SURFACE-GRAVITY DETERMINATIONS
FOR MAIN-SEQUENCE B STARS

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ABSTRACT

Hydrogen-line equivalent widths computed from model atmospheres have been used to obtain values of surface gravity for B stars by comparison with observed Hγ equivalent widths. The effects of stellar rotation and differing line-broadening theories on the deduced gravities are discussed. When the effects of rotation are allowed for and the Edmonds, Schlüter, and Wells semi-empirical Stark broadening theory is used, a mean surface gravity, $\log g = 4.25$, is found for B stars. This determination lies within 0.1 in the log of the values deduced from double-lined eclipsing binary systems and computed interior models for B stars.

I. INTRODUCTION

Since the suggestion by Russell and Stewart (1924) that stellar hydrogen-line broadening results primarily from the Stark effect, it has been known that the strength of the Balmer lines in B stars provides a measure of the electron pressure in these early-type atmospheres.

Owing to the almost complete ionization of hydrogen in the atmospheres of stars hotter than A0, the electron pressure is almost linearly dependent on the total pressure, which in turn is proportional to the surface gravity, g.

Therefore, it is possible to determine the surface gravity of a B star, given observed hydrogen-line contours and contours computed from realistic model stellar atmospheres. The surface gravity, combined with a determination of the effective temperature and composition, then locates the star in the H-R diagram. Accurate determinations of g from observation can therefore be of considerable importance. However, the validity of the computed hydrogen-line profiles depends both on the correctness of the line-broadening theory and on the model atmospheres.

Recently, the computed models have become increasingly realistic as various authors have calculated accurate non-gray models of early-type stars. In addition to computing traditional models with relevant continuum opacity sources, members of the Princeton and Harvard-Smithsonian stellar-atmosphere groups have, over the last few years, been able to compute accurately the effects of line-blanketing and departures from LTE, while Collins (1963, 1965, 1967) has been able to discuss quantitatively the effects of rotation on the stellar continuum and lines.

Coincidentally, the progress over the last decade on the difficult problem of describing hydrogen-line broadening has also been significant. Several investigators have attempted to make use of the model-atmosphere and line-broadening theory to predict emergent line profiles. Specifically, we mention the work of Cayrel and Traving (1960) on the solar hydrogen lines; Searle and Oke (1962), and Underhill (1962) on stellar hydrogen lines; and, most recently and most accurately, Stienon (1964a) and Mihalas (1964) on the stellar hydrogen lines.

The gravities determined for main-sequence B stars can be compared with the gravities deduced from studies of double-lined eclipsing binary systems and with those computed from available stellar interior models in this mass range. Stienon (1964b) has found that his computations yield surface gravities of the order of log g = 3.8 for representative main-sequence

stars. This is smaller by about 0.3 to 0.4 than the main-sequence surface gravities expected from interiors and eclipsing binaries. Similarly, a comparison with eclipsing binary data (Olson, private communication) indicates discrepancies of the same order when equivalent widths of Mihalas (1964) are used.

Finally, it should be stated that recently several investigators, including the present authors and K. Swamy (private communication), have found it impossible to reproduce the hydrogen-line equivalent widths calculated by Mihalas to better than 10 to 15 per cent. The source of this discrepancy is not known, but we feel encouraged by the agreement of our computation both with other investigators' and with observations.

In an attempt to understand and reduce these large discrepancies, we shall rediscuss the problem of the surface-gravity determinations in view of the most recent advances in both the model-atmosphere computations and the theory of hydrogen-line broadening.

In our determinations of g, we make use of the observed relations between $H\gamma$ equivalent width, $W(H\gamma)$, and spectral type and $W(H\gamma)$ and unreddened (U-B) color, $(U-B)_0$. Model atmospheres provide a calibration of $(U-B)_0$ and spectral type in terms of the effective temperature T_{eff}. This calibration allows us to compare directly the observed $W(H\gamma)$ relations to a theoretical relation between $W(H\gamma)$ and T_{eff} as computed from the models for a grid of g values and thus choose a value for g. The uncertainties in this procedure arise from the following sources: (1) the broadening theory for hydrogen lines; (2) the model calibration of the T_{eff} - $(U-B)_0$ and the T_{eff} - spectral-type relations; and (3) the effects of stellar rotation on the line strengths, colors, and spectral types. In the next three sections, we discuss each of these difficulties.

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II. RECENT ADVANCES IN HYDROGEN-LINE BROADENING

Recently, important advances have been made in the theory of Stark broadening. Originally, it was assumed that the only particles contributing to the broadening were the heavier, and hence lower velocity, positively charged ions surrounding the absorbing atom. However, Margenau and Lewis (1959) pointed out that for sufficiently strong interactions (i.e., sufficiently large distances from the line center) the electrons must also be considered quasi-static broadeners.

Griem (1964, 1967) has estimated that the number of quasi-static perturbers may be represented as:

$$N_s = N_e \left(1 + \frac{2}{\sqrt{\pi}} \int_0^{Y_1} y^{\frac{1}{2}} e^{-y} \, dy \right) \quad , \tag{1}$$

where we have assumed that all electrons are from singly ionized atoms, and $Y_1 = |\Delta\lambda| / \Delta\lambda_G$, where $\Delta\lambda_G$ is an estimated cutoff parameter. The electrons are 50 per cent quasi-static at about $\Delta\lambda \sim 1.15\, \Delta\lambda_G$.

However, discrepancies between this theory and recent laboratory measurements have lead Edmonds, Schlüter, and Wells (1967) (hereinafter referred to as ESW) to suggest a different cutoff. They choose a form:

$$N_s = N_e \left(1.5 + 0.5\, \frac{Z-1}{Z+1} \right) \quad , \qquad Z = \frac{7\Delta\lambda}{\Delta\lambda_{ESW}} \tag{2}$$

where

$$\Delta\lambda_{ESW} \approx \frac{1}{2}\, \Delta\lambda_G \quad . \tag{3}$$

The electrons are 50 per cent quasi-static at $Z = 1$ or $\Delta\lambda \sim 14\,\Delta\lambda_G$. Since $\Delta\lambda_G$ depends inversely on the square of the upper quantum number, both theories predict that essentially all electrons are quasi-static in the formation of the wings of the Balmer lines above $H\epsilon$.

