NASA CONTRACTOR REPORT

NASA CR-978

NASA CR-
c. 1

PLANE-STRESS ANALYSIS OF AN EDGE-STIFFENED RECTANGULAR PLATE, TAKING INTO ACCOUNT BENDING AND SHEAR STIFFNESS OF THE STIFFENERS

by Yu-wen Hsu and Charles Libove

Prepared by
SYRACUSE UNIVERSITY RESEARCH INSTITUTE
Syracuse, N. Y.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1968

NASA CR-978

PLANE-STRESS ANALYSIS OF AN EDGE-STIFFENED RECTANGULAR PLATE, TAKING INTO ACCOUNT BENDING AND SHEAR STIFFNESS OF THE STIFFENERS

~~By~~ Yu-wen Hsu and Charles Libove

~~Issued by Originator as Report No.~~ SURI-ME1065-678

~~Prepared under Grant No. NsG-385 by~~

SYRACUSE UNIV~~ERSITY~~ RES~~EARCH~~ INST~~ITUTE~~
~~Syracuse, N.Y.~~

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

PLANE-STRESS ANALYSIS OF AN EDGE-STIFFENED RECTANGULAR PLATE, TAKING INTO ACCOUNT BENDING AND SHEAR STIFFNESS OF THE STIFFENERS

By Yu-wen Hsu and Charles Libove
Syracuse University

SUMMARY

A plane-stress analysis, by means of Fourier series, is presented for an isotropic or orthotropic elastic rectangular plate bounded by four uniform edge stiffeners and subjected to any prescribed temperature distribution and boundary loads.

This analysis is related to an earlier one, in which the flexural stiffness of the edge stiffeners was assumed to be negligible and the plate was assumed to be attached to the stiffeners along the latter's centroidal axes. In the present analysis, both the extensional and flexural (including transverse shear) stiffnesses of the stiffeners are considered, and the possibility is included that the plate is attached to the stiffeners along lines which are offset from their centroidal axes. At each corner the junction between the two meeting stiffeners is assumed to consist of a hinge and a coil spring. By varying the stiffness of the coil spring, any degree of joint rigidity, from that of a pure hinge to a perfectly rigid joint, can be simulated.

Using this analysis, numerical results were obtained for a number of specific cases involving prescribed force loading or prescribed temperature distributions.

As a check on the validity of the method, stresses were measured on a doubly symmetric edge-stiffened square plate subjected to stiffener-end loads. Good agreement was obtained between the measured and computed values of the plate and stiffener stresses.

INTRODUCTION

The rectangular plate with four edge-stiffeners is one of the basic elements in aircraft structures. The wing skin and spars, for example, are usually composed of such elements, with the stiffeners provided by spar caps, rib caps, and shear-web uprights. A ring- and stringer-stiffened cylindrical shell used for the interstage structure of a launch vehicle may also be considered to be made up of a number of edge-stiffened rectangular plates. Although the plate is usually curved in this case, the curvature effect may be negligible if the distance between two neighboring stiffeners is sufficiently small compared to the radius of curvature.

In the present paper, an elastic plane-stress analysis of this basic unit, the rectangular plate with four edge stiffeners, is carried out by means of Fourier series, for the case of any prescribed temperature distribution, constant through thickness, and any equilibrium system of prescribed loads applied to the outer periphery of the stiffeners. The loads may include shear flows, running tensions, and stiffener end-tensions. This structure and loading are shown schematically in figure 1.

The present analysis is an extension of an earlier one (ref.1), in which the flexural stiffness of the edge stiffeners was assumed to be negligible, the plate was assumed to be integrally attached to the stiffeners along the latter's centroidal axes, and the applied shear flow loadings were assumed to be acting along the stiffener centroidal axes. In the present analysis, both the extensional and flexural (including transverse shear) stiffnesses of the stiffeners are considered, and (as shown in fig. 1) the possibility is included that the line of attachment between stiffener and plate and the line of action of the external applied shear flows are offset from the stiffener centroidal axis.

The plate may be isotropic or orthotropic, with elastic constants that are independent of position and, if orthotropic, with axes of elastic symmetry parallel to the edges. The four edge stiffeners are integrally attached to the plate and are uniform. At each corner, the junction between the two meeting stiffeners is assumed to consist of a hinge and a coil spring, as shown in figure 2. By varying the stiffness of the coil spring any degree of joint rigidity, from that of a pure hinge to a perfectly rigid joint, can be simulated.

The analysis starts with the most general case, in which no symmetry is assumed in either the structure, the loading, or the temperature distribution. A number of special cases with various symmetries and several limiting cases with zero or infinite flexural stiffness for the stiffeners are then obtained by reduction of the general case. Some of the limiting cases are physically equivalent to problems considered in earlier papers (refs. 1 and 2), and the equations for these cases are found to be equivalent to those obtained in the earlier papers.

Using the present analysis, computations were made for a number of specific cases involving prescribed forces or prescribed temperature distribution. Curves of plate and stiffener stresses are presented showing the influence of flexural and transverse shear stiffness of the stiffeners, eccentricity (with respect to stiffener axis) of the line of attachment between plate and stiffener, and type of joint (hinged or rigid) at the corner where the stiffeners meet.

As a check on the validity of the theoretical results, an experiment was conducted on an edge-stiffened square plate under stiffener end loads, and a comparison, showing generally good agreement, is presented of the measured and computed plate stresses, stiffener tensions and stiffener bending moments.

More detailed descriptions of the structure and loading will be given in the following sections of this report, along with the results of the analysis, the results of calculations, the details of the experiment, discussion and concluding remarks.

The symbols are defined when they are first used, and the more important ones are compiled for ease of reference in appendix A.

The details of the analysis are given in appendices B and C. A detailed reading of these appendices is not required for the understanding and use of the results presented in the main body of this paper.

Acknowledgement. This investigation was conducted at Syracuse University with the financial assistance of the National Aeronautics and Space Administration under research grant NsG-385. The computations were supported in part by the National Science Foundation grant GP-1137.

DETAILED DESCRIPTION OF STRUCTURE

Dimensions and Coordinate System. The plate and stiffener combination is shown schematically in figure 1. The plate has a length of a and a width of b, measured from lines of attachment to the stiffeners. Any point in the plate is identified by its coordinates x and y in a Cartesian reference frame whose origin is at one corner of the plate and whose axes coincide with two adjacent edges, as shown in the figure.

Stiffeners. The stiffeners are assumed to be uniform and will be treated as beams with shearing deformations permitted. That is, plane sections will be assumed to remain plane, but not necessarily perpendicular to the stiffener axis. The cross-sectional areas are denoted by A_1, A_2, A_3, and A_4 for the stiffeners located at $x = 0$, $x = a$, $y = 0$, and $y = b$ respectively. Similarly, E_1, E_2, E_3, E_4 will denote the Young's moduli of the stiffeners; G_1, G_2, G_3, G_4 their shear moduli; I_1, I_2, I_3, I_4 their cross-sectional moments of inertia about centroidal axes perpendicular to the plane of the plate; and A_{s1}, A_{s2}, A_{s3}, A_{s4} their effective cross-sectional areas for computing transverse shear stiffness in bending parallel to the plate. Thus, E_1I_1, E_2I_2, E_3I_3, E_4I_4 will denote the bending stiffenesses of the stiffeners located at $x = 0$, $x = a$, $y = 0$, and $y = b$ respectively; and G_1A_{s1}, G_2A_{s2}, G_3A_{s3}, G_4A_{s4} will denote the corresponding transverse shear stiffnesses.

The offset distance between the centroidal axis of a stiffener and its line of attachment to the plate is denoted by t_1^i, t_2^i, t_3^i or t_4^i, as shown in figure 1. Similarly the offset distance to the line of action of an external shear-flow loading is denoted by t_1^o, t_2^o, t_3^o or t_4^o in figure 1. If a stiffener is joined to the plate along several lines (e.g., two rows of rivets), the inner line will be regarded as the line of atachment for purposes of the present analysis. Any portion of the plate outside this line can be regarded as part of the stiffener.

Stiffener junctions. As shown in figure 2, two stiffeners meeting at a corner are assumed to be joined by a hinge and a coil spring, the hinge coinciding with the corner of the plate and therefore offset from the stiffener axes. The coil spring stiffnesses are denoted by k_1, k_2, k_3, and k_4 (moment per radian) for the corners located at (o,o), (a,o), (a,b), and (o,b), respectively. The ends of a coil spring at any corner are assumed to be attached to the cross-sections, rather than the axes, of the two stiffeners meeting at the corner.

In view of the fact that shearing deformations are being considered for the stiffeners, the relative rotation of the end cross-sections of two meeting stiffeners is not necessarily the same as the relative rotation of their axes. By setting the coil-spring stiffness equal to zero or infinity, one can simulate a hinge or a rigid junction, respectively.

Loading. The assumed loading is shown in figure 1. It consists of forces P'_y, P''_y, etc. applied to the centroids of the end cross sections of the stiffeners, and distributed tensions $N_1(y)$, $N_2(y)$, $N_3(x)$, $N_4(x)$ and shear flows $q_1(y)$, $q_2(y)$, $q_3(x)$, $q_4(x)$ applied externally along the stiffeners. The distributed tensions and shear flows have dimensions of force per unit length. The loading as a whole is assumed to constitute an equilibrium system.

Thermal strains. The temperature distribution, and hence the thermal deformations corresponding to unrestrained thermal expansion of each infinitesimal plate element or each infinitely thin stiffener slice, are assumed to be known. The notation for these thermal deformations is indicated partially in figure 3. It is as follows: In the plate the thermal strains are $e_x(x,y)$ and $e_y(x,y)$ in the x- and y- directions, respectively. There is no thermal shear strain in the plate relative to the x and y axes, because these axes are parallel to the directions of elastic symmetry of the plate. If the stiffeners were cut into infinitely thin slices by means of sections perpendicular to their axes, then, in view of the assumption that plane cross sections remain plane, the thermal deformation of each slice could be described by means of two quantities: the strain and the curvature of the centroidal fiber of the slice. The former will be denoted by $e_1(y)$, $e_2(y)$, $e_3(x)$, and $e_4(x)$ for the stiffeners located at $x = 0$, $x = a$, $y = 0$, $y = b$, respectively. The latter (the thermal curvatures, not shown in figure 3) will be denoted by $\kappa_1(y)$, $\kappa_2(y)$, $\kappa_3(x)$, $\kappa_4(x)$, respectively and will be considered positive if they correspond to elongation of the inner fibers. If there is no variation of temperature through the depth of the stiffeners, these κ's will be zero.

The thermal deformations are assumed to be measured relative to some datum temperature distribution for which the plate is stress-free and the stiffener cross sections free of resultant thrust and bending moment.

Notation for total strains. The total strains (thermal plus elastic) at the stiffener axes are denoted by $\varepsilon_1(y)$, $\varepsilon_2(y)$, $\varepsilon_3(x)$ and $\varepsilon_4(x)$ for the stiffeners located at $x = 0$, $x = a$, $y = 0$ and $y = b$ respectively. The total normal strains in the plate are represented by $\varepsilon_x(x,y)$ in the x-direction and $\varepsilon_y(x,y)$ in the y-direction. The plate shear strain is symbolized by $\gamma_{xy}(x,y)$.

Notation for internal forces. Figure 4 indicates the notation employed for the internal forces in the stiffeners and plate. $P_1(y)$, $P_2(y)$, $P_3(x)$, $P_4(x)$ denote the cross-sectional tensions in the stiffeners located at $x = 0$, $x = a$, $y = 0$, $y = b$, respectively.

The corresponding stiffener bending moments (about centroidal axes) are denoted by $M_1(y)$, $M_2(y)$, $M_3(x)$, $M_4(x)$, and the transverse shears by $V_1(y)$, $V_2(y)$, $V_3(x)$, $V_4(x)$. The plate normal-stress resultants (force per unit length) are represented by $N_x(x,y)$ and $N_y(x,y)$ and the shear-stress resultant by $N_{xy}(x,y)$.

The corner moments produced by the coil springs at stiffener junctions are denoted by $\overline{M}_1$, $\overline{M}_2$, $\overline{M}_3$ and $\overline{M}_4$ (see fig. 4) for the plate corners (0,0), (a,0), (a,b) and (0,b), respectively. As implied in figure 4, a corner moment is considered to be positive if it corresponds to a reduction of the angle between the two neighboring stiffener end cross sections on which it acts. Because of the possible eccentric mutual reactions, $V_1(0)$, $V_1(b)$, etc., at the stiffener ends (see fig. 4) the corner moments $\overline{M}_1$, $\overline{M}_2$, etc. are in general not identical to the limiting values of the stiffener bending moments $M_1(0)$, $M_3(0)$, $M_2(0)$, $M_3(a)$, etc. as the stiffener ends are approached. Instead, they are related to each other by equations (B77).

<u>Stress-strain relations</u>. With the above notations established, the assumed stress-strain relations for the components of the structure can now be described. For the stiffeners they are as follows:

$$\varepsilon_\alpha = e_\alpha + \frac{P_\alpha}{A_\alpha E_\alpha} \quad (\alpha = 1,2,3,4) \tag{1}$$

with the Young's moduli E_1 and E_2 independent of Y, E_3 and E_4 independent of x. Equation (1) gives the strains along the axes of the stiffeners. The strains in the stiffeners along their lines of attachment to the plate are obtained by adding to these the strains due to bending moment and temperature variation across the depth of the stiffener. Thus the stiffener strains along the lines of attachment are

$$e_\alpha + \frac{P_\alpha}{A_\alpha E_\alpha} + \left(\frac{M_\alpha}{E_\alpha I_\alpha} + \kappa_\alpha\right) t^i_\alpha \qquad (\alpha = 1,2,3,4) \tag{2}$$

for the stiffeners at x = 0, x = a, y = 0, y = b, respectively.

The plate stress-strain-displacement relations are taken to be

$$\frac{\partial u}{\partial x} = \varepsilon_x = e_x + C_1 N_x - C_3 N_y$$

$$\frac{\partial v}{\partial y} = \varepsilon_y = e_y + C_2 N_y - C_3 N_x$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy} = C_4 N_{xy} \tag{3}$$

where u(x,y) and v(x,y) are the x-wise and y-wise displacement components, and the compliances C_1, C_2, C_3 and C_4 are independent of x and y. If the plate is homogeneous and isotropic, with thickness h, Young's modulus E, and Poisson's ratio ν, then

$$C_1 = C_2 = \frac{1}{Eh}$$

$$C_3 = \frac{\nu}{Eh}$$

$$C_4 = \frac{2(1 + \nu)}{Eh}$$

In order to describe the assumed moment-curvature relations for the stiffeners, it will be convenient to introduce a notation for the displacements of the stiffeners in direction perpendicular to their axes. Let $u_1^*(y)$ and $u_2^*(y)$ denote the x-wise displacements of points along the axes of the stiffeners located at $x = 0$ and $x = a$, respectively; and similarly let $v_3^*(x)$ and $v_4^*(x)$ denote the y-wise displacements of the stiffeners along $y = 0$ and $y = b$, respectively. Then the curvatures of the stiffener axes can be represented by $d^2u_1^*/dy^2$, $d^2u_2^*/dy^2$, $d^2v_3^*/dx^2$ and $d^2v_4^*/dx^2$, and the following relationships are assumed between these curvatures and the forces and moments acting on the stiffeners:

$$\left.\begin{aligned}
\frac{d^2u_1^*}{dy^2} &= -\frac{M_1(y)}{E_1I_1} - \frac{1}{G_1A_{s1}}\,[N_x(0,y) - N_1(y)] - \kappa_1(y) \\
-\frac{d^2u_2^*}{dy^2} &= -\frac{M_2(y)}{E_2I_2} - \frac{1}{G_2A_{s2}}\,[N_x(a,y) - N_2(y)] - \kappa_2(y) \\
\frac{d^2v_3^*}{dx^2} &= -\frac{M_3(x)}{E_3I_3} - \frac{1}{G_3A_{s3}}\,[N_y(x,0) - N_3(x)] - \kappa_3(x) \\
-\frac{d^2V_4^*}{dx^2} &= -\frac{M_4(x)}{E_4I_4} - \frac{1}{G_4A_{s4}}\,[N_y(x,b) - N_4(x)] - \kappa_4(x)
\end{aligned}\right\} \quad (4)$$

The first term on the right side of each equation represents the curvature due to bending moment, the second the curvature of the stiffener axis due to rate of change of transverse shear, and the last the curvature produced by temperature variation across the stiffener. The stiffeners are assumed to be so constituted that transverse shear strain arises only from transverse shear force; that is, the stiffener temperature distribution per se does not destroy the normality between stiffener cross sections and stiffener axis.

In anticipation of the subsequent imposition of continuity conditions between stiffener and plate edge, it may be noted, parenthetically, that the curvature along any constant x line in the plate is given by

$$\frac{\partial^2 u}{\partial y^2} = C_4 \frac{\partial N_{xy}}{\partial y} - \frac{\partial e_y}{\partial x} - C_2 \frac{\partial N_y}{\partial x} + C_3 \frac{\partial N_x}{\partial x} \tag{5a}$$

and along any constant y line by

$$\frac{\partial^2 v}{\partial x^2} = C_4 \frac{\partial N_{xy}}{\partial x} - \frac{\partial e_x}{\partial y} - C_1 \frac{\partial N_x}{\partial y} + C_3 \frac{\partial N_y}{\partial y} \tag{5b}$$

These expressions can easily be derived from equations (3).

At the corners, where the stiffeners meet, the relative rotation of the stiffener axes, positive for a reduction of the angle between them, is

$$\left.\begin{array}{ll} -\dfrac{\bar{M}_1}{k_1} + \dfrac{V_1(o)}{G_1 A_{s1}} + \dfrac{V_3(o)}{G_3 A_{s3}} & \text{at } x = 0,\ y = 0; \\[2ex] -\dfrac{\bar{M}_2}{k_2} + \dfrac{V_2(o)}{G_2 A_{s2}} - \dfrac{V_3(a)}{G_3 A_{s3}} & \text{at } x = a,\ y = 0; \end{array}\right\} (A)$$

$$- \frac{\bar{M}_3}{k_3} - \frac{V_2(b)}{G_2 A_{s2}} - \frac{V_4(a)}{G_4 A_{s4}} \qquad \text{at} \quad x = a, \; y = b;$$

and

$$- \frac{\bar{M}_4}{k_4} - \frac{V_1(b)}{G_1 A_{s1}} + \frac{V_4(0)}{G_4 A_{s4}} \qquad \text{at} \quad x = 0, \; y = b;$$

The "angle changes of the plate" at the corresponding corners are given by

$$\left.\begin{array}{l} C_4 (N_{xy})_{x=0,\; y=0} \\ - C_4 (N_{xy})_{x=a,\; y=0} \\ C_4 (N_{xy})_{x=a,\; y=b} \\ \text{and} \qquad - C_4 (N_{xy})_{x=0,\; y=b} \end{array}\right\} \text{(B)}$$

respectively. In the subsequent analysis (Appendix B) the two sets of angle changes, (A) and (B), will have to be equated.

SERIES EXPANSIONS FOR PRESCRIBED LOADS AND THERMAL STRAINS

The results of the present analysis, to be discussed shortly, consists of formulas for the stiffener and plate stresses, stiffener moments and transverse shears, in terms of the given loading and thermal-strain distribution. However, the loading and thermal strains do not appear explicitly in these formulas, it is rather the Fourier coefficients of these quantities that are required. In anticipation of this requirement, it is assumed that the given distributed loadings can be expressed in Fourier series of the following form, with known coefficients:

$$N_1(y) = \sum_{n=1}^{N} B'_n \sin\left(\frac{n\pi y}{b}\right)$$

$$N_2(y) = \sum_{n=1}^{N} B''_n \sin\left(\frac{n\pi y}{b}\right) \tag{6}$$

$$N_3(x) = \sum_{m=1}^{M} B'''_m \sin\left(\frac{m\pi x}{a}\right)$$

$$N_4(x) = \sum_{m=1}^{M} B''''_m \sin\left(\frac{m\pi x}{a}\right)$$

$$q_1(y) = \sum_{n=0}^{N} Q'_n \cos\left(\frac{n\pi y}{b}\right)$$

$$q_2(y) = \sum_{n=0}^{N} Q''_n \cos\left(\frac{n\pi y}{b}\right)$$

$$q_3(x) = \sum_{m=0}^{M} Q'''_m \cos\left(\frac{m\pi x}{a}\right)$$

$$q_4(x) = \sum_{m=0}^{M} Q''''_m \cos\left(\frac{m\pi x}{a}\right) \tag{7}$$

The known curvatures of the stiffeners arising from the known temperature variation across the stiffeners are also assumed to be representable by Fourier series with known coefficients, as follows:

$$\kappa_1(y) = \sum_{n=1}^{N} K'_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\kappa_2(y) = \sum_{n=1}^{N} K''_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\kappa_3(x) = \sum_{m=1}^{M} K'''_m \sin\left(\frac{m\pi x}{a}\right) \tag{8}$$

$$\kappa_4(x) = \sum_{m=1}^{M} K''''_m \sin\left(\frac{m\pi x}{a}\right)$$

Similarly, certain thermal-strain differences and certain first derivatives of the plate thermal strains at boundary are assumed to be known in the form of Fourier series, as follows:

$$\left.\begin{aligned}
e_1(y) - e_y(0,y) &= \sum_{n=1}^{N} T_n{}' \sin\left(\frac{n\pi y}{b}\right) \\
e_2(y) - e_y(a,y) &= \sum_{n=1}^{N} T_n{}'' \sin\left(\frac{n\pi y}{b}\right) \\
e_3(x) - e_x(x,0) &= \sum_{m=1}^{M} T_m{}''' \sin\left(\frac{m\pi x}{a}\right) \\
e_4(x) - e_x(x,b) &= \sum_{m=1}^{M} T_m{}'''' \sin\left(\frac{m\pi x}{a}\right)
\end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned}
\left(\frac{\partial e_y}{\partial x}\right)_{x=0} &= \sum_{n=1}^{N} L_n{}' \sin\left(\frac{n\pi y}{b}\right) \\
\left(\frac{\partial e_y}{\partial x}\right)_{x=a} &= \sum_{n=1}^{N} L_n{}'' \sin\left(\frac{n\pi y}{b}\right) \\
\left(\frac{\partial e_x}{\partial y}\right)_{y=0} &= \sum_{n=1}^{M} L_m''' \sin\left(\frac{m\pi x}{a}\right) \\
\left(\frac{\partial e_x}{\partial y}\right)_{y=b} &= \sum_{m=1}^{M} L_m'''' \sin\left(\frac{m\pi x}{a}\right)
\end{aligned}\right\} \quad (10)$$

Finally, $\partial^2 e_y/\partial x^2 + \partial^2 e_x/\partial y^2$ is assumed to be known and representable by the following series in the open region $0<x<a$, $0<y<b$:

$$\frac{\partial^2 e_y}{\partial x^2} + \frac{\partial^2 e_x}{\partial y^2} = \sum_{m=1}^{M} \sum_{n=1}^{N} T_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (11)$$

Finite upper limits M and N are shown for the summation indexes in equations (6) to (11) in expectation of the fact that it will normally be necessary to use truncated rather than infinite series for practical computational reasons.

The Fourier coefficients appearing in equations (6) to (11) can be determined from the usual definitions. For example,

$$B_n' = \frac{2}{b} \int_0^b N_1(y) \sin \frac{n\pi y}{b} \, dy$$

$$Q_n' = \frac{2-\delta_{no}}{b} \int_0^b q_1(y) \cos \frac{n\pi y}{b} \, dy$$

$$T_n' = \frac{2}{b} \int_0^b [e_1(y) - e_y(0,y)] \sin \frac{n\pi y}{b} \, dy$$

$$T_{mn} = \frac{4}{ab} \int_0^b \int_0^a \left(\frac{\partial^2 e_y}{\partial x^2} + \frac{\partial^2 e_x}{\partial y^2} \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dxdy \tag{12}$$

where δ_{no} is Kronecker's delta. Integration by parts in equation (12) gives the following alternate formula which permits T_{mn} to be evaluated from the first derivatives of e_y and e_x instead of the second derivatives:

$$T_{mn} = - \frac{m\pi}{a} \frac{4}{ab} \int_0^b \int_0^a \frac{\partial e_y}{\partial x} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dxdy$$

$$- \frac{n\pi}{b} \frac{4}{ab} \int_0^a \int_0^b \frac{\partial e_x}{\partial y} \cos \frac{n\pi y}{b} \sin \frac{m\pi x}{a} \, dydx \tag{13}$$

Equation (13) can be used for discontinuous e_y or e_x provided that $\partial e_y/\partial x$ and $\partial e_x/\partial y$ are regarded to be infinite, in the manner of the Dirac delta function, at the loci of points of discontinuity. If e_y and e_x are continuous in the closed region $0 \leq x \leq a$, $0 \leq y \leq b$, further integration by parts gives

$$T_{mn} = -\frac{m\pi}{a}\frac{4}{ab}\int_0^b [e_y(a,y)\cos m\pi - e_y(0,y)]\sin\frac{n\pi y}{b}\,dy$$

$$-(\frac{m\pi}{a})^2\frac{4}{ab}\int_0^b\int_0^a e_y(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy$$

$$-\frac{n\pi}{b}\frac{4}{ab}\int_0^a [e_x(x,b)\cos n\pi - e_x(x,0)]\sin\frac{m\pi x}{a}\,dx$$

$$-(\frac{n\pi}{b})^2\frac{4}{ab}\int_0^a\int_0^b e_x(x,y)\sin\frac{n\pi y}{b}\sin\frac{m\pi x}{a}\,dydx \qquad (14)$$

RESULTS OF ANALYSIS

The analysis in Appendix B gives equations for the plate stress resultants, the stiffener tensions, bending moments and transverse shears, mainly in the form of Fourier series. In the first of the following subsections, these equations are pointed out, that is, their location in Appendix B is given. Subsection (2) tells how the Fourier coefficients and other unknowns appearing in these equations can be computed and gives the basic equations for the general case and the simplified form of the basic equations for various symmetrical and antisymmetrical cases. Subsection (3) provides simplified basic equations for several limiting cases (zero eccentricity between plate edge and stiffener axis, zero or infinite flexural and shear stiffness for the stiffeners). The subsections (4) and (5) describe and give results for twenty-two numerical examples, showing the effect of varying selected parameters. The last subsection (6) provides a discussion for the numerical results and indicates the importance of the parameters that are considered.

(1) Equations for the plate and stiffener stresses. The equations for the plate stress resultants, the stiffener tensions, bending moments, and transverse shears are given in Appendix B and are summarized in Table 1.

TABLE 1. SUMMARY OF PLATE AND STIFFENER STRESS RESULTANTS

<table>
<tr><th>QUANTITY</th><th>SYMBOL</th><th>EQUATION</th><th>REGION OF VALIDITY</th></tr>
<tr><td>Plate Shear Stress Resultants</td><td>N_{xy}</td><td>(B34)</td><td>Entire Plate</td></tr>
<tr><td rowspan="3">Plate Normal Stress Resultants in y-direction</td><td rowspan="3">N_y</td><td>(B24)</td><td>Entire Plate, excluding edges</td></tr>
<tr><td>(B25)-(B28)</td><td>Edges, excluding corners</td></tr>
<tr><td>Last four equations of (B133)</td><td>Corners</td></tr>
</table>

TABLE 1 (continued). SUMMARY OF PLATE AND STIFFENER STRESS RESULTANTS

QUANTITY	SYMBOL	EQUATION	REGION OF VALIDITY
Plate Normal Stress Resultants in x-direction	N_x	(B29)	Entire Plate, excluding edges
		(B30)-(B33)	Edges, excluding corners
		First four equations of (B133)	Corners
Stiffener Transverse Shear Forces	V_i (i=1,2,3,4)	(B15)	Entire Stiffener
Stiffener Axial Tensions	P_i (i=1,2,3,4)	(B13)	Entire Stiffener, excluding ends
	$P_i(0), P_i(b)$ (i=1,2) $P_j(0), P_j(a)$ (j=3,4)	(B76)	Stiffener ends (x=0+, y=0+; x=a-, y=b-)
Corner Moments Produced By Coil Springs	$\tilde{M}_i$ (i=1,2,3,4)	(B129)-(B132)	
Stiffener Bending Moments	M_i (i=1,2,3,4)	(B14)	Entire Stiffener, excluding ends
	$M_i(0), M_i(b)$ (i=1,2) $M_j(0), M_j(a)$ (j=1,2)	(B77)	Stiffener ends (x=0+, y=0+, x=a-, y=b-)

Table 1 is self-explanatory. However it may be worthwhile calling attention explicitly to certain aspects of the equations cited in it. For example, it is noted that equation (B24) given for the plate stress resultant N_y, is not valid along the plate edges. Special equations, (B25) to (B28) are therefore given in Table 1 for the plate stress resultant N_y along the plate edges. These special equations are, in turn, not valid at the plate corners, and other equations, (B133), are therefore indicated for the plate corners. Analogous remarks apply to the plate stress resultant N_x, the stiffener axial tensions, and the stiffener bending moments. It was noted earlier that because of the mutual reactions $V_1(b)$, $V_4(0)$, etc. at the stiffener junctions (see fig. 4), the limiting internal values of the stiffener bending moments, namely $M_4(0)$, $M_1(b)$, etc., are not necessarily equal to the corner coil spring moments $\bar{M}_4$, etc. For the same reason the limiting values of the stiffener tensions, $P_4(0)$, $P_1(b)$,etc., are not in general equal to the externally applied stiffener end loads P_x''', P_y'', etc.

Evaluation of series coefficients and other basic unknowns. In order to use the equations listed in Table 1 for numerical calculation of stresses, etc., one must first evaluate the Fourier coefficients c_n', c_n'', c_m''', c_m'''', g_m', g_m'', g_n''', g_n'''', j_{mn}, s_n', s_n'', s_m''', s_m'''', b_n', b_n'', b_m''', b_m'''', and v_n', v_n'', v_m''', v_m'''' appearing in them, as well as the corner moments $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, $\bar{M}_4$, and the stiffener end shears $v_1(0)$, $v_1(b)$, etc. The first eight groups of coefficients, namely, c_n' through g_n'''', and the four corner moments $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, $\bar{M}_4$, are the key to the calculation of the other unknowns. Once these key unknowns are evaluated, the following sequence of steps will lead to the remaining unknowns: From equations (B111) to (B118) compute the stiffener end shears $v_1(0)$, $v_1(b)$, etc.; from equations (B76) the stiffener end tensions; from equations (B77) the stiffener end moments; and from equations (B92) and (B94) to (B99) the remaining Fourier coefficients for the plate stresses (j_{mn}), stiffener tensions (s_n', etc.), stiffener bending moments (b_n', etc.) and stiffener transverse shears (v_n', etc.). Thus the basic objective in the rest of this section is to describe the calculation procedure for the key unknowns, c_n' through g_n'''', $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, and $\bar{M}_4$.

These key unknowns are defined by various systems of simultaneous equations in Appendix B, depending upon the type of symmetry of the structure and loading and the type of connection at the stiffener junctions. These various systems of equations are summarized in Table 2. Those cases which have been studied numerically, and for which computed results will be given subsequently, are indicated in the table by heavily outlined boxes. The box with a double border in the upper left corner of Table 2 contains the most general equations; the equations for all the other cases represented in Table 2 can be obtained by reduction from this most general case.

TABLE 2. EQUATIONS FOR COMPUTING c's, g's AND $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, $\bar{M}_4$

PROPERTIES OF SYMMETRY	CORNER CONDITIONS (SAME TYPE AT ALL CORNERS)		
	ELASTIC-JOINTED	HINGE-JOINTED	RIGID-JOINTED
NONE (general case)	(B119) to (B126) and (B129) to (B132)	(B119) to (B126) with $\bar{M}_1=\bar{M}_2=\bar{M}_3=\bar{M}_4=0$	(B119) to (B126) and (B129) to (B132) with $k_1=k_2=k_3=k_4=\infty$
SINGLE SYMMETRY: Structure and loading symmetrical about $y=\frac{b}{2}$.	(B135) to (B142)	(B135) to (B140) with $\bar{M}_1=\bar{M}_2=0$	(B135) to (B142) with $k_1=k_2=\infty$
DOUBLE SYMMETRY: Structure and loading symmetrical about $y=\frac{b}{2}$ and $x=\frac{a}{2}$.	(B149) to (B152); $\bar{M}$ given by (B148)	(B144) to (B147) with $\bar{M}=0$	(B144) to (B148) with $k = \infty$
ENTIRE SYMMETRY: Square plate; structure and loading symmetrical about $y=\frac{b}{2}$, $x=\frac{a}{2}$, and the plate diagonals.	(B157), (B158); $\bar{M}$ given by (B156)	(B154), (B155) with $\bar{M}=0$	(B154) to (B156) with $k = \infty$
SINGLE ANTISYMMETRY: Structure symmetrical about $y=\frac{b}{2}$; loading antisymmetrical about $y=\frac{b}{2}$.	(B160) to (B167)	(B160) to (B165) with $\bar{M}_1=\bar{M}_2=0$	(B160) to (B167) with $k_1=k_2=\infty$
DOUBLE ANTISYMMETRY: Structure symmetrical about $y=\frac{b}{2}$, $x=\frac{a}{2}$; loading antisymmetrical about $y=\frac{b}{2}$, $x=\frac{a}{2}$.	(B169) to (B173)	(B169) to (B172) with $\bar{M}=0$	(B169) to (B173) with $k = \infty$
ENTIRE ANTISYMMETRY: Square plate; structure symmetrical about $y=\frac{b}{2}$, $x=\frac{a}{2}$, and plate diagonals; loading antisymmetrical about $y=\frac{b}{2}$, $x=\frac{a}{2}$, but symmetrical about plate diagonals.	(B178) and (B179); $\bar{M}$ given by (B177)	(B175) and (B176) with $\bar{M}=0$	(B178) and (B179) $\bar{M}$ given by (B177) with $k = \infty$

A procedure for solving the most general equations, those in the double-bordered box, follows:

(a). Solve equations (B119) to (B126) simultaneously, for c's and g's in terms of the other four unknowns $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ and $\bar{M}_4$. (An iterative procedure, such as the Gauss-Seidel iteration method, will probably be advisable in this step, especially for large M and N.)

(b). Substitute the results from (a) into equations (B129) to (B132) to eliminate the c's and g's, and thus obtain four equations containing only the $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ and $\bar{M}_4$ as unknowns.

(c). Obtain the values of the $\bar{M}_1$, $\bar{M}_2$ etc., by solving these four equations simultaneously.

(d). Substitute the values of $\bar{M}_1$, $\bar{M}_2$ etc., into the results obtained in (a) to compute the values of c's and g's.

For the other cases represented in the table, some simplifications of this procedure is possible. For example, for the case of a non-symmetrical structure with purely hinge-jointed stiffeners, $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ and $\bar{M}_4$ vanish; hence, as indicated in the table, equations (B119) to (B126) alone are sufficient for a solution for each of the c's and g's, while equations (B129) to (B132) become unnecessary. Thus steps (b) to (d) of the above procedure can be eliminated.

The case of mixed corner conditions (i.e., some joints purely hinged, others not) is not included in Table 2. However, such a case can be handled with the general equations in upper-left box of Table 2, by merely equating to zero the appropriate corner moment ($\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ or $\bar{M}_4$) and the corresponding spring constant (k_1, k_2, k_3 or k_4) for every joint that is hinged. In addition, for each hinged corner one of the four equations (B129) to (B132) must be omitted from the system in accordance with the following scheme:

Eq. (B129) omitted if corner (0,0) is purely hinged.

Eq. (B130) omitted if corner (a,0) is purely hinged.

Eq. (B131) omitted if corner (a,b) is purely hinged.

Eq. (B132) omitted if corner (0,b) is purely hinged.

These equations relate the angle change of a plate corner to the angle change of the coil spring at that corner.* The equation to be omitted develops the indeterminate form $\bar{M}/k = 0/0$ in one of its terms when the corner moment and spring stiffness are equated to zero. If the indeterminacy is removed by first multiplying through by the spring constant, the equation merely re-states that the corner moment vanishes.

For the case of single symmetry, single antisymmetry, and double antisymmetry, the procedure is similar to the one just described, except that smaller systems of equations are involved.

For the cases of double symmetry, entire symmetry, and entire antisymmetry in Table 2, a further simplification was made for

---

*This significance of equations (B129) to (B132) is more readily apparent from their earlier form, (B128).

$k \neq \infty$: The equations corresponding to (B129) to (B132) were solved explicitly for the corner moments (they are all numerically equal in these cases) in terms of the c's and g's, and the corner moments were then eliminated in the equations corresponding to (B119) to (B126). Therefore for these cases the equations which are given for the c's and g's do not involve corner moments, and the c's and g's can therefore be computed immediately in terms of known quantities. When $k = \infty$ (rigid-jointed stiffeners), however, it so happens that, for the case of double symmetry and entire symmetry, the equations corresponding to (B129) to (B132) can not be solved explicitly for the corner moments, and the above simplification is not possible.

(3) Limiting cases. A simplification results in the equations of Table 2 if some limiting cases of the structure are considered. The simplified equations for several such limiting cases are derived in Appendix B and listed in Table 3. The equations in column A of Table 3 were developed for the purpose of the numerical work, the results of which will be presented later. The equations in the remaining columns were developed for the purpose of comparison with earlier work of others. Limiting case D corresponds to the case considered in reference 1; limiting cases B and C correspond to certain special cases in reference 2. Thus a partial check on the correctness of the present analysis is obtained by noting that equations (B198) to (B201) in table 3 are equivalent

to equations (B61) to (B64) of reference 1, respectively, and equations (B191), (B192) and (B193) in Table 3 are equivalent to equations (B57), (E34) and (E35) of reference 2, respectively.

TABLE 3. EQUATIONS FOR COMPUTING c's, g's AND $\bar{M}$ FOR SEVERAL LIMITING CASES

PROPERTIES OF SYMMETRY	LIMITING CASE			
	(A) Zero eccentricity between plate edges and stiffener centroidal axes	(B) Two opposite stiffeners have infinite bending and shear stiffness, two other stiffeners have zero bending stiffness (including limiting case A and $K'_n=K''''_m=0$ for all n and m)	(C) All four stiffeners have infinite bending and shearing stiffnesses (including limiting case A and $K'_n=0$ for all n)	(D) All four stiffeners have zero bending stiffness (including limiting case A)
None				(B198) to (B201) and (B196)
Double Symmetry		(B191)		
Entire Symmetry	(B180) to (B182)		(B192) to (B194)	
Entire Anti-symmetry	(B183) to (B185)			

(4) <u>Description of numerical examples.</u> The foregoing general results were used to obtain numerical stress data for twenty-two illustrative problems. These problems are of three types: thermal-stress problems involving a continuous "pillow-shaped" temperature distribution, without any applied loads; "shear lag" problems, involving the diffusion of loads from the stiffener ends into the plate; and thermal-stress problems involving a discontinuity in temperature between the stiffeners and plate. The three groups of problems are shown schematically in figure 5.

In all these problems, the plate is square (b=a) and isotropic, with Young's modulus E, Poisson's ratio $\nu = 0.3$ and thickness h. The four stiffeners are identical, with cross-sectional area A, effective shear cross-sectional area A_s and bending moment of inertia I. The stiffeners are assumed to have the same Young's modulus as the plate and to have no temperature variation through their depth. The structure, loadings, and thermal strains in these problems are symmetrical about both center lines, namely x and y = $\frac{a}{2}$, and also about the plate diagonals. This kind of symmetry was referred to as entire symmetry in tables 2 and 3.

In the thermal stress problems, the stiffeners are at a uniform temperature of zero, while the plate has either a "pillow-shaped" temperature distribution of the form $\theta \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{a}\right)$ (fig. 5(a)) or a constant temperature of θ (fig. 5(c)). Thus θ denotes the temperature rise of the plate center relative to the edges.

In the shear-lag problem (fig. 5(b)), the temperature is uniform and the loading consists of identical tension loads of magnitude P applied to the end cross section of the stiffeners.

Additional information about the numerical examples is given in Table 4, which consitutes a summary description of all the twenty-two cases analyzed numerically. As shown in Table 4, the stiffeners were assumed to be either rigidly joined ($k = \infty$) or purely hinged ($k = 0$) at their junctions. A range of values of the stiffener flexibility parameter ha^3/I was used; high values of this parameter represent fairly flexible stiffeners, and zero represents stiffeners which are perfectly rigid under bending moment. The area ratio parameter $4ah/\pi^2A$ was assumed to have the same value 1.0 for all the twenty-two cases. In two of the numerical examples (Nos. 7 and 14 in Table 4), some offset ($t^i = 0.0272a$) was assumed between the stiffener axes and the lines of attachment of stiffener to plate; in the remaining examples this offset was taken as zero. In six of the cases, deflections due to transverse shear in the stiffners were neglected by the device of equating the effective shear area A_s of the stiffeners to infinity; these cases are identified by $A/A_s = 0$ in Table 4. In the remaining cases, shear deformations of the stiffeners were considered, and the effective shear area A_s was taken equal to the effective stretching area A, as an approximation. A value of A/A_s slightly greater than 1.0 would probably have been more appropriate. However, since the appropriate value of A/A_s varies with the shape of the stiffener cross section, and since the effects of transverse shear were expected to be small in any case, the value 1.0 was selected as a reasonably good one to indicate the order of magnitude of the effects of transverse shear deformations.

Table 4 also shows the value, namely 39, of the upper summation indexes M and N used in the calculations. The size of M indicates the size of the system of simultaneous equations that has to be solved (e.g., 40 equations for M = 39). The value M = N = 39 was chosen after trial calculations with M and N as high as 49, because it gave sufficient accuracy without excessive computation time (average computing time = 7 minutes per problem on the IBM 7074 for solving the simultaneous equations and computing the stresses).

The last two columns of Table 4 give the main equations employed in solving the listed twenty-two numerical examples and the figures in which the numerical results are plotted.

It is worth mentioning that the solution to the problem of discontinuous temperature distribution (no external load, constant plate temperature θ, zero stiffener temperature) was obtained very easily from the solution to the shear lag problem by means of a superposition technique, as indicated in Table 4. This superposition technique is described in Appendix C.

(5) Results of numerical examples. The numerical results of the twenty-two example problems listed in Table 4 are presented

TABLE 4. LIST OF PROBLEMS SOLVED AS NUMERICAL EXAMPLES

Problem Number	Classifications	Stiffener Joint	$\frac{ha^3}{I}$	$\frac{t^i}{a}$	$\frac{4ah}{\pi^2 A}$	$\frac{A}{A_s}$	ν	M (=N)	Equations Employed	Results
1	PILLOW-SHAPED TEMPERATURE DISTRIBUTION[1]	RIGID	110,000	0	1	1	.3	39	(B180) to (B182)	figs. 9(b)(i), 9(c)(i)
2		"	10,000	0	"	1	"	"	"	figs. 9(a), 9(b)(i), 9(d)
3		"	500	0	"	1	"	"	"	figs. 9(b)(ii), 12
4		"	0	0	"	1	"	"	"	figs. 9(b)(ii) 9(c)(ii)
5		"	110,000	0	"	0	"	"	"	fig. 9(c)(i)
6		"	0	0	"	0	"	"	"	fig. 9(c)(ii)
7		"	10,000	.0272	"	1	"	"	(B154) to (B156)	fig. 9(d)
8		HINGED	10,000	0	"	1	"	"	(B180) (B181)	fig. 9(a)
9	SHEAR LAG[2]	RIGID	110,000	0	"	1	"	"	(B180) to (B182)	figs. 10(a)(i) 10(b)(i)
10		"	10,000	0	"	1	"	"	"	figs. 10(b)(i) 10(d)
11		"	500	0	"	1	"	"	"	fig. 10(b)(ii)
12		"	0	0	"	1	"	"	"	figs. 10(a)(ii), 10(b)(ii), 10(c)(i)

TABLE 4. (Continued). LIST OF PROBLEMS SOLVED AS NUMERICAL EXAMPLES

Problem Number	Classifications	Stiffener Joint	$\frac{ha^3}{I}$	$\frac{t^i}{a}$	$\frac{4ah}{\pi^2 A}$	$\frac{A}{A_s}$	ν	M (=N)	Equations Employed	Results
13		RIGID	0	0	1	0	.3	39	(B180) to (B182)	figs. 10(a)(iii), 10(c)(i).
14	SHEAR LAG[2]	"	10,000	.0272	"	1	"	"	(B154) to (B156)	fig. 10(d).
15		HINGED	110,000	0	"	1	"	"	(B180) (B181)	figs. 10(a)(i), 10(b)(iii), 10(c)(ii).
16		"	0	0	"	1	"	"	"	figs. 10(a)(ii), 10(b)(iii), 10(c)(iii).
17		"	110,000	0	"	0	"	"	"	fig. 10(c)(ii).
18		"	0	0	"	0	"	"	"	figs. 10(a)(iii), 10(c)(iii).
19	DISCONTINUOUS TEMPERATURE DISTRIBUTION[3]	RIGID	110,000	0	"	1	"	"	USING SUPERPOSITION METHOD	fig. 11(a)(i).
20		"	500	0	"	1	"	"		figs. 11(a)(i), 12.
21		"	0	0	"	1	"	"		figs. 11(a)(ii), 11(b).
22		"	0	0	"	0	"	"		fig. 11(b).

NOTE: All problems are for an entirely symmetric square plate.

1. No external load; plate temperature $= \theta \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$; stiffener temperature = 0.
2. A tension load p applied at each stiffener end. Uniform temperature throughout the plate.
3. No external load; plate temperature = θ; stiffener temperature = 0.

in the form of dimensionless plots of the plate stress resultants N_x, N_y, N_{xy} as functions of x for fixed values of y, the stiffener tension $P_3(x)$, stiffener bending moment $M_3(x)$, and stiffener transverse shear $V_3(x)$. In view of the symmetry which exists about the plate center lines and diagonals, these results plotted over the range $0 \leqq x/a \leqq 0.5$, $0 \leqq y/a \leqq 0.5$ suffice to describe the results for the entire structure. All stresses, tensions, moments and shears were computed at x/a intervals of 0.02 in the region $x/a = 0$ to 0.1, and 0.05 in the region $x/a = 0.1$ to 0.5. In the y-directions y/a intervals of 0.1 were used.

The results are presented in figures 9 - 12. In these figures the results are grouped in different ways in order to show the effect of varying one or another of the parameters involved. Because of these groupings, the same set of results may appear in more than one place, as indicated by the last column of Table 4. The figures and the significant parameter which is varied in each are described below:

(A) Figure 9 gives numerical results for the pillow-shaped temperature distribution problems, with

(i) Figure 9(a) showing effect of stiffener joint rigidity (k);

(ii) Figure 9(b) showing effect of stiffener bending stiffness parameter (ha^3/I);

(iii) Figure 9(c) showing effect of stiffener transverse shear stiffness parameter (A/A_s);

(iv) Figure 9(d) showing effect of eccentricity between the stiffener axes and the lines of attachment of stiffener to plate (t^i/a).

(B) Figure 10 gives numerical results for the shear-lag problems; with

(i) Figure 10(a) showing effect of stiffener joint rigidity (k);

(ii) Figure 10(b) showing effect of stiffener bending stiffness parameter (ha^3/I);

(iii) Figure 10(c) showing effect of stiffener transverse shear stiffness parameter (A/A_s);

(iv) Figure 10(d) showing effect of eccentricity between the stiffener axes and the lines of attachment of stiffener to plate (t^i/a).

(C) Figure 11 gives numerical results for the discontinuous temperature distribution problems, with

(i) Figure 11(a) showing effect of stiffener bending rigidity parameter (ha^3/I);

(ii) Figure 11(b) showing effect of stiffener transverse shear stiffness parameter (A/A_s).

(D) Figure 12 gives a comparison between the pillow-shaped temperature distribution and the discontinuous temperature distribution.

(6) Discussion of numerical results. The typical difference in effect between rigid and hinged joints for the stiffener connections can be seen in figure 9(a) for the pillow-shaped temperature distri-

bution and figure 10(a)(i) for the shear-lag problem. The plate stress resultants N_x and N_y are only slightly affected by the type of joint rigidity, with the effect being a localized one near the corners. The shear stress resultant N_{xy} is more markedly affected, but again the effect is significant only near the corners. As is to be expected, the hinged joint permits larger values of N_{xy} at the corner than does the rigid joint. Because of shear deformations in the stiffeners, N_{xy} is not zero at the corners even for rigid-jointed stiffeners.

As far as the stiffener stresses are concerned, the effect of joint rigidity is seen to be negligible for the stiffener tensions, but significant (as is to be expected) for the stiffener bending moments and shears. The effect on the stiffener bending moments and shears is most noticeable at the stiffener ends, of course. However, the effect propagates along the length of the stiffener towards the center a distance which depends on the stiffener flexural stiffness. In figure 10(a)(i), for example, with $ha^3/I =$ 110,000, denoting a fairly low flexural stiffness, the stiffener shear and bending moment are essentially the same for hinged and rigid joints when x/a is greater than 0.1. On the other hand, figure 9(a), for a higher flexural stiffness ($ha^3/I = 10{,}000$), shows the stiffener shear and bending moment differing significantly for hinged and rigid joints over a greater length of stiffener.

For the limiting case of infinite stiffener flexural stiffness ($ha^3/I = 0$), it should, of course, make no difference to the plate if the stiffener junctions are rigid or hinged. This expectation is borne out by the computed results in figures 10(a)(ii) and (iii), where a single set of curves represents the plate stress resultants for both rigid and hinged joints. In this case the stiffener stresses, except for the bending moment, are also unaffected by the type of stiffener junction. The stiffener bending moments for hinged and rigid joints are seen to differ only by a constant when the stiffener flexural stiffness is infinite (figs. 10(a)(ii) and (iii)).

When the stiffeners are infinitely rigid in both flexure and shear ($ha^3/I = 0$, $A/A_s = 0$) and the loading is that which corresponds to the shear-lag problem (that is, equal tensions at the stiffener ends), it is to be expected that all the requirements of equilibrium and compatibility can be satisfied by a homogeneous state of biaxial tension in the plate with zero shear, and uniform tension along the length of each stiffener. This expectation is borne out by the computed results in figure 10(a) (iii). The plate stresses and stiffener tension shown there agree well with those that would be obtained by a simple direct and exact calculation based on considerations of equality of strain between stiffener and plate and overall equilibrium of each stiffener. This agreement constitutes a check on the correctness of the equations and method of the present paper.

The effect of stiffener flexural stiffness is demonstrated in figure 9(b) for the pillow-shaped temperature distribution, figure 10(b) for the shear-lag problem, and figure 11(a) for the discontinuous temperature distribution problem. Except in figure 10(b)(iii), the stiffener junctions are taken as rigid.

For the pillow-shaped temperature distribution problem with rigid-jointed stiffeners, the plate stress resultants N_x, N_y and N_{xy} change only slightly as the stiffener flexibility parameter ha^3/I varies from 110,000 to 10,000, as shown in figure 9(b)(i). The change, however, is more pronounced in figure 9(b)(ii), where the parameter ha^3/I is varied from 500 to the limiting value of 0. These changes, whether slight or pronounced, are seen to be most significant near plate corners and plate edges. The irregular wiggles appearing in figure 9(b)(i) along the line $y/a = 0$ are believed to be not present in actuality but caused by slow convergence of the series for the stresses along plate edges.

By comparing figures 9(b)(i) and (ii) of the present paper with figure 5(c) of reference 1 (in which stiffener flexural stiffness was assumed negligible), it can be seen that for fairly large values of the stiffener flexibility parameter, say $ha^3/I \gtrapprox 10{,}000$, the analysis neglecting stiffener flexural stiffness (ref. 1) is sufficiently accurate for all of the plate stresses except those near the plate corners. For smaller values of ha^3/I, on the other hand, the present type of analysis appears to be needed in estimating plate thermal stresses even in the interior region of the plate.

