GENERAL MOTORS CORPORATION

TEDDY—A TWO-DIMENSIONAL LAGRANGIAN CODE FOR DETERMINING THE WAVE PROPAGATION IN ELASTIC-PLASTIC MEDIA

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I. INTRODUCTION

The problem set up here is the solution, by finite difference methods, of wave propagation in compressible media in two space dimensions, with either rectangular or cylindrical geometry. Lagrangian grid representation is used. For this system the motion of the medium is described with reference to a mesh attached to the material. This results in a limitation of the method, and serious difficulties arise when physical situations involve severe distortions of the original mesh.

The program can be used with either a fluid or elastic-perfectly plastic solid. Either the polynomial or the Tillotson[1] equation of state can be used for describing the hydrodynamic component of the stress. Other equations of state may be incorporated, as well as equations of state to describe the behavior of other media (for example, the explosives).

Several two-dimensional time-dependent Lagrangian codes have been developed, such as TENSOR,[2] HEMP,[3] PIPE,[4] and RAVE I.[5] The method outlined here follows that formulated earlier by Wilkins (HEMP)[3] and Herrmann (RAVE I).[5] In order to make the present report self-contained the equations of motion are given in the following section, Section II, and the finite difference derivations are given fully in Section III. (Additional equations for material rotation are given in Appendix A.) The program itself is described in Section IV, and example problems with printed and plotted outputs are also given there. Finally, flow charts for the program are given in Appendix B and complete listings of the Fortran source decks are presented in Appendix C.

II. EQUATIONS OF MOTION

The governing equations of motion in their Lagrangian form are as follows.

A. CONSERVATION OF MASS

$$\dot{\rho} + \rho\,(\dot{e}^{xx} + \dot{e}^{yy} + \dot{e}^{zz}) = 0$$

where ρ is the medium density, e^{ij} the strain, and the dot indicates a time derivative. With the present nomenclature x, y, z can either be rectangular coordinates or cylindrical coordinate systems. For the axial symmetric problem the cylindrical coordinate system is used, in which case x denotes the radial coordinate, and z the axial coordinate.

The strain rates are defined as

$$\dot{e}^{ij} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i}\right)$$

where u^i indicates particle velocity in direction i, e.g.

$$u^x = \frac{\partial x}{\partial t}$$

and

$$\left[\frac{\partial u^i}{\partial x^j}\right] = \begin{bmatrix} \frac{\partial u^x}{\partial x} & 0 & \frac{\partial u^x}{\partial z} \\ 0 & (\alpha-1)\,\frac{u^x}{x} & 0 \\ \frac{\partial u^z}{\partial x} & 0 & \frac{\partial u^z}{\partial x} \end{bmatrix}$$

Here α is a coefficient which takes on a value unity for motion in the x-z plane and 2 for axially symmetric motion.

B. CONSERVATION OF MOMENTUM

$$\rho a^x = \frac{\partial t^{xx}}{\partial x} + \frac{\partial t^{xz}}{\partial z} + (\alpha - 1)\frac{t^{xx} - t^{yy}}{x}$$

$$\rho a^z = \frac{\partial t^{xz}}{\partial x} + \frac{\partial t^{zz}}{\partial z} + (\alpha - 1)\frac{t^{xz}}{x}$$

where a^i is the acceleration in direction i , and t^{ij} are the stress components.

C. CONSERVATION OF ENERGY

The rate of change of internal energy/unit mass $\dot{\varepsilon}$, is given by

$$\rho\,\dot{\varepsilon} = p\frac{\dot{\rho}}{\rho} + \rho\,{}^{d}\dot{\varepsilon}$$

where

$$p = -\frac{1}{3}(t^{xx} + t^{yy} + t^{zz})$$

$$\rho\,{}^{d}\dot{\varepsilon} = 2\,{}^{d}t^{rr}\,{}^{d}\dot{e}^{rr} + 2\,{}^{d}t^{rz}\,{}^{d}\dot{e}^{rz} + 2\,{}^{d}t^{zz}\,{}^{d}\dot{e}^{zz} + {}^{d}t^{rr}\,{}^{d}\dot{e}^{zz} + {}^{d}t^{zz}\,{}^{d}\dot{e}^{rr}$$

$${}^{d}t^{ij} = t^{ij} + p$$

$${}^{d}\dot{e}^{ij} = \dot{e}^{ij} + \frac{1}{3}\frac{\dot{\rho}}{\rho}\delta^{ij}$$

$$\delta^{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

so that

$${}^{d}t^{xx} + {}^{d}t^{yy} + {}^{d}t^{zz} = 0$$

$${}^{d}\dot{e}^{xx} + {}^{d}\dot{e}^{yy} + {}^{d}\dot{e}^{zz} = 0$$

Since during a given time step an element of the body rotates it is necessary to correct the stress so as to refer to the fixed coordinate system (see Appendix A). The objective stress rates are defined by

$$\overset{\nabla}{t}{}^{xx} = \dot{t}^{xx} - 2w_{xz}t^{xz}$$

$$\overset{\nabla}{t}{}^{xz} = \dot{t}^{xz} + w_{xz}(t^{xx} - t^{zz})$$

$$\overset{\nabla}{t}{}^{zz} = \dot{t}^{zz} + 2w_{xz}t^{xz}$$

where

$$w_{xz} = \frac{1}{2}\left(\frac{\partial u^x}{\partial z} - \frac{\partial u^z}{\partial x}\right)$$

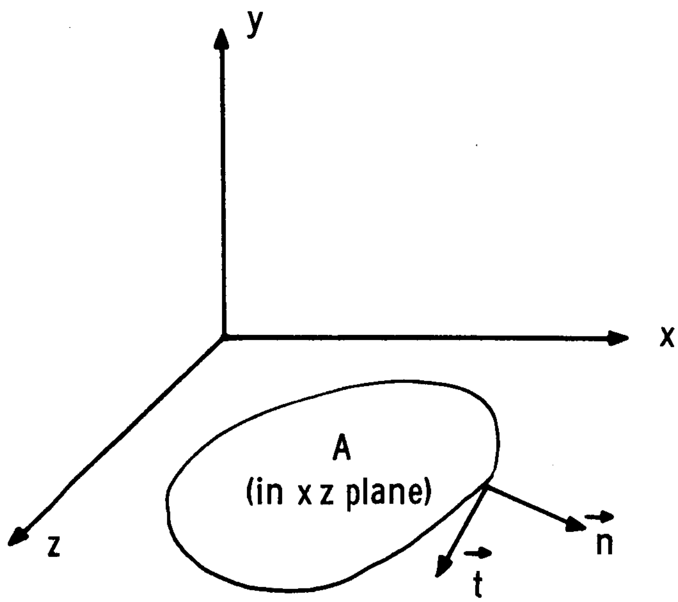
The above together with an equation of state are the required equations for solution of wave propagation problems. With given initial and boundary conditions specific problems can be solved within the limit of the Lagrangian code described.

III. FINITE-DIFFERENCE EQUATIONS

A. CONSERVATION OF MOMENTUM

The partial derivatives are set into finite difference form by the following method.

Integral definition of the partial derivatives



$$\frac{\partial \psi}{\partial x} = \lim_{A \to O} \frac{\oint_s F(\vec{n} \cdot \vec{i})\, d\ell}{A}$$

$$\frac{\partial \psi}{\partial z} = \lim_{A \to O} \frac{\oint_s F(\vec{n} \cdot \vec{k})\, d\ell}{A}$$

where s is the boundary of area A, ℓ an arc length, $\vec{n}$ the normal vector and $\vec{t}$ the tangent vector to the arc all lying in xz plane. These are related to the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ along the axes x, y, z respectively by the relations

$$\vec{t} = \vec{i}\,\frac{\partial x}{\partial s} + \vec{k}\,\frac{\partial z}{\partial s}$$

$$\vec{n} = \vec{i}\,\frac{\partial x}{\partial n} + \vec{k}\,\frac{\partial z}{\partial n}$$

$$= \vec{j} \times \vec{t}$$

$$= \vec{j} \times \left(\vec{i}\,\frac{\partial x}{\partial s} + \vec{k}\,\frac{\partial z}{\partial s}\right)$$

$$= \vec{i}\,\frac{\partial z}{\partial s} - \vec{k}\,\frac{\partial x}{\partial s}$$

On substitution we find that

$$\frac{\partial \psi}{\partial x} = \lim_{A \to O} \frac{\oint_s \psi \frac{\partial z}{\partial s}\, ds}{A}$$

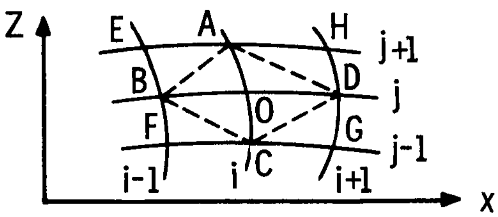
and

$$\frac{\partial \psi}{\partial z} = \lim_{A \to O} \frac{-\oint_s \psi \frac{\partial x}{\partial s}\, ds}{A}$$

Returning to the equations of motion

$$\frac{1}{\rho}\,\frac{\partial t^{xx}}{\partial x} = \lim_{A \to O} \frac{\oint t^{xx} \frac{\partial z}{\partial s}\, ds}{\rho A}$$

etc.



Applying these to quadrilateral ABCD,

$$\left(\frac{1}{\rho}\,\frac{\partial t^{xx}}{\partial x}\right)_0 = \frac{-1}{(\rho A)_0}\left\{(t^{xx})_1(z_D - z_A) + (t^{zz})_2(z_A - z_B) + (t^{xx})_3(z_B - z_C) + (t^{xx})_4(z_C - z_D)\right\}$$

$$\left(\frac{1}{\rho}\,\frac{\partial t^{xz}}{\partial z}\right)_0 = \frac{1}{(\rho A)_0}\left\{(t^{xz})_1(x_D - x_A) + (t^{xz})_2(x_A - x_B) + (t^{xz})_3(x_B - x_C) + (t^{xz})\,(x_C - x_D)\right\}$$

The term (ρA) appearing in these equations is required at 0, where ρ and A are quantities averaged over the meshes 1, 2, 3 and 4. This term is well represented by the expression

$$(A\rho)_o = A_{I1}\rho_1 + A_{I2}\rho_2 + A_{I3}\rho_3 + A_{I4}\rho_4$$

where A_{I1} is the area of triangle AOD, A_{I2} of AOB, A_{I3} of BOC, and A_{I4} of COD.

The last term on the right of the momentum equation is written

$$\left(\frac{t^{xx} - t^{yy}}{\rho x}\right)_o = \frac{1}{4}\left\{\frac{\left[(t^{xx})_1 - (t^{yy})_1\right]}{\rho_1 \bar{x}_{I1}} + \frac{\left[(t^{xx})_2 - (t^{yy})_2\right]}{\rho_2 \bar{x}_{I2}} + \frac{\left[(t^{xx})_3 - (t^{yy})_3\right]}{\rho_3 \bar{x}_{I3}} + \frac{\left[(t^{xx})_4 - (t^{yy})_4\right]}{\rho_4 \bar{x}_{I4}}\right\}$$

where

$$\bar{x}_{I1} = \frac{1}{3}(x_O + x_D + x_A)$$

$$\bar{x}_{I2} = \frac{1}{3}(x_O + x_A + x_B)$$

$$\bar{x}_{I3} = \frac{1}{3}(x_O + x_B + x_C)$$

$$\bar{x}_{I4} = \frac{1}{3}(x_O + x_C + x_D)$$

Similarly for the second momentum equation

$$\left(\frac{1}{\rho}\frac{\partial t^{xz}}{\partial x}\right)_o = \frac{-1}{(\rho A)_o}\left\{(t^{xz})_1(z_D - z_A) + (t^{xz})_2(z_A - z_B) + (t^{xz})_3(z_B - z_C) + (t^{xz})_4(z_C - z_D)\right\}$$

$$\left(\frac{1}{\rho}\frac{\partial t^{zz}}{\partial Z}\right)_o = \frac{1}{(\rho A)_o}\left\{(t^{zz})_1(x_D - x_A) + (t^{zz})_2(x_A - x_B) + (t^{zz})_3(x_B - x_C) + (t^{zz})_4(x_C - x_D)\right\}$$

and

$$\left(\frac{t^{zz}}{\rho x}\right)_o = \frac{1}{4}\left\{\frac{(t^{xz})_1}{\rho_1 \bar{x}_{I1}} + \frac{(t^{xz})_3}{\rho_2 \bar{x}_{I2}} + \frac{(t^{xz})_3}{\rho_3 \bar{x}_{I3}} + \frac{(t^{xz})_4}{\rho_4 \bar{x}_{I4}}\right\}$$

Substituting these finite difference quantities in the momentum equation enables us to evaluate the accelerations a^x and a^z:

$$(a^x)^n_o = -\frac{(t^{xx})^n_1 (z^n_D - z^n_A) + (t^{xx})^n_2 (z^n_A - z^n_B) + (t^{xx})^n_3 (z^n_B - z^n_C) + (t^{xx})^n_4 (z^n_C - z^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(t^{xz})^n_1 (x^n_D - x^n_A) + (t^{xz})^n_2 (x^n_A - x^n_B) + (t^{xz})^n_3 (x^n_B - x^n_C) + (t^{xz})^n_4 (x^n_C - x^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(\alpha - 1)}{4}\left\{\frac{\left[(t^{xx})^n_1 - (t^{yy})^n_1\right]}{\rho^n_1 \bar{x}^n_{I1}} + \frac{\left[(t^{xx})^n_2 - (t^{yy})^n_2\right]}{\rho^n_2 \bar{x}^n_{I2}} + \frac{\left[(t^{xx})^n_3 - (t^{yy})^n_3\right]}{\rho^n_3 \bar{x}^n_{I3}}\right.$$

$$\left. + \frac{\left[(t^{xx})^n_4 - (t^{yy})^n_4\right]}{\rho^n_4 \bar{x}^n_{I4}}\right\}$$

$$(a^z)^n_o = -\frac{(t^{xz})^n_1 (z^n_D - z^n_A) + (t^{xz})^n_2 (z^n_A - z^n_B) + (t^{xz})^n_3 (z^n_B - z^n_C) + (t^{xz})^n_4 (z^n_C - z^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(t^{zz})^n_1 (x^n_D - x^n_A) + (t^{zz})^n_2 (x^n_A - x^n_B) + (t^{zz})^n_3 (x^n_B - x^n_C) + (t^{zz})^n_4 (x^n_C - x^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(\alpha - 1)}{4}\left\{\frac{(t^{xz})^n_1}{\rho^n_1 \bar{x}^n_{I1}} + \frac{(t^{xz})^n_2}{\rho^n_2 \bar{x}^n_{I2}} + \frac{(t^{xz})^n_3}{\rho^n_3 \bar{x}^n_{I3}} + \frac{(t^{xz})^n_4}{\rho^n_4 \bar{x}^n_{I4}}\right\}$$

The areas in these equations are given by

$$A_{I1}^n = \frac{1}{2}\left\{\left(x_D^n - x_O^n\right)\left(z_A^n - z_O^n\right) - \left(z_D^n - z_O^n\right)\left(x_A^n - x_O^n\right)\right\}$$

$$A_{I2}^n = \frac{1}{2}\left\{\left(x_A^n - x_O^n\right)\left(z_B^n - z_O^n\right) - \left(z_A^n - z_O^n\right)\left(x_B^n - x_O^n\right)\right\}$$

$$A_{I3}^n = \frac{1}{2}\left\{\left(x_B^n - x_O^n\right)\left(z_C^n - z_O^n\right) - \left(z_B^n - z_O^n\right)\left(x_C^n - x_O^n\right)\right\}$$

$$A_{I4}^n = \frac{1}{2}\left\{\left(x_C^n - x_O^n\right)\left(z_D^n - z_O^n\right) - \left(z_C^n - z_O^n\right)\left(x_D^n - x_O^n\right)\right\}$$

Knowing the accelerations, the velocities and the coordinates of the vertices at time (n+1) may be evaluated

$$\left(u^x\right)_O^{n+1/2} = \left(u^x\right)_O^{n-1/2} + 1/2\left(\Delta t^{n+1/2} + \Delta t^{n-1/2}\right)\left(a^x\right)_O^n$$

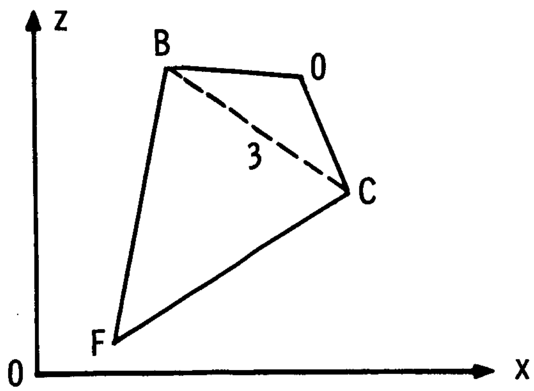
$$\left(u^z\right)_O^{n+1/2} = \left(u^z\right)_O^{n-1/2} + 1/2\left(\Delta t^{n+1/2} + \Delta t^{n-1/2}\right)\left(a^z\right)_O^n$$

and

$$x_O^{n+1} = x_O^n + \Delta t^{n+1/2}\left(u^x\right)_O^{n+1/2}$$

$$z_O^{n+1} = z_O^n + \Delta t^{n+1/2}\left(u^z\right)_O^{n+1/2}$$

B. CONTINUITY EQUATION



Knowing the positions of the vertices for the quadrilaterals at time (n+1) the densities may be calculated, e.g.,

$$\rho_3^{n+1} = \frac{M_3}{A_{I3}^{n+1}\left(\bar{x}_{I3}^{n+1}\right)^{\alpha-1} + A_{\Phi 3}^{n+1}\left(\bar{x}_{\Phi 3}^{n+1}\right)^{\alpha-1}}$$

where

$$A_{I3}^{n+1} = \frac{1}{2}\left\{\left(x_B^{n+1} - x_o^{n+1}\right)\left(z_C^{n+1} - z_o^{n+1}\right) - \left(z_B^{n+1} - z_o^{n+1}\right)\left(x_C^{n+1} - x_o^{n+1}\right)\right\}$$

$$A_{\Phi 3}^{n+1} = \frac{1}{2}\left\{\left(x_C^{n+1} - x_F^{n+1}\right)\left(z_B^{n+1} - z_F^{n+1}\right) - \left(z_C^{n+1} - z_F^{n+1}\right)\left(x_B^{n+1} - x_F^{n+1}\right)\right\}$$

$$\bar{x}_{I3}^{n+1} = \frac{1}{3}\left(x_B^{n+1} + x_C^{n+1} + x_o^{n+1}\right)$$

$$\bar{x}_{\Phi 3}^{n+1} = \frac{1}{3}\left(x_C^{n+1} + x_B^{n+1} + x_F^{n+1}\right)$$

M_3 is given by

$$M_3 = \rho_3^o\left\{A_{I3}^o\left(\bar{x}_{I3}^o\right)^{\alpha-1} + A_{\Phi 3}^o\left(\bar{x}_{\Phi 3}^o\right)^{\alpha-1}\right\}$$

since mass is conserved within each zone.

Using the evaluated density

$$\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2} = \frac{2}{\Delta t^{n+1/2}} \frac{\rho_3^{n+1} - \rho_3^n}{\rho_3^{n+1} + \rho_3^n}$$

$$\left(\frac{\Delta\rho}{\rho^2}\right)_3^{n+1/2} = 4\,\frac{\rho_3^{n+1} - \rho_3^n}{\left(\rho_3^{n+1} + \rho_3^n\right)^2}$$

and

$$\eta_3^{n+1} = \frac{\rho_3^{n+1}}{\rho_3^o}$$

C. ENERGY EQUATION

This energy equation in finite difference form becomes

$$\varepsilon_3^{n+1} - \varepsilon_3^n = \frac{1}{2}\left(p_3^{n+1} + p_3^n\right)\left(\frac{\Delta\rho}{\rho^2}\right)_3 + \Delta_q\varepsilon_3 + \Delta^d\varepsilon_3$$

$$\Delta_q\varepsilon_3 = \frac{1}{2}\left(q_3^{n+1/2} + q_3^{n-1/2}\right)\left(\frac{\Delta\rho}{\rho^2}\right)_3$$

$$\Delta^d\varepsilon_3 = \Delta_e^d\varepsilon_3 + \Delta_q^d\varepsilon_3$$

$$\Delta_e^d\varepsilon_3 = \frac{\Delta t^{n+1/2}}{\rho_3^o\left(\eta_3^{n+1} + \eta_3^n\right)}\left\{\left[\left(d_t xx\right)_3^{n+1} + \left(d_t xx\right)_3^n\right]\left[2\left(d_{\dot{e}} xx\right)_3^{n+1/2} + \left(d_{\dot{e}} zz\right)_3^{n+1/2}\right] + \left[\left(d_t zz\right)_3^{n+1} + \left(d_t zz\right)_3^n\right]\left[2\left(d_{\dot{e}} zz\right)_3^{n+1/2} + \left(d_{\dot{e}} xx\right)_3^{n+1/2}\right] + \left[\left(d_t xz\right)_3^{n+1} + \left(d_t xz\right)_3^n\right]\left[2\left(d_{\dot{e}} xz\right)_3^{n+1/2}\right]\right\}$$

and

$$\Delta_q^d\varepsilon_3 = \frac{\Delta t^{n+1/2}}{\rho_3^o\left(\eta_3^{n+1} + \eta_3^n\right)}\left\{\left[\left(d_q xx\right)_3^{n+1/2} + \left(d_q xx\right)_3^{n-1/2}\right]\left[2\left(d_{\dot{e}} xx\right)_3^{n+1/2} + \left(d_{\dot{e}} zz\right)_3^{n+1/2}\right] + \left[\left(d_q zz\right)_3^{n+1/2} + \left(d_q zz\right)_3^{n-1/2}\right]\left[2\left(d_{\dot{e}} zz\right)_3^{n+1/2} + \left(d_{\dot{e}} xx\right)_3^{n+1/2}\right] + \left[\left(d_q xz\right)_3^{n+1/2} + \left(d_q xz\right)_3^{n-1/2}\right]\left[2\left(d_{\dot{e}} xz\right)_3^{n+1/2}\right]\right\}$$

where

$$\left(t^{xx}\right)^{n+1} = -\left(p^{n+1} + q^{n+1/2}\right) + \left(d_t^{xx}\right)^{n+1} + \left(d_q^{xx}\right)^{n+1/2}$$

$$\left(t^{yy}\right)^{n+1} = -3\left(p^{n+1} + q^{n+1/2}\right) - \left(t^{xx}\right)^{n+1} - \left(t^{zz}\right)^{n+1}$$

$$\left(t^{zz}\right)^{n+1} = -\left(p^{n+1} + q^{n+1/2}\right) + \left(d_t^{zz}\right)^{n+1} + \left(d_q^{zz}\right)^{n+1/2}$$

$$\left(t^{xz}\right)^{n+1} = \left(d_t^{xz}\right)^{n+1} + \left(d_q^{xz}\right)^{n+1/2}$$

q being an artificial viscosity to be introduced later.

Using the definition derived earlier for partial derivative, namely

$$\frac{\partial u^x}{\partial x} = \lim_{A \to O} \frac{\oint_s u^x \frac{\partial z}{\partial s} \, ds}{A}$$

the deviatoric strain rates can be evaluated. The required partial derivatives for quadrilateral OBFC are

$$\left(\frac{\partial u^x}{\partial x}\right)_3^{n+1/2} = \frac{-2}{A_3^{n+1} + A_3^n} \left\{ \frac{\left[\left(u^x\right)_C^{n+1/2} + \left(u^x\right)_O^{n+1/2}\right]}{2} \frac{\left[z_C^{n+1} + z_C^n - z_O^{n+1} - z_O^n\right]}{2} \right.$$

$$+ \frac{\left[\left(u^x\right)_O^{n+1/2} + \left(u^x\right)_B^{n+1/2}\right]}{2} \frac{\left[z_O^{n+1} + z_O^n - z_B^{n+1} - z_B^n\right]}{2}$$

$$+ \frac{\left[\left(u^x\right)_B^{n+1/2} + \left(u^x\right)_F^{n+1/2}\right]}{2} \frac{\left[z_B^{n+1} + z_B^n - z_F^{n+1} - z_F^n\right]}{2}$$

$$\left. + \frac{\left[\left(u^x\right)_F^{n+1/2} + \left(u^x\right)_C^{n+1/2}\right]}{2} \frac{\left[z_F^{n+1} + z_F^n - z_C^{n+1} - z_C^n\right]}{2} \right\}$$

$$= \frac{-1}{2\left(A_3^{n+1} + A_3^n\right)} \left\{ \left[\left(u^x\right)_O^{n+1/2} - \left(u^x\right)_F^{n+1/2} \right] \left[z_C^{n+1} + z_C^n - z_B^{n+1} - z_B^n \right] - \left[\left(u^x\right)_C^{n+1/2} - \left(u^x\right)_B^{n+1/2} \right] \left[z_O^{n+1} + z_O^n - z_F^{n+1} - z_F^n \right] \right\}$$

$$\left(\frac{\partial u^z}{\partial z}\right)_3^{n+1/2} = \frac{1}{2\left(A_3^{n+1} + A_3^n\right)} \left[\left(u^z\right)_O^{n+1/2} - \left(u^z\right)_F^{n+1/2} \right] \left[x_C^{n+1} + x_C^n - x_B^{n+1} - x_B^n \right] - \left[\left(u^z\right)_C^{n+1/2} - \left(u^z\right)_B^{n+1/2} \right] \left[x_O^{n+1} + x_O^n - x_F^{n+1} - x_F^n \right]$$

$$\left(\frac{\partial u^z}{\partial x}\right)_3^{n+1/2} = \frac{-1}{2\left(A_3^{n+1} + A_3^n\right)} \left\{ \left[\left(u^z\right)_O^{n+1/2} - \left(u^z\right)_F^{n+1/2} \right] \left[z_C^{n+1} + z_C^n - z_B^{n+1} - z_B^n \right] - \left[\left(u^z\right)_C^{n+1/2} - \left(u^z\right)_B^{n+1/2} \right] \left[z_O^{n+1} + z_O^n - z_F^{n+1} - z_F^n \right] \right\}$$

and

$$\left(\frac{\partial u^x}{\partial z}\right)_3^{n+1/2} = \frac{1}{2\left(A_3^{n+1} + A_3^n\right)} \left\{ \left[\left(u^x\right)_O^{n+1/2} - \left(u^x\right)_F^{n+1/2} \right] \left[x_C^{n+1} + x_C^n - x_B^{n+1} - x_B^n \right] - \left[\left(u^x\right)_C^{n+1/2} - \left(u^x\right)_B^{n+1/2} \right] \left[x_O^{n+1} + x_O^n - x_F^{n+1} - x_F^n \right] \right\}$$

Substituting in the expression for the deviatoric strain rate and spin

$$\left(d_e xx\right)_3^{n+1/2} = \frac{-1}{2\left(A_3^{n+1} + A_3^n\right)} \left\{ \left[\left(u^x\right)_O^{n+1/2} - \left(u^x\right)_F^{n+1/2} \right] \left[z_C^{n+1} + z_C^n - z_B^{n+1} - z_B^n \right] - \left[(u^x)_C^{n+1/2} - (u^x)_B^{n+1/2} \right] \left[z_O^{n+1} + z_O^n - z_F^{n+1} - z_F^n \right] \right\} + \frac{1}{3}\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2}$$

$$\left(d_e^{\cdot}yy\right)_3^{n+1/2} = -\left\{\left(d_e^{\cdot}xx\right)_3^{n+1/2} + \left(d_e^{\cdot}zz\right)_3^{n+1/2}\right\}$$

$$\left(d_e^{\cdot}zz\right)_3^{n+1/2} = \frac{1}{2(A_3^{n+1}+A_3^n)}\left\{\left[(u^z)_O^{n+1/2} - (u^z)_F^{n+1/2}\right]\left[x_C^{n+1} + x_C^n - x_B^{n+1} - x_B^n\right]\right.$$

$$\left. - \left[(u^z)_C^{n+1/2} - (u^z)_B^{n+1/2}\right]\left[x_o^{n+1} + x_O^n - x_F^{n+1} - x_F^n\right]\right\} + \frac{1}{3}\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2}$$

$$\left(d_e^{\cdot}xz\right)_3^{n+1/2} = \frac{1}{4(A_3^{n+1}+A_3^n)}\left\{\left[(u^x)_O^{n+1/2} - (u^x)_F^{n+1/2}\right]\left[x_C^{n+1} + x_C^n - x_B^{n+1} - x_B^n\right]\right.$$

$$- \left[(u^x)_C^{n+1/2} - (u^x)_B^{n+1/2}\right]\left[x_O^{n+1} + x_O^n - x_F^{n+1} - x_F^n\right]$$

$$- \left[(u^z)_O^{n+1/2} - (u^z)_F^{n+1/2}\right]\left[z_C^{n+1} + z_C^n - z_B^{n+1} - z_B^n\right]$$

$$\left. + \left[(u^z)_C^{n+1/2} - (u^z)_B^{n+1/2}\right]\left[z_O^{n+1} + z_O^n - z_F^{n+1} - z_F^n\right]\right\}$$

and

$$(w^{xz})_3^{n+1/2} = \frac{1}{4(A_3^{n+1}+A_3^n)}\left\{\left[(u^x)_O^{n+1/2} - (u^x)_F^{n+1/2}\right]\left[x_C^{n+1} + x_C^n - x_B^{n+1} - x_B^n\right]\right.$$

$$- \left[(u^x)_C^{n+1/2} - (u^x)_B^{n+1/2}\right]\left[x_O^{n+1} + x_O^n - x_F^{n+1} - x_F^n\right]$$

$$+ \left[(u^z)_O^{n+1/2} - (u^z)_F^{n+1/2}\right]\left[z_C^{n+1} + z_C^n - z_B^{n+1} - z_B^n\right]$$

$$\left. - \left[(u^z)_C^{n+1/2} - (u^z)_B^{n+1/2}\right]\left[z_O^{n+1} + z_O^n - z_F^{n+1} - z_F^n\right]\right\}$$

D. ARTIFICIAL VISCOSITY AND STABILITY CONDITION

Numerical calculations which include the effect of a moving boundary, such as a shock wave, become complicated because shocks move relative to the grid system. Furthermore, the motion of the discontinuity is not known in advance but is governed by the differential equations and the Rankine-Hugoniot equations. As a result, the treatment of shocks requires lengthy calculations at each step in time.

A scheme which overcomes this deficiency calls for the inclusion of an artificial viscosity in the calculations.(6) This smears out the shock so that the discontinuity varies rapidly but continuously (typically about three grids). Shocks are readily recognized in the numerical results as near discontinuities which move through the media at nearly the correct speed and across which the pressure, density, etc. have nearly the correct values, provided the thickness of the shock layer is small in comparison with other physically relevant dimensions of the system. The inclusion of the artificial viscosity term does require a shorter time step for stability; however, this is not serious provided the artificial viscosity is only sufficient to produce a shock thickness comparable with the grid size.

The artificial viscosity used in the present calculations are composed of two terms. Following von Neumann and Richtmyer(6) the first term is a bulk viscosity coefficient dependent on the dilatation so that stresses are negligible in areas of moderate gradient. The second term is to provide damping in the sonic range, giving a composite term

$$q_3^{n+1/2} = \frac{1}{2}\,\rho_3^{o}\left(\eta_3^{n+1} + \eta_3^{n}\right)\left\{C_1^2\left(\frac{A_3^{n+1} + A_3^{n}}{2}\right)\left[\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2}\right]^2 + C_2\left(\frac{A_3^{n+1} + A_3^{n}}{2}\right)^{1/2} C_3^{n}\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2}\right\}$$

if

$$\left(\frac{\dot{\rho}}{\rho}\right)_3^{n+1/2} > 0$$

otherwise

$$q_3^{n+1/2} = 0$$

where $C_1 \simeq 2.0$ and $C_2 \simeq 0.1$.

Viscous stress deviator, analogous to the viscous pressure, may also be introduced. The following are listed within the program.

$$\left(d_q xx\right)_3^{n+1/2} = \frac{1}{2} C_3 \rho_3^o \left(\eta_3^{n+1} + \eta_3^n\right) C_3^n \left(\frac{A_3^{n+1} + A_3^n}{2}\right)^{1/2} \left(d_{\dot{e}} xx\right)_3^{n+1/2}$$

$$\left(d_q yy\right)_3^{n+1/2} = -\left[\left(d_q xx\right)_3^{n+1/2} + \left(d_q zz\right)_3^{n+1/2}\right]$$

$$\left(d_q zz\right)_3^{n+1/2} = \frac{1}{2} C_3 \rho_3^o \left(\eta_3^{n+1} + \eta_3^n\right) C_3^n \left(\frac{A_3^{n+1} + A_3^n}{2}\right)^{1/2} \left(d_{\dot{e}} zz\right)_3^{n+1/2}$$

$$\left(d_q xz\right)_3^{n+1/2} = \frac{1}{2} C_3 \rho_3^o \left(\eta_3^{n+1} + \eta_3^n\right) C_3^n \left(\frac{A_3^{n+1} + A_3^n}{2}\right)^{1/2} \left(d_{\dot{e}} xz\right)_3^{n+1/2}$$

In the above equations C_1, C_2, C_3 are constants and the velocity of sound to be used, C_3^n, is approximated by

$$C_3^n = a_o + b_o \left(p_3^n\right)^{\epsilon}$$

a_o, b_o, ϵ being constants.

The stability criterion used for the two-dimensional case is

$$\Delta t^{n+3/2} = \left. \frac{K\delta}{\sqrt{\left(C_3^n\right)^2 + 4C_1^2 \delta^2 \left|\frac{\dot{\rho}}{\rho}\right|^2}} \right| \text{minimum over } j$$

for $|\dot{\rho}/\rho| > 0$, otherwise

$$\Delta t^{n+3/2} = \frac{K\delta}{C_3^n}$$

where δ is the area of zone 3 divided by the longest diagonal, K an artificial stability criterion.

If the calculated time step

$$\Delta t^{n+3/2} > (1.1)\Delta t^{n+1/2}$$

then

$$\Delta t^{n+3/2} = (1.1)\Delta t^{n+1/2}$$

is used as the time step.

E. BOUNDARY CONDITIONS

At the boundaries the equations of motion must allow for either a free or fixed surface. Along fixed surfaces the material may slide, but it may not move normal to the wall. To calculate the accelerations along the wall, phantom zones are considered to exist on the opposite side of the fixed surface, for which the stresses t^{xx}, t^{yy}, t^{zz}, density ρ , radius $\bar{\bar{x}}$, are equal, while the shear stress reflects antisymmetrically.

Consider the motion of point O along the fixed boundary on the z-axis. The coordinate position of point B is given by

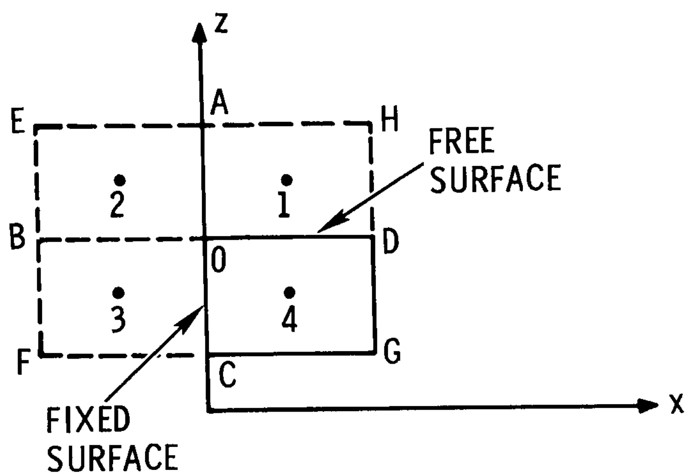
$$x_B = 2x_O - x_D \, , \; z_B = z_D$$

Similarly for a fixed boundary along the x-axis coordinates of C are

$$x_C = x_A \quad z_C = 2z_O - z_A \; .$$

Phantom zones on the opposing side of a free surface are given zero stresses and density.

Exactly the same scheme can be used for corner zones, with fixed-fixed, free-fixed, or free-free boundary without any ambiguity. As an example, consider the corner zone on the Z-axis.



$$(a^x)_0^n = 0 \qquad z_B = z_D$$

$$(u^x)_0^{n+1/2} = 0 \qquad x_B = 2x_O - x_D$$

$$(t^{zz})_1^n = (t^{zz})_2^n = 0 \qquad A_{I1}^n = A_{I2}^n = 0$$

$$(t^{zz})_3^n = (t^{zz})_4^n \qquad A_{I3}^n = A_{I4}^n$$

$$(t^{xz})_1^n = -(t^{xz})_2^n = 0 \qquad \rho_1^n \bar{x}_{I1} = \rho_2^n \bar{x}_{I2} = 0$$

$$(t^{xz})_4^n = -(t^{xz})_3^n \qquad \rho_3^n \bar{x}_{I3} = \rho_4^n \bar{x}_{I4}$$

$$\rho_3 = \rho_4$$

For these conditions

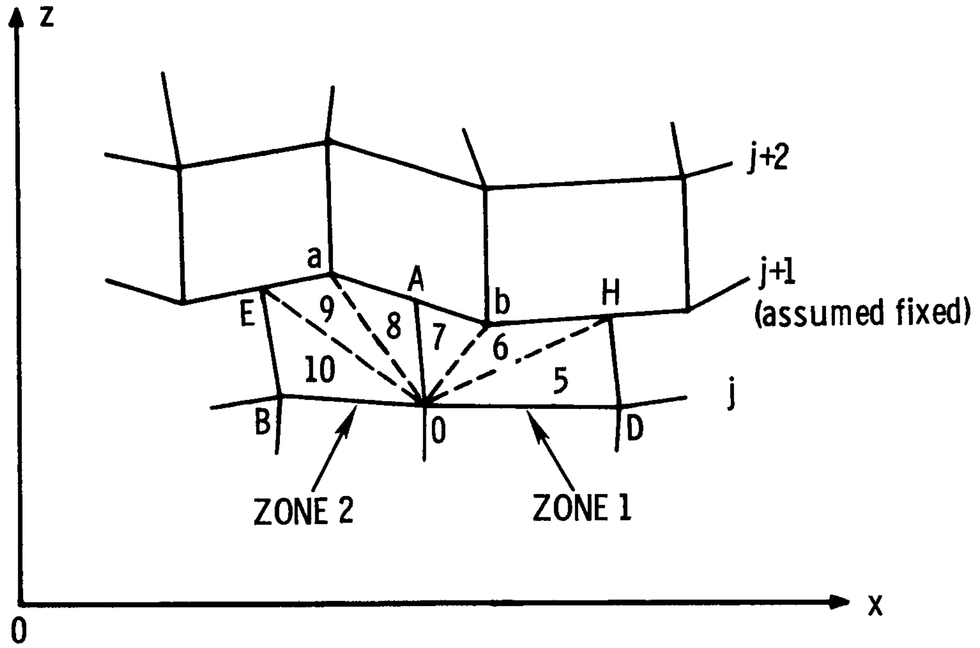
$$(a^z)_0^n = -\frac{(t^{xz})_4^n (z_C^n - z_D^n)}{A_{I4}^n \rho_4^n} + \frac{(t^{zz})_4^n (x_C^n - x_D^n)}{A_{I4}^n \rho_4^n}$$

Interfaces between materials require no special treatment, unless the materials are allowed to slide relative to each other. This case is treated below.