Unfortunately, there are significant differences between the emergent contours predicted from the two theories for $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$. Since we are not in a position to make a choice between these two theories, we will present results from both.

III. THE MODEL ATMOSPHERES AND THE CHOICE OF A SCALE OF EFFECTIVE TEMPERATURES

The first step in which the model atmospheres enter is in the computation of the theoretical relation between $W(H\gamma)$ and T_{eff} for a grid of g values.

Recent work by the Princeton group (Morton and Adams 1968; Hickock and Morton 1968; Mihalas and Morton 1965; and Mihalas 1966) has improved on earlier computations (Mihalas 1964; Strom and Avrett 1965), in that these authors have taken into account the effects on B-star models of blanketing by strong lines in the ultraviolet. The overall effect of blanketing on the optical-depth regions in which the continuum is formed is basically to raise the temperature, since at continuum $\tau \gtrsim 0.3$, the lines are optically thick and no radiation can escape at the line frequencies. In order to preserve flux constancy, the source function and thus the temperature must be increased at these depths. This "backwarming effect" results in a model structure that at $\tau \gtrsim 0.3$ closely simulates the behavior of an unblanketed model of higher effective temperature. As a result, computations of the Princeton group show that the emergent fluxes longward of 3000 $\overset{\bullet}{A}$ for the blanketed models of effective temperature $T - \Delta T$ are matched by an unblanketed model of higher effective temperature T. In Figure 1 we plot against T the values of ΔT deduced by these authors. Deviations of individual points from our smoothed curve are to greater than 300 °K. Since only a few blanketed models have been published, we shall use this graph to relate values of T_{eff} for unblanketed models to the true values of T_{eff}. This will allow us to make use of the much more extensive grid available for

unblanketed models in order to compute the Balmer line profiles and equiva-
lent widths. Of course, we realize that for $\tau \lesssim 0.3$, this procedure will
result in our choosing incorrect values for the temperature and pressure.
However, since the main contribution to the hydrogen-line equivalent widths
will come from the wings, which are basically formed at $\tau \gtrsim 0.3$, the errors
introduced by our approximation should not be severe.

The next stage of our analysis in which the models enter is in the cali-
bration of $(U-B)_0$ and spectral type in terms of T_{eff}. Morton and Adams
(1968) have recently published such calibrations. Their procedure for assign-
ing a value of T_{eff} to stars earlier than B4 is briefly as follows: (1) the
observed Balmer jumps, D, for stars of known MK spectral type as obtained
by Chalonge and Divan (1952) and the calibration of the spectral type - (U-V)
relation (Johnson 1963) determine a relation between D and (U-V); (2) the
theoretical D - T_{eff} relation as determined from the blanketed model atmos-
pheres relates T_{eff} and (U-V); and (3) the T_{eff} - (U-V) and spectral-type -
(U-V) relations allow the calibration of spectral type in terms of T_{eff}. For
stars later than B8, they adopt the calibration of Mihalas (1966) which is
based on a comparison of observed and model-predicted (B-V) colors. For
stars between spectral types B4 and B8, they plot $1/T_{eff}$ against (U-V) and
read off values of T_{eff} at the (U-V) values corresponding to the intermediate
types.

We believe that the models used by Morton and Adams (1968) in determin-
ing the T_{eff} scale are the most realistic available at present. However, two
difficulties arise in their procedure for determining stellar effective temper-
atures: (1) the absolute calibration of the observed Balmer jumps, and (2)
the effects of rotation on the spectral-type - (U-V), spectral-type - (B-V),
and D - (U-V) relations. These latter effects will be discussed in the following
section.

The scale of D as measured by Chalonge and Divan (1952) gives a value of D for Vega of 0.516. More recent work by Bahner (1963) suggests that D be increased to 0.54. D. Hayes of UCLA (private communication) believes 0.56 to be the most accurate choice for D on the basis of his recent recalibration of Vega. At earlier types, the Chalonge and Divan (1952) scale might be expected to yield an accurate value of D independent of any absolute calibration problems. However, the D determined by these authors for ϵ Ori (B0 Ia) shows the same disparity with Hayes' suggested scale as does Vega. We used Oke's (1964) scanner observations corrected to match the new absolute scale of Hayes in order to obtain this result. However, we note that Bahner's (1963) observations when placed on Hayes' scale would come close to matching the value of D measured by Chalonge. This confusing situation clearly requires further observational study. As an upper estimate to the errors in the value of D we have corrected the scale of Morton and Adams (1968) to account for the change of D from 0.516 to 0.56. We must therefore decrease their values of T_{eff} by the amounts listed in Table 1 (Column 3). The full correction arising from the change in D was applied for stars earlier than B4. For stars later than B8, since the Morton-Adams scale is based on (B-V) observations, this correction was set to 0. For intermediate types, a linear interpolation was used to establish the correction.

IV. THE EFFECTS OF STELLAR ROTATION ON THE OBSERVED LINE STRENGTHS, COLORS, AND SPECTRAL TYPES

Since the mean rotational velocity for B stars is approximately 200 km/sec, or about 1/3 the breakup velocity, the effects of rotation on the structure of these atmospheres are not insignificant on the high velocity side of the mean. Collins and his collaborators (Collins 1963, 1965, 1967; Collins and Harrington 1966) have examined the effects of rotation on B-star atmospheres. Qualitatively, we expect that for a rotating star the surface gravity will be a function of position on the star, having its highest and lowest values of the poles and equator, respectively. From an application of Von Zeipel's theorem, we expect the radiative flux, which is proportional to T^4, to be a direct

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function of the local surface gravity. Hence, the temperatures in the atmosphere of a rotating star are highest near the pole (and very close to the value of T_{eff} for a non-rotating star) and lowest at the equator.

The effective surface gravity in the case of a rotating star viewed pole-on is almost unchanged from that of a non-rotating star having the same mass. However, the effective gravity of the same rotating star viewed equator-on is reduced compared to a similar non-rotator. In both cases, the mean T_{eff} is reduced compared to that of a non-rotator, the lowering of T_{eff} being more pronounced when the star is viewed equator-on.