The effect of stiffener flexural stiffness on the stiffener stresses for the pillow-shaped temperature distribution problem is slight for the stiffener tensions but may be significant for the stiffener bending moments and transverse shears (see, for example, fig. 9(b)(c)). It should be noted that because of finite stiffener flexural stiffness, the stiffener end tensions are not zero, despite the absence of external stiffener end loads.

Corresponding to figure 9(b)(i) for the pillow-shaped temperature distribution problem, figure 10(b)(i) shows the effect of stiffener flexural stiffness for the shear-lag problem by comparing plate and stiffener stresses for ha^3/I = 110,000 and 10,000. It is noticed that for the same variation of the stiffener flexibility parameter ha^3/I the change of plate stresses is more significant for the shear-lag problem than for the pillow-shaped temperature distribution problem. The effect is again more significant near plate corners.

Comparing figures 10(b) of the present paper to the corresponding figure in reference 1 (fig. 6(c)), it is observed that the analysis in reference 1, which neglects flexural stiffness of the stiffeness, is applicable only for fairly high values of ha^3/I, say $ha^3/I \geqq 110{,}000$, in the central region of the plate. At the plate corners, considerable difference is observed between the results of reference 1 and the present results. Most importantly, the plate corner shear stress resultant N_{xy} is finite in figure 10(b) but infinite in the result given in reference 1. A noteworthy effect of stiffener flexural stiffness is to cause the limiting internal stiffener tension at the stiffener end to be different from the externally applied stiffener end load. This is evident from the fact that $P_3(x)/P$ is not equal to unity at $x/a = 0$ in figures 10.

Because the shear deformation of a beam is usually insignificant compared to the deformation due to bending, it is to be expected that the effect of finite transverse shear stiffness of the stiffeners on the plate and stiffener stresses will be small for plates with stiffeners of practical size. This expectation is supported by the results in figure 9(c)(i) for the pillow-shaped temperature distribution problem with rigid-jointed stiffeners and figure 10(c)(ii) for the shear-lag problem with hinge-jointed stiffeners. In both figures the value of $ha^3/I = 110{,}000$ is used, which corresponds to the stiffeners of fairly low (but practical) flexural stiffness. The alteration of plate stresses in these two figures, due to varying the stiffener shear stiffness parameter A/A_s from 1.0 to 0, is seen to be very small and localized near plate corners. The alteration of stiffener tensions and shears is also insignificant and localized near stiffener ends. As far as stiffener bending moments are concerned, figure 9(c)(i) again shows a small and localized (near stiffener ends) change when A/A_s is varied from 1.0 to 0. Figure 10(c)(ii), however, exhibits a significant and non-localized change of bending moment for the shear-lag problem with hinge-jointed stiffeners.

It is known that when the stiffeners are more and more stiff in bending the stiffener shear stiffness will play a more and more important role in stiffener lateral deflection. Consequently, for a plate with stiffeners of very high bending stiffness, the effect of stiffener shear stiffness on the plate and stiffener stresses may be expected to be highly significant. This expectation is confirmed by the results in figures 9(c)(ii), 10(c)(i), 10(c)(iii) and 11(b). For all these figures the stiffener flexural stiffness parameter ha^3/I is taken as the limiting value of 0. It is noticed that, in this case, the effect of varying the stiffener shear stiffness is by no means a localized one near plate corners as in the case of low stiffener flexural stiffness. Rather, the effect on the plate normal stress N_y of the pillow-shaped temperature distribution problem (fig. 9(c)(ii)) is seen to be more significant in the central region of the plate than near the corners.

It should be mentioned that the dashed curves in figures 10(c)(i) and (iii) are not computed results but are the exact results obtained by a simple direct calculation based on considerations of equality of strain between stiffener and plate and overall equilibrium of each stiffener, as mentioned previously. The computed results can be found in figure 10(a)(iii).

It is interesting to see that figures 9(d) and 10(d) exhibit a pronounced effect of the eccentricity between stiffener axes and plate edges on the plate stresses. This effect is seen to be distributed throughout the plate and highly amplified near plate corners. In figure 10(d), for the shear-lag problem, the maximum plate normal stresses N_x and N_y are changed enormously near the corners by introducing the eccentricity between stiffener axes and plate edges. Owing to this important change, it is concluded that in shear-lag problems especially, a careful study of the eccentricity between stiffener axes and plate edges is needed for a safe and economical design whenever such an eccentricity exists, and the plate stresses should be computed with the inclusion of this eccentricity.

The effect of this eccentricity on the stiffener stresses is less important as indicated in figures 9(d) and 10(d).

It is an attractive idea to try to replace a non-uniform temperature distribution by an equivalent uniform one. Figure 12, however, discourages such an idea, for the two types of temperature distributions are seen to produce entirely different types of stress distributions. A careful consideration of the actual temperature distribution in a plate for a practical problem is therefore advised.

EXPERIMENTAL INVESTIGATION

A limited experimental investigation to confirm the theoretical approach was felt to be desirable. Therefore an experiment was performed corresponding to the entirely symmetric shear-lag problem of figure 5(b) with rigid-jointed stiffeners. In the actual experiment tension loads were applied along one diagonal at a time, and the strains that would be produced by simultaneous loading at all four corners as in figure 5(b), were deduced by appropriate rotation and superposition. In the following subsections are given the detailed description of the test specimen, the test procedure, and a comparison of the experimental and computed results.

Detailed description of test specimen. The test specimen was made of 7075-T6 aluminum alloy. It consisted of a 15 3/4" x 15 3/4" x 1/16" square plate sandwiched between two identical four-sided square stiffener frames. The assembled specimen is shown in figure 7(a), which also show the corner grip fittings used in applying tension loads along the diagonals. The stiffener frames are shown in figure 8(a), and the manner in which the plate was sandwiched between them is shown in figure 8(b). The stiffener frames were machined from a solid plate so that near-perfect joint rigidity was automatically achieved without any special fittings. The two stiffener frames were tightly bolted to the plate by 1/8" diameter steel bolts at 15/32" spacing along the centerline of the stiffeners. As shown in figure 8(b), narrow strip washers were inserted between plate and stiffener frames in order to achieve approximately a line of attachment between plate and stiffener along the latter's centroidal axis. Figure 7(b) shows in detail one of the corner grip fittings.

The strain gage types and locations are shown in figure 7(a). Those gages distributed over the upper left quadrant of the plate were used for measuring plate normal stresses, those in the lower right quadrant for plate shear stresses, and those on the stiffeners for stiffener tensions and bending moments. Strain gages were used on one side of the plate only.

Experimental procedure. The Young's modulus and Poisson's ratio of the material were first determined by means of tension tests on two specimens cut in mutually perpendicular directions, from the plates out of which the stiffeners were machined. The following values of Young's modulus and Poisson's ratio were obtained, with no significant difference between the two orientations:

$$E = 10.66 \times 10^6 \text{ psi}$$

$$\nu = 0.33$$

These values were employed in the two-dimensional plane stress stress-strain relations in order to convert the measured plate strains to stresses.

The main specimen itself was tested in a universal tension testing machine between heads which gripped one pair of diagonally opposite grip fittings. Tension load was gradually applied and strain gage readings were recorded at 400-pound intervals until a maximum load of 2000 pounds was reached. The specimen was then unloaded and strain readings taken at the same loads during the unloading process in order to ascertain that the material had not been stressed beyond the elastic limit. For each gage, a straight line was fitted to the strain-versus-load data and the slope of this straight line was used to define the experimental value of strain per unit of applied load. Except for some gages in regions of low strain, there was relatively little scatter of the test points from a straight line. The same procedure was repeated with load applied along the other diagonal. By rotation and superposition, strains corresponding to the simultaneous application of equal loads at all four corners were obtained.

Comparison of experimental and computed results. For purpose of the calculations, the plate was considered to end at the bolt line (i.e., at the centerline of the stiffeners), thus making the dimension "a" 15 inches. The stiffener cross-sectional area was assumed to consist of all the shaded areas in figure 8(b), thus it included the stiffener area proper, the strip washer cross section, and the portion of the plate outside the bolt line, resulting in a value of A equal to 7/16 square inch. For computing the moment of inertia of the stiffener, the neutral axis for flexure was considered to coincide with the bolt axis (that is the very slight shift in neutral axis due to the plate area outside the bolt line was neglected), making I equal to 0.01899 in.^{4}, and $t^i = 0$. All four stiffener joints were considered as rigid. The stiffener effective shear area for computing shear stiffness was assumed as equal to the stiffener cross-sectional area, thus making $A/A_s = 1.0$.

On this basis, calculations were made for the plate stresses, stiffener tensions, and stiffener bending moments in the experimental specimen. The computed results are represented by the solid curves in figure (13). The small circles, triangles, and squares represent the corresponding experimental results.

The agreement between theory and experiment is seen to be good.

CONCLUDING REMARKS

A plane-stress analysis, together with some experimental confirmation, has been presented for a linearly elastic edge-stiffened rectangular plate subject to any equilibrium system of boundary loads and any temperature distribution, on the assumption that the elastic constants are independent of the temperature. The analysis is by means of Fourier series.

This problem had been considered earlier (ref. 1), with the stiffeners idealized as having finite extensional stiffness but negligible flexural stiffness and as being attached to the plate edges along their centroidal axes. In the present work, these idealizations have been dropped in favor of the following more realistic assumptions regarding the stiffeners: (i) The stiffeners have finite bending and transverse shear stiffness as well as finite extensional stiffness. (ii) At each corner, where two stiffeners meet, their end cross sections are joined by a coil spring to simulate any degree of joint rigidity from fully hinged to fully clamped. (iii) There may be eccentricity between the stiffener centroidal axes and the lines of attachment of stiffener to plate; similarly, there may be eccentricity between the stiffener centroidal axes and the lines of action of externally applied shear-flow loadings. (iv) There may be temperature variation across the stiffener depth, producing a thermal curvature as well as a thermal strain of the stiffener axis.

Numerical examples have been worked out in detail in order to investigate the effect of the first three of these assumptions on the plate and stiffener stresses. Three types of loading were considered in these examples: a thermal loading consisting of a "pillow-shaped" temperature-rise distribution over the plate, a thermal loading consisting of a zero temperature rise for the stiffeners and a non-zero uniform temperature rise for the plate, and a force loading consisting of equal external tensions at the ends of all four stiffeners. In all of these numerical examples, the structure was square with symmetry about each centerline and each diagonal.

The numerical examples revealed that finite stiffener flexural and shear stiffness, joint rigidity at the corners, and eccentricity between stiffener axis and line of attachment between stiffener and plate can all have a significant effect on the plate stresses, especially near the corners, and on the stiffener tensions. The inclusion of these elements also leads to stiffener shears and bending moments which, of course, would otherwise not be present at all.

Certain limiting cases of the present problem correspond to certain special cases of the problem in reference 2. As a check, the equations of the present work for a number of these limiting cases were shown to agree with the corresponding equations of reference 2.

It is expected that the analysis and numerical results of the present paper, combined with engineering judgment, may provide qualitative and quantitative information of use to stress analysts and structural designers.

APPENDIX A

SYMBOLS

Remarks. (i) The subscript 1, 2, 3, or 4 on a symbol for a stiffener-related quantity (excluding corner moments) identifies the stiffener location as x=0, x=a, y=0, or y=b, respectively. Such symbols when appearing without subscript indicate the common value of the quantities these symbols represent. (ii) The Fourier coefficients of known quantities (loads, thermal strains, etc.) and the initially unknown stiffener-related quantities (stiffener tensions, bending moments, etc.) are generally represented by capital letters, while the Fourier coefficients of the initially unknown plate-related quantities (internal stresses, etc.) are generally represented by small letters. (iii) Those symbols used in Appendix B for the combination of certain known quantities and Fourier coefficients are only defined where they are first used but not complied in this appendix.

a	plate dimension in x direction; see figure 1.
a_{mn}	F.C.* for the stress function F(x,y); see equations (B11) and (B85).
a'_n, a''_n, a'''_m, a''''_m	F.C. for F(0,y), F(a,y), F(x,0), F(x,b), respectively; see equations (B12) and (B84).

*Here and in the rest of the list the symbols F.C. stand for Fourier coefficients in series expansions.

A_1, A_2, A_3, A_4	stiffener cross-sectional areas.
A_{s1}, A_{s2}, A_{s3}, A_{s4}	stiffener effective cross-sectional areas for computing transverse shear stiffness in bending parallel to the plate.
b	plate dimension in y direction; see figure 1.
b_{mn}	F.C. for $\partial^3F/\partial y^3$; see equations (B39) and (B73).
b'_n, b''_n, b'''_m, b''''_m	F.C. for stiffener bending moments $M_1(y)$, $M_2(y)$, $M_3(x)$, $M_4(x)$, respectively; see equations (B14) and (B98).
B'_n, B''_n, B'''_m, B''''_m	F.C. for $N_1(y)$, $N_2(y)$, $N_3(x)$, $N_4(x)$, respectively; see equations (6).
c_{mn}	F.C. for $N_y(x,y)$; see equations (B24) and (B66).
c'_n, c''_n, c'''_m, c''''_m	F.C. for $N_y(0,y)$, $N_y(a,y)$, $N_y(x,0)$, $N_y(x,b)$, respectively; see equations (B25) to (B28).
C_1, C_2, C_3, C_4	plate compliances; see equations (3).
d_{mn}	F.C. for $\partial^3F/\partial x^3$; see equations (B38) and (B72).
e_{mn}	F.C. for $\partial^4F/\partial x^4$; see equations (B35) and (B69).
$e_1(y)$, $e_2(y)$, $e_3(x)$, $e_4(x)$	stiffener thermal strains; see figure 3.

$e_x(x,y)$, $e_y(x,y)$ — plate thermal strains; see figure 3.

E_{mn} — $C_2(m\pi/a)^4 + (C_4 - 2C_3)(m\pi/a)^2(n\pi/b)^2 + C_1(n\pi/b)^4$.

E_1, E_2, E_3, E_4 — Young's moduli for stiffeners.

$F(x,y)$ — stress function for plate; see equations (B4).

g_{mn} — F.C. for $N_x(x,y)$; see equations (B29) and (B67).

g'_m, g''_m, g'''_n, g''''_n — F.C. for $N_x(x,0)$, $N_x(x,b)$, $N_x(0,y)$, $N_x(a,y)$, respectively; see equations (B30) to (B33).

G_1, G_2, G_3, G_4 — moduli of rigidity for stiffeners.

h — thickness when plate is isotropic.

i_{mn} — F.C. for $\partial^4F/\partial y^4$; see equations (B36) and (B70).

I_1, I_2, I_3, I_4 — stiffener cross-sectional moments of inertia about centroidal axes perpendicular to the plane of the plate.

j_{mn} — F.C. for $-N_{xy}(x,y)$; see equations (B34), (B92), (B94) and (B95).

k_1, k_2, k_3, k_4 — spring stiffnesses (moment per radian) of the coil springs located at (0,0), (a,0), (a,b), and (0,b), respectively; see figure 2.

k — common value of the above when all are equal.

K'_n, K''_n, K'''_m, K''''_m — F.C. for $\kappa_1(y)$, $\kappa_2(y)$, $\kappa_3(x)$, $\kappa_4(x)$, respectively; see equations (8).

L'_n, L''_n, L'''_m, L''''_m	F.C. for $(\partial e_y/\partial x)_{x=0}$, $(\partial e_y/\partial x)_{x=a}$, $(\partial e_x/\partial y)_{y=0}$, $(\partial e_x/\partial y)_{y=b}$, respectively; see equations (10).
m, n, p, q	summation indexes (integers).
M	upper limit on m and p.
$M_1(y)$, $M_2(y)$, $M_3(x)$, $M_4(x)$	stiffener bending moments about centroidal axes; see figure 4.
$\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, $\bar{M}_4$	corner moments produced by the coil springs at stiffener junctions (0,0), (a,0), (a,b), and (0,b), respectively; see figure 4.
$\bar{M}$	common value of the above when all are equal in magnitude.
n	summation index (integer).
N	upper limit on n and q.
$N_1(y)$, $N_2(y)$, $N_3(x)$, $N_4(x)$	external running tensions, force per unit length; see figure 1.
$N_x(x,y)$, $N_y(x,y)$, $N_{xy}(x,y)$	plate stress-resultants, force per unit length; see figure 4.
p	summation index (integer)
P_{mn}	F.C. for $\partial^4 F/\partial x^2 \partial y^2$; see equations (B37) and (B71).

$P_1(y)$, $P_2(y)$, $P_3(x)$,

$P_4(x)$ stiffener cross-sectional tensions; see figure 4.

P'_y, P''_y, P'''_y, P''''_y,

P'_x, P''_x, P'''_x, P''''_x stiffener end loads indicated in figure 1.

P common value of the above when all are equal in magnitude.

q summation index (integer).

$q_1(y)$, $q_2(y)$, $q_3(x)$,

$q_4(x)$ external shear-flow loadings; see figure 1.

Q'_n, Q''_n, Q'''_m, Q''''_m F.C. for $q_1(y)$, $q_2(y)$, $q_3(x)$, $q_4(x)$, respectively; see equations (7).

s'_n, s''_n, s'''_m, s''''_m F.C. for the stiffener cross-sectional tensions; see equations (B13) and (B99).

$t_1^{\,i}$, $t_2^{\,i}$, $t_3^{\,i}$,

$t_4^{\,i}$ offset distances between stiffener centroidal axes and plate edges; see figure 1.

$t_1^{\,o}$, $t_2^{\,o}$, $t_3^{\,o}$,

$t_4^{\,o}$ offset distances between the lines of action of external shear-flow loadings and the stiffener centroidal axes; see figure 1.

t'_n, t''_n, t'''_m, t''''_m F.C. for the first derivatives of the stiffener cross-sectional tensions; see equations (B21) and (B63).

T_{mn}	F.C. for $\partial^2 e_y/\partial x^2 + \partial^2 e_x/\partial y^2$; see equations (11) to (14).
T'_n, T''_n, T'''_m, T''''_m	F.C. for thermal strain discontinuities between stiffeners and plate edges; see equations (9).
u,v	x and y components of displacements in plate.
$u_1{*}(y)$, $u_2{*}(y)$	x-wise displacements of points along the axes of the stiffeners located at x=0 and x=a, respectively.
v	plate displacement component in y-direction.
$v_3{*}(x)$, $v_4{*}(x)$	y-wise displacements of points along the axes of the stiffeners located at y=0 and y=b, respectively.
v'_n, v''_n, v'''_m, v''''_m	F.C. for the stiffener transverse shears; see equations (B15), (B96) and (B97).
$V_1(y)$, $V_2(y)$, $V_3(x)$, $V_4(x)$	stiffener transverse shears; see figure 4.
w_{mn}	F.C. for $\partial^3 F/\partial x \partial y^2$; see equations (B40) and (B74).
w'_n, w''_n, w'''_m, w''''_m	F.C. for the first derivatives of the stiffener bending moments; see equations (B22) and (B64).
x_{mn}	F.C. for $\partial^3 F/\partial x^2 \partial y$; see equations (B41) and (B75).
x,y	Cartesian coordinates; see figure 1.

y	Cartesian coordinate; see figure 1.
z'_n, z''_n, z'''_m, z''''_m	F.C. for the first derivatives of the stiffener transverse shears; see equations (B23) and (B65).
α	coefficient of thermal expansion of plate and stiffeners in numerical examples.
δ_{ij}	Kronecker's delta, unity when both subscripts are equal, zero otherwise.
$\varepsilon_x(x,y)$, $\varepsilon_y(x,y)$, $\gamma_{xy}(x,y)$	plate total strains; see equations (3).
$\varepsilon_1(y)$, $\varepsilon_2(y)$, $\varepsilon_3(x)$, $\varepsilon_4(x)$	stiffener total strains; see equations (1).
$\kappa_1(y)$, $\kappa_2(y)$, $\kappa_3(x)$, $\kappa_4(x)$	stiffener thermal curvatures due to variation of temperature through the depth of the stiffeners.
θ	temperature rise of plate center relative to the stiffeners, used in numerical examples.
ν	Poisson's ratio when plate is istropic.

APPENDIX B

THEORETICAL ANALYSIS

In this appendix are given the method and details of analysis for the problem described in the main body of this report.

The analytical approach is similar to that of reference 1. However, in reference 1 the flexural stiffness of the stiffeners was assumed to be negligible. This single assumption simplified the analysis considerably, for the externally applied running tensions were then transmitted directly to the plate edges, and therefore the boundary values of the plate normal stress were known. The dropping of this assumption in the present analysis adds considerably to the complexity of the problem. It not only makes the plate normal stresses along the boundary unknown, but also introduces unknown corner moments at the stiffener junctions if these junctions are not hinged. To compensate for the increase in the number of unknowns, it is now necessary to invoke additional conditions of compatible deformation, which were not required in reference 1. These are the requirements that (a) the curvature of a stiffener and the curvature of the plate edge to which it is attached must be equal, and (b) the change of angle between two stiffener axes meeting at a corner must be equal to the shear strain of the plate at that corner. These conditions lead to as many additional equations as there are additional unknowns.

The details follow.

Basic equations. With u(x,y) and v(x,y) denoting the x- and y-components of infinitesimal displacement, the strain-displacement relations for the plate are (also see eq. (3))

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{B1}$$

Equations (B1) imply the following compatibility condition on the strains

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_x}{\partial y^2} - \frac{\partial^2 \varepsilon_y}{\partial x^2} = 0 \tag{B2}$$

The plate equilibrium equations, namely

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \qquad \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{B3}$$

imply the existence of a stress function F(x,y) such that

$$N_x = \frac{\partial^2 F}{\partial y^2} \qquad N_y = \frac{\partial^2 F}{\partial x^2} \qquad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{B4}$$

Elimination of the strains in equation (B2) by use of equations (3) and then the stresses by use of equations (B4) leads to the following form of the compatibility condition, in which account is already taken of the equilibrium and stress-strain relations:

$$C_2 \frac{\partial^4 F}{\partial x^4} + (C_4 - 2C_3)\frac{\partial^4 F}{\partial x^2 \partial y^2} + C_1 \frac{\partial^4 F}{\partial y^4} + \frac{\partial^2 e_y}{\partial x^2} + \frac{\partial^2 e_x}{\partial y^2} = 0 \tag{B5}$$

Considering now infinitesimal lengths of the stiffeners as free bodies, and utilizing equations (B4) to express N_{xy}, N_x and N_y at the plate edges in terms of F, one obtains the following equilibrium equations governing the longitudinal variations of the stiffener cross-sectional tensions, bending moments, and transverse shears:

$$\left.\begin{aligned}
&\frac{dP_1}{dy} - \left(\frac{\partial^2 F}{\partial x \partial y}\right)_{x=0} - q_1(y) = 0 \\
&\frac{dP_2}{dy} + \left(\frac{\partial^2 F}{\partial x \partial y}\right)_{x=a} + q_2(y) = 0 \\
&\frac{dP_3}{dx} - \left(\frac{\partial^2 F}{\partial x \partial y}\right)_{y=0} - q_3(x) = 0 \\
&\frac{dP_4}{dx} + \left(\frac{\partial^2 F}{\partial x \partial y}\right)_{y=b} + q_4(x) = 0
\end{aligned}\right\} \quad (B6)$$

$$\left.\begin{aligned}
&\frac{dM_1}{dy} - t_1^i\left(\frac{\partial^2 F}{\partial x \partial y}\right)_{x=0} + q_1(y)\,t_1^o - V_1(y) = 0 \\
&\frac{dM_2}{dy} + t_2^i\left(\frac{\partial^2 F}{\partial x \partial y}\right)_{x=a} - q_2(y)\,t_2^o - V_2(y) = 0 \\
&\frac{dM_3}{dx} - t_3^i\left(\frac{\partial^2 F}{\partial x \partial y}\right)_{y=0} + q_3(x)\,t_3^o - V_3(x) = 0 \\
&\frac{dM_4}{dx} + t_4^i\left(\frac{\partial^2 F}{\partial x \partial y}\right)_{y=b} - q_4(x)\,t_4^o - V_4(x) = 0
\end{aligned}\right\} \quad (B7)$$

$$\left.\begin{aligned}
&\frac{dV_1}{dy} + \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=0} - N_1(y) = 0 \\
&\frac{dV_2}{dy} + \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=a} - N_2(y) = 0 \\
&\frac{dV_3}{dx} + \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=0} - N_3(x) = 0 \\
&\frac{dV_4}{dx} + \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=b} - N_4(x) = 0
\end{aligned}\right\} \quad (B8)$$

The sign convention for the stiffener tensions, bending moments and shears is shown in figure 4.

Integral attachment between the stiffeners and the plate edges implies equality of their longitudinal strains along the lines of attachment and leads to the following additional set of conditions, in which account is taken of the strains of the stiffeners due to bending and non-uniform temperature distribution across the stiffeners:

$$\frac{P_1(y)}{A_1E_1} + \left[\frac{M_1(y)}{E_1I_1} + K_1(y)\right]t_1^i + e_1(y) = \varepsilon_y(0,y) = (e_y + C_2N_y - C_3N_x)_{x=0}$$

$$\frac{P_2(y)}{A_2E_2} + \left[\frac{M_2(y)}{E_2I_2} + K_2(y)\right]t_2^i + e_2(y) = \varepsilon_y(a,y) = (e_y + C_2N_y - C_3N_x)_{x=a}$$

$$\frac{P_3(x)}{A_3E_3} + \left[\frac{M_3(x)}{E_3I_3} + K_3(x)\right]t_3^i + e_3(x) = \varepsilon_x(x,0) = (e_x + C_1N_x - C_3N_y)_{y=0}$$

$$\frac{P_4(x)}{A_4E_4} + \left[\frac{M_4(x)}{E_4I_4} + K_4(x)\right]t_4^i + e_4(x) = \varepsilon_x(x,b) = (e_x + C_1N_x - C_3N_y)_{y=b}$$

In these equations, the terms on the right-hand side are from equations (3). Substitution of equations (B4) yields

$$\left.\begin{aligned}
&\frac{P_1(y)}{A_1E_1} + \left[\frac{M_1(y)}{E_1I_1} + K_1(y)\right]t_1^i + [e_1(y) - e_y(0,y)] - C_2\left(\frac{\partial^2F}{\partial x^2}\right)_{x=0} + C_3\left(\frac{\partial^2F}{\partial y^2}\right)_{x=0} = 0\\
&\frac{P_2(y)}{A_2E_2} + \left[\frac{M_2(y)}{E_2I_2} + K_2(y)\right]t_2^i + [e_2(y) - e_y(a,y)] - C_2\left(\frac{\partial^2F}{\partial x^2}\right)_{x=a} + C_3\left(\frac{\partial^2F}{\partial y^2}\right)_{x=a} = 0
\end{aligned}\right|$$

$$\left.\begin{aligned}
&\frac{P_3(x)}{A_3E_3}+\left[\frac{M_3(x)}{E_3I_3}+K_3(x)\right]t_3^i+\left[e_3(x)-e_x(x,0)\right]-C_1\left(\frac{\partial^2F}{\partial y^2}\right)_{y=0}+C_3\left(\frac{\partial^2F}{\partial x^2}\right)_{y=0}=0\\
&\frac{P_4(x)}{A_4E_4}+\left[\frac{M_4(x)}{E_4I_4}+K_4(x)\right]t_4^i+\left[e_4(x)-e_x(x,b)\right]-C_1\left(\frac{\partial^2F}{\partial y^2}\right)_{y=b}+C_3\left(\frac{\partial^2F}{\partial x^2}\right)_{y=b}=0
\end{aligned}\right\}\quad (B9)$$

Integral attachment between the stiffeners and plate edges also implies equality of their curvatures. With the use of this condition, equations (4) and (5) together with (B4) lead to the following equations of compatible curvatures between plate edges and stiffeners:

$$\left.\begin{aligned}
&(C_3-C_4)\left(\frac{\partial^3F}{\partial x\partial y^2}\right)_{x=0}-C_2\left(\frac{\partial^3F}{\partial x^3}\right)_{x=0}-\left(\frac{\partial e_y}{\partial x}\right)_{x=0}+\frac{M_1(y)}{E_1I_1}\\
&\qquad+\frac{1}{G_1A_{s1}}\left[\left(\frac{\partial^2F}{\partial y^2}\right)_{x=0}-N_1(y)\right]+K_1(y)=0\\
&(C_3-C_4)\left(\frac{\partial^3F}{\partial x\partial y^2}\right)_{x=a}-C_2\left(\frac{\partial^3F}{\partial x^3}\right)_{x=a}-\left(\frac{\partial e_y}{\partial x}\right)_{x=a}-\frac{M_2(y)}{E_2I_2}\\
&\qquad-\frac{1}{G_2A_{s2}}\left[\left(\frac{\partial^2F}{\partial y^2}\right)_{x=a}-N_2(y)\right]-K_2(y)=0\\
&(C_3-C_4)\left(\frac{\partial^3F}{\partial x^2\partial y}\right)_{y=0}-C_1\left(\frac{\partial^3F}{\partial y^3}\right)_{y=0}-\left(\frac{\partial e_x}{\partial y}\right)_{y=0}+\frac{M_3(x)}{E_3I_3}\\
&\qquad+\frac{1}{G_3A_{s3}}\left[\left(\frac{\partial^2F}{\partial x^2}\right)_{y=0}-N_3(x)\right]+K_3(x)=0\\
&(C_3-C_4)\left(\frac{\partial^3F}{\partial x^2\partial y}\right)_{y=b}-C_1\left(\frac{\partial^3F}{\partial y^3}\right)_{y=b}-\left(\frac{\partial e_x}{\partial y}\right)_{y=b}-\frac{M_4(x)}{E_4I_4}\\
&\qquad-\frac{1}{G_4A_{s4}}\left[\left(\frac{\partial^2F}{\partial x^2}\right)_{y=b}-N_4(x)\right]-K_4(x)=0
\end{aligned}\right\}\quad (B10)$$

The problem can now be stated essentially as follows: Solve equations (B5) to (B9) and (B10) for F, P_i, M_i and V_i (i = 1,2,3,4) subjected to boundary conditions arising from the prescribed forces at the stiffener ends and the prescribed distributed loadings N_1 through N_4 and q_1 through q_4. In the following sections a formal solution to this problem will be obtained in terms of Fourier series.

<u>Series assumptions for F(x,y), P_i, M_i and V_i.</u> In the region $o<x<a$, $o<y<b$ of the plate, excluding the edges (x = o, a) and (y = o, b), the stress function F(x,y) will be assumed to be representable by the double Fourier series

$$F(x,y) = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \tag{B11}$$

with as yet unknown coefficients. Equation (B11) is, of course, not valid at the edges; however, there the values of F can be represented by the single Fourier series

$$\left.\begin{aligned} F(0,y) &= \sum_{n=1}^{N} a_n' \sin\left(\frac{n\pi y}{b}\right) \\ F(a,y) &= \sum_{n=1}^{N} a_n'' \sin\left(\frac{n\pi y}{b}\right) \end{aligned}\quad (0<y<b) \quad \begin{aligned} F(x,0) &= \sum_{m=1}^{M} a_m''' \sin\left(\frac{m\pi x}{a}\right) \\ F(x,b) &= \sum_{m=1}^{M} a_m'''' \sin\left(\frac{m\pi x}{a}\right) \end{aligned}\quad (0<x<a)\right\} \tag{B12}$$

Equations (B12) are again not valid at plate corners. At the corners F is assumed to be single-valued. It will be seen later that the corner values of F do not have to be determined.

Similarly, the stiffener tensions, bending moments, and transverse shears will be assumed in the following form:

$$\left.\begin{aligned} P_1(y) &= \sum_{n=1}^{N} s_n' \sin\left(\frac{n\pi y}{b}\right) \\ P_2(y) &= \sum_{n=1}^{N} s_n'' \sin\left(\frac{n\pi y}{b}\right) \end{aligned}\quad (0<y<b) \quad \begin{aligned} P_3(x) &= \sum_{m=1}^{M} s_m''' \sin\left(\frac{m\pi x}{a}\right) \\ P_4(x) &= \sum_{m=1}^{M} s_m'''' \sin\left(\frac{m\pi x}{a}\right) \end{aligned}\quad (0<x<a)\right\} \tag{B13}$$

$$\left.\begin{aligned} M_1(y) &= \sum_{n=1}^{N} b_n' \sin\left(\frac{n\pi y}{b}\right) \\ M_2(y) &= \sum_{n=1}^{N} b_n'' \sin\left(\frac{n\pi y}{b}\right) \end{aligned}\quad (0<y<b) \quad \begin{aligned} M_3(x) &= \sum_{m=1}^{M} b_m''' \sin\left(\frac{m\pi x}{a}\right) \\ M_4(x) &= \sum_{m=1}^{M} b_m'''' \sin\left(\frac{m\pi x}{a}\right) \end{aligned}\quad (0<x<a)\right\} \tag{B14}$$

$$\left.\begin{aligned} V_1(y) &= \sum_{n=0}^{N} v_n' \cos\left(\frac{n\pi y}{b}\right) \\ V_2(y) &= \sum_{n=0}^{N} v_n'' \cos\left(\frac{n\pi y}{b}\right) \end{aligned}\right\} (0 \leq y \leq b)$$

$$\left.\begin{aligned} V_3(x) &= \sum_{m=0}^{M} v_m''' \cos\left(\frac{m\pi x}{a}\right) \\ V_4(x) &= \sum_{m=0}^{M} v_m'''' \cos\left(\frac{m\pi x}{a}\right) \end{aligned}\right\} (0 \leq x \leq a) \qquad \text{(B15)}$$

The coefficients in the series in equations (B11) to (B15) are related to the left-hand sides through the usual formulas:

$$a_{mn} = \frac{4}{ab}\int_0^b\int_0^a F(x,y)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)dx\,dy \qquad \text{(B16)}$$

$$a_n' = \frac{2}{b}\int_0^b F(0,y)\sin\left(\frac{n\pi y}{b}\right)dy\,, \quad \text{etc.} \qquad \text{(B17)}$$

$$s_n' = \frac{2}{b}\int_0^b P_1(y)\sin\left(\frac{n\pi y}{b}\right)dy\,, \quad \text{etc.} \qquad \text{(B18)}$$

$$b_n' = \frac{2}{b}\int_0^b M_1(y)\sin\left(\frac{n\pi y}{b}\right)dy\,, \quad \text{etc.} \qquad \text{(B19)}$$

$$v_n' = \frac{2-\delta_{no}}{b}\int_0^b V_1(y)\cos\left(\frac{n\pi y}{b}\right)dy\,, \quad \text{etc.} \qquad \text{(B20)}$$

<u>Series for the derivatives of F(x,y), P_i, M_i and V_i (i = 1,2,3,4).</u> The derivatives appearing in equations (B4) to (B9) and (B10) will also be assumed expressible in series as follows:

$$\left.\begin{aligned} \frac{dP_1}{dy} &= \sum_{n=0}^{N} t_n' \cos\left(\frac{n\pi y}{b}\right) \\ \frac{dP_2}{dy} &= \sum_{n=0}^{N} t_n'' \cos\left(\frac{n\pi y}{b}\right) \end{aligned}\right\} (0 \leq y \leq b)$$

$$\left.\begin{aligned} \frac{dP_3}{dx} &= \sum_{m=0}^{M} t_m''' \cos\left(\frac{m\pi x}{a}\right) \\ \frac{dP_4}{dx} &= \sum_{m=0}^{M} t_m'''' \cos\left(\frac{m\pi x}{a}\right) \end{aligned}\right\} (0 \leq x \leq a) \qquad \text{(B21)}$$

$$\left.\begin{aligned}
\frac{dM_1}{dy} &= \sum_{n=0}^{N} w_n' \cos\left(\frac{n\pi y}{b}\right) \\
& \qquad\qquad (0<y<b) \\
\frac{dM_2}{dy} &= \sum_{n=0}^{N} w_n'' \cos\left(\frac{n\pi y}{b}\right) \\
\frac{dM_3}{dx} &= \sum_{m=0}^{M} w_m''' \cos\left(\frac{m\pi x}{a}\right) \\
& \qquad\qquad (0<x<a) \\
\frac{dM_4}{dx} &= \sum_{m=0}^{M} w_m'''' \cos\left(\frac{m\pi x}{a}\right)
\end{aligned}\right\} \quad \text{(B22)}$$

$$\left.\begin{aligned}
\frac{dV_1}{dy} &= \sum_{n=1}^{N} z_n' \sin\left(\frac{n\pi y}{b}\right) \\
& \qquad\qquad (0<y<b) \\
\frac{dV_2}{dy} &= \sum_{n=1}^{N} z_n'' \sin\left(\frac{n\pi y}{b}\right) \\
\frac{dV_3}{dx} &= \sum_{m=1}^{M} z_m''' \sin\left(\frac{m\pi x}{a}\right) \\
& \qquad\qquad (0<x<a) \\
\frac{dV_4}{dx} &= \sum_{m=1}^{M} z_m'''' \sin\left(\frac{m\pi x}{a}\right)
\end{aligned}\right\} \quad \text{(B23)}$$

$$N_y = \frac{\partial^2 F}{\partial x^2} = \sum_{m=1}^{M} \sum_{n=1}^{N} c_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \qquad \begin{pmatrix} 0<x<a \\ 0<y<b \end{pmatrix} \qquad \text{(B24)}$$

$$(N_y)_{x=0} = \left(\frac{\partial^2 F}{\partial x^2}\right)_{x=0} = \sum_{n=1}^{N} c_n' \sin\left(\frac{n\pi y}{b}\right) \qquad (0<y<b) \qquad \text{(B25)}$$

$$(N_y)_{x=a} = \left(\frac{\partial^2 F}{\partial x^2}\right)_{x=a} = \sum_{n=1}^{N} c_n'' \sin\left(\frac{n\pi y}{b}\right) \qquad (0<y<b) \qquad \text{(B26)}$$

$$(N_y)_{y=0} = \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=0} = \sum_{m=1}^{M} c_m''' \sin\left(\frac{m\pi x}{a}\right) \qquad (0<x<a) \qquad \text{(B27)}$$

$$(N_y)_{y=b} = \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=b} = \sum_{m=1}^{M} c_m'''' \sin\left(\frac{m\pi x}{a}\right) \qquad (0<x<a) \qquad \text{(B28)}$$

$$N_x = \frac{\partial^2 F}{\partial y^2} = \sum_{m=1}^{M}\sum_{n=1}^{N} g_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0<y<b \end{pmatrix} \quad (B29)$$

$$(N_x)_{y=0} = \left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0} = \sum_{m=1}^{M} g_m' \sin\left(\frac{m\pi x}{a}\right) \quad (0<x<a) \quad (B30)$$

$$(N_x)_{y=b} = \left(\frac{\partial^2 F}{\partial y^2}\right)_{y=b} = \sum_{m=1}^{M} g_m'' \sin\left(\frac{m\pi x}{a}\right) \quad (0<x<a) \quad (B31)$$

$$(N_x)_{x=0} = \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=0} = \sum_{n=1}^{N} g_n''' \sin\left(\frac{n\pi y}{b}\right) \quad (0<y<b) \quad (B32)$$

$$(N_x)_{x=a} = \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=a} = \sum_{n=1}^{N} g_n'''' \sin\left(\frac{n\pi y}{b}\right) \quad (0<y<b) \quad (B33)$$

$$-N_{xy} = \frac{\partial^2 F}{\partial x \partial y} = \sum_{m=0}^{M}\sum_{n=0}^{N} j_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0\le x\le a \\ 0\le y\le b \end{pmatrix} \quad (B34)$$

$$\frac{\partial^4 F}{\partial x^4} = \sum_{m=1}^{M}\sum_{n=1}^{N} e_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0<y<b \end{pmatrix} \quad (B35)$$

$$\frac{\partial^4 F}{\partial y^4} = \sum_{m=1}^{M}\sum_{n=1}^{N} i_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0<y<b \end{pmatrix} \quad (B36)$$

$$\frac{\partial^4 F}{\partial x^2 \partial y^2} = \sum_{m=1}^{M} \sum_{n=1}^{N} p_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0<y<b \end{pmatrix} \tag{B37}$$

$$\frac{\partial^3 F}{\partial x^3} = \sum_{m=0}^{M} \sum_{n=1}^{N} d_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0 \le x \le a \\ 0<y<b \end{pmatrix} \tag{B38}$$

$$\frac{\partial^3 F}{\partial y^3} = \sum_{m=1}^{M} \sum_{n=0}^{N} b_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0 \le y \le b \end{pmatrix} \tag{B39}$$

$$\frac{\partial^3 F}{\partial x \partial y^2} = \sum_{m=0}^{M} \sum_{n=1}^{N} w_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0 \le x \le a \\ 0<y<b \end{pmatrix} \tag{B40}$$

$$\frac{\partial^3 F}{\partial x^2 \partial y} = \sum_{m=1}^{M} \sum_{n=0}^{N} x_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \quad \begin{pmatrix} 0<x<a \\ 0 \le y \le b \end{pmatrix} \tag{B41}$$

where

$$t_n' = \frac{2-\delta_{n0}}{b} \int_0^b \left(\frac{dP_1}{dy}\right) \cos\left(\frac{n\pi y}{b}\right) dy \,, \quad \text{etc.} \tag{B42}$$

$$w_n' = \frac{2-\delta_{n0}}{b} \int_0^b \left(\frac{dM_1}{dy}\right) \cos\left(\frac{n\pi y}{b}\right) dy \,, \quad \text{etc.} \tag{B43}$$

$$z_n' = \frac{2}{b} \int_0^b \left(\frac{dV_1}{dy}\right) \sin\left(\frac{n\pi y}{b}\right) dy \,, \quad \text{etc.} \tag{B44}$$

$$c_{mn} = \frac{4}{ab}\int_0^a\int_0^b \left(\frac{\partial^2 F}{\partial x^2}\right) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B45}$$

$$c_n' = \frac{2}{b}\int_0^b \left(\frac{\partial^2 F}{\partial x^2}\right)_{x=0} \sin\left(\frac{n\pi y}{b}\right) dy \tag{B46}$$

$$c_n'' = \frac{2}{b}\int_0^b \left(\frac{\partial^2 F}{\partial x^2}\right)_{x=a} \sin\left(\frac{n\pi y}{b}\right) dy \tag{B47}$$

$$c_m''' = \frac{2}{a}\int_0^a \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=0} \sin\left(\frac{m\pi x}{a}\right) dx \tag{B48}$$

$$c_m'''' = \frac{2}{a}\int_0^a \left(\frac{\partial^2 F}{\partial x^2}\right)_{y=b} \sin\left(\frac{m\pi x}{a}\right) dx \tag{B49}$$

$$g_{mn} = \frac{4}{ab}\int_0^a\int_0^b \left(\frac{\partial^2 F}{\partial y^2}\right) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B50}$$

$$g_m' = \frac{2}{a}\int_0^a \left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0} \sin\left(\frac{m\pi x}{a}\right) dx \tag{B51}$$

$$g_m'' = \frac{2}{a}\int_0^a \left(\frac{\partial^2 F}{\partial y^2}\right)_{y=b} \sin\left(\frac{m\pi x}{a}\right) dx \tag{B52}$$

$$g_n''' = \frac{2}{b}\int_0^b \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=0} \sin\left(\frac{n\pi y}{b}\right) dy \tag{B53}$$

$$g_n'''' = \frac{2}{b}\int_0^b \left(\frac{\partial^2 F}{\partial y^2}\right)_{x=a} \sin\left(\frac{n\pi y}{b}\right) dy \tag{B54}$$

$$j_{mn} = \frac{(2-\delta_{m0})(2-\delta_{n0})}{ab}\int_0^a\int_0^b \left(\frac{\partial^2 F}{\partial x \partial y}\right) \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B55}$$

$$e_{mn} = \frac{4}{ab}\int_0^a\int_0^b \left(\frac{\partial^4 F}{\partial x^4}\right) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B56}$$

$$i_{mn} = \frac{4}{ab}\int_0^a\int_0^b \left(\frac{\partial^4 F}{\partial y^4}\right) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B57}$$

$$p_{mn} = \frac{4}{ab}\int_0^a\int_0^b \left(\frac{\partial^4 F}{\partial x^2 \partial y^2}\right) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B58}$$

$$d_{mn} = \frac{(2-\delta_{mo})2}{ab}\int_0^a\int_0^b \left(\frac{\partial^3 F}{\partial x^3}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B59}$$

$$b_{mn} = \frac{2(2-\delta_{no})}{ab}\int_0^a\int_0^b \left(\frac{\partial^3 F}{\partial y^3}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B60}$$

$$w_{mn} = \frac{(2-\delta_{mo})2}{ab}\int_0^a\int_0^b \left(\frac{\partial^3 F}{\partial x \partial y^2}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B61}$$

$$x_{mn} = \frac{2(2-\delta_{no})}{ab}\int_0^a\int_0^b \left(\frac{\partial^3 F}{\partial x^2 \partial y}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) dy\, dx \tag{B62}$$

The coefficients appearing in the series for the derivatives (equations (B21) to (B41)) are , of course, not independent of the coefficients in the series for the basic quantities (equations (B11) to (B15)). The former can be expressed in terms of the latter by means of integrations by parts in the right-hand sides of equations (B42) to (B62). For example, from equations (B42)

$$\left.\begin{aligned}
t_n' &= \frac{2-\delta_{no}}{b}\left[P_1(b)\cos(n\pi) - P_1(0) + \frac{n\pi}{b}\int_0^b P_1(y)\sin\left(\frac{n\pi y}{b}\right)dy\right] \\
&= \frac{2-\delta_{no}}{b}\left[P_1(b)\cos(n\pi) - P_1(0)\right] + \frac{n\pi}{b} s_n' \\
t_n'' &= \frac{2-\delta_{no}}{b}\left[P_2(b)\cos(n\pi) - P_2(0)\right] + \frac{n\pi}{b} s_n'' \\
t_m''' &= \frac{2-\delta_{mo}}{a}\left[P_3(a)\cos(m\pi) - P_3(0)\right] + \frac{m\pi}{a} s_m''' \\
t_m'''' &= \frac{2-\delta_{mo}}{a}\left[P_4(a)\cos(m\pi) - P_4(0)\right] + \frac{m\pi}{a} s_m''''
\end{aligned}\right\} \tag{B63}$$

Similarly, from equations (B43)

$$\left.\begin{aligned}
w_n' &= \frac{2-\delta_{no}}{b}[M_1(b)\cos(n\pi) - M_1(0)] + \frac{n\pi}{b} b_n' \\
w_n'' &= \frac{2-\delta_{no}}{b}[M_2(b)\cos(n\pi) - M_2(0)] + \frac{n\pi}{b} b_n'' \\
w_m''' &= \frac{2-\delta_{mo}}{a}[M_3(a)\cos(m\pi) - M_3(0)] + \frac{m\pi}{a} b_m''' \\
w_m'''' &= \frac{2-\delta_{mo}}{a}[M_4(a)\cos(m\pi) - M_4(0)] + \frac{m\pi}{a} b_m''''
\end{aligned}\right\} \quad \text{(B64)}$$

and from equations (B44)

$$\left.\begin{aligned}
z_n' &= -\frac{n\pi}{b} v_n' \\
z_n'' &= -\frac{n\pi}{b} v_n'' \\
z_m''' &= -\frac{m\pi}{a} v_m''' \\
z_m'''' &= -\frac{m\pi}{a} v_m''''
\end{aligned}\right\} \quad \text{(B65)}$$

Similarly, two partial integrations with respect to x in equation (B45) give

$$c_{mn} = \left(\frac{m\pi}{a}\right)\frac{2}{a}[a_n' - (-1)^m a_n''] - \left(\frac{m\pi}{a}\right)^2 a_{mn} \quad \text{(B66)}$$

Two with respect to y in equation (B50) give

$$g_{mn} = \left(\frac{n\pi}{b}\right)\frac{2}{b}[a_m''' - (-1)^n a_m''''] - \left(\frac{n\pi}{b}\right)^2 a_{mn} \quad \text{(B67)}$$

In equation (B55) partial integration with respect to x, followed by partial integration with respect to y in both of the resulting terms, gives

$$j_{mn} = \frac{(2-\delta_{m0})(2-\delta_{n0})}{ab}\left[(-1)^{m+n}F(a,b) - (-1)^m F(a,0) - (-1)^n F(0,b) + F(0,0)\right] + \left(\frac{m\pi}{a}\right)\frac{2-\delta_{n0}}{b}\left[(-1)^n a_m'''' - a_m'''\right] + \left(\frac{n\pi}{b}\right)\frac{2-\delta_{m0}}{a}\left[(-1)^m a_n'' - a_n'\right] + \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) a_{mn} \qquad \text{(B68)}$$

in which single valuedness of F at the corners has been assumed. Proceeding in a similar fashion with the right-hand sides of equations (B56) to (B62) one obtains

$$e_{mn} = \left(\frac{m\pi}{a}\right)\frac{2}{a}\left[c_n' - (-1)^m c_n''\right] - \left(\frac{m\pi}{a}\right)^3 \frac{2}{a}\left[a_n' - (-1)^m a_n''\right] + \left(\frac{m\pi}{a}\right)^4 a_{mn} \qquad \text{(B69)}$$

$$i_{mn} = \left(\frac{n\pi}{b}\right)\frac{2}{b}\left[g_m' - (-1)^n g_m''\right] - \left(\frac{n\pi}{b}\right)^3 \frac{2}{b}\left[a_m''' - (-1)^n a_m''''\right] + \left(\frac{n\pi}{b}\right)^4 a_{mn} \qquad \text{(B70)}$$

$$p_{mn} = \frac{4}{ab}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\left[(-1)^{m+n}F(a,b) - (-1)^m F(a,0) - (-1)^n F(0,b) + F(0,0)\right] + \frac{2}{b}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\left[(-1)^n a_m'''' - a_m'''\right] + \frac{2}{a}\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)\left[(-1)^m a_n'' - a_n'\right] + \left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 a_{mn} \qquad \text{(B71)}$$

$$d_{mn} = \frac{2-\delta_{m0}}{a}\left[(-1)^m c_n'' - c_n'\right] - \left(\frac{m\pi}{a}\right)^2 \frac{2}{a}\left[(-1)^m a_n'' - a_n'\right] - \left(\frac{m\pi}{a}\right)^3 a_{mn} \qquad \text{(B72)}$$

$$b_{mn} = \frac{2-\delta_{n0}}{b}\left[(-1)^n g_m'' - g_m'\right] - \left(\frac{n\pi}{b}\right)^2 \frac{2}{b}\left[(-1)^n a_m'''' - a_m'''\right] - \left(\frac{n\pi}{b}\right)^3 a_{mn} \qquad \text{(B73)}$$

$$w_{mn} = -\left(\frac{n\pi}{b}\right) j_{mn} \qquad \text{(B74)}$$

$$\chi_{mn} = -\left(\frac{m\pi}{a}\right) \dot{j}_{mn} \tag{B75}$$

It should be noted here that the stiffener end tensions $P_1(o)$, $P_1(b)$, $P_2(o)$, $P_2(b)$, etc. appearing in equations (B63) are the limiting values of $P_1(y)$, $P_2(y)$ etc. as stiffener ends are approached. They are in general not equal to the externally applied stiffener end loads P'_y, P''_y, P'''_y, P''''_y, etc. because of the mutual reaction forces existing at the points where stiffeners meet (see fig. 4). The $P_1(o)$, $P_1(b)$, etc. are related to the applied loads P'_y, P''_y, etc. by the following equations:

$$\left.\begin{aligned}
P_1(o) &= P_y' - V_3(o) & P_2(o) &= P_y''' + V_3(a) \\
P_1(b) &= P_y'' - V_4(o) & P_2(b) &= P_y'''' + V_4(a) \\
P_3(o) &= P_x' - V_1(o) & P_4(o) &= P_x''' + V_1(b) \\
P_3(a) &= P_x'' - V_2(o) & P_4(a) &= P_x'''' + V_2(b)
\end{aligned}\right\} \text{(B76)}$$

Similarly, the stiffener end bending moments $M_1(o)$, $M_1(b)$, $M_2(o)$, $M_2(b)$, etc. in equations (B64) are the limiting values of $M_1(y)$, $M_2(y)$, etc. as the stiffener ends are approached. When there is essentricity of attachment between plate edges (therefore corner hinges) and stiffener axes, these stiffener end moments will not be equal to the corner moments $\overline{M}_1$, $\overline{M}_2$, etc. produced by the coil springs.

