THE UNIVERSITY OF MICHIGAN
ANN ARBOR, MICHIGAN

SEMIANNUAL PROGRESS REPORT NO. 2

ON

FREQUENCY MULTIPLICATION IN HIGH-ENERGY ELECTRON BEAMS

This report covers the period April 1, 1967 to October 1, 1967

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Project 08400

RESEARCH GRANT NO. NGR 23-005-183
OFFICE OF SPACE SCIENCE AND APPLICATIONS
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

February, 1968

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1. General Introduction (R. J. Lomax)

The basic objectives of this research program are to investigate the following:

a. Interaction of high-energy electron beams with quasi-optical circuits and plasmas (Cerenkov radiation).

b. Electron beam-plasma interactions using cold-cathode discharges.

c. Nonlinear electron beam-plasma theory.

d. Instabilities in non-Maxwellian plasmas.

This report covers the second six months of effort on the program which was initiated on October 1, 1966. The following sections of the report summarize the status of the various projects and include discussions of results and plans for future investigations.

2. Relativistic Electron-Beam Device

Supervisor: J. E. Rowe

Staff: G. T. Konrad

The object of this phase of the program is the generation of millimeter wavelength radiation by use of a tightly bunched relativistic electron beam. The experimental setup was described briefly in the first progress report.

During the last period the 400 ℓ/sec ion pump was received and attached to the vacuum chamber. A vacuum in the low 10^{-8} Torr region could be maintained during operation of the device. The PIG discharge that had existed in the past when an axial magnetic focusing field was applied to the device was no longer observed. A breakdown along the high-voltage bushing occurred in the vicinity of 80 kv, however. This threshold rose with time, so that it is believed that substantially higher voltages can be applied after an appropriate aging-in period.

The main difficulty was encountered with beam transmission through the high-voltage anode. It is believed that the hole in the quartz Cerenkov coupler is too small for the beam to pass through without some electrons striking the quartz and charging it to a negative potential. This manifests itself during operation in a very erratic beam transmission. It was therefore decided to increase the tunnel diameter in the quartz cone by 50 percent and to deposit a metallic layer a few Ångstroms thick along the length of the tunnel so that the electrons striking the cone may be drained off.

At the present time these modifications are in progress and experimental work will be resumed as soon as they are completed.

3. Beam-Plasma Interactions

Supervisor: R. J. Lomax

Staff: J. D. Gillanders

Our new beam-plasma tube has recently been completed. In the initial tests a tenuous beam was generated by the plasma electron gun. The gun seems to work best in the pressure range around 100 microns with the 5000-volt power supply presently available. The tests indicate that a higher voltage might improve the operation at lower pressures. A 10,000-volt supply is now being repaired for use in further tests.

An absolute instability has been investigated using the computer program. The instability occurs in the backward-wave region at about 1.045 times the plasma frequency (ω_p) for the following conditions: cyclotron frequency 0.707 ω_p, beam-to-plasma density ratio 0.001, and beam microperveance 1. This instability was found and identified by using Briggs's criterion[1]. The computer program was used to trace the roots of the dispersion equation as the imaginary part of the frequency approached negative infinity for fixed real part. A saddle point was located in the neighborhood of $Re(\beta) = 1.135$ and $Im(\beta) = 0.187$. The roots from one side of the saddle point accumulate at a point on the positive imaginary axis, while those from the other side approach negative imaginary infinity as $Im(\omega)$ approaches negative infinity, thus identifying this as an absolute instability.

The effect of collisions on both this instability and the convective instabilities is being investigated. If the absolute instability is damped out by the collisions before the convective instability is completely damped out, there is a possibility of stable backward-wave gain; if not, then only oscillations can be obtained in this region. The results have been delayed due to difficulties in the computer program.

4. Time-Dependent Nonlinear Analysis of Electron-Beam Plasma Interaction

Supervisor: J. E. Rowe

Staff: A. Lin

The beam-plasma system will be represented by velocity-modulated electron-beam disks continuously injected into warm electron plasma disks with an immobile neutralizing ion background. The initial conditions for the plasma will be taken to correspond to a homogeneous plasma with a drifting Maxwellian distribution of thermal velocities. To maintain

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1. Briggs, R. J., Electron-Stream Interaction with Plasmas, The M.I.T. Press, Cambridge, Mass; 1964.

the neutrality of the plasma, the beam density will be taken to be small compared to the plasma density. With the development of the high-speed large memory computers, it has become possible to follow the behavior of thousands of disks in a self-consistent field. The positions and velocities of the disks in the interaction region will be stored and advanced stepwise in time as follows:

1. From the position of each disk, the electric field acting on each disk is calculated.

2. Newton's equation of motion is integrated for a short time step, using the local electric field and assuming its constancy through the time step.

