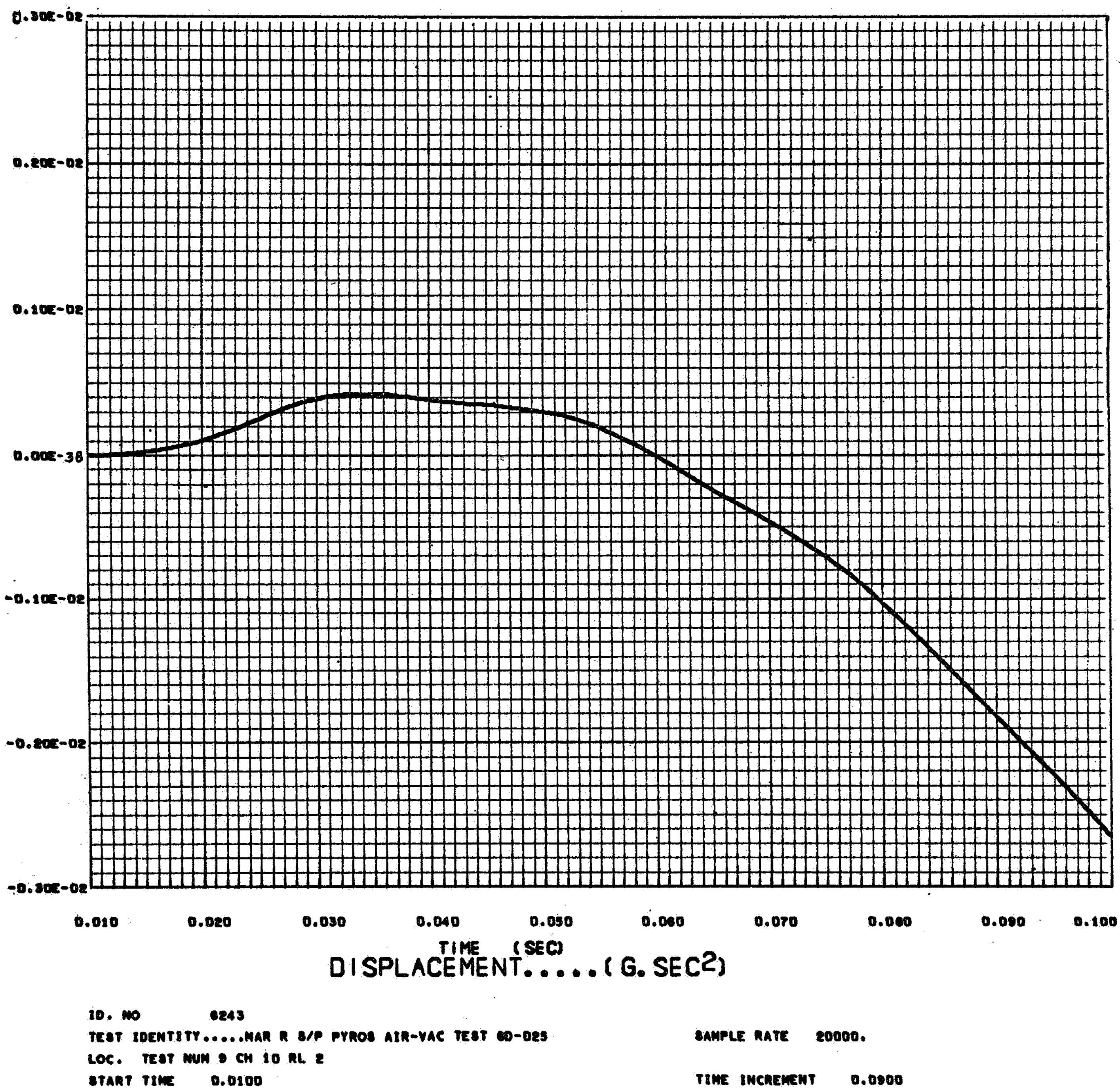


Fig. 6. Sample of displacement plot

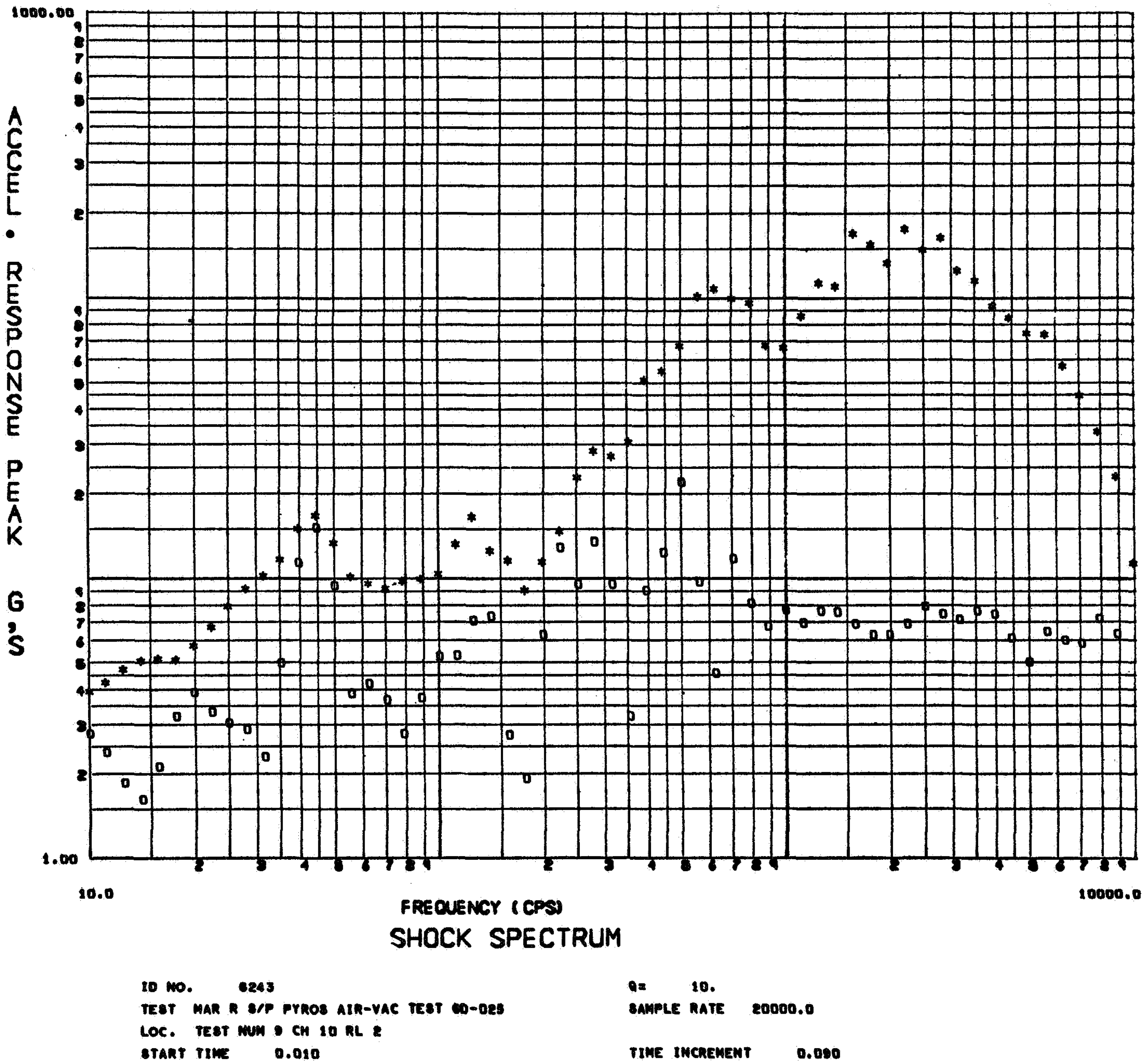


Fig. 7. Sample of shock-spectrum plot, Q = 10

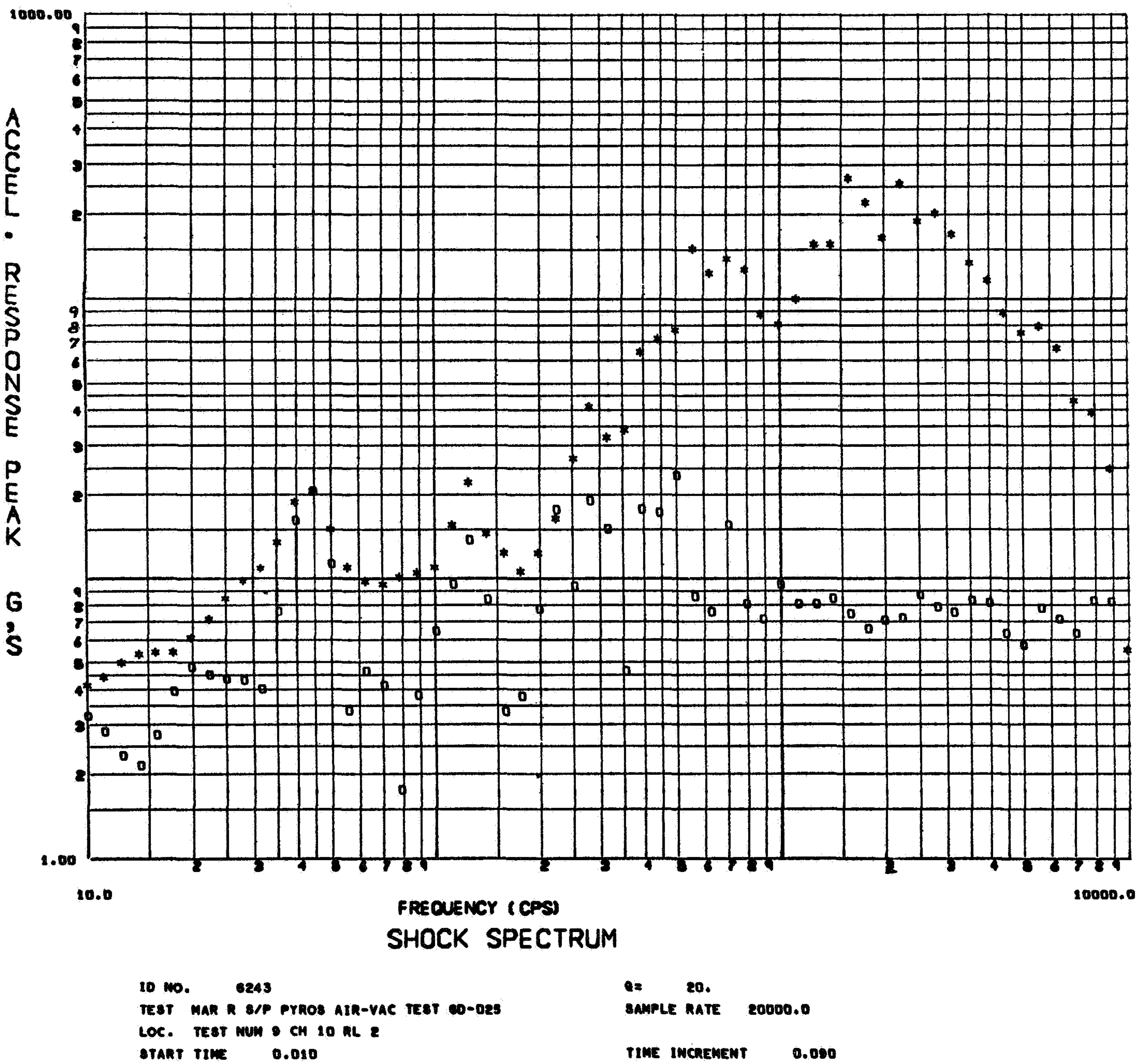


Fig. 8. Sample of shock-spectrum plot, Q = 20

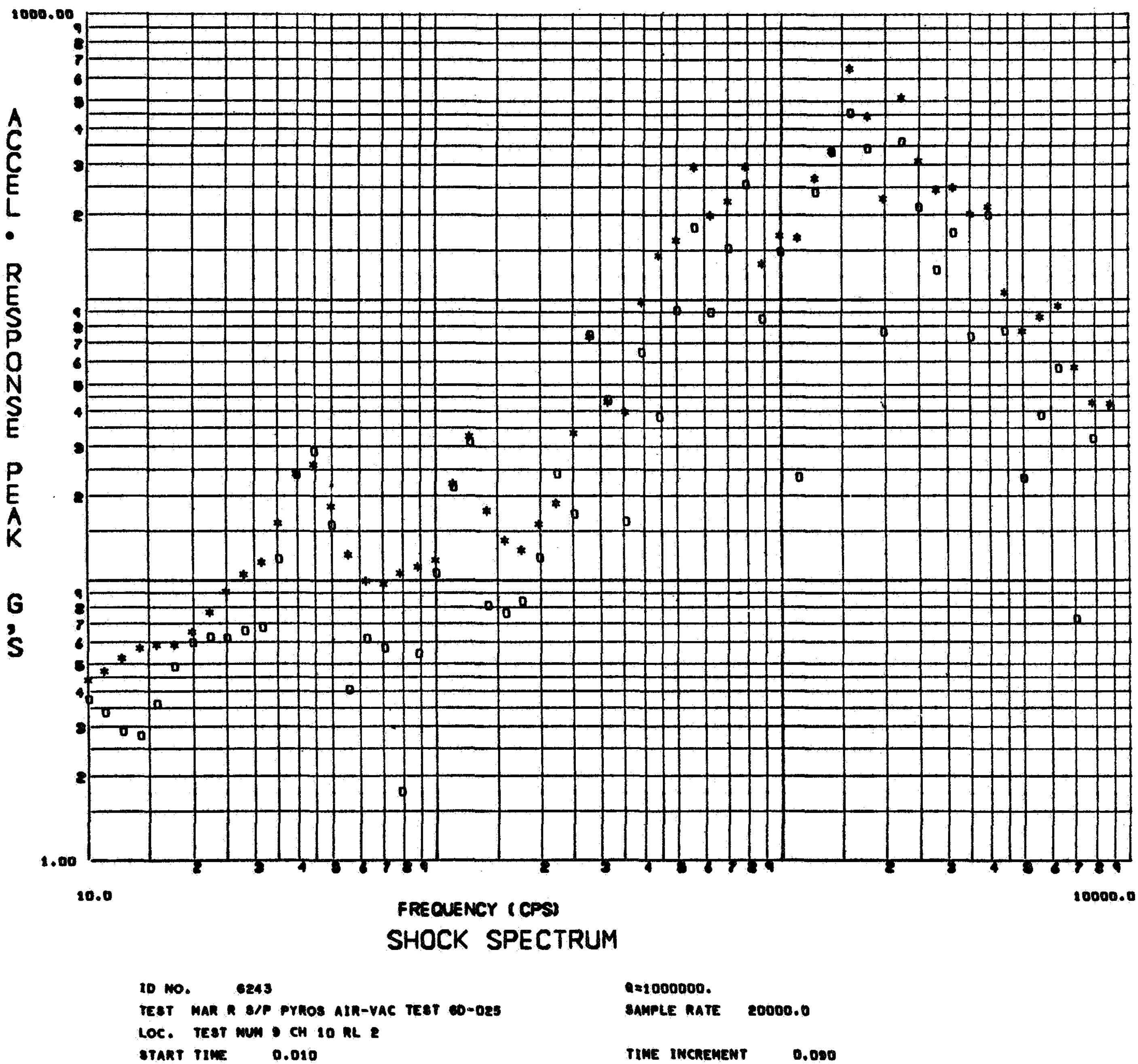


Fig. 9. Sample of shock-spectrum plot, Q = 1,000,000

```
TEST IDENTITY.....MAR R S/P PYROS AIR-VAC TEST 6D-025                                6243A00
TEST NUM 9 CH 10 RL 2                          10.00     3     20                    6243A01
 0.395E 01 0.424E 01 0.475E 01 0.510E 01 0.515E 01 0.512E 01 0.574E 01    6243A 2
 0.672E 01 0.801E 01 0.921E 01 0.103E 02 0.117E 02 0.151E 02 0.169E 02    6243A 3
 0.134E 02 0.101E 02 0.961E 01 0.933E 01 0.981E 01 0.999E 01 0.104E 02    6243A 4
 0.133E 02 0.167E 02 0.127E 02 0.116E 02 0.916E 01 0.115E 02 0.148E 02    6243A 5