The two sets of moments are related as follows:

$$\left.\begin{aligned}
M_1(0) &= \bar{M}_1 - t_1^i V_3(0) & \qquad M_2(0) &= \bar{M}_2 + t_2^i V_3(a) \\
M_1(b) &= \bar{M}_4 - t_1^i V_4(0) & \qquad M_2(b) &= \bar{M}_3 + t_2^i V_4(a) \\
M_3(0) &= \bar{M}_1 - t_3^i V_1(0) & \qquad M_4(0) &= \bar{M}_4 + t_4^i V_1(b) \\
M_3(a) &= \bar{M}_2 - t_3^i V_2(0) & \qquad M_4(a) &= \bar{M}_3 + t_4^i V_2(b)
\end{aligned}\right\} \tag{B77}$$

<u>Boundary values of F</u>. From equations (B4) (using subscripts on F now for convenience to denote partial differentiations),

$$F_{yy}(0,y) = N_x(0,y)$$

Therefore

$$F_y(0,y) = F_y(0,0) + \int_0^y N_x(0,y')\,dy'$$

and

$$F(0,y) = F(0,0) + y F_y(0,0) + \int_0^y \int_0^{y'} N_x(0,y'')\,dy''\,dy' \tag{B78}$$

Substitution of y = b in equation (B78) gives

$$F_y(0,0) = \frac{1}{b}\left[F(0,b) - F(0,0) - \int_0^b \int_0^{y'} N_x(0,y'')\,dy''\,dy'\right]$$

which result,substituted back into equation (B78) gives

$$\begin{aligned}
F(0,y) = F(0,0) &+ \frac{y}{b}\left[F(0,b) - F(0,0) - \int_0^b \int_0^{y'} N_x(0,y'')\,dy''\,dy'\right] \\
&+ \int_0^y \int_0^{y'} N_x(0,y'')\,dy''\,dy'
\end{aligned} \tag{B79}$$

Thus the variation of F along the edge x = o has been expressed in terms of two constants F(o,o), F(o,b) and the boundary stress resultant $N_x(o,y)$. Replacing $N_x(o,y)$ by its series expansion, equation (B32), and carrying out the integrations indicated in equation (B79) give

$$F(o,y) = F(o,o) + \frac{y}{b}[F(o,b) - F(o,o)] - \sum_{n=1}^{N} g_n'''\left(\frac{b}{n\pi}\right)^2 \sin\left(\frac{n\pi y}{b}\right) \tag{B80}$$

Going through a similar procedure for each of the remaining edges, one obtains

$$F(a,y) = F(a,o) + \frac{y}{b}[F(a,b) - F(a,o)] - \sum_{n=1}^{N} g_n''''\left(\frac{b}{n\pi}\right)^2 \sin\left(\frac{n\pi y}{b}\right) \tag{B81}$$

$$F(x,o) = F(o,o) + \frac{x}{a}[F(a,o) - F(o,o)] - \sum_{m=1}^{M} c_m'''\left(\frac{a}{m\pi}\right)^2 \sin\left(\frac{m\pi x}{a}\right) \tag{B82}$$

$$F(x,b) = F(o,b) + \frac{x}{a}[F(a,b) - F(o,b)] - \sum_{m=1}^{M} c_m''''\left(\frac{a}{m\pi}\right)^2 \sin\left(\frac{m\pi x}{a}\right) \tag{B83}$$

Substituting equations (B80) to (B83) into equations (B17) and carrying out the integrations one has

$$\begin{aligned}
a_n' &= \frac{2}{n\pi}[F(o,o) - (-1)^n F(o,b)] - \left(\frac{b}{n\pi}\right)^2 g_n''' \\
a_n'' &= \frac{2}{n\pi}[F(a,o) - (-1)^n F(a,b)] - \left(\frac{b}{n\pi}\right)^2 g_n'''' \\
a_m''' &= \frac{2}{m\pi}[F(o,o) - (-1)^m F(a,o)] - \left(\frac{a}{m\pi}\right)^2 c_m''' \\
a_m'''' &= \frac{2}{m\pi}[F(o,b) - (-1)^m F(a,b)] - \left(\frac{a}{m\pi}\right)^2 c_m''''
\end{aligned} \tag{B84}$$

Thus, the Fourier coefficients in equations (B12), and therefore the boundary values of F, have been expressed in terms of the four unknown constants F(o,o), F(o,b), F(a,o), F(a,b) and the unknown coefficients g_n''', g_n'''', c_m''', c_m'''' related to stresses along plate edges.

<u>Substitution of series into the basic equations</u>. Through equation (B63) to (B75) all the unknown coefficients in the derivative series are expressed in terms of the basic unknowns a_{mn}; s_n', s_n'', s_m''', s_m''''; b_n', b_n'', b_m''', b_m''''; v_n', v_n'', v_m''', v_m''''; c_n', c_n'', c_m''', c_m''''; g_m', g_m'', g_n''', g_n''''; and F(a,b), F(a,o), F(o,b), F(o,o); and in terms of the end values of P_i, M_i and V_i (i = 1, 2, 3, 4). Relationships among these basic unknowns will now be obtained by substituting the assumed series into the basic equations (B5) to (B9) and (B10).

Considering first equation (B5), substituting into it the series expansions from equations (11) and (B35) to (B37), and eliminating e_{mn}, i_{mn}, and p_{mn} through equations (B69) to (B71), one obtains:

$$C_2\left\{\left(\frac{m\pi}{a}\right)\frac{2}{a}\left[c_n'-(-1)^m c_n''\right]-\left(\frac{m\pi}{a}\right)^3\frac{2}{a}\left[a_n'-(-1)^m a_n''\right]+\left(\frac{m\pi}{a}\right)^4 a_{mn}\right\}$$

$$+(C_4-2C_3)\left\{\frac{4}{ab}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\left[(-1)^{m+n}F(a,b)-(-1)^m F(a,o)-(-1)^n F(o,b)+F(o,o)\right]\right.$$

$$+\frac{2}{b}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\left[(-1)^n a_m''''-a_m'''\right]+\frac{2}{a}\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)\left[(-1)^m a_n''-a_n'\right]$$

$$\left.+\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 a_{mn}\right\}+C_1\left\{\left(\frac{n\pi}{b}\right)\frac{2}{b}\left[g_m'-(-1)^n g_m''\right]\right.$$

$$\left.-\left(\frac{n\pi}{b}\right)^3\frac{2}{b}\left[a_m'''-(-1)^n a_m''''\right]+\left(\frac{n\pi}{b}\right)^4 a_{mn}\right\}+T_{mn}=0$$

Solving this equation for a_{mn} and eliminating a_n', a_n'', a_m''', and a_m'''' through equations (B82), one obtains

$$a_{mn} = \frac{4}{mn\pi^2}\left[(-1)^{m+n}F(a,b) - (-1)^m F(a,0) - (-1)^n F(0,b) + F(0,0)\right]$$

$$-\frac{1}{E_{mn}}\Big\{ T_{mn} + \frac{2}{a}\left(\frac{m\pi}{a}\right)\left[c_n' - (-1)^m c_n''\right]C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)\left[g_m' - (-1)^n g_m''\right]C_1$$

$$+ \frac{2}{a}\left(\frac{m\pi}{a}\right)\left(\frac{b}{n\pi}\right)^2\left[g_n''' - (-1)^m g_n''''\right]\left[\left(\frac{m\pi}{a}\right)^2 C_2 + \left(\frac{n\pi}{b}\right)^2 (C_4 - 2C_3)\right]$$

$$+ \frac{2}{b}\left(\frac{n\pi}{b}\right)\left(\frac{a}{m\pi}\right)^2\left[c_m''' - (-1)^n c_m''''\right]\left[\left(\frac{n\pi}{b}\right)^2 C_1 + \left(\frac{m\pi}{a}\right)^2 (C_4 - 2C_3)\right]\Big\} \tag{B85}$$

where

$$E_{mn} = C_2\left(\frac{m\pi}{a}\right)^4 + (C_4 - 2C_3)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + C_1\left(\frac{n\pi}{b}\right)^4 \tag{B86}$$

Thus, through the compatibility equation, the unknown a_{mn} have been expressed in terms of a smaller class of unknowns, namely the c_n', c_n'', c_m''', c_m'''', g_m', g_m'', g_n''', and g_n''''.

Turning now to the equilibrium equations for the stiffener tensions, (B6), substituting the series from equations (B21), (B34), and (7) and utilizing equations (B63), one obtains the relationships

$$\frac{2-\delta_{n0}}{b}\left[(-1)^n P_1(b) - P_1(0)\right] + \left(\frac{n\pi}{b}\right)S_n' - Q_n' - \sum_{m=0}^{M} j'_{mn} = 0 \qquad (n = 0, 1, 2, \cdots, N) \tag{B87}$$

$$\frac{2-\delta_{n0}}{b}\left[(-1)^n P_2(b) - P_2(0)\right] + \left(\frac{n\pi}{b}\right)S_n'' + Q_n'' + \sum_{m=0}^{M} j'_{mn}(-1)^m = 0 \qquad (n = 0, 1, 2, \cdots, N) \tag{B88}$$

$$\frac{2-\delta_{m0}}{a}\left[(-1)^m P_3(a) - P_3(0)\right] + \left(\frac{m\pi}{a}\right)S_m''' - Q_m''' - \sum_{n=0}^{N} j'_{mn} = 0 \qquad (m = 0, 1, 2, \cdots, M) \tag{B89}$$

$$\frac{2-\delta_{mo}}{a}\left[(-1)^m P_4(a) - P_4(o)\right] + \left(\frac{m\pi}{a}\right) S_m'''' + Q_m'''' + \sum_{n=o}^{N} \grave{j}_{mn}(-1)^n = 0 \qquad (m = 0, 1, 2, \cdots, M) \tag{B90}$$

The equations of this group with n = o and m = o will be written separately. From equation (B68), in conjunction with (B84) and (B85), it is first noted that

$$\grave{j}_{oo} = \frac{1}{ab}\left[F(o,o) - F(a,o) - F(o,b) + F(a,b)\right] \tag{B91}$$

$$\left.\begin{aligned} \grave{j}_{on} &= \frac{1}{a}\left(\frac{b}{n\pi}\right)\left(g_n''' - g_n''''\right) \qquad \text{for } n \neq o \\ \grave{j}_{mo} &= \frac{1}{b}\left(\frac{a}{m\pi}\right)\left(c_m''' - c_m''''\right) \qquad \text{for } m \neq o \end{aligned}\right\} \tag{B92}$$

Therefore equations (B87) and (B88) for n = o, (B89) and (B90) for m = o give

$$P_1(b) - P_1(o) - bQ_o' - b\grave{j}_{oo} + \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)\left(c_m'''' - c_m'''\right) = 0 \tag{B93a}$$

$$P_2(b) - P_2(o) + bQ_o'' + b\grave{j}_{oo} - \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)\left(c_m'''' - c_m'''\right)(-1)^m = 0 \tag{B93b}$$

$$P_3(a) - P_3(o) - aQ_o''' - a\grave{j}_{oo} + \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)\left(g_n'''' - g_n'''\right) = 0 \tag{B93c}$$

$$P_4(a) - P_4(o) + aQ_o'''' + a\grave{j}_{oo} - \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)\left(g_n'''' - g_n'''\right)(-1)^n = 0 \tag{B93d}$$

These equations serve to establish four different forms of expression for j_{oo}, as follows:

$$j_{00} = -Q_0' + \frac{1}{b}\left[P_1(b) - P_1(0) + \sum_{m=1}^{M} \left(\frac{a}{m\pi}\right)\left(c_m'''' - c_m'''\right)\right] \quad \text{(B94a)}$$

$$j_{00} = -Q_0'' - \frac{1}{b}\left[P_2(b) - P_2(0) - \sum_{m=1}^{M} \left(\frac{a}{m\pi}\right)(-1)^m\left(c_m'''' - c_m'''\right)\right] \quad \text{(B94b)}$$

$$j_{00} = -Q_0''' + \frac{1}{a}\left[P_3(a) - P_3(0) + \sum_{n=1}^{N} \left(\frac{b}{n\pi}\right)\left(g_n'''' - g_n'''\right)\right] \quad \text{(B94c)}$$

$$j_{00} = -Q_0'''' - \frac{1}{a}\left[P_4(a) - P_4(0) - \sum_{n=1}^{N} \left(\frac{b}{n\pi}\right)(-1)^n\left(g_n'''' - g_n'''\right)\right] \quad \text{(B94d)}$$

All the four expressions will be used at different times in the later analyses.

Equations (B92) and any one of (B94) give expressions for those unknown j_{mn} having at least one subscript zero. An expression for those j_{mn} with neither subscript zero can be obtained by substituting into equation (B68) the expressions for a_n', a_n'', a_m''', a_m'''' and a_{mn} from equations (B84) and (B85). The result is:

$$\begin{aligned} \underset{\substack{m\neq 0\\ n\neq 0}}{j_{mn}} = &-\frac{mn\pi^2}{abE_{mn}}\left\{ T_{mn} + \frac{2}{a}\left(\frac{m\pi}{a}\right)\left[c_n' - (-1)^m c_n''\right]C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)\left[g_m' - (-1)^n g_m''\right]C_1\right\} \\ &+ \frac{1}{E_{mn}}\left\{\frac{2}{b}\left(\frac{m\pi}{a}\right)^3\left[c_m''' - (-1)^n c_m''''\right]C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3\left[g_n''' - (-1)^m g_n''''\right]C_1\right\} \end{aligned} \quad \text{(B95)}$$

The equations of equilibrium for the stiffener transverse shears, (B8), will now be considered. Substituting the series from equations (B23), (B27), (B28), (B32), (B33) and (6) into equations (B8), and considering equation (B65), one obtains:

$$
\left.
\begin{aligned}
v_n' &= \left(\frac{b}{n\pi}\right)\left[g_n''' - B_n'\right] && (n \neq 0)\\
v_n'' &= \left(\frac{b}{n\pi}\right)\left[g_n'''' - B_n''\right] && (n \neq 0)\\
v_m''' &= \left(\frac{a}{m\pi}\right)\left[c_m''' - B_m'''\right] && (m \neq 0)\\
v_m'''' &= \left(\frac{a}{m\pi}\right)\left[c_m'''' - B_m''''\right] && (m \neq 0)
\end{aligned}
\right\} \qquad \text{(B96)}
$$

Meanwhile, from equations (B15)

$$V_1(0) = \sum_{n=0}^{N} v_n' = v_0' + \sum_{n=1}^{N} v_n', \quad etc.$$

It therefore follows that

$$
\left.
\begin{aligned}
v_0' &= V_1(0) - \sum_{n=1}^{N} \left(\frac{b}{n\pi}\right)\left[g_n''' - B_n'\right]\\
v_0'' &= V_2(0) - \sum_{n=1}^{N} \left(\frac{b}{n\pi}\right)\left[g_n'''' - B_n''\right]\\
v_0''' &= V_3(0) - \sum_{m=1}^{M} \left(\frac{a}{m\pi}\right)\left[c_m''' - B_m'''\right]\\
v_0'''' &= V_4(0) - \sum_{m=1}^{M} \left(\frac{a}{m\pi}\right)\left[c_m'''' - B_m''''\right]
\end{aligned}
\right\} \qquad \text{(B97)}
$$

Similarly, equations (B7), with the various terms replaced by their series expressions from equations (7), (B34), (B15) and (B22) and w_n', w_n'', w_m''', w_m'''' then eliminated through equations (B64), give

$$\frac{2-\delta_{n0}}{b}\left[M_1(b)(-1)^n - M_1(0)\right] + \left(\frac{n\pi}{b}\right)b_n' - t_1^i \sum_{m=0}^{M} j_{mn}' + t_1^o Q_n' - v_n' = 0$$

$$\frac{2-\delta_{n0}}{b}\left[M_2(b)(-1)^n - M_2(0)\right] + \left(\frac{n\pi}{b}\right)b_n'' + t_2^i \sum_{m=0}^{M} (-1)^m j_{mn}' - t_2^o Q_n'' - v_n'' = 0$$

$$\frac{2-\delta_{m0}}{a}\left[M_3(a)(-1)^m - M_3(0)\right] + \left(\frac{m\pi}{a}\right)b_m''' - t_3^i \sum_{n=0}^{N} j_{mn}' + t_3^o Q_m''' - v_m''' = 0$$

$$\frac{2-\delta_{m0}}{a}\left[M_4(a)(-1)^m - M_4(0)\right] + \left(\frac{m\pi}{a}\right)b_m'''' + t_4^i \sum_{n=0}^{N} (-1)^n j_{mn}' - t_4^o Q_m'''' - v_m'''' = 0$$

Thus, with the use of equations (B92), (B95), (B96), and (B97) one obtains

$$\left.\begin{aligned}
b_n' &= \frac{2}{n\pi}[M_1(0)-(-1)^n M_1(b)] + \frac{t_1^i}{a}\left(\frac{b}{n\pi}\right)^2 (g_n''' - g_n'''') - t_1^i \sum_{m=1}^{M} \left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\Big\{ T_{mn} \\
&\quad + \frac{2}{a}\left(\frac{m\pi}{a}\right)[C_n' - (-1)^m C_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m' - (-1)^n g_m'']C_1 \Big\} \\
&\quad + t_1^i \left(\frac{b}{n\pi}\right) \sum_{m=1}^{M} \frac{1}{E_{mn}} \Big\{ \frac{2}{b}\left(\frac{m\pi}{a}\right)^3 [C_m''' - (-1)^n C_m''''] C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3 [g_n''' - (-1)^m g_n''''] C_1 \Big\} \\
&\quad + \left(\frac{b}{n\pi}\right)^2 (g_n''' - B_n') - t_1^o \left(\frac{b}{n\pi}\right) Q_n' \\
b_n'' &= \frac{2}{n\pi}[M_2(0)-(-1)^n M_2(b)] - \frac{t_2^i}{a}\left(\frac{b}{n\pi}\right)^2 (g_n''' - g_n'''') + t_2^i \sum_{m=1}^{M} \left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}\Big\{ T_{mn} \\
&\quad + \frac{2}{a}\left(\frac{m\pi}{a}\right)[C_n' - (-1)^m C_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m' - (-1)^n g_m'']C_1 \Big\} \\
&\quad - t_2^i \left(\frac{b}{n\pi}\right) \sum_{m=1}^{M} \frac{(-1)^m}{E_{mn}} \Big\{ \frac{2}{b}\left(\frac{m\pi}{a}\right)^3 [C_m''' - (-1)^n C_m''''] C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3 [g_n''' - (-1)^m g_n''''] C_1 \Big\} \\
&\quad + \left(\frac{b}{n\pi}\right)^2 (g_n'''' - B_n'') + t_2^o \left(\frac{b}{n\pi}\right) Q_n'' \\
b_m''' &= \frac{2}{m\pi}[M_3(0)-(-1)^m M_3(a)] + \frac{t_3^i}{b}\left(\frac{a}{m\pi}\right)^2 (C_m''' - C_m'''') - t_3^i \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\Big\{ T_{mn} \\
&\quad + \frac{2}{a}\left(\frac{m\pi}{a}\right)[C_n' - (-1)^m C_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m' - (-1)^n g_m'']C_1 \Big\} \\
&\quad + t_3^i \left(\frac{a}{m\pi}\right) \sum_{n=1}^{N} \frac{1}{E_{mn}} \Big\{ \frac{2}{b}\left(\frac{m\pi}{a}\right)^3 [C_m''' - (-1)^n C_m''''] C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3 [g_n''' - (-1)^m g_n''''] C_1 \Big\} \\
&\quad + \left(\frac{a}{m\pi}\right)^2 (C_m''' - B_m''') - t_3^o \left(\frac{a}{m\pi}\right) Q_m''' \\
b_m'''' &= \frac{2}{m\pi}[M_4(0)-(-1)^m M_4(a)] - \frac{t_4^i}{b}\left(\frac{a}{m\pi}\right)^2 (C_m''' - C_m'''') + t_4^i \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}\Big\{ T_{mn} \\
&\quad + \frac{2}{a}\left(\frac{m\pi}{a}\right)[C_n' - (-1)^m C_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m' - (-1)^n g_m'']C_1 \Big\} \\
&\quad - t_4^i \left(\frac{a}{m\pi}\right) \sum_{n=1}^{N} \frac{(-1)^n}{E_{mn}} \Big\{ \frac{2}{b}\left(\frac{m\pi}{a}\right)^3 [C_m''' - (-1)^n C_m''''] C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3 [g_n''' - (-1)^m g_n''''] C_1 \Big\} \\
&\quad + \left(\frac{a}{m\pi}\right)^2 (C_m'''' - B_m'''') + t_4^o \left(\frac{a}{m\pi}\right) Q_m''''
\end{aligned}\right\} \quad \text{(B98)}$$

Equations (B9) can now be used to establish the expressions for the Fourier coefficients s_n', s_n'', s_m''', s_m'''' in terms of the same class of basic unknowns c_n', c_n'', c_m''', c_m'''', g_m', g_m'', g_n''', and g_n''''. Using series expansions from equations (9), (B13), (B14), (B25) to (B28) and (B30) to (B33) and eliminating b_n', b_n'', b_m''', b_m'''' through equations (B98) the following expressions for the s_n', s_n'', s_m''' and s_m'''' result:

$$\left.\begin{aligned}
\frac{s_n'}{A_1E_1} &= -\frac{t_1^i}{E_1I_1}\left(\frac{2}{n\pi}\right)[M_1(0)-(-1)^n M_1(b)] - T_n' - t_1^i K_n' + \frac{(t_1^i)^2}{E_1I_1}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}T_{mn} + \frac{t_1^i}{E_1I_1}\left(\frac{b}{n\pi}\right)^2 B_n' \\
&\quad + \frac{t_1^i t_1^o}{E_1I_1}\left(\frac{b}{n\pi}\right)Q_n' + C_2 c_n' - \left[C_3 + \frac{(t_1^i)^2}{E_1I_1}\frac{1}{a}\left(\frac{b}{n\pi}\right)^2 + \frac{t_1^i}{E_1I_1}\left(\frac{b}{n\pi}\right)^2\right]g_n''' + \frac{(t_1^i)^2}{E_1I_1}\frac{1}{a}\left(\frac{b}{n\pi}\right)^2 g_n'''' \\
&\quad + \frac{(t_1^i)^2}{E_1I_1}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left\{\frac{2}{a}\left(\frac{m\pi}{a}\right)[c_n'-(-1)^m c_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m'-(-1)^n g_m'']C_1\right\} \\
&\quad - \frac{(t_1^i)^2}{E_1I_1}\left(\frac{b}{n\pi}\right)\sum_{m=1}^{M}\frac{1}{E_{mn}}\left\{\frac{2}{b}\left(\frac{m\pi}{a}\right)^3[c_m'''-(-1)^n c_m'''']C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3[g_n'''-(-1)^m g_n'''']C_1\right\} \\
\frac{s_n''}{A_2E_2} &= -\frac{t_2^i}{E_2I_2}\left(\frac{2}{n\pi}\right)[M_2(0)-(-1)^n M_2(b)] - T_n'' - t_2^i K_n'' - \frac{(t_2^i)^2}{E_2I_2}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}T_{mn} + \frac{t_2^i}{E_2I_2}\left(\frac{b}{n\pi}\right)^2 B_n'' \\
&\quad - \frac{t_2^i t_2^o}{E_2I_2}\left(\frac{b}{n\pi}\right)Q_n'' + C_2 c_n'' + \frac{(t_2^i)^2}{E_2I_2}\frac{1}{a}\left(\frac{b}{n\pi}\right)^2 g_n''' - \left[C_3 + \frac{(t_2^i)^2}{E_2I_2}\frac{1}{a}\left(\frac{b}{n\pi}\right)^2 + \frac{t_2^i}{E_2I_2}\left(\frac{b}{n\pi}\right)^2\right]g_n'''' \\
&\quad - \frac{(t_2^i)^2}{E_2I_2}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}\left\{\frac{2}{a}\left(\frac{m\pi}{a}\right)[c_n'-(-1)^m c_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m'-(-1)^n g_m'']C_1\right\} \\
&\quad - \frac{(t_2^i)^2}{E_2I_2}\left(\frac{b}{n\pi}\right)\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\left\{\frac{2}{b}\left(\frac{m\pi}{a}\right)^3[c_m'''-(-1)^n c_m'''']C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3[g_n'''-(-1)^m g_n'''']C_1\right\} \\
\frac{s_m'''}{A_3E_3} &= -\frac{t_3^i}{E_3I_3}\left(\frac{2}{m\pi}\right)[M_3(0)-(-1)^m M_3(a)] - T_m''' - t_3^i K_m''' + \frac{(t_3^i)^2}{E_3I_3}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}T_{mn} + \frac{t_3^i}{E_3I_3}\left(\frac{a}{m\pi}\right)^2 B_m''' \\
&\quad + \frac{t_3^i t_3^o}{E_3I_3}\left(\frac{a}{m\pi}\right)Q_m''' + C_1 g_m' - \left[C_3 + \frac{(t_3^i)^2}{E_3I_3}\frac{1}{b}\left(\frac{a}{m\pi}\right)^2 + \frac{t_3^i}{E_3I_3}\left(\frac{a}{m\pi}\right)^2\right]c_m''' + \frac{t_3^i}{E_3I_3}\frac{1}{b}\left(\frac{a}{m\pi}\right)^2 c_m'''' \\
&\quad + \frac{(t_3^i)^2}{E_3I_3}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left\{\frac{2}{a}\left(\frac{m\pi}{a}\right)[c_n'-(-1)^m c_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m'-(-1)^n g_m'']C_1\right\} \\
&\quad - \frac{(t_3^i)^2}{E_3I_3}\left(\frac{a}{m\pi}\right)\sum_{n=1}^{N}\frac{1}{E_{mn}}\left\{\frac{2}{b}\left(\frac{m\pi}{a}\right)^3[c_m'''-(-1)^n c_m'''']C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3[g_n'''-(-1)^m g_n'''']C_1\right\} \\
\frac{s_m''''}{A_4E_4} &= -\frac{t_4^i}{E_4I_4}\left(\frac{2}{m\pi}\right)[M_4(0)-(-1)^m M_4(a)] - T_m'''' - t_4^i K_m'''' - \frac{(t_4^i)^2}{E_4I_4}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}T_{mn} + \frac{t_4^i}{E_4I_4}\left(\frac{a}{m\pi}\right)^2 B_m'''' \\
&\quad - \frac{t_4^i t_4^o}{E_4I_4}\left(\frac{a}{m\pi}\right)Q_m'''' + C_1 g_m'' + \frac{(t_4^i)^2}{E_4I_4}\frac{1}{b}\left(\frac{a}{m\pi}\right)^2 c_m''' - \left[C_3 + \frac{(t_4^i)^2}{E_4I_4}\frac{1}{b}\left(\frac{a}{m\pi}\right)^2 + \frac{t_4^i}{E_4I_4}\left(\frac{a}{m\pi}\right)^2\right]c_m'''' \\
&\quad - \frac{(t_4^i)^2}{E_4I_4}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}\left\{\frac{2}{a}\left(\frac{m\pi}{a}\right)[c_n'-(-1)^m c_n'']C_2 + \frac{2}{b}\left(\frac{n\pi}{b}\right)[g_m'-(-1)^n g_m'']C_1\right\} \\
&\quad + \frac{(t_4^i)^2}{E_4I_4}\left(\frac{a}{m\pi}\right)\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\left\{\frac{2}{b}\left(\frac{m\pi}{a}\right)^3[c_m'''-(-1)^n c_m'''']C_2 + \frac{2}{a}\left(\frac{n\pi}{b}\right)^3[g_n'''-(-1)^m g_n'''']C_1\right\}
\end{aligned}\right\} \quad \text{(B99)}$$

Reduction in number of simultaneous equations. Equations (B87) to (B90) with the n = o and m = o equations excluded, and equations (B10) with their various terms replaced by series expansions, can now be used to obtain eight systems of simultaneous equations in which the c_n', c_n'', c_m''', c_m'''', g_m', g_m'', g_n''', and g_n'''' are the only unknowns ($V_1(o)$, $V_1(b)$, $\overline{M}_1$, $\overline{M}_2$, etc. are assumed to be known for the time being; their expressions, in terms of the c's and g's, will be given later). The first four of these systems of equations are obtained from equations (B87) to (B90) when s_n', s_n'', s_m''', s_m'''', j_{mn}, $P_1(o)$, $P_1(b)$, etc. and $M_1(o)$, $M_1(b)$, etc. are eliminated with the aid of equations (B76), (B77), (B95) and (B99). They are as follows:

$$D_{Mn}' c_n' - F_{Mn}' c_n'' - G_{Mn}' g_n''' + H_{Mn}' g_n''''$$

$$= -\frac{2}{b}\left(\frac{n\pi}{b}\right)^2\left[\frac{A_1(t_1^i)^2}{I_1}+1\right]C_1 \sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left[g_m' - (-1)^n g_m''\right]$$

$$+\frac{2}{b}\left[\frac{A_1(t_1^i)^2}{I_1}+1\right]C_2 \sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}}\left[c_m''' - (-1)^n c_m''''\right] - R_n'$$

$$-\frac{2}{b}\left[\frac{A_1(t_1^i)^2}{I_1}+1\right]\left[V_3(o) - (-1)^n V_4(o)\right] + \frac{A_1(t_1^i)^2}{I_1}\frac{2}{b t_1^i}\left[\overline{M}_1 - (-1)^n \overline{M}_4\right] \qquad \text{(B100)}$$

$$(n = 1, 2, \cdots N)$$

$$D''_{Mn} c'_n - F''_{Mn} c''_n - G''_{Mn} g'''_n + H''_{Mn} g''''_n$$

$$= -\frac{2}{b}\left(\frac{n\pi}{b}\right)^2\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]C_1 \sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}\left[g'_m-(-1)^n g''_m\right]$$

$$+\frac{2}{b}\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]C_2 \sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^3\frac{(-1)^m}{E_{mn}}\left[c'''_m-(-1)^n c''''_m\right]+R''_n$$

$$-\frac{2}{b}\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]\left[V_3(a)-(-1)^n V_4(a)\right]-\frac{A_2(t_2^i)^2}{I_2}\frac{2}{b t_2^i}\left[\bar{M}_2-(-1)^n\bar{M}_3\right] \qquad \text{(B101)}$$

$$(n=1,2,\cdots,N)$$

$$D'''_{Nm} g'_m - F'''_{Nm} g''_m - G'''_{Nm} c'''_m + H'''_{Nm} c''''_m$$

$$= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left[\frac{A_3(t_3^i)^2}{I_3}+1\right]C_2 \sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[c'_n-(-1)^m c''_n\right]$$

$$+\frac{2}{a}\left[\frac{A_3(t_3^i)^2}{I_3}+1\right]C_1 \sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}}\left[g'''_n-(-1)^m g''''_n\right]-R'''_m$$

$$-\frac{2}{a}\left[\frac{A_3(t_3^i)^2}{I_3}+1\right]\left[V_1(0)-(-1)^m V_2(0)\right]+\frac{A_3(t_3^i)^2}{I_3}\frac{2}{a t_3^i}\left[\bar{M}_1-(-1)^m\bar{M}_2\right] \qquad \text{(B102)}$$

$$(m=1,2,\cdots,M)$$

$$D_{N_m}^{''''} g_m' - F_{N_m}^{''''} g_m'' - G_{N_m}^{''''} c_m''' + H_{N_m}^{''''} c_m''''$$

$$= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left[\frac{A_4(t_4^i)^2}{I_4}+1\right]C_2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}\left[c_n' - (-1)^m c_n''\right]$$

$$+\frac{2}{a}\left[\frac{A_4(t_4^i)^2}{I_4}+1\right]C_1\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^3\frac{(-1)^n}{E_{mn}}\left[g_n''' - (-1)^m g_n''''\right] + R_m''''$$

$$-\frac{2}{a}\left[\frac{A_4(t_4^i)^2}{I_4}+1\right]\left[V_1(b) - (-1)^m V_2(b)\right] - \frac{A_4(t_4^i)^2}{I_4}\frac{2}{a t_4^i}\left[\bar{M}_4 - (-1)^m \bar{M}_3\right]$$

$$(m=1,2,\cdots,M) \qquad \text{(B103)}$$

where

$$D_{M_n}' = C_2\left(\frac{n\pi}{b}\right)\left\{A_1E_1 + \left[\frac{A_1(t_1^i)^2}{I_1}+1\right]\frac{2}{a}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\right\}$$

$$D_{M_n}'' = \left[\frac{A_2(t_2^i)^2}{I_2}+1\right]C_2\left(\frac{n\pi}{b}\right)\frac{2}{a}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}$$

$$D_{N_m}''' = C_1\left(\frac{m\pi}{a}\right)\left\{A_3E_3 + \left[\frac{A_3(t_3^i)^2}{I_3}+1\right]\frac{2}{b}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\right\}$$

$$D_{N_m}'''' = \left[\frac{A_4(t_4^i)^2}{I_4}+1\right]C_1\left(\frac{m\pi}{a}\right)\frac{2}{b}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}$$

$$F_{M_n}' = \left[\frac{A_1(t_1^i)^2}{I_1}+1\right]C_2\left(\frac{n\pi}{b}\right)\frac{2}{a}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}$$

$$F_{M_n}'' = C_2\left(\frac{n\pi}{b}\right)\left\{A_2E_2 + \left[\frac{A_2(t_2^i)^2}{I_2}+1\right]\frac{2}{a}\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\right\}$$

$$F_{N_m}''' = \left[\frac{A_3(t_3^i)^2}{I_3}+1\right]C_1\left(\frac{m\pi}{a}\right)\frac{2}{b}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}$$

$$F_{N_m}'''' = C_1\left(\frac{m\pi}{a}\right)\left\{A_4E_4 + \left[\frac{A_4(t_4^i)^2}{I_4}+1\right]\frac{2}{b}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\right\}$$

$$G_{Mn}^{\prime}=A_1E_1C_3\left(\frac{n\pi}{b}\right)+\frac{A_1(t_1^i)^2}{I_1}\left(\frac{b}{t_1^i}\right)\left(\frac{1}{n\pi}\right)+\left[\frac{A_1(t_1^i)^2}{I_1}+1\right]\left[\frac{1}{a}\left(\frac{b}{n\pi}\right)+C_1\left(\frac{n\pi}{b}\right)^3\frac{2}{a}\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]$$

$$G_{Mn}^{\prime\prime}=\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]\left[\frac{1}{a}\left(\frac{b}{n\pi}\right)+C_1\left(\frac{n\pi}{b}\right)^3\frac{2}{a}\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]$$

$$G_{Nm}^{\prime\prime\prime}=A_3E_3C_3\left(\frac{m\pi}{a}\right)+\frac{A_3(t_3^i)^2}{I_3}\left(\frac{a}{t_3^i}\right)\left(\frac{1}{m\pi}\right)+\left[\frac{A_3(t_3^i)^2}{I_3}+1\right]\left[\frac{1}{b}\left(\frac{a}{m\pi}\right)+C_2\left(\frac{m\pi}{a}\right)^3\frac{2}{b}\sum_{n=1}^{N}\frac{1}{E_{mn}}\right]$$

$$G_{Nm}^{\prime\prime\prime\prime}=\left[\frac{A_4(t_4^i)^2}{I_4}+1\right]\left[\frac{1}{b}\left(\frac{a}{m\pi}\right)+C_2\left(\frac{m\pi}{a}\right)^3\frac{2}{b}\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]$$

$$H_{Mn}^{\prime}=\left[\frac{A_1(t_1^i)^2}{I_1}+1\right]\left[\frac{1}{a}\left(\frac{b}{n\pi}\right)+C_1\left(\frac{n\pi}{b}\right)^3\frac{2}{a}\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]$$

$$H_{Mn}^{\prime\prime}=A_2E_2C_3\left(\frac{n\pi}{b}\right)+\frac{A_2(t_2^i)^2}{I_2}\left(\frac{b}{t_2^i}\right)\left(\frac{1}{n\pi}\right)+\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]\left[\frac{1}{a}\left(\frac{b}{n\pi}\right)+C_1\left(\frac{n\pi}{b}\right)^3\frac{2}{a}\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]$$

$$H_{Nm}^{\prime\prime\prime}=\left[\frac{A_3(t_3^i)^2}{I_3}+1\right]\left[\frac{1}{b}\left(\frac{a}{m\pi}\right)+C_2\left(\frac{m\pi}{a}\right)^3\frac{2}{b}\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]$$

$$H_{Nm}^{\prime\prime\prime\prime}=A_4E_4C_3\left(\frac{m\pi}{a}\right)+\frac{A_4(t_4^i)^2}{I_4}\left(\frac{a}{t_4^i}\right)\left(\frac{1}{m\pi}\right)+\left[\frac{A_4(t_4^i)^2}{I_4}+1\right]\left[\frac{1}{b}\left(\frac{a}{m\pi}\right)+C_2\left(\frac{m\pi}{a}\right)^3\frac{2}{b}\sum_{n=1}^{N}\frac{1}{E_{mn}}\right]$$

$$R_n^{\prime}=-\frac{2}{b}\left[P_y^{\prime}-(-1)^nP_y^{\prime\prime}\right]-Q_n^{\prime}+\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}T_{mn}-A_1E_1\left(\frac{n\pi}{b}\right)T_n^{\prime}$$

$$-A_1E_1t_1^i\left(\frac{n\pi}{b}\right)K_n^{\prime}+\frac{A_1(t_1^i)^2}{I_1}\left[\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}T_{mn}+\frac{b}{t_1^i}\left(\frac{1}{n\pi}\right)B_n^{\prime}+\frac{t_1^o}{t_1^i}Q_n^{\prime}\right]$$

$$R_n^{\prime\prime}=-\frac{2}{b}\left[P_y^{\prime\prime\prime}-(-1)^nP_y^{\prime\prime\prime\prime}\right]+Q_n^{\prime\prime}-\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}T_{mn}-A_2E_2\left(\frac{n\pi}{b}\right)T_n^{\prime\prime}$$

$$-A_2E_2t_2^i\left(\frac{n\pi}{b}\right)K_n^{\prime\prime}-\frac{A_2(t_2^i)^2}{I_2}\left[\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}T_{mn}-\frac{b}{t_2^i}\left(\frac{1}{n\pi}\right)B_n^{\prime\prime}+\frac{t_2^o}{t_2^i}Q_n^{\prime\prime}\right]$$

$$R_m^{\prime\prime\prime}=-\frac{2}{a}\left[P_x^{\prime}-(-1)^mP_x^{\prime\prime}\right]-Q_m^{\prime\prime\prime}+\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}T_{mn}-A_3E_3\left(\frac{m\pi}{a}\right)T_m^{\prime\prime\prime}$$

$$-A_3E_3t_3^i\left(\frac{m\pi}{a}\right)K_m^{\prime\prime\prime}+\frac{A_3(t_3^i)^2}{I_3}\left[\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}T_{mn}+\frac{a}{t_3^i}\left(\frac{1}{m\pi}\right)B_m^{\prime\prime\prime}+\frac{t_3^o}{t_3^i}Q_m^{\prime\prime\prime}\right]$$

$$R_m^{\prime\prime\prime\prime}=-\frac{2}{a}\left[P_x^{\prime\prime\prime}-(-1)^mP_x^{\prime\prime\prime\prime}\right]+Q_m^{\prime\prime\prime\prime}-\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}T_{mn}-A_4E_4\left(\frac{m\pi}{a}\right)T_m^{\prime\prime\prime\prime}$$

$$-A_4E_4t_4^i\left(\frac{m\pi}{a}\right)K_m^{\prime\prime\prime\prime}-\frac{A_4(t_4^i)^2}{I_4}\left[\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}T_{mn}-\frac{a}{t_4^i}\left(\frac{1}{m\pi}\right)B_m^{\prime\prime\prime\prime}+\frac{t_4^o}{t_4^i}Q_m^{\prime\prime\prime\prime}\right]$$

The remaining four systems of equations are obtained from equations (B10). First, the series expansions from equations (6), (8), (10), (B14), (B27), (B28), (B32), (B33), and (B38) to (B41) are substituted into equations (B10) which results in

$$\left.\begin{aligned}
&(C_3-C_4)\sum_{m=0}^{M} w_{mn} - C_2\sum_{m=0}^{M} d_{mn} - L'_n + \frac{1}{E_1I_1}b'_n + \frac{1}{G_1A_{s1}}(g'''_n - B'_n) + K'_n = 0\\
&(C_3-C_4)\sum_{m=0}^{M}(-1)^m w_{mn} - C_2\sum_{m=0}^{M}(-1)^m d_{mn} - L''_n - \frac{1}{E_2I_2}b''_n - \frac{1}{G_2A_{s2}}(g''''_n - B''_n) - K''_n = 0\\
&(C_3-C_4)\sum_{n=0}^{N} x_{mn} - C_1\sum_{n=0}^{N} b_{mn} - L'''_m + \frac{1}{E_3I_3}b'''_m + \frac{1}{G_3A_{s3}}(c'''_m - B'''_m) + K'''_m = 0\\
&(C_3-C_4)\sum_{n=0}^{N}(-1)^n x_{mn} - C_1\sum_{n=0}^{N}(-1)^n b_{mn} - L''''_m - \frac{1}{E_4I_4}b''''_m - \frac{1}{G_4A_{s4}}(c''''_m - B''''_m) - K''''_m = 0
\end{aligned}\right\} \quad (B104)$$

Then x_{mn}, w_{mn}, d_{mn}, b_{mn}, and b'_n through b''''_m are replaced by their expressions in equations (B72) to (B75) and (B98), after which a_{mn}, j_{mn}, and a'_n through a''''_m are eliminated by means of equations (B85), (B92), (B95) and (B84). The following four systems of equations result:

$$I'_{Mn}c'_n - K'_{Mn}c''_n - P'_{Mn}g'''_n + S'_{Mn}g''''_n$$

$$= -\frac{2}{ab}\left(\frac{n\pi}{b}\right)C_1\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2 - \frac{b}{a}\frac{C_2}{C_4}(m\pi)^2 - \frac{abt_1^i}{C_4E_1I_1}\right]\left[g'_m - (-1)^n g''_m\right]$$

$$+\frac{2}{ab}\left(\frac{n\pi}{b}\right)C_2\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left[\frac{a}{b}\frac{C_1}{C_4}(n\pi)^2 - \frac{b}{a}\frac{C_2}{C_4}(m\pi)^2 - \left(\frac{b}{a}\right)^2\left(\frac{m}{n}\right)^2\frac{abt_1^i}{C_4E_1I_1}\right]\left[c'''_m - (-1)^n c''''_m\right]$$

$$-Z'_n + \left(\frac{2}{n\pi}\right)\frac{bt_1^i}{C_4E_1I_1}\left[V_3(0) - (-1)^n V_4(0)\right] - \left(\frac{2}{n\pi}\right)\frac{b}{C_4E_1I_1}\left[\bar{M}_1 - (-1)^n\bar{M}_4\right] \qquad (B105)$$

$$(n = 1, 2, \cdots, N)$$

$$I''_{Mn} c'_n - K''_{Mn} c''_n - P''_{Mn} g'''_n + S''_{Mn} g''''_n$$

$$= -\frac{2}{ab}\left(\frac{n\pi}{b}\right) C_1 \sum_{m=1}^{M} \left(\frac{m\pi}{a}\right) \frac{(-1)^m}{E_{mn}} \left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2 - \frac{b}{a}\frac{C_1}{C_4}(m\pi)^2 - \frac{abt_2^i}{C_4E_2I_2}\right]\left[g'_m-(-1)^n g''_m\right]$$

$$+\frac{2}{ab}\left(\frac{n\pi}{b}\right) C_2 \sum_{m=1}^{M} \left(\frac{m\pi}{a}\right) \frac{(-1)^m}{E_{mn}} \left[\frac{a}{b}\frac{C_1}{C_4}(n\pi)^2 - \frac{b}{a}\frac{C_3}{C_4}(m\pi)^2 - \left(\frac{b}{a}\right)^2\left(\frac{m}{n}\right)^2\frac{abt_2^i}{C_4E_2I_2}\right]\left[c'''_m-(-1)^n c''''_m\right]$$

$$+Z''_n + \left(\frac{2}{n\pi}\right)\frac{bt_2^i}{C_4E_2I_2}\left[V_3(a)-(-1)^n V_4(a)\right] + \left(\frac{2}{n\pi}\right)\frac{b}{C_4E_2I_2}\left[\bar{M}_2-(-1)^n\bar{M}_3\right] \qquad (B106)$$

$(n=1,2,----,N)$

$$I'''_{Nm} g'_m - K'''_{Nm} g''_m - P'''_{Nm} c'''_m + S'''_{Nm} c''''_m$$

$$= -\frac{2}{ab}\left(\frac{m\pi}{a}\right) C_2 \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right) \frac{1}{E_{mn}} \left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2 - \frac{a}{b}\frac{C_1}{C_4}(n\pi)^2 - \frac{abt_3^i}{C_4E_3I_3}\right]\left[c'_n-(-1)^m c''_n\right]$$

$$+\frac{2}{ab}\left(\frac{m\pi}{a}\right) C_1 \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right) \frac{1}{E_{mn}} \left[\frac{b}{a}\frac{C_2}{C_4}(m\pi)^2 - \frac{a}{b}\frac{C_3}{C_4}(n\pi)^2 - \left(\frac{a}{b}\right)^2\left(\frac{n}{m}\right)^2\frac{abt_3^i}{C_4E_3I_3}\right]\left[g'''_n-(-1)^m g''''_n\right]$$

$$-Z'''_m + \left(\frac{2}{m\pi}\right)\frac{at_3^i}{C_4E_3I_3}\left[V_1(0)-(-1)^m V_2(0)\right] - \left(\frac{2}{m\pi}\right)\frac{a}{C_4E_3I_3}\left[\bar{M}_1-(-1)^m\bar{M}_2\right] \qquad (B107)$$

$(m=1,2,----,M)$

$$I''''_{Nm} g'_m - K''''_{Nm} g''_m - P''''_{Nm} c'''_m + S''''_{Nm} c''''_m$$

$$= -\frac{2}{ab}\left(\frac{m\pi}{a}\right) C_2 \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right) \frac{(-1)^n}{E_{mn}} \left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2 - \frac{a}{b}\frac{C_1}{C_4}(n\pi)^2 - \frac{abt_4^i}{C_4E_4I_4}\right]\left[c'_n-(-1)^m c''_n\right]$$

$$+\frac{2}{ab}\left(\frac{m\pi}{a}\right) C_1 \sum_{n=1}^{N} \left(\frac{n\pi}{b}\right) \frac{(-1)^n}{E_{mn}} \left[\frac{b}{a}\frac{C_2}{C_4}(m\pi)^2 - \frac{a}{b}\frac{C_3}{C_4}(n\pi)^2 - \left(\frac{a}{b}\right)^2\left(\frac{n}{m}\right)^2\frac{abt_4^i}{C_4E_4I_4}\right]\left[g'''_n-(-1)^m g''''_n\right]$$

$$+Z''''_m + \left(\frac{2}{m\pi}\right)\frac{at_4^i}{C_4E_4I_4}\left[V_1(b)-(-1)^m V_2(b)\right] + \left(\frac{2}{m\pi}\right)\frac{a}{C_4E_4I_4}\left[\bar{M}_4-(-1)^m\bar{M}_3\right] \qquad (B108)$$

$(m=1,2,----,M)$

in which

$$I'_{Mn}=\frac{b}{a}\frac{C_2}{C_4}+\frac{2}{a^2}C_2\left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2-\frac{abt_1^i}{C_4E_1I_1}\right]\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}-2\frac{b}{a}\frac{C_2}{C_4}\sum_{m=1}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$I''_{Mn}=\frac{b}{a}\frac{C_2}{C_4}+\frac{2}{a^2}C_2\left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2-\frac{abt_2^i}{C_4E_2I_2}\right]\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}-2\frac{b}{a}\frac{C_2}{C_4}\sum_{m=1}^{M}(-1)^m\left[C_2\left(\frac{m\pi}{a}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$I'''_{Nm}=\frac{a}{b}\frac{C_1}{C_4}+\frac{2}{b^2}C_1\left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2-\frac{abt_3^i}{C_4E_3I_3}\right]\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}-2\frac{a}{b}\frac{C_1}{C_4}\sum_{n=1}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$I''''_{Nm}=\frac{a}{b}\frac{C_1}{C_4}+\frac{2}{b^2}C_1\left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2-\frac{abt_4^i}{C_4E_4I_4}\right]\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}-2\frac{a}{b}\frac{C_1}{C_4}\sum_{n=1}^{N}(-1)^n\left[C_1\left(\frac{n\pi}{b}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$K'_{Mn}=\frac{b}{a}\frac{C_2}{C_4}+\frac{2}{a^2}C_2\left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2-\frac{abt_1^i}{C_4E_1I_1}\right]\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}-2\frac{b}{a}\frac{C_2}{C_4}\sum_{m=1}^{M}(-1)^m\left[C_2\left(\frac{m\pi}{a}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$K''_{Mn}=\frac{b}{a}\frac{C_2}{C_4}+\frac{2}{a^2}C_2\left[\frac{a}{b}\frac{(C_3-C_4)}{C_4}(n\pi)^2-\frac{abt_2^i}{C_4E_2I_2}\right]\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}-2\frac{b}{a}\frac{C_2}{C_4}\sum_{m=1}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$K'''_{Nm}=\frac{a}{b}\frac{C_1}{C_4}+\frac{2}{b^2}C_1\left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2-\frac{abt_3^i}{C_4E_3I_3}\right]\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}-2\frac{a}{b}\frac{C_1}{C_4}\sum_{n=1}^{N}(-1)^n\left[C_1\left(\frac{n\pi}{b}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$K''''_{Nm}=\frac{a}{b}\frac{C_1}{C_4}+\frac{2}{b^2}C_1\left[\frac{b}{a}\frac{(C_3-C_4)}{C_4}(m\pi)^2-\frac{abt_4^i}{C_4E_4I_4}\right]\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}-2\frac{a}{b}\frac{C_1}{C_4}\sum_{n=1}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^4\frac{1}{E_{mn}}-1\right]$$

$$P'_{Mn}=\frac{b}{a}\frac{(C_3-C_4)}{C_4}+2\frac{b}{a}\frac{(C_3-C_4)}{C_4}C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}-2\frac{b}{a}\frac{C_1C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}$$
$$-\frac{abt_1^i}{C_4E_1I_1}\left[\left(\frac{b}{a}\right)^2\left(\frac{1}{n\pi}\right)^2+\frac{2}{a^2}C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\frac{1}{E_{mn}}+\frac{b^2}{at_1^i}\left(\frac{1}{n\pi}\right)^2\right]-\frac{b}{C_4G_1A_{s1}}$$

$$P''_{Mn}=\frac{b}{a}\frac{(C_3-C_4)}{C_4}+2\frac{b}{a}\frac{(C_3-C_4)}{C_4}C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}-2\frac{b}{a}\frac{C_1C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}$$
$$-\frac{abt_2^i}{C_4E_2I_2}\left[\left(\frac{b}{a}\right)^2\left(\frac{1}{n\pi}\right)^2+\frac{2}{a^2}C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]$$

$$P'''_{Nm}=\frac{a}{b}\frac{(C_3-C_4)}{C_4}+2\frac{a}{b}\frac{(C_3-C_4)}{C_4}C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{1}{E_{mn}}-2\frac{a}{b}\frac{C_1C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}$$
$$-\frac{abt_3^i}{C_4E_3I_3}\left[\left(\frac{a}{b}\right)^2\left(\frac{1}{m\pi}\right)^2+\frac{2}{b^2}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\frac{1}{E_{mn}}+\frac{a^2}{bt_3^i}\left(\frac{1}{m\pi}\right)^2\right]-\frac{a}{C_4G_3A_{s3}}$$

$$P''''_{Nm}=\frac{a}{b}\frac{(C_3-C_4)}{C_4}+2\frac{a}{b}\frac{(C_3-C_4)}{C_4}C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{(-1)^m}{E_{mn}}-2\frac{a}{b}\frac{C_1C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}$$
$$-\frac{abt_4^i}{C_4E_4I_4}\left[\left(\frac{a}{b}\right)^2\left(\frac{1}{m\pi}\right)^2+\frac{2}{b^2}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]$$

$$S'_{Mn}=\frac{b}{a}\frac{(C_3-C_4)}{C_4}+2\frac{b}{a}\frac{(C_3-C_4)}{C_4}C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}-2\frac{b}{a}\frac{C_1C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{(-1)^m}{E_{mn}}$$

$$-\frac{abt_1^i}{C_4E_1I_1}\left[\left(\frac{b}{a}\right)^2\left(\frac{1}{n\pi}\right)^2+\frac{2}{a^2}C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]$$

$$S''_{Mn}=\frac{b}{a}\frac{(C_3-C_4)}{C_4}+2\frac{b}{a}\frac{(C_3-C_4)}{C_4}C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}-2\frac{b}{a}\frac{C_1C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}$$

$$-\frac{abt_2^i}{C_4E_2I_2}\left[\left(\frac{b}{a}\right)^2\left(\frac{1}{n\pi}\right)^2+\frac{2}{a^2}C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1}^{M}\frac{1}{E_{mn}}+\frac{b^2}{at_2^i}\left(\frac{1}{n\pi}\right)^2\right]-\frac{b}{C_4G_2A_{s2}}$$

$$S'''_{Nm}=\frac{a}{b}\frac{(C_3-C_4)}{C_4}+2\frac{a}{b}\frac{(C_3-C_4)}{C_4}C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}-2\frac{a}{b}\frac{C_1C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^n}{E_{mn}}$$

$$-\frac{abt_3^i}{C_4E_3I_3}\left[\left(\frac{a}{b}\right)^2\left(\frac{1}{m\pi}\right)^2+\frac{2}{b^2}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]$$

$$S''''_{Nm}=\frac{a}{b}\frac{(C_3-C_4)}{C_4}+2\frac{a}{b}\frac{(C_3-C_4)}{C_4}C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{1}{E_{mn}}-2\frac{a}{b}\frac{C_1C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}$$

$$-\frac{abt_4^i}{C_4E_4I_4}\left[\left(\frac{a}{b}\right)^2\left(\frac{1}{m\pi}\right)^2+\frac{2}{b^2}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1}^{N}\frac{1}{E_{mn}}+\frac{a^2}{bt_4^i}\left(\frac{1}{m\pi}\right)^2\right]-\frac{a}{C_4G_4A_{s4}}$$

$$Z'_n=\frac{b}{a}\sum_{m=1}^{M}\left[\frac{(C_3-C_4)}{C_4}\left(\frac{n\pi}{b}\right)^2-\frac{C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\right]\frac{m\pi}{E_{mn}}T_{mn}-\frac{b}{C_4}L'_n+\frac{b}{C_4}K'_n$$

$$-\frac{abt_1^i}{C_4E_1I_1}\left[\frac{1}{a^2}\sum_{m=1}^{M}\frac{m\pi}{E_{mn}}T_{mn}+\frac{b^2}{at_1^i}\left(\frac{1}{n\pi}\right)^2B'_n+\frac{b}{a}\frac{t_1^o}{t_1^i}\left(\frac{1}{n\pi}\right)Q'_n\right]-\frac{b}{C_4G_1A_{s1}}B'_n$$

$$Z''_n=-\frac{b}{a}\sum_{m=1}^{M}\left[\frac{(C_3-C_4)}{C_4}\left(\frac{n\pi}{b}\right)^2-\frac{C_2}{C_4}\left(\frac{m\pi}{a}\right)^2\right](-1)^m\frac{m\pi}{E_{mn}}T_{mn}+\frac{b}{C_4}L''_n+\frac{b}{C_4}K''_n$$

$$+\frac{abt_2^i}{C_4E_2I_2}\left[\frac{1}{a^2}\sum_{m=1}^{M}(-1)^m\frac{m\pi}{E_{mn}}T_{mn}-\frac{b^2}{at_1^i}\left(\frac{1}{n\pi}\right)^2B''_n+\frac{b}{a}\frac{t_2^o}{t_2^i}\left(\frac{1}{n\pi}\right)Q''_n\right]-\frac{b}{C_4G_2A_{s2}}B''_n$$

$$Z'''_m=\frac{a}{b}\sum_{n=1}^{N}\left[\frac{(C_3-C_4)}{C_4}\left(\frac{m\pi}{a}\right)^2-\frac{C_1}{C_4}\left(\frac{n\pi}{b}\right)^2\right]\frac{n\pi}{E_{mn}}T_{mn}-\frac{a}{C_4}L'''_m+\frac{a}{C_4}K'''_m$$

$$-\frac{abt_3^i}{C_4E_3I_3}\left[\frac{1}{b^2}\sum_{n=1}^{N}\frac{n\pi}{E_{mn}}T_{mn}+\frac{a^2}{bt_3^i}\left(\frac{1}{m\pi}\right)^2B'''_m+\frac{a}{b}\frac{t_3^o}{t_3^i}\left(\frac{1}{m\pi}\right)Q'''_m\right]-\frac{a}{C_4G_3A_{s3}}B'''_m$$

$$Z''''_m=-\frac{a}{b}\sum_{n=1}^{N}\left[\frac{(C_3-C_4)}{C_4}\left(\frac{m\pi}{a}\right)^2-\frac{C_1}{C_4}\left(\frac{n\pi}{b}\right)^2\right](-1)^n\frac{n\pi}{E_{mn}}T_{mn}+\frac{a}{C_4}L''''_m+\frac{a}{C_4}K''''_m$$

$$+\frac{abt_4^i}{C_4E_4I_4}\left[\frac{1}{b^2}\sum_{n=1}^{N}(-1)^n\frac{n\pi}{E_{mn}}T_{mn}-\frac{a^2}{bt_4^i}\left(\frac{1}{m\pi}\right)^2B''''_m+\frac{a}{b}\frac{t_4^o}{t_4^i}\left(\frac{1}{m\pi}\right)Q''''_m\right]-\frac{a}{C_4G_4A_{s4}}B''''_m$$

Equations (B100) to (B103) and (B105) to (B108) contain 4(M+N) simultaneous equations which serve to determine the 4(M+N) unknowns, namely, c_n', c_n'', c_m''', c_m'''', g_m', g_m'', g_n''', and g_n''''. These eight equations could be further reduced to four equations if equations (B100) and (B101) together with equations (B105) and (B106) were solved for each c_n', c_n'', g_n''' and g_n'''' in terms of all the g_m', g_m'', c_m''', and c_m''''. In place of equations (B102), (B103), (B107) and (B108) a new set of 4M simultaneous equations would be obtained, involving only 4M unknowns (g_m', g_m'', c_m''' and c_m'''). While fewer unknowns would have to be solved with the use of such reduced 4M simultaneous equations, the former 4(M+N) simultaneous equation, (B100) to (B103) and (B105) to (B108), will be retained and employed in this report because of its greater simplicity in the form of the coefficients.