F. SLIDING INTERFACE

The problem of interest in the present study involves the pulse propagating in a plate on impact by a gaseous debris. It is important in this case to allow the two materials to slide relative to one another, otherwise the gaseous interface zones become excessively distorted in a very short time. We follow the approach initiated by Wilkins,[3, 7] which considers a simple decoupling of the grid points at the interface. The upper material interface is considered as a fixed boundary for a given time step Δt . During this time the equations of motion for the sliding material are the same as those given for motion along a fixed boundary. The fixed boundary is then advanced in time using the force field of the sliding boundary next to it. The new position of the fixed boundary provides a new boundary for the sliding material.

(1) Point A on the slide line (j+1)



In the diagram above, the acceleration of A along the fixed boundary ab is

$$(a^{ab})^n_A = \left[\frac{\left(p^n_1 + q^{n-1/2}_1\right)\left(z^n_O - z^n_H\right) + \left(p^n_2 + q^{n-1/2}_2\right)\left(z^n_E - z^n_O\right)}{\rho^n_1\left(A^n_6 + A^n_7\right) + \rho^n_2\left(A^n_8 + A^n_9\right)}\right]\cos\theta_{ab}$$

$$-\left[\frac{\left(p^n_1 + q^{n-1/2}_1\right)\left(x^n_O - x^n_H\right) + \left(p^n_2 + q^{n-1/2}_2\right)\left(x^n_E - x^n_O\right)}{\rho^n_1\left(A^n_6 + A^n_7\right) + \rho^n_2\left(A^n_8 + A^n_9\right)}\right]\sin\theta_{ab}$$

where

$$\sin\theta_{ab} = \frac{z^n_b - z^n_a}{\sqrt{\left(x^n_b - x^n_a\right)^2 + \left(z^n_b - z^n_a\right)^2}}$$

$$\cos\theta_{ab} = \frac{x^n_b - x^n_a}{\sqrt{\left(x^n_b - x^n_a\right)^2 + \left(z^n_b - z^n_a\right)^2}}$$

The velocities and position of A after a time increment (A moves to *A) for the above restrictions are given by:

$$(u^x)^{n+1/2}_{*A} = (u^x)^{n-1/2}_O + \frac{1}{2}\left(\Delta t^{n+1/2} + \Delta t^{n-1/2}\right)\left(a^{ab}\cos\theta_{ab}\right)^n_A$$

$$(u^z)^{n+1/2}_{*A} = (u^z)^{n-1/2}_O + \frac{1}{2}\left(\Delta t^{n+1/2} + \Delta t^{n-1/2}\right)\left(a^{ab}\sin\theta_{ab}\right)^n_A$$

$$x^{n+1}_{*A} = x^n_A + \Delta t^{n+1/2}\,(u^x)^{n+1/2}_{*A}$$

$$z^{n+1}_{*A} = z^n_A + \Delta t^{n+1/2}\,(u^z)^{n+1/2}_{*A}$$

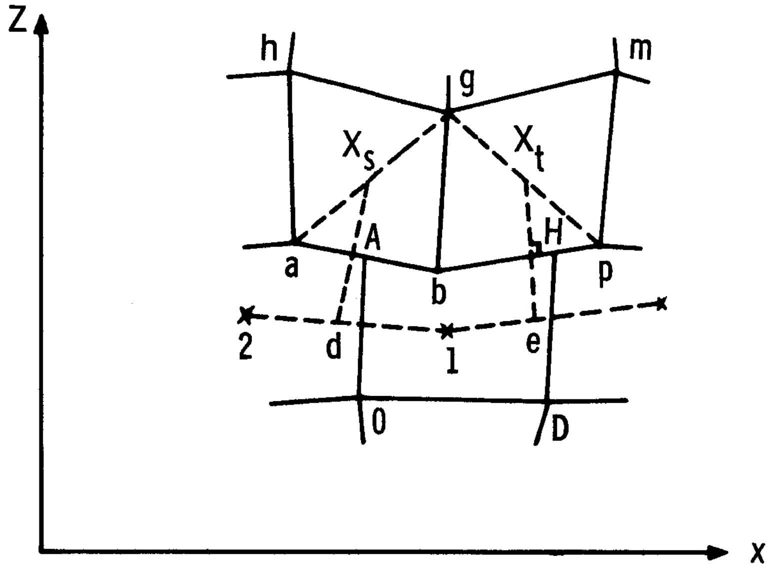
Here the velocities at point A at time (n-1/2) are assumed to be the same as the velocities of point O. Also the point *A may pass to another line segment.

The slope of line O*A is given by

$$\frac{z_{*A}^{n+1} - z_O^{n+1}}{x_{*A}^{n+1} - x_O^{n+1}} = (\tan \theta_{O*A})^{n+1} = (\tan \theta_{OA})^{n+1}$$

To advance in time points on the slide line.

The motion of points a, b, etc. are calculated assuming that the grid points are accelerated by the forces from below as well as above the interface. Forces from below the interface are calculated by the scheme given below.



Given the points 1, 2, x_s, etc. where

$$\bar{x}_1^n = \frac{1}{4}(x_O^n + x_D^n + x_H^n + x_A^n)$$

$$\bar{z}_1^n = \frac{1}{4}(z_O^n + z_D^n + z_H^n + z_A^n)$$

$$\bar{x}_2^n = \frac{1}{4}(x_O^n + x_A^n + x_E^n + x_B^n)$$

$$\bar{z}_2^n = \frac{1}{4}(z_O^n + z_A^n + z_E^n + z_B^n)$$

$$\bar{x}_S^n = \frac{1}{4}(x_a^n + x_b^n + x_g^n + x_h^n)$$

$$\bar{z}_S^n = \frac{1}{4}(z_a^n + z_b^n + z_g^n + z_h^n)$$

and the slope m_2 of the line through X_S perpendicular to ab, i.e.

$$m_2 = -\cot\theta_{ab}$$

$$= -\frac{x_b^n - x_a^n}{z_b^n - z_a^n}$$

which bisects the line 12 at d. In order to calculate the coordinates of point d, it is necessary to find between which points d lies.

For each point, i.e. 1, 2, etc., calculate

$$x_2^1 - x_S^1 = (x_2 - x_O)\cos\theta_{ab} - (z_2 - z_S)\sin\theta_{ab}$$

Test consecutive values of $(x_i^1 - x_S^1)$ until a change in sign is found. The points 1 and 2 between which d lies will then have been located. Coordinates of point d can now be calculated.

$$x_d^n = \frac{\left(m_{12}\bar{x}_2^n - m_2\bar{x}_s^n + \bar{z}_s^n - \bar{z}_2^n\right)}{m_{12} - m_2}$$

$$z_d^n = \frac{m_2\left[\bar{z}_2^n - m_{12}\left(\bar{x}_2^n - \bar{x}_s^n\right)\right] - m_{12}\bar{z}_s^n}{m_2 - m_{12}}$$

$$m_{12} = \frac{\bar{z}_1^n - \bar{z}_2^n}{\bar{x}_1^n - \bar{x}_2^n}$$

If $|m_{12}| < 10^{-4}$ and $|m_2| > 10^4$, $x_d = \bar{x}_S^n$ and $z_d = \bar{z}_2^n$.

The stresses at d are calculated as follows:

$$p_d^n + q_d^{n-1/2} = \frac{\left(p_1^n + q_1^{n-1/2}\right)\left|d - X_2\right| + \left(p_2^n + q_2^{n-1/2}\right)\left|d - X_1\right|}{\left|X_1 - X_2\right|}$$

where

$$\left|d - X_2\right| = \sqrt{\left(x_d^n - \bar{x}_2^n\right)^2 + \left(z_d^n - \bar{z}_2^n\right)^2}$$

$$\left|d - X_1\right| = \sqrt{\left(x_d^n - \bar{x}_1^n\right)^2 + \left(z_d^n - \bar{z}_2^n\right)^2}$$

$$\left|X_1 - X_2\right| = \sqrt{\left(\bar{x}_1^n - \bar{x}_2^n\right)^2 + \left(\bar{z}_1^n - \bar{z}_2^n\right)^2}$$

Similarly the stresses at e are found by following the same steps.

The accelerations at point b are calculated parallel and perpendicular to ap in the following manner:

$$(\ddot{r})^n = -\frac{(t^{xx})_t^n(z_p^n - z_g^n) + (t^{xx})_s^n(z_g^n - z_a^n) - (p_d^n + q_d^{n-1/2})(z_a^n - z_b^n) - (p_e^n + q_e^{n-1/2})(z_b^n - z_p^n)}{A_{It}^n\rho_t^n + A_{Is}^n\rho_s^n}$$

$$+ \frac{(t^{xz})_t^n(x_p^n - x_g^n) + (t^{xz})_s^n(x_g^n - x_a^n)}{A_{It}^n\rho_t^n + A_{Is}^n\rho_s^n}$$

$$+ \frac{(\alpha-1)}{2}\left\{\frac{(t^{xx})_t^n - (t^{yy})_t^n}{\rho_t^n\,\bar{x}_{It}^n} + \frac{(t^{xx})_s^n - (t^{yy})_s^n}{\rho_s^n\,\bar{x}_{Is}^n}\right\}$$

and

$$(g)^n = -\frac{(t^{xz})_t^n(z_p^n - z_g^n) + (t^{xz})_s^n(z_g^n - z_a^n)}{A_{It}^n\rho_t^n + A_{Is}^n\rho_s^n}$$

$$+\frac{(t^{zz})_t^n(x_p^n - x_g^n) + (t^{zz})_s^n(x_g^n - x_a^n) - (p_d^n + q_d^{n-1/2})(x_a^n - x_b^n) - (p_e^n + q_e^{n-1/2})(x_b^n - x_p^n)}{A_{It}^n\rho_t^n + A_{Is}^n\rho_s^n}$$

$$+\frac{(\alpha - 1)}{2}\left\{\frac{(t^{xz})_t^n}{\rho_t^n\,\bar{x}_t^n} + \frac{(t^{xz})_s^n}{\rho_s^n\,\bar{x}_s^n}\right\}$$

where

$$\bar{x}_t^n = \frac{1}{3}(x_b^n + x_p^n + x_g^n)$$

$$\bar{x}_s^n = \frac{1}{3}(x_a^n + x_b^n + x_g^n)$$

The accelerations of b parallel and perpendicular to ap are given by

$$(R)_b^n = (r)^n(\cos\theta_{ap})^n + (g)^n(\sin\theta_{ap})^n \quad \text{parallel to ap}$$

$$(G)_b^n = \frac{M_t}{M_t + M_1}\left\{-(r)^n(\sin\theta_{ap})^n + (g)^n(\cos\theta_{ap})^n\right\} \quad \text{perpendicular to ap}$$

where

$$(\sin\theta_{ap})^n = \frac{z_p^n - z_a^n}{\sqrt{(z_p^n - z_a^n)^2 + (x_p^n - x_a^n)^2}}$$

$$(\cos\theta_{ap})^n = \frac{x_p^n - x_a^n}{\sqrt{(z_p^n - z_a^n)^2 + (x_p^n - x_a^n)^2}}$$

Using the values calculated above for the accelerations, the velocities and position of point b at a later time are:

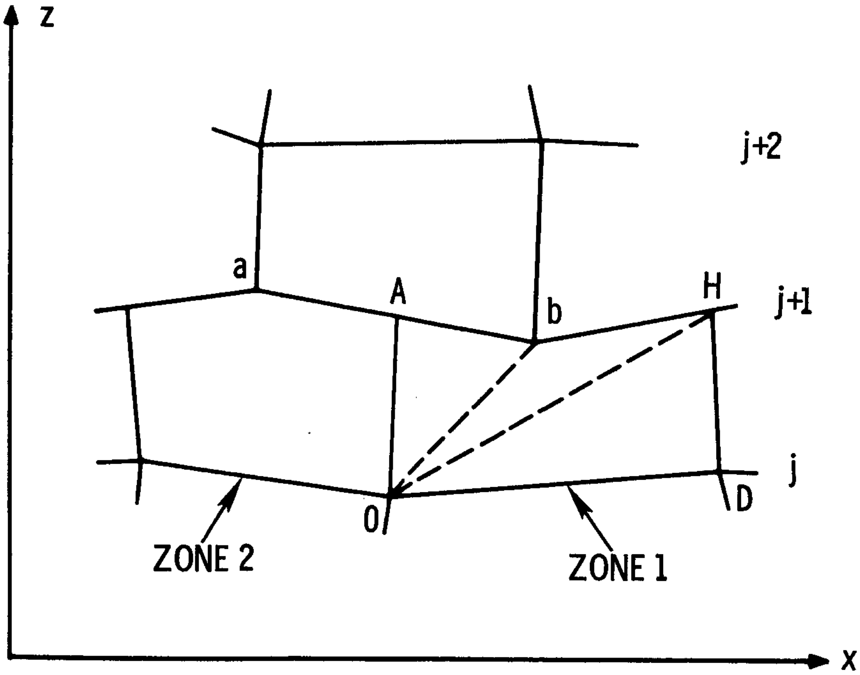
$$(u^x)_b^{n+1/2} = (u^x)_b^{n-1/2} + \frac{1}{2}(\Delta t^{n+1/2} + \Delta t^{n-1/2})\left[(R)_b^n (\cos\theta_{ap})^n - (G)_b^n (\sin\theta_{ap})^n\right]$$

$$(u^z)_b^{n+1/2} = (u^z)_b^{n-1/2} + \frac{1}{2}(\Delta t^{n+1/2} + \Delta t^{n-1/2})\left[(R)_b^n (\sin\theta_{ap})^n + (G)_b^n (\cos\theta_{ap})^n\right]$$

$$x_b^{n+1} = x_b^n + \Delta t^{n+1/2} (u^x)_b^{n+1/2}$$

$$z_b^{n+1} = z_b^n + \Delta t^{n+1/2} (u^z)_b^{n+1/2}$$

At time (n+1) it is necessary to search for points between which A lies and the coordinates of point A , as well as the densities and areas in zones 1, 2, etc., and 5, 6, 7, 8, 9, 10, etc. We begin with the knowledge of the coordinates of 0 and the slope of the line OA (see diagram below)



For each point a, b, p, etc. we calculate

$$(\tan\theta_{AOb})^{n+1} = \frac{(\tan\theta_{OA})^{n+1} - (\tan\theta_{Ob})^{n+1}}{1 + (\tan\theta_{OA})^{n+1} (\tan\theta_{Ob})^{n+1}}$$

where

$$(\tan \theta_{Ob})^{n+1} = \frac{z_b^{n+1} - z_O^{n+1}}{x_b^{n+1} - x_O^{n+1}}$$

with the following limitations.

(i) if $|\tan \theta_{OA}| > 10^4 \quad \tan \theta_{AOb} = \frac{1}{\tan \theta_{Ob}}$

(ii) if $|\tan \theta_{Ob}| > 10^4 \quad \tan \theta_{AOb} = -\frac{1}{\tan \theta_{OA}}$

(iii) if $|\tan \theta_{OA}| > 10^4$ and $|\tan \theta_{Ob}| > 10^4$, $\tan \theta_{AOb} = 0$

Test for $\tan \theta_{AOp}$ etc., i.e. consecutive points along the slide line, until a change of sign is found. The points where this will occur are the points between which A lies, i.e. a and b in the diagram.

The coordinates of A may now be found

$$x_A^{n+1} = \frac{\left[z_a^{n+1} - z_O^{n+1} + (\tan \theta_{OA})^{n+1} x_O^{n+1} - (\tan \theta_{ab})^{n+1} x_a^{n+1}\right]}{\left[(\tan \theta_{OA})^{n+1} - (\tan \theta_{ab})^{n+1}\right]}$$

$$z_A^{n+1} = \frac{(\tan \theta_{OA})^{n+1}\left[z_a^{n+1} - (\tan \theta_{ab})^{n+1}(x_a^{n+1} - x_O^{n+1})\right] - (\tan \theta_{ab})^{n+1} z_O^{n+1}}{\left[(\tan \theta_{OA})^{n+1} - (\tan \theta_{ab})^{n+1}\right]}$$

$$(\tan \theta_{ab})^{n+1} = \frac{z_b^{n+1} - z_a^{n+1}}{x_b^{n+1} - x_a^{n+1}}$$

Similarly for point H.

We can now calculate the areas enclosed by AODHb

$$A_5^{n+1} = \frac{1}{2}\left\{(x_D^{n+1} - x_O^{n+1})(z_H^{n+1} - z_O^{n+1}) - (z_D^{n+1} - z_O^{n+1})(x_H^{n+1} - x_O^{n+1})\right\}$$

$$A_6^{n+1} = \frac{1}{2}\left\{(x_H^{n+1}-x_O^{n+1})(z_b^{n+1}-z_O^{n+1})-(z_H^{n+1}-z_O^{n+1})(x_b^{n+1}-x_O^{n+1})\right\}$$

$$A_7^{n+1} = \frac{1}{2}\left\{(x_b^{n+1}-x_O^{n+1})(z_A^{n+1}-z_O^{n+1})-(z_b^{n+1}-z_O^{n+1})(x_A^{n+1}-x_O^{n+1})\right\}$$

The density of zone 1 is given by

$$\rho_1^{n+1} = \frac{M_1}{A_5^{n+1}(\bar{x}_5^{n+1})^{\alpha-1}+A_b^{n+1}(\bar{x}_b^{n+1})^{\alpha-1}+A_7^{n+1}(\bar{x}_7^{n+1})^{\alpha-1}}$$

where

$$\bar{x}_5^{n+1} = \frac{1}{3}(x_O^{n+1}+x_D^{n+1}+x_H^{n+1})$$

$$\bar{x}_6^{n+1} = \frac{1}{3}(x_O^{n+1}+x_H^{n+1}+x_b^{n+1})$$

$$\bar{x}_7^{n+1} = \frac{1}{3}(x_O^{n+1}+x_b^{n+1}+x_A^{n+1})$$

Coordinates at the center of the zone are taken to be

$$\bar{x}_1^{n+1} = \frac{1}{4}(x_O^{n+1}+x_D^{n+1}+x_H^{n+1}+x_A^{n+1})$$

$$\bar{z}_1^{n+1} = \frac{1}{4}(z_O^{n+1}+z_D^{n+1}+z_H^{n+1}+z_A^{n+1})$$

(Although these are not, in general, the center of the zone they are adequate to the current approximation.)

Similarly, the areas and densities in zone OAaEB, etc. are obtained.

G. EQUATION OF STATE

(1) Polynomial

The hydrodynamic pressure is assumed to be given by the relation

$$p = f_1(\rho) + f_2(\rho)\,\varepsilon$$

which in finite difference form is

$$p^{n+1} = f_1^{n+1} + f_2^{n+1} \varepsilon^{n+1}$$

Substituting in the energy equation

$$\varepsilon^{n+1} - \varepsilon^n = \frac{1}{2} (f_1^{n+1} + f_2^{n+1} \varepsilon^{n+1} + p^n) \frac{\Delta\rho}{\rho^2} + \Delta_q \varepsilon + \Delta^d \varepsilon$$

or

$$\varepsilon^{n+1} = \frac{\varepsilon^n + 1/2\,(f_1^{n+1} + p^n) \dfrac{\Delta\rho}{\rho^2} + \Delta_q \varepsilon + \Delta^d \varepsilon}{1 - 1/2\, f_2^{n+1} \dfrac{\Delta\rho}{\rho^2}}$$

On evaluating ε^{n+1} the pressure p^{n+1} can be calculated.

(2) Tillotson

In this equation of state the hydrodynamic pressure is given by

$$p = a + \left\{ \frac{b}{1 + \varepsilon/\varepsilon_o \eta^2} \right\} \rho\varepsilon + A\mu + B\mu^2$$

for

$\eta > 1$ for all $\varepsilon > o$ (Region II)

and

$\eta > \eta_s$ for all $\varepsilon < \varepsilon'_s$ (Region III)

and in Region IV

$$p = a\,\varepsilon_\rho + \left\{ \frac{b\varepsilon\rho}{1 + \varepsilon/\varepsilon_o \eta^2} + A\mu\, e^{\beta[1 - 1/\eta]} \right\} e^{-\alpha[1-1/\eta]^2}$$

for

$\eta_s < \eta < 1$ for $\varepsilon < \varepsilon'_s$

and

$\eta_s > \eta$ for all $\varepsilon > o$

In the above equations

$$\mu = \eta - 1$$

Two methods of solving for the pressure and energy are outlined below.

In Regions II and III, i.e. for

$$\eta_3^{n+1} > 1 \quad \text{and} \quad \varepsilon_3^{n+1} > 0$$

and

$$\eta_3^{n+1} > \eta_s \quad \text{for all} \quad \varepsilon_3^{n+1} < \varepsilon_s'$$

the equation of state can be written

$$p_3^{n+1} = \varepsilon_3^{n+1} \left(b_1 + \frac{b_2}{b_3 \varepsilon_3^{n+1} + 1} \right) + b_4$$

where

$$b_1 = a \eta_3^{n+1} \rho_3^o$$

$$b_2 = b \eta_3^{n+1} \rho_3^o$$

$$b_3 = \frac{1}{\varepsilon_o (\eta_3^{n+1})^2}$$

$$b_4 = A (\eta_3^{n+1} - 1) + B (\eta_3^{n+1} - 1)^2$$

Combining with the energy equation

$$p_3^{n+1} = a_1 \varepsilon_3^{n+1} + a_2$$

where

$$a_1 = \frac{2}{(\Delta\rho/\rho^2)_3^{n+1}}$$

$$a_2 = \frac{-2}{(\Delta\rho/\rho^2)_3^{n+1}} \left(\varepsilon_3^n + \Delta_q \varepsilon_3 + \Delta^d \varepsilon_3 \right) - p_3^n$$

we obtain

$$\varepsilon_3^{n+1} = \frac{-K_2 + \sqrt{K_2^2 - 4K_1K_3}}{2K_1}$$

where

$$K_1 = b_3 (a_1 - b_1)$$

$$K_2 = (a_1 - b_1) + b_3 (a_2 - b_4) - b_2$$

$$K_3 = a_2 - b_4$$

In Region IV i.e. where

$$\eta_s < \eta_3^{n+1} < 1 \text{ for } \varepsilon_3^{n+1} < \varepsilon'_s$$

and

$$\eta_s > \eta_3^{n+1} \text{ for all } \varepsilon_3^{n+1} > o$$

the equation of state can be written

$$p_3^{n+1} = b_1 \varepsilon_3^{n+1} + \frac{c_2 \varepsilon_3^{n+1}}{b_3 \varepsilon_3^{n+1} + 1} + c_3$$

where

$$c_1 = \frac{1}{\eta_3^{n+1}} - 1$$

$$c_2 = b \, \eta_3^{n+1} \rho_3^o \, e^{-\alpha c_1^2}$$

$$c_3 = A (\eta_3^{n+1} - 1) \, e^{-\beta c_1} \, e^{-\alpha c_1^2}$$

Combining with the energy equation

$$p_3^{n+1} = a_1 \varepsilon_3^{n+1} + a_2$$

we obtain

$$\varepsilon_3^{n+1} = \frac{-K_5 + \sqrt{K_5 - 4K_4K_6}}{2K_4}$$

where

$$K_4 = b_3 (a_1 - b_1)$$

$$K_5 = (a_1 - b_1) + b_3 (a_2 - c_3) - c_2$$

$$K_6 = a_2 - c_3$$

Alternatively an iterative procedure may be used

$$\tilde{\varepsilon}^{n+1} = \varepsilon^n + (p^n + q^{n-1/2}) \frac{\Delta\rho}{\rho^2}$$

$$\tilde{p}^{n+1} = p(\tilde{\varepsilon}^{n+1}, \rho^{n+1})$$

where $p(\varepsilon, \rho)$ are the equations of state given previously, and $\tilde{p}^{n+1}$, $\tilde{\varepsilon}^{n+1}$ are the first approximations to the pressure and energy respectively at time (n+1). The values assumed at time (n+1) are taken to be

$$\varepsilon^{n+1} = \tilde{\varepsilon}^{n+1} + 1/2\, (\tilde{p}^{n+1} + p^n) \frac{\Delta\rho}{\rho^2}$$

$$p^{n+1} = p(\tilde{\varepsilon}^{n+1}, \rho^{n+1})$$

(3) Elastic-Perfectly Plastic

The equations relating stress deviations to the deviatoric strain rate written in finite difference form leads to an implicit set of equations

$$\left({}^{d}_{e}t^{xx}\right)^{n+1} - \left({}^{d}_{e}t^{xx}\right)^{n} - \Delta t^{n+1/2} (w^{xz})^{n+1/2} \left\{ \left({}^{d}_{e}t^{xz}\right)^{n+1} + \left({}^{d}_{e}t^{xz}\right)^{n} \right\}$$

$$= 2\,\Delta t^{n+1/2}\, G^{n+1/2} \left({}^{d}\dot{e}^{xx}\right)^{n+1/2}$$

$$\left({}^{d}_{e}t^{yy}\right)^{n+1} - \left({}^{d}_{e}t^{yy}\right)^{n} = 2\,\Delta t^{n+1/2}\, G^{n+1/2}\, \left({}^{d}\dot{e}^{yy}\right)^{n+1/2}$$

$$\left({}^{d}_{e}t^{zz}\right)^{n+1} - \left({}^{d}_{e}t^{yy}\right)^{n} + \Delta t^{n+1/2} \left(w^{xz}\right)^{n+1/2} \left\{ \left({}^{d}_{e}t^{xz}\right)^{n+1} + \left({}^{d}_{e}t^{xz}\right)^{n} \right\}$$

$$= 2\,\Delta t^{n+1/2}\, G^{n+1/2}\, \left({}^{d}\dot{e}^{zz}\right)^{n+1/2}$$

$$\left({}^{d}_{e}t^{xz}\right)^{n+1} - \left({}^{d}_{e}t^{xz}\right)^{n} + 1/2\,\Delta t^{n+1/2} \left(w^{xz}\right)^{n+1/2} \left\{ \left({}^{d}_{e}t^{xx}\right)^{n+1} + \left({}^{d}_{e}t^{xx}\right)^{n} \right.$$

$$\left. - \left({}^{d}_{e}t^{zz}\right)^{n+1} - \left({}^{d}_{e}t^{zz}\right)^{n} \right\} = 2\,\Delta t^{n+1/2}\, G^{n+1/2}\, \left({}^{d}\dot{e}^{xz}\right)^{n+1/2}$$

If we choose the time increment $\Delta t^{n+1/2}$ sufficiently small so that

$$\left[\Delta t^{n+1/2} \left(w^{xz}\right)^{n+1/2}\right]^2 \ll 1$$

i.e. the rotation from t^n to t^{n+1} is small, then these equations may be expressed in a relatively simple explicit form. The rotation in time $\Delta t^{n+1/2}$ is

$$\Delta \emptyset = \Delta t^{n+1/2}\; w^{xz\; n+1/2}$$

and the stress deviators at t^{n+1} given approximately by

$${}^{d}_{e}t^{xx} = \left({}^{d}_{e}t^{xx}\right)^{n} + 2\Delta t^{n+1/2} G^{n+1/2} \left({}^{d}\dot{e}^{xx}\right)^{n+1/2} + 2\Delta\emptyset \left\{ \left({}^{d}_{e}t^{xz}\right)^{n} \right.$$

$$\left. + \Delta t^{n+1/2}\, G^{n+1/2} \left({}^{d}\dot{e}^{xz}\right)^{n+1/2} \right\}$$

$${}^{d}_{e}t^{yy} = \left({}^{d}_{e}t^{yy}\right)^{n} + 2\Delta t^{n+1/2} G^{n+1/2} \left({}^{d}\dot{e}^{yy}\right)^{n+1/2}$$

$${}^{d}_{e}t^{zz} = \left({}^{d}_{e}t^{zz}\right)^{n} + 2\Delta t^{n+1/2} G^{n+1/2} \left({}^{d}\dot{e}^{zz}\right)^{n+1/2} - 2\Delta\emptyset \left\{ \left({}^{d}_{e}t^{xz}\right)^{n} \right.$$

$$\left. + \Delta t^{n+1/2} G^{n+1/2} \left({}^{d}\dot{e}^{xz}\right)^{n+1/2} \right\}$$

$$\underset{e}{d}{}_t xz = \left(\underset{e}{d}{}_t xz\right)^n + 2\Delta t^{n+1/2} G^{n+1/2} \left(d_{\dot{e}} xz\right)^{n+1/2} - \Delta \phi \left\{ \left(\underset{e}{d}{}_t xx\right)^n \right.$$

$$\left. + \Delta t^{n+1/2} G^{n+1/2} \left(d_{\dot{e}} xx\right)^{n+1/2} - \left(\underset{e}{d}{}_t zz\right)^n - \Delta t^{n+1/2} G^{n+1/2} \left(d_{\dot{e}} zz\right)^{n+1/2} \right\}$$

These equations are valid only if the material remains elastic, and

$$\left(d_t xx\right)^{n+1} = \underset{e}{d}{}_t xx$$

$$\left(d_t yy\right)^{n+1} = \underset{e}{d}{}_t yy$$

$$\left(d_t zz\right)^{n+1} = \underset{e}{d}{}_t zz$$

$$\left(d_t xz\right)^{n+1} = \underset{e}{d}{}_t xz$$

If the material becomes plastic, i.e.

$$\bar{\bar{\Pi}} = \left\{ \left(\underset{e}{d}{}_t xx\right)^2 + \left(\underset{e}{d}{}_t yy\right)^2 + \left(\underset{e}{d}{}_t zz\right)^2 + 2\left(\underset{e}{d}{}_t xz\right)^2 \right\} > \frac{2}{3}\, y_o^2$$

or

$$\bar{\bar{\Pi}} = 2\left\{ \left(\underset{e}{d}{}_t xx\right)^2 + \left(\underset{e}{d}{}_t xx\right)\left(\underset{e}{d}{}_t zz\right) + \left(\underset{e}{d}{}_t zz\right)^2 + \left(\underset{e}{d}{}_t xz\right)^2 \right\} > \frac{2}{3}\, y_o^2$$

then

$$\left(d_t xx\right)^{n+1} = \sqrt{\frac{\frac{2}{3} y_o^2}{\bar{\bar{\Pi}}}} \left(\underset{e}{d}{}_t xx\right)$$

$$\left(d_t yy\right)^{n+1} = \sqrt{\frac{\frac{2}{3} y_o^2}{\bar{\bar{\Pi}}}} \left(\underset{e}{d}{}_t yy\right)$$

$$\left(d_t zz\right)^{n+1} = \sqrt{\frac{\frac{2}{3} y_o^2}{\bar{\bar{\Pi}}}} \left(\underset{e}{d}{}_t zz\right)$$

$$\left({}^{d}t^{xz}\right)^{n+1} = \sqrt{\frac{\frac{2}{3}\, y_o^{\;2}}{\bar{\bar{\text{II}}}}}\left({}^{d}_{e}t^{xz}\right)$$

The total stresses are given by

$$(t^{xx})^{n+1} = -(p^{n+1} + q^{n+1/2}) + ({}^{d}t^{xx})^{n+1} + ({}^{d}q^{xx})^{n+1/2}$$

$$(t^{yy})^{n+1} = -(p^{n+1} + q^{n+1/2}) + ({}^{d}t^{yy})^{n+1} + ({}^{d}q^{yy})^{n+1/2}$$

$$(t^{zz})^{n+1} = -(p^{n+1} + q^{n+1/2}) + ({}^{d}t^{zz})^{n+1} + ({}^{d}q^{zz})^{n+1/2}$$

and

$$(t^{xz})^{n+1} = ({}^{d}t^{xz})^{n+1} + ({}^{d}q^{xz})^{n+1/2}$$

where p is obtained from the hydrodynamic equation of state and the q's by the expressions for artificial viscosity.

IV. PROGRAM DESCRIPTION

A. COMPUTER CONFIGURATION

The available computer configuration greatly influenced the form the program took. The program was set up to run on an IBM 7040/44 computer with 32 K of core storage and eight tape drives on a single channel. The system monitor program, input, output, plotter results, and the chain feature of the loader reduce the available scratch tapes to three. One of these tapes is used to give the program a re-start capability.

An IBM 1401 computer prepares the input tapes and processes both the printed and plotted results. A CalComp Model 565 plotter is driven on-line by the IBM 1401 computer.

The two remaining scratch tapes were not used to provide auxiliary storage, since the reading and writing of data from core to tape and back again would make computation time excessive with the single-channel computer configuration.

The fact that all data must remain in the core at all times puts a severe limitation on the number of zones into which a problem may be divided. At present the program will accept up to a total of 600 zones, which may be spread over five plates.

B. MESH NUMBERING SCHEME

In order to promote a better understanding of the code itself, term definitions and the mesh numbering system will be explained. Figure 1 should be referred to during the following discussion.

TERM	DEFINITION
plate	an integral piece of material (for example, the target)

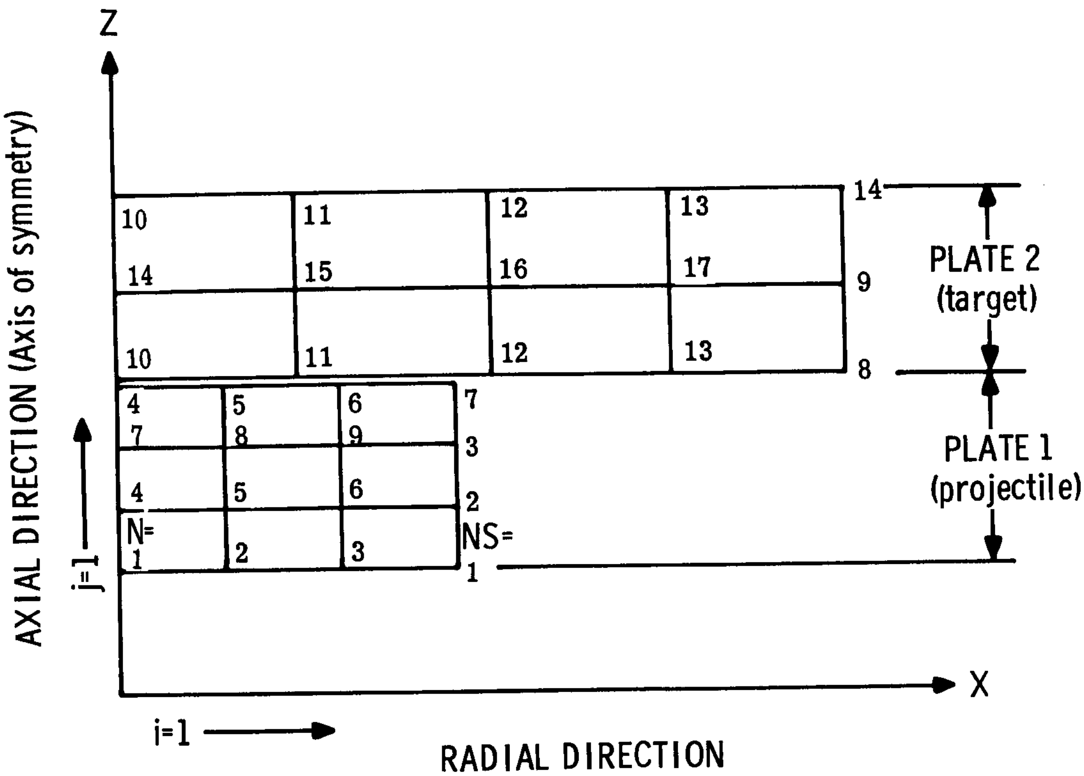


Figure 1 Mesh Numbering Scheme

zone	a subdivision of a plate whose boundaries are formed by the grid lines (sometimes referred to as a mesh)
mesh point	the point of intersection of two grid lines (also referred to as a station)

The zones themselves, with all their material properties, are numbered consecutively starting at the lower left corner, moving in the direction of increasing x, and then toward increasing z. The active zone indicator is N. The position and velocity of a zone are taken as the lower-left corner. All other zone properties are assumed to be at the center of the zone.

The right-hand and upper boundary mesh points are considered "special." Only position and velocity are stored. The active special station indicator is NS.

C. PROGRAM STRUCTURE

The program is divided into a main program and three links, and uses the chain feature of the loader. The function of each part is as follows:

a) Main program — directs sequence in which the links are called into core. Main program remains in core at all times.

b) Link 1 — reads all input, and prints it for verification. Generates problem configuration. Initializes all variables prior to start of integration.

c) Link 2 — integrates until time to output results or stop.

d) Link 3 — prints and plots results, and updates restart tape on signal from Link 2.

The program has been divided into a large number of subroutines to facilitate program modifications and checkout (see Appendixes B and C).

In an attempt to make the code as machine-independent as possible, FORTRAN IV was used wherever possible. As a result, there are only two machine-language subroutines in the entire code.