 0.230E 02 0.287E 02 0.275E 02 0.309E 02 0.516E 02 0.552E 02 0.680E 02    6243A 6
 0.102E 03 0.108E 03 0.100E 03 0.967E 02 0.681E 02 0.668E 02 0.869E 02    6243A 7
 0.114E 03 0.111E 03 0.170E 03 0.156E 03 0.134E 03 0.177E 03 0.148E 03    6243A 8
 0.165E 03 0.126E 03 0.116E 03 0.938E 02 0.854E 02 0.749E 02 0.744E 02    6243A 9
 0.572E 02 0.448E 02 0.334E 02 0.231E 02 0.113E 02 0.278E 01 0.239E 01    6243A10
 0.186E 01 0.162E 01 0.212E 01 0.320E 01 0.392E 01 0.332E 01 0.304E 01    6243A11
 0.288E 01 0.231E 01 0.500E 01 0.115E 02 0.152E 02 0.946E 01 0.387E 01    6243A12
 0.421E 01 0.367E 01 0.277E 01 0.375E 01 0.528E 01 0.530E 01 0.708E 01    6243A13
 0.737E 01 0.277E 01 0.192E 01 0.630E 01 0.129E 02 0.957E 01 0.135E 02    6243A14
 0.953E 01 0.321E 01 0.908E 01 0.124E 02 0.221E 02 0.973E 01 0.456E 01    6243A15