The scale of effective temperatures deduced by Morton and Adams (1968) refers to a mean value of T_{eff} at each spectral type. This mean value is of course affected by the range of rotational velocities at each type. For our purposes, we wish to compare hydrogen lines computed for non-rotating models with observations. We therefore must obtain values of T_{eff} and hydrogen-line strengths which refer to a zero-rotation sequence.

The effect on the colors and line strengths of increasing rotational velocity from zero to some value v for a B star of a given mass is to: (1) decrease the equivalent width of the hydrogen lines and (2) increase $(U-B)_0$, D, (U-V), and (B-V).

Collins has recently (1967) computed the effects of rotation on spectral type and concludes that except for velocities near breakup, line ratios used in defining spectral types are unaffected.

We would expect from the above discussion that: (1) in a plot of $W(H\gamma)$ against spectral type, stars having the highest values of v sin i will lie lowest in this diagram; (2) in a plot of $W(H\gamma)$ against $(U-B)_0$, stars having the highest values of v sin i will be displaced toward lower $W(H\gamma)$ and large values of $(U-B)_0$; (3) in a plot of (U-V) or (B-V) against spectral type, stars having the lowest values of v sin i will give the most negative values of (U-V) and (B-V); (4) since both (U-V) and D essentially measure the Balmer discontinuity, the D against (U-V) plot will be unaffected by rotation.

In Figure 2 we plot $W(H\gamma)$ (as determined from the work of Petrie (1964)) against spectral type. The symbols indicate stars having high and low values of $v \sin i$. The values of $v \sin i$ used have been obtained from the work of Abt and Hunter (1962), Sletteback (1949, 1954, 1956), Sletteback and Howard (1955), McNamara and Larsson (1962), and Anderson, Stoeckly, and Kraft (1966). There is a strong tendency for the stars having the highest values of $v \sin i$ to fall lowest in this diagram, as expected from (1). We therefore adopt as the observed relation between $W(H\gamma)$ and spectral type for non-rotating stars, the upper envelope as defined by the points plotted in Figure 2.

In Figure 3 we make a similar plot, but now with $(U-B)_0$ as the abcissa. As expected, the stars having the highest values of $v \sin i$ lie lowest in the diagram, and we again adopt the upper envelope of this plot for our $W(H\gamma)$ – $(U-B)_0$ relation for non-rotating stars.

In Figure 4 we plot $(U-V)$ against spectral type and discriminate again between the stars having high and low values of $v \sin i$. Note that the effects of interstellar reddening and increased rotation both result in increasing $(U-V)$ while leaving the spectral type unaffected. For the most part, the upper envelope in this plot is determined by the slow rotators, as expected. The values of $(U-V)$ adopted by Johnson (1963) as appropriate means are shown for each spectral type.

We recall from § III that the scale of T_{eff} as determined by Morton and Adams (1968) depends partly on the adopted relation between $(U-V)$ and spectral type. From Figure 4 we note that for spectral types later than B2, the Johnson mean values lie below the upper envelope. The relation between spectral type and T_{eff} for non-rotating stars should properly be obtained from the T_{eff} – $(U-V)$ relation calibrated by Morton and Adams (1968) and the upper envelope of the $(U-V)$ – spectral-type plot. By using the Johnson $(U-V)$ – spectral-type relation instead of the upper envelope of this relation, Morton and Adams (1968) have chosen values of T_{eff} for each spectral type that are too small. Owing to the intrinsic measuring accuracy of ± 0.01 mag in the determination of $(B-V)$ and the relative insensitivity of $(B-V)$ to T_{eff} for B stars, it will be impossible to argue in a definitive way regarding the

effects of rotation on the (B-V) - spectral-type relation. As a result we shall assume that for stars later than B8, the effects of rotation on the spectral-type (B-V) relation are negligible. This assumption does not significantly affect our final results regarding surface gravity. In Table 1 (Column 4) we indicate the amount by which the Morton and Adams values of T_{eff} must be increased.

In Figure 5 we plot against spectral type the values of T_{eff} deduced by Morton and Adams, the revision of their values as suggested by our discussion, and the effective temperatures recently determined from stellar-diameter measurements by Hanbury Brown, Davis, Allen, and Rome (1967). Note the marginally better agreement of the Hanbury Brown et al. T_{eff} determinations with our values between types B3 and B9. It should be mentioned that some of the stars for which diameters were measured are rapid rotators and therefore a correction to the T_{eff} values obtained by Hanbury Brown et al. may be necessary. Such a correction should increase the plotted values of T_{eff} by no more then a few percent.

V. THE DETERMINATION OF SURFACE GRAVITIES

After correcting the temperature scale of Morton and Adams (1968) for rotation effects and the new calibration of the Balmer discontinuity, we can now transform the W(Hγ) - spectral type and W(Hγ) - $(U-B)_0$ relations to plots of W(Hγ) against T_{eff}. These plots are almost identical.

From the model atmospheres, we compute a theoretical relation between W(Hγ) and T_{eff} for models having log g = 4.1 and 3.8. For these models, the fractional helium abundance by number is taken to be 0.09 and hydrogen by number is 0.91.

The techniques and programs of Strom and Avrett (1965) were used in all model calculations.

In Figures 6a and 6b we compare the W(Hγ) - T_{eff} relation obtained from the stars having the highest value of W(Hγ) at each spectral type (see Fig. 2) with the relations computed from the models and from the ESW semi-empirical broadening theory (6a) and the revised Griem theory (6b).

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From the ESW $W(H\gamma)$ values we deduce a mean log g for B stars of ~4.25. The Griem $W(H\gamma)$ values give a value of log g ~ 4.05. From the eclipsing binary systems considered reliable by Popper (as quoted by Morton and Adams (1968)), the mean surface gravity for main-sequence B stars is log g = 4.19.