One is a modified version of the IBM-supplied floating-point trap routine. It is used to set all underflows to zero, and to continue execution without an error message. However, on encountering the first overflow, a location-error message is printed, and the subroutine then transfers to a specified location in the main program. The code should run equally well with the standard IBM-supplied floating-point trap routine.

The other machine-language routine is a tape rewind subroutine to allow rewinding of a tape without first writing an end-of-file mark. This routine is used to initially rewind the plot tape (tape 2), and to rewind the chain tape (tape 0) to reduce the amount of time to load Link 2 after outputting results in Link 3.

All the plotting subroutines are made up of calls to the standard CALCOMP-supplied routines. However, CALCOMP does change its argument lists from time to time, so the calling sequences may be different from those at other installations.

D. PROGRAM FEATURES AND LIMITATIONS

Summarized below are some of the features incorporated into the code, along with some of its limitations and notes of caution.

1. Plane or cylindrically symmetric geometry may be specified.
2. Hydrodynamic or elastic=perfectly plastic media may be specified.
3. There is a choice of two equations of state:
   a) Polynomial
   b) Tillotson
4. Interface sliding is allowed between hydrodynamic and elastic-plastic plates and between two hydrodynamic plates, but not between two elastic-plastic plates.
5. If the interface between plates is not allowed to slide, then the zone width (radial component) in each plate must be the same.
6. To save computation time, if two complete rows of zones in the target plate are found to have no velocity, computation for this cycle is stopped, and computation for the next cycle is started.

7. The plates are not limited to rectangular shapes. By specifying the angle the boundaries make with respect to the X-axis, any quadrilateral shape may be used.
8. The "projectile" plate may be specified entirely as input. This allows the plate to take on any shape ("free-form").
9. Any individual plate must contain a minimum of four zones. That is, there must be two rows and two columns of zones in each plate.

At this writing not every combination of problem configurations has been tried, and some program errors may still exist.

E. INPUT DATA FORMAT

The input data format is listed out below; Figures 2 through 5 show samples of the four types of load sheets used.

```
C      DEFINITION OF INPUT QUANTITIES                                     FORMAT
C
C           CARD 1.                                                       (12A6)
C
C    TITLE  - 72 CHARACTERS OF ALPHA-NUMERICS FOR PROBLEM IDEN IFICATION.
C             PRINTS OUT ON FIRST SHEET OF OUTPUT, AND ON EACH PLOT.
C
C           CARD 2.                                                       (6E12.8)
C
C    STRTC  - STARTING TIME CYCLE.
C             IF     0.0 - SUBROUTINE INTLIZ CALLED.
C             IF NOT 0.0 - READS TAPE 4 FOR VALUES OF COMMON A D PROCEEDS.
C
C           CARD 3.                                                       (6E12.8)
C
C    CZ     - COEFFICIENT IN ARTIFICIAL VISCOSITY EQUATION
C    CL     - COEFFICIENT IN ARTIFICIAL VISCOSITY EQUATION
C    C3     - COEFFICIENT IN DEVIATORIC ARTIFICIAL VISCOSITY E UATION
C    C4     - COEFFICIENT IN DEVIATORIC ARTIFICIAL VISCOSITY E UATION
C    FK     - COEFFICIENT IN TIME INCREMENT EQUATION
C    SBAR   - UPPER LIMIT OF STRESS. COMPRESSION IS NEGATIVE.             (D/SQCM)
C             IF STRESS EXCEEDS THIS VALUE, PROGRAM WILL STOP.
C
C           CARD 4.                                                       (6E12.8)
C
C    TPRNT  - TIME INCREMENT FOR PRINTOUT AND PLOT                        (SEC)
C    TMAX   - PROBLEM TERMINATE TIME                                      (SEC)
C    PRNTC  - PRINT CYCLE INCREMENT.
C             ALLOWS PRINTOUT AND PLOT AFTER EACH PRNTC CYCLES  IF DESIRED.
C    WRTPC  - WRITE TAPE INCREMENT
C             UPDATES THE RESTART TAPE AFTER EACH WRTPC CYCLES+
C    STOPC  - STOP CYCLE.
C             ALLOWS PROGRAM TO BE TERMINATED AFTER STOPC CYCLES.
C
C           CARD 5 0.                                                     (12I6)
C
C    M      - NO. OF PLATES IN PROBLEM.
C    NBL    - BOUNDARY INDICATOR - LOWER X OF FIRST PLATE.
C             = 1 - FREE SURFACE
C             = 2 - FIXED SURFACE
C    NBU    - BOUNDARY INDICATOR - UPPER X OF LAST PLATE
C    IALF   - GEOMETRY INDICATOR
C             = 1 - PLANE
C             = 2 - CYLINDRICAL
C    NOSLID - BOUNDARY INTERFACE INDICATOR
C             = 0 - THE INTERFACES ARE ALLOWED TO SLIDE. ONLY LAST PLATE
```

```
C                    IS ALLOWED TO INCLUDE STRENGTH.
C               = 1 - NO RELATIVE MOVEMENT ALONG INTERFACE BOUNDARIES.
C                    MESH SIZE IN X-DIRECTION MUST BE SAME FOR ALL PLATES.
C  N1D      - ONE DIMENSIONAL CODE SIMULATOR INDICATOR
C               = 0 - NORMAL MODE - 2-D PROBLEM
C               = 1 - 1-D PROBLEM
C  NBQ      - ARTIFICIAL VISCOSITY INDICATOR
C               = 0 - EVALUATE VISCOSITY ONLY IN COMPRESSION
C               = 1 - ALWAYS EVALUATE VISCOSITY
C  NPLOT    - PLOT INDICATOR
C               = 0 OR BLANK - PLOT X VS. Z
C               = 1          - PLOT TXX1 VS. X AND Z
C               = 2          - PLOT TYY1 VS. X AND Z
C               = 3          - PLOT TZZ1 VS. X AND Z
C               = 4          - PLOT TXZ1 VS. X AND Z
C               = 5          - DRAW ALL 5 PLOTS
C               = 6          - SKIP PLOTS
C  NCRVS    - THE NUMBER OF COLUMNS (I VALUES) TO BE PLOTTED. MAX. OF 20.
C
C           CARD 5 1.  IF NCRVS = 0, OMIT THIS CARD.                        (12I6)
C
C  JCRVS(L),L=1,NCRVS
C           - THE COLUMN VALUES TO BE PLOTTED.
C
C           CARD 6 0.  THERE ARE 8 CARDS TO DEFINE EACH PLATE. THERE        (12I6)
C                      WILL BE  M  SETS OF THE FOLLOWING 8 CARDS.
C
C  IM       - NO. OF ZONES IN RADIAL (X) DIRECTION
C  JM       - NO. OF ZONES IN AXIAL  (Z) DIRECTION
C  NBZL     - INDICATOR FOR BOUNDARY CONDITION - LEFT  Z OF EACH PLATE
C  NBZR     - INDICATOR FOR BOUNDARY CONDITION - RIGHT Z OF EACH PLATE
C  MEQ      - INDICATOR FOR EQUATION-OF-STATE
C               = 1 - TILLOTSON
C               = 2 - POLYNOMIAL
C
C
C           CARD 6 1.                                                       (6E12.8)
C
C  XI       - INITIAL POSITION IN X-DIRECTION OF LOWER-LEFT CORNER          (CM)
C  XL       - WIDTH OF PLATE IN X-DIRECTION                                 (CM)
C  ZI       - INITIAL POSITION IN Z-DIRECTION OF UPPER-LEFT CORNER          (CM)
C  ZL       - LENGTH OF PLATE IN Z-DIRECTION                                (CM)
C  UXZ      - INITIAL VELOCITY IN X-DIRECTION                               (CM/SEC)
C  UZZ      - INITIAL VELOCITY IN Z-DIRECTION                               (CM/SEC)
C
C           CARD 6 2.                                                       (6E12.8)
C
C  UXINT    - INITIAL INTERFACE VELOCITY IN X-DIRECTION                     (CM/SEC)
C  UZINT    - INITIAL INTERFACE VELOCITY IN Z-DIRECTION                     (CM/SEC)
C  RHOI     - INITIAL VALUE OF MATERIAL DENSITY                             (G/CC)
C  TXXZ     - INITIAL VALUE OF STRESS IN X-DIRECTION                        (D/SQCM)
C  TYYZ     - INITIAL VALUE OF STRESS IN Y-DIRECTION                        (D/SQCM)
C  TZZZ     - INITIAL VALUE OF STRESS IN Z-DIRECTION                        (D/SQCM)
```

```
C
C         CARD 6 3.                                                     (6E12.8)
C
C  TXZZ   - INITIAL VALUE OF SHEAR STRESS                               (D/SQCM)
C  EZ     - INITIAL VALUE OF ENERGY                                     (ERG/GM)
C  PHIL   - ANGLE OF INCLINATION OF LEFT Z-BOUNDARY                     (DEG)
C  PHIR   - ANGLE OF INCLINATION OF RIGHT Z-BOUNDARY                    (DEG)
C  PSIL   - ANGLE OF INCLINATION OF LOWER X-BOUNDARY                    (DEG)
C  PSIU   - ANGLE OF INCLINATION OF UPPER X-BOUNDARY                    (DEG)
C
C         IF IM = 0, THE PLATE WILL BE --FREE-FORM-- IN SHAPE (EX. A
C                    DEBRIS CLOUD). ALL QUANTITIES NEEDED TO DEFINE THE
C                    PLATE ARE INPUTTED.
C
C                    ONLY ONE FREE-FORM PLATE PER PROBLEM.
C
C                    CARDS 6 1, 6 2, 6 3 ARE OMITTED AND REPLACED WITH
C                    THE FOLLOWING CARDS.
C
C                    THERE WILL BE JM SETS OF CARDS. EACH SET CONSISTS
C                    OF THREE TYPES OF CARDS, AND DEFINES A ROW.
C
C         CARD 1.                                                       (I6)
C
C  IM1(J) - J IS THE ROW INDICATOR. NO. OF ZONES IN ROW J. U7 TO 20 ROWS
C           ARE ALLOWED.
C
C         CARD 2.    EACH PAIR OF CARDS (ONE CARD 2 AND ONE  ARD 3)     (5E15.8)
C                    DEFINES A ZONE. THEREFORE, THERE WILL B  IM1 PAIRS
C                    OF CARDS.
C
C  UX(N)   - N IS THE ZONE COUNTER. RADIAL VELOCITY                     (CM/SEC)
C  UZ(N)   - AXIAL VELOCITY                                             (CM/SEC)
C  P(N)    - HYDRODYNAMIC PRESSURE IN THE ZONE                          (D/SQCM)
C  E(N)    - ENERGY                                                     (ERG/GM)
C  RHO(N)  - DENSITY                                                    (G/CC)
C
C         CARD 3.                                                       (5E15.8)
C
C  X(N)    - RADIAL POSITION                                            (CM)
C  Z(N)    - AXIAL POSITION                                             (CM)
C
C                    AFTER ALL ZONE-DEFINING CARDS HAVE BEEN READ IN,
C                    CARDS DEFINING THE POSITION AND VELOCIT ES OF THE
C                    RIGHT-HAND AND INTERFACE BOUNDARY MESH -OINTS ARE
C                    READ.
C
C         CARD 4.    EACH CARD DEFINES ONE BOUNDARY MESH POINT.         (5E15.8)
C
C  UXS(NS) - RADIAL VELOCITY                                            (CM/SEC)
C  UZS(NS) - AXIAL VELOCITY                                             (CM/SEC)
C  XS(NS)  - RADIAL POSITION                                            (CM)
C  ZS(NS)  - AXIAL POSITION                                             (CM)
```

```
C
C     THE FOLLOWING QUANTITIES DESCRIBE THE MATERIAL PROPERTIES OF EACH PLATE
C
C     FOR MEQ = 1
C          CARD 6 4.                                                    (6E12.8)
C
C  RHOZ    - DENSITY                                                    (G/CC)
C  SA      - SMALL A IN E.O.S.
C  SB      - SMALL B IN E.O.S.
C  CAPA    - BIG A IN E.O.S.                                            (D/SQCM)
C  CAPB    - BIG B IN E.O.S.                                            (D/SQCM)
C  ALPHA   - ALPHA IN E.O.S.
C
C     FOR MEQ = 2
C          CARD 6 4.                                                    (6E12.8)
C
C  RHOZ    - DENSITY                                                    (G/CC)
C  K0      - COEFFICINT OF ARG**0 IN F1
C  K1      - COEFFICINT OF ARG**1 IN F1
C  K2      - COEFFICINT OF ARG**2 IN F1
C  K3      - COEFFICINT OF ARG**3 IN F1
C  H0      - COEFFICINT OF ARG**0 IN F2
C
C     FOR MEQ = 1
C          CARD 6 5.                                                    (6E12.8)
C
C  BETA    - BETA  IN E.O.S.
C  EZERO   - E SUB ZERO IN E.O.S.                                       (ERG/GM)
C  RHOS    - RHO SUB S  IN E.O.S.
C  EPRS    - E SUB S  IN E.O.S.                                         (ERG/GM)
C
C     FOR MEQ = 2
C          CARD 6 5.                                                    (6E12.8)
C  H1      - COEFFICINT OF ARG**1 IN F2
C  H2      - COEFFICINT OF ARG**2 IN F2
C  H3      - COEFFICINT OF ARG**3 IN F2
C
C          CARD 6 6.                                                    (6E12.8)
C
C  CNO     - CONSTANT TERM IN SOUND SPEED EQUATION                      (CM/SEC)
C  CNA     - PRESSURE MULTIPLIER TERM IN SOUND SPEED EQUATION          (CMPS/MB)
C  CNEXP   - EXPONENT OF PRESSURE TERM IN SOUND SPEED EQUATION
C
C          CARD 6 7.                                                    (6E12.8)
C
C  G0      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                     (D/SQCM)
C  G1      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                     (D/SQCM)
C  G2      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                     (D/SQCM)
C  G3      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                     (D/SQCM)
C  CAPY    - YIELD STRENGTH                                             (D/SQCM)
C  UTS     - ULTIMATE TENSILE STRENGTH OF MATERIAL                      (D/SQCM)
C
C     END OF INPUT QUANTITIES.
```

SHEET 1 of ___

General Motors Corporation

IDENTIFICATION NUMBER

Card	Cols 1–12	Cols 13–24	Cols 25–36	Cols 37–48	Cols 49–60	Cols 61–72	Cols 73–80
	TITLE (L), L = 1, 12 (cols 1–72)						CARD 1
	STRTC						2
	C_Z	C_L	C_3	C_4	FK	SBAR – d/cm²	3
	TPRNT – sec	TMAX – sec	PRNTC	WRTPC	STOPC		4
	M / NBL	NBU / IALF	NOSLID / N1D	NBQ / NPLOT	NCRVS		5 0
Do Not Punch Unless Filled In.	JCRVS(1) / JCRVS(2)	JCRVS(3) / . . .					5 1
				/ JCRVS(2i)			5 2
	IM(I) / JM(I)	NBZL(I) / NBZR(I)	MEQ(I)				6 1 0
If Plate 1 is "Free-Form" (IM(I) = 0), these cards will be replaced by Type 3 & Type 4 cards	XI(I) – cm	XL(I) – cm	ZI(I) – cm	ZL(I) – cm	UXZ(I) – cm/sec	UZZ(I) – cm/sec	6 1 1
	UXINT(I) – cm/sec	UZINT(I) – cm/sec	RHOZ(I) – g/cc	TXXZ(I) – d/cm²	TYYZ(I) – d/cm²	TZZZ(I) – d/cm²	6 1 2
	TXZZ(I) – d/cm²	EZ(I) – erg/g	PHIL(I) – deg	PHIR(I) – deg	PSIL(I) – deg	PSIU(I) – deg	6 1 3
	RHO(I) – g/cc	a(I); k_0(I); γ(I)	b(I); k_1(I); CJP(I)	A(I); K_2(I); CJR(I)	B(I); k_3(I); CJD(I)	α(I); H_0(I); DETV(I)	6 1 4

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NOTE: CARDS 6_0 THRU 6_7 SHOULD BE REPEATED FOR EACH PLATE.

Figure 2 Type 1 Load Sheet

General Motors Corporation

SHEET 2 of __

1–12	13–24	25–36	37–48	49–60	61–72	IDENTIFICATION NUMBER (73–80)
β(1); H_1(1); CBRN(1)	E_0(1); H_2(1); X_D(1)	ρ_S(1); H_3(1); Z_D(1)	E_S(1)			CARD 615
CNO(1) - cm/sec	CNA(1)	CNEXP(1)				616
G_0(1)	G_1(1)	G_2(1)	G_3(1)	Y(1)	UTS(1) - d/cm²	617
IM(2) JM(2)	NBZL(2) NBZR(2)	MEQ(2)				620
XI(2) - cm	XL(2) - cm	ZI(2) - cm	ZL(2) - cm	UXZ(2) - cm/sec	UZZ(2) - cm/sec	621
UXINT(2) - cm/sec	UZINT(2) - cm/sec	RHOZ(2) - g/cc	TXXZ(2) - d/cm²	TYYZ(2) - d/cm²	TZZZ(2) - d/cm²	622
TXZZ(2) - d/cm²	EZ(2) - erg/g	PHIL(2) - deg	PHIR(2) - deg	PSIL(2) - deg	PSIU(2) - deg	623
RHO(2) - g/cc	a(2); k_0(2); γ(2)	μ(2); K_1(2); CJPC(2)	A(2); k_2(2); CJR(2)	B(2); k_3(2); CJD(2)	d(2); H_6(2); DETV(2)	624
β(2); H_1(2); CBRN(2)	E_0(2); H_2(2); X_D(2)	ρ_S(2); H_3(2); Z_D(2)	E_S(2)			625
CNO(2) - cm/sec	CNA(2)	CNEXP(2)				626
G_0(2)	G_1(2)	G_2(2)	G_3(2)	Y(2)	UTS(2) - d/cm²	627

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Figure 3 Type 2 Load Sheet

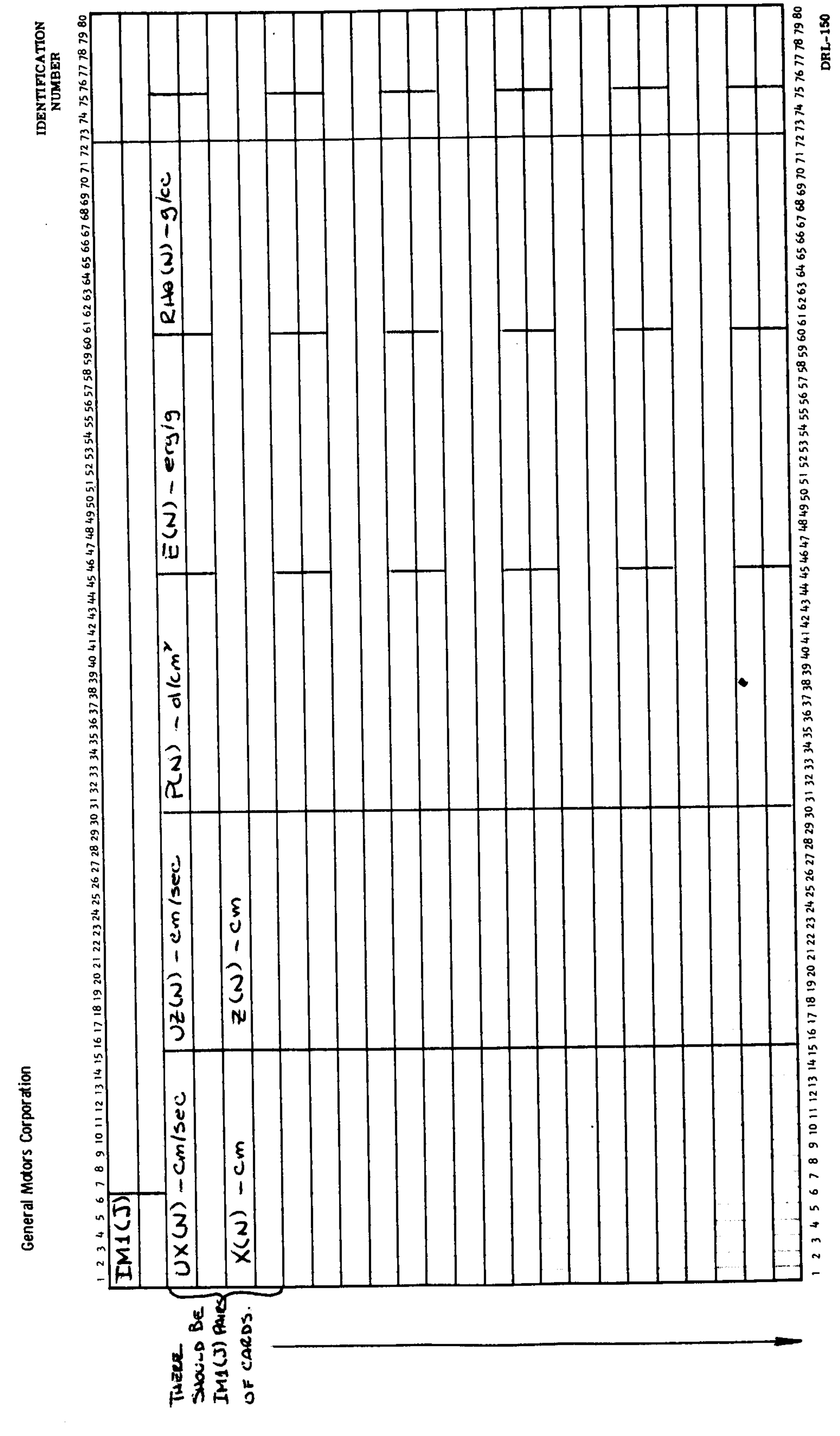


Figure 4 Type 3 Load Sheet

General Motors Corporation

IDENTIFICATION NUMBER

1–15	16–30	31–45	46–60	61–72	73–80
UXS (WS) - cm/sec	UZS (WS) - cm/sec	XS(WS) - cm	ZS(WS) - cm		

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

DRL-150

Figure 5 Type 4 Load Sheet

F. PROGRAM OUTPUT

1. Printed Output

a. At the start of a new problem, the input data read in from cards is printed out to serve as a check in case the results are not as expected. Figure 6 is an example.

b. Figure 7 is an example of the printed output at each mesh, and is self-explanatory.

2. Plotted Output

There are two different types of plots which may be drawn:

a. Mesh position (shape) at some instance in time

b. Stress vs position at a given time.

The stress plots are drawn for each plate on a separate graph. Figures 8 and 9 are examples of the position and stress plots.

G. EXAMPLE PROBLEMS

Three problems, to serve as examples, are solved and compared with analytical solutions here. They are as follows:

1. The first is for a one-dimensional shock-tube problem. The high-pressure and low-pressure initial conditions are as follows.

High Pressure	Low Pressure
$p = 48$ Mb	$p = 0$
$\rho = 0.8$ gm/cm^3	$\rho = 1.2$ gm/cm^3
$E = 90 \times 10^{12}$ ergs/gm	$E = 0$

Both materials behave as perfect gases for which the ratio of specific heats is 5/3.

Comparison of the analytical and computer solution for the wave at 1.0 and 1.5 μsec are given in Figure 10. It is seen that excellent agreement is obtained.

```
                                              INPUT DATA

                          DEBRIS CLOUD TEST

    C0      CL      C3      C4      FK      TPRNT-SEC       TMAX-SEC      SBAR-D/CSQ   START C PRINT C WRTP C  STOP C
  2.000  0.100  0.      0.      0.100  0.20000000E-06  0.20000000E-06 -0.99999998E 19     0.       0.    200.      0.

NO.PLATES X-LOW.B. X-UP.B. ALPHA SLDNG.IFCE VISC.IND.
       2        1       1      2        0        0

PLATE X-STATIONS Z-STATIONS X-LFT.B. Z-RGHT.B.  E.O.S.
   1          0          20        2        1      1
   2         20          15        2        1      1

PLATE    XI-CM            XL-CM           ZI-CM           ZL-CM          UXZ-CM/SEC      UZZ-CM/SEC      UXINT-CM/SEC    UZINT-CM/SEC
  2  0.               0.10000000E 02  0.52264996E 01  0.30000000E 00  0.              0.              0.              0.

PLATE   PHIL-DEG         PHIR-DEG        PSIL-DEG        PSIU-DEG
  2  0.90000000E 02  0.90000000E 02  0.              0.

PLATE  RHOZ-G/CC        TXXZ-D/CSQ      TYYZ-D/CSQ      TZZZ-D/CSQ      TXZZ-D/CSQ      EZ-ERG/GM
  2  0.27000000E 01  0.              0.              0.              0.              0.

PLATE  1 IS A DEBRIS CLOUD.

MESH   UX-CM/SEC        UZ-CM/SEC      STRESS-D/CSQ     E-ERG/G         RHO-G/CC         X-CM            Z-CM
   1  0.               0.16232680E 06  0.              0.14626053E 12  0.36368684E-02  0.              0.41100000E 00
   2  0.63592570E 05  0.11793858E 06  0.              0.40497578E 11  0.70473591E-02  0.13900000E 00  0.41100000E 00
   3  0.15490812E 06  0.52126053E 05  0.              0.29810278E 11  0.11584522E-01  0.27800000E 00  0.41100000E 00
   4  0.25979609E 06  0.39116723E 05  0.              0.33381018E 11  0.25427544E-02  0.71499999E 00  0.41100000E 00
   5  0.               0.23242365E 06  0.              0.14457737E 12  0.36168440E-02  0.              0.48049999E 00
   6  0.58597682E 05  0.17135757E 06  0.              0.41166542E 11  0.79036359E-02  0.13900000E 00  0.48049999E 00
   7  0.14597362E 06  0.12535089E 06  0.              0.29982896E 11  0.89543827E-02  0.27800000E 00  0.48049999E 00
   8  0.30838989E 06  0.10779782E 06  0.              0.36400917E 11  0.24543573E-02  0.71499999E 00  0.48049999E 00
   9  0.               0.32159433E 06  0.              0.14241594E 12  0.32867227E-02  0.              0.54999998E 00
  10  0.56538855E 05  0.24337544E 06  0.              0.44008555E 11  0.66583531E-02  0.13900000E 00  0.54999998E 00
  11  0.14711478E 06  0.18945691E 06  0.              0.30779859E 11  0.71246836E-02  0.27800000E 00  0.54999998E 00
  12  0.3423738[illegible]  0.18486100E 06  [illegible]   0.39568[illegible]  [illegible]22308168E-02  [illegible]99999E 00  0.549[illegible] 00

[illegible]E 07  [illegible]  0.  [illegible] 12  0.1[illegible]  [illegible]
 120  0.               0.279[illegible]E 07  0.          [illegible]01556E 11  0.2555[illegible]E-03  0.              [illegible]49484996E 01
 121  0.16088598E 06  0.28183420E 07  0.              0.49827397E 11  0.23898227E-03  0.13900000E 00  0.49484996E 01
 122  0.37095282E 06  0.28374710E 07  0.              0.66466905E 11  0.19923450E-03  0.27800000E 00  0.49484996E 01
 123  0.73494311E 06  0.28841899E 07  0.              0.10523855E 12  0.15266709E-03  0.71499999E 00  0.49484996E 01
 124  0.11725113E 07  0.29764786E 07  0.              0.14098421E 12  0.35736107E-04  0.12710000E 01  0.49484996E 01
 125  0.14597778E 07  0.30090224E 07  0.              0.14098421E 12  0.35736107E-04  0.18269999E 01  0.49484996E 01
 126  0.14597778E 07  0.30090224E 07  0.              0.14098421E 12  0.35736107E-04  0.23829999E 01  0.49484996E 01

POINT  UXS-CM/SEC       UZS-CM/SEC        XS-CM           ZS-CM
   1  0.53649670E 06  0.42494797E 05  0.12710000E 01  0.41100000E 00
   2  0.57374787E 06  0.10104062E 06  0.12710000E 01  0.48049999E 00
   3  0.60599460E 06  0.16805187E 06  0.12710000E 01  0.54999998E 00
   4  0.94631103E 06  0.24827040E 06  0.18269999E 01  0.61949997E 00
   5  0.10253670E 07  0.44339401E 06  0.23829999E 01  0.77849996E 00
   6  0.11366269E 07  0.67577897E 06  0.23829999E 01  0.10565000E 01
   7  0.15167652E 07  0.11083706E 07  0.23829999E 01  0.13345000E 01
   8  [illegible]      [illegible]      0.29389999E 01  [illegible]
   9  0.16025416E 07  0.15092859E 07  0.29389999E 01  0.18904999E 01
  10  0.18607950E 07  0.17616570E 07  0.29389999E 01  0.21684999E 01
  11  0.18968692E 07  0.19380253E 07  0.29389999E 01  0.24464999E 01
  12  0.19400256E 07  0.21224722E 07  0.29389999E 01  0.27244999E 01
  13  0.19625958E 07  0.22940918E 07  0.29389999E 01  0.30024998E 01
  14  0.19528162E 07  0.24096195E 07  0.29389999E 01  0.32804998E 01
  15  0.17105578E 07  0.25081730E 07  0.29389999E 01  0.35584998E 01
  16  0.17069826E 07  0.26368723E 07  0.29389999E 01  0.38364998E 01
  17  0.17021509E 07  0.27475747E 07  0.29389999E 01  0.41144997E 01
  18  0.14668726E 07  0.28153426E 07  0.29389999E 01  0.43924997E 01
  19  0.14586341E 07  0.29236888E 07  0.29389999E 01  0.46704996E 01
  20  0.14597778E 07  0.30090224E 07  0.29389999E 01  0.49484996E 01
  21  0.               0.28810549E 07  0.              0.52264996E 01
  22  0.16510378E 06  0.29044222E 07  0.13900000E 00  0.52264996E 01
  23  0.38835755E 06  0.29245874E 07  0.27800000E 00  0.52264996E 01
  24  0.74739373E 06  0.29681583E 07  0.71499999E 00  0.52264996E 01
  25  0.11008820E 07  0.30384203E 07  0.12710000E 01  0.52264996E 01
  26  0.11008820E 07  0.30384203E 07  0.18269999E 01  0.52264996E 01
  27  0.11008820E 07  0.30384203E 07  0.23829999E 01  0.52264996E 01
  28  0.11008820E 07  0.30384203E 07  0.29389999E 01  0.52264996E 01

                                          MATERIAL CONSTANTS

PLATE   RHO-G/CC          SA              SB              A               B              ALPHA            BETA            EZERO
   1  0.27000000E 01  0.50000000E 00  0.16300000E 01  0.75200000E 12  0.65000000E 12  0.50000000E 01  0.50000000E 01  0.50000000E 11

         RHOS             ES              CNO             CNA            CNEXP
      0.91000000E 00  0.30000000E 11  0.52800000E 06  0.44000000E 01  0.45000000E 00

          G0              G1              G2              G3              CAPY            UTS
      0.              0.              0.              0.              0.              0.10000000E 20

PLATE   RHO-G/CC          SA              SB              A               B              ALPHA            BETA            EZERO
   2  0.27000000E 01  0.50000000E 00  0.16300000E 01  0.75200000E 12  0.65000000E 12  0.50000000E 01  0.50000000E 01  0.50000000E 11

         RHOS             ES              CNO             CNA            CNEXP
      0.91000000E 00  0.30000000E 11  0.52800000E 06  0.44000000E 01  0.45000000E 00

          G0              G1              G2              G3              CAPY            UTS
      0.28630000E 12  0.58610000E 01  0.42670000E 01  0.              0.27600000E 10  0.12000000E 11

RESTART TAPE UPDATED AT CYCLE      98
```

Figure 6 Example of Printed Input Data

```
                              DEBRIS CLOUD TEST

 CYCLE       TIME-SEC       DELTA T-SEC
    98  0.20099201E-06  0.17736918E-08

  PER CYCLE ENERGIES - INTERNAL-ERG    KINETIC-ERG      TOTAL-ERG          RADIAL MOM.-G CM/SEC AXIAL MOM.-G CM/SEC
                      0.13242272E 11  0.18597794E 12  0.19922023E 12           0.65656913E 05        0.14474099E 06

ACCUMULATED ENERGIES - VISC.ST.-ERG    PLASTIC-ERG
                      0.92204328E 10  0.53013678E 11
PLATE Z    X       SIGMA X-D/CSQ   SIGMA Y-D/CSQ   SIGMA Z-D/CSQ   SIGMA XZ-D/CSQ     Q-D/CSQ          RHO-G/CC        ENERGY-ERG/GM
      ST.  ST.     X VEL.-CM/SEC   Z VEL.-CM/SEC       X-CM            Z-CM        Q X-D/CSQ          Q Z-D/CSQ       Q XZ-D/CSQ

  1    1    1   -0.10616871E 09 -0.10616871E 09 -0.10616871E 09  0.               0.                0.19709845E-02  0.10773091E 12
                 0.             -0.13754007E 06  0.              0.40930738E 00  0.                0.              0.

       1    2   -0.65543364E 08 -0.65543364E 08 -0.65543364E 08  0.               0.                0.41934414E-02  0.31259687E 11
                 0.88902756E 05 -0.49057465E 05  0.15455030E 00  0.41605540E 00  0.                0.              0.

       1    3   -0.90973737E 08 -0.90973737E 08 -0.90973737E 08  0.               0.                0.75541121E-02  0.24085669E 11
                 0.15224580E 06 -0.19856104E 05  0.30888072E 00  0.41362525E 00  0.                0.              0.

       1    4   -0.21970023E 08 -0.21970023E 08 -0.21970023E 08  0.               0.                0.16387303E-02  0.26813244E 11
                 0.26674557E 06 -0.31743041E 05  0.76795061E 00  0.41117065E 00  0.                0.              0.

       1    4    0.54682711E 06 -0.35619604E 05  0.13799244E 01  0.41108781E 00

       2    1   -0.13887767E 09 -0.13887767E 09 -0.13887767E 09  0.               0.                0.23712154E-02  0.11713500E 12
                 0.              0.22125749E 06  0.              0.52672721E 00  0.                0.              0.

       2    2   -0.89730697E 08 -0.89730697E 08 -0.89730697E 08  0.               0.                0.53133050E-02  0.33775391E 11
                 0.81543677E 05  0.16153804E 06  0.15331455E 00  0.51421955E 00  0.                0.              0.

       2    3   -0.77964517E 08 -0.77964517E 08 -0.77964517E 08  0.               0.                0.62307004E-02  0.25025659E 11
                 0.14548466E 06  0.13003949E 06  0.30730387E 00  0.50627167E 00  0.                0.              0.
```

```
      20    5   -0.12738908E 10 -0.12738908E 10 -0.12738908E 10  0.               0.39605458E 09  0.12059008E-03  0.14559071E 14
                 0.24105352E 07  0.45359393E 06  0.16511711E 01  0.51543037E 01  0.                0.              0.

      20    6   -0.73985686E 09 -0.73985686E 09 -0.73985686E 09  0.               0.14187053E 09  0.97899499E-04  0.12216468E 14
                 0.21017092E 07  0.29467254E 06  0.21642155E 01  0.51539557E 01  0.                0.              0.

      20    7   -0.30911336E 09 -0.30911336E 09 -0.30911336E 09  0.               0.34664475E 08  0.74359832E-04  0.73818173E 13
                 0.22228069E 07  0.30457271E 06  0.27550267E 01  0.51366759E 01  0.                0.              0.

      20    7    0.38474041E 07  0.36195002E 06  0.35537726E 01  0.51584222E 01

      21    1    0.              0.26895355E 04  0.              0.52268645E 01

  1   21    2    0.32582105E 06  0.25875733E 04  0.15247205E 00  0.52268541E 01

      21    3    0.41525090E 06  0.24867587E 04  0.31501610E 00  0.52268431E 01

      21    4    0.13818733E 07  0.15794278E 04  0.91693105E 00  0.52267586E 01

      21    5    0.24069640E 07  0.60488726E 03  0.16519144E 01  0.52266461E 01

      21    6    0.21074272E 07  0.47046787E 03  0.21647161E 01  0.52266052E 01

      21    7    0.22244897E 07  0.20162909E 03  0.27566153E 01  0.52265731E 01

      21    7    0.38474022E 07  0.67209696E 02  0.35537726E 01  0.52265083E 01

  2    1    1   -0.23931577E 10 -0.23463675E 10 -0.44902604E 10 -0.17388867E 08  0.38069416E 09  0.27095852E 01  0.31264142E 07
                 0.              0.26895355E 04  0.              0.52268645E 01  0.                0.              0.

       1    2   -0.18778176E 10 -0.18341485E 10 -0.34885065E 10 -0.26265162E 08  0.33373193E 09  0.27073592E 01  0.20671064E 07
                 0.              0.24502587E 04  0.50000000E 00  0.52268306E 01  0.                0.              0.

       1    3   -0.11574251E 10 -0.11488164E 10 -0.22245737E 10 -0.12684503E 08  0.11909564E 09  0.27049672E 01  0.95834139E 06
                -0.45982836E 02  0.17018348E 04  0.99999755E 00  0.52267442E 01  0.                0.              0.

       1    4   -0.72166921E 09 -0.72158350E 09 -0.14135468E 10 -0.10195242E 08  0.47895798E 08  0.27032357E 01  0.38928735E 06
                -0.41456840E 02  0.97621053E 03  0.14999974E 01  0.52266614E 01  0.                0.              0.
```

```
      14   19    0.              0.              0.              0.               0.                0.27000000E 01  0.
                 0.              0.              0.90000000E 01  0.54864995E 01  0.                0.              0.

      14   20    0.              0.              0.              0.               0.                0.27000000E 01  0.
                 0.              0.              0.95000000E 01  0.54864995E 01  0.                0.              0.

      14   20    0.              0.              0.10000000E 02  0.54864995E 01

 END OF 01563-2
```

