A Report on Session 12

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by

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Paper 12 A   Engineering Philosophy of Optimization in the Problem of Analytical Design of Optimal Controllers

by A. M. Letov   U.S.S.R.

In his paper Letov quotes Lord Russell with reference to the advantage the theorist has relative to an applied scientist. This has been expressed somewhat differently by Mark Twain and describes also the advantages of being a rapporteur. "To be good is to be noble; but to show others how to be good is nobler and less trouble."

The problem which Letov is raising is certainly a real one and one that is always faced in applying a mathematical theory to the solution of a concrete problem. Letov focuses his attention upon the problem in applying the theory of optimal control to the design and synthesis of control systems of how to identify good performance criteria (payoff functionals). This is certainly not the only difficulty in applying the theory of optimal control, but it is a central one and requires an answer. He views this as a problem within the theory and proposes a method for studying it.

I would like first to talk about a problem of a similar nature where the answer to the problem did come from theory. Liapunov was able to develop a mathematical theory of stability because he first gave precise definitions of the concepts of stability and asymptotic stability. A careful examination of the definitions themselves leads immediately to the question of whether or not this theory is of practical significance. Stability was defined for perturbations which are a sudden impulse which instantaneously displace the system from its equilibrium state and then no longer exist. By itself this should not satisfy the engineer but theory has shown that asymptotic stability (uniform asymptotic stability for nonautomous systems) implies a strong stability under perturbations that act not instantaneously but over all time. This was a consequence of the solution of the converse problem of stability. Liapunov's sufficiency conditions for asymptotic stability are also necessary. This does not answer all of the practical difficulties with the application of stability theory for we know, for example, that there are systems which are unstable which perform satisfactorily, and we also have the problem of how do we assure stability under perturbations of systems which act only over a finite period of time. Theory has provided some answers and with respect to the latter problem, this is included in Letov's secondary criteria.

The difficulty with defining optimal control to mean that control which minimizes a performance functional is that *per se* this may give no information about many practical characteristics of performance which are almost necessary criteria for satisfactory operation of the system and for feasibility of construction. Letov takes as one of his primary criteria that optimal control imply asymptotic stability and here theory has provided at least a sufficient condition. Asymptotic stability by itself is only a local property and this immediately suggests as a secondary criterion that the region of asymptotic stability be required to be sufficiently large. This brings out one of the more interesting aspects of Letov's paper and emphasizes a basic weakness in defining optimality in terms of the minimization of a functional. Simple examples show that although a properly selected performance criterion may imply asymptotic stability, the size of the region of asymptotic stability will depend upon the system being controlled. This means that one must give up, in general, the idea of selecting *a priori* performance functionals which will be "good" for all systems. Whether a performance criterion is "good" or not depends upon the system being controlled. The example considered by Letov in his paper illustrates this same point even more clearly.

Therefore we should perhaps give up this limited notion of optimality based on minimizing a performance functional and look for other definitions of optimality which, however, have the property that one can build around the concept a mathematical theory. It therefore might be useful to study the following type of converse problem: given a control system which satisfies criteria of the type which Letov proposes, which describe directly the performance of a system, can it be shown that this implies an optimality of a more general type?

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Paper 12 B Asymptotic Behaviour of Time-varying, Forced, Nonlinear, Discrete Systems

by G. P. Szegö Milan, Italy

The absolute stability of a nonautonomous system of difference equations (eq. 2.4) that is asymptotic in a restricted sense to a discrete Lurie system (eq. 2.3) is studied. Sufficient conditions for absolute attraction are obtained by Liapunov functions on the basis of some general results on asymptotically autonomous systems which are given in Section 5.

Section 4 on "Discrete Dynamical Systems" is a digression and has little relationship to the main problem of the paper. The section is concerned with autonomous systems whereas the remainder of the paper deals with nonautonomous systems. The results of this section, which are trivial, do not seem to be used elsewhere in the paper. The study of difference equations and discrete dynamical systems are both concerned with the study of the iterates of a transformation of the state space into itself. For discrete dynamical systems as defined here the transformation is topological and the two theorems in section 4 are obvious properties of the iterates of a transformation with an inverse. If in the definition of a discrete dynamical system I is taken to be the positive intergers (the motions define a semi-group), then the study of discrete dynamical systems and the study of difference equations are equivalent. The fact that the motions define a semi-group causes no difficulty in the development of a stability theory. This remark is made because one statement in the paper would seem to imply that there is some difference between studying discrete dynamical systems and difference equations.