The initial main sequence computed by Kelsall and Stromgren (1964) for stars having masses greater than $2M_{\odot}$ gives log g ~ 4.30 for a chemical composition X = 0.70, Y/Z = 14.0 (or, in terms of fractional abundances, 0.90 for hydrogen and 0.10 for helium, identical with our mix). It would appear, therefore, that the ESW broadening theory allows us to choose a value of log g more in accordance with the predictions of stellar-interior computations and observed values for eclipsing binaries. If the ESW theory is correct, then the predicted and directly observed values of log g for main-sequence B stars agree to about 0.1 in the log.

The major possible sources of error affecting this conclusion are:

1) <u>Uncertainties in the observed $W(H\gamma)$</u>. The results of Petrie (1964) appear to agree well with most other independent photographic observations of $H\gamma$. However, Stock (1956) finds from his photoelectric observations values of $W(H\gamma)$ larger than Petrie's by about 15 per cent. Such an increase, if correct, would predict a mean log g of ~4.4 from the Griem theory and ~4.6 from the ESW formulation. Owing to the general agreement with Petrie's scale of $W(H\gamma)$, we feel that his observations are probably the best available at this time. However, a considerable amount of work toward obtaining accurate hydrogen-line profiles and equivalent widths would seem desirable at this time.

2) <u>Uncertainties in the effective temperature scale</u>. We feel that a good estimate of the probable errors in the T_{eff} scale for stars between B3 and B9 is probably about ± 500°K. This error, in turn, introduces an error of about ± 0.1 in log g. These errors in T_{eff} will, of course, be reduced when absolute flux observations from rockets and satellites become available.

However, much more work could be done in providing accurate spectrophoto-
metric calibrations in the accessible regions of the spectrum and in com-
puting more detailed models that incorporate the effects of both line blanket-
ing and stellar rotation.

In conclusion, we feel that to within the present uncertainties in the
theory and observations, there is no really serious disagreement in the
gravities deduced from hydrogen-line equivalent widths and those determined
directly from eclipsing binary systems and interior models.

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TABLE 1

CORRECTIONS TO THE T_{eff} SCALE OF MORTON AND ADAMS

Spectral Type	T_{eff} (Morton & Adams)	ΔT (Abs. Calibration)	ΔT (Rotation)	T_{eff} (Revised)
B2	20500	-2000	0	18500
B3	17900	-1500	800	17200
B5	15600	-1000	600	15200
B6	14600	- 750	600	14450
B7	13600	- 450	600	13750
B8	12000	- 250	1200	12950
B9	10700	0	300	11000
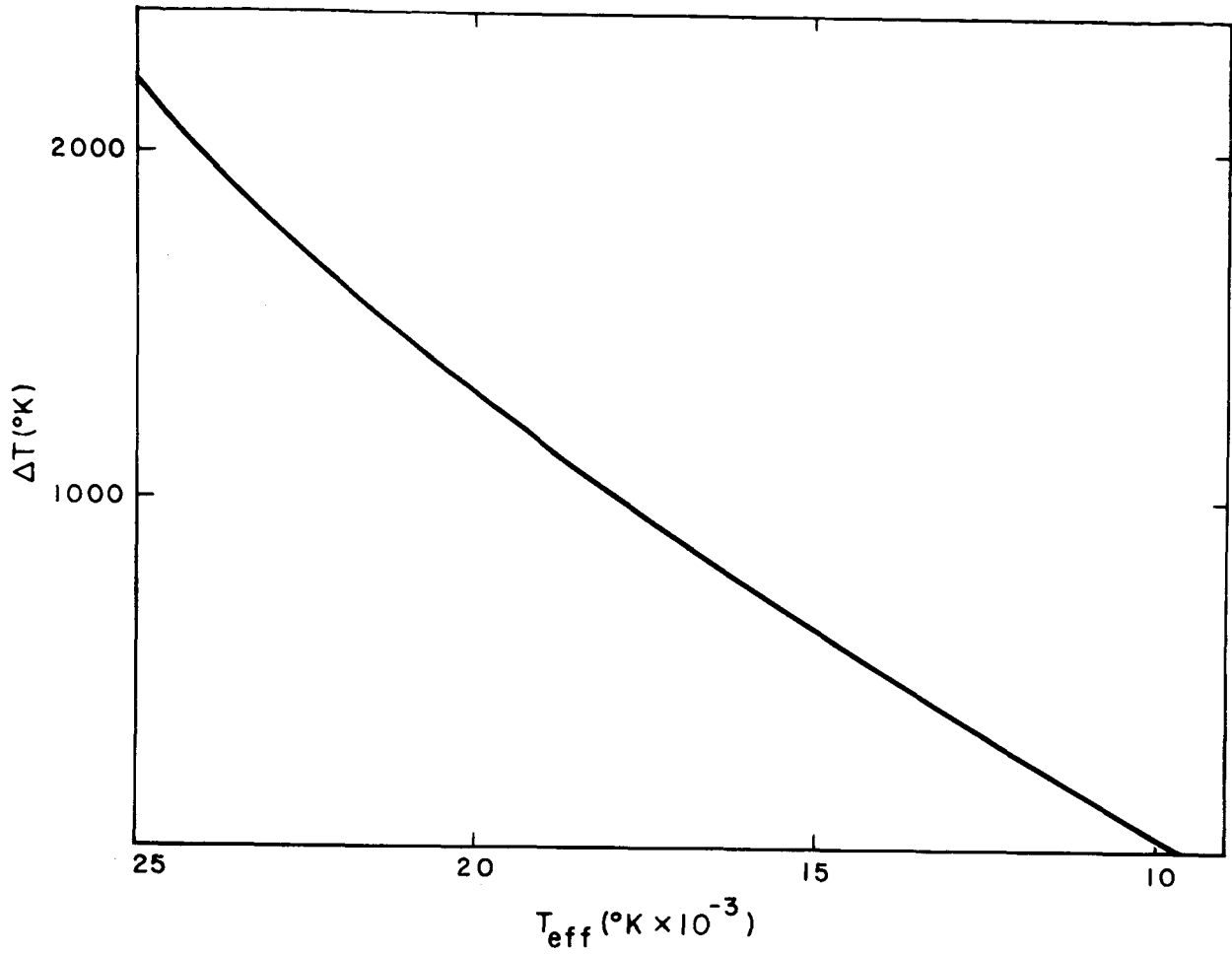
Fig. 1.— The difference ΔT in effective temperature between blanketed and unblanketed model atmospheres that predict the same emergent fluxes in the visible (Morton and Adams 1968).
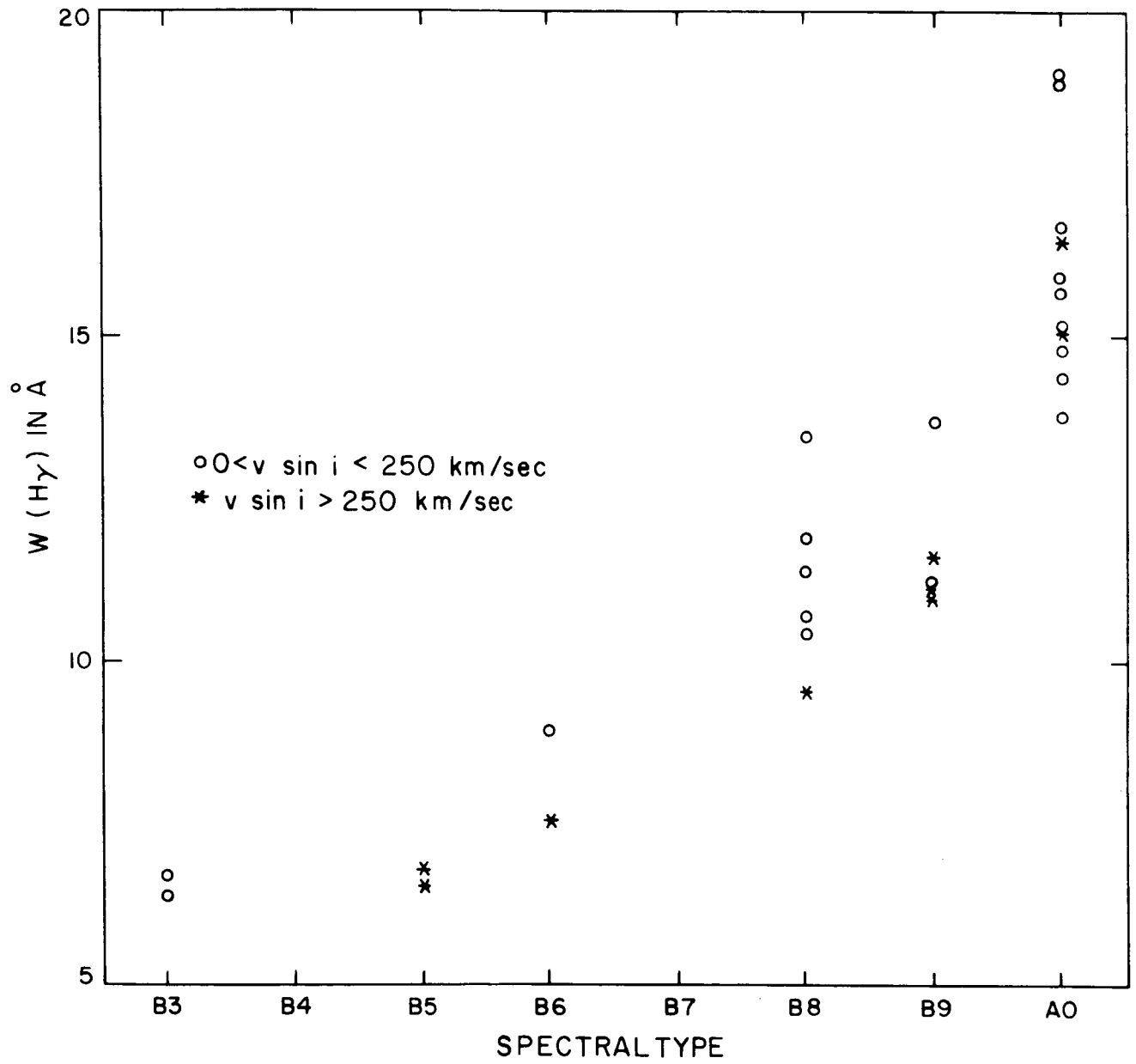
Fig. 2.—Observed Hγ equivalent widths for stars of differing v sin i plotted against spectral type. Stars having lowest v sin i have the highest W(Hγ).