 0.117E 02 0.812E 01 0.674E 01 0.767E 01 0.693E 01 0.760E 01 0.758E 01    6243A16
 0.684E 01 0.622E 01 0.627E 01 0.687E 01 0.793E 01 0.744E 01 0.710E 01    6243A17
 0.759E 01 0.739E 01 0.611E 01 0.499E 01 0.645E 01 0.598E 01 0.584E 01    6243A18
 0.716E 01 0.633E 01 0.755E 00-0.000E-19-0.000E-19-0.000E-19-0.000E-19    6243A19
     .         .         .         .         .         .         .           .
     .         .         .         .         .         .         .           .
     .         .         .         .         .         .         .           .
     .         .         .         .         .         .         .           .
     .         .         .         .         .         .         .           .
TEST IDENTITY.....MAR R S/P PYROS AIR-VAC TEST 6D-025                                6243C00
TEST NUM 9 CH 10 RL 2                          10.00     3     20                    6243C01
 0.441E 01 0.471E 01 0.526E 01 0.571E 01 0.586E 01 0.588E 01 0.657E 01    6243C 2
 0.770E 01 0.911E 01 0.105E 02 0.117E 02 0.161E 02 0.241E 02 0.260E 02    6243C 3
 0.185E 02 0.124E 02 0.994E 01 0.974E 01 0.106E 02 0.112E 02 0.119E 02    6243C 4
 0.223E 02 0.327E 02 0.177E 02 0.139E 02 0.128E 02 0.160E 02 0.190E 02    6243C 5
 0.336E 02 0.746E 02 0.434E 02 0.401E 02 0.982E 02 0.144E 03 0.162E 03    6243C 6
 0.295E 03 0.199E 03 0.223E 03 0.295E 03 0.133E 03 0.170E 03 0.165E 03    6243C 7
 0.270E 03 0.335E 03 0.654E 03 0.441E 03 0.228E 03 0.514E 03 0.308E 03    6243C 8
 0.244E 03 0.248E 03 0.201E 03 0.214E 03 0.106E 03 0.774E 02 0.868E 02    6243C 9
 0.946E 02 0.573E 02 0.430E 02 0.428E 02 0.149E-03 0.373E 01 0.335E 01    6243C10
 0.287E 01 0.278E 01 0.360E 01 0.492E 01 0.596E 01 0.625E 01 0.621E 01    6243C11