Figure 7 Example of Printed Output at Each Mesh

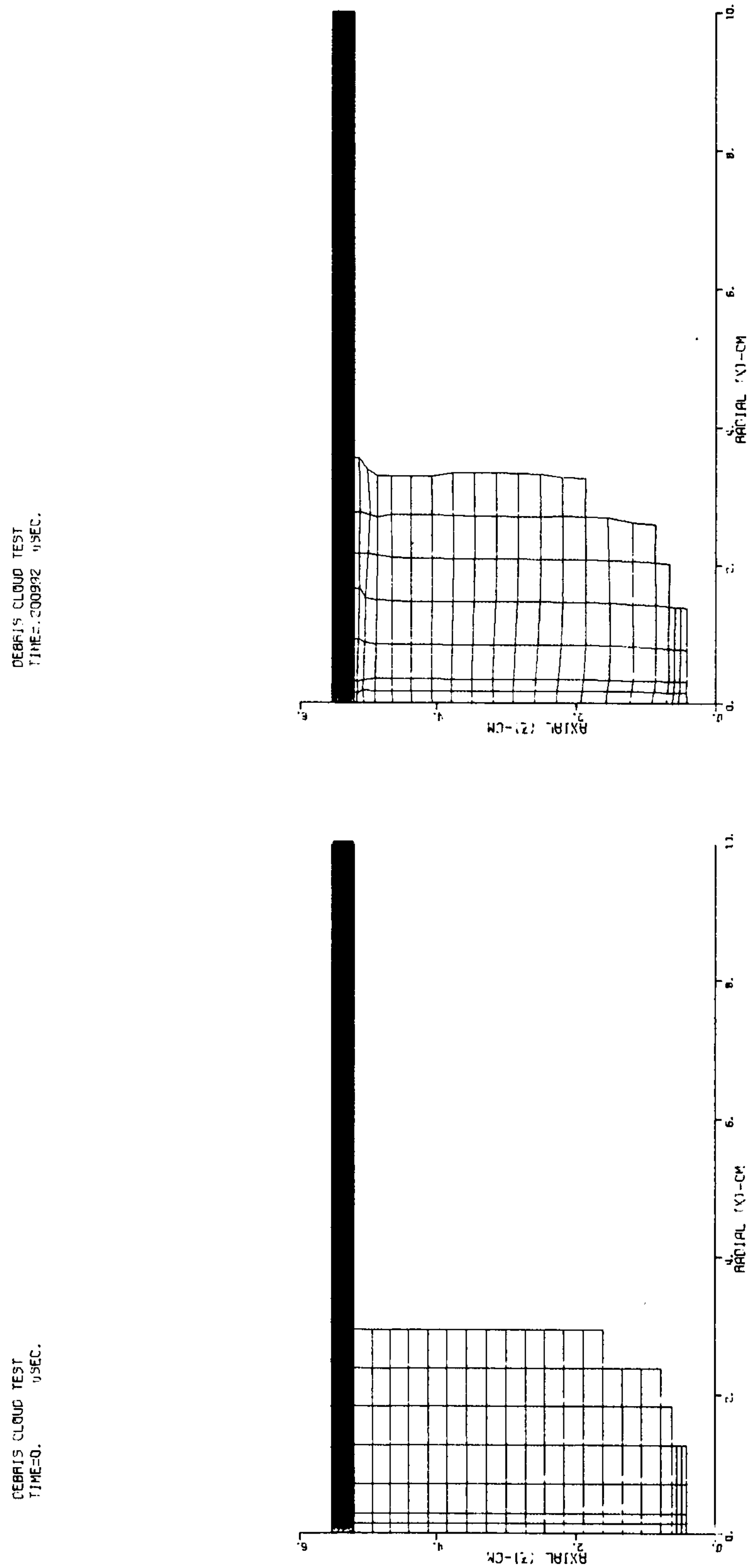


Figure 8 Example of Position Plot

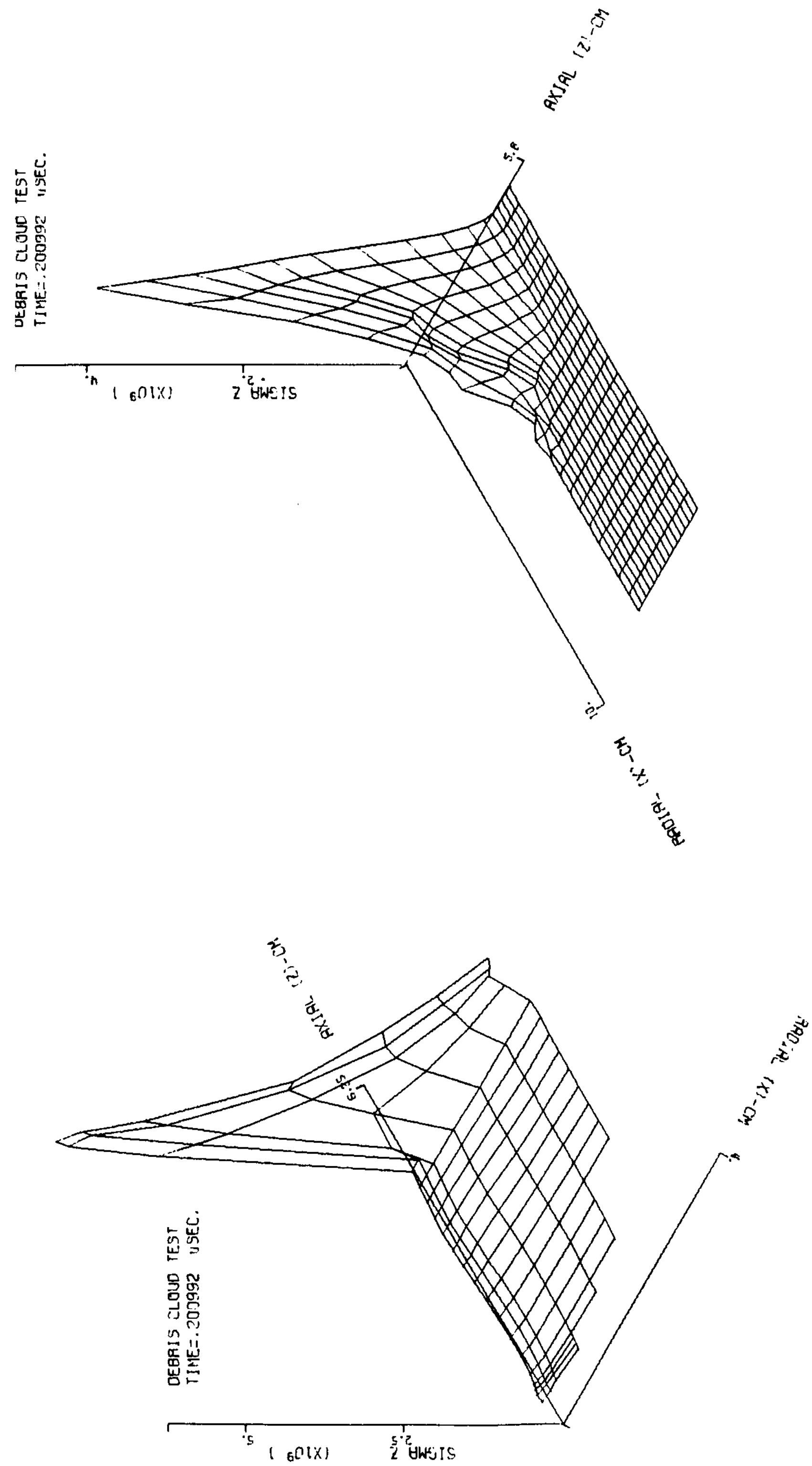
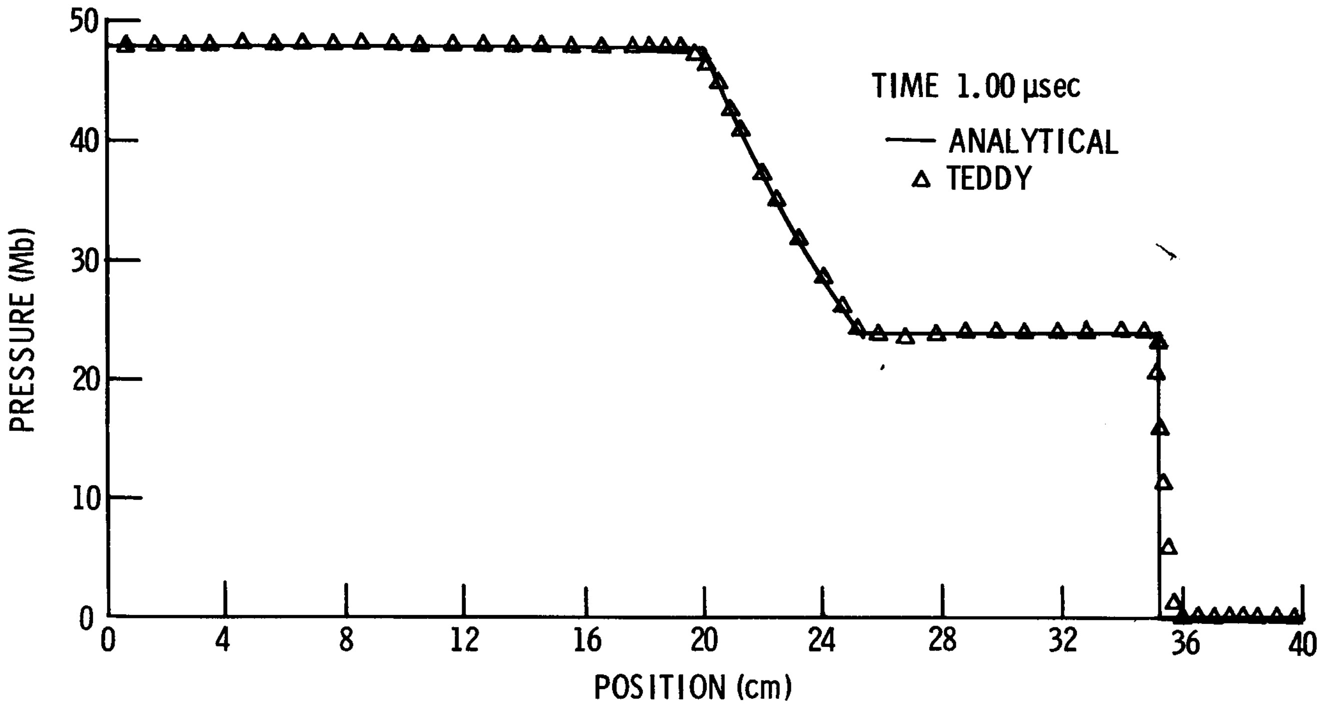


Figure 9 Example of Stress Plot



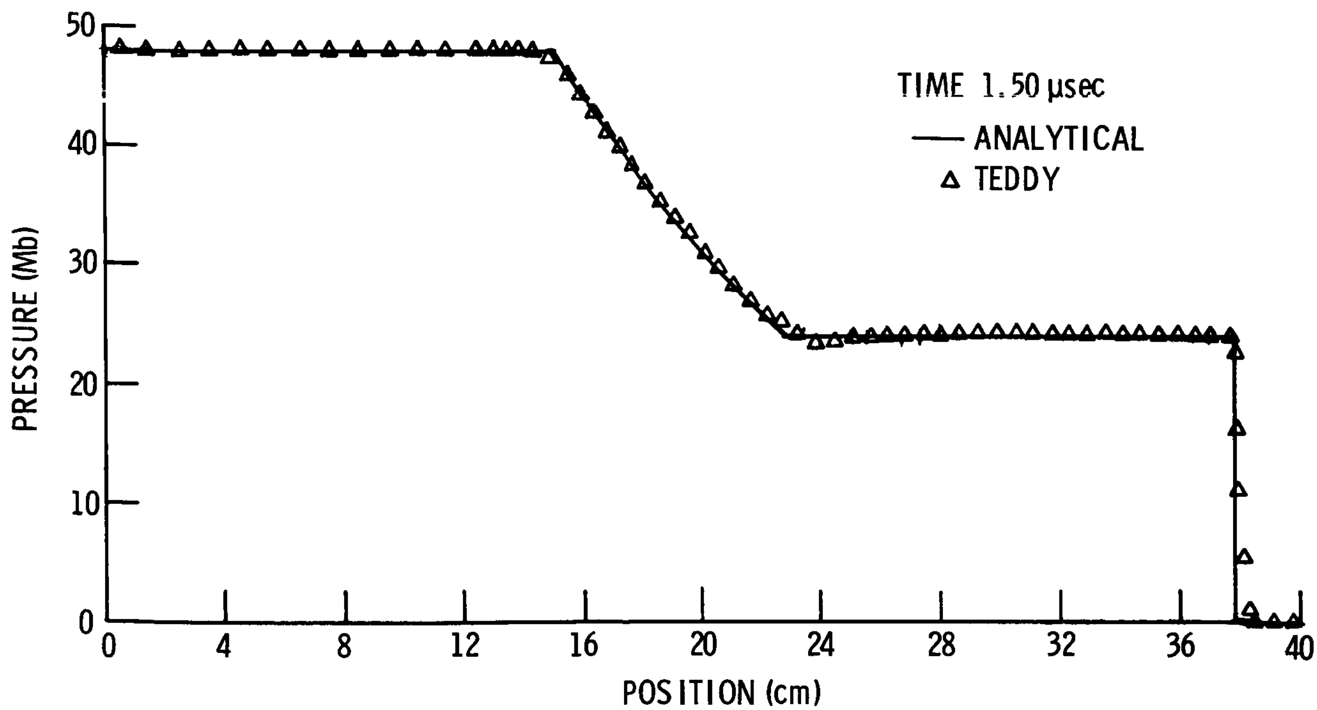


Figure 10 Example 1: Shock Tube Analysis

2. The second check solution is for an aluminum plate impacting a rigid wall at an impact velocity of 0.1262 km/sec. An elastic-plastic constitutive equation with $y = 2.76$ kb is used. Again good agreement is obtained, as shown in Figure 11.

3. The third check problem involves the expansion of a spherical cavity within a sphere subject to time-dependent internal pressure. Exact solutions have been given by Blake.[(8)] Although this can be treated as a one-dimensional problem, the computor code treats it as an axially symmetric two-dimensional problem. Comparison of the analytical and computer solutions shows very good agreement; see Figure 12.

While these and other problems have given satisfactory answers, it should be emphasized that care must be exercised in using the program for other problems. Undetected errors may still exist in the program despite efforts to eliminate all of them. Also, other conditions may require a different choice of viscosity coefficients and stability criterion.

H. PROGRAM FLOW CHARTS

The flow charts for the program are presented in Appendix B. Not all the subroutines comprising the 2-D Lagrangian Code have been flow charted. Those that are very short and/or straight-forward in nature have been omitted. The source deck listings will explain fully these routines.

Also, in those routines that involve large amounts of "book-keeping" logic, an idealized approach has been taken in the flow charts so that the basic principles underlying these routines is made clear.

I. FORTRAN SOURCE DECKS

Appendix C is a listing of the Fortran Source Decks. In this appendix, the block of COMMON, DIMENSION, and EQUIVALENCE statements that is common to all the Fortran programs and subprograms, except three, is listed only once. It is to be remembered, though, that except for subroutines QUAD, PMU, and AX2, it is an integral part of all other programs.

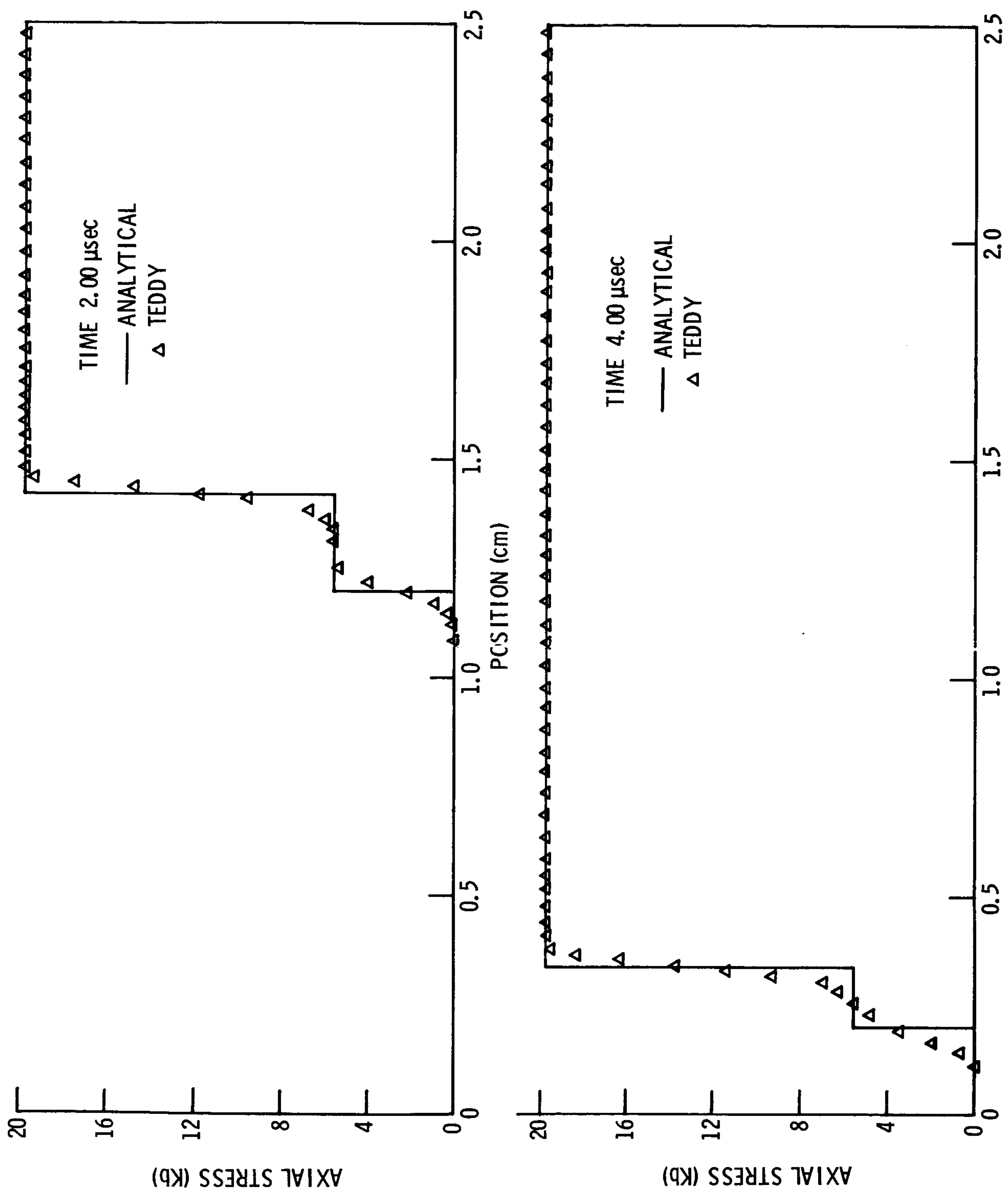
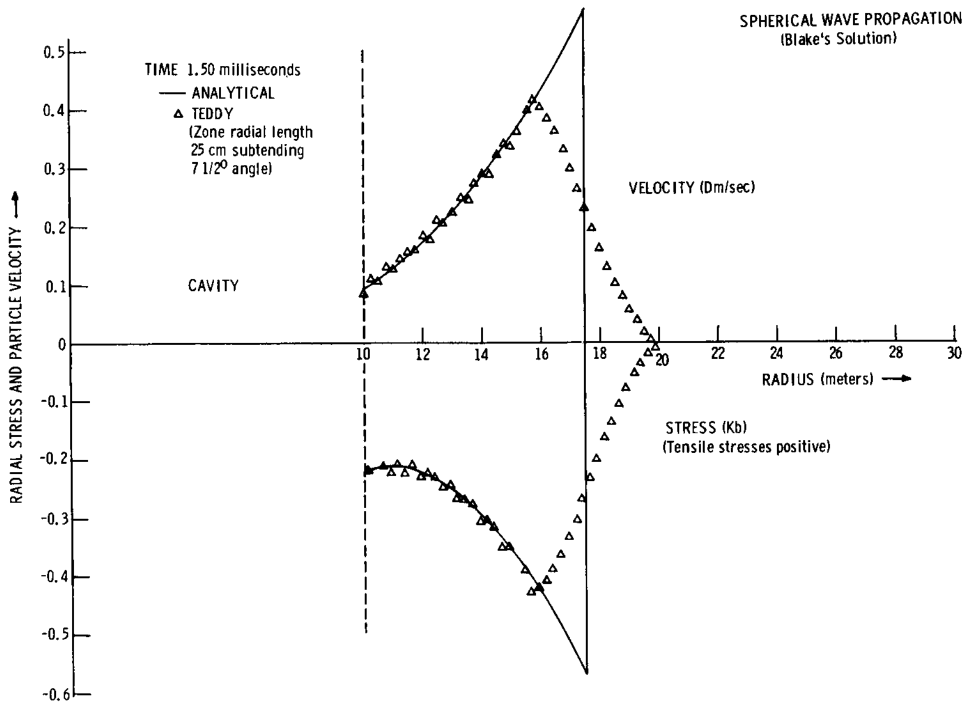
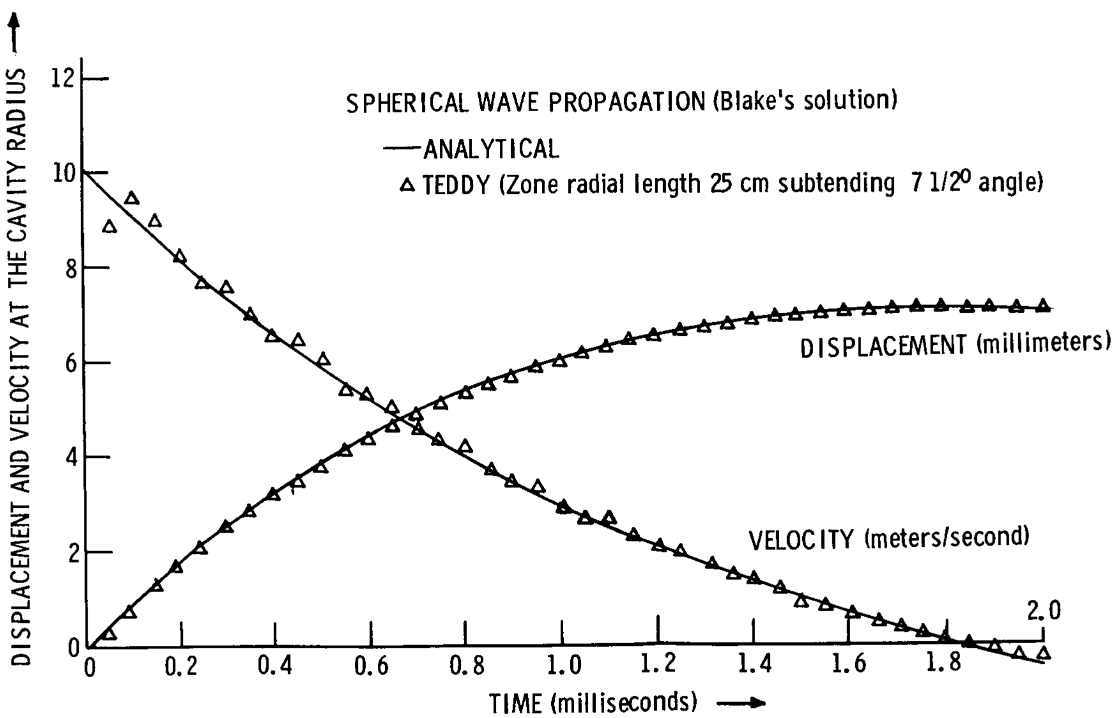


Figure 11 Example 2: Stress Wave Profiles due to the Elastic-Plastic Impact of an Aluminum Plate on a Rigid Wall



a. Velocity and Stress vs Radius



b. Velocity and Displacement vs Time

Figure 12 Example 3: Wave Propagation in an Elastic Medium Surrounding a Spherical Cavity Subjected to a Time-Varying Internal Pressure

REFERENCES

1. J.H. Tillotson, "Metallic Equations of State for Hypervelocity Impact," General Atomic Report GA-3216, July 18, 1962
2. G. Maenchen and S. Sack, "The Tensor Code," University of California, Lawrence Radiation Laboratory, UCRL-7316, April 9, 1963
3. M. L. Wilkins, "Calculation of Elastic-Plastic Flow," University of California, Lawrence Radiation Laboratory, UCRL-7322, April 19, 1963
4. C.S. Godfrey, D.J. Andrews, E.T. Trigg and M. Wilkins, "Prediction Calculations for Free Field Ground Motion," Air Force Weapons Laboratory Technical Documentary Report No. WL-TDR-64-27, May 1964
5. W. Herrmann and M. O'Brien, "RAVE I," Aeroelastic and Structures Research Laboratory, (Massachusetts Institute of Technology) Report No. 1021, June 1964
6. J. von Neumann and R.D. Richtmyer, "A Method for the Numerical Calculation of Hydrodynamic Shocks," J. Appl. Phys., Vol. 21, 1950, p. 232
7. M. L. Wilkins, Private correspondence
8. F.G. Blake, Jr., "Spherical Wave Propagation in Solid Media," J. of the Acoustical Society of America, Vol. 24, 1952, p. 211

APPENDIXES

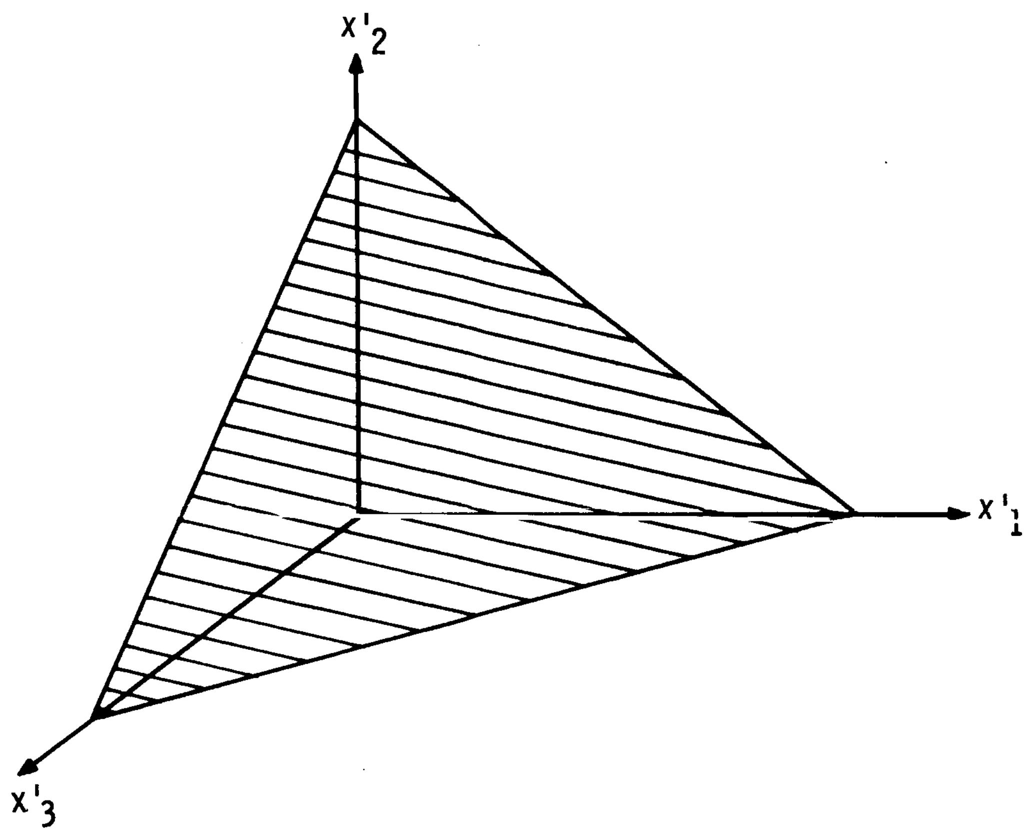
Appendix A: Material Rotation

Appendix B: Program Flow Charts

Appendix C: Listings of Fortran Source Decks

APPENDIX A
MATERIAL ROTATION

(1) Stresses at a Point



Consider the coordinate system whose axes lie in and normal to the shaded area above. Let $x_1\ x_2\ x_3$ be the new coordinates with direction cosines a_{ij} . The stresses in the new coordinate system are given by

$$t^{ij} = a_{il}\ a_{jm}(t^{lm})'$$

where $(t^{lm})'$ are the stresses referred to the coordinate system $x_1' x_2' x_3'$.
Similarly

$$(t^{ij})' = a_{il}\, a_{jm}\, t^{lm}$$

(2) Material Rotation

During a given time step an element of the body rotates which necessitates correcting the stresses to refer to the fixed coordinate system $x_1x_2x_3$. The system $x_1'x_2'x_3'$ participates in the instantaneous rotation in the neighborhood of the considered particle P. The coordinate dx_i of this particle changes at the rate $dv_i = \epsilon_{ijk}\, w_k\, dx_j'$ where w_k is the vorticity vector at P and ϵ_{ijk} is such that

$\epsilon_{ijk} = 0$ when any two indices are equal

$\epsilon_{ijk} = 1$ when i, j, k are an even permutation of 1, 2, 3

$\epsilon_{ijk} = -1$ when i, j, k are an odd permutation of 1, 2, 3.

At time t + dt

$$dx_i = dx'_i - \epsilon_{ijk}\, w_k\, dt\, dx'_j$$

$$= (\delta_{ij} - \epsilon_{ijk}\, w_k dt)\, dx'_j$$

in which case the coefficients in the transformation are

$$a_{ij} = \delta_{ij} - \epsilon_{ijk}\, w_k\, dt$$

During the same time step the improved component of stress has increased to

$$t^{ij} + \dot{t}^{ij}\, dt$$

where t^{ij} and $\dot{t}^{ij}$ are taken at particle P at time t.

Transforming

$$(t^{ij})' = a_{ip}\, a_{jq}\, (t^{pq} + \dot{t}^{pq}\, dt)$$

$$= (\delta_{ip} - \epsilon_{ipk}\, w_k\, dt)(\delta_{jq} - \epsilon_{jq\rho}\, w_\rho\, dt)(t^{pq} + \dot{t}^{pq}\, dt)$$

$$= t^{ij} + (\dot{t}^{ij} - t^{iq}\, \epsilon_{qj\rho}\, w_1 - t^{jp}\, \epsilon_{pik}\, w_k)\, dt$$

to within higher-order terms in dt . The contents in brackets represent a suitable definition of the objective stress rate tensor $\overset{\nabla}{t}{}^{ij}$

$$\overset{\nabla}{t}{}^{ij} = \dot{t}^{ij} - t^{iq} \epsilon_{qj\rho} w_{\rho} - t^{jp} \epsilon_{pik} w_{k}$$

Examples:

(1) For one dimensional motion Equations 6 reduces to

$$\overset{\nabla}{t}{}^{ii} = \dot{t}^{ii}$$

(2) For an axial-symmetric motion, using cylindrical coordinates (z axial and x radial coordinate)

$$\overset{\nabla}{t}{}^{xx} = \dot{t}^{xx} - 2w_{xz} t^{xz}$$

$$\overset{\nabla}{t}{}^{yy} = \dot{t}^{yy}$$

$$\overset{\nabla}{t}{}^{zz} = \dot{t}^{zz} + 2w_{xz} t^{xz}$$

$$\overset{\nabla}{t}{}^{xz} = \dot{t}^{xz} + w_{xz} (t^{xx} - t^{zz}) \quad .$$

where w_{xz} is the spin given by

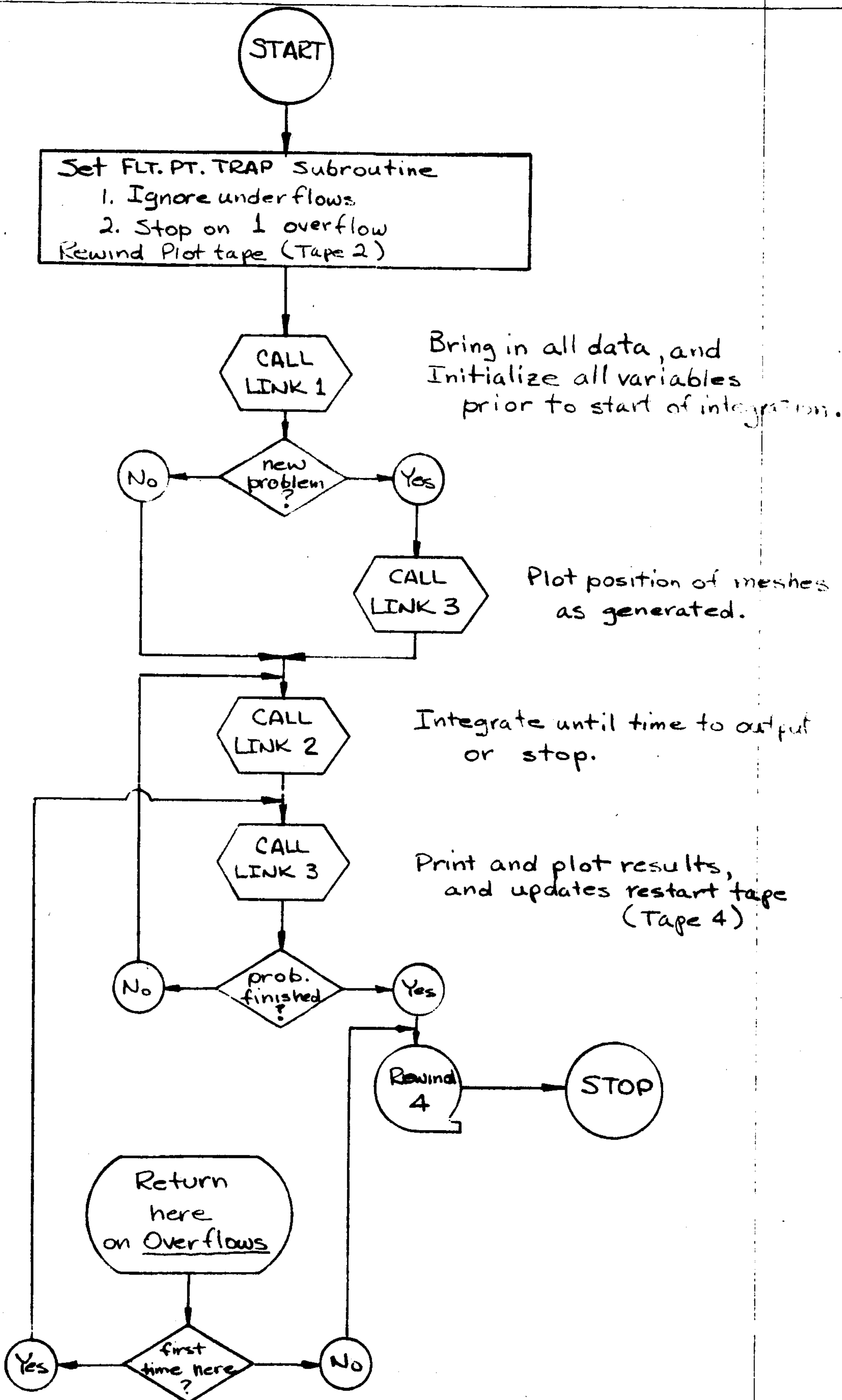
$$w_{xz} = \frac{1}{2} \left(\frac{\partial u^x}{\partial z} - \frac{\partial u^z}{\partial x} \right)$$

and u are the particle velocities in the direction indicated by the superscript.