This calculation gives a new position and velocity for each disk. With this model on hand, many interesting phenomena can be investigated, such as plasma effects on the bunching of the electron beam in the interaction region and the velocity spread of the electron beam at the collector.

5. The Study of Cyclotron Harmonic Instabilities

Supervisor: W. D. Getty

Staff: A. Singh

5.1 Threshold Conditions for Instability in the Experiment: The Approximate Dispersion Equation.

The objective of the experiment is to excite the electrostatic cyclotron harmonic wave in a neutralized electron beam. This wave has been shown by several authors[1-3] to be unstable for the case of an infinite, homogeneous electron beam. The analyses that present this result usually involve solving the Harris dispersion equation[4] by a digital computer, and so far results for only a few values of parameters have been published. For this reason a simplified expression for the instability threshold condition has been obtained which is useful for predicting whether or not observable instabilities may be expected for the values of parameters used in the experiment, and if so, what parallel wavelength should be observed. A method suggested by Morse[5] has been followed.

The general electrostatic dispersion equation is given by[1]

$$1 - \frac{\omega_p^2}{k^2} \sum_{-\infty}^{\infty} \left[k_{\parallel}^2 \frac{J_n^2(\mu)}{(\omega - n\omega_c)^2} + \frac{k_{\perp}^2}{2\omega_c} \left(\frac{J_{n-1}^2(\mu) - J_{n+1}^2(\mu)}{\omega - n\omega_c} \right) \right] = 0 \quad , \tag{5.1}$$

where $\mu = k_{\perp} v/\omega_c$, $k^2 = k_{\parallel}^2 + k_{\perp}^2$, and J_n are the Bessel functions of the first kind of order n. $k_{\parallel}$ and $k_{\perp}$ are the wave numbers, parallel and perpendicular, respectively, to the constant magnetic field B; v is the magnitude of the perpendicular component of the electron's

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1. Gruber, S. et al., "High-Frequency Velocity Space Instabilities", <u>Phys. Fluids</u>, vol. 8, No. 8, pp. 1504-1509; August, 1965.

2. Crawford, F. W. and Tataronis, J. A., "Absolute Instabilities of Perpendicularly Propagating Cyclotron Harmonic Plasma Waves", <u>Jour. Appl. Phys.</u>, vol. 36, No. 9, pp. 2930-2934; September, 1965.

3. Speck, C. E. and Bers, A., "Instabilities in Quasi-Static Waves Across B_o", Quarterly Progress Report No. 79, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., p. 113; October 15, 1965.

4. Harris, E. G., "Plasma Instabilities Associated with Anisotropic Energy Distributions", <u>Jour. Nucl. Energy</u>, Part C, vol. 2, p. 138; 1961.

5. Morse, D. L., Sperry Rand Research Center, Sudbury, Mass., unpublished work.

time-averaged velocity; ω_c is the electron cyclotron frequency; and ω_p is the plasma frequency. This dispersion equation is valid for a velocity distribution in which all electrons have zero parallel velocity and the magnitude of the perpendicular velocity is v for all electrons. Thus each electron executes simple circular gyrations about some fixed guiding center in its unperturbed orbit. The guiding centers are assumed to be uniformly distributed spatially.

In the experiment all electrons have a parallel velocity component equal to v_o. This is accounted for by a Doppler shift of the frequency ω by an amount equal to $k_{\|}v_o$. Thus any frequency calculated from the above dispersion equation is added to $k_{\|}v_o$ to find the frequency that would be observed in the laboratory reference frame.