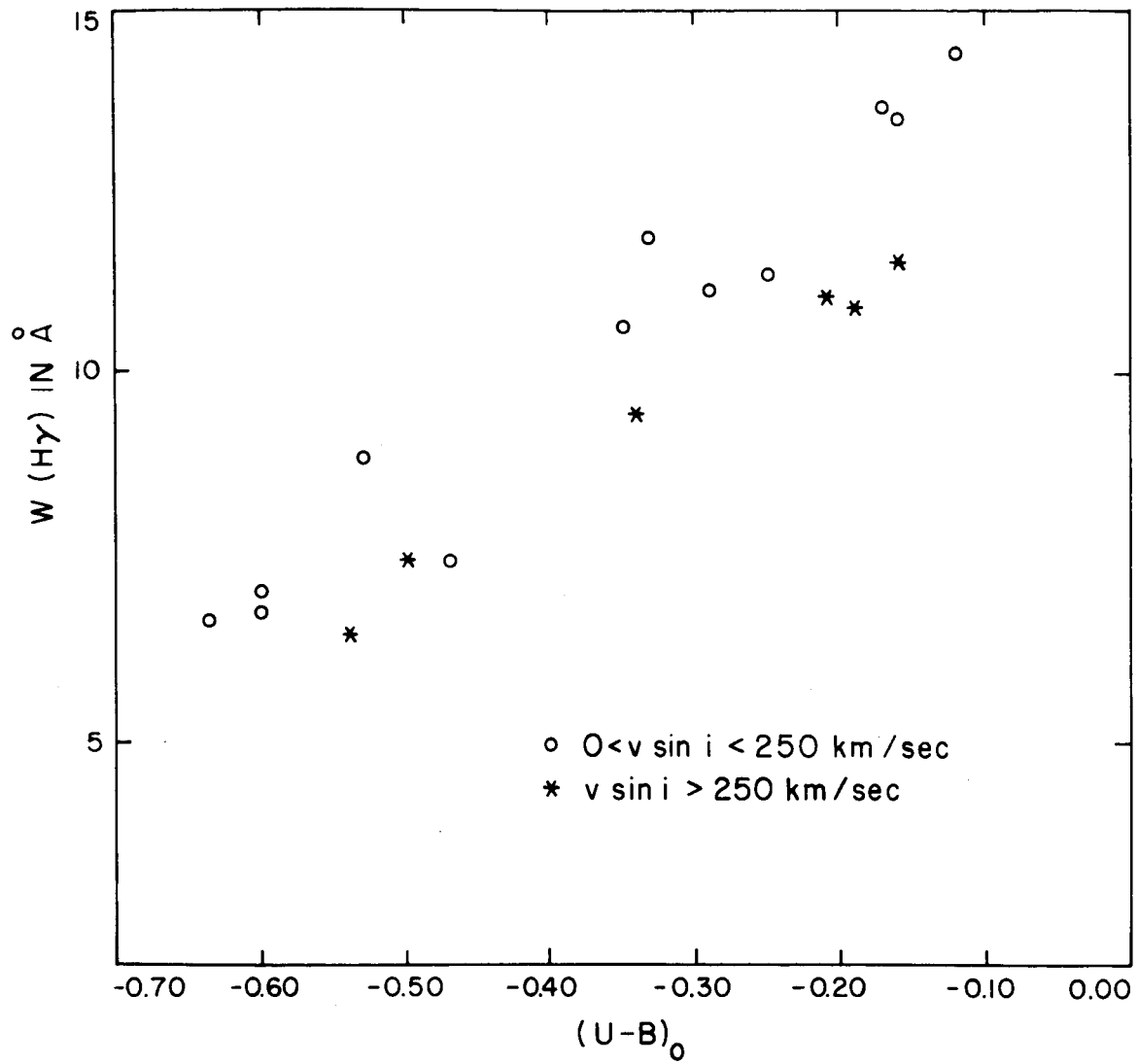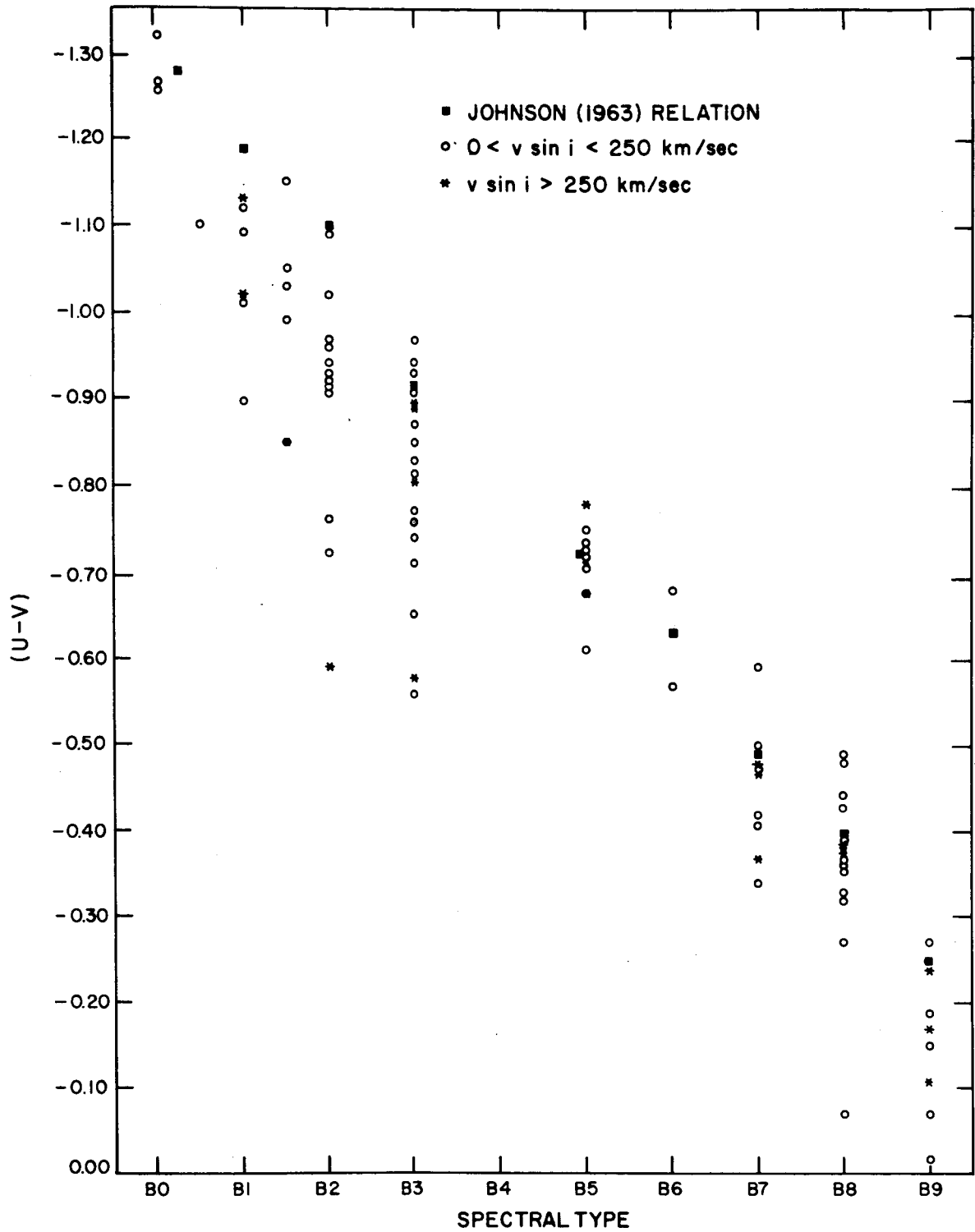Fig. 3.—Observed Hγ equivalent widths for stars of differing v sin i plotted against $(U-B)_0$. Stars having lowest v sin i have highest W(Hγ).