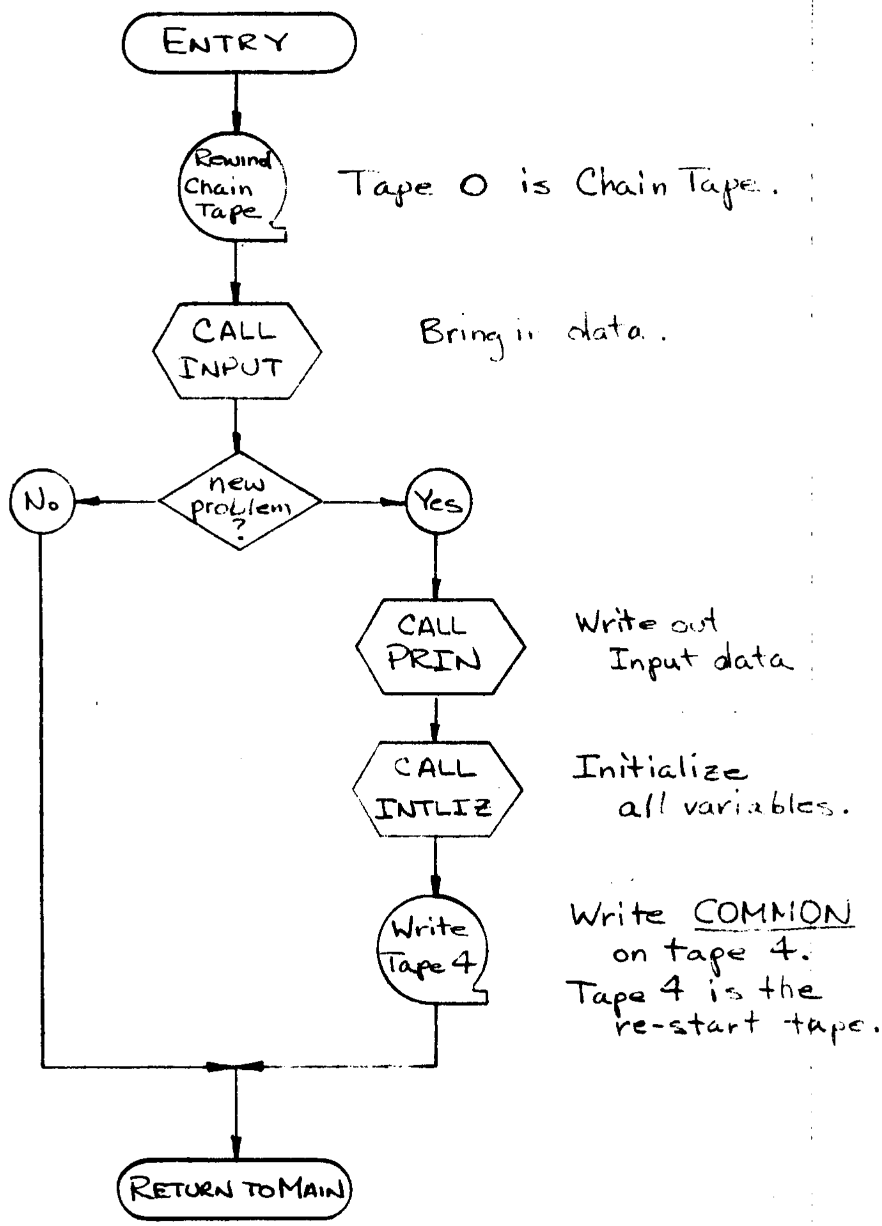
APPENDIX B
PROGRAM FLOW CHARTS

Flow Charts for the TEDDY Code Subroutines are presented in this Appendix as follows:

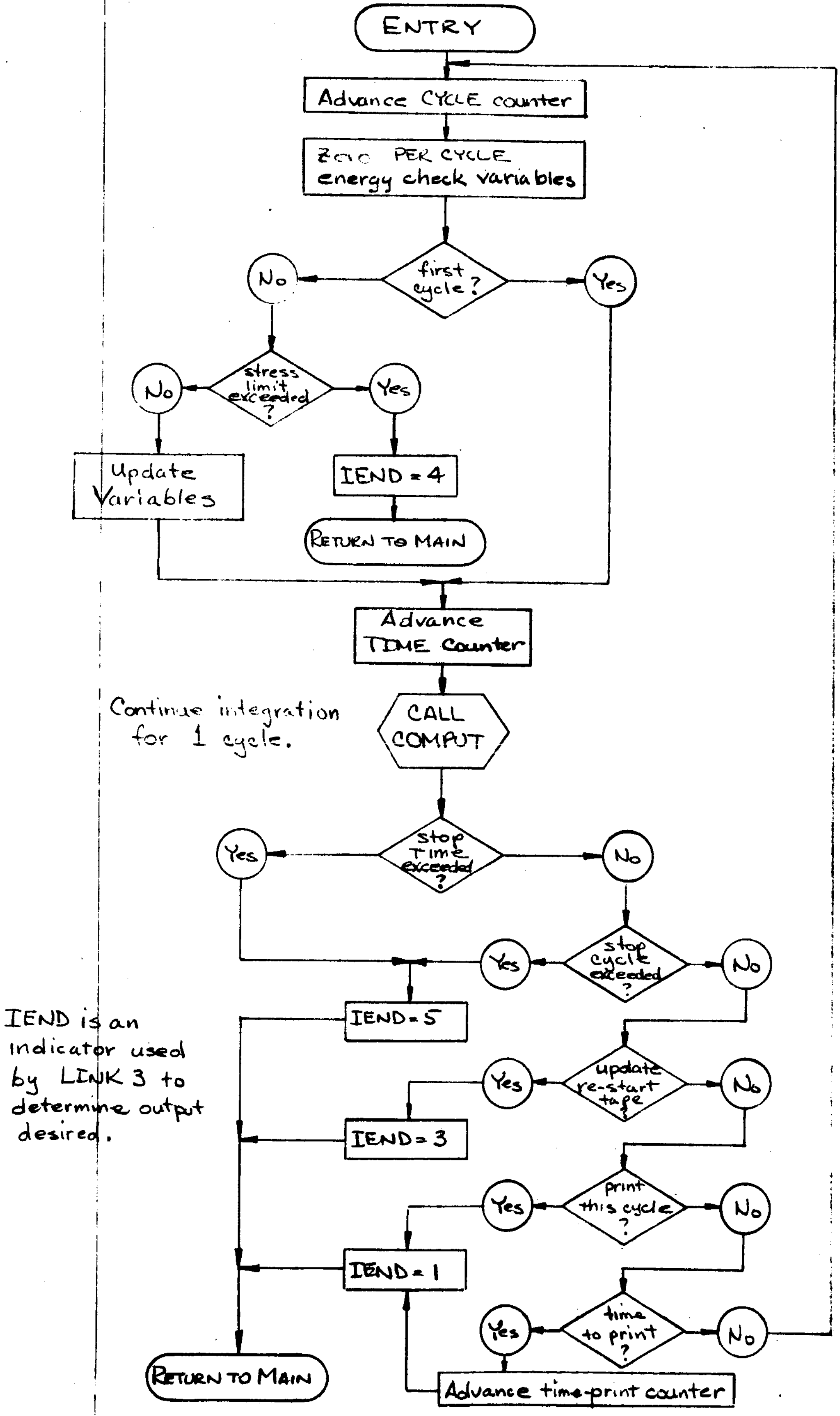
MAIN PROGRAM



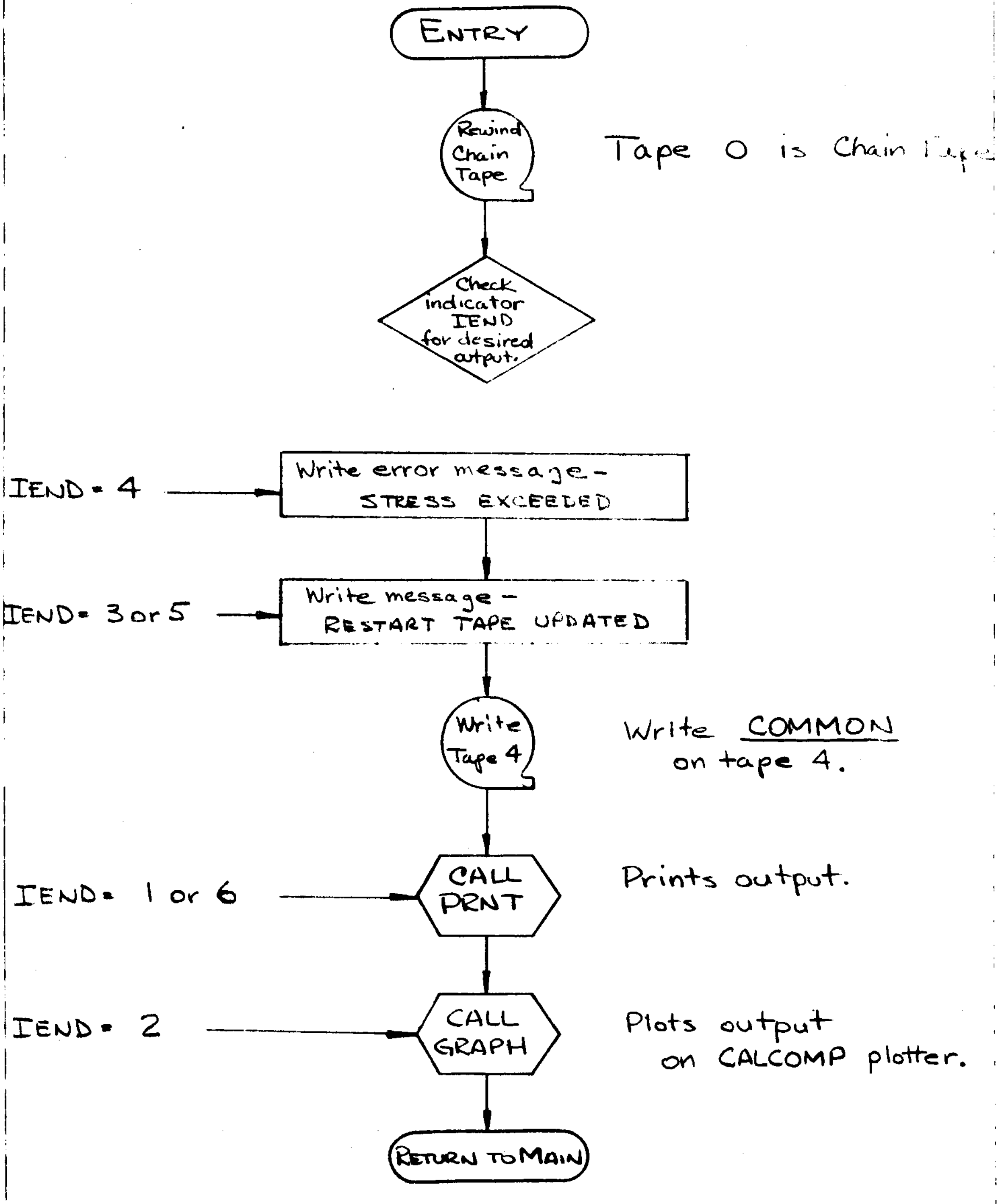
LINK 1


ENTRY
Rewind Chain Tape
Tape 0 is Chain Tape.
CALL INPUT
Bring in data.
new problem?
No
Yes
CALL PRIN
Write out Input data
CALL INTLIZ
Initialize all variables.
Write Tape 4
Write COMMON on tape 4. Tape 4 is the re-start tape.
RETURN TO MAIN

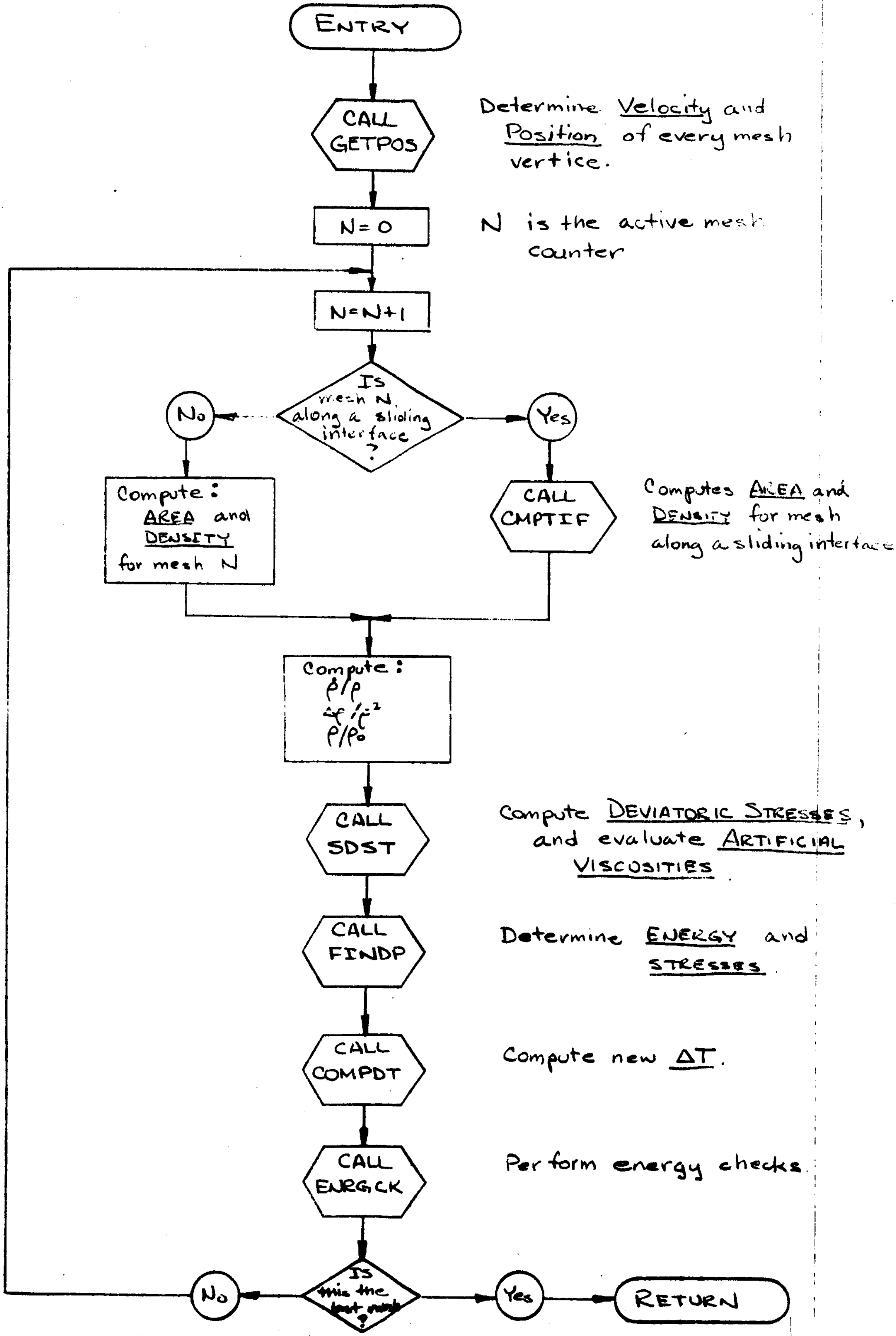
LINK 2


ENTRY
Advance CYCLE counter
Zero PER CYCLE energy check variables
first cycle ?
No
Yes
stress limit exceeded ?
No
Yes
Update Variables
IEND = 4
RETURN TO MAIN
Advance TIME Counter
Continue integration for 1 cycle.
CALL COMPUT
stop Time exceeded ?
Yes
No
stop cycle exceeded ?
Yes
No
IEND is an indicator used by LINK 3 to determine output desired.
IEND = 5
update re-start tape ?
Yes
No
IEND = 3
print this cycle ?
Yes
No
IEND = 1
time to print ?
Yes
No
RETURN TO MAIN
Advance time-print counter

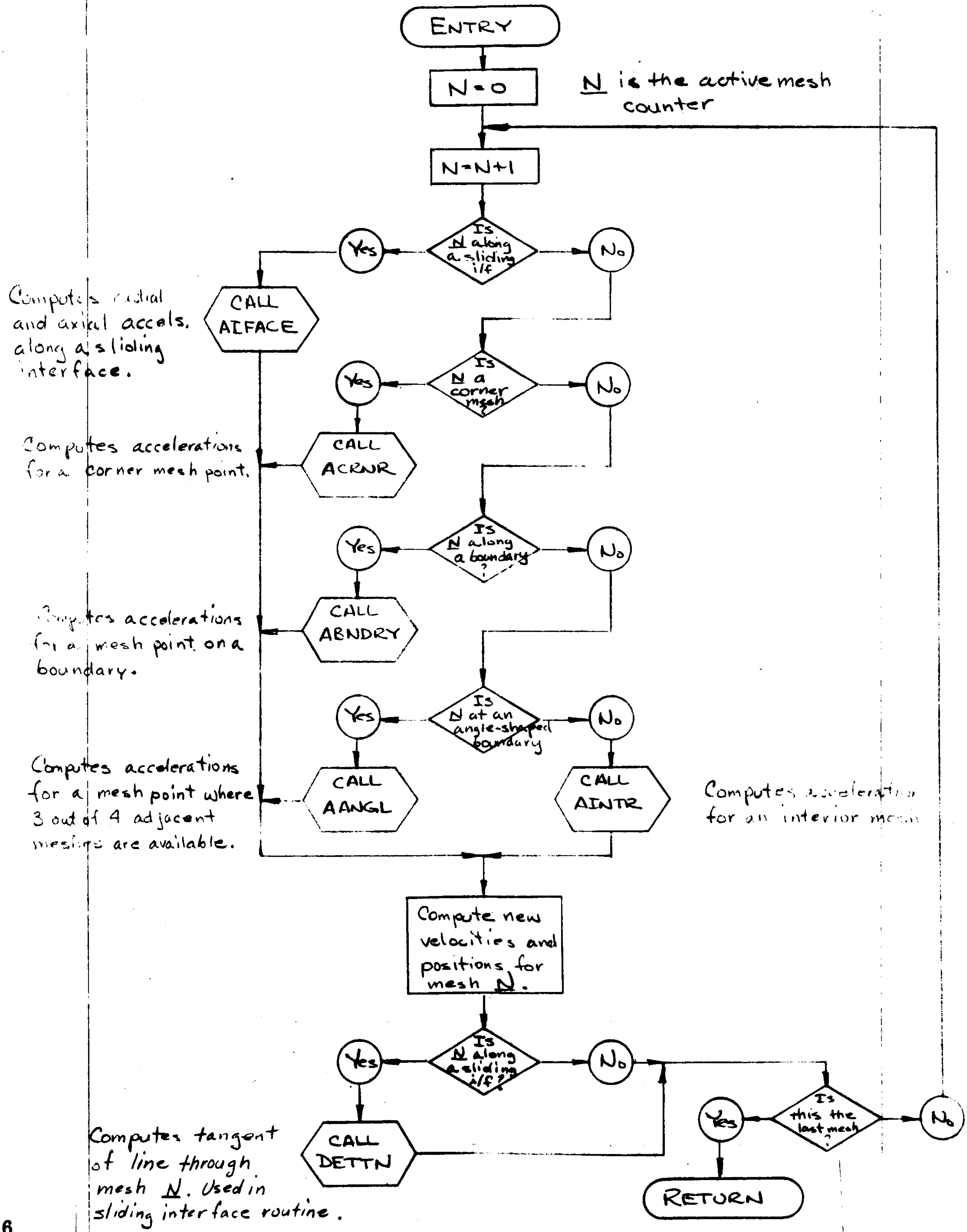
LINK 3


ENTRY
Rewind Chain Tape
Tape 0 is Chain Tape
Check indicator IEND for desired output.
IEND = 4
Write error message - STRESS EXCEEDED
IEND = 3 or 5
Write message - RESTART TAPE UPDATED
Write Tape 4
Write COMMON on tape 4.
IEND = 1 or 6
CALL PRNT
Prints output.
IEND = 2
CALL GRAPH
Plots output on CALCOMP plotter.
RETURN TO MAIN

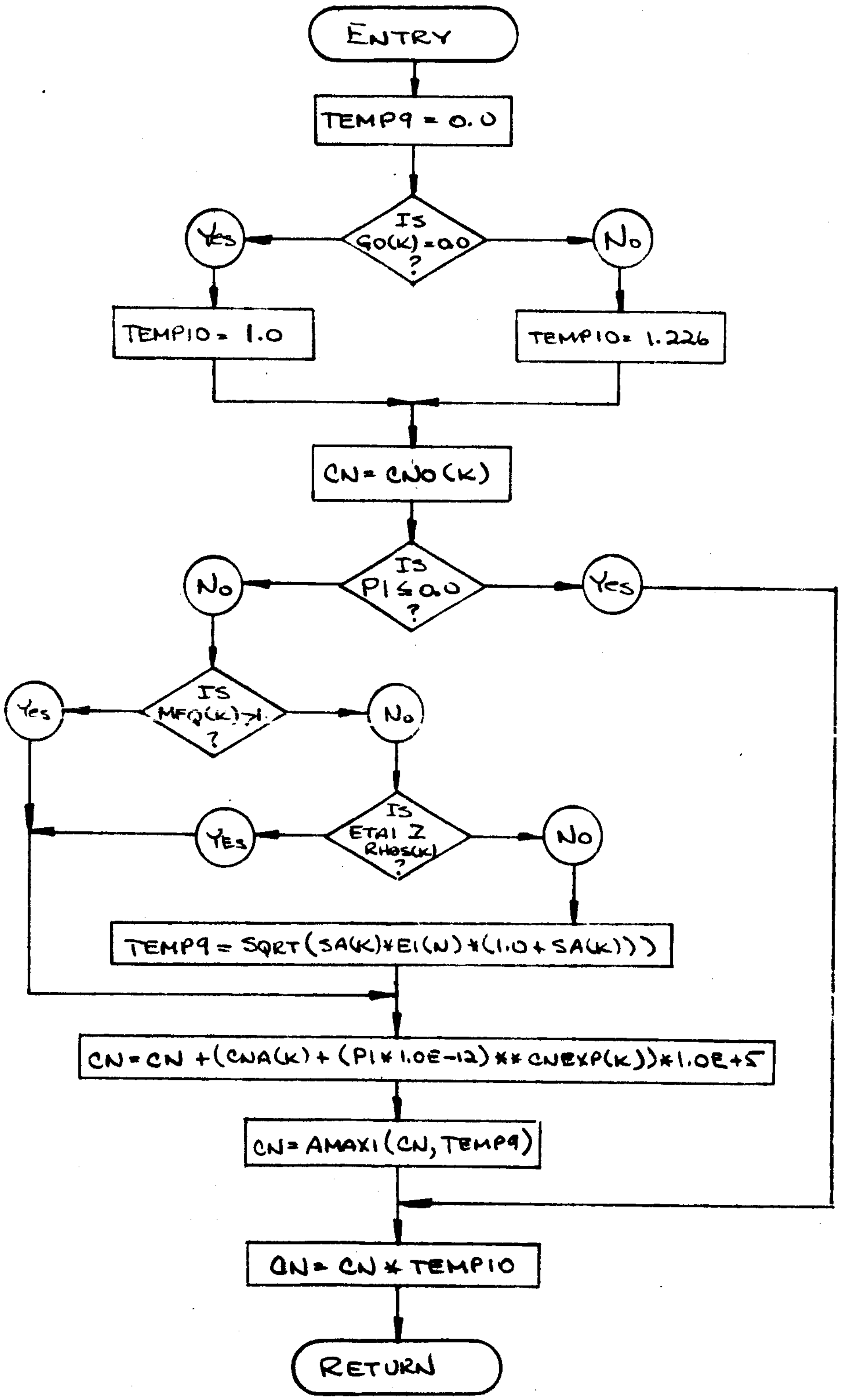
SUBROUTINE COMPUT



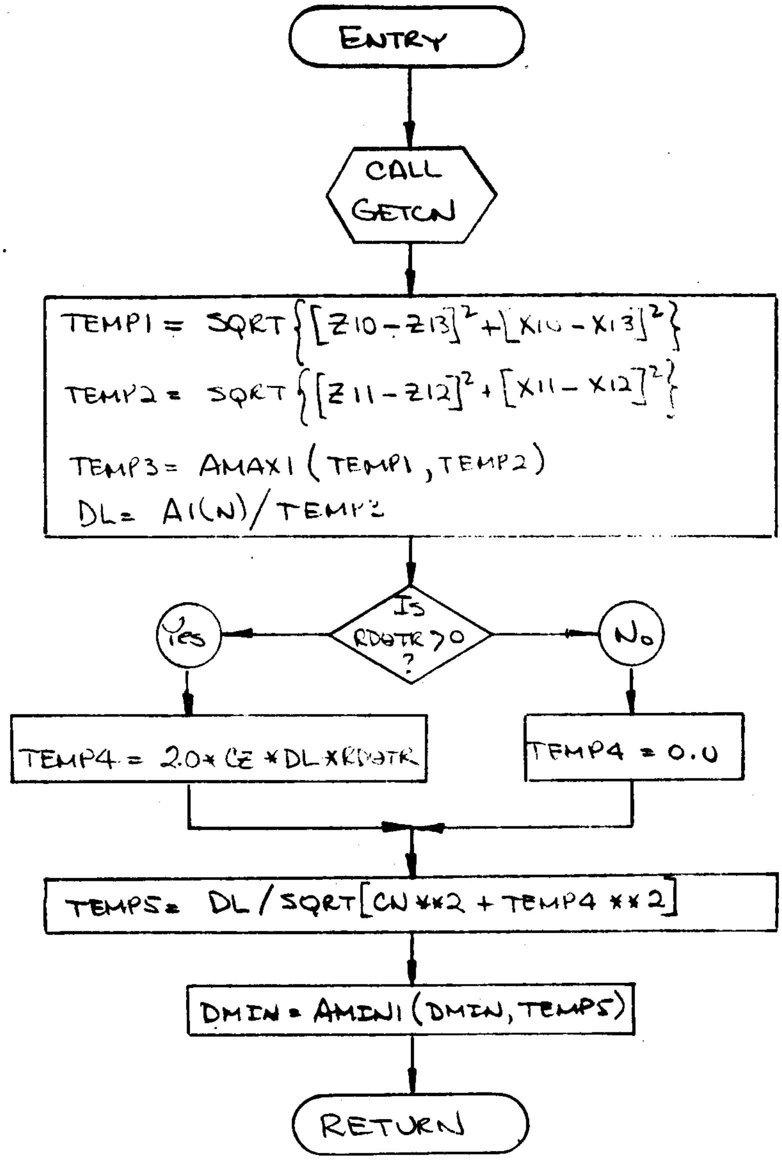
SUBROUTINE GETPOS



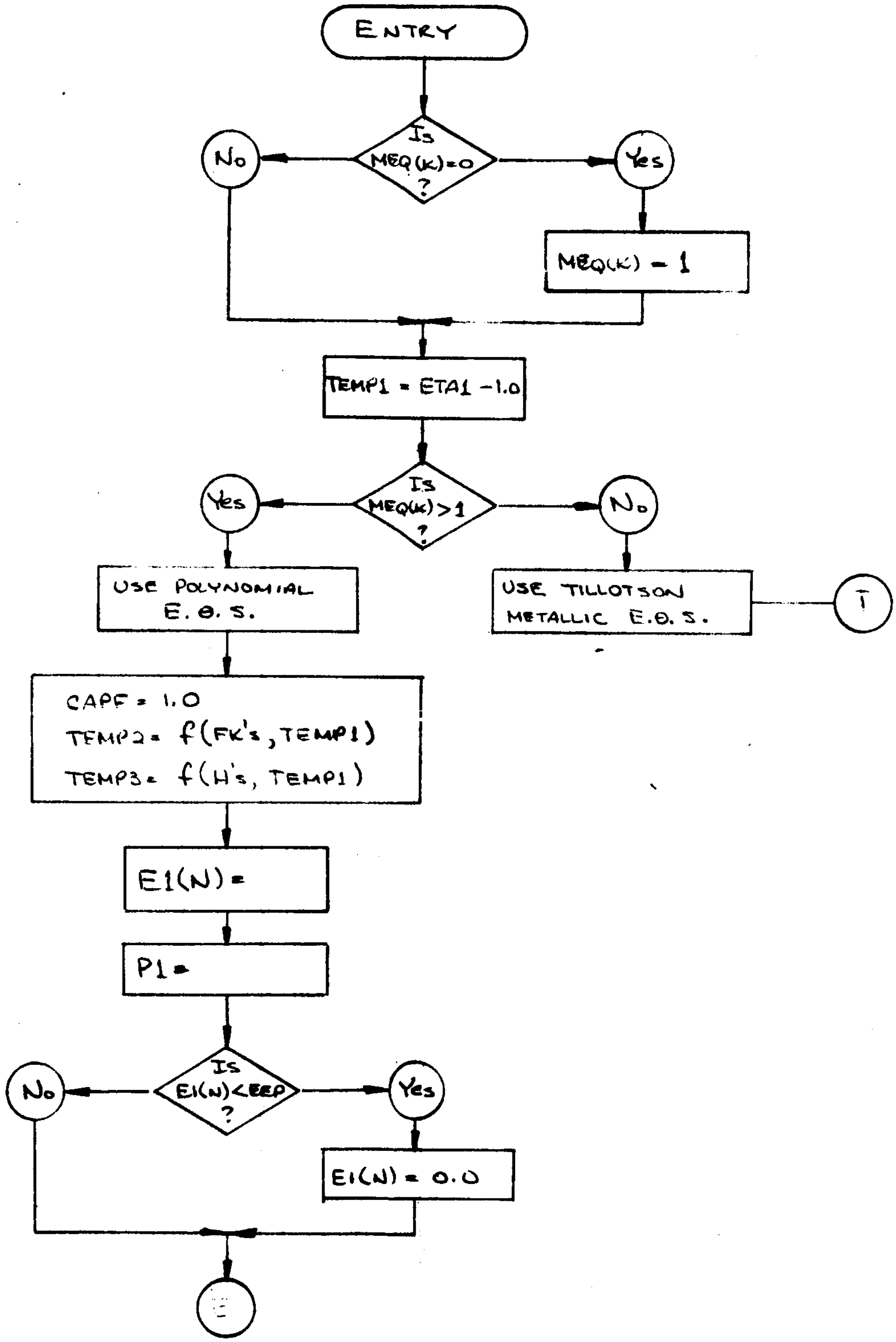
SUBROUTINE GETCN

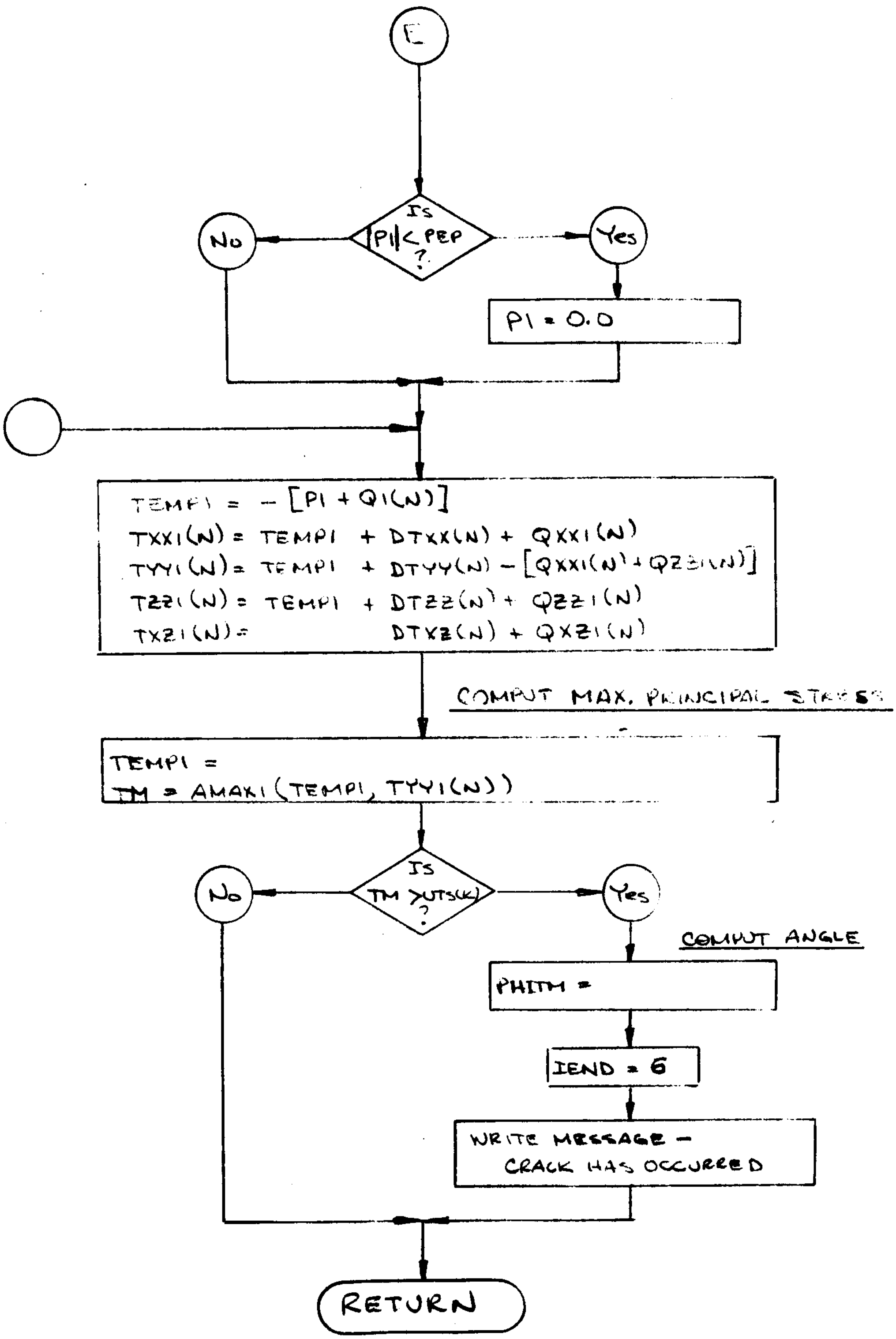


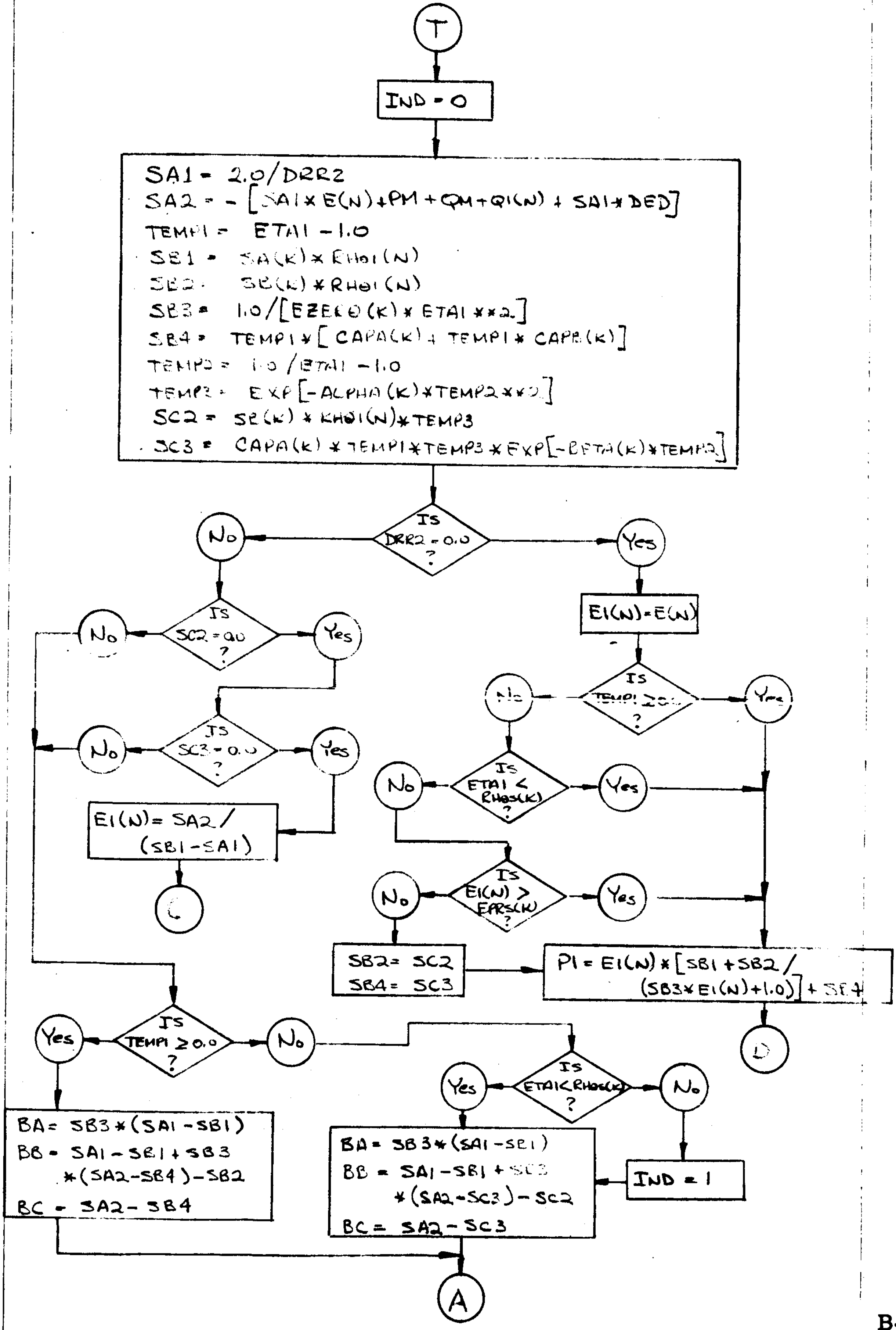
SUBROUTINE COMPDT

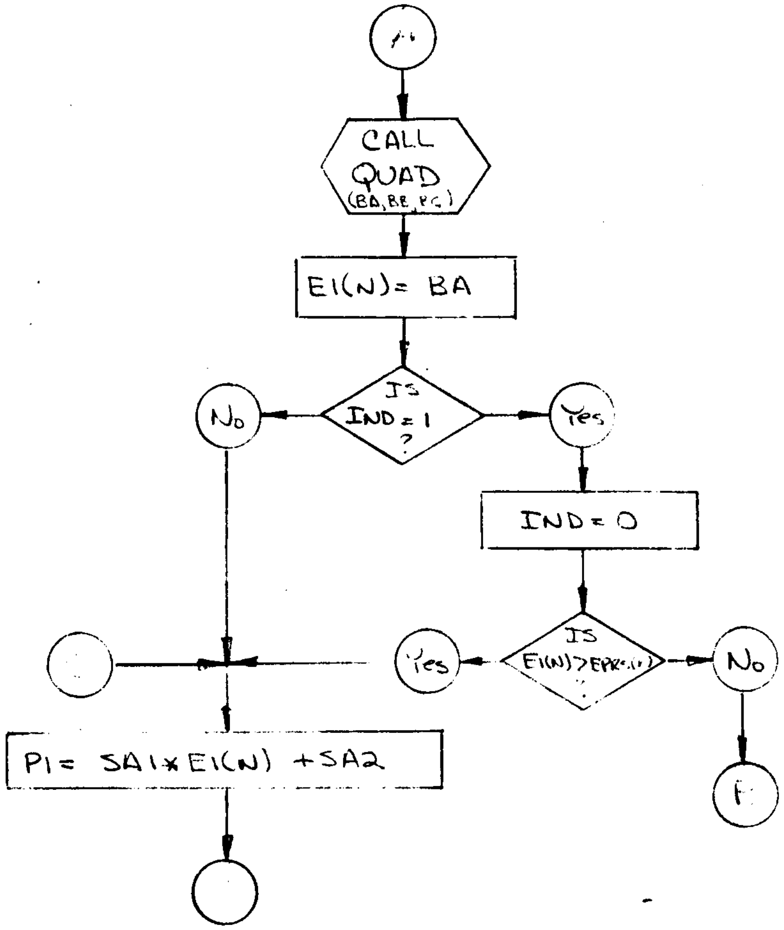


SUBROUTINE FINDP

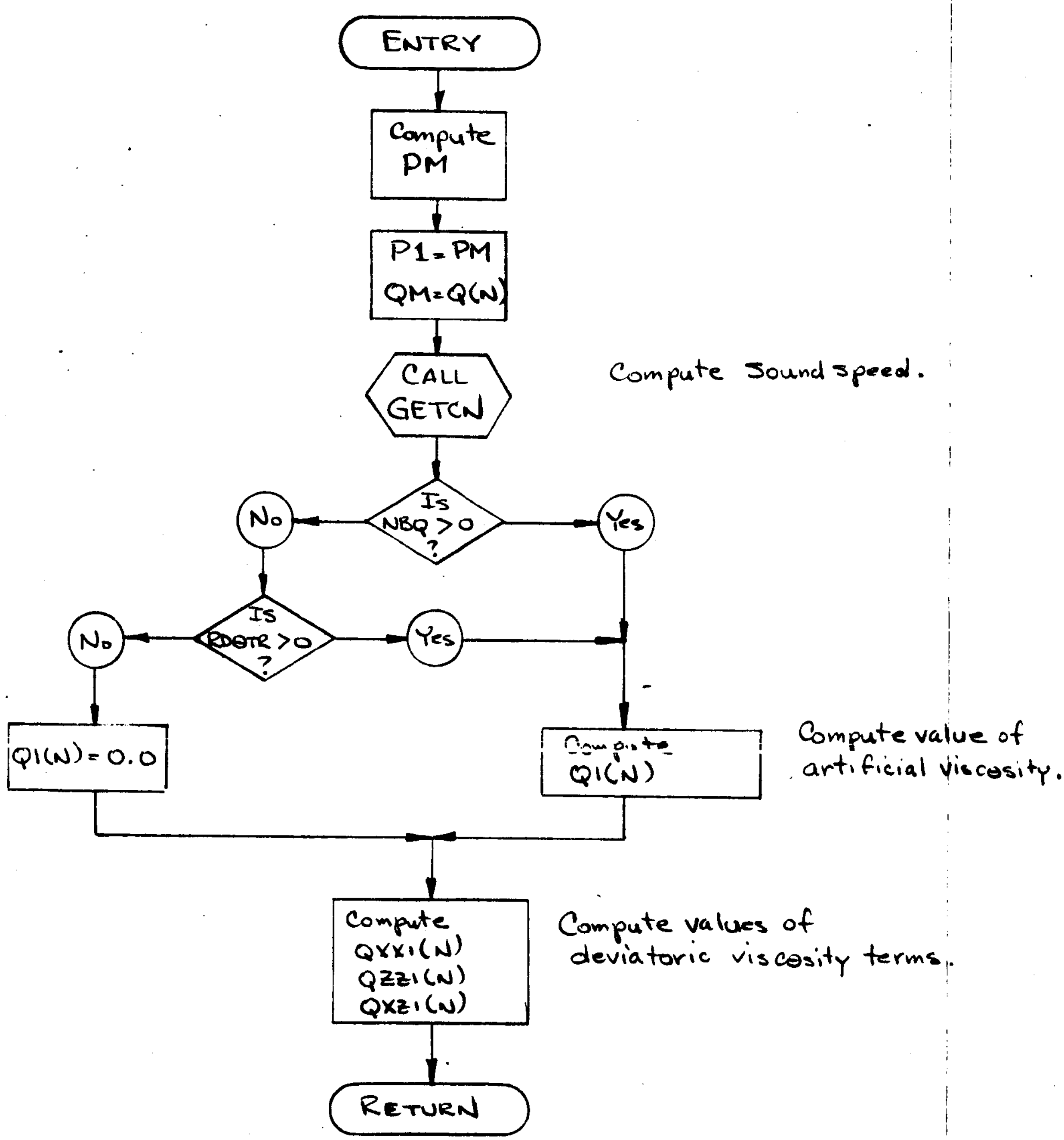



E
Is |P1| < PEP ?
No
Yes
P1 = 0.0
TEMP1 = − [P1 + Q1(N)]
TXX1(N) = TEMP1 + DTXX(N) + QXX1(N)
TYY1(N) = TEMP1 + DTYY(N) − [QXX1(N) + QZZ1(N)]
TZZ1(N) = TEMP1 + DTZZ(N) + QZZ1(N)
TXZ1(N) = DTXZ(N) + QXZ1(N)
COMPUT MAX. PRINCIPAL STRESS
TEMP1 =
TM = AMAX1 (TEMP1, TYY1(N))
Is TM > UTS(K) ?
No
Yes
COMPUT ANGLE
PHITM =
IEND = 6
WRITE MESSAGE — CRACK HAS OCCURRED
RETURN