Paper 12 C Stability of Two Classes of Non-linear Multivariable Control Systems

by W. G. Rae and G. D. S. MacLellan

Leicester, England

Lurie's technique for handling the problem of absolute stability is applied to a special case of a multiple nonlinear feedback system. The process is controlled by n feedback loops, each of which contains a nonlinear element and the feedback is of such a nature that the vector feedback function $f(y)$ has the property that $f(y) = (f_1(y_1), f_2(y_2), \cdots, f_n(y_n))$. As is pointed out by the authors, the two classes of problems considered are equivalent but the specific sufficient conditions that they give for the two classes of problems and also for special cases of each of these are undoubtedly useful.

This same treatment should carry over to the more general case when the nonlinear feedback has the property that the differential form $f(y) \cdot dy$ is exact. Reference should also have been made to S. Lefschetz, Stability of Nonlinear Control Systems, Academic Press, N.Y., 1965, 56-60. Note however that in this more general treatment by Lefschetz the assumption that $f(y) \cdot dy$ is exact needs to be stated explicitly.

The notation used in the matrix equation (4) is confusing since it contains what is later explained to be a vector on the right-hand side. What is meant is that the feedback matrix on the right-hand side has as its j-th column the vector $f_j^{(ij)}$.

Paper 12 D Theory of the Invariance and Autonomy in Multidimensional Essentially Non-linear Control Systems

by V. V. Pavlov Kiev, U.S.S.R.

The principal idea exploited in this paper is the following: if certain state variables represented by a vector X are not to depend upon disturbances F , then a necessary and sufficient condition for this is that its derivative $\dot{X}$ with respect to time should not depend upon F or upon anything else that depends upon F . This triviality is correct, since it is assumed implicitly that the initial conditions on X have the same property.

If this leads anywhere, no specific example is given to demonstrate that it does although general claims are made.

Paper 12 E On Estimation of Non-linear Transfer Systems from Plotted Step Responses

by Manfred Radtke Ilmenau, Germany

A method is proposed for determining by measurement the dynamic characteristics of a transmission element consisting of a single nonlinear characteristic with linear elements connected at the input and output. The formulas upon which the method is based depend upon the assumption that the non-linear characteristic is piecewise linear and that each of the linear elements can be characterized by two parameters - a dimension n and a time constant T . These restrictions are severe and the range of application of the method can be expected to be limited. For engineering applications it will always be necessary to check the model obtained by the method with the actual transmission element. Of course, the best that one can do is to check that at least in some respects the model conforms to the actual element.

Paper 12 F Energetic Stability of Nonlinear Systems

by J. Kudrewicz Warsaw, Poland

Input-output systems and feedback systems can be represented in terms of mappings of a function space into itself. The open loop gain and the cosine of the phase shift are defined in terms of the norm and the inner product. The feedback system is then said to be stable if zero excitation implies zero output. Here "zero" signal means a signal whose norm is zero; thus the interpretation of stability depends upon the norm selected for the space. In this particular paper the norm is taken to be the root-mean-square value of the signal and for this reason the elements of the function space are called "steady state signals". Sufficient conditions for stability can then be given in terms of the open loop gain and the open loop cosine of the phase shift. Applications to systems defined by differential equations and to sampled-data systems are discussed briefly. As the author points out, similar general studies of this type have been carried out by Zames and Sandberg, the difference being the way in which the function spaces are normed. Stability theorems are derived from basic inequalities satisfied by the norm and the inner product. The theory is neat and simple and yields results for nonlinear systems which are generalizations of well-known results for linear systems.

Paper 12 G Assynchronous Simultaneous Oscillations in Non-linear Systems

by D. M. Rosen Tel-Aviv, Israel

There is really not much that I have to say about this paper. The nonlinear problem is linearized by the use of a dual input describing function and criteria are given for determining the stability of double assynchronous oscillations in a system with a linear part and a single nonlinear element.

It would seem that a study of this problem in the light of what is now known about invariant manifolds in the theory of differential equations might lead to a better understanding of what is going on and could also provide a motivation for writing down the dual input describing function in much the same way as the original method of describing functions was motivated.

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By G. P. Szegö Milan, Italy

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