Fig. 4.— Observed (U-V) plotted against spectral type for stars of differing v sin i. In general, the upper envelope is determined by the stars having the lowest values of v sin i.
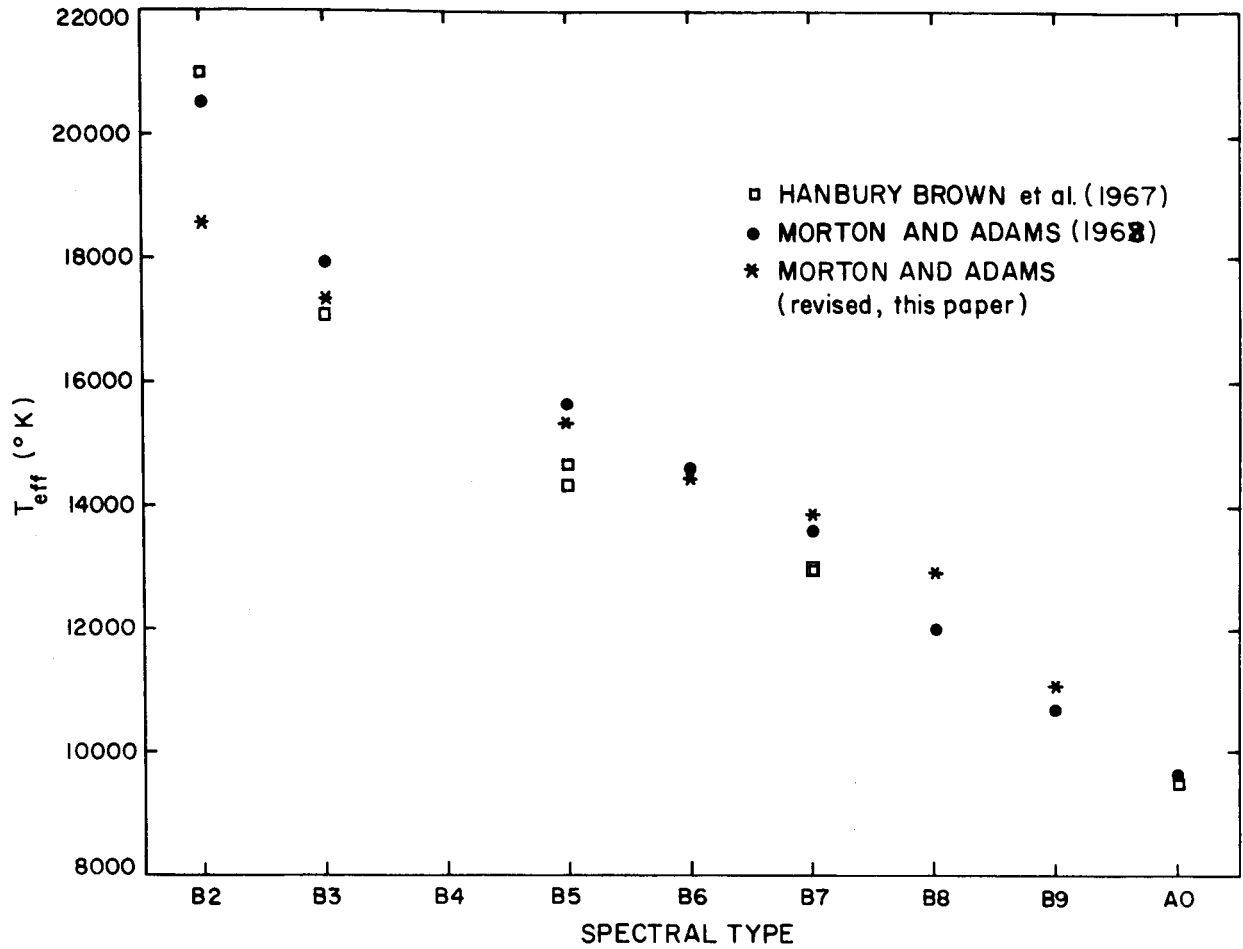
Fig. 5.—A comparison of the effective temperature scales deduced by
Morton and Adams (1968), the Morton and Adams scale as
revised in this paper, and the scale determined by Hanbury
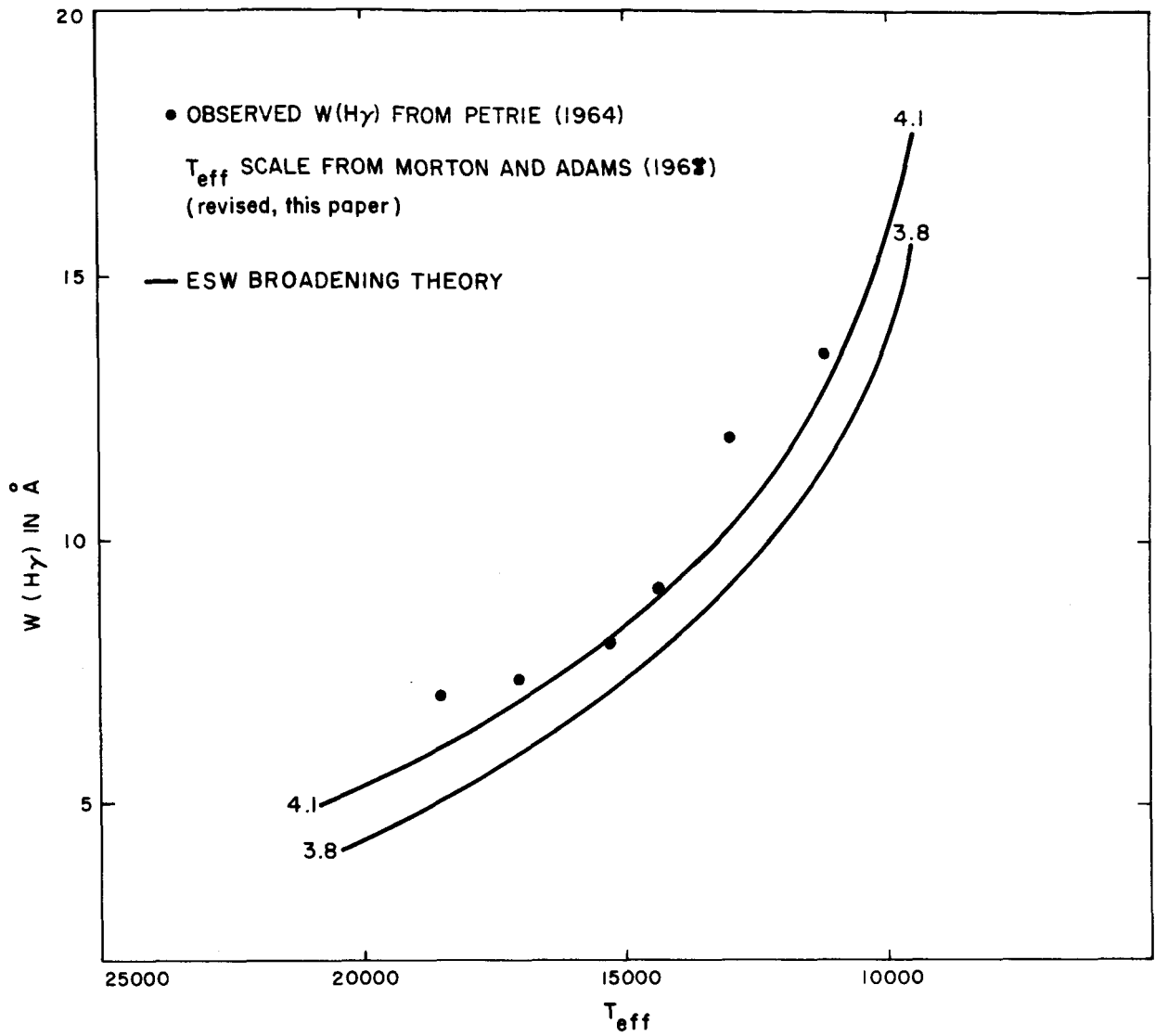Brown and his associates from stellar diameter measurements.