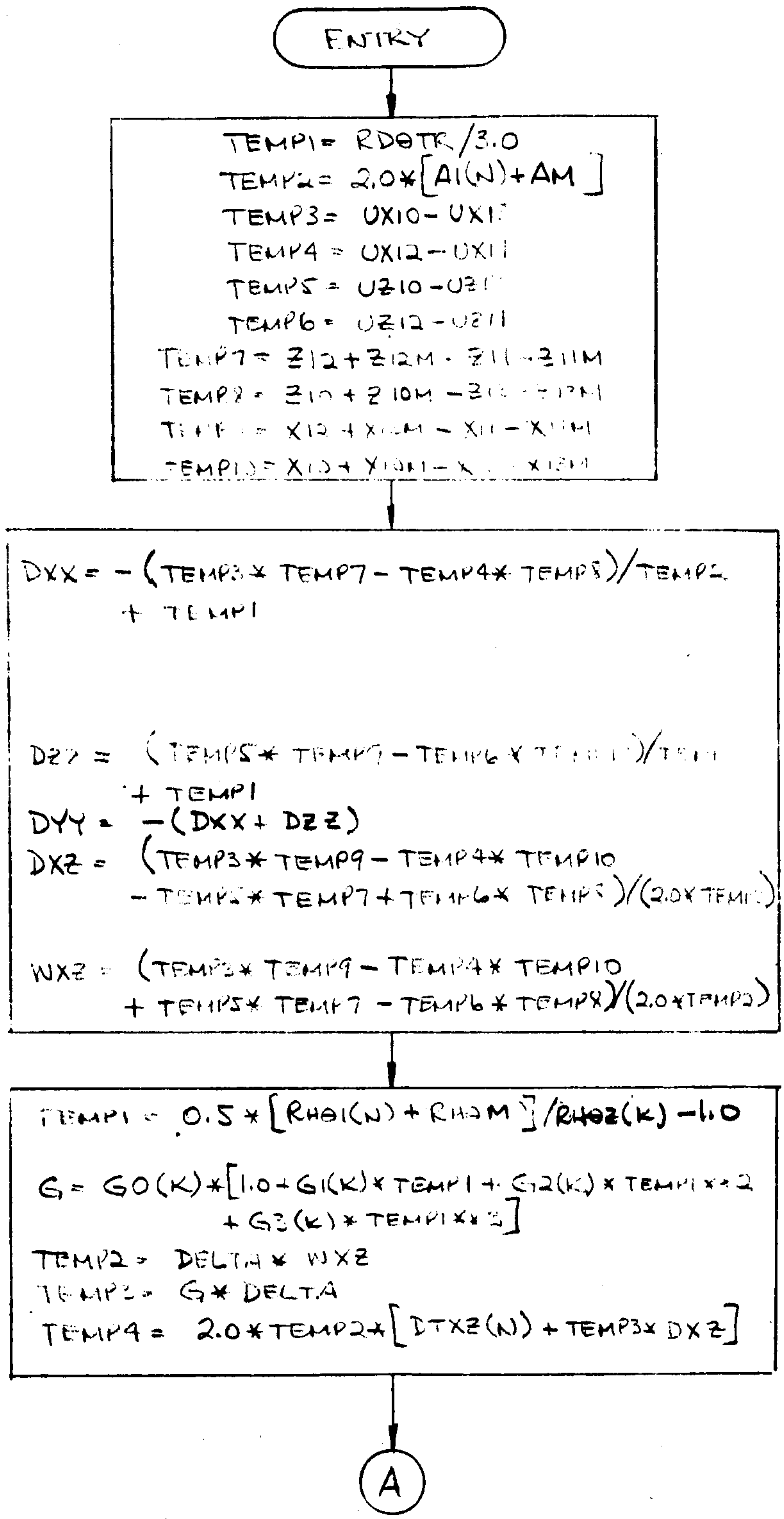

T
IND = 0
SA1 = 2.0/DRR2
SA2 = - [SA1 * E(N) + PM + QM + QI(N) + SA1 * DED]
TEMP1 = ETA1 - 1.0
SB1 = SA(K) * RHO1(N)
SB2 = SB(K) * RHO1(N)
SB3 = 1.0/[EZERO(K) * ETA1 ** 2]
SB4 = TEMP1 * [CAPA(K) + TEMP1 * CAPB(K)]
TEMP2 = 1.0 / ETA1 - 1.0
TEMP3 = EXP [-ALPHA(K) * TEMP2 ** 2]
SC2 = SE(K) * RHO1(N) * TEMP3
SC3 = CAPA(K) * TEMP1 * TEMP3 * EXP[-BETA(K) * TEMP2]
Is DRR2 = 0.0 ?
No
Yes
Is SC2 = 0.0 ?
Is SC3 = 0.0 ?
E1(N) = SA2/ (SB1 - SA1)
C
E1(N) = E(N)
Is TEMP1 ≥ 0.0 ?
Is ETA1 < RHOS(K) ?
Is E1(N) > EPRS(K) ?
SB2 = SC2
SB4 = SC3
P1 = E1(N) * [SB1 + SB2/ (SB3 * E1(N) + 1.0)] + SB4
D
Is TEMP1 ≥ 0.0 ?
Is ETA1 < RHOS(K) ?
IND = 1
BA = SB3 * (SA1 - SB1)
BB = SA1 - SB1 + SB3 * (SA2 - SB4) - SB2
BC = SA2 - SB4
BA = SB3 * (SA1 - SB1)
BB = SA1 - SB1 + SB3 * (SA2 - SC3) - SC2
BC = SA2 - SC3
A

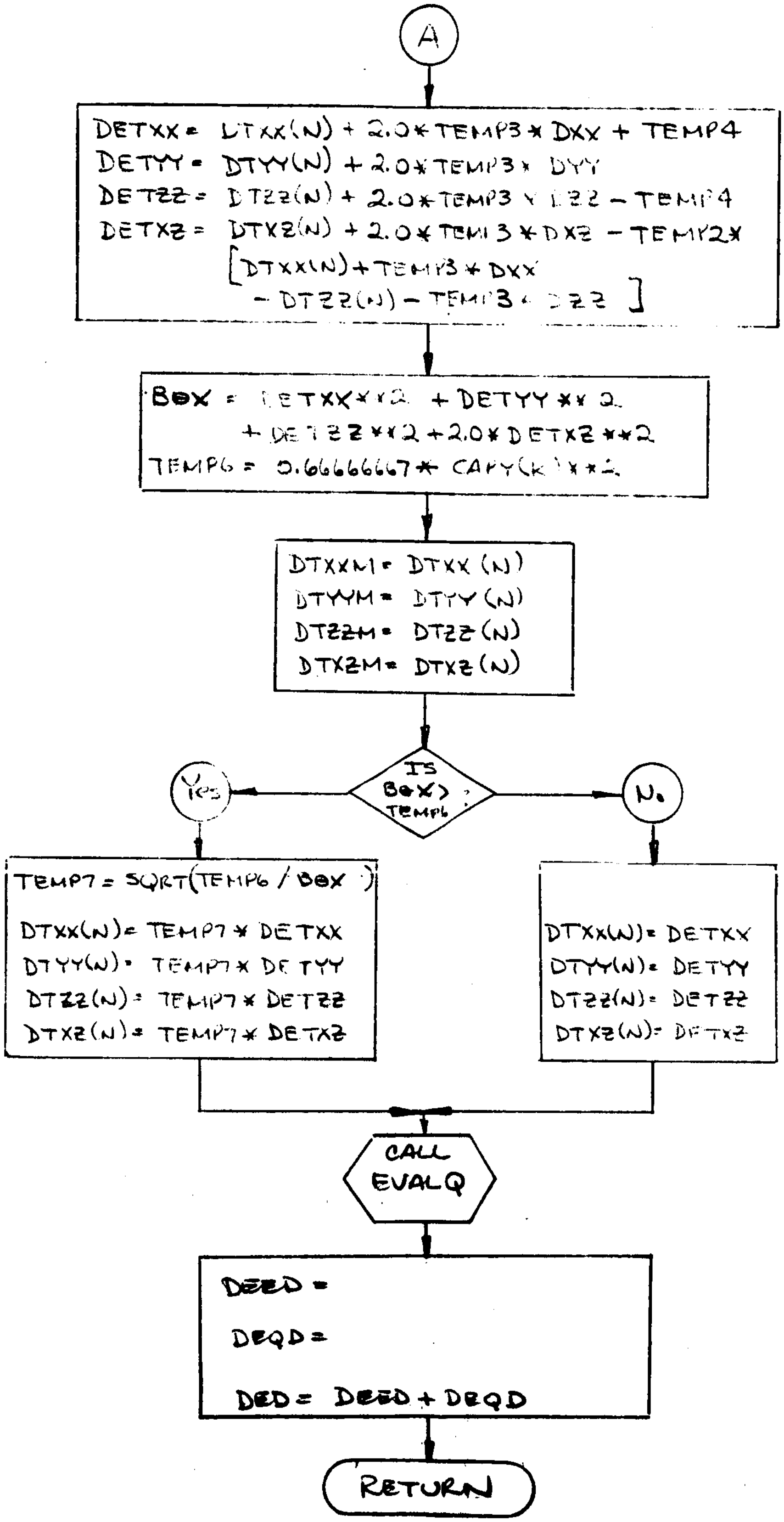

CALL
QUAD
(BA, BE, PC)
E1(N) = BA
IS
IND = 1
?
No
Yes
IND = 0
IS
E1(N) > EPR(I)
?
Yes
No
P1 = SA1 * E1(N) + SA2

SUBROUTINE EVALQ

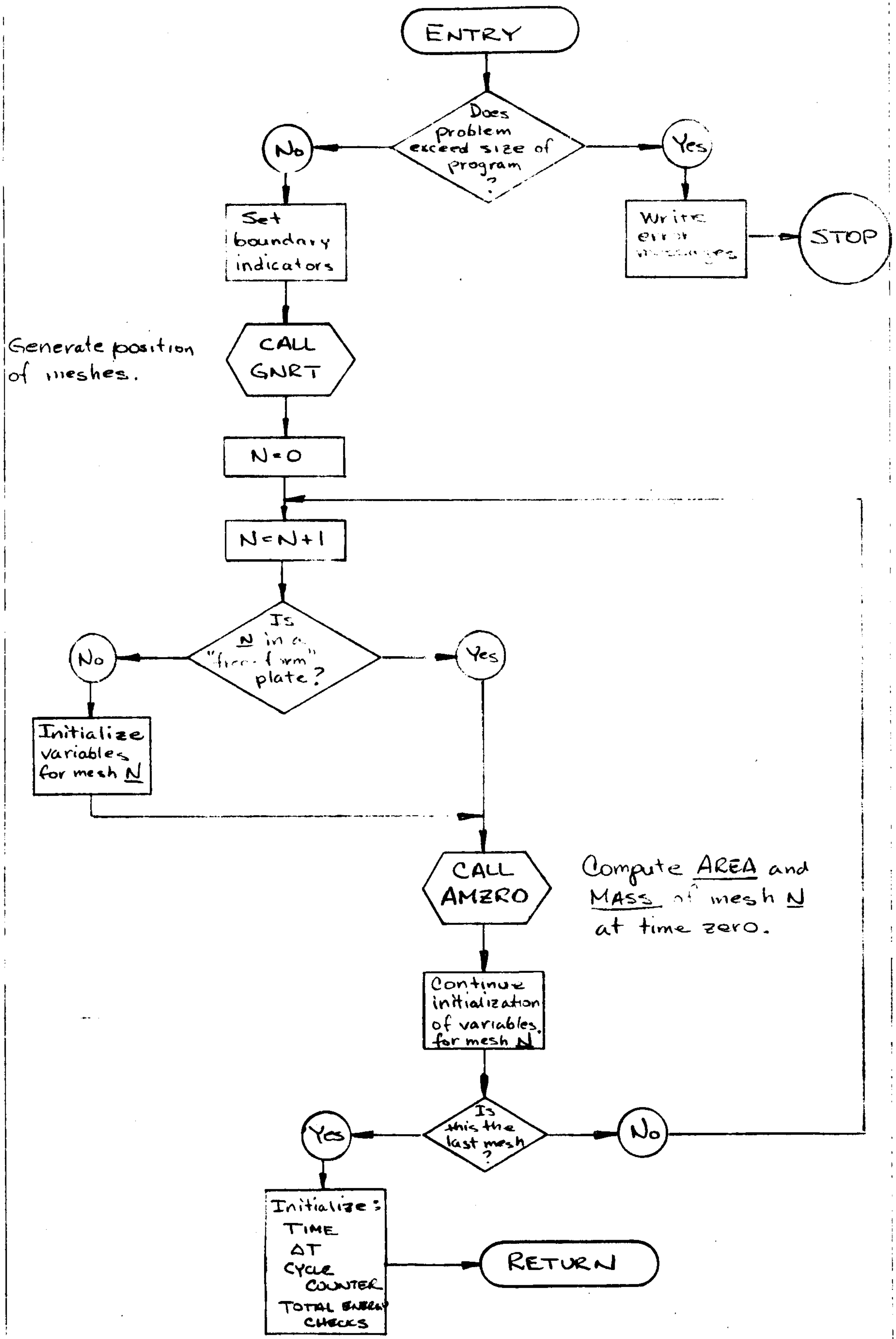


SUBROUTINE SDST

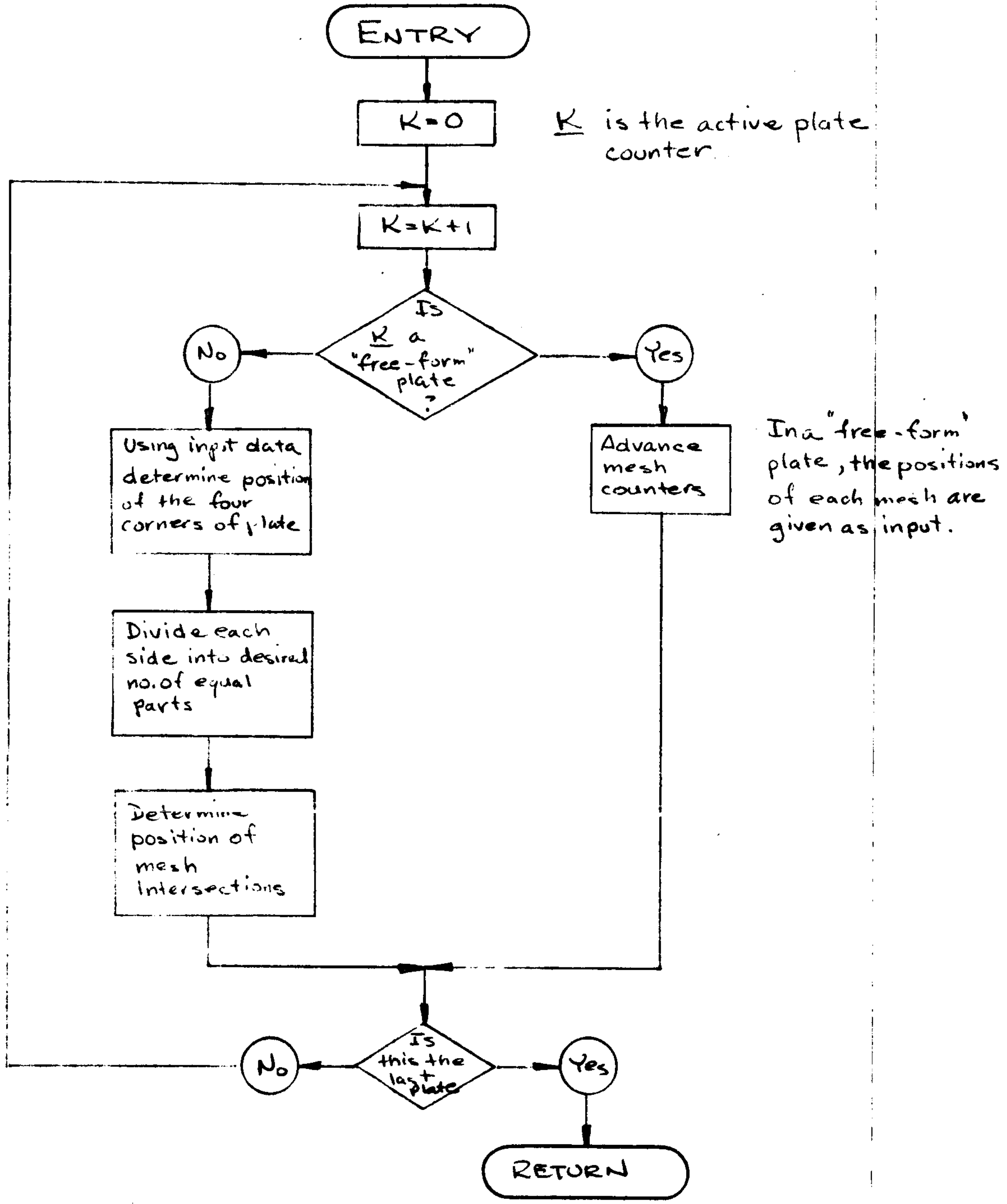



A
DETXX = DTXX(N) + 2.0*TEMP3*DXX + TEMP4
DETYY = DTYY(N) + 2.0*TEMP3*DYY
DETZZ = DTZZ(N) + 2.0*TEMP3*DZZ − TEMP4
DETXZ = DTXZ(N) + 2.0*TEMP3*DXZ − TEMP2*
[DTXX(N) + TEMP3*DXX
− DTZZ(N) − TEMP3*DZZ]
BOX = DETXX**2 + DETYY**2
+ DETZZ**2 + 2.0*DETXZ**2
TEMP6 = 0.66666667*CAPY(K)**2
DTXXM = DTXX(N)
DTYYM = DTYY(N)
DTZZM = DTZZ(N)
DTXZM = DTXZ(N)
IS BOX > TEMP6 ?
Yes
No
TEMP7 = SQRT(TEMP6 / BOX)
DTXX(N) = TEMP7*DETXX
DTYY(N) = TEMP7*DETYY
DTZZ(N) = TEMP7*DETZZ
DTXZ(N) = TEMP7*DETXZ
DTXX(N) = DETXX
DTYY(N) = DETYY
DTZZ(N) = DETZZ
DTXZ(N) = DETXZ
CALL EVALQ
DEED =
DEQD =
DED = DEED + DEQD
RETURN

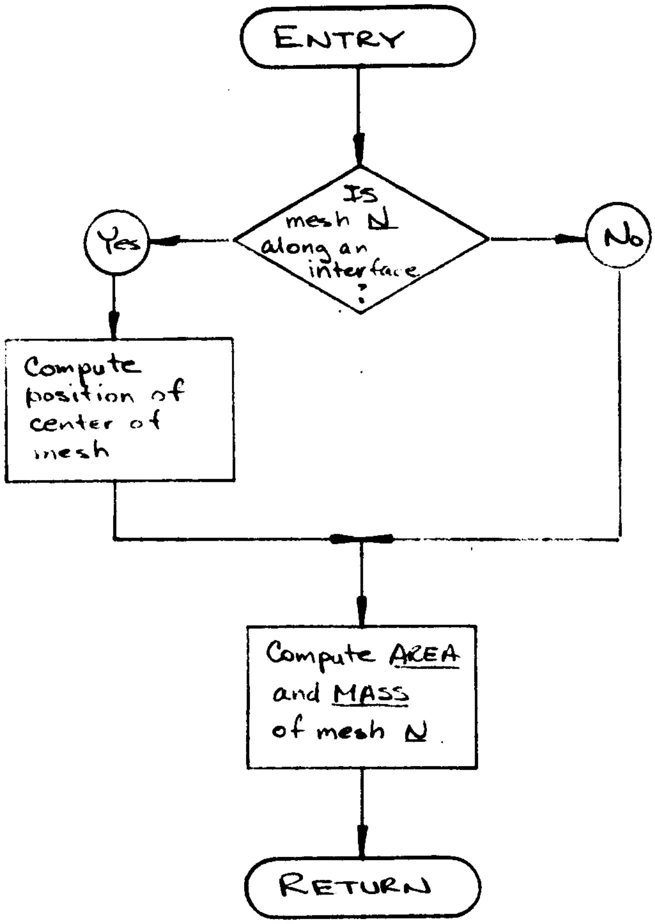
SUBROUTINE INTLIZ



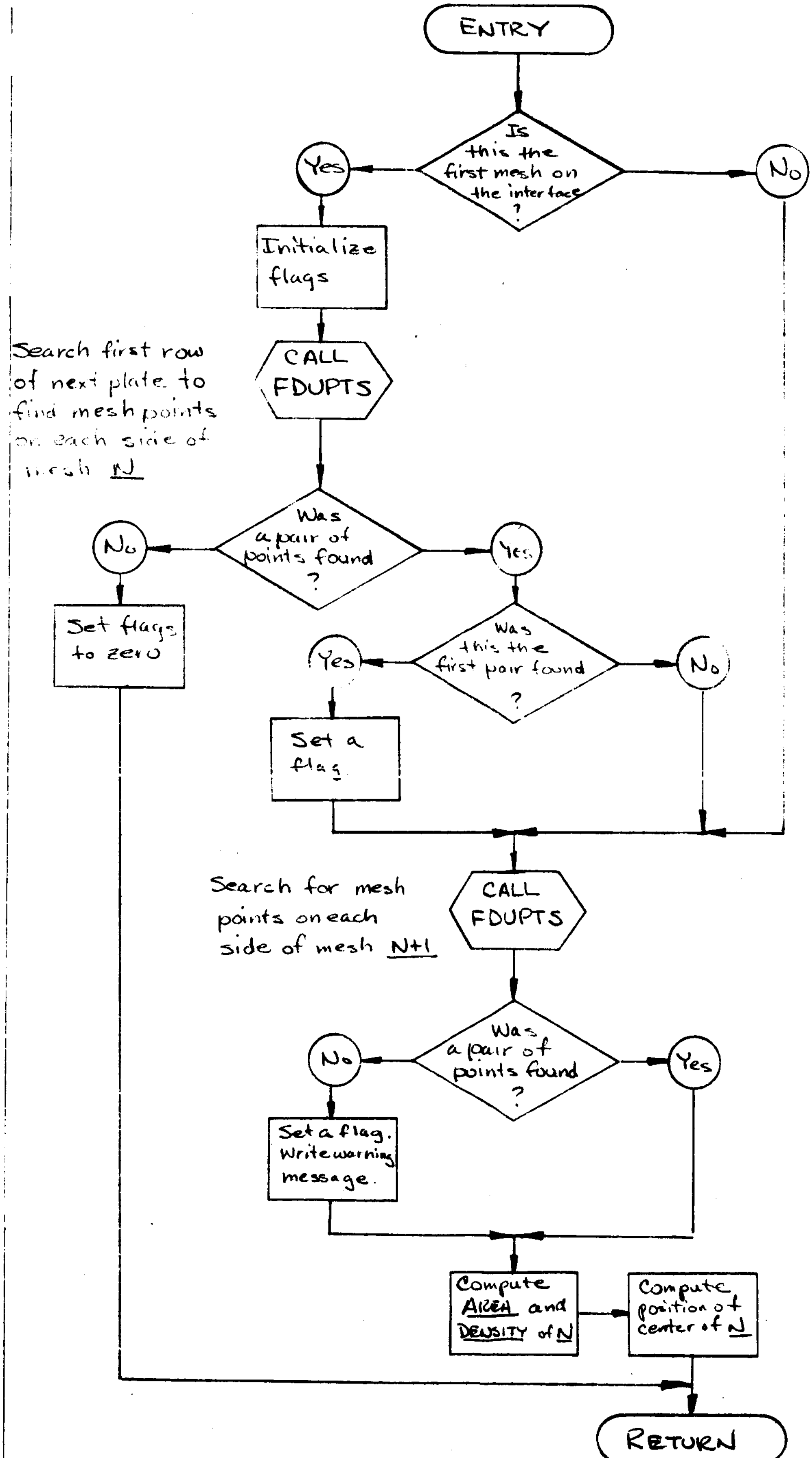
SUBROUTINE GNRT



SUBROUTINE AMZRO

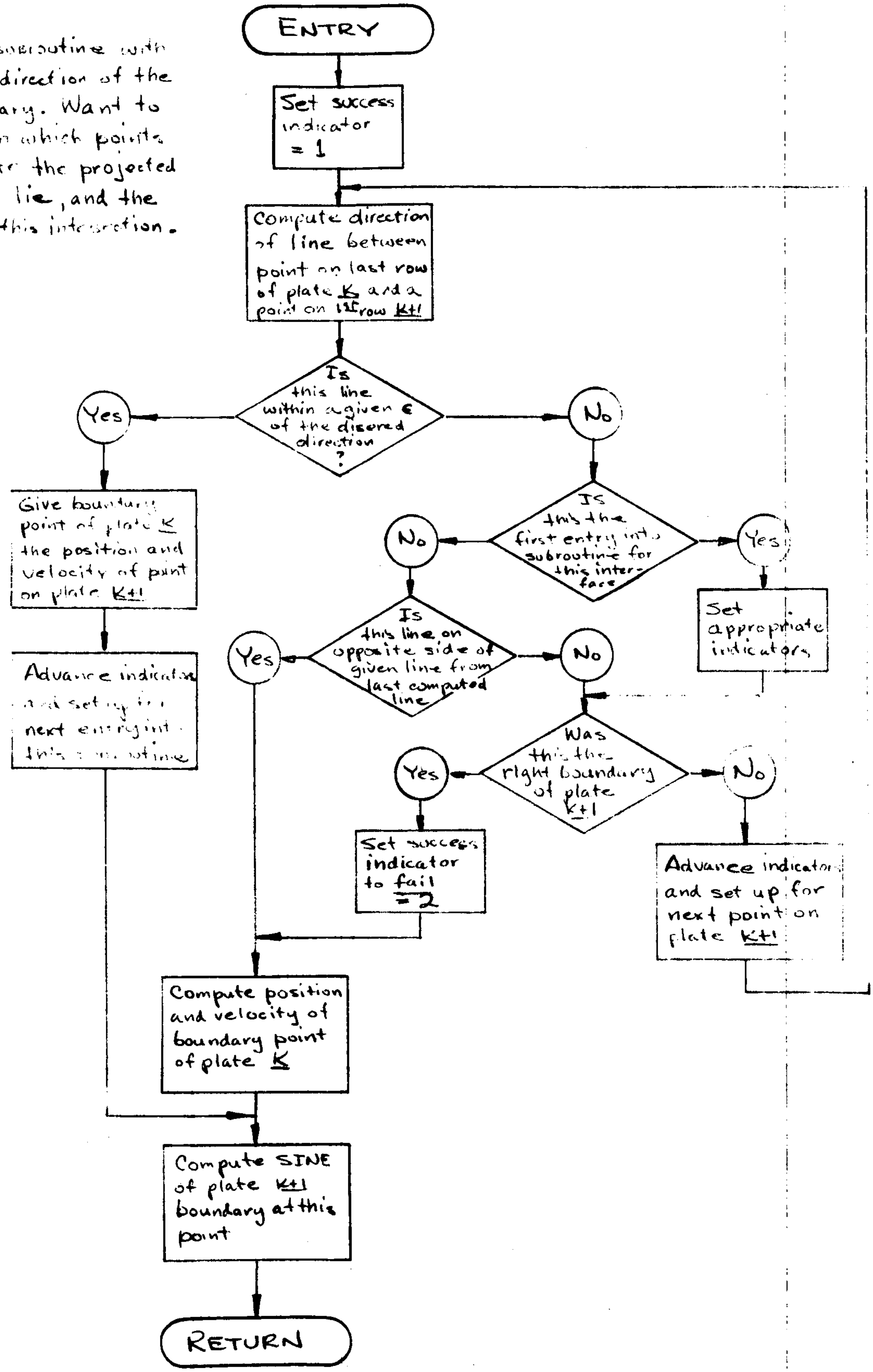


Subroutine CMPTIF

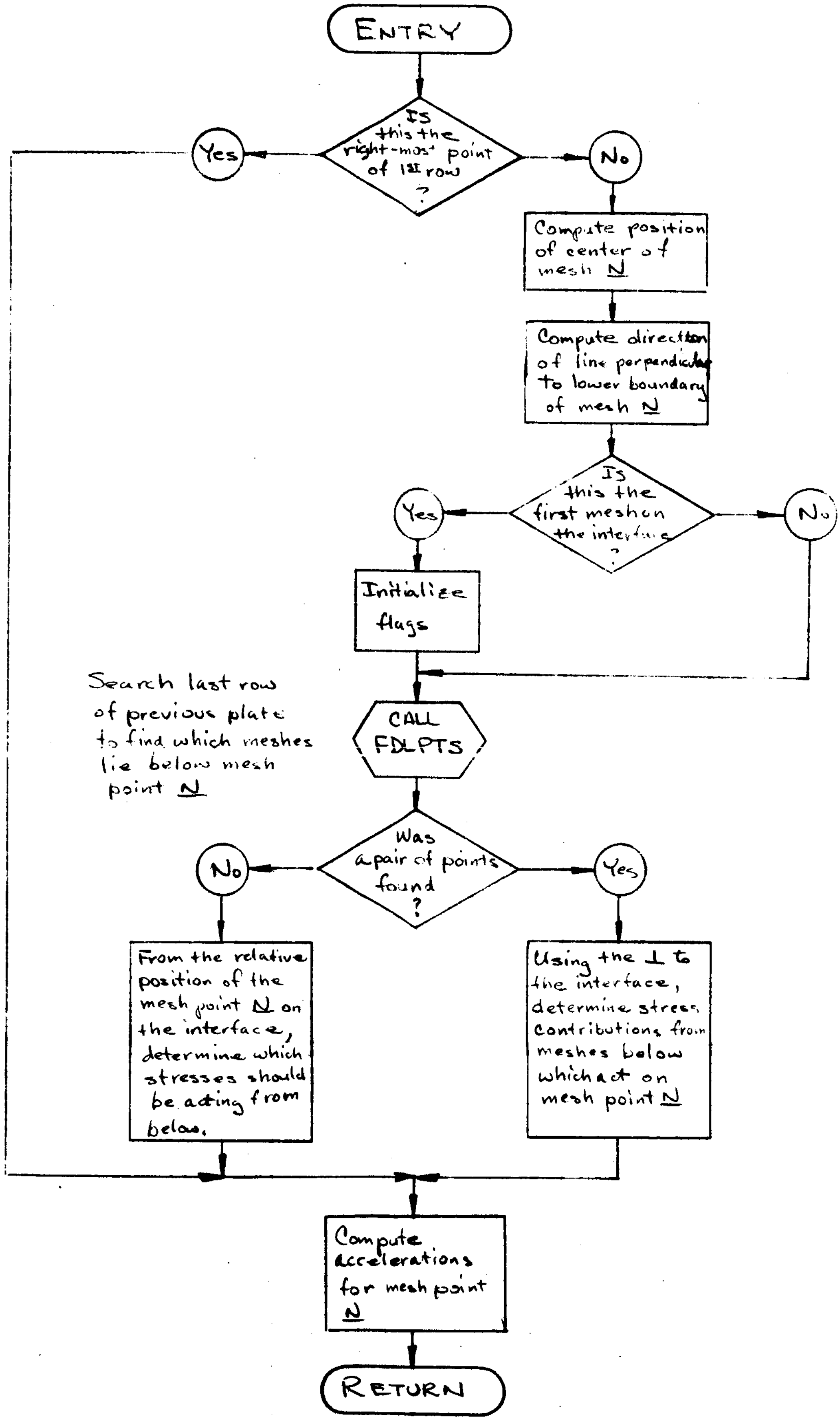


SUBROUTINE FDUPTS

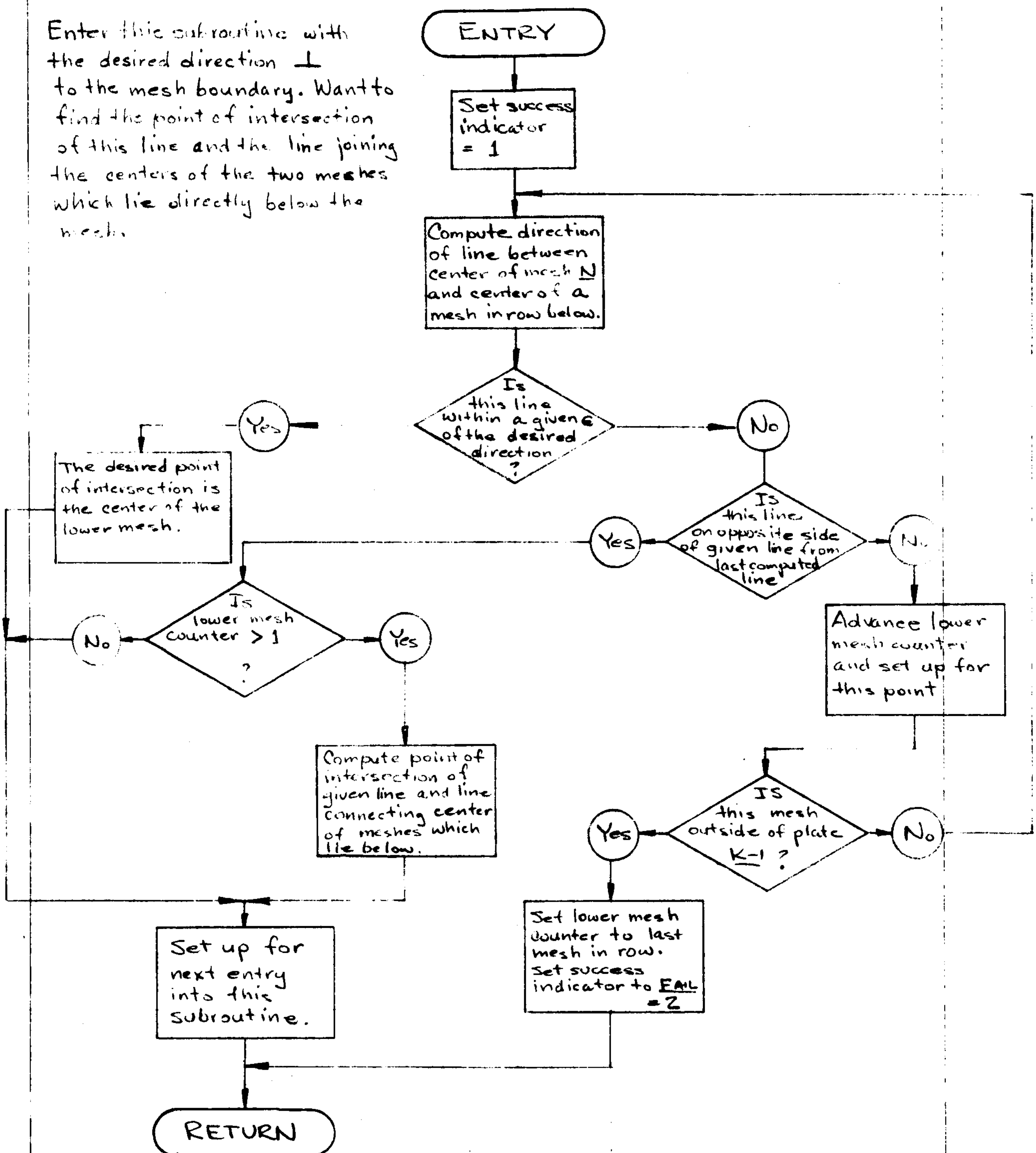
Enter this subroutine with the desired direction of the mesh boundary. Want to find between which points on next plate the projected point would lie, and the position of this intersection.