<u>End shears of stiffeners</u>. Moment equilibrium of the stiffener located at x = o will now be considered. Referring to figure 6, taking moments about point 0, and applying equations (B77), one obtains

$$-V_1(b)\cdot b + (\bar{M}_4 - \bar{M}_1) + t_1^i[V_3(0) - V_4(0)] + t_1^i\int_0^b N_{xy}(0,y)\,dy + t_1^o\int_0^b q_1(y)\,dy$$

$$-\int_0^b N_x(0,y)\,y\,dy + \int_0^b N_1(y)\,y\,dy = 0$$

Substituting the series expansions from equations (6), (7), (B32) and (B34) into this equation and carrying out the integrations indicated, one gets

$$-V_1(b) + \frac{t_1^i}{b} V_3(0) - \frac{t_1^i}{b} V_4(0) = \sum_{n=1}^{N} \left(\frac{b}{n\pi}\right) (-1)^n \left(B_n' - g_n'''\right) + t_1^i \sum_{m=0}^{M} j_{mo} - t_1^o Q_o' + \frac{1}{b}\left(\bar{M}_1 - \bar{M}_4\right) \tag{B109}$$

In view of equations (B76), equations (B94a) to (B94d) may be rewritten as follows:

$$j_{oo} = -Q_o' - \frac{1}{b}\left(P_y' - P_y''\right) - \frac{1}{b}\sum_{m=1}^{M} \frac{a}{m\pi}\left(C_m''' - C_m''''\right) + \frac{1}{b}\left[V_3(0) - V_4(0)\right] \tag{B110a}$$

$$j_{oo} = -Q_o'' + \frac{1}{b}\left(P_y''' - P_y''''\right) - \frac{1}{b}\sum_{m=1}^{M} (-1)^m \left(\frac{a}{m\pi}\right)\left(C_m''' - C_m''''\right) + \frac{1}{b}\left[V_3(a) - V_4(a)\right] \tag{B110b}$$

$$j_{oo} = -Q_o''' - \frac{1}{a}\left(P_x' - P_x''\right) - \frac{1}{a}\sum_{n=1}^{N} \frac{b}{n\pi}\left(g_n''' - g_n''''\right) + \frac{1}{a}\left[V_1(0) - V_2(0)\right] \tag{B110c}$$

$$j_{oo} = -Q_o'''' + \frac{1}{a}\left(P_x''' - P_x''''\right) - \frac{1}{a}\sum_{n=1}^{N} (-1)^n \frac{b}{n\pi}\left(g_n''' - g_n''''\right) + \frac{1}{a}\left[V_1(b) - V_2(b)\right] \tag{B110d}$$

Substitution of equations (B110a) and (B92) into equation (B109) yields

$$V_1(b) = \frac{1}{b}(\bar{M}_4 - \bar{M}_1) + \frac{t_1^i}{b}(P_y' - P_y'') + (t_1^i + t_1^o)Q_o' - \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)B_n' + \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)g_n''' \qquad \text{(B111)}$$

Writing the moment equilibrium equations for the stiffeners located at x = a, y = o, y = b, and going through a similar procedure but using equations (B110b), (B110c), (B110d), respectively, for eliminating j_{oo}, one obtains the following equations for $V_2(b)$, $V_3(a)$, and $V_4(a)$ analogous to (B111):

$$V_2(b) = \frac{1}{b}(\bar{M}_3 - \bar{M}_2) + \frac{t_2^i}{b}(P_y''' - P_y'''') - (t_2^i + t_2^o)Q_o'' - \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)B_n'' + \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)g_n'''' \qquad \text{(B112)}$$

$$V_3(a) = \frac{1}{a}(\bar{M}_2 - \bar{M}_1) + \frac{t_3^i}{a}(P_x' - P_x'') + (t_3^i + t_3^o)Q_o''' - \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)B_m''' + \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)C_m''' \qquad \text{(B113)}$$

$$V_4(a) = \frac{1}{a}(\bar{M}_3 - \bar{M}_4) + \frac{t_4^i}{a}(P_x''' - P_x'''') - (t_4^i + t_4^o)Q_o'''' - \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)B_m'''' + \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)C_m'''' \qquad \text{(B114)}$$

Equations (B111) to (B114) give the expressions for four of the eight stiffener end shears. The remaining four end shears, namely $V_1(o)$, $V_2(o)$, $V_3(o)$ and $V_4(o)$, can be obtained in a similar manner by taking moments about the other end of the stiffener, or can be obtained by using equations (B111) to (B114) and the equilibrium equations of stiffener lateral forces. The resulting expressions are

$$V_1(0) = \frac{1}{b}(\bar{M}_4 - \bar{M}_1) + \frac{t_1^i}{b}(\underline{P}_y' - \underline{P}_y'') + (t_1^i + t_1^o)Q_o' - \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)B_n' + \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)g_n''' \tag{B115}$$

$$V_2(0) = \frac{1}{b}(\bar{M}_3 - \bar{M}_2) + \frac{t_2^i}{b}(\underline{P}_y''' - \underline{P}_y'''') - (t_2^i + t_2^o)Q_o'' - \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)B_n'' + \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)g_n'''' \tag{B116}$$

$$V_3(0) = \frac{1}{a}(\bar{M}_2 - \bar{M}_1) + \frac{t_3^i}{a}(\underline{P}_x' - \underline{P}_x'') + (t_3^i + t_3^o)Q_o''' - \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)B_m''' + \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)C_m''' \tag{B117}$$

$$V_4(0) = \frac{1}{a}(\bar{M}_3 - \bar{M}_4) + \frac{t_4^i}{a}(\underline{P}_x''' - \underline{P}_x'''') - (t_4^i + t_4^o)Q_o'''' - \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)B_m'''' + \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)C_m'''' \tag{B118}$$

<u>Eliminating V_1(o), V_2(o), etc. in equations (B100) to (B103) and (B105) to (B108).</u> The expressions for stiffener end transverse shears, equations (B111) to (B118), can now be employed to eliminate the V_1(o), V_2(o), etc. involved in equations (B100) to (B103) and (B105) to (B108). These equations then take on the following form:

$$D'_{Mn} c'_n - F'_{Mn} c''_n - G'_{Mn} g'''_n + H'_{Mn} g''''_n$$

$$= -\frac{2}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right) C_1 \sum_{m=1}^{M} \frac{m\pi}{a}\frac{1}{E_{mn}}\left[g'_m - (-1)^n g''_m\right]$$

(B119)

$$+\frac{2}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\sum_{m=1}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right]\left[c'''_m - (-1)^n c''''_m\right]$$

$$+\frac{2}{ab}\left[\frac{A_1(t_1^i)^2}{I_1}\left(1+\frac{a}{t_1^i}\right)+1\right]\left[\bar{M}_1 - (-1)^n\bar{M}_4\right] - \frac{2}{ab}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\left[\bar{M}_2 - (-1)^n\bar{M}_3\right] - \bar{R}'_n$$

$$(n = 1, 2, \ldots\ldots, N)$$

$$D''_{Mn} c'_n - F''_{Mn} c''_n - G''_{Mn} g'''_n + H''_{Mn} g''''_n$$

$$= -\frac{2}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_2(t_2^i)^2}{I_2}+1\right) C_1 \sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{(-1)^m}{E_{mn}}\left[g'_m - (-1)^n g''_m\right]$$

(B120)

$$+\frac{2}{b}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)\sum_{m=1}^{M}(-1)^m\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right]\left[c'''_m - (-1)^n c''''_m\right]$$

$$-\frac{2}{ab}\left[\frac{A_2(t_2^i)^2}{I_2}\left(1+\frac{a}{t_2^i}\right)+1\right]\left[\bar{M}_2 - (-1)^n\bar{M}_3\right] + \frac{2}{ab}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)\left[\bar{M}_1 - (-1)^n\bar{M}_4\right] + \bar{R}''_n$$

$$(n = 1, 2, \ldots\ldots, N)$$

$$D_{Nm}^{'''} g_m' - F_{Nm}^{'''} g_m'' - G_{Nm}^{'''} c_m''' + H_{Nm}^{'''} c_m''''$$

$$= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)G_2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[c_n' - (-1)^m c_n''\right]$$

$$+\frac{2}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\sum_{n=1}^{N}\left[G_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right]\left[g_n''' - (-1)^m g_n''''\right]$$

$$+\frac{2}{ab}\left[\frac{A_3(t_3^i)^2}{I_3}\left(1+\frac{b}{t_3^i}\right)+1\right]\left[\bar{M}_1 - (-1)^m\bar{M}_2\right] - \frac{2}{ab}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\left[\bar{M}_4 - (-1)^m\bar{M}_3\right] - \bar{R}_m'''$$

$$(m = 1, 2, \ldots, M) \qquad \text{(B121)}$$

$$D_{Nm}^{''''} g_m' - F_{Nm}^{''''} g_m'' - G_{Nm}^{''''} c_m''' + H_{Nm}^{''''} c_m''''$$

$$= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_4(t_4^i)^2}{I_4}+1\right)G_2\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}\left[c_n' - (-1)^m c_n''\right]$$

$$+\frac{2}{a}\left(\frac{A_4(t_4^i)^2}{I_4}+1\right)\sum_{n=1}^{N}(-1)^n\left[G_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right]\left[g_n''' - (-1)^m g_n''''\right]$$

$$-\frac{2}{ab}\left[\frac{A_4(t_4^i)^2}{I_4}\left(1+\frac{b}{t_4^i}\right)+1\right]\left[\bar{M}_4 - (-1)^m\bar{M}_3\right] + \frac{2}{ab}\left(\frac{A_4(t_4^i)^2}{I_4}+1\right)\left[\bar{M}_1 - (-1)^m\bar{M}_2\right] + \bar{R}_m''''$$

$$(m = 1, 2, \ldots, M) \qquad \text{(B122)}$$

$$I'_{Mn}\, c'_n - K'_{Mn}\, c''_n - P'_{Mn}\, g'''_n + S'_{Mn}\, g''''_n$$

$$= -2\frac{G_1}{G_4}\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left[(G_3-G_4)\left(\frac{n\pi}{b}\right)^2 - G_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_1^i}{E_1I_1}\right]\left[g'_m-(-1)^n g''_m\right]$$

$$+\frac{2}{G_4}\sum_{m=1}^{M}\left\{G_2\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[G_1\left(\frac{n\pi}{b}\right)^2 - G_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_1^i}{E_1I_1}\right] + \left(\frac{a}{m\pi}\right)\left(\frac{b}{n\pi}\right)\frac{t_1^i}{E_1I_1}\right\}\left[c'''_m-(-1)^n c''''_m\right]$$

$$-\frac{2}{G_4}\,\frac{1}{E_1I_1}\left(\frac{b}{n\pi}\right)\left(\frac{t_1^i}{a}+1\right)\left[\bar{M}_1-(-1)^n\bar{M}_4\right] + \frac{2}{G_4}\,\frac{1}{E_1I_1}\left(\frac{b}{n\pi}\right)\frac{t_1^i}{a}\left[\bar{M}_2-(-1)^n\bar{M}_3\right] - \bar{Z}'_n \qquad \text{(B123)}$$

$$(n = 1, 2, \ldots, N)$$

$$I''_{Mn}\, c'_n - K''_{Mn}\, c''_n - P''_{Mn}\, g'''_n + S''_{Mn}\, g''''_n$$

$$= -2\frac{G_1}{G_4}\left(\frac{n\pi}{b}\right)\sum_{m=1}^{M}(-1)^m\left(\frac{m\pi}{a}\right)\frac{1}{E_{mn}}\left[(G_3-G_4)\left(\frac{n\pi}{b}\right)^2 - G_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_2^i}{E_2I_2}\right]\left[g'_m-(-1)^n g''_m\right]$$

$$+\frac{2}{G_4}\sum_{m=1}^{M}(-1)^m\left\{G_2\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[G_1\left(\frac{n\pi}{b}\right)^2 - G_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_2^i}{E_2I_2}\right] + \left(\frac{a}{m\pi}\right)\left(\frac{b}{n\pi}\right)\frac{t_2^i}{E_2I_2}\right\}\left[c'''_m-(-1)^n c''''_m\right]$$

$$+\frac{2}{G_4}\,\frac{1}{E_2I_2}\left(\frac{b}{n\pi}\right)\left(\frac{t_2^i}{a}+1\right)\left[\bar{M}_2-(-1)^n\bar{M}_3\right] - \frac{2}{G_4}\,\frac{1}{E_2I_2}\left(\frac{b}{n\pi}\right)\frac{t_2^i}{a}\left[\bar{M}_1-(-1)^n\bar{M}_4\right] + \bar{Z}''_n$$

$$(n = 1, 2, \ldots, N) \qquad \text{(B124)}$$

$$I'''_{Nm}\, g'_m - K'''_{Nm}\, g''_m - P'''_{Nm}\, c'''_m + S'''_{Nm}\, c''''_m$$

$$= -2\frac{G_2}{G_4}\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[(G_3-G_4)\left(\frac{m\pi}{a}\right)^2 - G_1\left(\frac{n\pi}{b}\right)^2 - \frac{t_3^i}{E_3I_3}\right]\left[c'_n-(-1)^m c''_n\right]$$

$$+\frac{2}{G_4}\sum_{n=1}^{N}\left\{G_1\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[G_2\left(\frac{m\pi}{a}\right)^2 - G_3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{a}{m\pi}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{t_3^i}{E_3I_3}\right] + \left(\frac{a}{m\pi}\right)\left(\frac{b}{n\pi}\right)\frac{t_3^i}{E_3I_3}\right\}\left[g'''_n-(-1)^m g''''_n\right]$$

$$-\frac{2}{G_4}\,\frac{1}{E_3I_3}\left(\frac{a}{m\pi}\right)\left(\frac{t_3^i}{b}+1\right)\left[\bar{M}_1-(-1)^m\bar{M}_2\right] + \frac{2}{G_4}\,\frac{1}{E_3I_3}\left(\frac{a}{m\pi}\right)\frac{t_3^i}{b}\left[\bar{M}_4-(-1)^m\bar{M}_3\right] - \bar{Z}'''_m$$

$$(m = 1, 2, \ldots, M) \qquad \text{(B125)}$$

$$I''''_{Nm}\, g'_m - K''''_{Nm}\, g''_m - P''''_{Nm}\, c'''_m + S''''_{Nm}\, c''''_m$$

$$= -2\frac{G_2}{G_4}\left(\frac{m\pi}{a}\right)\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)\frac{(-1)^n}{E_{mn}}\left[(G_3-G_4)\left(\frac{m\pi}{a}\right)^2 - G_1\left(\frac{n\pi}{b}\right)^2 - \frac{t_4^i}{E_4I_4}\right]\left[c'_n-(-1)^m c''_n\right]$$

$$+\frac{2}{G_4}\sum_{n=1}^{N}(-1)^n\left\{G_1\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\frac{1}{E_{mn}}\left[G_2\left(\frac{m\pi}{a}\right)^2 - G_3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{a}{m\pi}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{t_4^i}{E_4I_4}\right] + \left(\frac{a}{m\pi}\right)\left(\frac{b}{n\pi}\right)\frac{t_4^i}{E_4I_4}\right\}\left[g'''_n-(-1)^m g''''_n\right]$$

$$+\frac{2}{G_4}\,\frac{1}{E_4I_4}\left(\frac{a}{m\pi}\right)\left(\frac{t_4^i}{b}+1\right)\left[\bar{M}_4-(-1)^m\bar{M}_3\right] - \frac{2}{G_4}\,\frac{1}{E_4I_4}\left(\frac{a}{m\pi}\right)\frac{t_4^i}{b}\left[\bar{M}_1-(-1)^m\bar{M}_2\right] + Z''''_m$$

$$(m = 1, 2, \ldots, M) \qquad \text{(B126)}$$

in which

$$\bar{R}'_n = R'_n + \frac{2}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\left\{\frac{t_3^i}{a}\left(P'_x - P''_x\right) - (-1)^n\frac{t_4^i}{a}\left(P'''_x - P''''_x\right) + \left(t_3^i + t_3^\circ\right)Q'''_o\right.$$

$$\left. + (-1)^n\left(t_4^i + t_4^\circ\right)Q''''_o - \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)\left[B'''_m - (-1)^n B''''_m\right]\right\}$$

$$\bar{R}''_n = R''_n - \frac{2}{b}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)\left\{\frac{t_3^i}{a}\left(P'_x - P''_x\right) - (-1)^n\frac{t_4^i}{a}\left(P'''_x - P''''_x\right) + \left(t_3^i + t_3^\circ\right)Q'''_o\right.$$

$$\left. + (-1)^n\left(t_4^i + t_4^\circ\right)Q''''_o - \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)\left[B'''_m - (-1)^n B''''_m\right]\right\}$$

$$\bar{R}'''_m = R'''_m + \frac{2}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\left\{\frac{t_1^i}{b}\left(P'_y - P''_y\right) - (-1)^m\frac{t_2^i}{b}\left(P'''_y - P''''_y\right) + \left(t_1^i + t_1^\circ\right)Q'_o\right.$$

$$\left. + (-1)^m\left(t_2^i + t_2^\circ\right)Q''_o - \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)\left[B'_n - (-1)^m B''_n\right]\right\}$$

$$\bar{R}''''_m = R''''_m - \frac{2}{a}\left(\frac{A_4(t_4^i)^2}{I_4}+1\right)\left\{\frac{t_1^i}{b}\left(P'_y - P''_y\right) - (-1)^m\frac{t_2^i}{b}\left(P'''_y - P''''_y\right) + \left(t_1^i + t_1^\circ\right)Q'_o\right.$$

$$\left. + (-1)^m\left(t_2^i + t_2^\circ\right)Q''_o - \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)\left[B'_n - (-1)^m B''_n\right]\right\}$$

$$\bar{Z}'_n = Z'_n - \frac{2}{C_4}\,\frac{t_1^i}{E_1 I_1}\left(\frac{b}{n\pi}\right)\left\{\frac{t_3^i}{a}\left(P'_x - P''_x\right) - (-1)^n\frac{t_4^i}{a}\left(P'''_x - P''''_x\right) + \left(t_3^i + t_3^\circ\right)Q'''_o\right.$$

$$\left. + (-1)^n\left(t_4^i + t_4^\circ\right)Q''''_o - \sum_{m=1}^{M}\left(\frac{a}{m\pi}\right)\left[B'''_m - (-1)^n B''''_m\right]\right\}$$

$$\bar{Z}''_n = Z''_n + \frac{2}{C_4}\,\frac{t_2^i}{E_2 I_2}\left(\frac{b}{n\pi}\right)\left\{\frac{t_3^i}{a}\left(P'_x - P''_x\right) - (-1)^n\frac{t_4^i}{a}\left(P'''_x - P''''_x\right) + \left(t_3^i + t_3^\circ\right)Q'''_o\right.$$

$$\left. + (-1)^n\left(t_4^i + t_4^\circ\right)Q''''_o - \sum_{m=1}^{M}(-1)^m\left(\frac{a}{m\pi}\right)\left[B'''_m - (-1)^n B''''_m\right]\right\}$$

$$\bar{Z}_m''' = Z_m''' - \frac{2}{C_4}\frac{t_3^i}{E_3 I_3}\left(\frac{a}{m\pi}\right)\Big\{\frac{t_1^i}{b}\left(P_y' - P_y''\right) - (-1)^m \frac{t_2^i}{b}\left(P_y''' - P_y''''\right) + \left(t_1^i + t_1^\circ\right) Q_o'$$

$$+ (-1)^m \left(t_2^i + t_2^\circ\right) Q_o'' - \sum_{n=1}^{N}\left(\frac{b}{n\pi}\right)\left[B_n' - (-1)^m B_n''\right]\Big\}$$

$$\bar{Z}_m'''' = Z_m'''' + \frac{2}{C_4}\frac{t_4^i}{E_4 I_4}\left(\frac{a}{m\pi}\right)\Big\{\frac{t_1^i}{b}\left(P_y' - P_y''\right) - (-1)^m \frac{t_2^i}{b}\left(P_y''' - P_y''''\right) + \left(t_1^i + t_1^\circ\right) Q_o'$$

$$+ (-1)^m \left(t_2^i + t_2^\circ\right) Q_o'' - \sum_{n=1}^{N}(-1)^n\left(\frac{b}{n\pi}\right)\left[B_n' - (-1)^m B_n''\right]\Big\}$$

Thus the stiffener end shears, which were considered known in the earlier analysis for the sake of convenience, have now been eliminated from the basic equations. If the corner moments, $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ and $\bar{M}_4$ are now also taken into account as unknowns, equations (B119) to (B126) contain 4M + 4N + 4 unknowns, namely, c_n' through c_m'''', g_m' through g_n'''' and $\bar{M}_1$ through $\bar{M}_4$, but only 4M + 4N simultaneous equations. Four more equations are therefore required. These four equations are developed in the following subsection.

Corner moments. The centroidal axes of any two adjacent stiffeners are at 90° to each other before loading. Owing to the development of end shears and corner moments, a small change of this angle will occur during loading. This change of angle is given by expressions (A) in the main body. Similarly, the plate edges at each corner, which make an angle of 90° before loading, experience a small change of angle during loading because of the plate shear stress resultant N_{xy} at the corner. These angle changes are given by expressions (B) in the main body. Equating expressions (A) and (B) (because of integral attachment between stiffeners and plate), one obtains

$$\left.\begin{aligned}
-\frac{\bar{M}_1}{k_1} + \frac{V_1(0)}{G_1 A_{s1}} + \frac{V_3(0)}{G_3 A_{s3}} &= C_4 N_{xy}(0,0) \\
-\frac{\bar{M}_2}{k_2} + \frac{V_2(0)}{G_2 A_{s2}} - \frac{V_3(a)}{G_3 A_{s3}} &= -C_4 N_{xy}(a,0) \\
-\frac{\bar{M}_3}{k_3} - \frac{V_2(b)}{G_2 A_{s2}} - \frac{V_4(a)}{G_4 A_{s4}} &= C_4 N_{xy}(a,b) \\
-\frac{\bar{M}_4}{k_4} - \frac{V_1(b)}{G_1 A_{s1}} + \frac{V_4(0)}{G_4 A_{s4}} &= -C_4 N_{xy}(0,b)
\end{aligned}\right\} \quad \text{(B127)}$$

Substitution of the series expression, equation (B34), for the right side terms of these equations yields

$$-\frac{\bar{M}_1}{k_1} + \frac{V_1(o)}{G_1 A_{s1}} + \frac{V_3(o)}{G_3 A_{s3}} = -G_4\left[j_{oo} + \sum_{m=1}^{M} j_{mo} + \sum_{n=1}^{N} j_{on} + \sum_{m=1}^{M}\sum_{n=1}^{N} j_{mn}\right]$$

$$-\frac{\bar{M}_2}{k_2} + \frac{V_2(o)}{G_2 A_{s2}} - \frac{V_3(a)}{G_3 A_{s3}} = -G_4\left[j_{oo} + \sum_{m=1}^{M}(-1)^m j_{mo} + \sum_{n=1}^{N} j_{on} + \sum_{m=1}^{M}\sum_{n=1}^{N}(-1)^m j_{mn}\right]$$

(B128)

$$-\frac{\bar{M}_3}{k_3} - \frac{V_2(b)}{G_2 A_{s2}} - \frac{V_4(a)}{G_4 A_{s4}} = -G_4\left[j_{oo} + \sum_{m=1}^{M}(-1)^m j_{mo} + \sum_{n=1}^{N}(-1)^n j_{on} + \sum_{m=1}^{M}\sum_{n=1}^{N}(-1)^{m+n} j_{mn}\right]$$

$$-\frac{\bar{M}_4}{k_4} - \frac{V_1(b)}{G_1 A_{s1}} + \frac{V_4(o)}{G_4 A_{s4}} = G_4\left[j_{oo} + \sum_{m=1}^{M} j_{mo} + \sum_{n=1}^{N}(-1)^n j_{on} + \sum_{m=1}^{M}\sum_{n=1}^{N}(-1)^n j_{mn}\right]$$

Using equations (B111) to (B118) to eliminate $V_1(o)$, $V_2(o)$, etc. and equations (B92), (B110), (B95) to eliminate j_{oo}, j_{mo}, j_{on} and j_{mn}, one can convert equations (B128) to the following final form:

$$
\begin{aligned}
&\Theta'\bar{M}_1 - \Phi'\bar{M}_2 + \Lambda'\bar{M}_3 - \psi'\bar{M}_4 \\
&= -C_1C_4\frac{2}{b}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[g_m' - (-1)^n g_m''\right] - C_2C_4\frac{2}{a}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{1}{E_{mn}}\left[c_n' - (-1)^m c_n''\right] \\
&+\sum_{n=1}^{N}\frac{b}{n\pi}\left[\frac{1}{G_1A_{s1}} + \frac{C_4}{a} + 2C_1C_4\frac{1}{a}\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g_n''' - \frac{C_4}{a}\sum_{n=1}^{N}\frac{b}{n\pi}\left[1 + 2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g_n'''' \\
&+\sum_{m=1}^{M}\frac{a}{m\pi}\left[\frac{1}{G_3A_{s3}} + \frac{C_4}{b} + 2C_2C_4\frac{1}{b}\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{1}{E_{mn}}\right]c_m''' - \frac{C_4}{b}\sum_{m=1}^{M}\frac{a}{m\pi}\left[1 + 2C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]c_m'''' \\
&+\Gamma'
\end{aligned}
\tag{B129}
$$

$$
\begin{aligned}
&-\Phi'\bar{M}_1 + \Phi''\bar{M}_2 - \Lambda''\bar{M}_3 + \Lambda'\bar{M}_4 \\
&= C_1C_4\frac{2}{b}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^m}{E_{mn}}\left[g_m' - (-1)^n g_m''\right] + C_2C_4\frac{2}{a}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{(-1)^m}{E_{mn}}\left[c_n' - (-1)^m c_n''\right] \\
&-\frac{C_4}{a}\sum_{n=1}^{N}\frac{b}{n\pi}\left[1 + 2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g_n''' + \sum_{n=1}^{N}\frac{b}{n\pi}\left[\frac{1}{G_2A_{s2}} + \frac{C_4}{a} + 2C_1C_4\frac{1}{a}\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g_n'''' \\
&-\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\left[\frac{1}{G_3A_{s3}} + \frac{C_4}{b} + 2C_2C_4\frac{1}{b}\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{1}{E_{mn}}\right]c_m''' + \frac{C_4}{b}\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\left[1 + 2C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]c_m'''' \\
&+\Gamma''
\end{aligned}
\tag{B130}
$$

$$
\begin{aligned}
&\Lambda'\bar{M}_1 - \Lambda''\bar{M}_2 + \Lambda'''\bar{M}_3 - \psi'''\bar{M}_4 \\
&= -C_1C_4\frac{2}{b}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^{m+n}}{E_{mn}}\left[g_m' - (-1)^n g_m''\right] - C_2C_4\frac{2}{a}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{(-1)^{m+n}}{E_{mn}}\left[c_n' - (-1)^m c_n''\right] \\
&+\frac{C_4}{a}\sum_{n=1}^{N}(-1)^n\frac{b}{n\pi}\left[1 + 2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g_n''' - \sum_{n=1}^{N}(-1)^n\frac{b}{n\pi}\left[\frac{1}{G_2A_{s2}} + \frac{C_4}{a} + 2C_1C_4\frac{1}{a}\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g_n'''' \\
&+\frac{C_4}{b}\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\left[1 + 2C_2\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{(-1)^n}{E_{mn}}\right]c_m''' - \sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\left[\frac{1}{G_4A_{s4}} + \frac{C_4}{b} + 2C_2C_4\frac{1}{b}\left(\frac{m\pi}{a}\right)^4\sum_{n=1}^{N}\frac{1}{E_{mn}}\right]c_m'''' \\
&+\Gamma'''
\end{aligned}
\tag{B131}
$$

$$
\begin{aligned}
&-\Psi'\bar{M}_1 + \Lambda'\bar{M}_2 - \Psi'''\bar{M}_3 + \Psi''''\bar{M}_4 \\
&= G_1 G_4 \frac{2}{b} \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{m\pi}{a} \left(\frac{n\pi}{b}\right)^2 \frac{(-1)^n}{E_{mn}} \left[g_m' - (-1)^n g_m'' \right] + G_2 G_4 \frac{2}{a} \sum_{m=1}^{M} \sum_{n=1}^{N} \left(\frac{m\pi}{a}\right)^2 \frac{n\pi}{b} \frac{(-1)^n}{E_{mn}} \left[c_n' - (-1)^m c_n'' \right] \\
&- \sum_{n=1}^{N} (-1)^n \frac{b}{n\pi} \left[\frac{1}{G_1 A_{s1}} + \frac{G_4}{a} + 2G_1 G_4 \frac{1}{a} \left(\frac{n\pi}{b}\right)^4 \sum_{m=1}^{M} \frac{1}{E_{mn}} \right] g_n''' + \frac{G_4}{a} \sum_{n=1}^{N} (-1)^n \frac{b}{n\pi} \left[1 + 2G_1 \left(\frac{n\pi}{b}\right)^4 \sum_{m=1}^{M} \frac{(-1)^m}{E_{mn}} \right] g_n'''' \\
&- \frac{G_4}{b} \sum_{m=1}^{M} \frac{a}{m\pi} \left[1 + 2G_2 \left(\frac{m\pi}{a}\right)^4 \sum_{n=1}^{N} \frac{(-1)^n}{E_{mn}} \right] c_m''' + \sum_{m=1}^{M} \frac{a}{m\pi} \left[\frac{1}{G_4 A_{s4}} + \frac{G_4}{a} + 2G_2 G_4 \left(\frac{m\pi}{a}\right)^4 \sum_{n=1}^{N} \frac{1}{E_{mn}} \right] c_m'''' \\
&+ \Gamma''''
\end{aligned} \tag{B132}
$$

where

$$\Theta' = \frac{1}{k_1} + \frac{1}{b}\frac{1}{G_1 A_{s1}} + \frac{1}{a}\left(\frac{1}{G_3 A_{s3}} + \frac{G_4}{b}\right)$$

$$\Phi' = \frac{1}{a}\left(\frac{1}{G_3 A_{s3}} + \frac{G_4}{b}\right)$$

$$\Phi'' = \frac{1}{k_2} + \frac{1}{a}\frac{1}{G_3 A_{s3}} + \frac{1}{b}\left(\frac{1}{G_2 A_{s2}} + \frac{G_4}{a}\right)$$

$$\Lambda' = \frac{G_4}{ab}$$

$$\Lambda'' = \frac{1}{b}\left(\frac{1}{G_2 A_{s2}} + \frac{G_4}{a}\right)$$

$$\Lambda''' = \frac{1}{k_3} + \frac{1}{b}\frac{1}{G_2 A_{s2}} + \frac{1}{a}\left(\frac{1}{G_4 A_{s4}} + \frac{G_4}{b}\right)$$

$$\Psi' = \frac{1}{b}\left(\frac{1}{G_1 A_{s1}} + \frac{G_4}{a}\right)$$

$$\Psi''' = \frac{1}{a}\left(\frac{1}{G_4 A_{s4}} + \frac{G_4}{b}\right)$$

$$\Psi'''' = \frac{1}{k_4} + \frac{1}{a}\frac{1}{G_4 A_{s4}} + \frac{1}{b}\left(\frac{1}{G_1 A_{s1}} + \frac{G_4}{a}\right)$$

$$\Gamma' = \left(\frac{1}{G_1 A_{s1}}\frac{t_1^i}{b} - \frac{G_4}{b}\right)\left(P_y' - P_y''\right) + \left(\frac{1}{G_3 A_{s3}} + \frac{G_4}{b}\right)\frac{t_3^i}{a}\left(P_x' - P_x''\right) - \frac{G_4}{b}\frac{t_4^i}{a}\left(P_x''' - P_x''''\right)$$

$$+\left[\frac{1}{G_1 A_{s1}}\left(t_1^i + t_1^o\right) - G_4\right]Q_o' + \left(\frac{1}{G_3 A_{s3}} + \frac{G_4}{b}\right)\left(t_3^i + t_3^o\right)Q_o''' + \frac{G_4}{b}\left(t_4^i + t_4^o\right)Q_o'''' - \frac{1}{G_1 A_{s1}}\sum_{n=1}^{N}\frac{b}{n\pi}B_n'$$

$$-\left(\frac{1}{G_3 A_{s3}} + \frac{G_4}{b}\right)\sum_{m=1}^{M}\frac{a}{m\pi}B_m''' + \frac{G_4}{b}\sum_{m=1}^{M}\frac{a}{m\pi}B_m'''' - G_4\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\frac{n\pi}{b}\frac{1}{E_{mn}}T_{mn}$$

$$\Gamma'' = -\frac{G_4}{a}\frac{t_1^i}{b}\left(P_y' - P_y''\right) + \left(\frac{1}{G_2 A_{s2}} + \frac{G_4}{a}\right)\frac{t_2^i}{b}\left(P_y''' - P_y''''\right) - \left(\frac{1}{G_3 A_{s3}}\frac{t_3^i}{a} - \frac{G_4}{a}\right)\left(P_x' - P_x''\right)$$

$$-\frac{G_4}{a}\left(t_1^i + t_1^o\right)Q_o' - \left(\frac{1}{G_2 A_{s2}} + \frac{G_4}{a}\right)\left(t_2^i + t_2^o\right)Q_o'' - \left[\frac{1}{G_3 A_{s3}}\left(t_3^i + t_3^o\right) - G_4\right]Q_o''' + \frac{G_4}{a}\sum_{n=1}^{N}\frac{b}{n\pi}B_n'$$

$$-\left(\frac{1}{G_2 A_{s2}} + \frac{G_4}{a}\right)\sum_{n=1}^{N}\frac{b}{n\pi}B_n'' + \frac{1}{G_3 A_{s3}}\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}B_m''' + G_4\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\frac{n\pi}{b}\frac{(-1)^m}{E_{mn}}T_{mn}$$

$$\Gamma''' = -\left(\frac{1}{G_2 A_{s2}}\frac{t_2^i}{b} - \frac{G_4}{b}\right)\left(P_y''' - P_y''''\right) + \frac{G_4}{b}\frac{t_3^i}{a}\left(P_x' - P_x''\right) - \left(\frac{1}{G_4 A_{s4}} + \frac{G_4}{b}\right)\frac{t_4^i}{a}\left(P_x''' - P_x''''\right)$$

$$+\left[\frac{1}{G_2 A_{s2}}\left(t_2^i + t_2^o\right) - G_4\right]Q_o'' + \frac{G_4}{b}\left(t_3^i + t_3^o\right)Q_o''' + \left(\frac{1}{G_4 A_{s4}} + \frac{G_4}{b}\right)\left(t_4^i + t_4^o\right)Q_o''''$$

$$+\frac{1}{G_2 A_{s2}}\sum_{n=1}^{N}(-1)^n\frac{b}{n\pi}B_n'' - \frac{G_4}{b}\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}B_m''' + \left(\frac{1}{G_4 A_{s4}} + \frac{G_4}{b}\right)\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}B_m''''$$

$$-G_4\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\frac{n\pi}{b}\frac{(-1)^{m+n}}{E_{mn}}T_{mn}$$

$$
\begin{aligned}
\Gamma'''' = & -\left(\frac{1}{G_1 A_{s1}} + \frac{G_4}{a}\right)\frac{t_1^i}{b}\left(P_y' - P_y''\right) + \frac{G_4}{a}\frac{t_2^i}{b}\left(P_y''' - P_y''''\right) + \left(\frac{1}{G_4 A_{s4}}\frac{t_4^i}{a} - \frac{G_4}{a}\right)\left(P_z''' - P_z''''\right) \\
& -\left(\frac{1}{G_1 A_{s1}} + \frac{G_4}{a}\right)\left(t_1^i + t_1^o\right)Q_o' - \frac{G_4}{a}\left(t_2^i + t_2^o\right)Q_o'' - \left[\frac{1}{G_4 A_{s4}}\left(t_4^i + t_4^o\right) - G_4\right]Q_o'''' + \left(\frac{1}{G_1 A_{s1}} + \frac{G_4}{a}\right)\sum_{n=1}^{N}(-1)^n \frac{b}{n\pi} B_n' \\
& - \frac{G_4}{a}\sum_{n=1}^{N}(-1)^n \frac{b}{n\pi} B_n'' - \frac{1}{G_4 A_{s4}}\sum_{m=1}^{M}\frac{a}{m\pi} B_m'''' + G_4 \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m\pi}{a}\frac{n\pi}{b}\frac{(-1)^n}{E_{mn}} T_{mn}
\end{aligned}
$$

Equations (B129) to (B132) can be written in a matrix form and the left side will have a symmetric coefficient matrix for $\overline{M}_1$ through $\overline{M}_4$ which reads

$$
\begin{bmatrix}
\Theta' & -\Phi' & \Lambda' & -\Psi' \\
-\Phi' & \Phi'' & -\Lambda'' & \Lambda' \\
\Lambda' & -\Lambda'' & \Lambda''' & -\Psi''' \\
-\Psi' & \Lambda' & -\Psi''' & \Psi''''
\end{bmatrix}
$$

If the inverse of this matrix exists one can always find an expression for each of the corner moments $\overline{M}_1$, $\overline{M}_2$, $\overline{M}_3$, and $\overline{M}_4$, ~~in terms~~ of the g's and c's. These expressions can then be employed to eliminate all the corner moments appearing in equations (B119) to (B126). Eight new groups of equations result. These eight new groups of equations contain 4M + 4N simultaneous equations and exactly 4M + 4N unknowns. A solution for each of the unknowns is then possible. However, it is not intended in this paper to give the expressions for $\overline{M}_1$ through $\overline{M}_4$ and to eliminate them in equations (B119) to (B126). Although this can be done by straight forward algebraic operations, the process is very tedious. Later on, when some special cases are considered, equations (B119) to (B126) and (B129) to (B132) will be much simplified. In such cases, an expression for each of the $\overline{M}_1$ through $\overline{M}_4$ will be given.

Evaluation of plate stresses, stiffener tensions, bending moments and transverse shears. The foregoing analysis can now be concluded as follows: The values of c's, g's and $\overline{M}_1$ through $\overline{M}_4$ can be obtained from the system of simultaneous equations (B119) to (B126), and (B129) to (B132). With these values known, equations (B111) to (B118) give the stiffener end transverse shears, $V_1(o)$, $V_1(b)$, etc. The values of the stiffener end tensions, $P_1(o)$, $P_1(b)$, etc. and the stiffener end moments, $M_1(o)$, $M_1(b)$, etc. are then given by equations (B76) and (B77), respectively. Having determined the above quantities, equations (B98) give the Fourier coefficients b'_n, b''_n, b'''_m, b''''_m, equations (B99) the s'_n, s''_n, s'''_m, s''''_m, equations (B96) and (B97) the v'_n, v''_n, v'''_m, v''''_m, and equations (B92), (B95), and one of the equations (B94), the j_{mn}. Then equations (B13) can give the stiffener tensions, equations (B14) the stiffener bending moments, equations (B15) the stiffener transverse shears, and equations (B24) to (B34) the plate stresses except the normal stresses at plate corners. The evaluation of the corner values of N_x and N_y is given in the following subsection.