The approximate method consists of taking the terms n = 0, ±1, ±2 in the infinite sum that appears in Eq. 5.1. Since $J_{-n}(\mu) = (-1)^n J_n(\mu)$, terms up to $|n| = 2$ can be combined to obtain the approximate dispersion equation

$$\left(\frac{k_{\|}}{k_{\perp}}\right)^2 = \frac{f_2(y) - \omega_c^2/\omega_p^2}{\omega_c^2/\omega_p^2 - f_1(y)} , \qquad (5.2)$$

where

$$f_1(y) = \frac{J_o^2(\mu)}{y^2} + 2J_1^2(\mu)\,\frac{y^2+1}{(y^2-1)^2} + 2J_2^2(\mu)\,\frac{y^2+4}{(y^2-4)^2} ,$$

$$f_2(y) = [J_o^2(\mu) - J_2^2(\mu)]\,\frac{1}{y^2-1} + [J_1^2(\mu) - J_3^2(\mu)]\,\frac{2}{y^2-4}$$

and

$$y = \omega/\omega_c .$$

Equation 5.2 is a good approximation in the range $0 \leqq y < 2$. $k_\perp$ is considered to be fixed by the transverse dimensions of the plasma (if the plasma is a column of radius a, then $k_\perp \cong a^{-1}$). Equation 5.2 gives $k_\parallel$ as a function of normalized frequency y. There will be ranges of y where Eq. 5.2 gives no real solutions for $k_\parallel$. In these frequency bands the wave is cut off. Equation 5.2 can be used to plot $k_\parallel/k_\perp$ as a function of frequency in the range $0 \leqq y \leqq 2$, and the results obtained are very similar to the plots of $k_\parallel/k_\perp$ vs. y given by Gruber et al.[1]. For small values of ω_p/ω_c there are two poles of $k_\parallel/k_\perp$ in the range $0 \leqq y \leqq 1$. Between these two poles there is a cutoff region. As ω_p/ω_c increases these two poles move toward each other and finally coalesce and the cutoff frequency band between the poles disappears. This condition marks the onset of an absolute instability for which ω is complex for real values of $k_\parallel/k_\perp$. It may be predicted numerically by examining the behavior of $f_1(y)$ as a function of y.

The condition for poles of $k_\parallel/k_\perp$ is that

$$f_1(y) = \frac{\omega_c^2}{\omega_p^2} . \tag{5.3}$$

A plot of $f_1(y)$ vs. y reveals that $f_1(y)$ has two minima in the range $0 \leqq y \leqq 2$. For a given value of μ, $f_1(y)$ does not depend on ω_p^2/ω_c^2 and therefore one $f_1(y)$ curve can be plotted for each value of μ, and the threshold value of ω_p/ω_c can be found. In Fig. 5.1 the function $f_1(y)$ is plotted vs. y^2 for four values of μ. The threshold values for ω_c^2/ω_p^2 are given by the values of the function at its minima. Note that the minimum in the range $1 < y < 2$ corresponds to a larger value of ω_p^2/ω_c^2 than the one in the range $0 < y < 1$. The minimum of $f_1(y)$ in the frequency range $0 < y^2 < 1$ is used to obtain the threshold value of ω_p^2/ω_c^2 which marks