SUBROUTINE AIFACE



SUBROUTINE FDLPTS



APPENDIX C
LISTINGS OF FORTRAN SOURCE DECKS

The main program and subroutines are listed in this appendix as follows:

NOTE: The blocks of DIMENSION, COMMON, and EQUIVALENCE statements listed on pages C-11 and C-12 are common to all the subroutines except QUAD, PMU, and AX2. Although these blocks are repeated in the cards for the subroutines, they are listed here only once.

```
$IBFTC 2DMN00  LIST,REF
C   ******************** DECK 6A2  GMDRL ********************
C   2-D LAGRANGIAN -- MAIN PROGRAM FOR IBM 7040 VERSION OF CHAIN
C
C
C      DEFINITION OF INPUT QUANTITIES                                    FORMAT
C
C          CARD 1.                                                      (12A6)
C
C   TITLE  - 72 CHARACTERS OF ALPHA-NUMERICS FOR PROBLEM IDENTIFICATION.
C            PRINTS OUT ON FIRST SHEET OF OUTPUT, AND ON EACH PLOT.
C
C          CARD 2.                                                      (6E12.8)
C
C   STRTC  - STARTING TIME CYCLE.
C            IF      0.0 - SUBROUTINE INTLIZ CALLED.
C            IF NOT 0.0 - READS TAPE 4 FOR VALUES OF COMMON AND PROCEEDS.
C
C          CARD 3.                                                      (6E12.8)
C
C   CZ     - COEFFICIENT IN ARTIFICIAL VISCOSITY EQUATION
C   CL     - COEFFICIENT IN ARTIFICIAL VISCOSITY EQUATION
C   C3     - COEFFICIENT IN DEVIATORIC ARTIFICIAL VISCOSITY EQUATION
C   C4     - COEFFICIENT IN DEVIATORIC ARTIFICIAL VISCOSITY EQUATION
C   FK     - COEFFICIENT IN TIME INCREMENT EQUATION
C   SBAR   - UPPER LIMIT OF STRESS. COMPRESSION IS NEGATIVE.           (D/SQCM)
C            IF STRESS EXCEEDS THIS VALUE, PROGRAM WILL STOP.
C
C          CARD 4.                                                      (6E12.8)
C
C   TPRNT  - TIME INCREMENT FOR PRINTOUT AND PLOT                       (SEC)
C   TMAX   - PROBLEM TERMINATE TIME                                     (SEC)
C   PRNTC  - PRINT CYCLE INCREMENT.
C            ALLOWS PRINTOUT AND PLOT AFTER EACH PRNTC CYCLES, IF DESIRED.
C   WRTPC  - WRITE TAPE INCREMENT
C            UPDATES THE RESTART TAPE AFTER EACH WRTPC CYCLES.
C   STOPC  - STOP CYCLE.
C            ALLOWS PROGRAM TO BE TERMINATED AFTER STOPC CYCLES.
C
C          CARD 5 0.                                                    (12I6)
C
C   M      - NO. OF PLATES IN PROBLEM.
C   NBL    - BOUNDARY INDICATOR - LOWER X OF FIRST PLATE.
C            = 1 - FREE SURFACE
C            = 2 - FIXED SURFACE
C   NBU    - BOUNDARY INDICATOR - UPPER X OF LAST PLATE
C   IALF   - GEOMETRY INDICATOR
C            = 1 - PLANE
C            = 2 - CYLINDRICAL
C   NOSLID - BOUNDARY INTERFACE INDICATOR
C            = 0 - THE INTERFACES ARE ALLOWED TO SLIDE. ONLY LAST PLATE
C                  IS ALLOWED TO INCLUDE STRENGTH.
C            = 1 - NO RELATIVE MOVEMENT ALONG INTERFACE BOUNDARIES.
```

```
C                 MESH SIZE IN X-DIRECTION MUST BE SAME FOR ALL PLATES.
C   NID     - ONE DIMENSIONAL CODE SIMULATOR INDICATOR
C             = 0 - NORMAL MODE - 2-D PROBLEM
C             = 1 - 1-D PROBLEM
C   NBQ     - ARTIFICIAL VISCOSITY INDICATOR
C             = 0 - EVALUATE VISCOSITY ONLY IN COMPRESSION
C             = 1 - ALWAYS EVALUATE VISCOSITY
C   NPLOT   - PLOT INDICATOR
C             = 0 OR BLANK - PLOT X VS. Z
C             = 1          - PLOT TXX1 VS. X AND Z
C             = 2          - PLOT TYY1 VS. X AND Z
C             = 3          - PLOT TZZ1 VS. X AND Z
C             = 4          - PLOT TXZ1 VS. X AND Z
C             = 5          - DRAW ALL 5 PLOTS
C             = 6          - SKIP PLOTS
C   NCRVS   - THE NUMBER OF COLUMNS (I VALUES) TO BE PLOTTED. MAX. OF 20.
C
C          CARD 5 1.   IF NCRVS = 0, OMIT THIS CARD.                    (12I6)
C
C   JCRVS(L),L=1,NCRVS
C           - THE COLUMN VALUES TO BE PLOTTED.
C
C          CARD 6 0.   THERE ARE 8 CARDS TO DEFINE EACH PLATE. THERE     (12I6)
C                      WILL BE  M  SETS OF THE FOLLOWING 8 CARDS.
C
C   IM      - NO. OF ZONES IN RADIAL (X) DIRECTION
C   JM      - NO. OF ZONES IN AXIAL  (Z) DIRECTION
C   NBZL    - INDICATOR FOR BOUNDARY CONDITION - LEFT  Z OF EACH PLATE
C   NBZR    - INDICATOR FOR BOUNDARY CONDITION - RIGHT Z OF EACH PLATE
C   MEQ     - INDICATOR FOR EQUATION-OF-STATE
C             = 1 - TILLOTSON
C             = 2 - POLYNOMIAL
C             = 3 - POLYTROPIC (EXPLOSIVE)
C
C          CARD 6 1.                                                    (6E12.8)
C
C   XI      - INITIAL POSITION IN X-DIRECTION OF LOWER-LEFT CORNER      (CM)
C   XL      - WIDTH OF PLATE IN X-DIRECTION                             (CM)
C   ZI      - INITIAL POSITION IN Z-DIRECTION OF UPPER-LEFT CORNER      (CM)
C   ZL      - LENGTH OF PLATE IN Z-DIRECTION                            (CM)
C   UXZ     - INITIAL VELOCITY IN X-DIRECTION                           (CM/SEC)
C   UZZ     - INITIAL VELOCITY IN Z-DIRECTION                           (CM/SEC)
C
C          CARD 6 2.                                                    (6E12.8)
C
C   UXINT   - INITIAL INTERFACE VELOCITY IN X-DIRECTION                 (CM/SEC)
C   UZINT   - INITIAL INTERFACE VELOCITY IN Z-DIRECTION                 (CM/SEC)
C   RHOI    - INITIAL VALUE OF MATERIAL DENSITY                         (G/CC)
C   TXXZ    - INITIAL VALUE OF STRESS IN X-DIRECTION                    (D/SQCM)
C   TYYZ    - INITIAL VALUE OF STRESS IN Y-DIRECTION                    (D/SQCM)
C   TZZZ    - INITIAL VALUE OF STRESS IN Z-DIRECTION                    (D/SQCM)
C
C          CARD 6 3.                                                    (6E12.8)
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C
C   TXZZ    - INITIAL VALUE OF SHEAR STRESS                              (D/SQCM)
C   EZ      - INITIAL VALUE OF ENERGY                                    (ERG/GM)
C   PHIL    - ANGLE OF INCLINATION OF LEFT Z-BOUNDARY                    (DEG)
C   PHIR    - ANGLE OF INCLINATION OF RIGHT Z-BOUNDARY                   (DEG)
C   PSIL    - ANGLE OF INCLINATION OF LOWER X-BOUNDARY                   (DEG)
C   PSIU    - ANGLE OF INCLINATION OF UPPER X-BOUNDARY                   (DEG)
C
C         IF IM = 0, THE PLATE WILL BE --FREE-FORM-- IN SHAPE (EX. A
C                    DEBRIS CLOUD). ALL QUANTITIES NEEDED TO DEFINE THE
C                    PLATE ARE INPUTTED.
C
C                    ONLY ONE FREE-FORM PLATE PER PROBLEM.
C
C                    CARDS 6 1, 6 2, 6 3 ARE OMITTED AND REPLACED WITH
C                    THE FOLLOWING CARDS.
C
C                    THERE WILL BE JM SETS OF CARDS. EACH SET CONSISTS
C                    OF THREE TYPES OF CARDS, AND DEFINES A ROW.
C
C         CARD 1.                                                        (I6)
C
C   IM1(J) - J IS THE ROW INDICATOR. NO. OF ZONES IN ROW J. UP TO 20 ROWS
C            ARE ALLOWED.
C
C         CARD 2.    EACH PAIR OF CARDS (ONE CARD 2 AND ONE CARD 3)      (5E15.8)
C                    DEFINES A ZONE. THEREFORE, THERE WILL BE IM1 PAIRS
C                    OF CARDS.
C
C   UX(N)   - N IS THE ZONE COUNTER. RADIAL VELOCITY                     (CM/SEC)
C   UZ(N)   - AXIAL VELOCITY                                             (CM/SEC)
C   P(N)    - HYDRODYNAMIC PRESSURE IN THE ZONE                          (D/SQCM)
C   E(N)    - ENERGY                                                     (ERG/GM)
C   RHO(N)  - DENSITY                                                    (G/CC)
C
C         CARD 3.                                                        (5E15.8)
C
C   X(N)    - RADIAL POSITION                                            (CM)
C   Z(N)    - AXIAL POSITION                                             (CM)
C
C                    AFTER ALL ZONE-DEFINING CARDS HAVE BEEN READ IN,
C                    CARDS DEFINING THE POSITION AND VELOCITIES OF THE
C                    RIGHT-HAND AND INTERFACE BOUNDARY MESH POINTS ARE
C                    READ.
C
C         CARD 4.    EACH CARD DEFINES ONE BOUNDARY MESH POINT.          (5E15.8)
C
C   UXS(NS) - RADIAL VELOCITY                                            (CM/SEC)
C   UZS(NS) - AXIAL VELOCITY                                             (CM/SEC)
C   XS(NS)  - RADIAL POSITION                                            (CM)
C   ZS(NS)  - AXIAL POSITION                                             (CM)
C
C      THE FOLLOWING QUANTITIES DESCRIBE THE MATERIAL PROPERTIES OF EACH PLATE
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```
C
C      FOR MEQ = 1
C           CARD 6 4.                                                    (6E12.8)
C
C  RHOZ    - DENSITY                                                     (G/CC)
C  SA      - SMALL A IN E.O.S.
C  SB      - SMALL B IN E.O.S.
C  CAPA    - BIG A IN E.O.S.                                             (D/SQCM)
C  CAPB    - BIG B IN E.O.S.                                             (D/SQCM)
C  ALPHA   - ALPHA IN E.O.S.
C
C      FOR MEQ = 2
C           CARD 6 4.                                                    (6E12.8)
C
C  RHOZ    - DENSITY                                                     (G/CC)
C  K0      - COEFFICINT OF ARG**0 IN F1
C  K1      - COEFFICINT OF ARG**1 IN F1
C  K2      - COEFFICINT OF ARG**2 IN F1
C  K3      - COEFFICINT OF ARG**3 IN F1
C  H0      - COEFFICINT OF ARG**0 IN F2
C
C      FOR MEQ = 1
C           CARD 6 5.                                                    (6E12.8)
C
C  BETA    - BETA  IN E.O.S.
C  EZERO   - E SUB ZERO IN E.O.S.                                        (ERG/GM)
C  RHOS    - RHO SUB S  IN E.O.S.
C  EPRS    - E SUB S  IN E.O.S.                                          (ERG/GM)
C
C      FOR MEQ = 2
C           CARD 6 5.                                                    (6E12.8)
C  H1      - COEFFICINT OF ARG**1 IN F2
C  H2      - COEFFICINT OF ARG**2 IN F2
C  H3      - COEFFICINT OF ARG**3 IN F2
C
C           CARD 6 6.                                                    (6E12.8)
C
C  CNO     - CONSTANT TERM IN SOUND SPEED EQUATION                       (CM/SEC)
C  CNA     - PRESSURE MULTIPLIER TERM IN SOUND SPEED EQUATION            (CMPS/MB)
C  CNEXP   - EXPONENT OF PRESSURE TERM IN SOUND SPEED EQUATION
C
C           CARD 6 7.                                                    (6E12.8)
C
C  G0      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                      (D/SQCM)
C  G1      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                      (D/SQCM)
C  G2      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                      (D/SQCM)
C  G3      - COEFFICIENT IN MODULUS OF RIGIDITY FIT                      (D/SQCM)
C  CAPY    - YIELD STRENGTH                                              (D/SQCM)
C  UTS     - ULTIMATE TENSILE STRENGTH OF MATERIAL                       (D/SQCM)
C
C      END OF INPUT QUANTITIES.
C
C      ALL QUANTITIES DEFINED BELOW REFER TO INDIVIDUAL ZONES.
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C
C   DX      - INITIAL ZONE SIZE IN X-DIRECTION                                  (CM)
C   DZ      - INITIAL ZONE SIZE IN Z-DIRECTION                                  (CM)
C   X       - RADIAL POSITION AT TIME  N                                        (CM)
C   X1      - RADIAL POSITION AT TIME  N+1                                      (CM)
C   Z       - AXIAL POSITION AT TIME  N                                         (CM)
C   Z1      - AXIAL POSITION AT TIME  N+1                                       (CM)
C   UX      - RADIAL VELOCITY AT TIME  N-1/2                                    (CM/SEC)
C   UX1     - RADIAL VELOCITY AT TIME  N+1/2                                    (CM/SEC)
C   UZ      - AXIAL VELOCITY AT TIME  N-1/2                                     (CM/SEC)
C   UZ1     - AXIAL VELOCITY AT TIME  N+1/2                                     (CM/SEC)
C   RHO     - DENSITY AT TIME  N                                                (G/CC)
C   RHO1    - DENSITY AT TIME  N+1                                              (G/CC)
C   A       - AREA AT TIME  N                                                   (SQ CM)
C   A1      - AREA AT TIME  N+1                                                 (SQ CM)
C   TXX     - STRESS IN X-DIRECTION AT TIME  N                                  (D/SQCM)
C   TXX1    - STRESS IN X-DIRECTION AT TIME  N+1                                (D/SQCM)
C   TYY     - STRESS IN Y-DIRECTION AT TIME  N                                  (D/SQCM)
C   TYY1    - STRESS IN Y-DIRECTION AT TIME  N+1                                (D/SQCM)
C   TZZ     - STRESS IN Z-DIRECTION AT TIME  N                                  (D/SQCM)
C   TZZ1    - STRESS IN Z-DIRECTION AT TIME  N+1                                (D/SQCM)
C   TXZ     - SHEAR STRESS AT TIME  N                                           (D/SQCM)
C   TXZ1    - SHEAR STRESS AT TIME  N+1                                         (D/SQCM)
C   Q       - ARTIFICIAL VISCOSITY VALUE AT TIME  N                             (D/SQCM)
C   Q1      - ARTIFICIAL VISCOSITY VALUE AT TIME  N+1                           (D/SQCM)
C   QXX1    - X COMPONENT OF DEVIATORIC ARTIFICIAL VISCOSITY                    (D/SQCM)
C   QZZ1    - Z COMPONENT OF DEVIATORIC ARTIFICIAL VISCOSITY                    (D/SQCM)
C   QXZ1    - SHEAR COMP. OF DEVIATORIC ARTIFICIAL VISCOSITY                    (D/SQCM)
C   E       - ENERGY AT TIME N                                                  (ERG)
C   E1      - ENERGY AT TIME N+1                                                (ERG)
C   ZM      - MASS                                                              (GM)
C   DTXX    - DEVIATORIC STRESS IN X-DIRECTION AT TIME  N+1                     (D/SQCM)
C   DTYY    - DEVIATORIC STRESS IN Y-DIRECTION AT TIME  N+1                     (D/SQCM)
C   DTZZ    - DEVIATORIC STRESS IN Z-DIRECTION AT TIME  N+1                     (D/SQCM)
C   DTXZ    - DEVIATORIC SHEAR STRESS AT TIME  N+1                              (D/SQCM)
C   TM      - LARGEST TENSILE STRESS OF THE 3 PRINCIPAL STRESSES                (D/SQCM)
C   PHITM   - ANGLE OF ROTATION OF THE PRINCIPAL STRESS COORDINATES             (DEG)
C
C      THE FOLLOWING QUANTITIES REFER TO SPECIAL RIGHT AND INTER-FACE
C      BOUNDARY MESH POINTS.
C
C   XS      - RADIAL POSITION AT TIME  N                                        (CM)
C   XS1     - RADIAL POSITION AT TIME  N+1                                      (CM)
C   ZS      - AXIAL POSITION AT TIME  N                                         (CM)
C   ZS1     - AXIAL POSITION AT TIME  N+1                                       (CM)
C   UXS     - RADIAL VELOCITY AT TIME  N-1/2                                    (CM/SEC)
C   UXS1    - RADIAL VELOCITY AT TIME  N+1/2                                    (CM/SEC)
C   UZS     - AXIAL VELOCITY AT TIME  N-1/2                                     (CM/SEC)
C   UZS1    - AXIAL VELOCITY AT TIME  N+1/2                                     (CM/SEC)
C   TN      - TANGENT FOR SLIDING INTERFACE
C   SNAB    - SINE OF LINE AB FOR SLIDING INTERFACE
C
C      OTHER TERMS WHICH APPEAR IN COMMON
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C
C   NCYCLE - CYCLE COUNTER
C   TIME   - TIME                                                           (SEC)
C   DELPR  - SAME AS TPRNT                                                  (SEC)
C   DELTA  - TIME INCREMENT AT  N+1/2                                       (SEC)
C   DELTAM - TIME INCREMENT AT  N-1/2                                       (SEC)
C   DMIN   - USED IN TIME INCREMENT CALCULATION
C   ALF    - FLOATED IALF
C   IEND   - END OF PROBLEM INDICATOR
C   N      - ACTIVE ZONE SEQUENTIAL COUNTER
C   NS     - ACTIVE SPECIAL MESH SEQUENTIAL COUNTER
C   NSR    - RIGHT BOUNDARY MESH POINT INDICATOR
C   NMAX   - TOTAL NO. OF ZONES
C   NMAX1  - TOTAL NO. OF ZONES IN THE FREE-FORM PLATE
C   NSMAX  - TOTAL NO. OF SPECIAL MESH POINTS
C   I      - X-DIRECTION (COLUMN) INDICATOR
C   J      - Z-DIRECTION (ROW) INDICATOR
C   K      - PLATE INDICATOR
C   IMX    - IM OF ACTIVE PLATE
C   JMX    - JM OF ACTIVE PLATE
C   IMXM   - NO. OF ZONES IN LARGEST ROW IN FREE-FORM PLATE
C   LSTN   - ZONE NUMBER AFTER WHICH ALL ZONE VALUES ARE ZERO. STOP
C            CALCULATION OF PRESENT CYCLE. AND PROCEED TO NEXT CYCLE.
C   L      - INDICATOR IN ACCELERATION SUBROUTINES
C   LL     - INDICATOR IN ACCELERATION SUBROUTINES
C   P1     - HYDRODYNAMIC PRESSURE AT TIME  N+1                             (D/SQCM)
C   PM     - HYDRODYNAMIC PRESSURE AT TIME  N                               (D/SQCM)
C   RDOTR  - RHO DOT / RHO AT TIME  N+1/2
C   DRR2   - DELTA RHO / RHO**2 AT TIME  N+1/2
C   DED    - TERM IN ENERGY EQUATION
C   ETA    - RHO / RHOZ
C   ETA1   - RHO1 / RHOZ
C   AM     - A OF ACTIVE ZONE
C   RHOM   - RHO OF ACTIVE ZONE
C   EM     - ENERGY OF ACTIVE ZONE AT TIME  N
C   G      - MODULUS OF RIGIDITY                                            (D/SQCM)
C   DXX    - X COMPONENT OF STRETCHING DEVIATOR AND SPIN TENSOR (N+1/2)     (1/SEC)
C   DZZ    - Z COMPONENT OF STRETCHING DEVIATOR AND SPIN TENSOR (N+1/2)     (1/SEC)
C   DXZ    - SHEAR COMP. OF STRETCHING DEVIATOR AND SPIN TENSOR (N+1/2)     (1/SEC)
C   BOX    - SUM OF STRESS DEVIATORS                                        (D/SQCM)
C   XP     - TOTAL RADIAL MOMENTUM FOR EACH CYCLE                           (G/CMPS)
C   ZP     - TOTAL AXIAL  MOMENTUM FOR EACH CYCLE                           (G/CMPS)
C   XK     - TOTAL RADIAL KINETIC ENERGY FOR EACH CYCLE                     (ERG)
C   ZK     - TOTAL AXIAL  KINETIC ENERGY FOR EACH CYCLE                     (ERG)
C   TIE    - INTERNAL ENERGY FOR EACH CYCLE                                 (ERG)
C   TE     - TOTAL ENERGY FOR EACH CYCLE                                    (ERG)
C   TDEE   - ACCUMULATED TOTAL SPHERICAL STRESS WORK                        (ERG)
C   TDEQ   - ACCUMULATED TOTAL SPHERICAL VISCOUS STRESS WORK                (ERG)
C   TDEED  - ACCUMULATED TOTAL DEVIATORIC STRESS WORK                       (ERG)
C   TDEQD  - ACCUMULATED TOTAL DEVIATORICVISCOUS STRESS WORK                (ERG)
C   TDEPD  - ACCUMULATED TOTAL PLASTIC WORK                                 (ERG)
C   TDEQPD - ACCUMULATED TOTAL ENERGY                                       (ERG)
C   DEED   - DEVIATORIC STRESS WORK FOR ACTIVE ZONE                         (ERG)
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C   DEQD    - DEVIATORIC VISCOUS STRESS WORK FOR ACTIVE ZONE              (ERG)
C   UEP     - CALCULATION CUTOFF FOR VECOCITIES                           (CM/SEC)
C   REP     - CALCULATION CUTOFF FOR DENSITIES                            (G/CC)
C   EEP     - CALCULATION CUTOFF FOR ENERGIES                             (ERG/GM)
C   PEP     - CALCULATION CUTOFF FOR PRESSURES                            (D/SQCM)
C   NONE    - FLAG IN INTERFACE CALCULATION
C   XRT     - USED IN INTERFACE CALCULATION
C   ZRT     - USED IN INTERFACE CALCULATION
C   XH      - USED IN INTERFACE CALCULATION
C   ZH      - USED IN INTERFACE CALCULATION
C   NH      - USED IN INTERFACE CALCULATION
C   TNRT    - USED IN INTERFACE CALCULATION
C   TNR     - USED IN INTERFACE CALCULATION
C   TNP     - USED IN INTERFACE CALCULATION
C   LK      - USED IN INTERFACE CALCULATION
C   NFRST   - USED IN INTERFACE CALCULATION
C   NLST    - USED IN INTERFACE CALCULATION
C   NSLST   - USED IN INTERFACE CALCULATION
C   XSD     - USED IN INTERFACE CALCULATION
C   ZSD     - USED IN INTERFACE CALCULATION
C   TNNS    - USED IN INTERFACE CALCULATION
C   TN      - USED IN INTERFACE CALCULATION
C   NL      - USED IN INTERFACE CALCULATION
C   SINAB   - USED IN INTERFACE CALCULATION
C   COSAB   - USED IN INTERFACE CALCULATION
C   UXM     - USED IN INTERFACE CALCULATION
C   UZM     - USED IN INTERFACE CALCULATION
C   DSUM    - DELTA + DELTAM
C   AX      - ACCELERATION IN X-DIRECTION
C   AZ      - ACCELERATION IN Z-DIRECTION
C   NBXL    - INDICATOR FOR BOUNDARY CONDITION - LOWER X OF EACH PLATE
C   NBXU    - INDICATOR FOR BOUNDARY CONDITION - UPPER X OF EACH PLATE
C   AIN     - TEMPORARY STORAGE FOR AREA. ALSO FOR ACCELERATION.
C   AIN1    - TEMPORARY STORAGE FOR AREA.
C   AIN2    - TEMPORARY STORAGE FOR AREA.
C   N1      - WORKING STORAGE
C   N2      - WORKING STORAGE
C   N3      - WORKING STORAGE
C   X10     - WORKING STORAGE
C   Z10     - WORKING STORAGE
C   X11     - WORKING STORAGE
C   Z11     - WORKING STORAGE
C   X12     - WORKING STORAGE
C   Z12     - WORKING STORAGE
C   X13     - WORKING STORAGE
C   Z13     - WORKING STORAGE
C   X21     - WORKING STORAGE
C   Z21     - WORKING STORAGE
C   X22     - WORKING STORAGE
C   Z22     - WORKING STORAGE
C   X10M    - WORKING STORAGE
C   Z10M    - WORKING STORAGE
C   X11M    - WORKING STORAGE
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C    Z11M    - WORKING STORAGE
C    X12M    - WORKING STORAGE
C    Z12M    - WORKING STORAGE
C    X13M    - WORKING STORAGE
C    Z13M    - WORKING STORAGE
C    UX10    - WORKING STORAGE
C    UZ10    - WORKING STORAGE
C    UX11    - WORKING STORAGE
C    UZ11    - WORKING STORAGE
C    UX12    - WORKING STORAGE
C    UZ12    - WORKING STORAGE
C    UX13    - WORKING STORAGE
C    UZ13    - WORKING STORAGE
C    DTXXM   - WORKING STORAGE
C    DTYYM   - WORKING STORAGE
C    DTZZM   - WORKING STORAGE
C    DTXZM   - WORKING STORAGE
C    DL      - WORKING STORAGE
C    IFNT    - WORKING STORAGE
C    SPACE   - WORKING STORAGE
C    ISPACE  - WORKING STORAGE
C
C
      DIMENSION         X( 600),         X1( 600),         Z( 600),
     1   Z1( 600),      UX( 600),        UX1( 600),        UZ( 600),
     2   UZ1( 600),     RHO( 600),       RHO1( 600),       A( 600),
     3   A1( 600),      TXX( 600),       TXX1( 600),       TYY( 600),
     4   TYY1( 600),    TZZ( 600),       TZZ1( 600),       TXZ( 600),
     5   TXZ1( 600),    Q( 600),         Q1( 600),         E( 600),
     6   E1( 600),      DTXX( 600),      DTYY( 600),       DTZZ( 600),
     7   DTXZ( 600),    ZM( 600),        QXX1( 600),       QZZ1( 600),
     8   QXZ1( 600)
      DIMENSION         XS( 210),        XS1( 210),        ZS( 210),
     1   ZS1( 210),     UXS( 210),       UXS1( 210),       UZS( 210),
     2   UZS1( 210),    TN( 210),        SNAB( 210),
     3   NH( 42, 5),    XH( 42, 5),      ZH( 42, 5)
      DIMENSION         TITLE(12),       IM( 5),           JM( 5),
     1   NBZL( 5),      NBZR( 5),        XI( 5),           XL( 5),
     2   ZI( 5),        ZL( 5),          UXZ( 5),          UZZ( 5),
     3   UXINT( 5),     UZINT( 5),       RHOI( 5),         TXXZ( 5),
     4   TYYZ( 5),      TZZZ( 5),        TXZZ( 5),         EZ( 5),
     5   RHOZ( 5),      SA( 5),          SB( 5),           CAPA( 5),
     6   CAPB( 5),      ALPHA( 5),       BETA( 5),         EZERO( 5),
     7   RHOS( 5),      EPRS( 5),        CNO( 5),          CNA( 5),
     8   CNEXP( 5),     GO( 5),          G1( 5),           G2( 5),
     9   G3( 5),        CAPY( 5),        UTS( 5),          MEQ( 5)
      DIMENSION         PHIL( 5),        PHIR( 5),         PSIL( 5),
     1   PSIU( 5),      FKO( 5),         FK1( 5),          FK2( 5),
     2   FK3( 5),       HO( 5),          H1( 5),           H2( 5),
     3   H3( 5),        GAM( 5),         CJP( 5),          CJR( 5),
     4   CJD( 5),       DETV( 5),        CBRN( 5),         XD( 5),
     5   ZD( 5),        NFRST( 5),       NLST( 5),         NSLST( 5),
     6   NBXL( 5),      NBXU( 5)
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      DIMENSION          SPACE(10),        ISPACE(10),        JCRVS(20)
      DIMENSION          IM1( 50)
C
      COMMON    NCYCLE, STOPC,  PRNTC,  STRTC,  TIME,   TMAX,   TPRNT,
     1   DELPR, DELTA,  DELTAM, DMIN,   FK,     IEND,   NMAX,   NSMAX,
     2   SBAR,  N,      NS,     TITLE,  CZ,     CL,     M,      NBL,
     3   NBU,   IALF,   IM,     JM,     NBZL,   NBZR,   XI,     XL,
     4   ZI,    ZL,     UXZ,    UZZ,    UXINT,  UZINT,  RHOI,   TXXZ,
     5   TYYZ,  TZZZ,   TXZZ,   EZ,     RHOZ,   SA,     SB,     CAPA,
     6   CAPB,  ALPHA,  BETA,   EZERO,  RHOS,   EPRS,   CNO,    CNA,
     7   CNEXP, G0,     G1,     G2,     G3,     CAPY,   IMX,    JMX,
     8   NPLOT, CN,     P1,     RDOTR,  DRR2,   PM,     DED,    ETA,
     9   ETA1,  NONE,   ZRT,    XRT,    ZH,     XH,     TNRT,   TNR
      COMMON    TNP,    LK,     NL,     XSD,    ZSD,    TNNS,   AX,
     1   AZ,    DSUM,   AIN ,   COSAB,  SINAB,  NH,     ALF,    AIN1,
     2   AIN2,  TEMP1,  TEMP2,  TEMP3,  TEMP4,  TEMP5,  TEMP6,  TEMP7,
     3   TEMP8, TEMP9,  TEMP10, X10,    Z10,    X11,    Z11,    X12,
     4   Z12,   X13,    Z13,    X21,    Z21,    X22,    Z22,    LL,
     5   N2,    WRTPC,  L,      X10M,   Z10M,   X11M,   Z11M,   X12M,
     6   Z12M,  X13M,   Z13M,   UX10,   UZ10,   UX11,   UZ11,   UX12,
     7   UZ12,  UX13,   UZ13,   DTXXM,  DTYYM,  DTZZM,  DTXZM,  DL,
     8   NFRST, NLST,   IENT,   N1,     NSLST,  NBXL,   NBXU,   NBQ,
     9   NFN,   AM,     RHOM,   EM,     C3,     C4,     DXX,    DZZ
      COMMON    DXZ,    XP,     ZP,     XK,     ZK,     TIE,    TE,
     1   TDEE,  TDEQ,   TDEED,  TDEQD,  TDEPD,  TDEQPD, DEED,   DEQD,
     2   BOX,   G,      NCRVS,  JCRVS,  CAPF,   UEP,    REP,    EEP,
     3   PEP,   IJK,    NOSLID, MEQ,    PHIL,   PHIR,   PSIL,   PSIU,
     4   UTS
      COMMON    I,      J,      K,      X,      X1,     Z,      Z1,
     1   UX1,   UZ1,    RHO1,   A1,     TXX1,   TYY1,   TZZ1,   TXZ1,
     2   DTXX,  DTYY,   DTZZ,   DTXZ,   Q1,     QXX1,   QZZ1,   QXZ1,
     3   E1,    ZM,     XS,     XS1,    ZS,     ZS1,    UXS1,   UZS1,
     4   TN
      COMMON    SPACE,  ISPACE
      COMMON    IM1
C
      EQUIVALENCE        (UX,UX1),         (UZ,UZ1),          (UXS,UXS1),
     1   (UZS,UZS1),     (TXX,TXX1),       (TZZ,TZZ1),        (TYY,TYY1),
     2   (TXZ,TXZ1),     (E,E1),           (A,A1),            (RHO,RHO1),
     3   (Q,Q1),         (TN,SNAB)
      EQUIVALENCE        (SA,FK0,GAM),     (SB,FK1,CJP),      (CAPA,FK2,CJR),
     1   (CAPB,FK3,CJD), (ALPHA,H0,DETV),(BETA,H1,CBRN),  (EZERO,H2,XD),
     2   (RHOS,H3,ZD)
      EQUIVALENCE        (ISPACE(4),NSR)
C
C     SET TRAP ROUTINE TO IGNORE UNDERFLOWS, BUT TRANSFER TO STATEMENT
C     200 ON ANY OVERFLOW.
C
      NTMS = 0
      ASSIGN 200 TO ITRA
      CALL FPT (.TRUE.,ITRA)
C
C     SETUP FOR UTILITY TAPES USED IN OTHER LINKS
```

```
C
      CALL RWND2
      IFAKE = 1
      IF (IFAKE .EQ. 1) GO TO 140
      READ (5,100)
  100 FORMAT ( 1H  )
      WRITE(6,100)
C
C     LINK1 READS INPUT CARDS, WRITES INPUT VALUES.
C           IF A NEW PROBLEM, INITIALIZES ALL VARIABLES AND WRITES TAPE 4.
C           IF A CONTINUATION, READS TAPE 4.
C
  140 CALL CHAIN(3)
C
C     PLOT SHAPE AT CYCLE 0.
C
      IF (STRTC .GT. 0.0) GO TO 150
      DO 142 L=1,NMAX
      X1(L) = X(L)
      Z1(L) = Z(L)
  142 CONTINUE
      DO 144 L=1,NSMAX
      XS1(L) = XS(L)
      ZS1(L) = ZS(L)
  144 CONTINUE
      IPLOT = NPLOT
      NPLOT = 0
      IEND = 2
      CALL CHAIN(2)
      NPLOT = IPLOT
C