<u>Corner values of N_x and N_y.</u> By using the first of equations (B9) and letting y = o, one obtains

$$\frac{P_1(o)}{A_1E_1} + \left[\frac{M_1(o)}{E_1I_1} + \chi_1(o)\right]t_1^i + e_1(o) = e_y(o,o) + C_2N_y(o,o) - C_3N_x(o,o)$$

Similarly, one can write

$$\frac{P_2(b)}{A_2E_2} + \left[\frac{M_2(b)}{E_2I_2} + \chi_2(b)\right]t_2^i + e_2(b) = e_y(a,b) + C_2N_y(a,b) - C_3N_x(a,b)$$

$$\frac{P_3(a)}{A_3E_3} + \left[\frac{M_3(a)}{E_3I_3} + \chi_3(a)\right]t_3^i + e_3(a) = e_x(a,o) + C_1N_x(a,o) - C_3N_y(a,o)$$

$$\frac{P_4(o)}{A_4E_4} + \left[\frac{M_4(o)}{E_4I_4} + \chi_4(o)\right]t_4^i + e_4(o) = e_x(o,b) + C_1N_x(o,b) - C_3N_y(o,b)$$

$$\frac{P_1(b)}{A_1E_1} + \left[\frac{M_1(b)}{E_1I_1} + \chi_1(b)\right]t_1^i + e_1(b) = e_y(o,b) + C_2N_y(o,b) - C_3N_x(o,b)$$

$$\frac{P_2(0)}{A_2E_2} + \left[\frac{M_2(0)}{E_2I_2} + \mathcal{X}_2(0)\right]t_2^i + e_2(0) = e_y(a,0) + G_2N_y(a,0) - G_3N_x(a,0)$$

$$\frac{P_3(0)}{A_3E_3} + \left[\frac{M_3(0)}{E_3I_3} + \mathcal{X}_3(0)\right]t_3^i + e_3(0) = e_x(0,0) + G_1N_x(0,0) - G_3N_y(0,0)$$

$$\frac{P_4(a)}{A_4E_4} + \left[\frac{M_4(a)}{E_4I_4} + \mathcal{X}_4(a)\right]t_4^i + e_4(a) = e_x(a,b) + G_1N_x(a,b) - G_3N_y(a,b)$$

The above eight equations serve to establish the corner values of the plate stress resultants N_x and N_y in terms of the known thermal strains, and the end values of the stiffener bending moments and tensions. Solving the above equations one obtains,

$$N_x(0,0) = \frac{1}{G_1G_2-G_3^2}\left\{G_3\left[\frac{P_1(0)}{A_1E_1} + \left(\frac{M_1(0)}{E_1I_1} + \mathcal{X}_1(0)\right)t_1^i + e_1(0) - e_y(0,0)\right]\right.$$
$$\left. + G_2\left[\frac{P_3(0)}{A_3E_3} + \left(\frac{M_3(0)}{E_3I_3} + \mathcal{X}_3(0)\right)t_3^i + e_3(0) - e_x(0,0)\right]\right\}$$

$$N_x(a,0) = \frac{1}{G_1G_2-G_3^2}\left\{G_3\left[\frac{P_2(0)}{A_2E_2} + \left(\frac{M_2(0)}{E_2I_2} + \mathcal{X}_2(0)\right)t_2^i + e_2(0) - e_y(a,0)\right]\right.$$
$$\left. + G_2\left[\frac{P_3(a)}{A_3E_3} + \left(\frac{M_3(a)}{E_3I_3} + \mathcal{X}_3(a)\right)t_3^i + e_3(a) - e_x(a,0)\right]\right\}$$

$$N_x(0,b) = \frac{1}{G_1G_2-G_3^2}\left\{G_3\left[\frac{P_1(b)}{A_1E_1} + \left(\frac{M_1(b)}{E_1I_1} + \mathcal{X}_1(b)\right)t_1^i + e_1(b) - e_y(0,b)\right]\right.$$
$$\left. + G_2\left[\frac{P_4(0)}{A_4E_4} + \left(\frac{M_4(0)}{E_4I_4} + \mathcal{X}_4(0)\right)t_4^i + e_4(0) - e_x(0,b)\right]\right\}$$

$$
\left.
\begin{aligned}
N_x(a,b) &= \frac{1}{G_1G_2-G_3^2}\left\{G_3\left[\frac{P_2(b)}{A_2E_2}+\left(\frac{M_2(b)}{E_2I_2}+\chi_2(b)\right)t_2^i+e_2(b)-e_y(a,b)\right]\right. \\
&\quad \left.+G_2\left[\frac{P_4(a)}{A_4E_4}+\left(\frac{M_4(a)}{E_4I_4}+\chi_4(a)\right)t_4^i+e_4(a)-e_x(a,b)\right]\right\} \\
N_y(0,0) &= \frac{1}{G_1G_2-G_3^2}\left\{G_1\left[\frac{P_1(0)}{A_1E_1}+\left(\frac{M_1(0)}{E_1I_1}+\chi_1(0)\right)t_1^i+e_1(0)-e_y(0,0)\right]\right. \\
&\quad \left.+G_3\left[\frac{P_3(0)}{A_3E_3}+\left(\frac{M_3(0)}{E_3I_3}+\chi_3(0)\right)t_3^i+e_3(0)-e_x(0,0)\right]\right\} \\
N_y(a,0) &= \frac{1}{G_1G_2-G_3^2}\left\{G_1\left[\frac{P_2(0)}{A_2E_2}+\left(\frac{M_2(0)}{E_2I_2}+\chi_2(0)\right)t_2^i+e_2(0)-e_y(a,0)\right]\right. \\
&\quad \left.+G_3\left[\frac{P_3(a)}{A_3E_3}+\left(\frac{M_3(a)}{E_3I_3}+\chi_3(a)\right)t_3^i+e_3(a)-e_x(a,0)\right]\right\} \\
N_y(0,b) &= \frac{1}{G_1G_2-G_3^2}\left\{G_1\left[\frac{P_1(b)}{A_1E_1}+\left(\frac{M_1(b)}{E_1I_1}+\chi_1(b)\right)t_1^i+e_1(b)-e_y(0,b)\right]\right. \\
&\quad \left.+G_3\left[\frac{P_4(0)}{A_4E_4}+\left(\frac{M_4(0)}{E_4I_4}+\chi_4(0)\right)t_4^i+e_4(0)-e_x(0,b)\right]\right\} \\
N_y(a,b) &= \frac{1}{G_1G_2-G_3^2}\left\{G_1\left[\frac{P_2(b)}{A_2E_2}+\left(\frac{M_2(b)}{E_2I_2}+\chi_2(b)\right)t_2^i+e_2(b)-e_y(a,b)\right]\right. \\
&\quad \left.+G_3\left[\frac{P_4(a)}{A_4E_4}+\left(\frac{M_4(a)}{E_4I_4}+\chi_4(a)\right)t_4^i+e_4(a)-e_x(a,b)\right]\right\}
\end{aligned}
\right\}\text{(B133)}
$$

Special case: symmetry about $y = \frac{b}{2}$. If the structure, loading and thermal strains are symmetrical about the line $y = \frac{b}{2}$, then a corresponding symmetry obtains in the stress function F and in the plate and stiffener stresses. Consequently one may set

$$\left.\begin{aligned}
&A_3 = A_4\ ;\ I_3 = I_4\ ;\ t_3^{i} = t_4^{i}\ ;\ t_3^{o} = t_4^{o}\ ;\ k_1 = k_4\ ;\ k_2 = k_3\ ; \\
&E_3 = E_4\ ;\ G_3 = G_4\ ;\ A_{S3} = A_{S4}\ ;\ Q_n' = Q_n'' = 0 \text{ for } n \text{ even}\ ; \\
&Q_m''' = -Q_m'''' \text{ for all } m\ ;\ B_n' = B_n'' = 0 \text{ for } n \text{ even}\ ;\ B_m''' = B_m'''' \\
&\text{for all } m\ ;\ P_y' = P_y''\ ;\ P_y''' = P_y''''\ ;\ P_x' = P_x'''\ ;\ P_x'' = P_x''''\ ; \\
&T_{mn} = 0 \text{ for } n \text{ even}\ ;\ T_n' = T_n'' = 0 \text{ for } n \text{ even}\ ; \\
&T_m''' = T_m'''' \text{ for all } m\ ;\ c_n' = c_n'' = g_n''' = g_n'''' = 0 \text{ for } n \\
&\text{even}\ ;\ g_m' = g_m'' \text{ for all } m\ ;\ c_m''' = c_m'''' \text{ for all } m\ ; \\
&\bar{M}_1 = \bar{M}_4\ ;\ \bar{M}_2 = \bar{M}_3\ ;\ \text{etc.}
\end{aligned}\right\} \quad \text{(B134)}$$

In place of equations (B119) to (B126) the following system results:

$$D'_{Mn}c'_n - F'_{Mn}c''_n - G'_{Mn}g'''_n + H'_{Mn}g''''_n$$
$$= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)C_1\sum_{m=1}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}g'_m + \frac{4}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)$$
$$\cdot\sum_{m=1}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right]c'''_m + \frac{4}{ab}\left[\frac{A_1(t_1^i)^2}{I_1}\left(1+\frac{a}{t_1^i}\right)+1\right]\bar{M}_1$$
$$-\frac{4}{ab}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\bar{M}_2 - \bar{R}'_n \qquad (n=1,3,---,N)$$
(B135)

$$D''_{Mn}c'_n - F''_{Mn}c''_n - G''_{Mn}g'''_n + H''_{Mn}g''''_n$$
$$= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)C_1\sum_{m=1}^{M}\frac{m\pi}{a}\frac{(-1)^m}{E_{mn}}g'_m + \frac{4}{b}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)$$
$$\cdot\sum_{m=1}^{M}(-1)^m\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right]c'''_m + \frac{4}{ab}\left[\frac{A_2(t_2^i)^2}{I_2}+1\right]\bar{M}_1$$
$$-\frac{4}{ab}\left[\frac{A_2(t_2^i)^2}{I_2}\left(1+\frac{a}{t_2^i}\right)+1\right]\bar{M}_2 + \bar{R}''_n \qquad (n=1,3,---,N)$$
(B136)

$$(D'''_{Nm} - F'''_{Nm})g'_m - (G'''_{Nm} - H'''_{Nm})c'''_m$$
$$= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)C_2\sum_{n=1,3,\cdots}^{N}\frac{n\pi}{b}\frac{1}{E_{mn}}\left[c'_n-(-1)^m c''_n\right]$$
$$+\frac{2}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\sum_{n=1,3,\cdots}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right]\left[g'''_n-(-1)^m g''''_n\right]$$
$$+\frac{2}{ab}\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\left[\bar{M}_1 - (-1)^m\bar{M}_2\right] - \bar{R}'''_m \qquad (m=1,2,\cdots,M)$$
(B137)

$$I'_{Mn} c'_n - K'_{Mn} c''_n - P'_{Mn} g'''_n + S'_{Mn} g''''_n$$

$$= -4 \frac{C_1}{C_4} \frac{n\pi}{b} \sum_{m=1}^{M} \frac{m\pi}{a} \frac{1}{E_{mn}} \left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_1^i}{E_1 I_1} \right] g'_m$$

$$+ \frac{4}{C_4} \sum_{m=1}^{M} \left\{ C_2 \frac{m\pi}{a} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2 \left(\frac{b}{n\pi}\right)^2 \frac{t_1^i}{E_1 I_1} \right] \right.$$

$$\left. + \frac{a}{m\pi} \frac{b}{n\pi} \frac{t_1^i}{E_1 I_1} \right\} c'''_m$$

$$- \frac{4}{C_4} \frac{1}{E_1 I_1} \frac{b}{n\pi} \left(\frac{t_1^i}{a} + 1 \right) \bar{M}_1 + \frac{4}{C_4} \frac{1}{E_1 I_1} \frac{b}{n\pi} \frac{t_1^i}{a} \bar{M}_2 - \bar{Z}'_n \quad (n = 1, 3, \cdots, N) \qquad \text{(B138)}$$

$$I''_{Mn} c'_n - K''_{Mn} c''_n - P''_{Mn} g'''_n + S''_{Mn} g''''_n$$

$$= -4 \frac{C_1}{C_4} \frac{n\pi}{b} \sum_{m=1}^{M} (-1)^m \frac{m\pi}{a} \frac{1}{E_{mn}} \left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_2^i}{E_2 I_2} \right] g'_m$$

$$+ \frac{4}{C_4} \sum_{m=1}^{M} (-1)^m \left\{ C_2 \frac{m\pi}{a} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2 \left(\frac{b}{n\pi}\right)^2 \frac{t_2^i}{E_2 I_2} \right] \right.$$

$$\left. + \frac{a}{m\pi} \frac{b}{n\pi} \frac{t_2^i}{E_2 I_2} \right\} c'''_m \qquad \text{(B139)}$$

$$- \frac{4}{C_4} \frac{1}{E_2 I_2} \frac{b}{n\pi} \left[\frac{t_2^i}{a} \bar{M}_1 - \left(\frac{t_2^i}{a} + 1 \right) \bar{M}_2 \right] + \bar{Z}''_n \quad (n = 1, 3, \cdots, N)$$

$$
\begin{aligned}
&(I_{Nm}''' - K_{Nm}''') g_m' - (P_{Nm}''' - S_{Nm}''') c_m''' \\
&= -2 \frac{C_2}{C_4} \frac{m\pi}{a} \sum_{n=1,3,\ldots}^{N} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[(C_3 - C_4)\left(\frac{m\pi}{a}\right)^2 - C_1\left(\frac{n\pi}{b}\right)^2 - \frac{t_3^i}{E_3 I_3} \right] \left[c_n' - (-1)^m c_n'' \right] \\
&\quad + \frac{2}{C_4} \sum_{n=1,3,\ldots}^{N} \left\{ C_1 \frac{m\pi}{a} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[C_2\left(\frac{m\pi}{a}\right)^2 - C_3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{a}{m\pi}\right)^2 \left(\frac{n\pi}{b}\right)^2 \frac{t_3^i}{E_3 I_3} \right] \right. \\
&\qquad\qquad \left. + \frac{a}{m\pi} \frac{b}{n\pi} \frac{t_3^i}{E_3 I_3} \right\} \left[g_n''' - (-1)^m g_n'''' \right] \\
&\quad - \frac{2}{C_4} \frac{1}{E_3 I_3} \frac{a}{m\pi} \left[\bar{M}_1 - (-1)^m \bar{M}_2 \right] - \bar{Z}_m''' \qquad (m = 1, 2, \cdots M)
\end{aligned}
\tag{B140}
$$

It should be noted that in this special case equations (B121) are identical to equations (B122) and equations (B125) are identical to equations (B126).

The eight systems (or 4M + 4N) of equations, (B119) to (B126), have now been reduced to six (or 2M + 2N + 2). A similar procedure also reduces the four equations, (B129) to (B132), to the following two equations, because the symmetry makes equation (B131) identical to (B130) and (B132) identical to (B129):

$$
\begin{aligned}
&\left(\frac{1}{k_1}+\frac{1}{a}\frac{1}{G_3A_{s3}}\right)\bar{M}_1-\frac{1}{a}\frac{1}{G_3A_{s3}}\bar{M}_2 \\
&=-C_1C_4\frac{4}{b}\sum_{m=1}^{M}\sum_{n=1,3\ldots}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}g'_m+\sum_{m=1}^{M}\frac{a}{m\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{m\pi}{a}\right)^4\sum_{n=1,3\ldots}^{N}\frac{1}{E_{mn}}\right]c'''_m \\
&\quad-C_2C_4\frac{2}{a}\sum_{m=1}^{M}\sum_{n=1,3\ldots}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{1}{E_{mn}}\left[c'_n-(-1)^m c''_n\right]+\sum_{n=1,3\ldots}^{N}\frac{b}{n\pi}\left[\frac{1}{G_1A_{s1}}+\frac{C_4}{a}+2C_1C_4\frac{1}{a}\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g'''_n \\
&\quad-\frac{C_4}{a}\sum_{n=1,3\ldots}^{N}\frac{b}{n\pi}\left[1+2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g''''_n+\Gamma' \qquad (B141)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{a}\frac{1}{G_3A_{s3}}\bar{M}_1+\left(\frac{1}{k_2}+\frac{1}{a}\frac{1}{G_3A_{s3}}\right)\bar{M}_2 \\
&=C_1C_4\frac{4}{b}\sum_{m=1}^{M}\sum_{n=1,3\ldots}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{(-1)^m}{E_{mn}}g'_m-\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{m\pi}{a}\right)^4\sum_{n=1,3\ldots}^{N}\frac{1}{E_{mn}}\right]c'''_m \\
&\quad+C_2C_4\frac{2}{a}\sum_{m=1}^{M}\sum_{n=1,3\ldots}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{(-1)^m}{E_{mn}}\left[c'_n-(-1)^m c''_n\right]-\frac{C_4}{a}\sum_{n=1,3\ldots}^{N}\frac{b}{n\pi}\left[1+2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g'''_n \\
&\quad+\sum_{n=1,3\ldots}^{N}\frac{b}{n\pi}\left[\frac{1}{G_2A_{s2}}+\frac{C_4}{a}+2C_1C_4\frac{1}{a}\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g''''_n+\Gamma'' \qquad (B142)
\end{aligned}
$$

Equations (B135) to (B142) constitute 2M + 2N + 4 simultaneous equations with exactly the same number of unknowns, namely, c'_n, c''_n, c'''_m, g'_m, g'''_n, g''''_n, and $\bar{M}_1$, $\bar{M}_2$. The other basic unknowns, c''''_m, g''_m and $\bar{M}_3$, $\bar{M}_4$ are given in the equations (B134).

<u>Special case: symmetry about $y = \frac{b}{2}$ and $x = \frac{a}{2}$.</u> A further reduction in the number of simultaneous equations results if the

structure, loading, and thermal strains are symmetrical about both centerlines, $y = \frac{b}{2}$ and $x = \frac{a}{2}$. In this case one may write, in addition to equations (B134), the following conditions:

$$\left.\begin{aligned}
&A_1 = A_2;\ I_1 = I_2;\ t_1^i = t_2^i;\ t_1^o = t_2^o;\ k_1 = k_2 = k_3 = k_4 = k;\ E_1 = E_2; \\
&G_1 = G_2,\ A_{s1} = A_{s2};\ Q_m''' = Q_m'''' = 0 \text{ for } m \text{ even};\ Q_n' = -Q_n'' \text{ for } n \text{ odd}; \\
&B_m''' = B_m'''' = 0 \text{ for } m \text{ even};\ B_n' = B_n'' \text{ for } n \text{ odd};\ P_y' = P_y'' = P_y''' = P_y''''; \\
&P_x' = P_x'' = P_x''' = P_x'''';\ \bar{M}_1 = \bar{M}_2 = \bar{M}_3 = \bar{M}_4 = \bar{M};\ T_{mn} = 0 \text{ for } m \text{ or } n \text{ even}; \\
&T_m''' = T_m'''' = 0 \text{ for } m \text{ even};\ T_n' = T_n'' \text{ for } n \text{ odd}; \\
&c_m''' = c_m'''' = g_m' = g_m'' = 0 \text{ for } m \text{ even};\ c_n' = c_n'' \text{ for } n \text{ odd}; \\
&g_n''' = g_n'''' \text{ for } n \text{ odd; etc.}
\end{aligned}\right\} \quad \text{(B143)}$$

Using the above equations and observing that the equations (B135), (138) and (B141) are now identical to equations (B136), (B139) and (B142), respectively, one may reduce equations (B135) to (B142) to the following systems:

$$(D'_{Mn} - F'_{Mn})\,c'_n - (G'_{Mn} - H'_{Mn})\,g'''_n$$

$$= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)C_1 \sum_{m=1,3,\cdots}^{M} \frac{m\pi}{a}\frac{1}{E_{mn}}\,g'_m$$

$$+\frac{4}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\sum_{m=1,3,\cdots}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right]c'''_m$$

$$+\frac{4}{ab}\,\frac{a}{t_1^i}\,\frac{A_1(t_1^i)^2}{I_1}\,\bar{M} - \bar{R}'_n$$

$$(n = 1, 3, \cdots, N) \qquad \text{(B144)}$$

$$(D'''_{Nm} - F'''_{Nm})\,g'_m - (G'''_{Nm} - H'''_{Nm})\,c'''_m$$

$$= -\frac{4}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)C_2 \sum_{n=1,3,\cdots}^{N} \frac{n\pi}{b}\frac{1}{E_{mn}}\,c'_n$$

$$+\frac{4}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\sum_{n=1,3,\cdots}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right]g'''_n$$

$$+\frac{4}{ab}\,\frac{b}{t_3^i}\,\frac{A_3(t_3^i)^2}{I_3}\,\bar{M} - \bar{R}'''_m$$

$$(m = 1, 3, \cdots, M) \qquad \text{(B145)}$$

$$(I'_{Mn} - K'_{Mn})\,c'_n - (P'_{Mn} - S'_{Mn})\,g'''_n$$

$$= -4\,\frac{C_1}{C_4}\,\frac{n\pi}{b}\sum_{m=1,3,\cdots}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_1^i}{E_1 I_1}\right]g'_m$$

$$+\frac{4}{C_4}\sum_{m=1,3,\cdots}^{M}\left\{C_2\,\frac{m\pi}{a}\,\frac{n\pi}{b}\,\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_1^i}{E_1 I_1}\right]\right.$$

$$\left. + \frac{a}{m\pi}\,\frac{b}{n\pi}\,\frac{t_1^i}{E_1 I_1}\right\}c'''_m$$

$$-\frac{4}{C_4}\,\frac{1}{E_1 I_1}\,\frac{b}{n\pi}\,\bar{M} - \bar{Z}'_n$$

$$(n = 1, 3, \cdots, N) \qquad \text{(B146)}$$

$$(I_{Nm}''' - K_{Nm}''')\, g_m' - (P_{Nm}''' - S_{Nm}''')\, c_m'''$$

$$= -4 \frac{C_2}{C_4} \frac{m\pi}{a} \sum_{n=1,3,\cdots}^{N} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[(C_3 - C_4)\left(\frac{m\pi}{a}\right)^2 - C_1\left(\frac{n\pi}{b}\right)^2 - \frac{t_3^i}{E_3 I_3} \right] c_n'$$

$$+ \frac{4}{C_4} \sum_{n=1,3,\cdots}^{N} \left\{ C_1 \frac{m\pi}{a} \frac{n\pi}{b} \frac{1}{E_{mn}} \left[C_2\left(\frac{m\pi}{a}\right)^2 - C_3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{a}{m\pi}\right)^2\left(\frac{n\pi}{b}\right)^2 \frac{t_3^i}{E_3 I_3} \right] \right.$$

$$\left. + \frac{a}{m\pi} \frac{b}{n\pi} \frac{t_3^i}{E_3 I_3} \right\} g_n'''$$

$$- \frac{4}{C_1} \frac{1}{E_3 I_3} \frac{a}{m\pi} \bar{M} - Z_m''' \qquad (m = 1, 3, \cdots, M) \qquad \text{(B147)}$$

$$\frac{\bar{M}}{k} = -C_1 C_4 \frac{4}{b} \sum_{m=1,3,\cdots}^{M} \sum_{n=1,3,\cdots}^{N} \frac{m\pi}{a} \left(\frac{n\pi}{b}\right)^2 \frac{1}{E_{mn}} g_m'$$

$$+ \sum_{m=1,3,\cdots}^{M} \frac{a}{m\pi} \left[\frac{1}{G_3 A_{s3}} + 4 C_2 C_4 \frac{1}{b} \left(\frac{m\pi}{a}\right)^4 \sum_{n=1,3,\cdots}^{N} \frac{1}{E_{mn}} \right] c_m''' - C_2 C_4 \frac{4}{a} \sum_{m=1,3,\cdots}^{M} \sum_{n=1,3,\cdots}^{N} \left(\frac{m\pi}{a}\right)^2 \frac{n\pi}{b} \frac{1}{E_{mn}} c_n'$$

$$+ \sum_{n=1,3,\cdots}^{N} \frac{b}{n\pi} \left[\frac{1}{G_1 A_{s1}} + 4 C_1 C_4 \frac{1}{a} \left(\frac{n\pi}{b}\right)^4 \sum_{m=1,3,\cdots}^{M} \frac{1}{E_{mn}} \right] g_n''' + \Gamma' \qquad \text{(B148)}$$

The last equation may be used to eliminate the $\bar{M}$ involved in equations (B144) to (B147), provided that the spring constant k is not infinity. If this is done, the following system of equations result:

$$
-\frac{4}{b}C_1\sum_{p=1,3,\ldots}^{M}\frac{p\pi}{a}\left\{\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\frac{1}{E_{pn}}+kC_4\frac{4}{ab}\frac{a}{t_1^i}\frac{A_1(t_1^i)^2}{I_1}\sum_{q=1,3,\ldots}^{N}\left(\frac{q\pi}{b}\right)^2\frac{1}{E_{pq}}\right\}g_p'
$$

$$
+\sum_{p=1,3,\ldots}^{M}\left\{\frac{4}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\left[C_2\left(\frac{p\pi}{a}\right)^3\frac{1}{E_{pn}}-\frac{a}{p\pi}\right]\right.
$$

$$
\left.+k\frac{4}{ab}\frac{a}{t_1^i}\frac{A_1(t_1^i)^2}{I_1}\frac{a}{p\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{p\pi}{a}\right)^4\sum_{q=1,3,\ldots}^{N}\frac{1}{E_{pq}}\right]\right\}c_p'''
$$

$$
-\sum_{q=1,3,\ldots}^{N}\left\{k\frac{16C_4}{a^2b}\frac{a}{t_1^i}C_2\frac{A_1(t_1^i)^2}{I_1}\frac{q\pi}{b}\sum_{p=1,3,\ldots}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pq}}+\delta_{nq}\left(D_{Mn}'-F_{Mn}'\right)\right\}c_q'
$$

$$
+\sum_{q=1,3,\ldots}^{N}\left\{k\frac{4}{ab}\frac{a}{t_1^i}\frac{A_1(t_1^i)^2}{I_1}\frac{b}{q\pi}\left[\frac{1}{G_1A_{s1}}+4C_1C_4\frac{1}{a}\left(\frac{q\pi}{b}\right)^4\sum_{p=1,3,\ldots}^{M}\frac{1}{E_{pq}}\right]+\delta_{nq}\left(G_{Mn}'-H_{Mn}'\right)\right\}g_q'''
$$

$$
=\bar{R}_n'-k\frac{4}{ab}\frac{a}{t_1^i}\frac{A_1(t_1^i)^2}{I_1}\Gamma' \qquad (n=1,3,\cdots,N) \qquad \text{(B149)}
$$

$$
-\sum_{p=1,3,\ldots}^{M}\left\{k\frac{16C_4}{ab^2}\frac{b}{t_3^i}C_1\frac{A_3(t_3^i)^2}{I_3}\frac{p\pi}{a}\sum_{q=1,3,\ldots}^{N}\left(\frac{q\pi}{b}\right)^2\frac{1}{E_{pq}}+\delta_{mp}\left(D_{Nm}'''-F_{Nm}'''\right)\right\}g_p'
$$

$$
+\sum_{p=1,3,\ldots}^{M}\left\{k\frac{4}{ab}\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\frac{a}{p\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{p\pi}{a}\right)^4\sum_{q=1,3,\ldots}^{N}\frac{1}{E_{pq}}\right]\right.
$$

$$
\left.+\delta_{mp}\left(G_{Nm}'''-H_{Nm}'''\right)\right\}c_p'''
$$

$$
-\frac{4}{a}C_2\sum_{q=1,3,\ldots}^{N}\frac{q\pi}{b}\left\{\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\frac{1}{E_{mq}}+kC_4\frac{4}{ab}\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\sum_{p=1,3,\ldots}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pq}}\right\}c_q'
$$

$$
+\sum_{q=1,3,\ldots}^{N}\left\{\frac{4}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\left[C_1\left(\frac{q\pi}{b}\right)^3\frac{1}{E_{mq}}-\frac{b}{q\pi}\right]\right.
$$

$$
\left.+k\frac{4}{ab}\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\frac{b}{q\pi}\left[\frac{1}{G_1A_{s1}}+4C_1C_4\frac{1}{a}\left(\frac{q\pi}{b}\right)^4\sum_{p=1,3,\ldots}^{M}\frac{1}{E_{pq}}\right]\right\}g_q'''
$$

$$
=\bar{R}_m'''-k\frac{4}{ab}\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\Gamma' \qquad (m=1,3,\cdots,M) \qquad \text{(B150)}
$$

$$
\begin{aligned}
&-\sum_{p=1,3,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{p\pi}{a}\frac{n\pi}{b}\frac{1}{E_{pn}}\left[(C_3-C_4)\left(\frac{n\pi}{b}\right)^2-C_2\left(\frac{p\pi}{a}\right)^2-\frac{t_1^i}{E_1I_1}\right]\right.\\
&\qquad\left.-kC_1\frac{16}{b}\frac{1}{E_1I_1}\frac{p\pi}{a}\frac{b}{n\pi}\sum_{q=1,3,\cdots}^{N}\left(\frac{q\pi}{b}\right)^2\frac{1}{E_{pq}}\right\}g_p'\\
&+\frac{4}{C_4}\sum_{p=1,3,\cdots}^{M}\left\{C_2\frac{p\pi}{a}\frac{n\pi}{b}\frac{1}{E_{pn}}\left[C_1\left(\frac{n\pi}{b}\right)^2-C_3\left(\frac{p\pi}{a}\right)^2-\left(\frac{p\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_1^i}{E_1I_1}\right]+\frac{a}{p\pi}\frac{b}{n\pi}\frac{t_1^i}{E_1I_1}\right.\\
&\qquad\left.-k\frac{1}{E_1I_1}\frac{a}{p\pi}\frac{b}{n\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{p\pi}{a}\right)^4\sum_{q=1,3,\cdots}^{N}\frac{1}{E_{pq}}\right]\right\}c_p'''\\
&+\sum_{q=1,3,\cdots}^{N}\left\{kC_2\frac{16}{a}\frac{1}{E_1I_1}\frac{b}{n\pi}\frac{q\pi}{b}\sum_{p=1,3,\cdots}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pq}}-\delta_{nq}\left(I_{Mn}'-K_{Mn}'\right)\right\}c_q'\\
&-\sum_{q=1,3,\cdots}^{N}\left\{k\frac{4}{C_4}\frac{1}{E_1I_1}\frac{b}{n\pi}\frac{b}{q\pi}\left[\frac{1}{G_1A_{s1}}+4C_1C_4\frac{1}{a}\left(\frac{q\pi}{b}\right)^4\sum_{p=1,3,\cdots}^{M}\frac{1}{E_{pq}}\right]-\delta_{nq}\left(P_{Mn}'-S_{Mn}'\right)\right\}g_q'''\\
&=\bar{Z}_n'+k\frac{4}{C_1}\frac{1}{E_1I_1}\frac{b}{n\pi}\Gamma' \qquad (n=1,3,\cdots,N)
\end{aligned}
\tag{B151}
$$

$$
\begin{aligned}
&\sum_{p=1,3,\cdots}^{M}\left\{kC_1\frac{16}{b}\frac{1}{E_3I_3}\frac{a}{m\pi}\frac{p\pi}{a}\sum_{q=1,3,\cdots}^{N}\left(\frac{q\pi}{b}\right)^2\frac{1}{E_{pq}}-\delta_{mp}\left(I_{Nm}'''-K_{Nm}'''\right)\right\}g_p'\\
&-\sum_{p=1,3,\cdots}^{M}\left\{k\frac{4}{C_4}\frac{1}{E_3I_3}\frac{a}{m\pi}\frac{a}{p\pi}\left[\frac{1}{G_3A_{s3}}+4C_2C_4\frac{1}{b}\left(\frac{p\pi}{a}\right)^4\sum_{q=1,3,\cdots}^{N}\frac{1}{E_{pq}}\right]-\delta_{mp}\left(P_{Nm}'''-S_{Nm}'''\right)\right\}c_p'''\\
&-\sum_{q=1,3,\cdots}^{N}\left\{4\frac{C_2}{C_4}\frac{m\pi}{a}\frac{q\pi}{b}\frac{1}{E_{mq}}\left[(C_3-C_4)\left(\frac{m\pi}{a}\right)^2-C_1\left(\frac{q\pi}{b}\right)^2-\frac{t_3^i}{E_3I_3}\right]\right.\\
&\qquad\left.-kC_2\frac{16}{a}\frac{1}{E_3I_3}\frac{a}{m\pi}\frac{q\pi}{b}\sum_{p=1,3,\cdots}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pq}}\right\}c_q'\\
&+\frac{4}{C_4}\sum_{q=1,3,\cdots}^{N}\left\{C_1\frac{m\pi}{a}\frac{q\pi}{b}\frac{1}{E_{mq}}\left[C_2\left(\frac{m\pi}{a}\right)^2-C_3\left(\frac{q\pi}{b}\right)^2-\left(\frac{a}{m\pi}\right)^2\left(\frac{q\pi}{b}\right)^2\frac{t_3^i}{E_3I_3}\right]+\frac{a}{m\pi}\frac{b}{q\pi}\frac{t_3^i}{E_3I_3}\right.\\
&\qquad\left.-k\frac{1}{E_3I_3}\frac{a}{m\pi}\frac{b}{q\pi}\left[\frac{1}{G_1A_{s1}}+4C_1C_4\frac{1}{a}\left(\frac{q\pi}{b}\right)^4\sum_{p=1,3,\cdots}^{M}\frac{1}{E_{pq}}\right]\right\}g_q'''\\
&=\bar{Z}_m'''+k\frac{4}{C_4}\frac{1}{E_3I_3}\frac{a}{m\pi}\Gamma' \qquad (m=1,3,\cdots,M)
\end{aligned}
\tag{B152}
$$

When k approaches infinity, which implies rigid-jointed stiffeners, equation (B148) can no longer be employed to eliminate the $\bar{M}$ in equations (B144) to (B147). In such case no simplification for the basic equations is possible, and equations (B144) to (B148) should be solved simultaneously for the c's, g's and $\bar{M}$.

Special case: symmetry about x = a/2, y = b/2, and the plate diagonals. If the structure, loading and thermal strains are entirely symmetric, that is to say, symmetrical about the two centerlines x = a/2, y = b/2 and the plate diagonals, the following relations exist:

$$\left.\begin{aligned}
&a = b\,;\ A_1 = A_2 = A_3 = A_4 = A\,;\ I_1 = I_2 = I_3 = I_4 = I\,;\ t_1^i = t_2^i = t_3^i = t_4^i = t^i\,;\\
&t_1^o = t_2^o = t_3^o = t_4^o = t^o\,;\ k_1 = k_2 = k_3 = k_4 = k\,;\ E_1 = E_2 = E_3 = E_4 = E\,;\ C_1 = C_2\,;\\
&G_1 = G_2 = G_3 = G_4 = G\,;\ A_{s1} = A_{s2} = A_{s3} = A_{s4} = A_s\,;\ Q_n' = -Q_n'' = Q_n''' = -Q_n''''\\
&\text{for } n \text{ odd and } M = N\,;\ Q_n' = Q_n'' = Q_n''' = Q_n'''' = 0 \text{ for } n \text{ even};\\
&P_y' = P_y'' = P_y''' = P_y'''' = P_x' = P_x'' = P_x''' = P_x'''' = P\,;\\
&\bar{M}_1 = \bar{M}_2 = \bar{M}_3 = \bar{M}_4 = \bar{M}\,;\ T_{mn} = 0 \text{ for } m \text{ or } n \text{ even};\\
&B_n' = B_n'' = B_n''' = B_n'''' \text{ for } n \text{ odd and } M = N\,;\\
&B_n' = B_n'' = B_n''' = B_n'''' \text{ for } n \text{ even};\\
&T_n' = T_n'' = T_n''' = T_n'''' \text{ for } n \text{ odd and } M = n\,;
\end{aligned}\right\} \text{(B153)}$$

$$
\left.\begin{array}{l}
c_n' = c_n'' = c_n''' = c_n'''' = g_n' = g_n'' = g_n''' = g_n'''' = 0 \quad \text{for } n \text{ even;} \\
c_n' = c_n'' = g_n' = g_n'' \quad \text{for } n \text{ odd and } M = N; \\
c_m''' = c_m'''' = g_m''' = g_m'''' \quad \text{for } m \text{ odd and } M=N; \\
T_n' = T_n'' = T_n''' = T_n'''' = 0 \quad \text{for } n \text{ even, etc.}
\end{array}\right\}
$$

It is observed that when the above relations are considered, equations (B144) and (B146) are identical to equations (B145) and (B147), respectively. The system of equations (B144) to (B148) then reduce to the following form:

$$
\sum_{m=1,3\cdots}^{M} \left[C_1 \frac{4}{a}\left(\frac{n\pi}{a}\right)^2 \left(\frac{A(t^i)^2}{I} + 1\right) \frac{m\pi}{a} \frac{1}{E_{mn}} + \delta_{mn} \left(D_{Mn}' - F_{Mn}' \right) \right] g_m'
$$

$$
- \sum_{m=1,3\cdots}^{M} \left\{ \frac{4}{a} \left(\frac{A(t^i)^2}{I} + 1 \right) \left[C_1 \left(\frac{m\pi}{a}\right)^3 \frac{1}{E_{mn}} - \frac{a}{m\pi} \right] + \delta_{mn} \left(G_{Mn}' - H_{Mn}' \right) \right\} c_m'''
$$

$$
= \frac{4}{a^2} \frac{a}{t^i} \frac{A(t_i)^2}{I} \bar{M} - \bar{R}_n' \qquad (n = 1, 3, \cdots, M) \qquad \text{(B154)}
$$

$$\sum_{m=1,3,\cdots}^{M} \left\{ 4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{n\pi}{a}\right)^2 - C_1\left(\frac{m\pi}{a}\right)^2 - \frac{t^i}{EI} \right] + \delta_{mn}\left(I'_{Mn} - K'_{Mn} \right) \right\} g'_m$$

$$- \sum_{m=1,3,\cdots}^{M} \left\{ 4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{a}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{a}{n\pi}\right)^2\frac{t^i}{EI} \right] \right.$$

$$\left. + \frac{4}{C_4}\frac{a}{m\pi}\frac{a}{n\pi}\frac{t^i}{EI} + \delta_{mn}\left(P'_{Mn} - S'_{Mn} \right) \right\} c'''_m$$

$$= -\frac{4}{C_4}\frac{1}{EI}\frac{a}{n\pi}\bar{M} - \bar{Z}' \qquad (n = 1, 3, ---, M) \qquad \text{(B155)}$$

Meanwhile, equations (B153) can also be simplified as follows:

$$\frac{\bar{M}}{k} = -\frac{8}{a}C_1C_4 \sum_{m=1,3,}^{M}\sum_{n=1,3,}^{N} \frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2\frac{1}{E_{mn}}g'_m$$

$$+ 2\sum_{m=1,3,\cdot}^{M} \frac{a}{m\pi}\left[\frac{1}{GA_s} + 4C_1C_4\frac{1}{a}\left(\frac{m\pi}{a}\right)^4 \sum_{n=1,3\cdots}^{M}\frac{1}{E_{mn}} \right] c'''_m + \Gamma' \qquad \text{(B156)}$$

Again, if k approaches infinity equation (B156) can not be used to eliminate the $\overline{M}$ in equations (B154) and (B155) and therefore equations (B154) to (B156) have to be solved simultaneously for g'_m, c'''_m and $\overline{M}$. If k vanishes,(implying hinge-jointed stiffeners), equations (B154) and (B155) alone are solvable for g'_m and c'''_m by setting $\overline{M}$ equal to zero. Equation (B156) then becomes unnecessary. For any finite value of k equation (B156) can always be used to eliminate the $\overline{M}$ in equations (B154) and (B155). If this is done, one has

$$\sum_{m=1,3,\cdots}^{M}\left\{C_1\frac{4}{a}\frac{m\pi}{a}\left[\left(\frac{n\pi}{a}\right)^2\left(\frac{A_1(t^i)^2}{I}+1\right)\frac{1}{E_{mn}}+kC_4\frac{8}{a^2}\frac{a}{t^i}\frac{A(t^i)^2}{I}\sum_{q=1,3,\cdots}^{M}\left(\frac{q\pi}{a}\right)^2\frac{1}{E_{pq}}\right]\right.$$

$$\left.+\delta_{mn}\left(D'_{Mn}-F'_{Mn}\right)\right\}g'_m$$

$$-\sum_{m=1,3,\cdots}^{M}\left\{\frac{4}{a}\left(\frac{A(t^i)^2}{I}+1\right)\left[C\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}}-\frac{a}{m\pi}\right]+k\frac{8}{a^2}\frac{a}{t^i}\frac{A(t^i)^2}{I}\frac{a}{m\pi}\left[\frac{1}{GA_s}\right.\right.$$

$$\left.\left.+4C_1C_4\frac{1}{a}\left(\frac{m\pi}{a}\right)^4\sum_{q=1,3,\cdots}^{M}\frac{1}{E_{mq}}\right]+\delta_{mn}\left(G'_{Mn}-H'_{Mn}\right)\right\}c'''_m$$

$$=k\frac{4}{a^2}\frac{a}{t^i}\frac{A(t^i)^2}{I}\Gamma'-\bar{R}'_n \qquad (n=1,3,\cdots,M) \qquad \text{(B157)}$$

$$\sum_{m=1,3,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[(C_3-C_4)\left(\frac{n\pi}{a}\right)^2-C_1\left(\frac{m\pi}{a}\right)^2-\frac{t^i}{EI}\right]\right.$$

$$\left.-kC_1\frac{32}{a}\frac{1}{EI}\frac{a}{n\pi}\frac{m\pi}{a}\sum_{q=1,3,\cdots}^{M}\left(\frac{q\pi}{a}\right)^2\frac{1}{E_{mq}}+\delta_{mn}\left(I'_{Mn}-K'_{Mn}\right)\right\}g'_m$$

$$-\sum_{m=1,3,\cdots}^{M}\left\{4\frac{C_1}{C_1}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{a}\right)^2-C_3\left(\frac{m\pi}{a}\right)^2-\left(\frac{m\pi}{a}\right)^2\left(\frac{a}{n\pi}\right)^2\frac{t^i}{EI}\right]+\frac{4}{C_4}\frac{a}{m\pi}\frac{a}{n\pi}\frac{t^i}{EI}\right.$$

$$\left.-k\frac{8}{C_4}\frac{1}{EI}\frac{a}{m\pi}\frac{a}{n\pi}\left[\frac{1}{GA_s}+4C_1C_4\frac{1}{a}\left(\frac{m\pi}{a}\right)^4\sum_{q=1,3,\cdots}^{M}\frac{1}{E_{mq}}\right]+\delta_{mn}\left(P'_{Mn}-S'_{Mn}\right)\right\}c'''_m$$

$$=-k\frac{4}{C_4}\frac{1}{EI}\frac{a}{n\pi}\Gamma'-\bar{Z}' \qquad (n=1,3,\cdots,M) \qquad \text{(B158)}$$

<u>Special case: antisymmetry about</u> $y = \frac{b}{2}$. Similar to the symmetrical cases discussed above a simplification exists if the loadings and thermal

strains possess certain antisymmetrical properties but the structure possesses corresponding symmetrical properties. For example, if the structure is symmetrical about the centerline $y = \frac{b}{2}$ but the loadings and thermal strains are antisymmetric about the same centerline, one can write immediately the following relations:

$$\left.\begin{aligned}
&A_3 = A_4;\ I_3 = I_4;\ t_3^i = t_4^i;\ t_3^o = t_4^o;\ k_1 = k_4;\ k_2 = k_3;\ E_3 = E_4;\\
&G_3 = G_4;\ A_{s3} = A_{s4};\ Q_n' = Q_n'' = 0 \text{ for } n \text{ odd};\ Q_m''' = Q_m''''\\
&\text{for all } m;\ B_n' = B_n'' = 0 \text{ for } n \text{ odd};\ B_m''' = -B_m'''' \text{ for all } m;\\
&P_y' = -P_y'';\ P_y''' = -P_y'''';\ P_x' = -P_x''';\ P_x'' = -P_x'''';\\
&T_{mn} = 0 \text{ for } n \text{ odd};\ T_n' = T_n'' = 0 \text{ for } n \text{ odd};\\
&T_m''' = -T_m'''' \text{ for all } m;\ c_n' = c_n'' = g_n''' = g_n'''' = 0 \text{ for } n \text{ odd};\\
&g_m' = -g_m'' \text{ for all } m;\ c_m''' = -c_m'''' \text{ for all } m;\ \bar{M}_1 = -\bar{M}_4;\ \bar{M}_2 = -\bar{M}_3;\ \text{etc.}
\end{aligned}\right\} \quad \text{(B159)}$$

Substituting the above conditions into equations (B119) to (B126) and leaving out the equations (B122) and (B126), which are now identical to equations (B121) and (B125), respectively, one obtains the following simplified system of equations:

$$
\begin{aligned}
&D'_{Mn}\, c'_n - F'_{Mn}\, c''_n - G'_{Mn}\, g'''_m + H'_{Mn}\, g''''_n \\
&= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right) C_1 \sum_{m=1}^{M} \frac{m\pi}{a}\,\frac{1}{E_{mn}}\, g'_m \\
&\quad + \frac{4}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\sum_{m=1}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right] c'''_m \\
&\quad + \frac{4}{ab}\left[\frac{A_1(t_1^i)^2}{I_1}\left(1+\frac{a}{t_1^i}\right)+1\right]\bar{M}_1 - \frac{4}{ab}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\bar{M}_2 - \bar{R}'_n \qquad (n=2,4,\cdots,N)
\end{aligned}
\tag{B160}
$$

$$
\begin{aligned}
&D''_{Mn}\, c'_n - F''_{Mn}\, c''_n - G''_{Mn}\, g'''_n + H''_{Mn}\, g''''_n \\
&= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_2(t_2^i)^2}{I_2}+1\right) C_1 \sum_{m=1}^{M} \frac{m\pi}{a}\,\frac{(-1)^m}{E_{mn}}\, g'_m \\
&\quad + \frac{4}{b}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)\sum_{m=1}^{M}(-1)^m\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right] c'''_m \\
&\quad + \frac{4}{ab}\left(\frac{A_2(t_2^i)^2}{I_2}+1\right)\bar{M}_1 - \frac{4}{ab}\left[\frac{A_2(t_2^i)^2}{I_2}\left(1+\frac{a}{t_2^i}\right)+1\right]\bar{M}_2 + \bar{R}''_n \\
&\qquad (n=2,4,\cdots,N)
\end{aligned}
\tag{B161}
$$

$$
\begin{aligned}
&\left(D'''_{Nm} + F'''_{Nm}\right) g'_m - \left(G'''_{Nm} + H'''_{Nm}\right) c'''_m \\
&= -\frac{2}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right) C_2 \sum_{n=2,4,\ldots}^{N} \frac{n\pi}{b}\,\frac{1}{E_{mn}}\left[c'_n - (-1)^m c''_n\right] \\
&\quad + \frac{2}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\sum_{n=2,4,\ldots}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right]\left[g'''_n - (-1)^m g''''_n\right] \\
&\quad + \frac{2}{ab}\left[2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right) + \frac{b}{t_3^i}\,\frac{A_3(t_3^i)^2}{I_3}\right]\left[\bar{M}_1 - (-1)^m\bar{M}_2\right] - \bar{R}'''_m \\
&\qquad (m=1,2,\cdots,M)
\end{aligned}
\tag{B162}
$$

$$I'_{Mn}\, c'_n - K'_{Mn}\, c''_n - P'_{Mn}\, g'''_n + S'_{Mn}\, g''''_n$$

$$= -4\frac{C_1}{C_4}\frac{n\pi}{b}\sum_{m=1}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_1^i}{E_1 I_1}\right] g'_m$$

$$+ \frac{4}{C_4}\sum_{m=1}^{M}\frac{a}{m\pi}\frac{b}{n\pi}\left\{ C_2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_1^i}{E_1 I_1}\right] + \frac{t_1^i}{E_1 I_1}\right\} c'''_m$$

$$- \frac{4}{C_4}\frac{1}{E_1 I_1}\frac{b}{n\pi}\left(\frac{t_1^i}{a} + 1\right)\bar{M}_1 + \frac{4}{C_4}\frac{1}{E_1 I_1}\frac{b}{n\pi}\frac{t_1^i}{a}\bar{M}_2 - \bar{Z}'_n$$

$$(n = 2, 4, \cdots, N) \qquad \text{(B163)}$$

$$I''_{Mn}\, c'_n - K''_{Mn}\, c''_n - P''_{Mn}\, g'''_n + S''_{Mn}\, g''''_n$$

$$= -4\frac{C_1}{C_4}\frac{n\pi}{b}\sum_{m=1}^{M}(-1)^m\frac{m\pi}{a}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_2^i}{E_2 I_2}\right] g'_m$$

$$+ \frac{4}{C_4}\sum_{m=1}^{M}(-1)^m\frac{a}{m\pi}\frac{b}{n\pi}\left\{ C_2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_2^i}{E_2 I_2}\right] + \frac{t_2^i}{E_2 I_2}\right\} c'''_m$$

$$- \frac{4}{C_4}\frac{1}{E_2 I_2}\frac{b}{n\pi}\frac{t_2^i}{a}\bar{M}_1 + \frac{4}{C_4}\frac{1}{E_2 I_2}\frac{b}{n\pi}\left(\frac{t_2^i}{a} + 1\right)\bar{M}_2 + \bar{Z}''_n$$

$$(n = 2, 4, \cdots, N) \qquad \text{(B164)}$$

$$(I_{Nm}''' + K_{Nm}''')\, g_m' - (P_{Nm}''' + S_{Nm}''')\, c_m'''$$

$$= -2\,\frac{C_2}{C_4}\,\frac{m\pi}{a}\sum_{n=2,4}^{N}\frac{n\pi}{b}\,\frac{1}{E_{mn}}\left[(C_3' - C_4')\left(\frac{m\pi}{a}\right)^2 - C_1'\left(\frac{n\pi}{b}\right)^2 - \frac{t_3^i}{E_3 I_3}\right]\left[c_n' - (-1)^m c_n''\right]$$

$$+ \frac{2}{C_4}\sum_{n=2,4,\cdots}^{N}\frac{a}{m\pi}\,\frac{b}{n\pi}\left\{C_1'\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[C_2'\left(\frac{m\pi}{a}\right)^2 - C_3'\left(\frac{n\pi}{b}\right)^2\right.\right.$$

$$\left.\left. - \left(\frac{a}{m\pi}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{t_3^i}{E_3 I_3}\right] + \frac{t_3^i}{E_3 I_3}\right\}\left[g_n''' - (-1)^m g_n''''\right]$$

$$- \frac{2}{C_4}\,\frac{1}{E_3 I_3}\,\frac{a}{m\pi}\left(2\,\frac{t_3^i}{b} + 1\right)\left[\bar{M}_1 - (-1)^m \bar{M}_2\right] - \bar{Z}_m''' \qquad (m = 1, 2, \cdots, M) \qquad \text{(B165)}$$

Using equations (B159) and writing out the full expressions for Θ', Φ, etc., one can also rewrite equations (B129) and (B130) as follows:

$$\left(\frac{1}{k_1} + \frac{2}{b}\,\frac{1}{G_1 A_{s1}} + \frac{2C_4}{ab} + \frac{1}{a}\,\frac{1}{G_3 A_{s3}}\right)\bar{M}_1 - \left(\frac{1}{a}\,\frac{1}{G_3 A_{s3}} + \frac{2C_4}{ab}\right)\bar{M}_2$$

$$= -\frac{4}{b}\,C_1 C_4 \sum_{m=1}^{M}\sum_{n=2,4,\cdots}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\,g_m'$$

$$+ \sum_{m=1}^{M}\frac{a}{m\pi}\left[\frac{1}{G_3 A_{s3}} + 2\,\frac{C_4}{b} + \frac{4}{b}\,C_2 C_4'\left(\frac{m\pi}{a}\right)^4\sum_{n=2,4,\cdots}^{N}\frac{1}{E_{mn}}\right]c_m'''$$

$$+ \sum_{n=2,4,\cdots}^{N}\frac{b}{n\pi}\left[\frac{1}{G_1 A_{s1}} + \frac{C_4}{a} + \frac{2}{a}\,C_1 C_4\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right]g_n'''$$

$$- \frac{C_4}{a}\sum_{n=2,4,\cdots}^{N}\frac{b}{n\pi}\left[1 + 2C_1'\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right]g_n''''$$

$$- \frac{2}{a}\,C_2 C_4\sum_{m=1}^{M}\sum_{n=2,4,\cdots}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\,\frac{1}{E_{mn}}\left[c_n' - (-1)^m c_n''\right] + \Gamma'$$

(B166)

$$-\left(\frac{1}{a}\frac{1}{G_3A_{s3}}+\frac{2C_4}{ab}\right)\bar{M}_1 + \left(\frac{1}{k_2}+\frac{2}{b}\frac{1}{G_2A_{s2}}+\frac{2C_4}{ab}+\frac{1}{a}\frac{1}{G_3A_{s3}}\right)\bar{M}_2$$

$$= \frac{4}{b} C_1 C_4 \sum_{m=1}^{M} \sum_{n=2,4,\ldots}^{N} \frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2 \frac{(-1)^m}{E_{mn}}\, g'_m$$

$$- \sum_{m=1}^{M} (-1)^m \frac{a}{m\pi}\left[\frac{1}{G_3A_{s3}} + 2\frac{C_4}{b} + \frac{4}{b}C_2C_4\left(\frac{m\pi}{a}\right)^4 \sum_{n=2,4,\ldots}^{N} \frac{1}{E_{mn}}\right] c'''_m$$

$$- \frac{C_4}{a}\sum_{n=2,4,\ldots}^{N} \frac{b}{n\pi}\left[1+2C_1\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{(-1)^m}{E_{mn}}\right] g'''_n$$

$$+ \sum_{n=2,4,\ldots}^{N} \frac{b}{n\pi}\left[\frac{1}{G_2A_{s2}}+\frac{C_4}{a}+\frac{2}{a}C_1C_4\left(\frac{n\pi}{b}\right)^4\sum_{m=1}^{M}\frac{1}{E_{mn}}\right] g''''_n$$

$$+ \frac{2}{a}C_2C_4 \sum_{m=1}^{M}\sum_{n=2,4,\ldots}^{N} \left(\frac{m\pi}{a}\right)^2 \frac{n\pi}{b}\frac{(-1)^m}{E_{mn}}\left[c'_n - (-1)^m c''_n\right] + T'' \tag{B167}$$

while equations (B131) and (B132) become identical to the above two equations.

Equations (B160) to (B167) may be solved simultaneously for c'_n, c''_n, c'''_m, g'_m, g'''_n, g''''_n, and $\bar{M}_1$, $\bar{M}_2$, or one may use equations (B166) and (B167) to eliminate $\bar{M}_1$ and $\bar{M}_2$ in equations (B160) to (B165), and then solve for c'_n, c''_n, c'''_m, g'_m, g'''_n, and g''''_n .