 0.660E 01 0.676E 01 0.120E 02 0.240E 02 0.290E 02 0.158E 02 0.406E 01    6243C12
 0.622E 01 0.571E 01 0.176E 01 0.547E 01 0.106E 02 0.216E 02 0.312E 02    6243C13
 0.814E 01 0.761E 01 0.843E 01 0.121E 02 0.242E 02 0.173E 02 0.750E 02    6243C14
 0.440E 02 0.162E 02 0.651E 02 0.384E 02 0.911E 02 0.180E 03 0.901E 02    6243C15
 0.152E 03 0.255E 03 0.852E 02 0.147E 03 0.233E 02 0.240E 03 0.331E 03    6243C16
 0.455E 03 0.341E 03 0.765E 02 0.361E 03 0.211E 03 0.127E 03 0.171E 03    6243C17
 0.738E 02 0.198E 03 0.770E 02 0.230E 02 0.385E 02 0.567E 02 0.725E 01    6243C18
 0.319E 02 0.416E 02 0.275E 00-0.000E-19-0.000E-19-0.000E-19-0.000E-19    6243C19
```

Fig. 10. Sample of card output

REFERENCE

1. Lane, D. W., Digital Shock Spectrum Analysis by Recursive Filtering, Shock and Vibration Bulletin No. 33, Part II, February 1964, Department of Defense, The Pentagon, Washington, D. C.