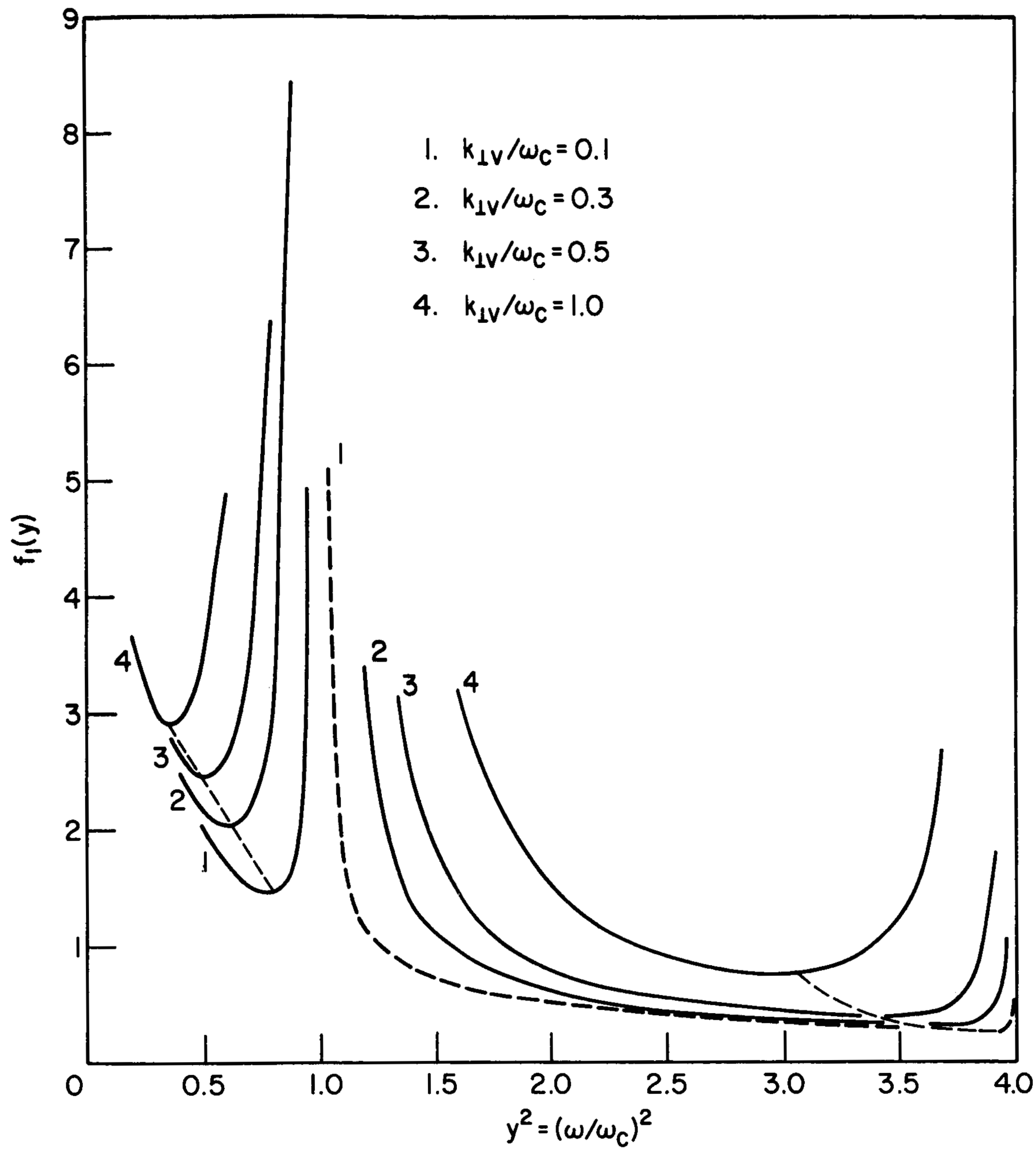


FIG. 5.1 PLOT OF $f_1(y)$ AS A FUNCTION OF y^2 FOR $0 \leqq y^2 \leqq 4$.

the onset of complex ω roots for real values of $k_{\parallel}/k_{\perp}$. This result is plotted in Fig. 5.2. The required ratio of ω_p/ω_c varies from 0.83 to 0.58 over the range of μ which was investigated.

5.2 Electron-Beam Parameters. In this approach attainment of the required ratio of ω_p/ω_c has been attempted by synthesis of a plasma. The electrons are injected into the plasma from a triode electron gun. They pass through the corkscrew[6] on their way from the gun to the drift tube region. Transverse energy is obtained in the corkscrew at the expense of parallel energy. The ions are obtained by trapping the ions that are generated by ionization of the residual gas. In this setup the electron density is limited by the beam current from the electron gun. It has been found that it is very difficult to obtain the required value of ω_p/ω_c with a conventional electron-gun source. The available range of ω_p/ω_c is limited by gun perveance and corkscrew windup ratio as will be shown in this section. It will probably be necessary to introduce additional background plasma into the experiment to obtain unstable operating conditions. Crawford[2] has shown that the presence of background plasma lowers the required ratio of ω_p/ω_c appreciably.

The electron beam is generated by an electron gun with the final accelerating potential V_o. The current from the electron gun is assumed to be $I = P_\mu \times 10^{-6} \times V_o^{3/2}$, where P_μ is the microperveance. After passing through the corkscrew, the perpendicular energy of the beam electrons is $1/2\ mv^2 = 2V_o\xi$ and the parallel energy is $1/2\ m_o^2 = eV_o(1-\xi)$.

6. Wingerson, R. C. et al., "Trapping and Loss of Charged Particles in a Perturbed Magnetic Field", Phys. Fluids, vol. 7, No. 9, pp. 1475-1484; September, 1964.