<u>Special case: antisymmetry about $x = \frac{a}{2}$, $y = \frac{b}{2}$.</u> If the structure is symmetrical about the centerlines $x = \frac{a}{2}$ and $y = \frac{b}{2}$ but the loading and thermal strains are antisymmetrical about the same centerlines, then, in addition to equations (B159), one can write

$$\left.\begin{aligned}
&A_1 = A_2\,;\ I_1 = I_2\,;\ t_1^i = t_2^i\,;\ t_1^o = t_2^o\,;\ k_1 = k_2 = k_3 = k_4 = k \\
&E_1 = E_2\,;\ G_1 = G_2\,;\ A_{s1} = A_{s2}\,;\ Q_m''' = Q_m'''' = 0 \text{ for } m \text{ odd}; \\
&Q_n' = Q_n'' \text{ for } n \text{ even};\ B_m''' = B_m'''' = 0 \text{ for } m \text{ odd}; \\
&B_n' = -B_n'' \text{ for } n \text{ even};\ P_y' = -P_y'' = -P_y''' = P_y''''\,; \\
&P_x' = -P_x'' = -P_x''' = P_x''''\,;\quad T_{mn} = 0 \text{ for } m \text{ odd} \\
&T_m''' = T_m'''' = 0 \text{ for } m \text{ odd};\ T_n' = -T_n'' \text{ for } n \text{ even}; \\
&g_m' = g_m'' = c_m''' = c_m'''' = 0 \text{ for } m \text{ odd};\ c_n' = -c_n'' \text{ for } n \text{ even}; \\
&g_n''' = -g_n'''' \text{ for } n \text{ even};\ \bar{M}_1 = -\bar{M}_2 = \bar{M}_4 = -\bar{M}_4 = \bar{M} \\
&\text{etc.}
\end{aligned}\right\} \quad \text{(B168)}$$

With the use of equations (B168), equations (B160) to (B167) may be reduced to the following simpler system:

$$(D'_{Mn} + F'_{Mn}) c'_n - (G'_{Mn} + H'_{Mn}) g'''_n$$
$$= -\frac{4}{b}\left(\frac{n\pi}{b}\right)^2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)C_1 \sum_{m=2,4,\cdots}^{M} \frac{m\pi}{a}\frac{1}{E_{mn}} g'_m$$
$$+ \frac{4}{b}\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)\sum_{m=2,4,\cdots}^{M}\left[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\right] c'''_m$$
$$+ \frac{4}{ab}\left[2\left(\frac{A_1(t_1^i)^2}{I_1}+1\right)+\frac{a}{t_1^i}\frac{A_1(t_1^i)^2}{I_1}\right]\bar{M} - \bar{R}'_n \qquad (n=2,4,\cdots,N)$$

(B169)

$$(D'''_{Nm} + F'''_{Nm}) g'_m - (G'''_{Nm} + H'''_{Nm}) c'''_m$$
$$= -\frac{4}{a}\left(\frac{m\pi}{a}\right)^2\left(\frac{A_3(t_3^i)^2}{I_1}+1\right)C_2 \sum_{n=2,4,\cdots}^{N} \frac{n\pi}{b}\frac{1}{E_{mn}} c'_n$$
$$+ \frac{4}{a}\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)\sum_{n=2,4,\cdots}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}} - \frac{b}{n\pi}\right] g'''_m$$
$$+ \frac{4}{ab}\left[2\left(\frac{A_3(t_3^i)^2}{I_3}+1\right)+\frac{b}{t_3^i}\frac{A_3(t_3^i)^2}{I_3}\right]\bar{M} - \bar{R}'''_m \qquad (m=2,4,\cdots,M)$$

(B170)

$$(I'_{Mn} + K'_{Mn}) c'_n - (P'_{Mn} + S'_{Mn}) g'''_n$$
$$= -4\frac{C_1}{C_4}\frac{n\pi}{b}\sum_{m=2,4,\cdots}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2 - \frac{t_1^i}{E_1 I_1}\right] g'_m$$
$$+ \frac{4}{C_4}\sum_{m=2,4,\cdots}^{M}\frac{a}{m\pi}\frac{b}{n\pi}\left\{C_2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2\right.\right.$$
$$\left.\left. - \left(\frac{m\pi}{a}\right)^2\left(\frac{b}{n\pi}\right)^2\frac{t_1^i}{E_1 I_1}\right] + \frac{t_1^i}{E_1 I_1}\right\} c'''_m$$
$$-\frac{4}{C_4}\frac{1}{E_1 I_1}\frac{b}{n\pi}\left(2\frac{t_1^i}{a}+1\right)\bar{M} - \bar{Z}'_n \qquad (n=2,4,\cdots,N)$$

(B171)

$$(I'''_{Nm} + K'''_{Nm})g'_m - (P'''_{Nm} + S'''_{Nm})c'''_m$$

$$= -4\frac{C_2}{C_4}\frac{m\pi}{a}\sum_{n=2,4,\cdots}^{N}\frac{n\pi}{b}\frac{1}{E_{mn}}\left[(C_3 - C_4)\left(\frac{m\pi}{a}\right)^2 - C_1\left(\frac{n\pi}{b}\right)^2 - \frac{t_3^i}{E_3 I_3}\right]c'_n$$

$$+ \frac{4}{C_4}\sum_{n=2,4,\cdots}^{N}\frac{a}{m\pi}\frac{b}{n\pi}\left\{C_1\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\left[C_2\left(\frac{m\pi}{a}\right)^2 - C_3\left(\frac{n\pi}{b}\right)^2\right.\right.$$

$$\left.\left. - \left(\frac{a}{m\pi}\right)^2\left(\frac{n\pi}{b}\right)^2\frac{t_3^i}{E_3 I_3}\right] + \frac{t_3^i}{E_3 I_3}\right\}g'''_n$$

$$- \frac{4}{C_4}\frac{1}{E_3 I_3}\frac{a}{m\pi}\left(2\frac{t_3^i}{b} + 1\right)\bar{M} - \bar{Z}'''_m \qquad (m = 2, 4, \cdots, M) \qquad (B172)$$

$$\left(\frac{1}{k} + \frac{2}{b}\frac{1}{G_1 A_{s1}} + \frac{4C_4}{ab} + \frac{2}{a}\frac{1}{G_3 A_{s3}}\right)\bar{M}$$

$$= -\frac{4}{b}C_1 C_4\sum_{m=2,4,\cdots}^{M}\sum_{n=2,4,\cdots}^{N}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}g'_m$$

$$+ \sum_{m=2,4,\cdots}^{M}\frac{a}{m\pi}\left[\frac{1}{G_3 A_{s3}} + 2\frac{C_4}{b} + \frac{4}{b}C_2 C_4\left(\frac{m\pi}{a}\right)^4\sum_{n=2,4,\cdots}^{N}\frac{1}{E_{mn}}\right]c'''_m$$

$$- \frac{4}{a}C_2 C_4\sum_{m=2,4,\cdots}^{M}\sum_{n=2,4,\cdots}^{N}\left(\frac{m\pi}{a}\right)^2\frac{n\pi}{b}\frac{1}{E_{mn}}c'_n$$

$$+ \sum_{n=2,4,\cdots}^{N}\frac{b}{n\pi}\left[\frac{1}{G_1 A_{s1}} + 2\frac{C_4}{a} + \frac{4}{a}C_1 C_4\left(\frac{n\pi}{b}\right)^4\sum_{m=2,4,\cdots}^{M}\frac{1}{E_{mn}}\right]g'''_n + \Gamma' \qquad (B173)$$

Having established the equations (B175) to (B179), the method of solving these equations for the key unknowns is essentially the same as before. As in the previous special cases, the last equation may be used to eliminate the $\overline{M}$ in equation (B175) to (B178).

Special case: antisymmetry about $x = \frac{a}{2}$, $y = \frac{b}{2}$, symmetry about plate diagonals. If one imposes the symmetry of the structure, loading and thermal strains about the plate diagonals into the last special case, it can be shown that

$$\left.\begin{array}{l}
a = b\ ;\ A_1 = A_2 = A_3 = A_4 = A\ ;\ t_1^i = t_2^i = t_3^i = t_4^i = t^i \\
t_1^o = t_2^o = t_3^o = t_4^o = t^o\ ;\ k_1 = k_2 = k_3 = k_4 = k\ ; \\
E_1 = E_2 = E_3 = E_4 = E\ ;\ G_1 = G_2 = G_3 = G_4 = G\ ; \\
A_{s1} = A_{s2} = A_{s3} = A_{s4} = A_s\ ;\ Q_m''' = Q_m'''' = B_m''' = B_m'''' = 0 \\
\text{for } m \text{ odd};\ Q_n' = Q_n'' = B_n' = B_n'' = 0 \text{ for } n \text{ odd};\\
Q_m''' = Q_m'''' = Q_m' = Q_m'' \text{ for all } m \text{ and } M = N\ ; \\
B_m''' = -B_m'''' = B_m' = -B_m'' \text{ for all } m \text{ and } M = N\ ; \\
P_y' = -P_y'' = -P_y''' = P_y'''' = P_x' = -P_x'' = -P_x''' = P_x'''' = P\ ; \\
T_{mn} = 0 \text{ for } m \text{ or } n \text{ odd};\ T_m''' = T_m'''' = 0 \text{ for } m \text{ odd};\ T_n' = T_n'' = 0 \\
\text{for } n \text{ odd};\ T_m' = -T_m'' = T_m''' = -T_m'''' \text{ for } m \text{ even and} \\
M = N\ ;\ g_m' = g_m'' = c_m''' = c_m'''' = 0 \text{ for } m \text{ odd};\ c_n' = c_n'' = g_n''' = g_n'''' = 0 \\
\text{for } n \text{ odd};\ g_m' = -g_m'' = c_m' = -c_m'' \text{ for } m \text{ even and } M = N; \\
c_m''' = -c_m'''' = g_m''' = -g_m'''' \text{ for } m \text{ even and } M = N\ ; \\
\bar{M}_1 = -\bar{M}_2 = \bar{M}_3 = -\bar{M}_4 = \bar{M}\ ;\ etc.
\end{array}\right\}\ \text{(B174)}$$

Substituting the above relations into equations (B169) to (B173) and discarding duplicated equations, one obtains

$$\sum_{m=2,4,\cdots}^{M}\left[\frac{4}{a}C_1\left(\frac{A(t^i)^2}{I}+1\right)\frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2\frac{1}{E_{mn}}+\delta_{mn}\left(D'_{Mn}+F'_{Mn}\right)\right]g'_m$$

$$-\sum_{m=2,4,\cdots}^{M}\left\{\frac{4}{a}\left(\frac{A(t^i)^2}{I}+1\right)\left[C_1\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}}-\frac{a}{m\pi}\right]+\delta_{mn}\left(G'_{Mn}+H'_{Mn}\right)\right\}c'''_m$$

$$=\frac{4}{a^2}\left[2\left(\frac{A(t^i)^2}{I}+1\right)+\frac{a}{t^i}\frac{A(t^i)^2}{I}\right]\bar{M}-\bar{R}'_n$$

$$(n=2,4,\cdots,M)$$

(B175)

$$\sum_{m=2,4,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[(C_3-C_4)\left(\frac{n\pi}{a}\right)^2-C_1\left(\frac{m\pi}{a}\right)^2-\frac{t^i}{EI}\right]+\delta_{mn}\left(I'_{Mn}+K'_{Mn}\right)\right\}g'_m$$

$$-\sum_{m=2,4,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{a}\right)^2-C_3\left(\frac{m\pi}{a}\right)^2-\left(\frac{m\pi}{a}\right)^2\left(\frac{a}{n\pi}\right)^2\frac{t^i}{EI}\right]\right.$$

$$\left.+\frac{4}{C_4}\frac{a}{m\pi}\frac{a}{n\pi}\frac{t^i}{EI}+\delta_{mn}\left(P'_{Mn}+S'_{Mn}\right)\right\}c'''_m$$

$$=-\frac{4}{C_4}\frac{1}{EI}\frac{a}{n\pi}\left(2\frac{t^i}{a}+1\right)\bar{M}-\bar{Z}'_n$$

$$(n=2,4,\cdots,M)$$

(B176)

$$
\begin{aligned}
&\left(\frac{1}{k}+\frac{4}{a}\frac{1}{G\,A_s}+\frac{4C_4}{a^2}\right)\overline{M} \\
&= -\frac{8}{a}C_1C_4\sum_{m=2,4,\cdots}^{M}\sum_{n=2,4,\cdots}^{M}\frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2\frac{1}{E_{mn}}g_m' \\
&\quad + 2\sum_{m=2,4,\cdots}^{M}\frac{a}{m\pi}\left[\frac{1}{GA_s}+2\,\frac{C_4}{a}+\frac{4}{a}C_1C_4\left(\frac{m\pi}{a}\right)^4\sum_{n=2,4,\cdots}^{M}\frac{1}{E_{mn}}\right]c_m''' + \Gamma'
\end{aligned}
$$

(B177)

Equations (B175) to (B177) may be solved simultaneously for g_m', c_m''' and $\overline{M}$. One may also use equation (B182) to eliminate $\overline{M}$ in equations (B175) and (B176), which results in

$$
\begin{aligned}
&\sum_{m=2,4,\cdots}^{M}\Bigg[\frac{4}{a}C_1\left(\frac{A(t^i)^2}{I}+1\right)\frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2\frac{1}{E_{mn}} + \frac{32}{a^3}C_1C_4\,\Pi'\,\frac{m\pi}{a}\sum_{q=2,4,\cdots}^{M}\left(\frac{q\pi}{a}\right)^2\frac{1}{E_{mq}} \\
&\qquad\qquad + \delta_{mn}\left(D_{Mn}' + F_{Mn}'\right)\Bigg]g_m' \\
&-\sum_{m=2,4,\cdots}^{M}\Bigg\{\frac{4}{a}\left(\frac{A(t^i)^2}{I}+1\right)\left[C_1\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}}-\frac{a}{m\pi}\right] \\
&\qquad + \frac{8}{a^2}\Pi'\frac{a}{m\pi}\left[\frac{1}{GA_s}+\frac{2C_4}{a}+\frac{4}{a}C_1C_4\left(\frac{m\pi}{a}\right)^4\sum_{q=2,4,\cdots}^{M}\frac{1}{E_{mn}}\right]+\delta_{mn}\left(G_{Mn}'+H_{Mn}'\right)\Bigg\}c_m''' \\
&= \frac{4}{a^2}\Pi'\Gamma' - \overline{R}_n' \qquad\qquad (n=2,4,\cdots,M)
\end{aligned}
$$

(B178)

$$
\begin{aligned}
&\sum_{m=2,4,\cdots}^{M}\Big\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\Big[(C_3-C_4)\Big(\frac{n\pi}{a}\Big)^2-C_1\Big(\frac{m\pi}{a}\Big)^2-\frac{t^i}{EI}\Big] \\
&\qquad -\frac{8}{a}C_1C_4\Pi''\frac{a}{n\pi}\frac{m\pi}{a}\sum_{g=2,4,\cdots}^{M}\Big(\frac{g\pi}{a}\Big)^2\frac{1}{E_{mg}}+\delta_{mn}(I'_{Mn}+K'_{Mn})\Big\}g'_m \\
&-\sum_{m=2,4,\cdots}^{M}\Big\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\Big[C_1\Big(\frac{n\pi}{a}\Big)^2-C_3\Big(\frac{m\pi}{a}\Big)^2-\Big(\frac{m\pi}{a}\Big)^2\Big(\frac{a}{n\pi}\Big)^2\frac{t^i}{EI}\Big]+\frac{4}{C_4}\frac{a}{m\pi}\frac{a}{n\pi}\frac{t^i}{EI} \\
&-2\Pi''\frac{a}{n\pi}\frac{a}{m\pi}\Big[\frac{1}{GA_s}+2\frac{C_4}{a}+\frac{4}{a}C_1C_4\Big(\frac{m\pi}{a}\Big)^4\sum_{g=2,4,\cdots}^{M}\frac{1}{E_{mg}}\Big]+\delta_{mn}(P'_{Mn}+S'_{Mn})\Big\}c'''_m \\
&=-\Pi''\frac{a}{n\pi}\Gamma'-\bar{Z}'_n \qquad\qquad (n=2,4,\cdots,M) \qquad\qquad \text{(B179)}
\end{aligned}
$$

where

$$\Pi' = \frac{2\Big(\dfrac{A(t^i)^2}{I}+1\Big)+\dfrac{a}{t^i}\dfrac{A(t^i)^2}{I}}{\dfrac{1}{k}+\dfrac{4}{a}\dfrac{1}{GA_s}+\dfrac{4C_4}{a^2}}$$

$$\Pi'' = \frac{\dfrac{4}{C_4}\dfrac{1}{EI}\Big(2\dfrac{t^i}{a}+1\Big)}{\dfrac{1}{k}+\dfrac{4}{a}\dfrac{1}{GA_s}+\dfrac{4C_4}{a^2}}$$

<u>Limiting case of negligible t^i's</u>. If the plate is attached to or very close to the centroidal axes of the stiffeners, the values of t_1^i, t_2^i, etc. may be considered zero. Consequently, all the equations obtained above can be simplified. For example, equations (B154) to (B156) of the entirely symmetrical case can now be written in the

following form with D'_{Mn}, F'_{Mn}, G'_{Mn}, H'_{Mn}, etc., expanded into their full expressions:

$$
\begin{aligned}
&\sum_{m=1,3,\cdots}^{M}\left\{\frac{4}{a}C_1\frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2\frac{1}{E_{mn}}+\delta_{mn}C_1\frac{n\pi}{a}\left[AE+\frac{4}{a}\sum_{p=1,3,\cdots}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\right]\right\}g'_m \\
&-\sum_{m=1,3,\cdots}^{M}\left\{\frac{4}{a}\left[C_1\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}}-\frac{a}{m\pi}\right]+\delta_{mn}\frac{n\pi}{a}\left[C_3AE+\frac{4}{a}C_1\left(\frac{n\pi}{a}\right)^2\sum_{p=1,3,\cdots}^{M}\frac{1}{E_{pn}}\right]\right\}c'''_m \\
&=-\frac{4}{a}P+Q'_n-\frac{n\pi}{a}\sum_{m=1,3,\cdots}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}T_{mn}+AE\frac{n\pi}{a}T'_n+\frac{4}{a}\sum_{m=1,3,\cdots}^{M}\frac{a}{m\pi}B'_m
\end{aligned}
\tag{B180}
$$

$$(n=1,3,\cdots,M)$$

$$
\begin{aligned}
&\sum_{m=1,3,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[(C_3-C_4)\left(\frac{n\pi}{a}\right)^2-C_1\left(\frac{m\pi}{a}\right)^2\right]\right. \\
&\qquad\left.+\delta_{mn}4\frac{C_1}{C_4}\left(\frac{n\pi}{a}\right)^2\sum_{p=1,3,\cdots}^{M}\left[C_1\left(\frac{n\pi}{a}\right)^2-C_3\left(\frac{p\pi}{a}\right)^2\right]\frac{1}{E_{pn}}\right\}g'_m \\
&-\sum_{m=1,3,\cdots}^{M}\left\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{a}\right)^2-C_3\left(\frac{m\pi}{a}\right)^2\right]\right. \\
&\qquad\left.+\delta_{mn}\left\{4\frac{C_1}{C_4}\left(\frac{n\pi}{a}\right)^2\sum_{p=1,3,\cdots}^{M}\left[(C_3-C_4)\left(\frac{n\pi}{a}\right)^2-C_1\left(\frac{p\pi}{a}\right)^2\right]\frac{1}{E_{pn}}-\frac{a}{C_4}\left[\frac{1}{EI}\left(\frac{a}{n\pi}\right)^2+\frac{1}{GA_s}\right]\right\}\right\}c'''_m \\
&=-\frac{4}{C_4}\frac{1}{EI}\frac{a}{n\pi}\bar{M}-\frac{a}{C_4}\sum_{m=1,3,\cdots}^{M}\left[(C_3-C_4)\left(\frac{m\pi}{a}\right)^2-C_1\left(\frac{m\pi}{a}\right)^2\right]\frac{m\pi}{a}\frac{1}{E_{mn}}T_{mn} \\
&\qquad+\frac{a}{C_4}L'_n-\frac{a}{C_4}K'_n+\frac{a}{C_4}\left[\frac{1}{EI}\left(\frac{a}{n\pi}\right)^2+\frac{1}{GA_s}\right]B'_n+\frac{at^\circ}{C_4EI}\frac{a}{n\pi}Q'_n
\end{aligned}
$$

$$(n=1,3,\cdots,M) \tag{B181}$$

$$\frac{\bar{M}}{k} = -\frac{8}{a} C_1 C_4 \sum_{m=1,3,\cdots}^{M} \sum_{n=1,3,\cdots}^{M} \frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2 \frac{1}{E_{mn}} g_m'$$

$$+ 2 \sum_{m=1,3,\cdots}^{M} \frac{a}{m\pi} \left[\frac{1}{GA_s} + C_1 C_4 \frac{4}{a}\left(\frac{m\pi}{a}\right)^4 \sum_{n=1,3,\cdots}^{M} \frac{1}{E_{mn}}\right] c_m'''$$

$$- C_4 \sum_{m=1,3,\cdots}^{M} \sum_{n=1,3,\cdots}^{M} \frac{m\pi}{a} \frac{n\pi}{a} \frac{1}{E_{mn}} T_{mn} - \frac{2}{GA_s} \sum_{n=1,3,\cdots}^{M} \frac{a}{n\pi} B_n' \tag{B182}$$

Similarly, equations (B175) to (B177) of the entirely anti-symmetrical case become

$$\sum_{m=2,4,\cdots}^{M} \left\{\frac{4}{a} C_1 \frac{m\pi}{a}\left(\frac{n\pi}{a}\right)^2 \frac{1}{E_{mn}} + \delta_{mn} C_1 \frac{n\pi}{a}\left[AE + \frac{4}{a}\sum_{p=2,4,\cdots}^{M}\left(\frac{p\pi}{a}\right)^2 \frac{1}{E_{pn}}\right]\right\} g_m'$$

$$- \sum_{m=2,4,\cdots}^{M} \left\{ \frac{4}{a}\left[C_1 \left(\frac{m\pi}{a}\right)^3 \frac{1}{E_{mn}} - \frac{a}{m\pi}\right] \right.$$

$$\left. + \delta_{mn} \frac{n\pi}{a}\left[C_3 AE + \frac{2}{a}\left(\frac{a}{n\pi}\right)^2 + \frac{4}{a} C_1 \left(\frac{n\pi}{a}\right)^2 \sum_{p=2,4,\cdots}^{M} \frac{1}{E_{pn}}\right]\right\} c_m'''$$

$$= \frac{8}{a^2}\bar{M} + \frac{4}{a}P - \frac{n\pi}{a}\sum_{m=2,4,\cdots}^{M} \frac{m\pi}{a}\frac{1}{E_{mn}} T_{mn} + AE\,\frac{n\pi}{a} T_n' \tag{B183}$$

$$+ Q_n' - 4\frac{t^\circ}{a} Q_o' + \frac{4}{a}\sum_{m=2,4,\cdots}^{M} \frac{a}{m\pi} B_m' \qquad (n = 2, 4, \cdots, M)$$

$$
\begin{aligned}
&\sum_{m=2,4,\cdots}^{M} \Big\{ 4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\Big[(C_3-C_4)\Big(\frac{n\pi}{a}\Big)^2 - C_1\Big(\frac{m\pi}{a}\Big)^2\Big] \\
&\qquad + \delta_{mn}\, 4\frac{C_1}{C_4}\Big\{\frac{1}{2} + \Big(\frac{n\pi}{a}\Big)^2 \sum_{p=2,4,\cdots}^{M}\Big[C_1\Big(\frac{n\pi}{a}\Big)^2 - C_3\Big(\frac{p\pi}{a}\Big)^2\Big]\frac{1}{E_{pn}}\Big\}\Big\} g_m' \\
&-\sum_{m=2,4,\cdots}^{M} \Big\{ 4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}\Big[C_1\Big(\frac{n\pi}{a}\Big)^2 - C_3\Big(\frac{m\pi}{a}\Big)^2\Big] \\
&\qquad + \delta_{mn}\Big\{2\frac{C_3-C_4}{C_4} + 4\frac{C_1}{C_4}\Big(\frac{n\pi}{a}\Big)^2 \sum_{p=2,4,\cdots}^{M}\Big[(C_3-C_4)\Big(\frac{n\pi}{a}\Big)^2 - C_1\Big(\frac{p\pi}{a}\Big)^2\Big]\frac{1}{E_{pn}} - \frac{a}{C_4}\Big[\frac{1}{EI}\Big(\frac{a}{n\pi}\Big)^2 + \frac{1}{GA_s}\Big]\Big\}\Big\} c_m''' \\
&= -\frac{4}{C_4}\frac{1}{EI}\frac{a}{n\pi}\bar{M} - \frac{a}{C_4}\sum_{m=2,4,\cdots}^{M}\Big[(C_3-C_4)\Big(\frac{n\pi}{a}\Big)^2 - C_1\Big(\frac{m\pi}{a}\Big)^2\Big]\frac{m\pi}{a}\frac{1}{E_{mn}}T_{mn} + \frac{a}{C_4}L_n' \\
&\quad - \frac{a}{C_4}K_n' + \frac{a}{C_4}\Big[\frac{1}{EI}\Big(\frac{a}{n\pi}\Big)^2 + \frac{1}{GA_s}\Big]B_n' + \frac{at^{\circ}}{C_4 EI}\frac{a}{n\pi}Q_n' \qquad (n=2,4,\cdots,M) \qquad \text{(B184)}
\end{aligned}
$$

$$
\begin{aligned}
&\Big(\frac{1}{k} + \frac{4}{a}\frac{1}{GA_s} + \frac{4C_4}{a^2}\Big)\bar{M} \\
&= -\frac{8}{a}C_1C_4\sum_{m=2,4,\cdots}^{M}\sum_{n=2,4,\cdots}^{M}\frac{m\pi}{a}\Big(\frac{n\pi}{a}\Big)^2\frac{1}{E_{mn}}g_m' \\
&\quad + 2\sum_{m=2,4,\cdots}^{M}\frac{a}{m\pi}\Big[\frac{1}{GA_s} + 2\frac{C_4}{a} + \frac{4}{a}C_1C_4\Big(\frac{m\pi}{a}\Big)^4\sum_{n=2,4,\cdots}^{M}\frac{1}{E_{mn}}\Big]c_m''' \\
&\quad - 2\frac{C_4}{a}P - C_4\sum_{m=2,4,\cdots}^{M}\sum_{n=2,4,\cdots}^{M}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}T_{mn} \\
&\quad + C_4\Big(2\frac{t^{\circ}}{a}\frac{a}{C_4GA_s} + 2\frac{t^{\circ}}{a} - 1\Big)Q_0' - 2\Big(\frac{1}{GA_s} + \frac{C_4}{a}\Big)\sum_{m=2,4,\cdots}^{M}\frac{a}{m\pi}B_m' \qquad \text{(B185)}
\end{aligned}
$$

Limiting case of large stiffener bending and shearing stiffnesses. The case in which some edges of the plate are forced to remain straight can be handled by allowing the bending and shear stiffness of the corresponding stiffeners to approach infinity. Inasmuch as the simplification of the equations developed above for this limiting case is straightforward (one merely replaces by zero all terms involving the inverse of the bending or shearing stiffness of those stiffeners to be held straight), the simplified equations will not be given except for two special cases, both with $K_n' = K_n'' = K_m''' = K_m'''' = 0$: (a) Double symmetric, two opposite stiffeners perfectly rigid in flexure and shear, the other two stiffeners perfectly flexible, $t_3^i = 0$. (b) Entirely symmetric, all four stiffeners perfectly rigid in flexure and shear. These two cases were selected because they correspond to problems solved by another approach in reference 2 and therefore provide the opportunity for a check.

For case (a), substitute

$$\frac{1}{E_1 I_1} = \frac{1}{G_1 A_{s1}} = 0, \qquad t_3^i = 0 \quad , \quad \text{and} \quad K_n' = K_m''' = 0$$

into equations (B144), (B146), and (B145) to obtain

$$\frac{4}{b} C_1 \left(\frac{n\pi}{b}\right)^2 \sum_{m=1,3\cdots}^{M} \frac{m\pi}{a} \frac{1}{E_{mn}} g_m' + C_2 \frac{n\pi}{b} \left[A_1 E_1 + \frac{4}{a} \sum_{m=1,3\cdots}^{M} \left(\frac{m\pi}{a}\right)^2 \frac{1}{E_{mn}}\right] c_n'$$

$$- \frac{n\pi}{b} \left[C_3 A_1 E_1 + \frac{4}{a} C_1 \left(\frac{n\pi}{b}\right)^2 \sum_{m=1,3\cdots}^{M} \frac{1}{E_{mn}}\right] g_n'''$$

$$= \frac{4}{b} \sum_{m=1,3\cdots}^{M} \left[C_2 \left(\frac{m\pi}{a}\right)^3 \frac{1}{E_{mn}} - \frac{a}{m\pi}\right] c_m''' - \bar{R}_n' \qquad (n = 1, 3, \cdots N) \tag{B186}$$

$$4\frac{C_1}{C_4}\frac{n\pi}{b}\sum_{m=1,3,\ldots}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}\left[(C_3-C_4)\left(\frac{n\pi}{b}\right)^2-C_2\left(\frac{m\pi}{a}\right)^2\right]g_m'$$

$$+\,4\frac{b}{a}\frac{C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\left\{\sum_{m=1,3,\ldots}^{M}\left[C_1\left(\frac{n\pi}{b}\right)^2-C_3\left(\frac{m\pi}{a}\right)^2\right]\frac{1}{E_{mn}}\right\}c_n'$$

$$-\,4\frac{b}{a}\frac{C_1}{C_4}\left[(C_3-C_4)\left(\frac{n\pi}{b}\right)^4\sum_{m=1,3,\ldots}^{M}\frac{1}{E_{mn}}-C_2\left(\frac{n\pi}{b}\right)^2\sum_{m=1,3,\ldots}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\right]g_n'''$$

$$=4\frac{C_2}{C_4}\sum_{m=1,3,\ldots}^{M}\frac{m\pi}{a}\frac{n\pi}{b}\frac{1}{E_{mn}}\left[C_1\left(\frac{n\pi}{b}\right)^2-C_3\left(\frac{m\pi}{a}\right)^2\right]c_m'''-\bar{Z}_n' \qquad \text{(B187)}$$

$$(n=1,3,\cdots,N)$$

$$C_1\frac{m\pi}{a}\left[A_3E_3+\frac{4}{b}\sum_{n=1,3,\ldots}^{N}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\right]g_m'+\frac{4}{a}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1,3,\ldots}^{N}\frac{n\pi}{b}\frac{1}{E_{mn}}c_n'$$

$$-\,\frac{4}{a}\sum_{n=1,3,\ldots}^{N}\left[C_1\left(\frac{n\pi}{b}\right)^3\frac{1}{E_{mn}}-\frac{b}{n\pi}\right]g_n'''$$

$$=\frac{m\pi}{a}\left[C_3A_3E_3+\frac{4}{b}C_2\left(\frac{m\pi}{a}\right)^2\sum_{n=1,3,\ldots}^{N}\frac{1}{E_{mn}}\right]c_m'''-\bar{R}_m''' \qquad \text{(B188)}$$

$$(m=1,3,\cdots,M)$$

In equations (B147), first let $t_3^i = t_3^o = o$, then multiply each side by E_3I_3, and finally let $E_3I_3 = o$ and $\bar{M} = o$ to obtain

$$c_m''' = B_m''' \qquad (m=1,3,\cdots,M)$$

Substituting this last equation into (B186), (B187) and solving for c_n' and g_n''', the following equations result:

$$c_n' = \frac{1}{A^*}\left[\sum_{m=1,3,\ldots}^{M} G_{mn}^* g_m' + S_n^*\right] \qquad \text{(B189)}$$

$$g_n''' = \frac{1}{A_n^*}\left[\sum_{m=1,3..}^{M} F_{mn}^* g_m' + T_n^*\right] \tag{B190}$$

in which

$$A_n^* = 4\frac{b}{a}\frac{C_2}{C_4}\left(\frac{n\pi}{b}\right)^3\Big\{C_1\Big[A_1E_1 + \frac{4}{a}\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big]\Big[(C_3-C_4)\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\frac{1}{E_{pn}}$$

$$- C_2\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big] - \Big[C_3A_1E_1 + \frac{4}{a}C_1\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\frac{1}{E_{pn}}\Big]$$

$$\cdot\Big[C_1\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\frac{1}{E_{pn}} - C_3\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big]\Big\}$$

$$G_{mn}^* = -4\frac{C_1}{C_4}\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\Big\{\frac{4}{a}C_1\Big[(C_3-C_4)\left(\frac{n\pi}{b}\right)^4\sum_{p=1,3..}^{M}\frac{1}{E_{pn}} - C_2\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big]$$

$$-\Big[C_3A_1E_1 + \frac{4}{a}C_1\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\frac{1}{E_{pn}}\Big]\Big[(C_3-C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2\Big]\Big\}$$

$$S_n^* = 4\frac{b}{a}\frac{C_1}{C_4}\Big[(C_3-C_4)\left(\frac{n\pi}{b}\right)^4\sum_{m=1,3..}^{M}\frac{1}{E_{mn}} - C_2\left(\frac{n\pi}{b}\right)^2\sum_{m=1,3..}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\Big]$$

$$\cdot\Big\{\frac{4}{b}\sum_{m=1,3..}^{M}\Big[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\Big]B_m''' - \bar{R}_n'\Big\} - \frac{n\pi}{b}\Big[C_3A_1E_1 + \frac{4}{a}C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1,3..}^{M}\frac{1}{E_{mn}}\Big]$$

$$\cdot\Big\{4\frac{C_2}{C_4}\frac{n\pi}{b}\sum_{m=1,3..}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}\Big[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2\Big]B_m''' - \bar{Z}_n'\Big\}$$

$$F_{mn}^* = -4\frac{C_1}{C_4}C_2\frac{m\pi}{a}\left(\frac{n\pi}{b}\right)^2\frac{1}{E_{mn}}\Big\{\frac{4}{a}\left(\frac{n\pi}{b}\right)^2\Big[C_1\left(\frac{n\pi}{b}\right)^2\sum_{p=1,3..}^{M}\frac{1}{E_{pn}} - C_3\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big]$$

$$-\Big[A_1E_1 + \frac{4}{a}\sum_{p=1,3..}^{M}\left(\frac{p\pi}{a}\right)^2\frac{1}{E_{pn}}\Big]\Big[(C_3-C_4)\left(\frac{n\pi}{b}\right)^2 - C_2\left(\frac{m\pi}{a}\right)^2\Big]\Big\}$$

$$T_n^* = 4\frac{b}{a}\frac{C_2}{C_4}\left(\frac{n\pi}{b}\right)^2\Big[C_1\left(\frac{n\pi}{b}\right)^2\sum_{m=1,3..}^{M}\frac{1}{E_{mn}} - C_3\sum_{m=1,3..}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\Big]$$

$$\cdot\Big\{\frac{4}{b}\sum_{m=1,3..}^{M}\Big[C_2\left(\frac{m\pi}{a}\right)^3\frac{1}{E_{mn}} - \frac{a}{m\pi}\Big]B_m''' - \bar{R}_n'\Big\} - C_2\frac{n\pi}{b}\Big[A_1E_1 + \frac{4}{a}\sum_{m=1,3..}^{M}\left(\frac{m\pi}{a}\right)^2\frac{1}{E_{mn}}\Big]$$

$$\cdot\Big\{4\frac{C_2}{C_4}\frac{n\pi}{b}\sum_{m=1,3..}^{M}\frac{m\pi}{a}\Big[C_1\left(\frac{n\pi}{b}\right)^2 - C_3\left(\frac{m\pi}{a}\right)^2\Big]B_m''' - \bar{Z}_n'\Big\}$$

Equations (B189) and (B190) can now be substituted into equations (B188), which become

$$\sum_{p=1,3\cdots}^{M}\{\frac{4}{a}C_2(\frac{m\pi}{a})^2\sum_{n=1,3\cdots}^{N}\frac{G_{pn}^{*}}{A_n^{*}}\frac{n\pi}{b}\frac{1}{E_{mn}}-\frac{4}{a}\sum_{n=1,3\cdots}^{N}\frac{F_{pn}^{*}}{A_n^{*}}[C_1(\frac{n\pi}{b})^3\frac{1}{E_{mn}}-\frac{b}{n\pi}]$$

$$+\delta_{mp}C_1\frac{m\pi}{a}[A_3E_3+\frac{4}{b}\sum_{n=1,3\cdots}^{N}(\frac{n\pi}{b})^2\frac{1}{E_{mn}}]\}g_p'$$

$$=\frac{m\pi}{a}[C_3A_3E_3+\frac{4}{b}C_2(\frac{m\pi}{a})^2\sum_{n=1,3\cdots}^{N}\frac{1}{E_{mn}}]B_m'''-\bar{R}_m'''-\frac{4}{a}C_2(\frac{m\pi}{a})^2\sum_{n=1,3\cdots}^{N}\frac{n\pi}{b}\frac{1}{E_{mn}}\frac{S_n^{*}}{A_n^{*}}$$

$$+\frac{4}{a}\sum_{n=1,3\cdots}^{N}[C_1(\frac{m\pi}{a})^3\frac{1}{E_{mn}}-\frac{b}{n\pi}]\frac{T_n^{*}}{A_n^{*}}\qquad(m=1,3,\cdots,M)\qquad\text{(B191)}$$

Equations (B191) can now be compared to equations (D57) of reference 2. Although different notations are used, they are found to be equivalent.

For case (b), let

$$\frac{1}{EI}=\frac{1}{GA_s}=0\quad\text{and}\quad K_n'=0$$

Substituting into equations (B154) to (B156), one has

$$\sum_{m=1,3..}^{M}\{C_1\frac{4}{a}\frac{m\pi}{a}(\frac{n\pi}{a})^2\frac{1}{E_{mn}}+\delta_{mn}C_1\frac{n\pi}{a}[AE+\frac{4}{a}\sum_{p=1,3..}^{M}(\frac{p\pi}{a})^2\frac{1}{E_{pn}}]\}g_m'$$

$$-\sum_{m=1,3..}^{M}\{\frac{4}{a}[C_1(\frac{m\pi}{a})^3\frac{1}{E_{mn}}-\frac{a}{m\pi}]+\delta_{mn}[C_3AE\frac{n\pi}{a}+C_1\frac{4}{a}(\frac{n\pi}{a})^3\sum_{p=1,3..}^{M}\frac{1}{E_{pn}}]\}C_m'''$$

$$=\frac{4}{a}P+Q_n'-\frac{n\pi}{a}\sum_{m=1,3..}^{M}\frac{m\pi}{a}\frac{1}{E_{mn}}T_{mn}+AE\frac{n\pi}{a}T_n'+\frac{4}{a}\sum_{m=1,3..}^{M}\frac{a}{m\pi}B_m' \qquad \text{(B192)}$$

$$(n=1,3\cdots,M)$$

$$\sum_{m=1,3..}^{M}\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}[(C_3-C_4)(\frac{n\pi}{a})^2-C_1(\frac{m\pi}{a})^2]$$

$$+\delta_{mn}4\frac{C_1}{C_4}[(C_3-C_4)(\frac{n\pi}{b})^2\sum_{p=1,3..}^{M}(\frac{p\pi}{a})^2\frac{1}{E_{pn}}-\sum_{p=1,3..}^{M}[C_1(\frac{p\pi}{a})^4\frac{1}{E_{pn}}-1]]\}g_m'$$

$$-\sum_{m=1,3..}^{M}\{4\frac{C_1}{C_4}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}[C_1(\frac{n\pi}{a})^2-C_3(\frac{m\pi}{a})^2]$$

$$+\delta_{mn}4\frac{C_1}{C_4}[(C_3-C_4)(\frac{n\pi}{a})^4\sum_{p=1,3..}^{M}\frac{1}{E_{pn}}-C_1(\frac{n\pi}{a})^2\sum_{p=1,3..}^{M}(\frac{p\pi}{a})^2\frac{1}{E_{pn}}]\}C_m'''$$

$$=-\frac{a}{C_4}\sum_{m=1,3..}^{M}[(C_3-C_4)(\frac{n\pi}{a})^2-C_1(\frac{m\pi}{a})^2]\frac{m\pi}{a}\frac{1}{E_{mn}}T_{mn}+\frac{a}{C_4}L_n' \qquad \text{(B193)}$$

$$(n=1,3\cdots,M)$$

$$\frac{\bar{M}}{k}=-\frac{8}{a}C_1C_4\sum_{m=1,3..}^{M}\sum_{n=1,3..}^{M}\frac{m\pi}{a}(\frac{n\pi}{a})^2\frac{1}{E_{mn}}g_m'+\frac{8}{a}C_1C_4\sum_{m=1,3..}^{M}\sum_{n=1,3..}^{M}(\frac{m\pi}{a})^3\frac{1}{E_{mn}}C_m'''$$

$$-C_4\sum_{m=1,3..}^{M}\sum_{n=1,3..}^{M}\frac{m\pi}{a}\frac{n\pi}{a}\frac{1}{E_{mn}}T_{mn} \qquad \text{(B194)}$$

For $\overline{M} = o$, which corresponds to the case of hinge-jointed stiffeners, equations (B192) and (B193) are found to be equivalent to equations (E34) and (E35) of reference 2, where identical limiting and special conditions were assumed.

Limiting case of small bending stiffness. The case in which the stiffener bending stiffnesses are zero and $t_\alpha^i = t_\alpha^o = o$ $(\alpha=1,2,3,4)$ was analyzed in reference 1. The same case may be obtained as a limiting case of the present analysis by considering the stiffener junctions to be hinged and allowing the stiffener flexural stiffnesses to approach zero. The resulting equations can be compared with those of reference 1 to provide another check on the correctness of the present analysis.

To arrive at the limiting case just described, let

$$\overline{M}_\alpha = t_\alpha^i = t_\alpha^o = 0 \qquad (\alpha = 1,2,3,4) \tag{B195}$$

in equations (B123) to (B126), multiply by E_1I_1 in (B123), E_2I_2 in (B124), E_3I_3 in (B125) and E_4I_4 in (B126), and finally set

$$E_\alpha I_\alpha = 0 \qquad (\alpha = 1,2,3,4)$$

The following simple relations result:

$$\left.\begin{aligned} g_n''' &= B_n' \\ g_n'''' &= B_n'' \\ C_m''' &= B_m''' \\ C_m'''' &= B_m'''' \end{aligned}\right\} \qquad (B196)$$

Substituting (B195) and (B196) into equations (B111) to (B118) one has

$$V_1(0) = V_1(b) = V_2(0) = V_2(b) = V_3(0) = V_3(a) = V_4(0) = V_4(a) = 0,$$

consequently, from equations (B76)

$$\left.\begin{aligned} &P_y' = P_1(0), \quad P_y'' = P_1(b), \quad P_y''' = P_2(0), \quad P_y'''' = P_2(b), \\ &P_x' = P_3(0), \quad P_x'' = P_3(a), \quad P_x''' = P_4(0), \quad P_x'''' = P_4(a). \end{aligned}\right\} \qquad (B197)$$

By virtue of equations (B195) to (B197), equations (B119) to (B122) degenerate to the following system:

$$\bar{C}_n' \left[A_1 E_1 + \frac{2}{a} \sum_{m=1}^{M} \left(\frac{m\pi}{a} \right)^2 \frac{1}{E_{mn}} \right] - \bar{C}_n'' \left[\frac{2}{a} \sum_{m=1}^{M} (-1)^m \left(\frac{m\pi}{a} \right)^2 \frac{1}{E_{mn}} \right]$$

$$= U_n' - \frac{2}{b} \left(\frac{n\pi}{b} \right)^2 \sum_{m=1}^{M} \frac{1}{E_{mn}} \left[\bar{g}_m' - (-1)^n \bar{g}_m'' \right] \qquad (n = 1, 2, \ldots, N) \qquad \text{(B198)}$$

$$-\bar{C}_n' \left[\frac{2}{a} \sum_{m=1}^{M} (-1)^m \left(\frac{m\pi}{a} \right)^2 \frac{1}{E_{mn}} \right] + \bar{C}_n'' \left[A_2 E_2 + \frac{2}{a} \sum_{m=1}^{M} \left(\frac{m\pi}{a} \right)^2 \frac{1}{E_{mn}} \right]$$

$$= U_n'' + \frac{2}{b} \left(\frac{n\pi}{b} \right)^2 \sum_{m=1}^{M} \frac{(-1)^m}{E_{mn}} \left[\bar{g}_m' - (-1)^n \bar{g}_m'' \right] \qquad (n = 1, 2, \ldots, N) \qquad \text{(B199)}$$

$$\bar{g}_m' \left[A_3 E_3 + \frac{2}{b} \sum_{n=1}^{N} \left(\frac{n\pi}{b} \right)^2 \frac{1}{E_{mn}} \right] - \bar{g}_m'' \left[\frac{2}{b} \sum_{n=1}^{N} (-1)^n \left(\frac{n\pi}{b} \right)^2 \frac{1}{E_{mn}} \right]$$

$$= U_m''' - \frac{2}{a} \left(\frac{m\pi}{a} \right)^2 \sum_{n=1}^{N} \frac{1}{E_{mn}} \left[\bar{C}_n' - (-1)^m \bar{C}_n'' \right] \qquad (m = 1, 2, \ldots, M) \qquad \text{(B200)}$$

$$-\bar{g}_m' \left[\frac{2}{b} \sum_{n=1}^{N} (-1)^n \left(\frac{n\pi}{b} \right)^2 \frac{1}{E_{mn}} \right] + \bar{g}_m'' \left[A_4 E_4 + \frac{2}{b} \sum_{n=1}^{N} \left(\frac{n\pi}{b} \right)^2 \frac{1}{E_{mn}} \right]$$

$$= U_m'''' + \frac{2}{a} \left(\frac{m\pi}{a} \right)^2 \sum_{n=1}^{N} \frac{(-1)^n}{E_{mn}} \left[\bar{C}_n' - (-1)^m \bar{C}_n'' \right] \qquad (m = 1, 2, \ldots, M) \qquad \text{(B201)}$$

where

$$\bar{c}'_n = c'_n G_2 \frac{n\pi}{b} \quad , \quad \bar{c}''_n = c''_n G_2 \frac{n\pi}{b}$$

$$\bar{g}'_m = g'_m G_1 \frac{m\pi}{a} \quad , \quad \bar{g}''_m = g''_m G_1 \frac{m\pi}{a}$$

and

$$U'_n = Q'_n + \frac{2}{b}\left[P_1(0) - (-1)^n P_1(b)\right] + \frac{b}{an\pi}\left(B'_n - B''_n\right) + A_1 E_1 \frac{n\pi}{b}\left(G_3 B'_n + T'_n\right) - \sum_{m=1}^{M} K_{mn}$$

$$U''_n = -Q''_n + \frac{2}{b}\left[P_2(0) - (-1)^n P_2(b)\right] - \frac{b}{an\pi}\left(B'_n - B''_n\right) + A_2 E_2 \frac{n\pi}{b}\left(G_3 B''_n + T''_n\right) + \sum_{m=1}^{M} (-1)^m K_{mn}$$

$$U'''_m = Q'''_m + \frac{2}{a}\left[P_3(0) - (-1)^m P_3(a)\right] + \frac{a}{bm\pi}\left(B'''_m - B''''_m\right) + A_3 E_3 \frac{m\pi}{a}\left(G_3 B'''_m + T'''_m\right) - \sum_{n=1}^{N} K_{mn}$$

$$U''''_m = -Q''''_m + \frac{2}{a}\left[P_4(0) - (-1)^m P_4(a)\right] - \frac{a}{bm\pi}\left(B'''_m - B''''_m\right) + A_4 E_4 \frac{m\pi}{a}\left(G_3 B''''_m + T''''_m\right) + \sum_{n=1}^{N} (-1)^n K_{mn}$$

with K_{mn} given by

$$K_{mn} = \frac{1}{E_{mn}}\left\{ \frac{mn\pi^2}{ab} T_{mn} - \frac{2}{b}\left(\frac{m\pi}{a}\right)^3 G_2\left[B'''_m - (-1)^n B''''_m\right] - \frac{2}{a}\left(\frac{n\pi}{b}\right)^3 G_1\left[B'_n - (-1)^m B''_n\right]\right\}$$

In reference 1 the plate external distributed tensions are assumed to be transmitted directly to the sheet edges. This condition is now given by equations (B196). The last four equations, (B198) to (B201), are found to be identical to equations (B61) to (B64) of reference 1 except that the loading terms R_n', R_n'', R_m''', R_m'''' in reference 1 are now denoted by U_n', U_n'', U_m''', and U_m'''', respectively. This coincidence is considered as another check on the correctness of the equations presented in this paper.

APPENDIX C

METHOD OF SUPERPOSITION

In this appendix it will be shown that the shear-lag problem (Figure 5(b)) and the discontinuous-temperature-distribution problem (Figure 5(c)) are very closely related. In particular, it will be shown that the stresses for either one of these problems can be obtained by superimposing a very simple stress distribution on the stresses of the other problem. For simplicity this will be demonstrated only for the case of the entirely symmetric structure with isotropic plate and stiffeners and plate of the same Young's modulus; the same reasoning can be extended with no difficulty to nonsymmetric structures.

The argument is developed with the aid of figure 14. Problem A represents the discontinuous-temperature-distribution problem, in which the stiffeners have a temperature rise T of zero, the plate a uniform temperature rise of θ. Problem B represents the same structure with the same temperature rises, but in addition stiffener end tensions of magnitude $\alpha\theta AE$. These tensions in Problem B are so chosen as to produce stiffener strains of magnitude $\alpha\theta$ in complete compatibility with the plate strains of the same magnitude. In view of this compatibility, the plate stresses are all zero in Problem B, while the stiffener tensions are uniform and of magnitude $\alpha\theta AE$. Problem C represents the shear-lag problem of figure 5(b) with stiffener end tensions of magnitude $-\alpha\theta AE$. It is easily seen that by superimposing the loads and temperature rises of Problems B and C, one arrives at the loading condition (purely thermal) of Problem A. Consequently, if the stresses for the shear-lag problem (Problem C) are known, one immediately obtains the stresses for the discontinuous-temperature-distribution problem (Problem A) by superimposing the very simple stress distribution of Problem B.