Fig. 6a.— Observed Hγ equivalent width — T_{eff} relation compared with
the theoretical relations obtained from the models having
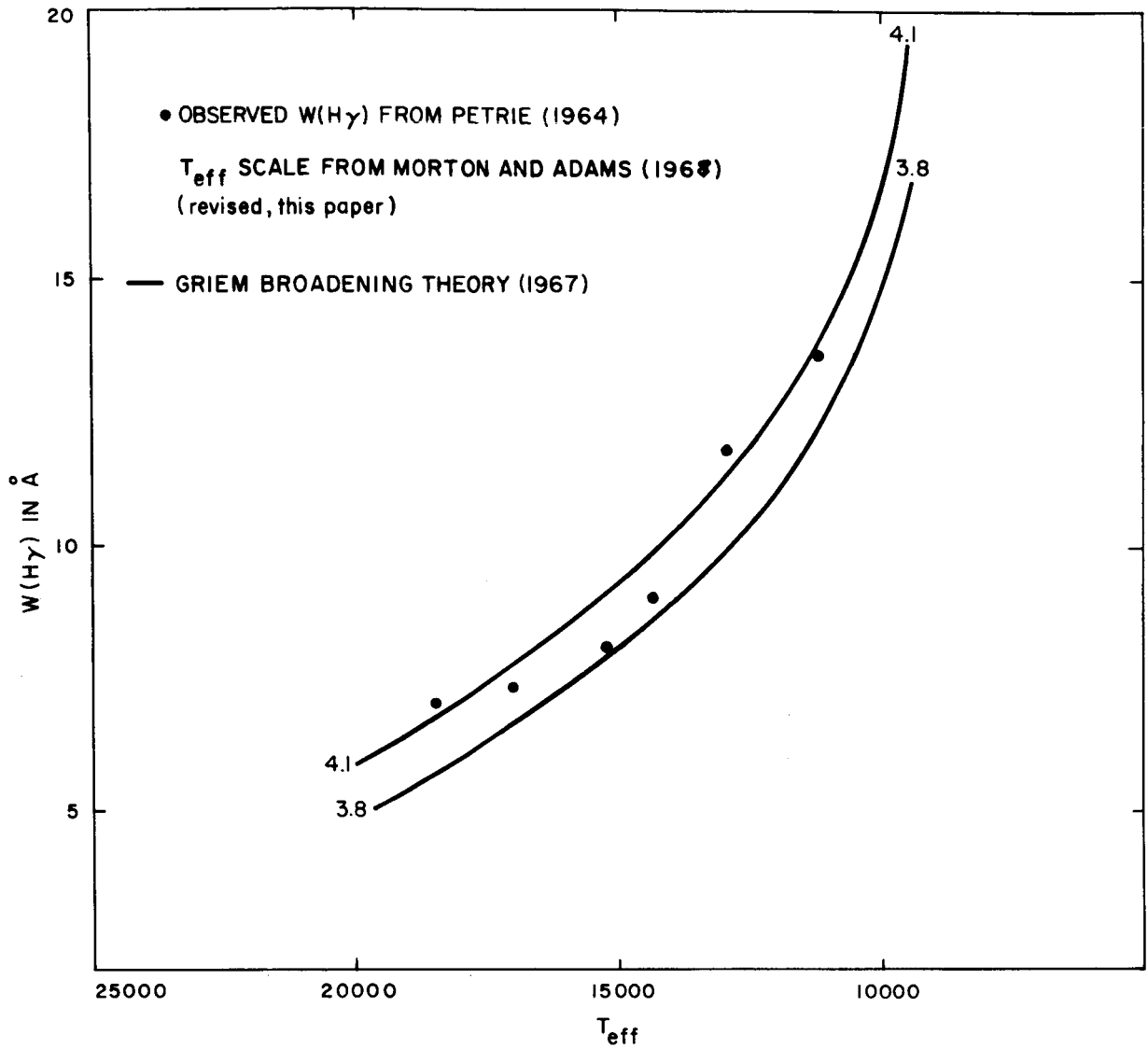log g = 3.8 and 4.1 and the ESW broadening theory.

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Fig. 6b. — Observed Hγ equivalent width — T_{eff} relation compared with the theoretical relations obtained from the models having log g = 3. 8 and 4. 1 and the Griem broadening theory.

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