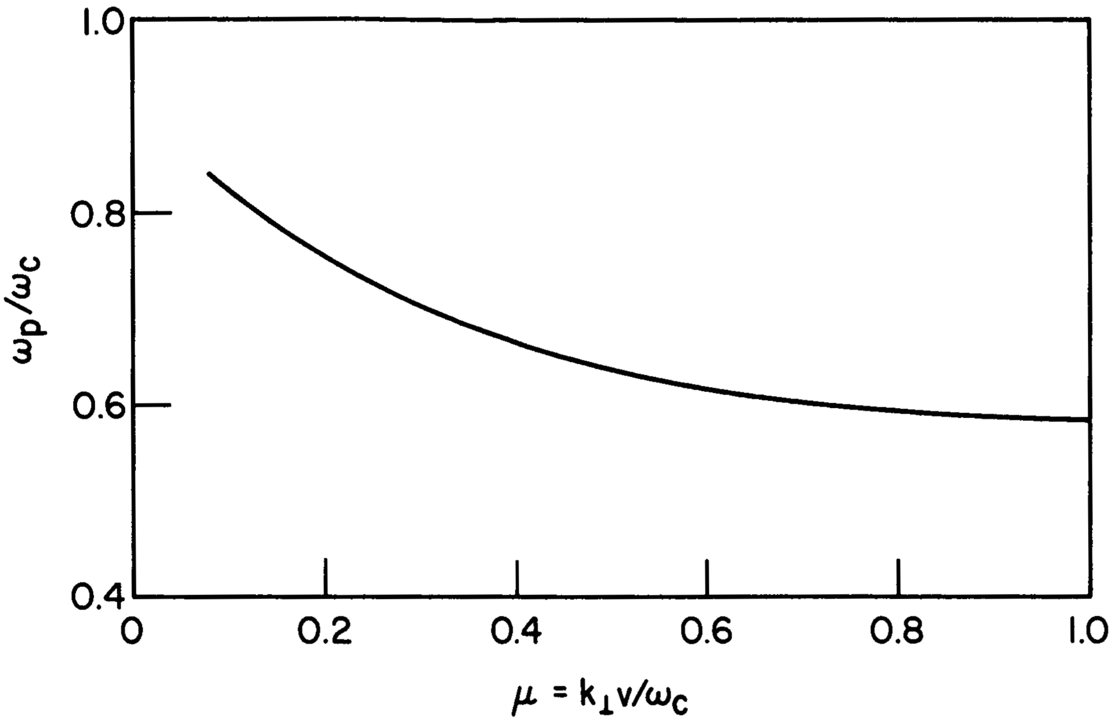


FIG. 5.2 THRESHOLD VALUES OF ω_p/ω_c FOR THE ONSET OF THE ABSOLUTE INSTABILITY AS A FUNCTION OF $k_\perp v/\omega_c$.

The windup ratio is given by ξ. In the drift region the beam is assumed to have a radius given by a and the axial drift velocity is v_o. The electrons are assumed to be uniformly distributed across the beam cross section. The electron Larmor radius ρ_L is assumed to be small compared to the beam radius a.

Using the beam model described above the ratio ω_p/ω_c is found to be given by

$$\frac{\omega_p}{\omega_c} = \left[2\pi\epsilon_o\left(\frac{2e}{m}\right)^{1/2}\right]^{-1/2} \frac{P_\mu^{1/2} \times 10^{-3}}{\xi^{1/2}(1 - \xi)^{1/4}} \frac{\rho_L}{a}$$

$$\cong 0.174 \frac{P_\mu^{1/2}}{\xi^{1/2}(1 - \xi)^{1/4}} \frac{\rho_L}{a} \ .$$

The parameter μ is given by

$$\mu = \frac{k_\perp v}{\omega_c} = 2.405 \frac{\rho_L}{a} \ ,$$

where it is assumed that $k_\perp = 2.405a^{-1}$. A third parameter of interest is the slope of the dispersion curve $k_{\|}/k_\perp$ after shifting the observer's frame of reference to the laboratory frame. Using the equation $\omega' = \omega + k_{\|}v_o$ for the frequency in the laboratory frame results in

$$\frac{\omega'}{\omega_c} = \frac{\omega}{\omega_c} + \frac{k_{\|}}{k_\perp}\left(\frac{1 - \xi}{\xi}\right)^{1/2} 2.405\left(\frac{\rho_L}{a}\right) \ .$$