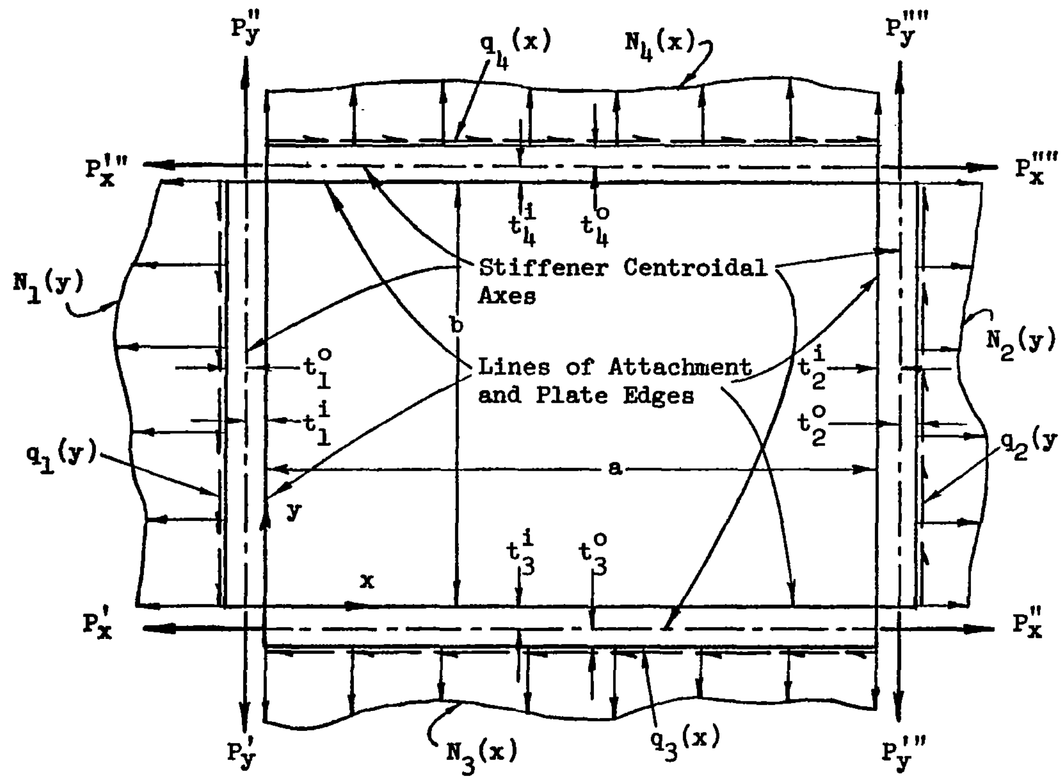


Figure 1.- Structure and loading

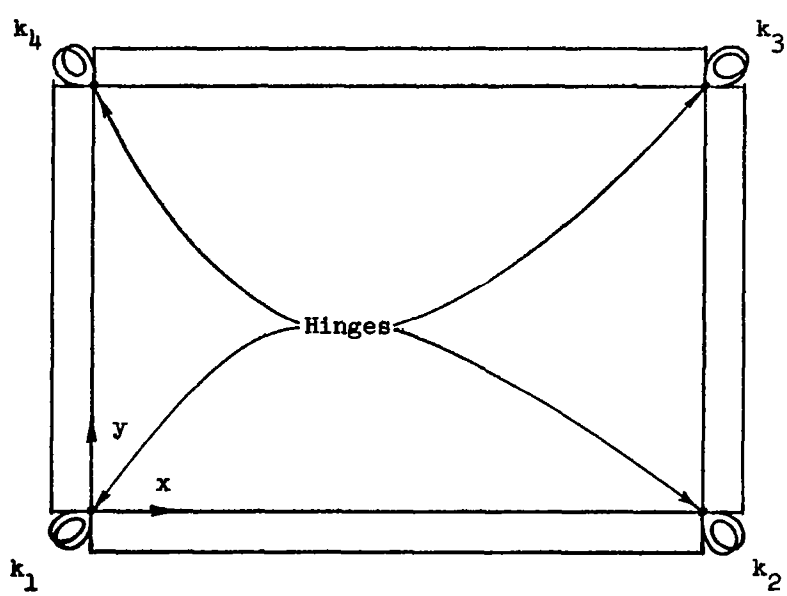


Figure 2.- Corner condition

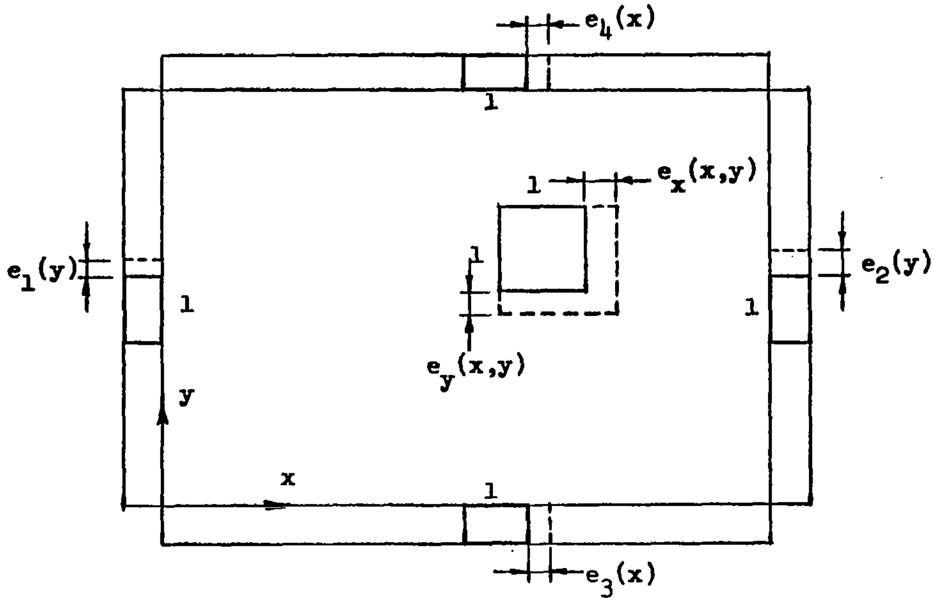


Figure 3.- Thermal strains

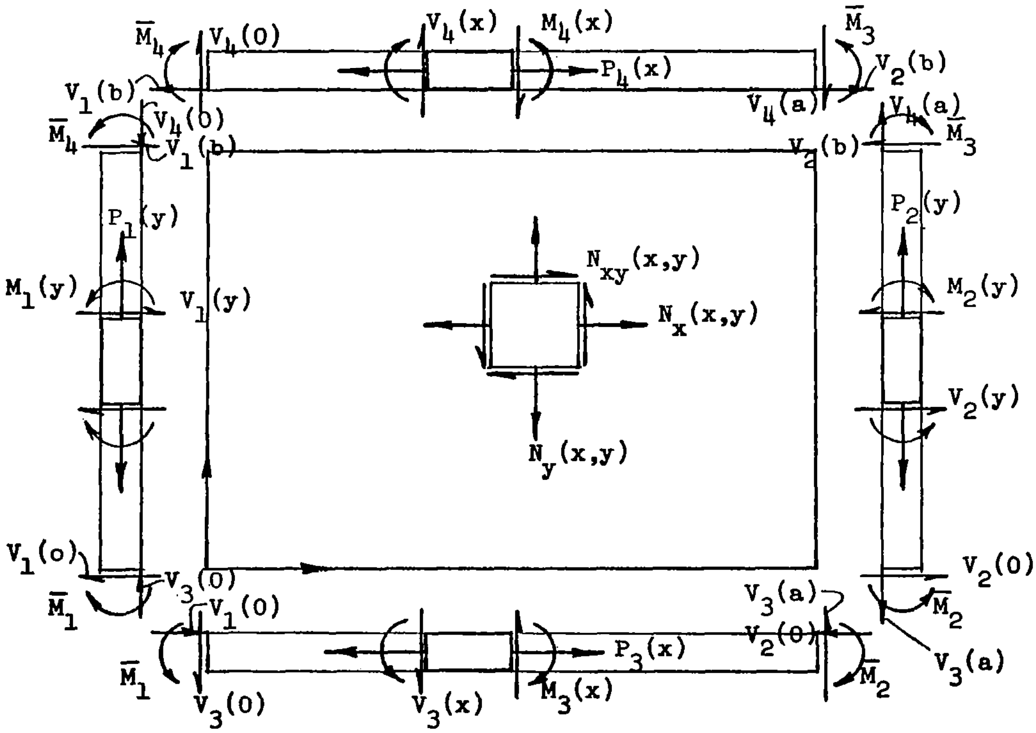
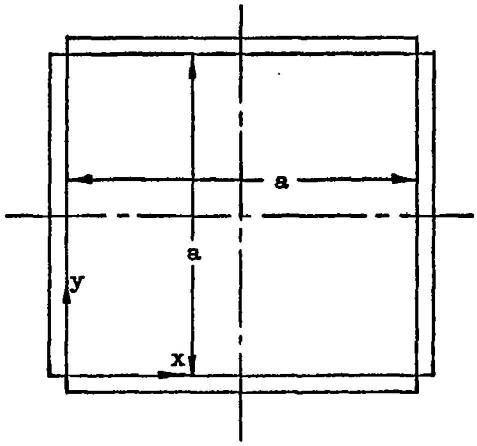
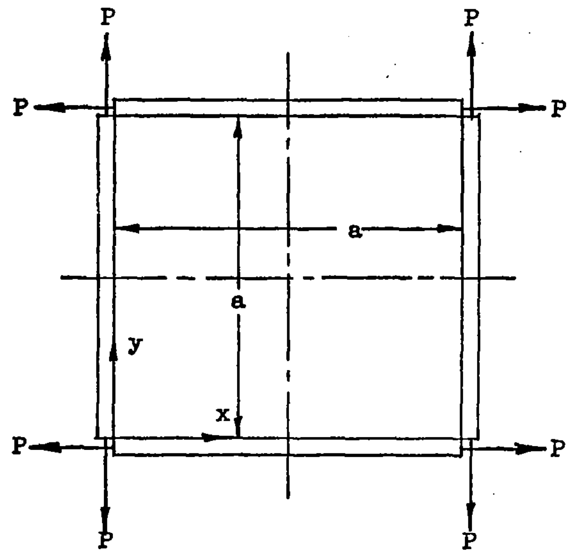


Figure 4.- Plate and stiffener forces and stiffener bending moments and transverse shears (externally applied loadings omitted).



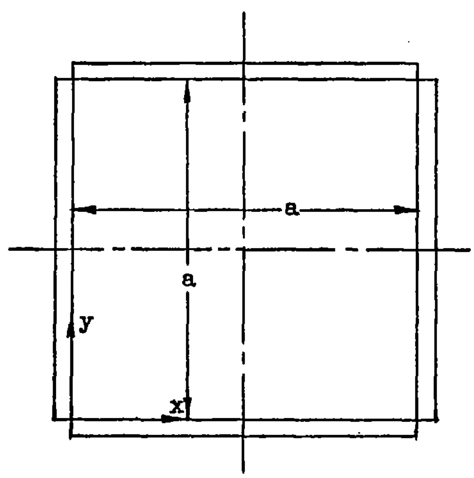
Stiffener temperature=0.
Plate temperature
= $\theta \sin(\pi x/a) \sin(\pi y/a)$.
Coefficient of expansion=α.

(a) Pillow-shaped temperature distribution.



Stiffener temperature=
plate temperature=constant.

(b) Shear lag.



Stiffener temperature=0.
Plate temperature=θ.
Coefficient of expansion=α.

(c) Discontinuous temperature distribution.

Figure 5.- Problems considered for numerical examples

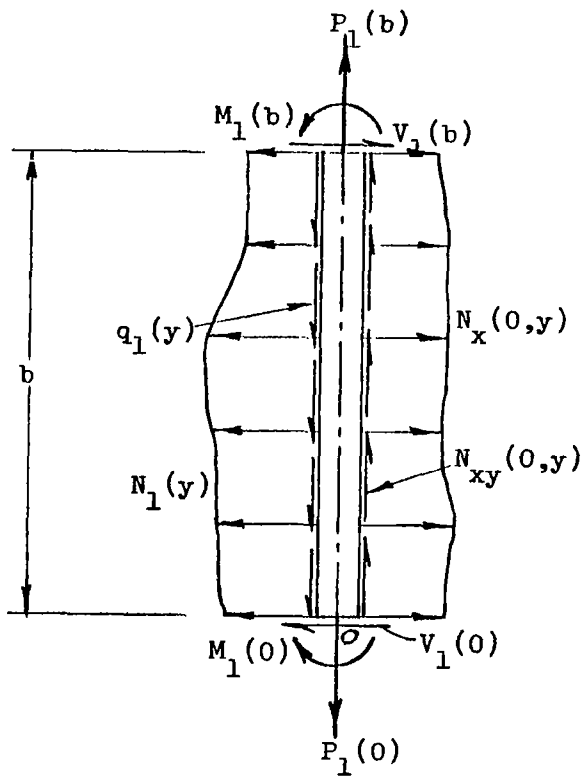
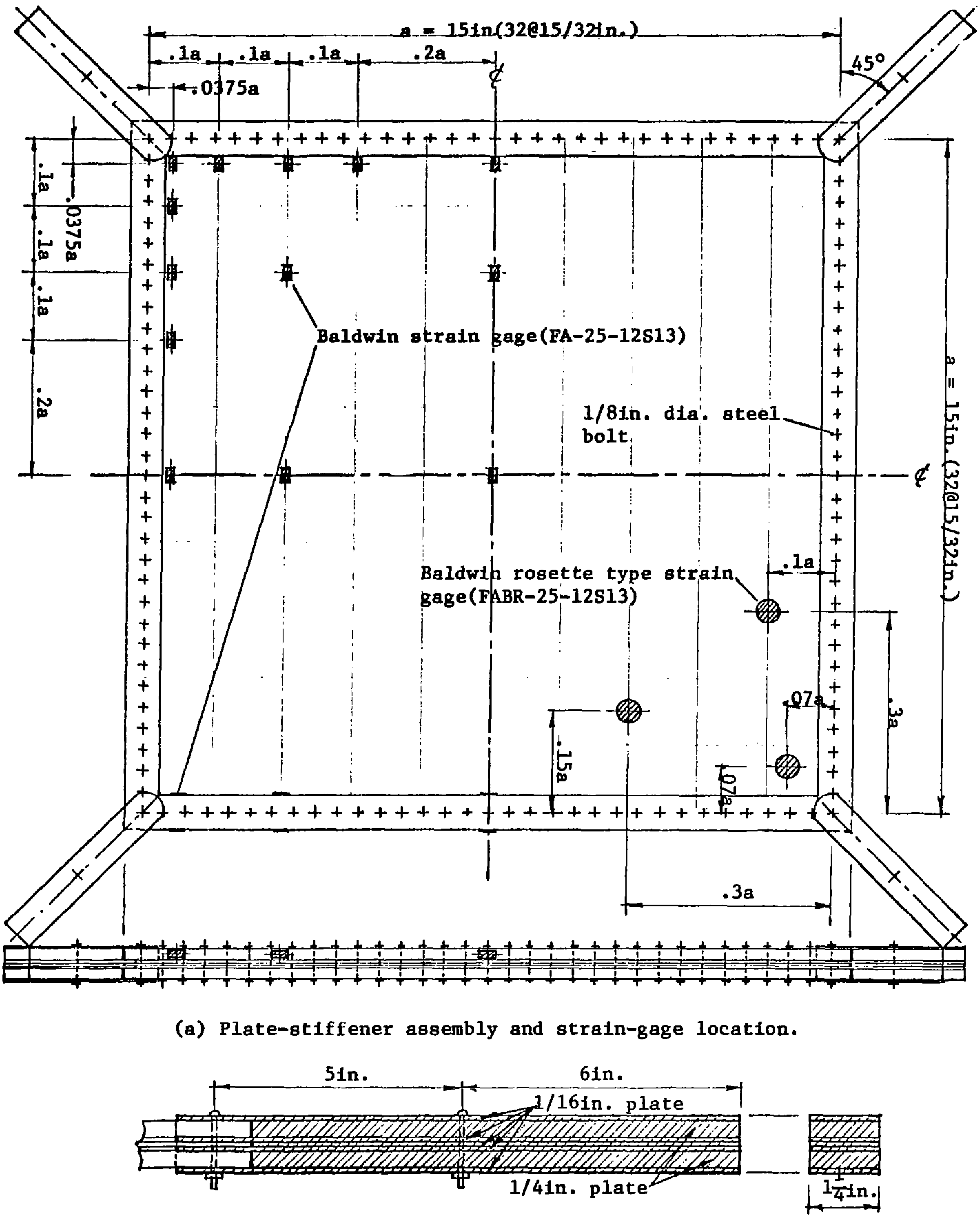


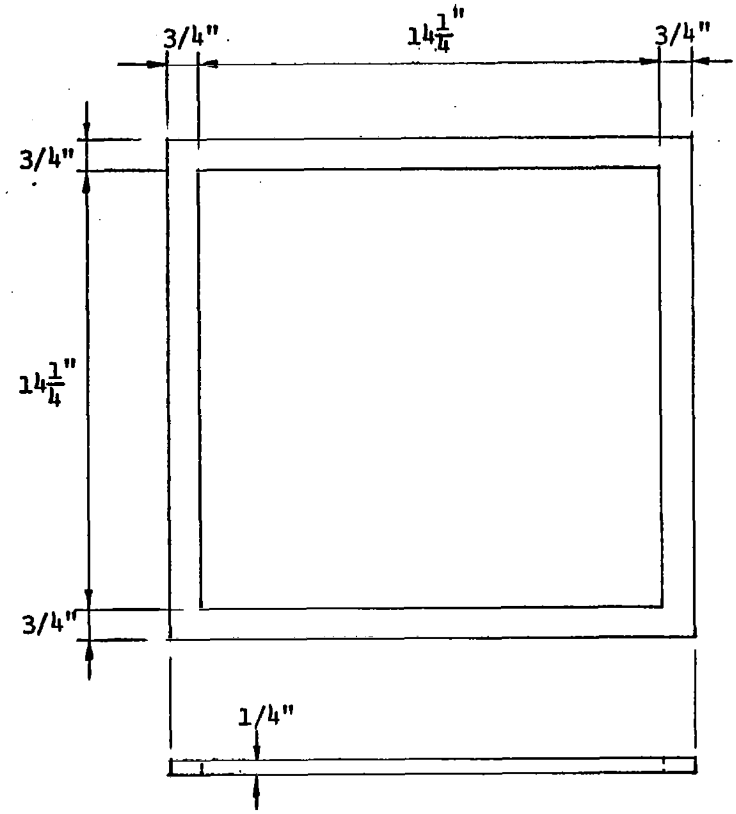
Figure 6.- Free body diagram of the stiffener at x = 0.



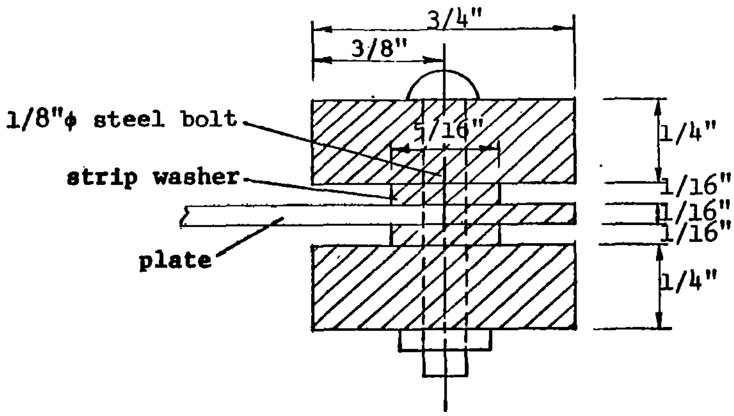
(a) Plate-stiffener assembly and strain-gage location.

(b) Grip fitting detail(shaded area represents grip fitting).

Figure 7.- Experimental specimen.



(a) One stiffener frame.



(b) Plate-stiffener connection.

Figure 8.- Stiffener details.

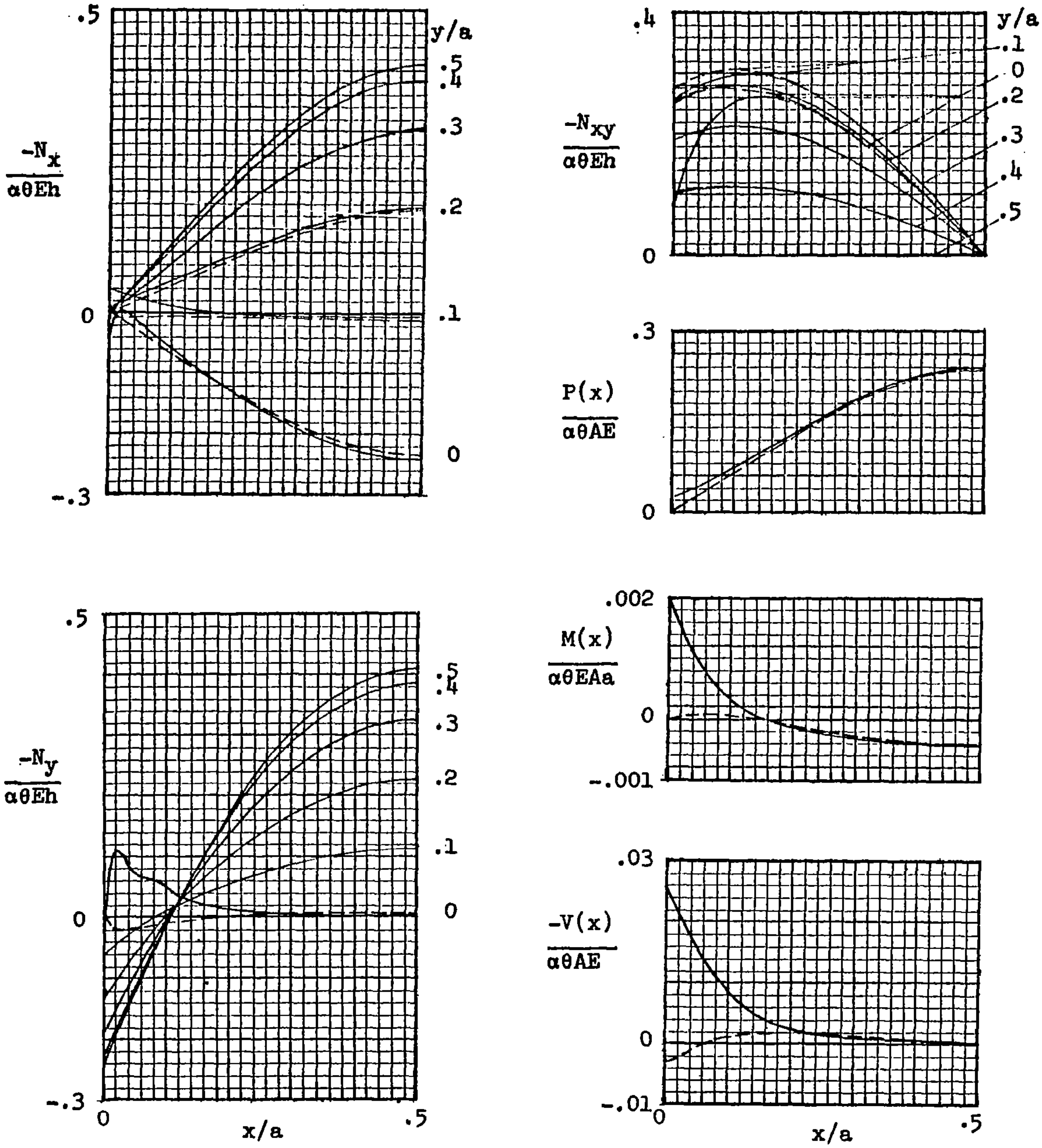


Figure 9.- Dimensionless plate stresses, stiffener tensions, bending moments and transverse shears due to pillow-shaped temperature distribution; $\nu = 0.3$; $4ah/\pi^2 A = 1.0$.

(a) Comparison of results for rigid-jointed stiffeners(solid curves) and hinge-jointed stiffeners(dashed curves); $ha^3/I = 10{,}000$; $A/A_s = 1.0$; $t^i/a = 0$. **(problems 2 and 8)**

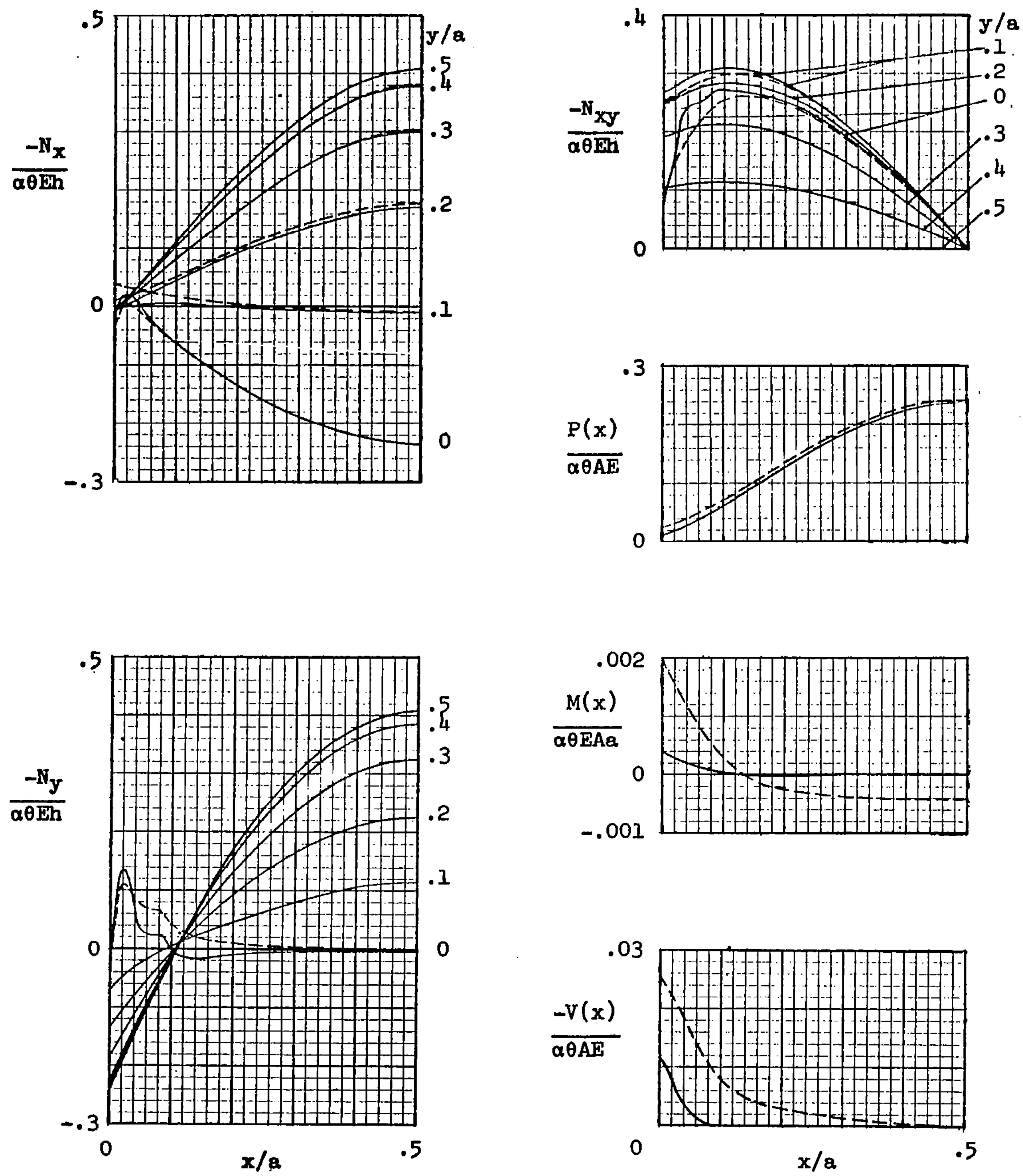


Figure 9.- Continued.

(b) Comparison of results for different stiffener bending rigidities; $A/A_s = 1.0$; $t^i/a = 0$; rigid-jointed stiffeners.

(i) $ha^3/I = 110{,}000$(solid curves) and $ha^3/I = 10{,}000$(dashed curves).

(problems 1 and 2)

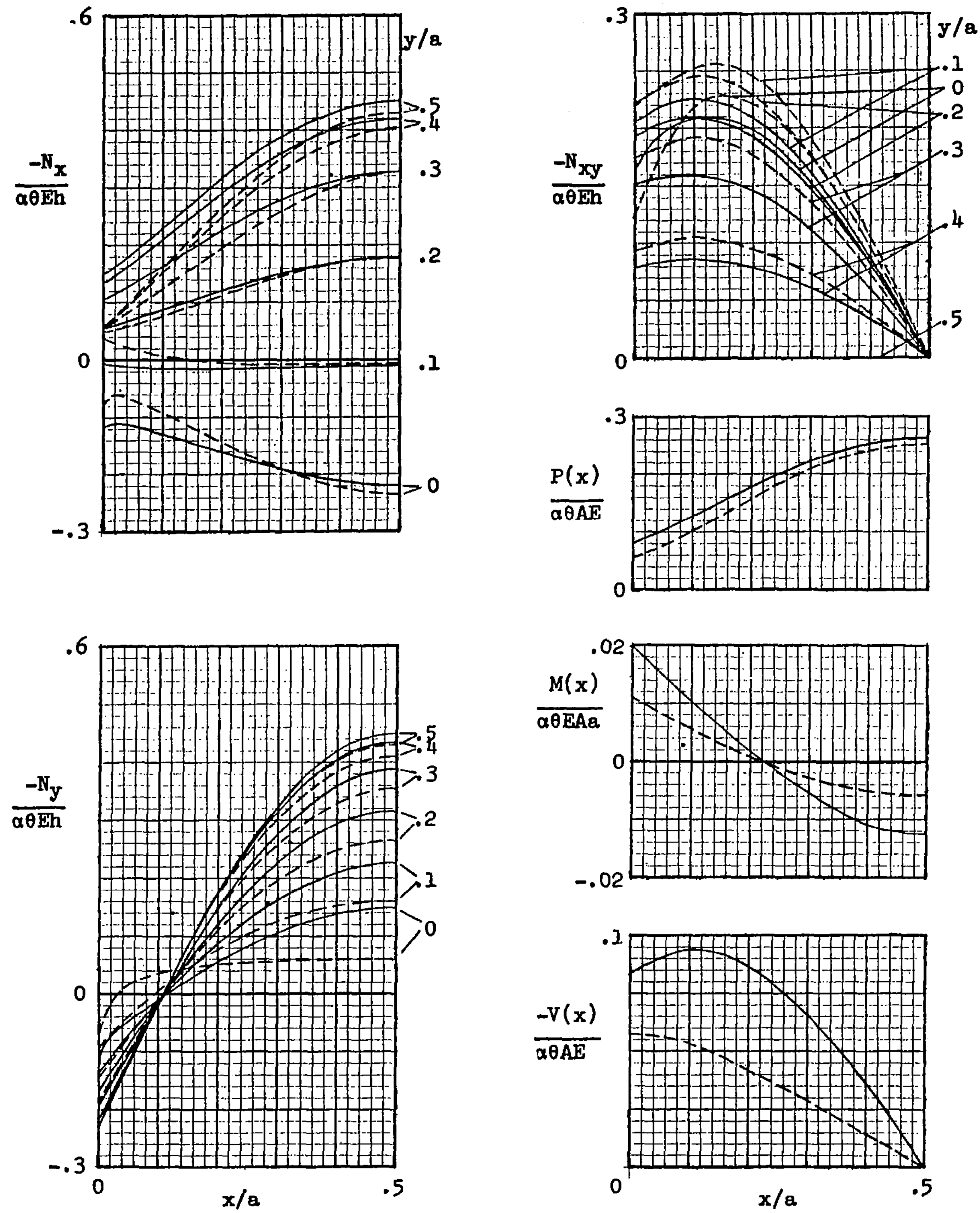


Figure 9(b).- Continued.

(ii) $ha^3/I = 0$(solid curves) and $ha^3/I = 500$(dashed curves). (problems 4 and 3)

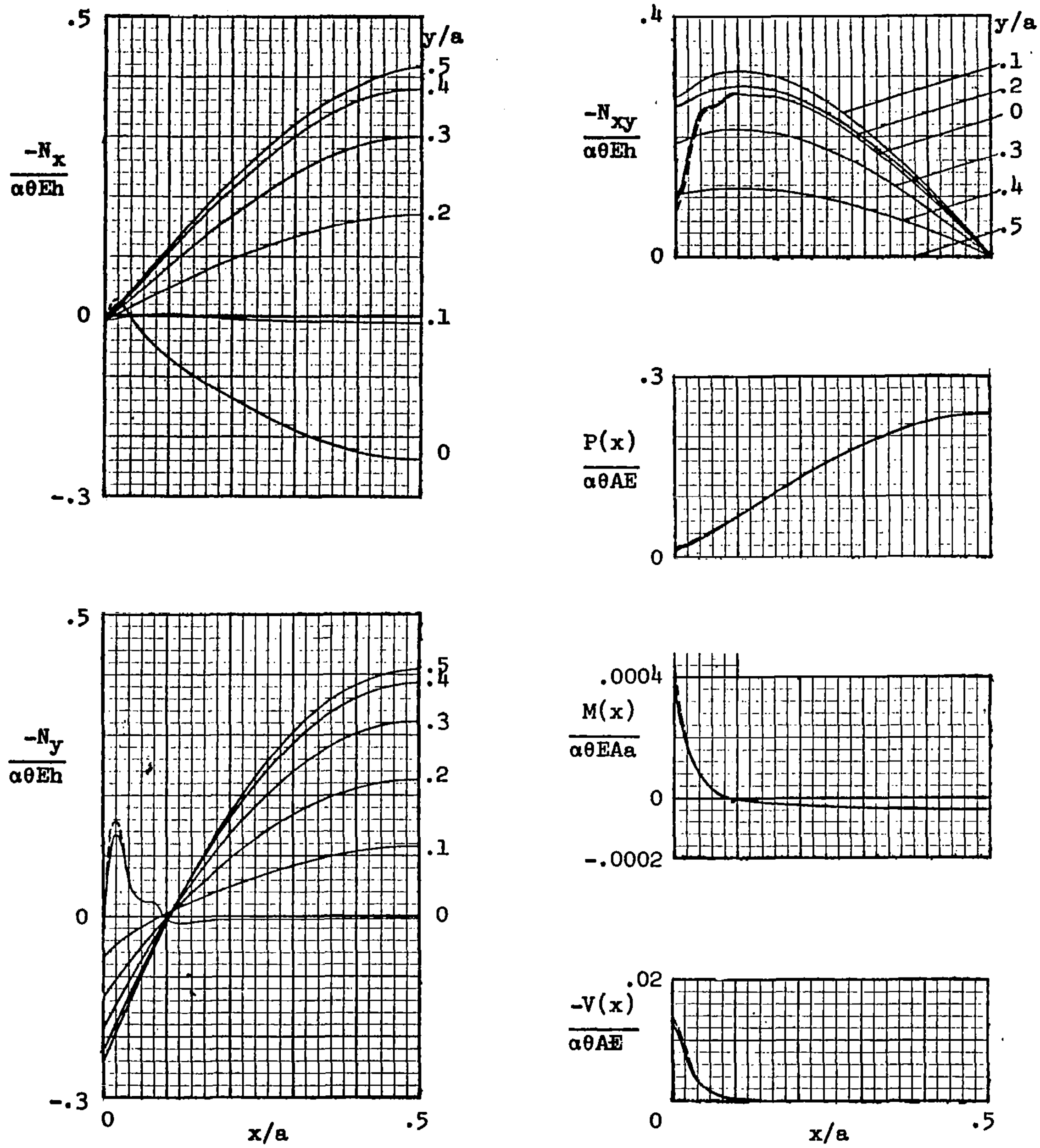


Figure 9.- Continued.

(c) Comparison of results for finite stiffener transverse shear stiffness (A/A_s = 1.0 , solid curves) and infinite stiffener transverse shear stiffness(A/A_s = 0, dashed curves); t^1/a = 0; rigid-jointed stiffeners.

(i) ha^3/I = 110,000 . (problems 1 and 5)

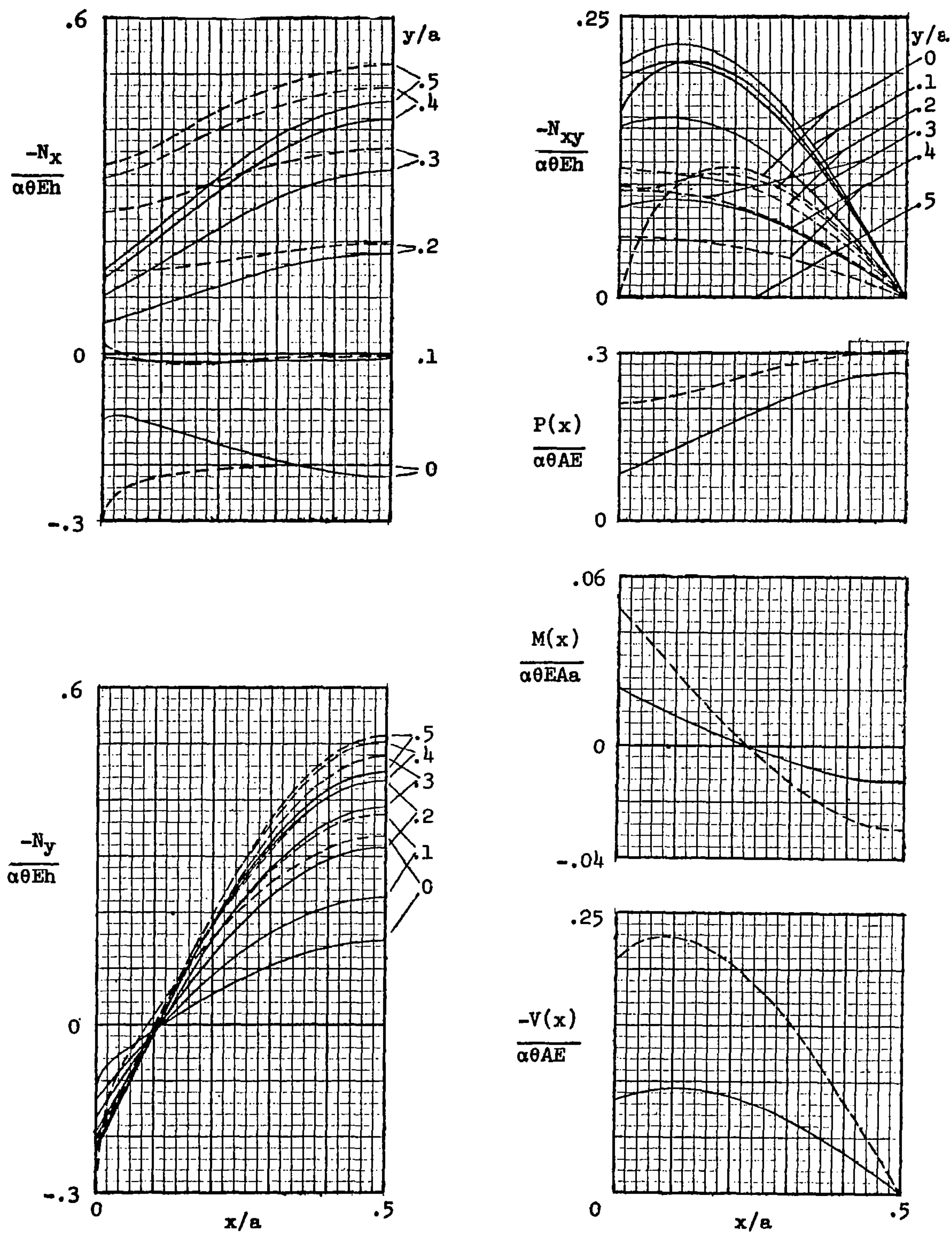


Figure 9(c).- Continued.

(ii) $ha^3/I = 0$. (problems 4 and 6)

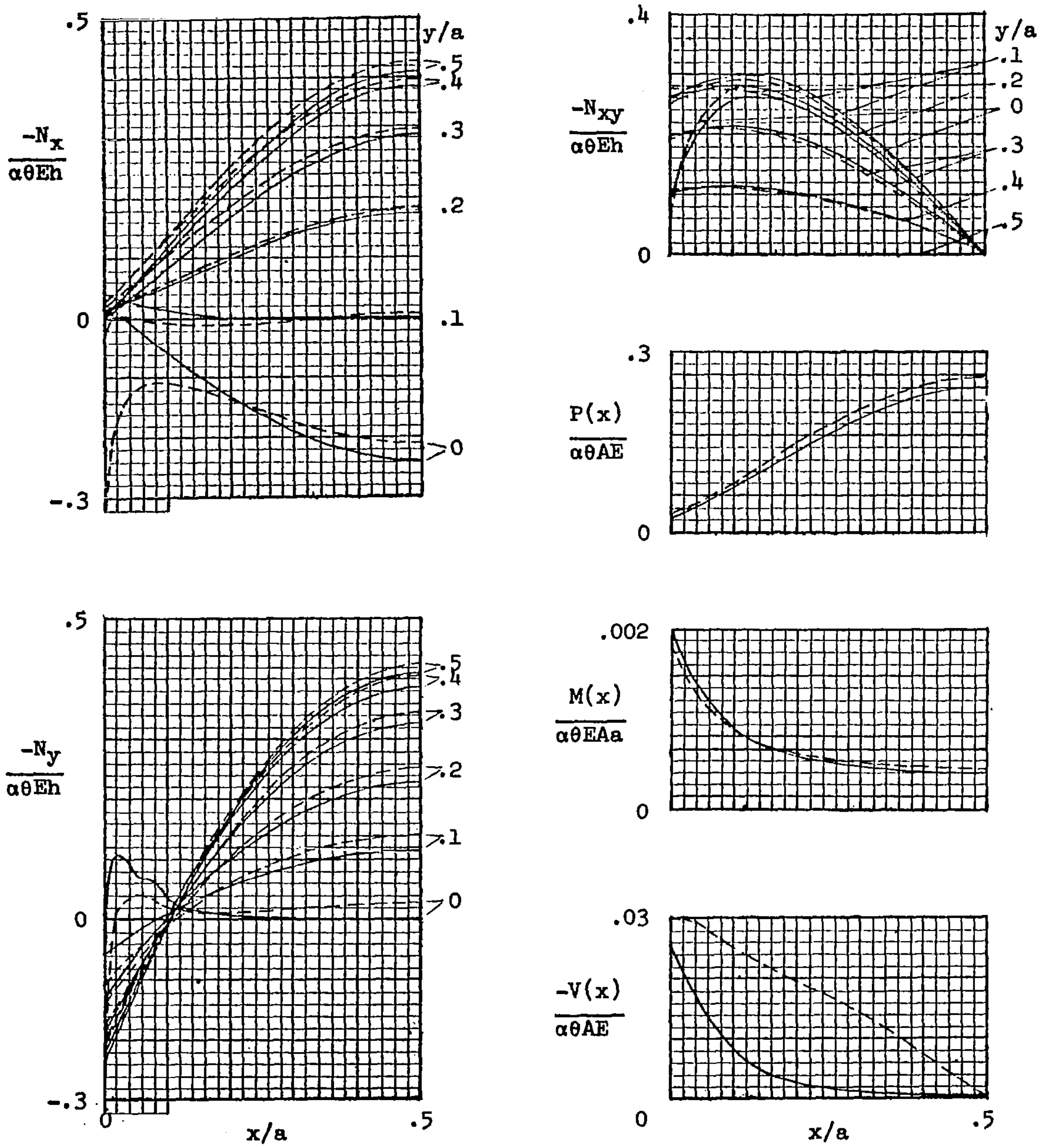


Figure 9.- Continued.

(d) Comparison of results for zero eccentricity(t^i/a = 0, solid curves) and finite eccentricity(t^i/a = .0272, dashed curves) between stiffener centroidal axes and plate edges; ha^3/I = 10,000; A/A_s = 1.0; rigid-jointed stiffeners. (problems 2 and 7)

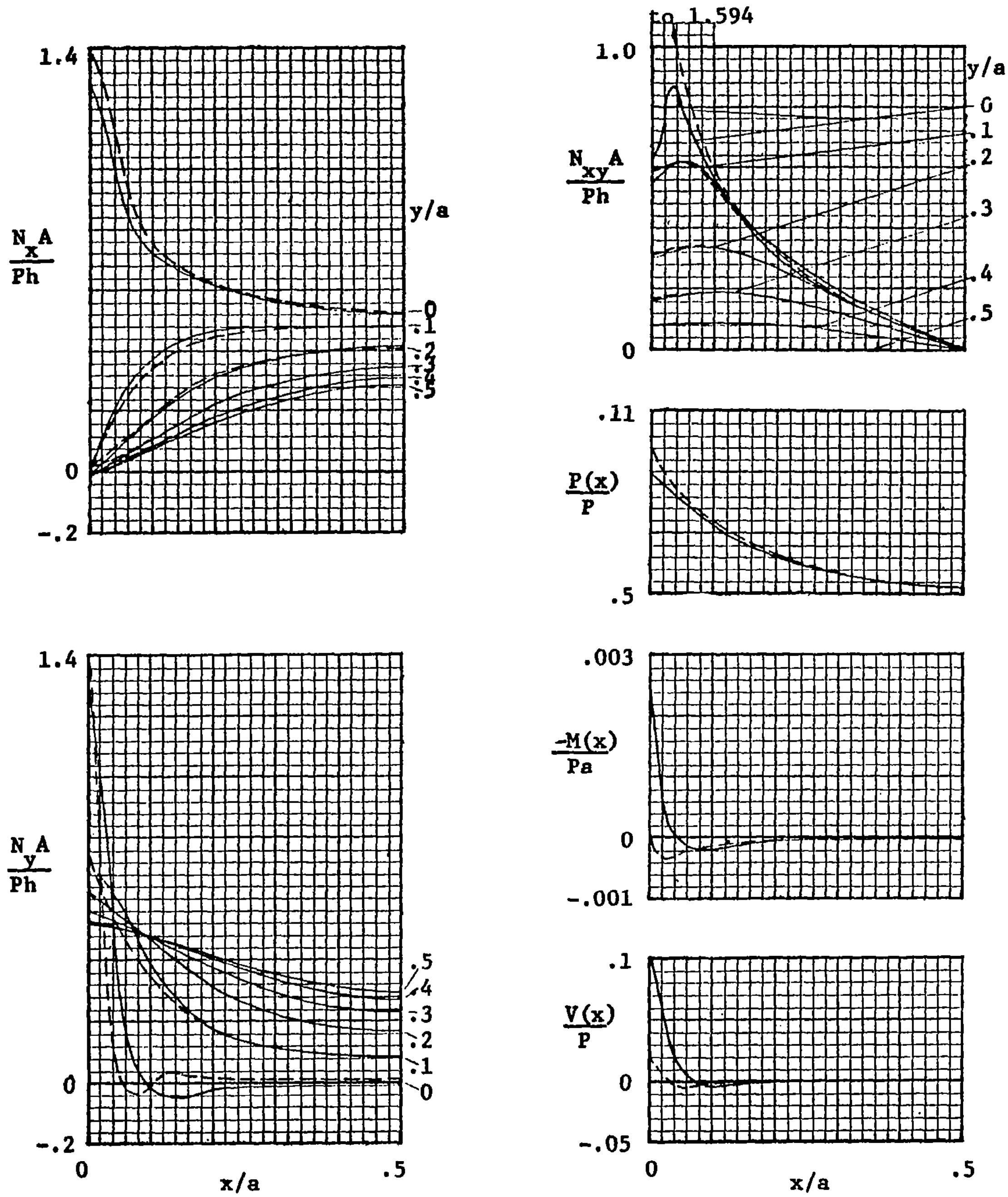


Figure 10.- Dimensionless plate stresses, stiffener tensions, bending moments and transverse shears due to stiffener end forces(shear lag problem); $\nu = 0.3$; $4ah/\pi^2 A = 1.0$.

(a) Comparison of results for rigid-jointed stiffeners(solid curves) and hinge-jointed stiffeners(dashed curves); $t^i/a = 0$; and

(i) $ha^3/I = 110{,}000$; $A/A_s = 1.0$, (problem 9 and 15)

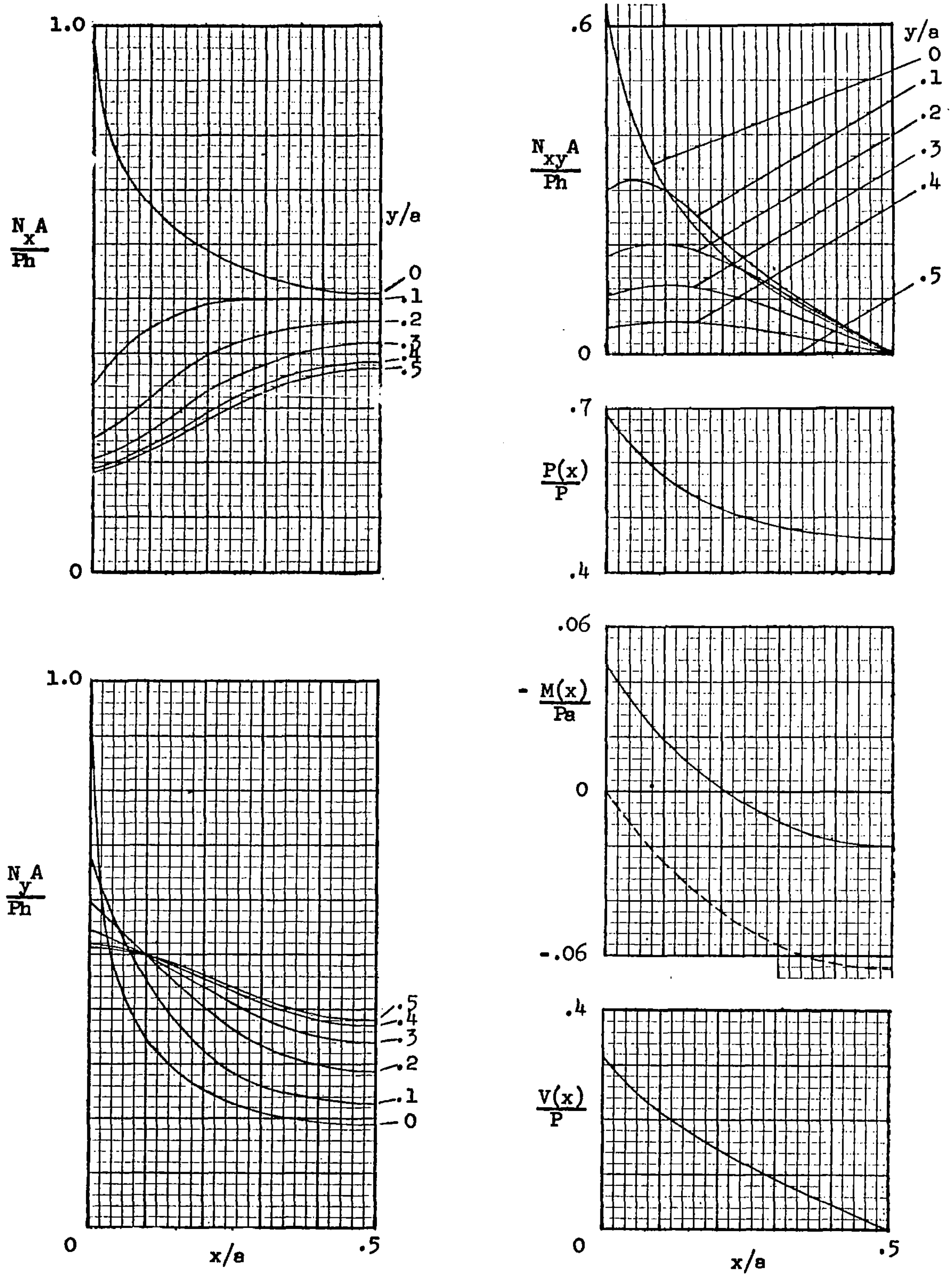


Figure 10(a).- continued.

(ii) $ha^3/I = 0$; $A/A_s = 1.0$, (problems 12 and 16)

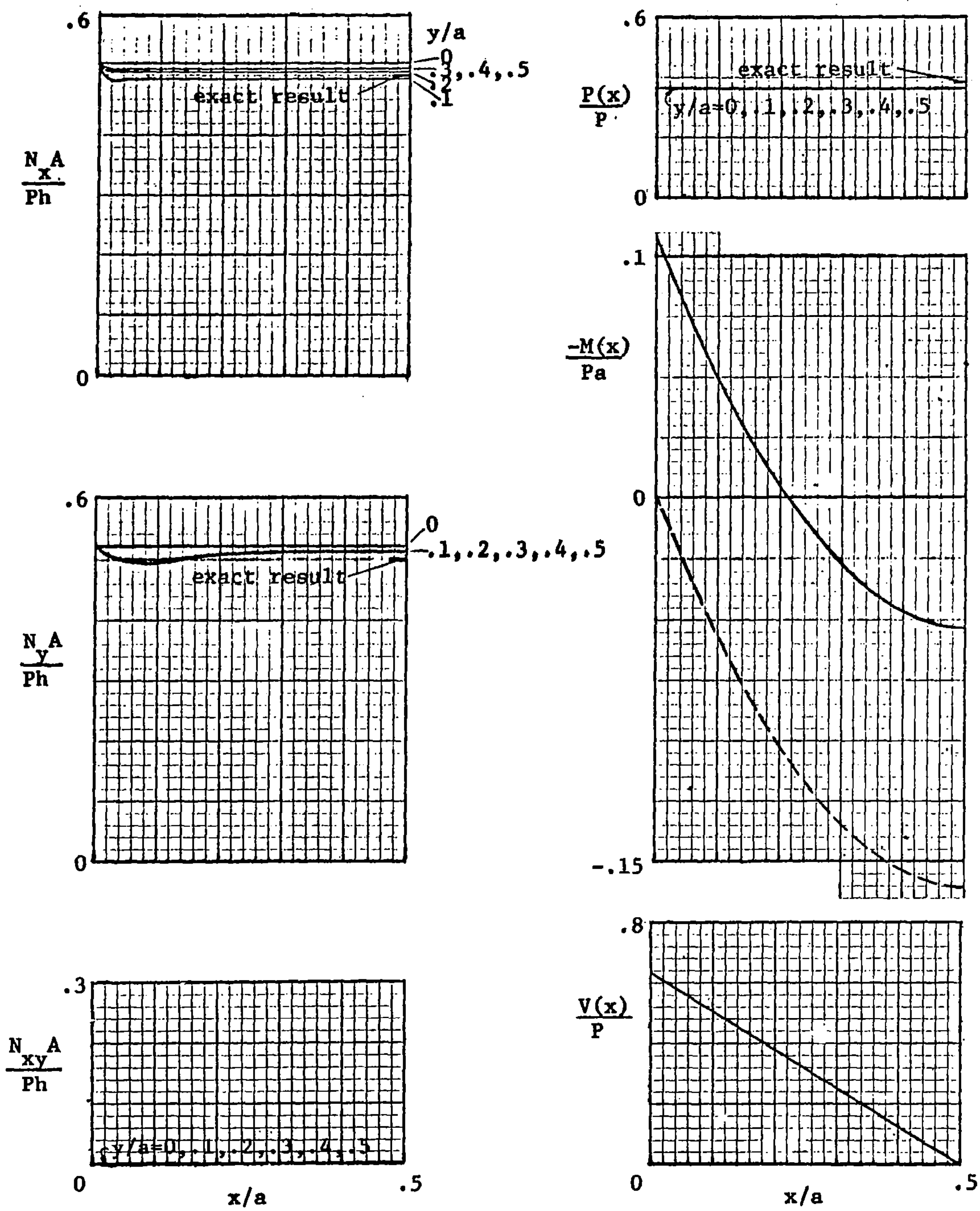


Figure 10(a).- continued.

(iii) $ha^3/I = 0$; $A/A_s = 0$. (problems 13 and 18)

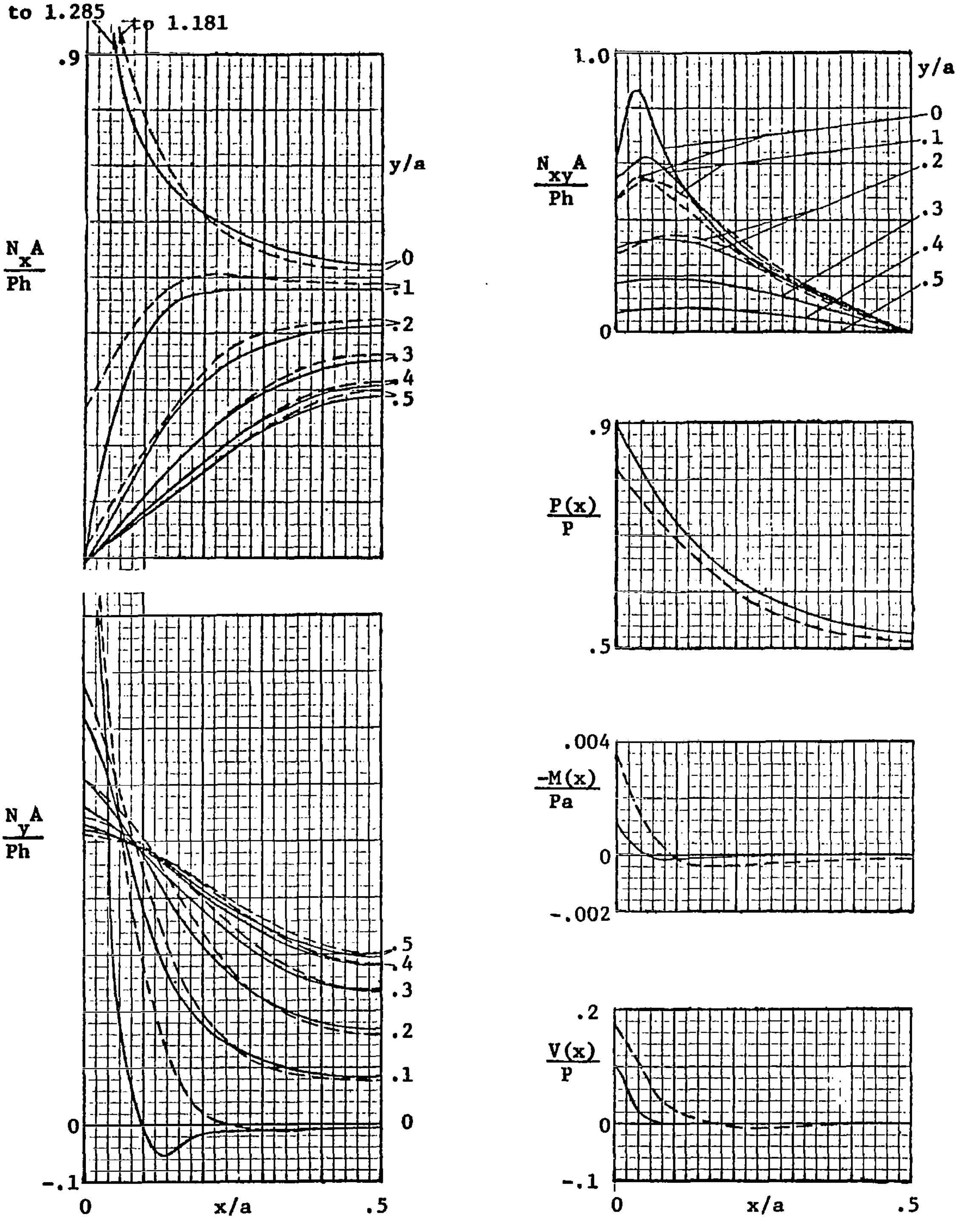


Figure 10.- continued.

(b) Comparison of results for different stiffener bending rigidities; $A/A_s = 1.0$; $t^i/a = 0$.

(i) $ha^3/I = 110{,}000$ (solid curves) and $ha^3/I = 10{,}000$ (dashed curves); rigid-jointed stiffeners. (problems 9 and 10)

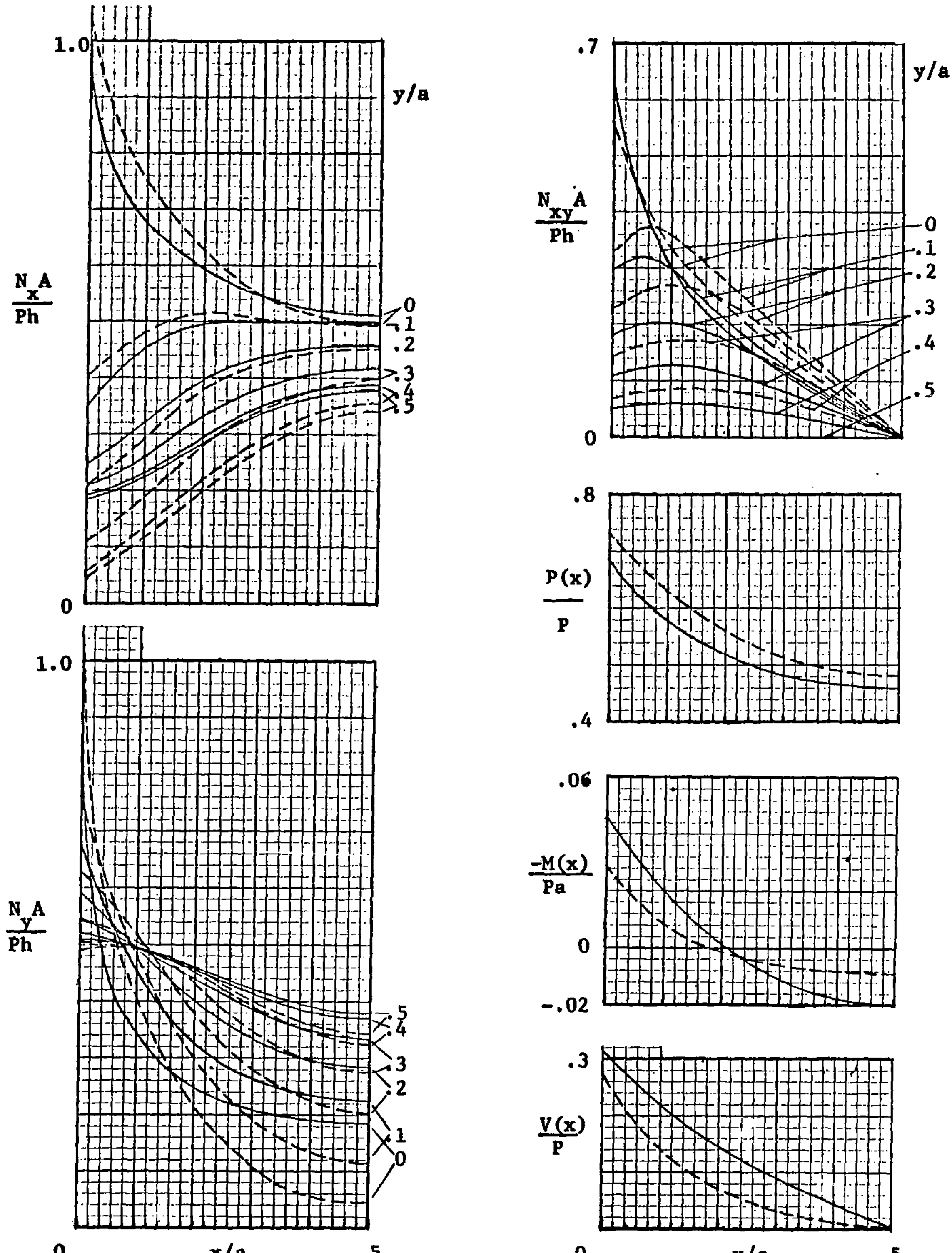


Figure 10(b).- continued.

(ii) $ha^3/I = 0$(solid curves) and $ha^3/I = 500$(dashed curves); rigid-jointed stiffeners. (problems 12 and 11)

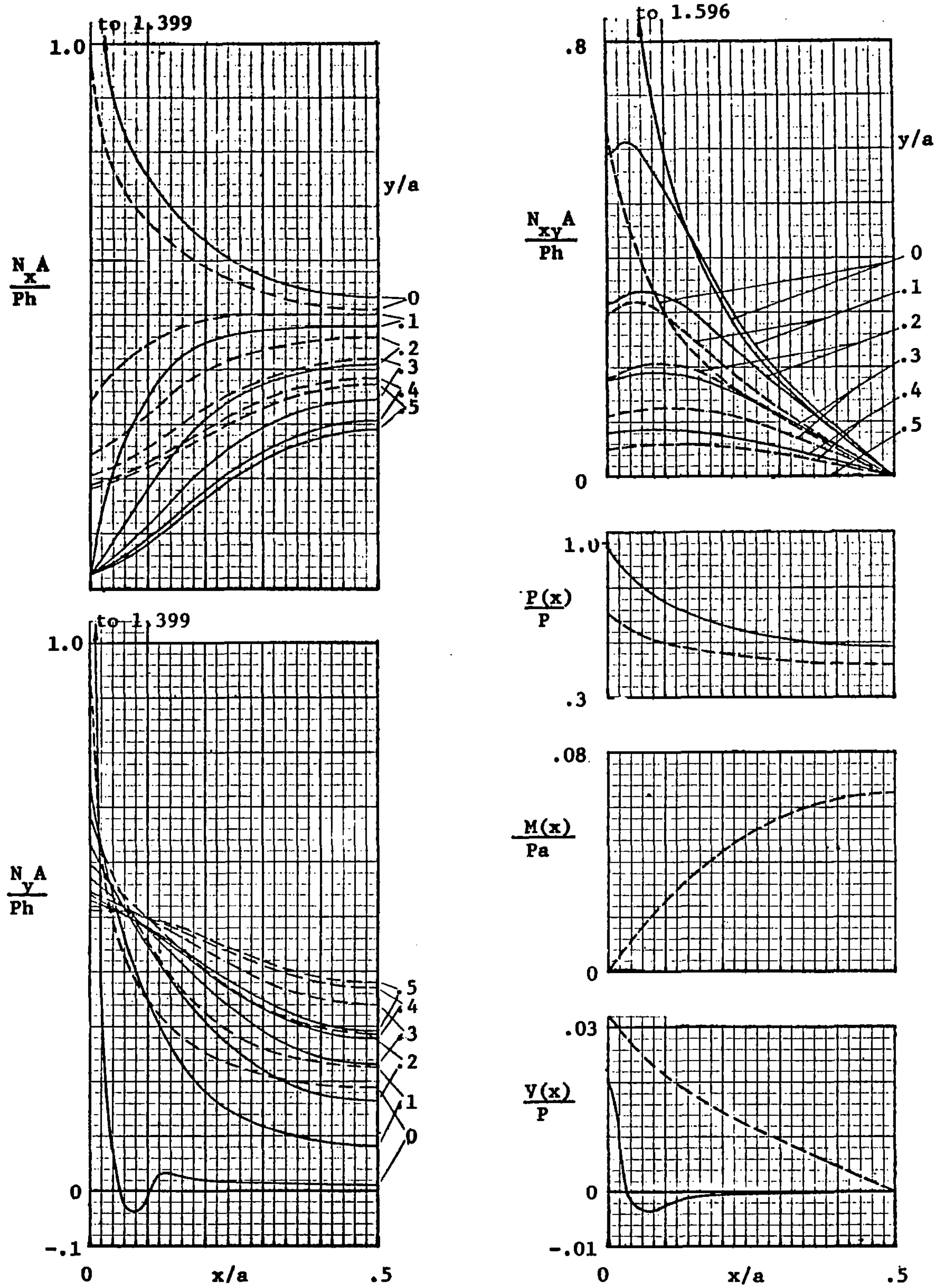


Figure 10(b).- continued.

(iii) ha^3/I = 110,000(solid curves) and ha^3/I = 0(dashed curves); hinge-jointed stiffeners. (problems 15 and 16)

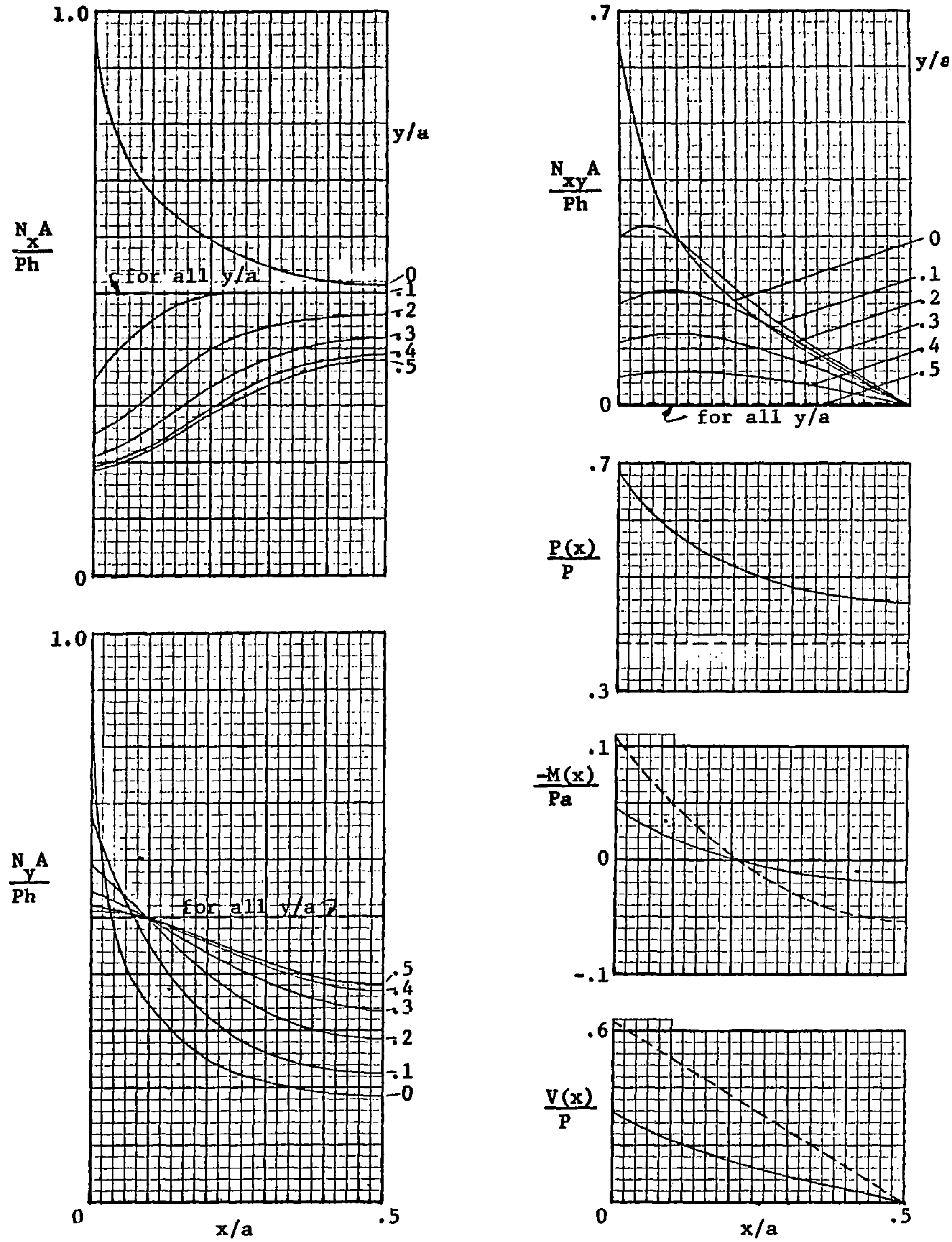


Figure 10.- continued.

(c) Comparison of results for finite stiffener transverse shear stiffness (A/A_s = 1.0, solid curves) and infinite stiffener transverse shear stiffness(A/A_s = 0, dashed curves); t^1/a = 0; and

(i) ha^3/I = 0; rigid-jointed stiffeners. (problems 12 and 13)

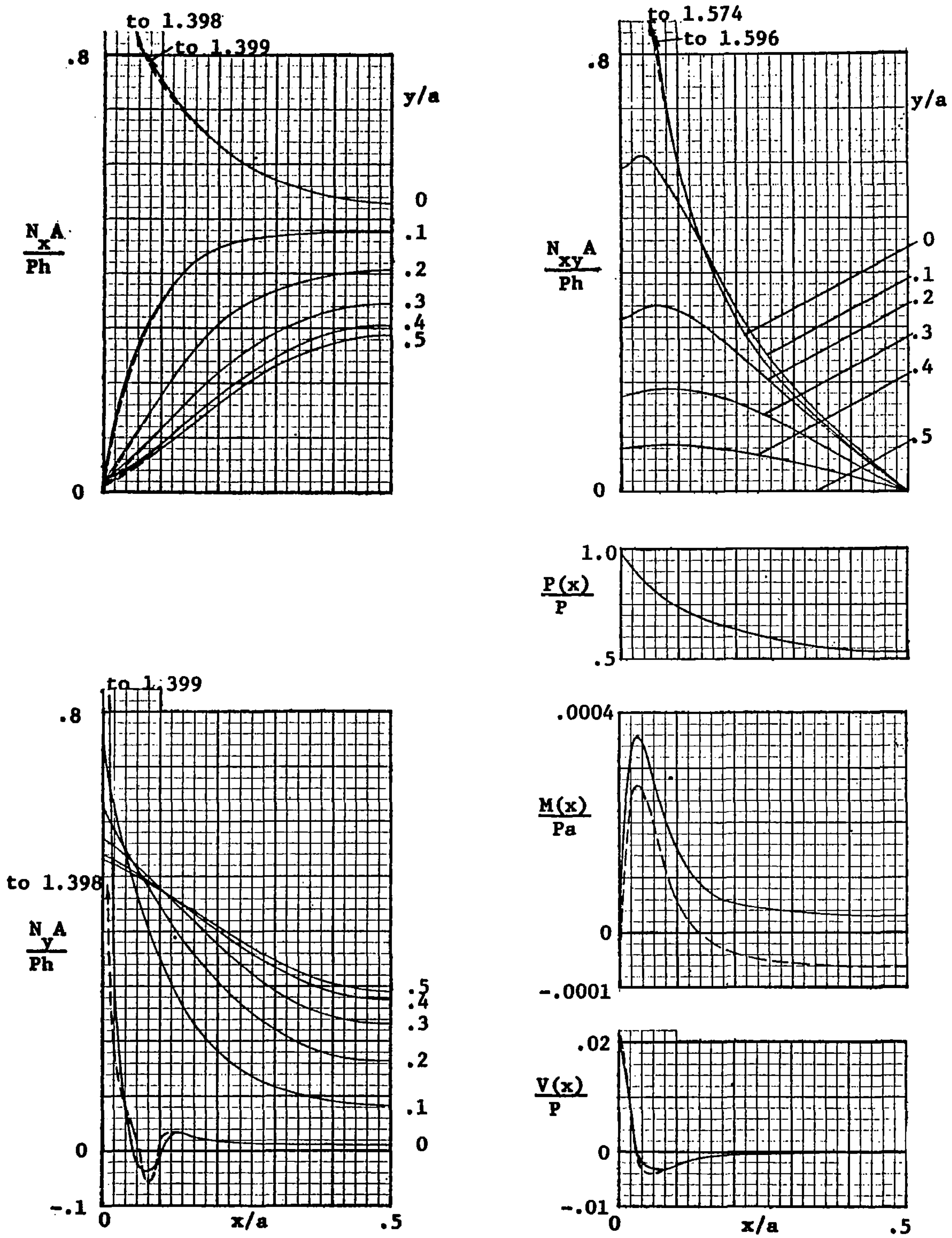


Figure 10(c).- continued.

(ii) $ha^3/I = 110{,}000$; hinge-jointed stiffeners. (problems 15 and 17)

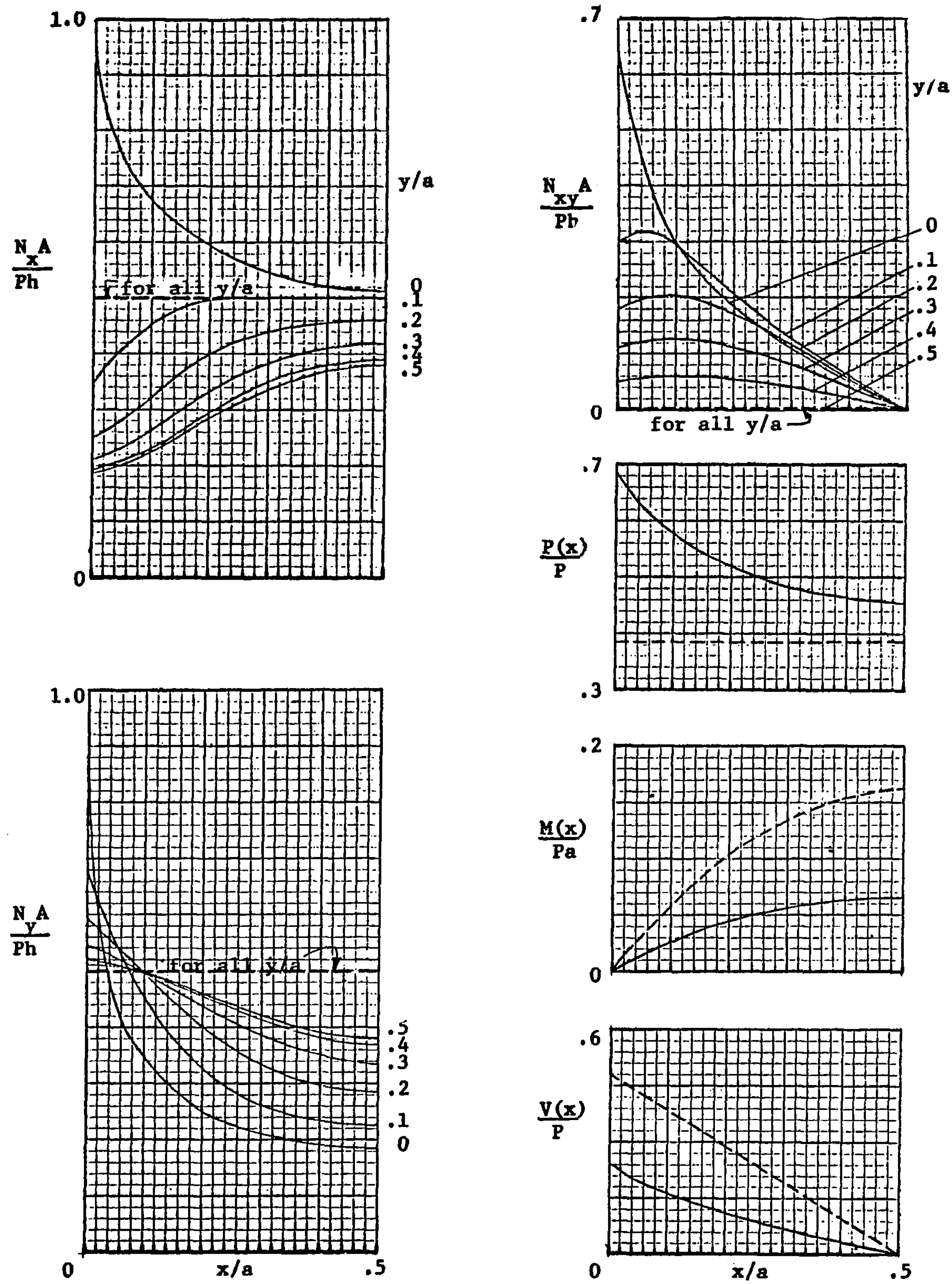


Figure 10(c).- continued.

(iii) $ha^3/I = 0$; hinge-jointed stiffeners. (problems 16 and 18)

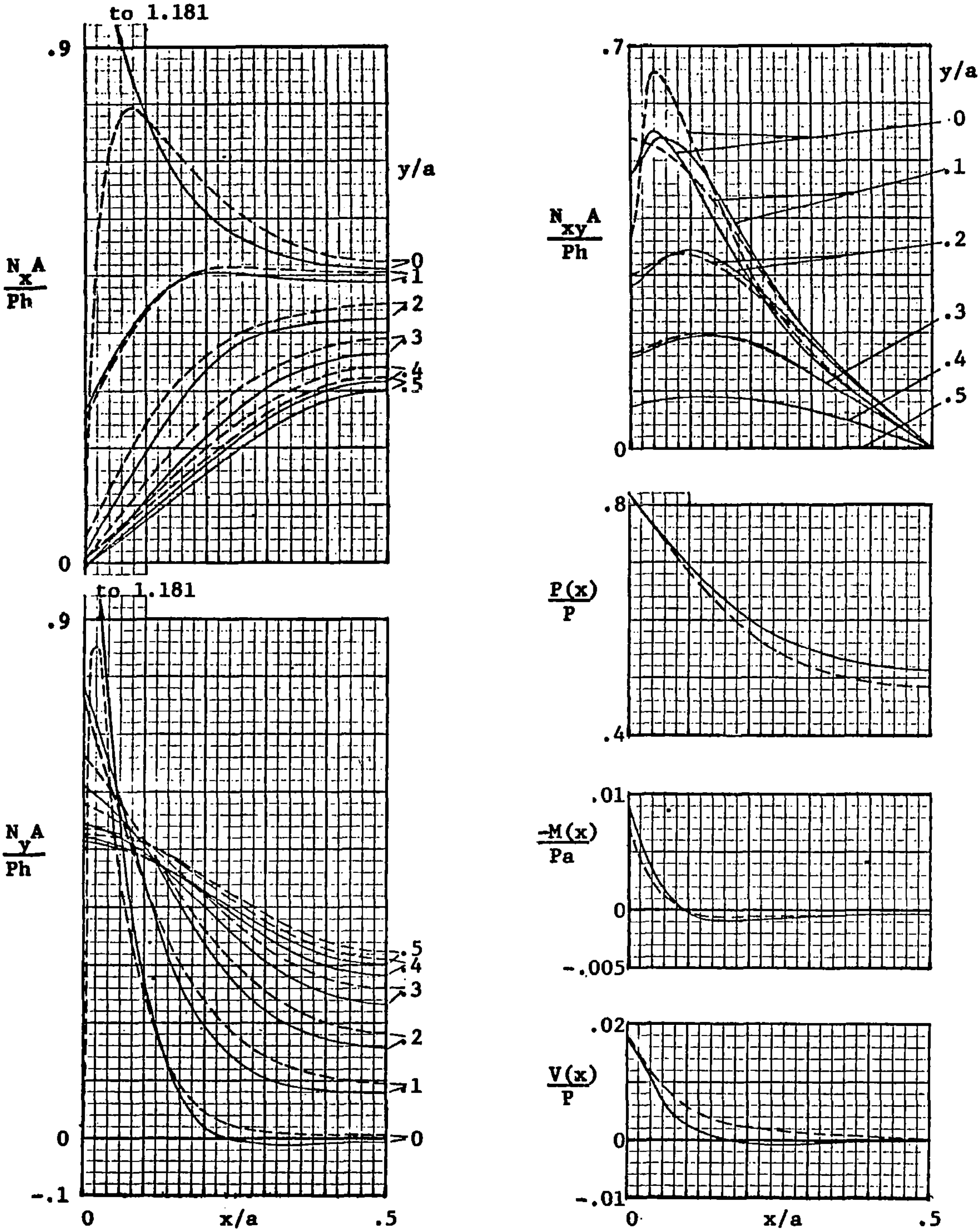


Figure 10.- continued.

(d) Comparison of results for zero eccentricity(t^1/a = 0, solid curves) and finite eccentricity(t^1/a = .0272, dashed curves) between stiffener centroidal axes and plate edges; ha^3/I = 10,000; A/A_s = 1.0; rigid jointed stiffeners. (problems 10 and 14)

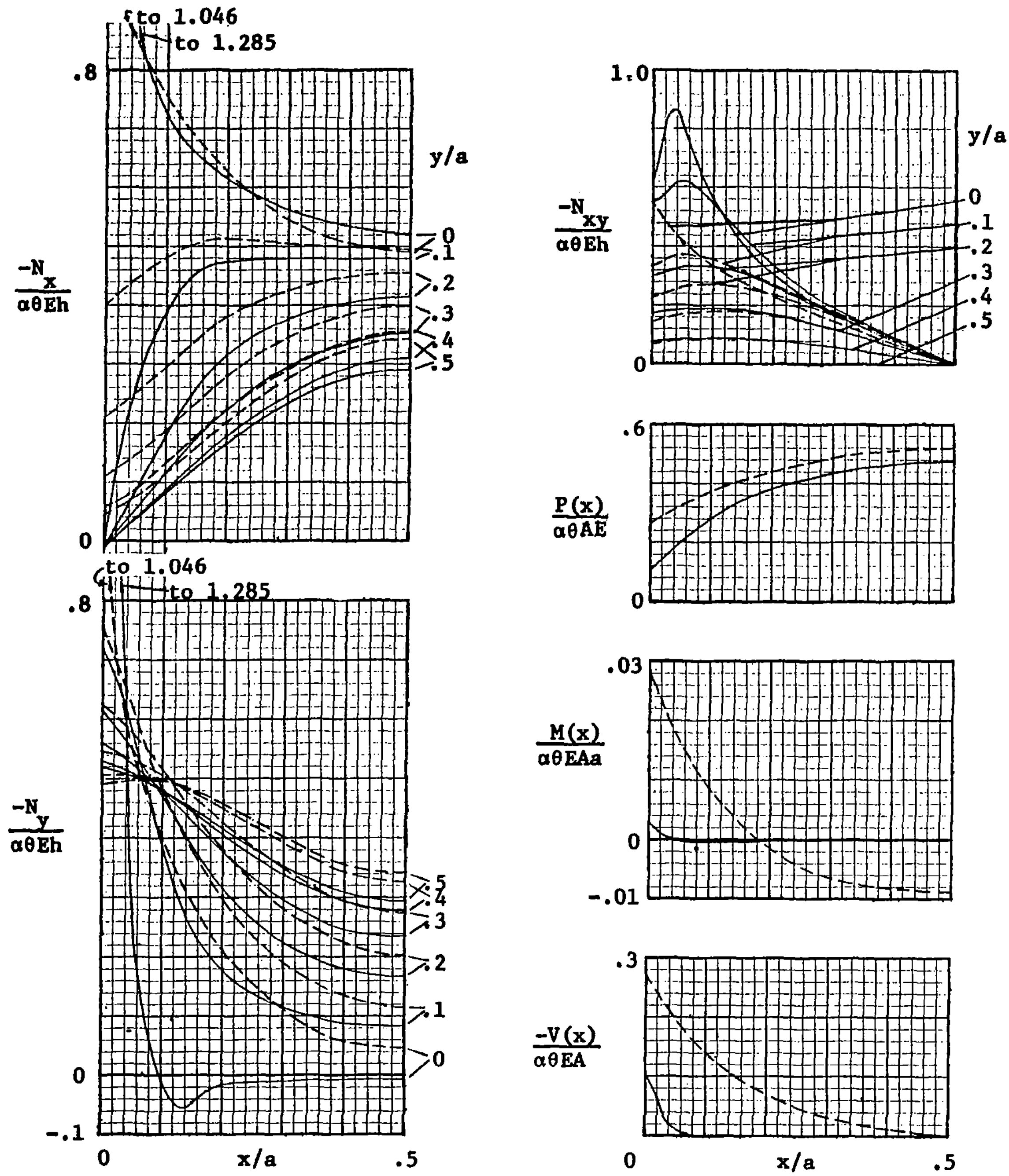


Figure 11.- Dimensionless plate stresses, stiffener tensions, bending moments and transverse shears due to discontinuous temperature distribution; $\nu = .3$; $4ah/\pi^2 A = 1.0$; $t^1/a = 0$; rigid-jointed stiffeners.

(a) Comparison of results for different stiffener bending rigidities, $A/A_s = 1.0$.

(i) $ha^3/I = 110{,}000$(solid curves) and $ha^3/I = 500$(dashed curves); (problems 19 and 20)

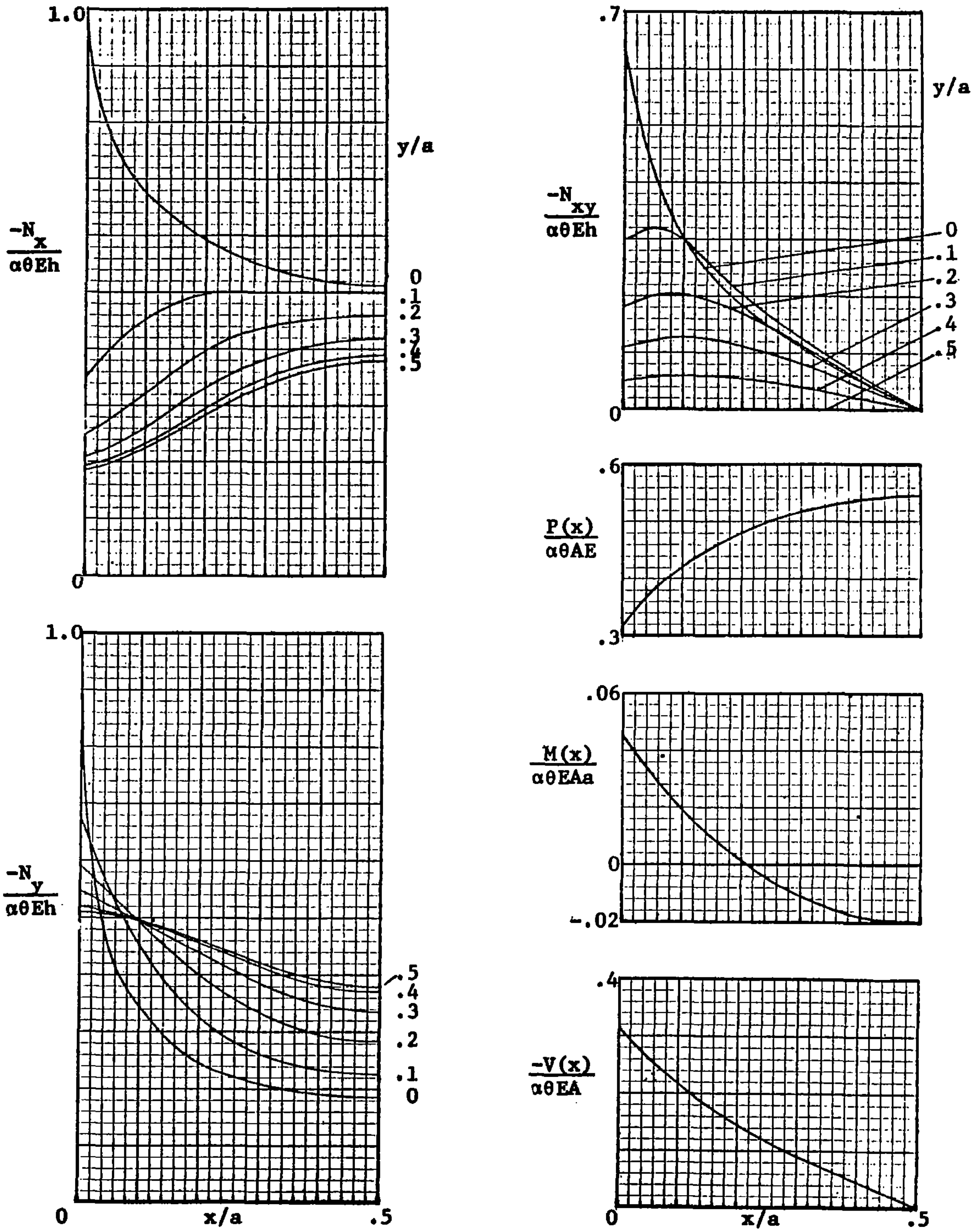


Figure 11(a).- continued.

(ii) $ha^3/I = 0$. (problem 21)

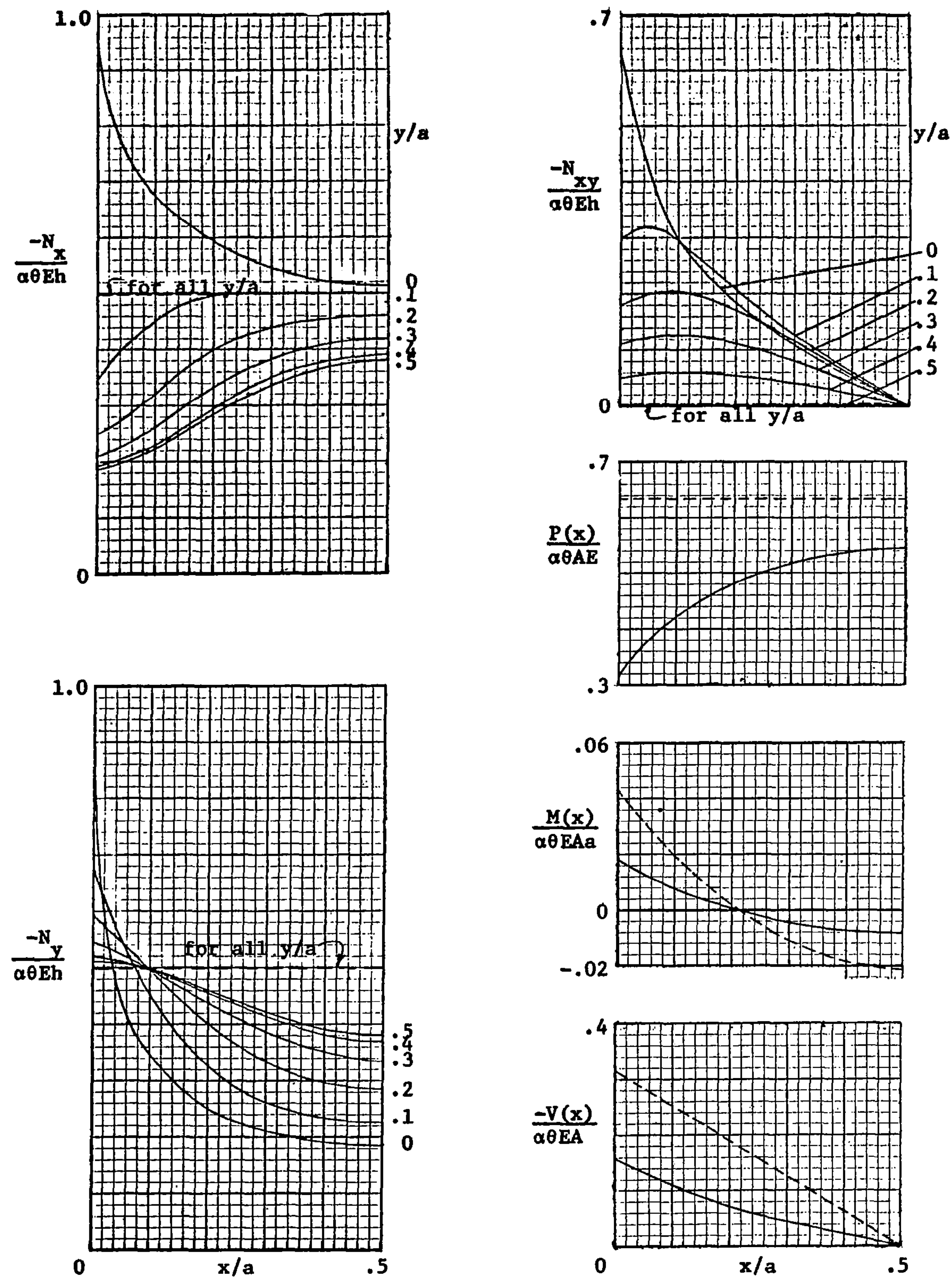


Figure 11.- continued.

(b) comparison of results for finite stiffener transverse shear stiffness(A/A_s = 1.0, solid curves) and infinite stiffener transverse shear stiffness(A/A_s = 0, dashed curves); ha^3/I = 0. (problems 21 and 22)

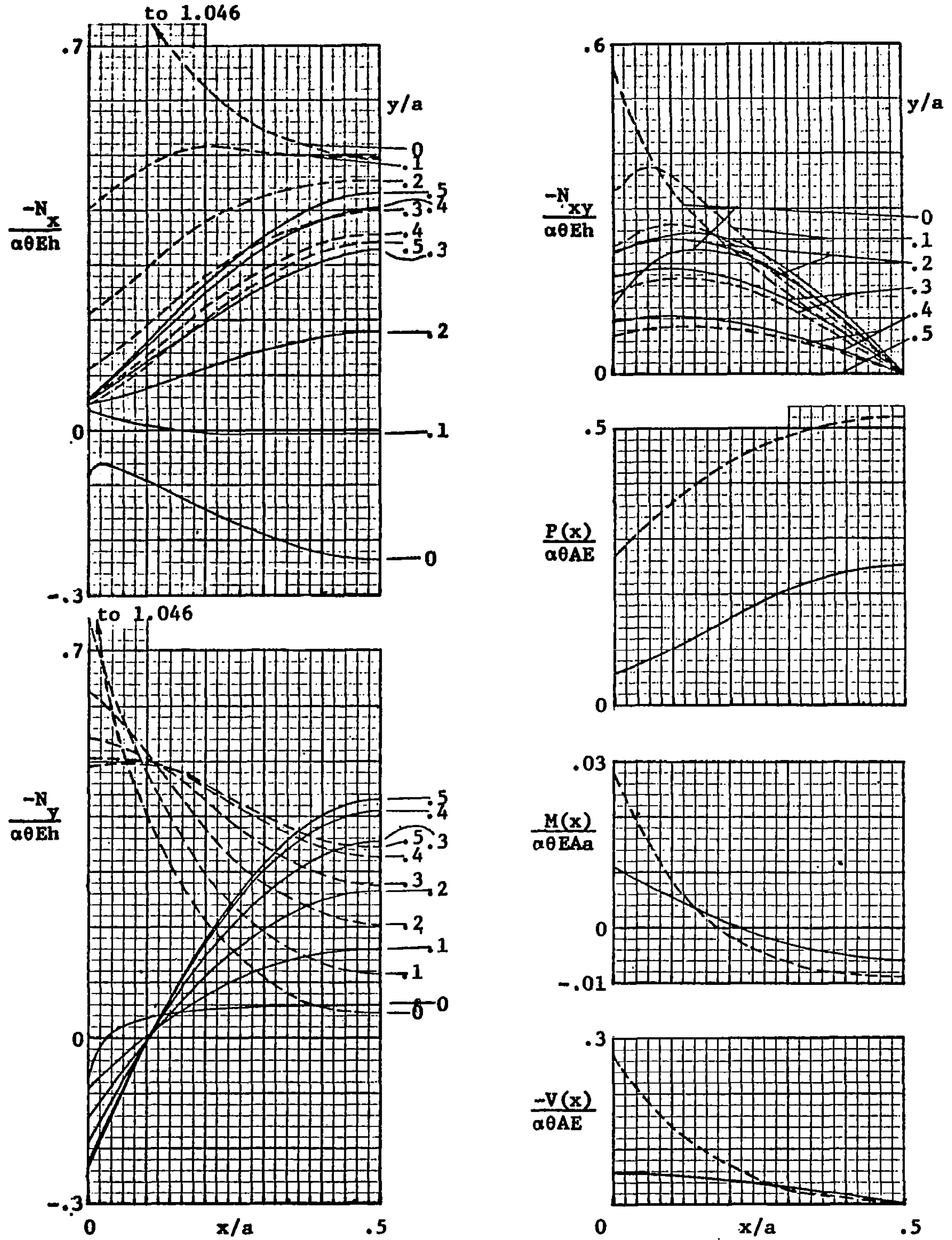


Figure 17.- Comparison of dimensionless plate stresses, stiffener tensions, bending moments and transverse shears for pillow shaped temperature distribution(solid curves) and discontinuous temperature distributio (dashed curves); $ha^3/I = 500$; $4ah/\pi^2 A = 1.0$; $A/A_s = 1.0$; $t^1/a = 0$; $\nu = .3$; rigid-jointed stiffeners. (problems 3 and 20)

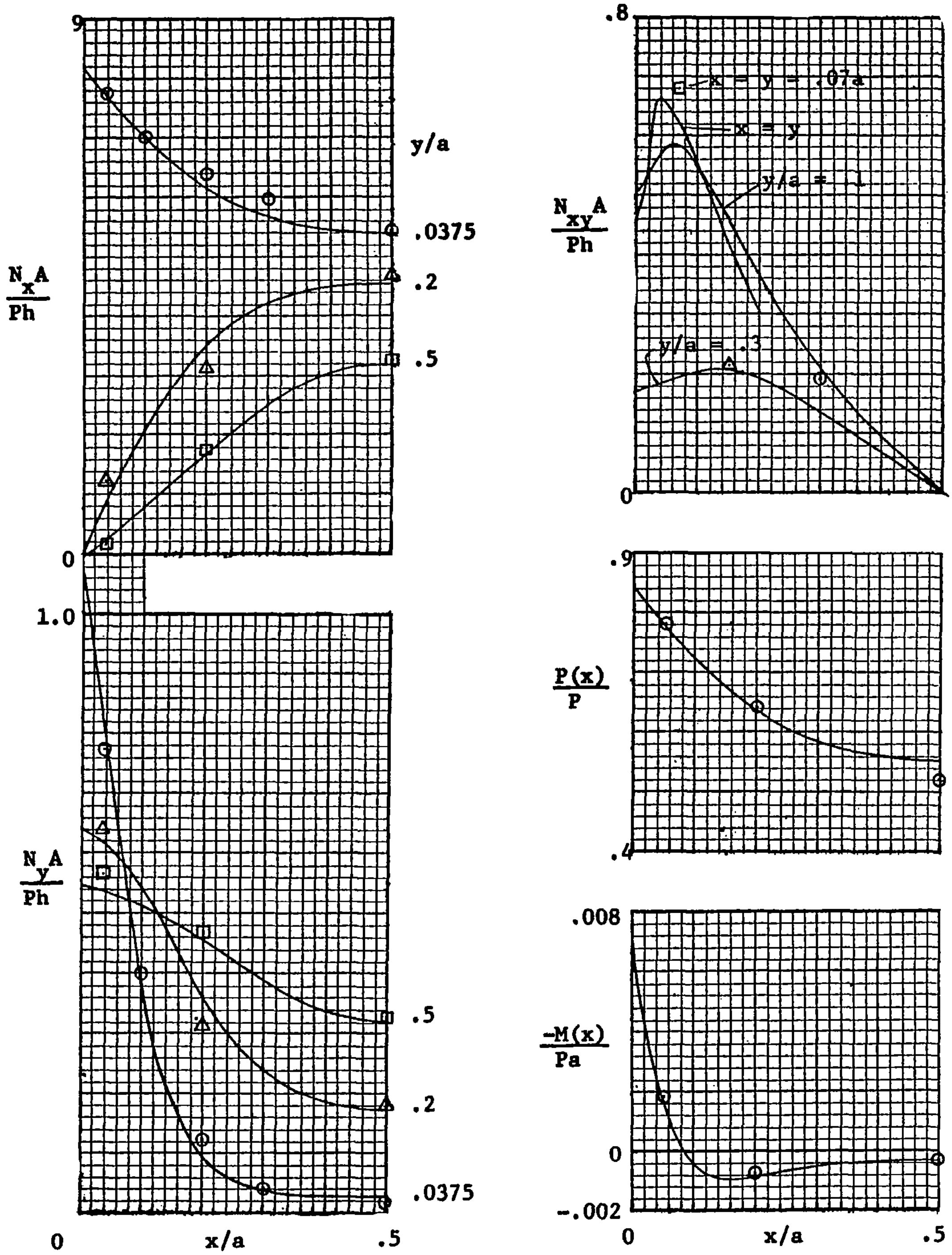


Figure 13.- Comparison of experimental results(small circles, triangles, and squares) and computed results(solid curves).

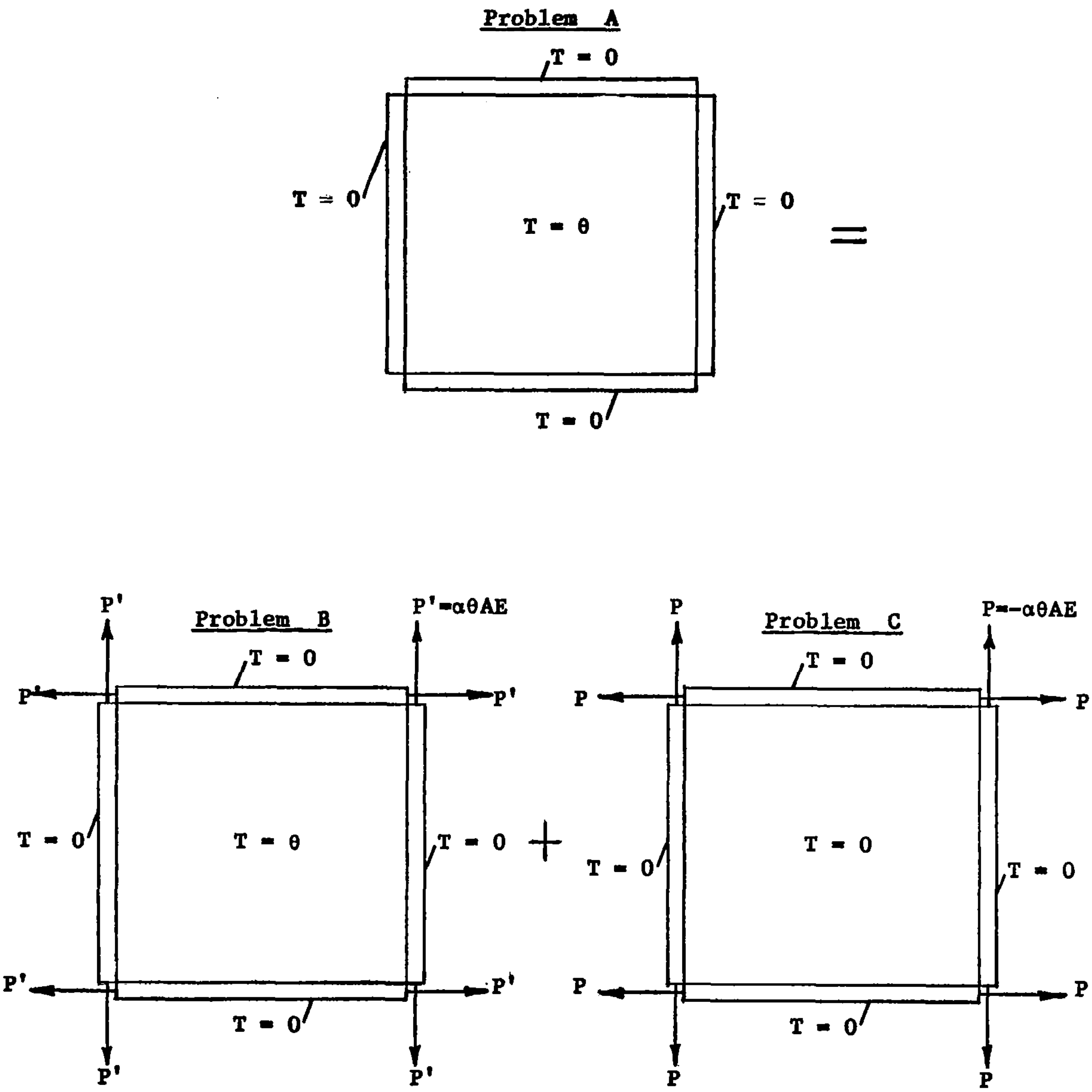


Figure 14.- Method of superposition.

REFERENCES

1. Libove, C., D. Panchal, and F. Dunn, "Plane-Stress Analysis of an Edge-Stiffened Rectangular Plate Subjected to Loads and Temperature Gradients." NASA TN D-2505, December, 1964.

2. Lin, C., "Plane-Stress Analysis of an Edge-Stiffened Rectangular Plate Subjected to Boundary Loads, Boundary Displacements, and Temperature Gradients." Master's thesis, Syracuse University, June, 1967.

NASA-Langley, 1968 —— 32 CR-978