After transformation to the laboratory reference frame the lines corresponding to constant values of ω/ω_c have a slope in a plot of ω'/ω_c given by

$$(\text{slope})^{-1} = 2.405 \left(\frac{1 - \xi}{\xi} \right)^{1/2} \left(\frac{\rho_L}{a} \right) .$$

As the slope is decreased the frequency at which the instability appears in the range $0 < \omega' < \omega_c$ approaches the value of ω_c. Experimentally it is preferable to keep the slope as large as possible to keep the wavelength of the instability as short as possible. This requirement must be compromised with the requirement of a large value of ω_p/ω_c.

The application of the above equations to the experiment leads to severe restrictions on the value of ξ. The maximum value that ρ_L/a can have is approximately 0.2 if the model of a homogeneous plasma with internal energy is to be approached. The maximum value of P_μ that can be obtained with a rectilinear-flow electron gun is approximately 3 micropervs. In order to obtain a ratio of ω_p/ω_c of 0.5 with these parameters, a windup ratio of 0.9998, or essentially 100 percent transfer of energy, is required. It is impractical to achieve this with a corkscrew, and therefore the corkscrew is being combined with an adiabatic jump in the axial magnetic flux density to increase the overall energy transfer ratio. A magnetic flux density increase by a factor of ξ^{-1}, where ξ is the corkscrew windup ratio, should give total energy transfer and a decrease in beam radius by the factor $\xi^{1/2}$.

5.3 Experimental Results. The electron gun used in the experiment has been improved and operated under conditions of good beam transmission from cathode to collector. The beam collector current is 25 ma at an accelerating voltage of 625 volts. The microperveance is slightly greater than 1.0 for this current and voltage. The corkscrew must be modified to account for the change in beam voltage and an increase in axial magnetic field from 100 gauss to 285 gauss. This electron gun will be used with the redesigned corkscrew and the magnetic-field jump to obtain an increased beam internal energy.

6. Nonlinear Lagrangian Analysis of Beam-Plasma Systems (J. E. Rowe)

A two-dimensional nonlinear system of Lagrangian equations has been developed for the analysis of beam-plasma interactions. This analysis includes the effects of radial motion of the charges and finite axial magnetic focusing fields. The theory is particularly oriented to the calculation of fundamental and harmonic currents in beam-plasma and two-beam systems. The equations have been programmed and checked out on a large-scale digital computer neglecting the effects of space-charge fields. Recently the space-charge field equations have been developed and are currently being checked out on the computer. It is anticipated that the entire system will be checked out and running soon. At that time calculations will be made for system parameters of interest.

7. Harmonic Current Generation in a Beam-Plasma System

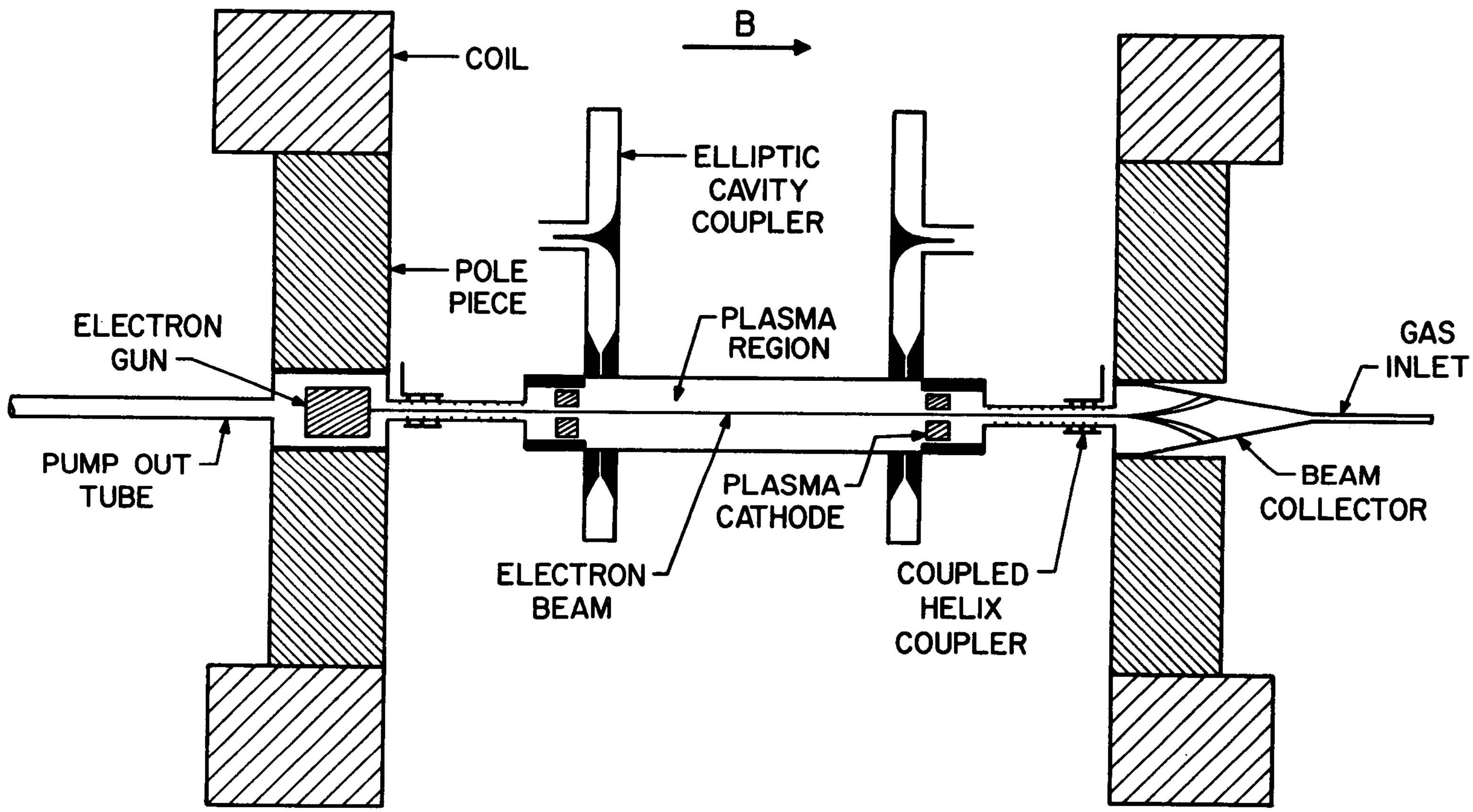
Supervisor: J. E. Rowe

Staff: G. T. Konrad

This portion of the program concerns itself with the study of harmonic generation and coupling schemes in beam-plasma systems. The theoretical aspect of the work consists of the derivation and solution of large-signal equations describing beam-plasma interactions including higher harmonic r-f current components. One-dimensional, as well as two-dimensional, analyses are under consideration. The plasma column is regarded as a transmission line with lumped circuit components which can be determined in terms of the various plasma parameters. Most of the equations have been derived during the past period. During the coming period it is planned to modify existing digital computer programs for solution of the equations.

The experimental work is to be conducted on the device shown in Fig. 7.1. The xenon plasma is generated by means of two hot-cathode Penning discharges. The electron beam passes through short helical slow-wave structures before entering the plasma and after leaving it. By this means the electron beam can be premodulated before the beam-plasma interaction takes place. Two sets of couplers are used. One set consists of coupled-helix couplers placed around the short sections of slow-wave structure, while the second set consists of elliptic cavity couplers placed directly around the plasma column. Under certain operating conditions of the plasma it should be possible to transmit an r-f wave across the plasma column, yielding a fairly tight method of coupling to the beam-plasma system. Being broadband structures, the elliptic cavity couplers are expected to work well at the fundamental r-f frequency, as well as at the first few harmonics. It is the object of the experimental work to study the harmonic generation in the beam-plasma system and compare these results with the theoretical predictions. In addition, the two methods of coupling are to be analyzed and compared.

The experimental device had already been assembled during the past period. Upon attempting to operate it, it was found that the gun cathode poisoned very rapidly due to outgassing of some organic material in its vicinity. It was therefore necessary to rebuild the device. This work is expected to be completed early during the coming period so that testing can commence at that time.



BEAM-PLASMA INTERACTION DEVICE

FIG. 7.1 BEAM-PLASMA INTERACTION DEVICE.