C     LINK2 INTEGRATES UNTIL TIME TO OUTPUT RESULTS. DETERMINES WHEN
C           PROBLEM IS FINISHED.
C
  150 CALL CHAIN(1)
C
C     LINK3 PRINTS AND PLOTS RESULTS.
C
  160 CALL CHAIN(2)
C
C     CHECK TO SEE IF PROBLEM FINISHED.
C
      IF (IEND .LT. 4) GO TO 150
  180 REWIND 4
      CALL EXIT
C
  200 IEND = 5
      NTMS = NTMS+1
      IF (NTMS .GT. 1) GO TO 180
      GO TO 160
      END
```

```
$IBFTC LNK110  LIST,REF
C  2-D LAGRANGIAN -- LINK1 FOR IBM 7040 CHAIN
C
      DIMENSION            X( 600),          X1( 600),         Z( 600),
     1  Z1( 600),          UX( 600),         UX1( 600),        UZ( 600),
     2  UZ1( 600),         RHO( 600),        RHO1( 600),       A( 600),
     3  A1( 600),          TXX( 600),        TXX1( 600),       TYY( 600),
     4  TYY1( 600),        TZZ( 600),        TZZ1( 600),       TXZ( 600),
     5  TXZ1( 600),        Q( 600),          Q1( 600),         E( 600),
     6  E1( 600),          DTXX( 600),       DTYY( 600),       DTZZ( 600),
     7  DTXZ( 600),        ZM( 600),         QXX1( 600),       QZZ1( 600),
     8  QXZ1( 600)
      DIMENSION            XS( 210),         XS1( 210),        ZS( 210),
     1  ZS1( 210),         UXS( 210),        UXS1( 210),       UZS( 210),
     2  UZS1( 210),        TN( 210),         SNAB( 210),
     3  NH( 42, 5),        XH( 42, 5),       ZH( 42, 5)
      DIMENSION            TITLE(12),        IM( 5),           JM( 5),
     1  NBZL( 5),          NBZR( 5),         XI( 5),           XL( 5),
     2  ZI( 5),            ZL( 5),           UXZ( 5),          UZZ( 5),
     3  UXINT( 5),         UZINT( 5),        RHOI( 5),         TXXZ( 5),
     4  TYYZ( 5),          TZZZ( 5),         TXZZ( 5),         EZ( 5),
     5  RHOZ( 5),          SA( 5),           SB( 5),           CAPA( 5),
     6  CAPB( 5),          ALPHA( 5),        BETA( 5),         EZERO( 5),
     7  RHOS( 5),          EPRS( 5),         CNO( 5),          CNA( 5),
     8  CNEXP( 5),         G0( 5),           G1( 5),           G2( 5),
     9  G3( 5),            CAPY( 5),         UTS( 5),          MEQ( 5)
      DIMENSION            PHIL( 5),         PHIR( 5),         PSIL( 5),
     1  PSIU( 5),          FK0( 5),          FK1( 5),          FK2( 5),
     2  FK3( 5),           H0( 5),           H1( 5),           H2( 5),
     3  H3( 5),            GAM( 5),          CJP( 5),          CJR( 5),
     4  CJD( 5),           DETV( 5),         CBRN( 5),         XD( 5),
     5  ZD( 5),            NFRST( 5),        NLST( 5),         NSLST( 5),
     6  NBXL( 5),          NBXU( 5)
      DIMENSION            SPACE(10),        ISPACE(10),       JCRVS(20)
C
      COMMON    NCYCLE,  STOPC,   PRNTC,   STRTC,   TIME,    TMAX,    TPRNT,
     1  DELPR,  DELTA,   DELTAM,  DMIN,    FK,      IEND,    NMAX,    NSMAX,
     2  SBAR,   N,       NS,      TITLE,   CZ,      CL,      M,       NBL,
     3  NBU,    IALF,    IM,      JM,      NBZL,    NBZR,    XI,      XL,
     4  ZI,     ZL,      UXZ,     UZZ,     UXINT,   UZINT,   RHOI,    TXXZ,
     5  TYYZ,   TZZZ,    TXZZ,    EZ,      RHOZ,    SA,      SB,      CAPA,
     6  CAPB,   ALPHA,   BETA,    EZERO,   RHOS,    EPRS,    CNO,     CNA,
     7  CNEXP,  G0,      G1,      G2,      G3,      CAPY,    IMX,     JMX,
     8  NPLOT,  CN,      P1,      RDOTR,   DRR2,    PM,      DED,     ETA,
     9  ETA1,   NONE,    ZRT,     XRT,     ZH,      XH,      TNRT,    TNR
      COMMON    TNP,     LK,      NL,      XSD,     ZSD,     TNNS,    AX,
     1  AZ,     DSUM,    AIN ,    COSAB,   SINAB,   NH,      ALF,     AIN1,
     2  AIN2,   TEMP1,   TEMP2,   TEMP3,   TEMP4,   TEMP5,   TEMP6,   TEMP7,
     3  TEMP8,  TEMP9,   TEMP10,  X10,     Z10,     X11,     Z11,     X12,
     4  Z12,    X13,     Z13,     X21,     Z21,     X22,     Z22,     LL,
     5  N2,     WRTPC,   L,       X10M,    Z10M,    X11M,    Z11M,    X12M,
     6  Z12M,   X13M,    Z13M,    UX10,    UZ10,    UX11,    UZ11,    UX12,
     7  UZ12,   UX13,    UZ13,    DTXXM,   DTYYM,   DTZZM,   DTXZM,   DL,
```

```
      8  NFRST,   NLST,    IENT,    N1,      NSLST,   NBXL,    NBXU,    NBQ,
      9  NFN,     AM,      RHOM,    EM,      C3,      C4,      DXX,     DZZ
       COMMON     DXZ,     XP,      ZP,      XK,      ZK,      TIE,     TE,
      1  TDEE,    TDEQ,    TDEED,   TDEQD,   TDEPD,   TDEQPD,  DEED,    DEQD,
      2  BOX,     G,       NCRVS,   JCRVS,   CAPF,    UEP,     REP,     EEP,
      3  PEP,     IJK,     NOSLID,  MEQ,     PHIL,    PHIR,    PSIL,    PSIU,
      4  UTS
       COMMON     I,       J,       K,       X,       X1,      Z,       Z1,
      1  UX1,     UZ1,     RHO1,    A1,      TXX1,    TYY1,    TZZ1,    TXZ1,
      2  DTXX,    DTYY,    DTZZ,    DTXZ,    Q1,      QXX1,    QZZ1,    QXZ1,
      3  E1,      ZM,      XS,      XS1,     ZS,      ZS1,     UXS1,    UZS1,
      4  TN
       COMMON       SPACE,   ISPACE
C
       EQUIVALENCE          (UX,UX1),          (UZ,UZ1),          (UXS,UXS1),
      1  (UZS,UZS1),        (TXX,TXX1),        (TZZ,TZZ1),        (TYY,TYY1),
      2  (TXZ,TXZ1),        (E,E1),            (A,A1),            (RHO,RHO1),
      3  (Q,Q1),            (TN,SNAB)
       EQUIVALENCE          (SA,FK0,GAM),      (SB,FK1,CJP),      (CAPA,FK2,CJR),
      1  (CAPB,FK3,CJD),    (ALPHA,H0,DETV),(BETA,H1,CBRN),       (EZERO,H2,XD),
      2  (RHOS,H3,ZD)
C
       DIMENSION            ZZ(15778)
C
       EQUIVALENCE          (ZZ(1),NCYCLE)
C
C      REWIND CHAIN TAPE (TAPE 0)
C
       CALL RWNDO
C
C      READ INPUT DATA FROM CARDS
C
       CALL INPUT
C
C      CHECK VALUE OF STARTING CYCLE -
C            = 0, INITIALIZE VARIABLES, AND WRITE ON TAPE 4.
C        NOT = 0, EXIT.
C
       IF ( STRTC .GT. 0.0) GO TO 150
C
C      WRITE OUT INPUT VALUES
C
       CALL PRIN
       CALL INTLIZ
       WRITE (4) ZZ
  150  IEND = 0
       CALL CHNXIT
       END
```

```
$IBFTC INPT21  LIST,REF
      SUBROUTINE INPUT                                                  INPT 100
C
C  2-D LAGRANGIAN -- READS INPUT DATA FROM CARDS
C
      DIMENSION          ZZ(15778)
C
      EQUIVALENCE        (ZZ(1),NCYCLE)
      EQUIVALENCE        (ISPACE(4),NSR)
      EQUIVALENCE        (ISPACE(9),N1D)
C
      READ (5,120) (TITLE(L),L=1,12)                                    INPT 110
  120 FORMAT (12A6)                                                     INPT 120
      READ (5,140) CSTRT                                                INPT 130
  140 FORMAT (6E12.8)                                                   INPT 140
C
C     CHECK VALUE OF STARTING CYCLE. IF NOT = 0, READ COMMON FROM TAPE 4.
C
      IF (CSTRT .LE. 0.0) GO TO 170                                     INPT 150
      READ (4) ZZ                                                       INPT 160
  170 STRTC = CSTRT                                                     INPT 170
      READ (5,140) CZ,CL,C3,C4,FK,SBAR                                  INPT 171
      READ (5,140) DELPR,TMAX,PRNTC,WRTPC,STOPC                         INPT 180
      READ (5,200) M,NBL,NBU,IALF,NOSLID,N1D,NBQ,NPLOT,NCRVS            INPT 190
  200 FORMAT (12I6)                                                     INPT 200
      IF ((NPLOT .EQ. 0) .OR. (NPLOT .GE. 6)) GO TO 210                 INPT 201
      IF (NCRVS .EQ. 0) GO TO 210                                       INPT 202
      READ (5,200)        (JCRVS(L),L=1,NCRVS)                          INPT 203
  210 IF (STRTC .GT. 0.0) RETURN                                        INPT 210
      DO 290 L=1,M                                                      INPT 211
      READ (5,200) IM(L),JM(L),NBZL(L),NBZR(L),MEQ(L)                   INPT 220
      IF (IM(L) .GT. 0) GO TO 230                                       INPT 221
C
C     INPUT FOR THIS PLATE IS IN A SPECIAL FORM
C
      CALL SPCLIN                                                       INPT 222
      GO TO 250                                                         INPT 223
C
  230 READ (5,140) XI(L),XL(L),ZI(L),ZL(L),UXZ(L),UZZ(L),UXINT(L),UZINT(INPT 230
     1L),RHOI(L),TXXZ(L),TYYZ(L),TZZZ(L),TXZZ(L),EZ(L)                  INPT 240
     2,PHIL(L),PHIR(L),PSIL(L),PSIU(L)                                  INPT 241
  250 READ (5,140) RHOZ(L),SA(L),SB(L),CAPA(L),CAPB(L),ALPHA(L),BETA(L),INPT 250
     1EZERO(L),RHOS(L),EPRS(L)                                          INPT 260
      READ (5,140) CNO(L),CNA(L),CNEXP(L)                               INPT 270
      READ (5,140) GO(L),G1(L),G2(L),G3(L),CAPY(L),UTS(L)               INPT 280
  290 CONTINUE                                                          INPT 290
      RETURN                                                            INPT 300
      END                                                               INPT 310
```

```
$IBFTC SPCIN3  LIST,REF
      SUBROUTINE SPCLIN
C
C  2-D LAGRANGIAN -- READS IN DATA WHEN PLATE INPUT IS IN A SPECIAL FORM
      EQUIVALENCE        (ISPACE(4),NSR)
      EQUIVALENCE        (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
C
      NS  = 0
      N   = 0
      IMXM = 0
C
C     DETERMINE MESH NO. OF FIRST MESHES IN THIS PLATE.
C
      LL = L-1
      IF (LL .LT. 1) GO TO 150
      DO 140 I=1,LL
      N = N+IM(I)*JM(I)
      NS = NS+IM(I)+JM(I)+1
  140 CONTINUE
C
  150 JMX = JM(L)
      DO 400 J=1,JMX
      READ (5,160) IM1(J)
  160 FORMAT ( 12I6 )
      IMX = IM1(J)
      IMXM =  MAX0(IMXM,IMX)
      DO 300 I=1,IMX
      N = N+1
      READ (5,220) UX(N),UZ(N),TXX(N),E(N),RHO(N),X(N),Z(N)
  220 FORMAT ( 5E15.8 )
      TXX(N) = -TXX(N)
      TYY(N) = TXX(N)
      TZZ(N) = TXX(N)
C
  300 CONTINUE
C
  400 CONTINUE
      NMAX1  = N
      NSMAX  = JMX+IMXM+1
C
C     READ POSITION AND VELOCITIES OF RIGHT-SIDE SPECIAL STATIONS
C
      DO 500 I=1,NSMAX
      NS = NS+1
      READ (5,220) UXS(NS),UZS(NS),XS(NS),ZS(NS)
  500 CONTINUE
      READ (5,220) UXINT(L),UZINT(L)
C
      PHIL(L) = 90.0
      PHIR(L) = 90.0
      PSIL(L) =  0.0
      PSIU(L) =  0.0
      RETURN
      END
```

```
$IBFTC PRIN20  LIST,REF
      SUBROUTINE PRIN
C
C  2-D LAGRANGIAN -- PRINTS INPUT DATA READ IN SUBR. INPUT
C
      EQUIVALENCE          (ISPACE(4),NSR)
      EQUIVALENCE          (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
      EQUIVALENCE          (ISPACE(9),N1D)
C
      WRITE (6,120) (TITLE(L),L=1,12)
  120 FORMAT (1H1,42X,47HA.H.JONES  MATERIAL SCIENCE GROUP  GM DRL  X470
     1/1H0,61X,10HINPUT DATA//1H0,29X,12A6 )
      WRITE (6,160) CZ,CL,C3,C4,FK,DELPR,TMAX,SBAR,STRTC,PRNTC,WRTPC,STO
     1PC,M,NBL,NBU, IALF,NOSLID,NBQ
  160 FORMAT (1H0, 2X, 2HCO, 5X, 2HCL, 5X, 2HC3, 5X, 2HC4, 5X, 2HFK, 6X,
     1 9HTPRNT-SEC, 8X, 8HTMAX-SEC, 6X,10HSBAR-D/CSQ , 3X, 7HSTART C, 1X
     2, 7HPRINT C, 1X, 6HWRTP C, 2X, 6HSTOP C /1H ,F6.3,F7.3,F7.3,F7.3,F
     37.3,E16.8,E16.8,E16.8,F7.0,F8.0,F7.0,F8.0 /1H0, 9HNO.PLATES, 1X, 8
     4HX-LOW.B., 1X, 7HX-UP.B., 1X, 5HALPHA, 1X,10HSLDNG.IFCE, 1X, 8H1-D
     5 IND., 1X, 9HVISC.IND. /1H , 6X,I2, 6X,I1, 8X,I1, 6X,I1, 6X,I1, 9X
     6,I1, 8X,I1 )
      WRITE (6,220)
  220 FORMAT (1H0,5HPLATE, 1X,10HX-STATIONS, 1X,10HZ-STATIONS, 1X, 8HX-L
     1FT.B., 1X, 9HZ-RGHT.B., 2X, 6HE.O.S. /)
      DO 270 L=1,M
      WRITE (6,260) L,IM(L),JM(L),NBZL(L),NBZR(L),MEQ(L)
  260 FORMAT (1H , 2X,I2, 5X,I6, 5X,I6, 7X,I1, 8X,I1, 7X, I1)
  270 CONTINUE
      WRITE (6,290)
  290 FORMAT (1H0,5HPLATE, 4X, 5HXI-CM,11X, 5HXL-CM,11X, 5HZI-CM,11X, 5H
     1ZL-CM, 9X,10HUXZ-CM/SEC, 6X,10HUZZ-CM/SEC, 5X,12HUXINT-CM/SEC, 4X,
     212HUZINT-CM/SEC /)
      DO 360 L=1,M
      IF (IM(L) .LE. 0) GO TO 360
      WRITE (6,350) L,XI(L),XL(L),ZI(L),ZL(L),UXZ(L),UZZ(L),UXINT(L),
     1UZINT(L)
  350 FORMAT (1H ,1X,I2,8E16.8)
  360 CONTINUE
      WRITE (6,362)
  362 FORMAT (1H0,5HPLATE, 3X, 8HPHIL-DEG, 8X, 8HPHIR-DEG, 8X, 8HPSIL-DE
     1G, 8X, 8HPSIU-DEG / )
      DO 366 L=1,M
      IF (IM(L) .LE. 0) GO TO 366
      WRITE (6,350) L,PHIL(L),PHIR(L),PSIL(L),PSIU(L)
  366 CONTINUE
      WRITE (6,380)
  380 FORMAT (1H0,5HPLATE, 2X, 9HRHOZ-G/CC, 7X,10HTXXZ-D/CSQ, 6X,10HTYYZ
     1-D/CSQ, 6X,10HTZZZ-D/CSQ, 6X,10HTXZZ-D/CSQ, 6X, 9HEZ-ERG/GM    /)
      DO 420 L=1,M
      IF (IM(L) .LE. 0) GO TO 420
      WRITE (6,350) L,RHOI(L),TXXZ(L),TYYZ(L),TZZZ(L),TXZZ(L),EZ(L)
  420 CONTINUE
      DO 431 L=1,M
      IF (IM(L) .GT. 0) GO TO 431
      WRITE (6,421) L
```

```
421 FORMAT (1H0, 6HPLATE ,I2,19H IS A DEBRIS CLOUD. /1H0,     4HMESH,
   13X, 9HUX-CM/SEC, 7X, 9HUZ-CM/SEC, 6X,12HSTRESS-D/CSQ, 6X, 7HE-ERG/
   2G, 9X, 8HRHO-G/CC,10X, 4HX-CM,12X, 4HZ-CM / )
    N = 0
    LL = L-1
    IF (LL .LT. 1) GO TO 424
    DO 423 I=1,LL
    N = N+IM(I)*JM(I)
423 CONTINUE
424 JMX = JM(L)
    DO 428 J=1,JMX
    IMX = IM1(J)
    DO 427 I=1,IMX
    N = N+1
    WRITE (6,426) N,UX(N),UZ(N),TXX(N),E(N),RHO(N),X(N),Z(N)
426 FORMAT(1H ,I3,8E16.8)
427 CONTINUE
428 CONTINUE
    WRITE (6,429)
429 FORMAT (1H0, 5HPOINT, 2X,10HUXS-CM/SEC, 6X,10HUZS-CM/SEC, 8X, 5HXS
   1-CM,11X, 5HZS-CM / )
    LL = JMX+IMXM+1
    DO 430 I=1,LL
    WRITE (6,426) I,UXS(I),UZS(I),XS(I),ZS(I)
430 CONTINUE
431 CONTINUE
    WRITE (6,440)
440 FORMAT (1H0,56X,18HMATERIAL CONSTANTS )
    DO 770 L=1,M
    MEQT = MEQ(L)
    GO TO (480,520,560),MEQT

480 WRITE (6,490)
490 FORMAT (1H0, 5HPLATE, 3X, 8HRHO-G/CC,11X, 2HSA,14X, 2HSB,14X, 1HA,
   115X, 1HB,13X, 5HALPHA,12X, 4HBETA,11X, 5HEZERO   )
    GO TO 590
520 WRITE (6,530)
530 FORMAT (1H0, 5HPLATE, 3X, 8HRHO-G/CC,11X, 2HK0,14X, 2HK1,14X, 2HK2
   1,14X, 2HK3,14X, 2HH0,14X, 2HH1,14X, 2HH2   )
    GO TO 590
560 WRITE (6,570)
570 FORMAT (1H0, 5HPLATE, 3X, 8HRHO-G/CC,10X, 5HGAMMA,11X, 3HCJP,12X,
   15HCJRHO,12X, 3HCJD,13X, 4HDETV,12X, 4HCBRN,13X, 2HXD     )
590 WRITE (6,350) L,RHOZ(L),SA(L),SB(L),CAPA(L),CAPB(L),ALPHA(L),BETA(
   1L),EZERO(L)
    GO TO (620,660,690),MEQT
620 WRITE (6,630)
630 FORMAT (1H0, 10X, 4HRHOS,13X, 2HES,13X, 3HCN0,13X, 3HCNA,12X, 5HCN
   1EXP   )
    GO TO 710
660 WRITE (6,670)
670 FORMAT (1H0, 11X, 2HH3, 29X, 3HCN0,13X, 3HCNA,12X, 5HCNEXP    )
    GO TO 710
690 WRITE (6,700)
700 FORMAT (1H0, 11X, 2HZD, 29X, 3HCN0,13X, 3HCNA,12X, 5HCNEXP    )
710 WRITE (6,720)    RHOS(L),EPRS(L),CN0(L),CNA(L),CNEXP(L)
```

```
  720 FORMAT (1H , 3X, 8E16.8)
      WRITE (6,740)
  740 FORMAT (1H0,           11X, 2HG0,14X, 2HG1,14X, 2HG2,14X, 2HG3,13X,
     14HCAPY,13X, 3HUTS    )
      WRITE (6,720)    G0(L),G1(L),G2(L),G3(L),CAPY(L),UTS(L)
  770 CONTINUE
C
C     CONVERT ALL ANGLES FROM DEGREES TO RADIANS.
C
      DO 830 L=1,M
      PHIL(L) = PHIL(L)/57.2957795
      PHIR(L) = PHIR(L)/57.2957795
      PSIL(L) = PSIL(L)/57.2957795
      PSIU(L) = PSIU(L)/57.2957795
  830 CONTINUE
      RETURN
      END
```

```
$IBFTC INTL50  LIST,REF
      SUBROUTINE INTLIZ                                                 INTL 100
C
C  2-D LAGRANGIAN -- INITIALIZES ALL VARIABLES PRIOR TO CYCLE 1.
C
      EQUIVALENCE        (ISPACE(4),NSR)
      EQUIVALENCE        (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
      EQUIVALENCE        (ISPACE(7),IMP),(ISPACE(8),IDP)
C
      NMAX = 600
      NSMAX = 210
      K = 5
C
C     CHECK TO SEE IF PROBLEM EXCEEDS SIZE OF PROGRAM
C
      IF (M .GT. K) GO TO  20
      N = 0
      NS = 0
      DO  10 I=1,M
      IF (IM(I) .GT. 0) GO TO 9
      N = N+NMAX1
      NS= NS+JM(I)+IMXM+1
      GO TO 10
    9 N = N + IM(I)*JM(I)
      NS = NS + IM(I) + JM(I) + 1
   10 CONTINUE
      IF (N .GT. NMAX) GO TO  40
      IF (NS .GT. NSMAX) GO TO  60
      GO TO  110
C
C     WRITE ERROR MESSAGES
C
   20 WRITE (6, 30) K

   30 FORMAT (1H0,35HERROR - YOU MAY NOT HAVE MORE THAN ,I2, 8H PLATES.)
      GO TO  80
   40 WRITE (6, 50) NMAX
   50 FORMAT (1H0,35HERROR - YOU MAY NOT HAVE MORE THAN ,I4, 7H ZONES.)
      GO TO  80
   60 WRITE (6, 70) NSMAX
   70 FORMAT (1H0,41HERROR - NO. OF SPECIAL STATIONS EXCEEDED ,I4, 2H .)
   80 CALL EXIT
C
C     SET CUTOFF VALUES FOR VELOCITY, DENSITY, ENERGY, AND PRESSURE
C
  110 UEP = 1.0
      REP = 1.0E-6
      EEP = 1.0E+3
      PEP = 1.0E+6
C
C     SET UP BOUNDARY CONDITION INDICATORS
C              1 = FREE
C              2 = FIXED
C
      IF (IALF .EQ. 1) GO TO 120
```

```
C
      DO 111 L=1,M
  111 NBZL(L) = 2
  120 NBXL(1) = NBL
      IF (M - 2) 130,200,150
  130 NBXU(1) = NBU
      GO TO 230
  150 K = M-1
      DO 190 L=2,K
      NBXL(L) = 1
      NBXU(L) = 1
      NFRST(L) = 0
      NLST(L)  = 9999
  190 CONTINUE
  200 NBXU(1) = 1
      NBXL(M) = 1
      NBXU(M) = NBU
      NFRST(1) = 0
      NLST(1)  = 9999
C
C     GENERATE POSITION OF THE MESHES
C
  230 CALL GNRT
C
C     INITIALIZE VARIABLES FOR EACH PLATE
C
      DMIN = 1.0
      ALF = IALF
      N = 0
      NS = 0
      IF (IM(1) .GT. 0) GO TO 270
      JMX = JM(1)
      N3 = JMX+IMXM+1
      DO 260 N2=1,N3
  260 SNAB(N2) = 0.0
      N2 = N3-IM1(JMX)
      GO TO 280
  270 N2 = JM(1)+1
      N3 = N2+IM(1)
C
  280 DO 810 K=1,M
      IFLG = 0
      IMP = 0
      IDP = 0
      IF (K .LE. 2) GO TO 300
      N2 = N2 + JM(K-1)+IM(K-1)+1
  300 JMX=JM(K)
```

```
C
      DO 720 J=1,JMX
      NS = NS + 1
      NSR = NS
      IMX = IM(K)
      IMP = IMX
      IF (IMX .GT. 0) GO TO 440
      IMX = IM1(J)
      IFLG = 1
      IF (IDP .LT. 0) NSR = NSR+IDP
      IF (J .LT. JMX) GO TO 400
      IMP = 0
      IDP = 0
      GO TO 460
  400 IMP = IM1(J+1)
      IDP = IMP-IMX
      GO TO 460
  440 UXS(NS) = UXZ(K)
      UZS(NS) = UZZ(K)
C
  460 DO 710 I=1,IMX
      N = N + 1
      IF (IFLG .GT. 0) GO TO 462
      RHO(N) = RHOI(K)
      TXX(N) = TXXZ(K)
      TYY(N) = TYYZ(K)
      TZZ(N) = TZZZ(K)
      TXZ(N) = TXZZ(K)
      E(N) = EZ(K)
C
  462 CALL AMZRO
C
C     DETERMINE MESH NUMBERS WHICH SHOW RELATIVE PLACEMENT OF THE PLATES
C
      IF (K .EQ. 1) GO TO 480
      IF (J .NE. 1) GO TO 480
      IF (X(N)+1.0E-6 .LT. XS(N2)) GO TO 480
      IF (NFRST(K-1) .NE. 0) GO TO 470
      NFRST(K-1) = N
      GO TO 480
  470 IF (X(N)+ .0E-6 .LT. XS(N3)) GO TO 480
      IF (NLST(K-1) .NE. 9999) GO TO 480
      NLST(K-1) = N
C
  480 TEMP1 = SQRT((Z10-Z13)**2 + (X10-X13)**2)
      TEMP2 = SQRT((Z11-Z12)**2 + (X11-X12)**2)
      TEMP3 = AMAX1(TEMP1,TEMP2)
      DL =  A(N)/TEMP3
      TEMP5 = SQRT(UXZ(1)**2 + UZZ(1)**2)
      TEMP4 = DL/(CNO(K) + 0.5*TEMP5) /3.0
      DMIN = AMIN1(DMIN,TEMP4)
      IF (IFLG .GT. 0) GO TO 710
      IF ((K .EQ. 1) .OR. (J .GT. 1)) GO TO 500
      IF ((N .LT. NFRST(K-1)) .OR. (N .GT. NLST(K-1))) GO TO 500
      UX(N) = UXINT(K-1)
```

```
      UZ(N) = UZINT(K-1)
      IF (I .LT. IMX) GO TO 520
      UXS(NS) = UXINT(K-1)
      UZS(NS) = UZINT(K-1)
      GO TO 520
  500 UX(N) = UXZ(K)
      UZ(N) = UZZ(K)
  520 IF (J .LT. JMX) GO TO 710
      UXS(NS) = UXINT(K)
      UZS(NS) = UZINT(K)
      SNAB(NS) = SIN(PSIU(K))
C
  710 CONTINUE
C
  720 CONTINUE
C
      NS = NS+1
      IF (IFLG .GT. 0) GO TO 810
      UXS(NS) = UXINT(K)
      UZS(NS) = UZINT(K)
      SNAB(NS) = SIN(PSIU(K))
C
  810 CONTINUE
C
      NMAX = N
      NSMAX = NS
C
      TPRNT = DELPR
      DELTA = DMIN*FK
      DELTAM = DELTA
      TIME = 0.0
      NCYCLE = 0
      TDEE   = 0.0
      TDEQ   = 0.0
      TDEED  = 0.0
      TDEQD  = 0.0
      TDEPD  = 0.0
      TDEQPD = 0.0
      RETURN
      END
```

```
$IBFTC GNRT01  LIST,REF
      SUBROUTINE GNRT
C
C  2-D LAGRANGIAN - GENERATES POSITIONS OF MESHES FROM INPUT DATA
C                   THIS VERSION IS FOR THE COUPLED CAMEO-2-D SERIES.
C
      EQUIVALENCE        (ISPACE(4),NSR)
      EQUIVALENCE        (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
      EQUIVALENCE        (ISPACE(10),NS1)
C
      ONE = 0.9999995
      ZRO = 0.0000005
C
      N = 0
      NS = 0
      NS1 = 0
C
      DO 800 K=1,M
      IMX = IM(K)
      JMX = JM(K)
      IF (IMX .GT. 0) GO TO 160
C
      N = N+NMAX1
      NS1 = NS1+JMX+IMXM+1
      GO TO 800
C
  160 IF ((PSIU(K) .LE. 6.2831853) .OR.  (PSIL(K) .LE. 6.2831853)) GO TO
     1 200
      CALL DOME
      GO TO 800
C
  200 FIMX = IMX
      FJMX = JMX
      NS = NS1
      NS1 = NS1+JMX
      CPSL = COS(PSIL(K))
      IF (ABS(CPSL) .GT. ONE) CPSL = SIGN( 1.0,CPSL)
      IF (ABS(CPSL) .LT. ZRO) CPSL = 0.0
      SPSL = SIN(PSIL(K))
      IF (ABS(SPSL) .GT. ONE) SPSL = SIGN( 1.0,SPSL)
      IF (ABS(SPSL) .LT. ZRO) SPSL = 0.0
      CPSU = COS(PSIU(K))
      IF (ABS(CPSU) .GT. ONE) CPSU = SIGN( 1.0,CPSU)
      IF (ABS(CPSU) .LT. ZRO) CPSU = 0.0
      SPSU = SIN(PSIU(K))
      IF (ABS(SPSU) .GT. ONE) SPSU = SIGN( 1.0,SPSU)
      IF (ABS(SPSU) .LT. ZRO) SPSU = 0.0
      CPHL = COS(PHIL(K))
      IF (ABS(CPHL) .GT. ONE) CPHL = SIGN( 1.0,CPHL)
      IF (ABS(CPHL) .LT. ZRO) CPHL = 0.0
      SPHL = SIN(PHIL(K))
      IF (ABS(SPHL) .GT. ONE) SPHL = SIGN( 1.0,SPHL)
      IF (ABS(SPHL) .LT. ZRO) SPHL = 0.0
```

```
      CPHR = COS(PHIR(K))
      IF (ABS(CPHR) .GT. ONE) CPHR = SIGN( 1.0,CPHR)
      IF (ABS(CPHR) .LT. ZRO) CPHR = 0.0
      SPHR = SIN(PHIR(K))
      IF (ABS(SPHR) .GT. ONE) SPHR = SIGN( 1.0,SPHR)
      IF (ABS(SPHR) .LT. ZRO) SPHR = 0.0
C
C              (XIF,ZIF) - POSITION OF LOWER RIGHT CORNER OF PLATE K.
C              (XJ ,ZJ ) -             UPPER LEFT
C              (XJF,ZJF) -             UPPER RIGHT
C              (XI ,ZI ) -             LOWER LEFT
C
      XIF = XI(K)+XL(K)*CPSL
      ZIF = ZI(K)+XL(K)*SPSL
      XJ  = XI(K)+ZL(K)*CPHL
      ZJ  = ZI(K)+ZL(K)*SPHL
      XJF =((ZIF-ZJ)*CPSU*CPHR + XJ*SPSU*CPHR - XIF*CPSU*SPHR)/(SPSU*CPH
     1R - CPSU*SPHR)
      IF (ABS(PSIU(K)) .GT. ABS(PHIR(K))) GO TO 290
      ZJF = ZJ + (XJF-XJ)*SPSU/CPSU
      GO TO 300
  290 ZJF = ZIF + (ZJF-XIF)*SPHR/CPHR
C
C     OBJECT OF THE GAME IS TO DIVIDE EACH SIDE INTO IMX, OR JMX EQUAL PARTS.
C
  300 XUL = DSQRT((DBLE(XJF-XJ))**2 + (DBLE(ZJF-ZJ))**2)
      DXU = XUL/FIMX
      ZRL = DSQRT((DBLE(XJF-XIF))**2 +(DBLE(ZJF-ZIF))**2)
      DZR = ZRL/FJMX
      DXL = XL(K)/FIMX
      DZL = ZL(K)/FJMX
C
      DO 700 J=1,JMX
      NS = NS+1
      FJM1 = J-1
      XS(NS) = XIF+FJM1*CPHR*DZR
      ZS(NS) = ZIF+FJM1*SPHR*DZR
      TEMP1  = XI(K)+FJM1*CPHL*DZL
      TEMP2  = ZI(K)+FJM1*SPHL*DZL
      TNJ = (ZS(NS)-TEMP2)/(XS(NS)-TEMP1)
      IF (J .GT. 1) GO TO 560
      DO 550 L=1,IMX
      NS1 = NS1+1
      FIM1 = L-1
      XS1(L) = XI(K)+FIM1*CPSL*DXL
      ZS1(L) = ZI(K)+FIM1*SPSL*DXL
      XS(NS1)= XJ+FIM1*CPSU*DXU
      ZS(NS1)= ZJ+FIM1*SPSU*DXU
      TEMP3 = XS(NS1)-XS1(L)
      IF (ABS(TEMP3/ZL(K)) .GT. 1.0E-7) GO TO 540
      TN(L) = 1.0E+7
      GO TO 550
```

```
  540 TN(L) = (ZS(NS1)-ZS1(L))/TEMP3
  550 CONTINUE
C
  560 DO 600 I=1,IMX
      N = N+1
      FIM1 = I-1
      IF (ABS(TN(I)) .LT. 1.0E+7) GO TO 580
      X(N) = XS1(I)
      GO TO 590
  580 X(N) = (ZS1(I)-TEMP2+TEMP1*TNJ-XS1(I)*TN(I))/(TNJ-TN(I))
  590 Z(N) = TEMP2+(X(N)-TEMP1)*TNJ
  600 CONTINUE
C
  700 CONTINUE
      NS1 = NS1+1
      XS(NS1) = XJF
      ZS(NS1) = ZJF
C
  800 CONTINUE
C
      RETURN
      END
```

```
$IBFTC DOME00  LIST,REF
      SUBROUTINE DOME
C
C  2-D LAGRANGIAN -- GENERATES POSITION OF MESHES FOR ELLIPTICAL SURFACES
C                    OF REVOLUTION
C
      EQUIVALENCE       (ISPACE(4),NSR)
      EQUIVALENCE       (ISPACE(10),NS1)
C
      NW = N
      NS = NS1
      NS1 = NS1+JM(K)
C
      SAL = XI(K)
      SAU = XI(K)+XL(K)
      SBL = ZI(K)
      SBU = ZI(K)+ZL(K)
      DPHI =(PHIL(K)-PHIR(K))/FLOAT(IM(K))
C
      PHI = PHIL(K)+DPHI
      IMX = IM(K)
      DO 500 I=1,IMX
      PHI = PHI-DPHI
      IF (ABS(PHI-1.5707963) .GT. 0.3E-6) GO TO 250
      XE = 0.0
      ZE = SBL
      XF = 0.0
      ZF = SBU
      GO TO 300
  250 TANP = TAN(PHI)
      XE = (SAL*SBL)/SQRT(SBL**2+(SAL*TANP)**2)
      ZE = XE*TANP
      XF = (SAU*SBU)/SQRT(SBU**2+(SBU*TANP)**2)
      ZF = XF*TANP
  300 DXEF = (XF-XE)/FLOAT(JM(K))
      DZEF = (ZF-ZE)/FLOAT(JM(K))
      XW = XE-DXEF
      ZW = ZE-DZEF
C
      JMX = JM(K)
      DO 450 J=1,JMX
      XW = XW+DXEF
      ZW = ZW+DZEF
      N = I+IMX*(J-1)+NW
      X(N) = XW
      Z(N) = ZW
  450 CONTINUE
      NS1 = NS1+1
      XS(NS1) = XF
      ZS(NS1) = ZF
C
  500 CONTINUE
```

```
C
      PHI = PHIR(K)
      IF (ABS(PHI) .GT. 0.3E-7) GO TO 600
      XE = SAL
      ZE = 0.0
      XF = SAU
      ZF = 0.0
      GO TO 650
  600 TANP = TAN(PHI)
      XE = (SAL*SBL)/SQRT(SBL**2+(SAL*TANP)**2)
      ZE = XE*TANP
      XF = (SAU*SBU)/SQRT(SBU**2+(SBU*TANP)**2)
      ZF = XF*TANP
  650 DXEF = (XF-XE)/FLOAT(JM(K))
      DZEF = (ZF-ZE)/FLOAT(JM(K))
      XW = XE-DXEF
      ZW = ZE-DZEF
C
      JMX = JM(K)
      DO 750 J=1,JMX
      XW = XW+DXEF
      ZW = ZW+DZEF
      NS = NS+1
      XS(NS) = XW
      ZS(NS) = ZW
  750 CONTINUE
C
      NS1 = NS1+1
      XS(NS1) = XF
      ZS(NS1) = ZF
C
      PSIL(K) = 0.0
      PSIU(K) = 0.0
      RETURN
      END
```

```
$IBFTC AMZR10  LIST,REF
      SUBROUTINE AMZRO
C
C  2-D LAGRANGIAN -- COMPUTES AREA AND MASS OF EACH MESH AT TIME = 0.0
C
      EQUIVALENCE        (ISPACE(4),NSR)
      EQUIVALENCE        (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
      EQUIVALENCE        (ISPACE(7),IMP),(ISPACE(8),IDP)
C
      X10 = X(N)
      Z10 = Z(N)
      NW = N+1
      X11 = X(NW)
      Z11 = Z(NW)
      NW  = N+IMX
      X12 = X(NW)
      Z12 = Z(NW)
      NW = NW+1
      X13 = X(NW)
      Z13 = Z(NW)
C
      IF (J .GE. JMX) GO TO 300
C
      IF (I .LT. IMX) GO TO 200
C
      X11 = XS(NSR)
      Z11 = ZS(NSR)
C
  200 IF (I-IMP) 450,250,210
  210 NS = NS+1
      X12 = XS(NS)
      Z12 = ZS(NS)
  250 NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
      GO TO 450
C
  300 NS = NS+1
      X12 = XS(NS)
      Z12 = ZS(NS)
      NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
C
      IF (I .LT. IMX) GO TO 410
C
      X11 = XS(NSR)
      Z11 = ZS(NSR)
C
  410 NH(I,K) = N
      XH(I,K) = 0.25*(X10+X11+X12+X13)
      ZH(I,K) = 0.25*(Z10+Z11+Z12+Z13)
C
```

```
C       COMPUTE AREA AND MASS OF MESH  N.
C
  450 XIBAR = (X11+X12+X10)/3.0
      XOBAR = (X11+X12+X13)/3.0
      AIN1  = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      AON1  = 0.5*((X12-X13)*(Z11-Z13) - (Z12-Z13)*(X11-X13))
      A(N)  = AIN1 + AON1
      TEMP9 = AIN1*XIBAR**(IALF-1) + AON1*XOBAR**(IALF-1)
      ZM(N) = TEMP9*RHO(N)
C
      RETURN
      END
```

```
$IBFTC LNK211  LIST,REF
C  2-D LAGRANGIAN -- LINK2 FOR IBM 7040 CHAIN
C
      EQUIVALENCE       (ISPACE(2),LSTN)
C
      LSTN = NMAX
C
C     UPDATE VARIABLES EXCEPT ON FIRST CYCLE
C
   10 NCYCLE = NCYCLE + 1
      XP     = 0.0
      ZP     = 0.0
      XK     = 0.0
      ZK     = 0.0
      TIE    = 0.0
      TE     = 0.0
      IF (NCYCLE .EQ. 1) GO TO  270
      DO 160 N=1,NMAX
      X(N) = X1(N)
      Z(N) = Z1(N)
      IF (TXX1(N) .LE. SBAR) GO TO 370
      IF (TYY1(N) .LE. SBAR) GO TO 370
      IF (TZZ1(N) .LE. SBAR) GO TO 370
      IF (TXZ1(N) .LE. SBAR) GO TO 370
  160 CONTINUE
      DO 220 NS=1,NSMAX
      XS(NS) = XS1(NS)
      ZS(NS) = ZS1(NS)
  220 CONTINUE
      DELTAM = DELTA
      DELTA = FK * DMIN
      TEMP1 = 1.1*DELTAM
      IF (DELTA .GT. TEMP1) DELTA = TEMP1
  270 TIME = TIME + DELTA
C
C     COMPUTE FOR 1 CYCLE
C
      CALL COMPUT
C
C     CHECK FOR END OF PROBLEM, OR IF PRINT OUT DESIRED
C
      IF (IEND .EQ. 5) GO TO 360
      IF (TIME .LT. TMAX) GO TO 310
  290 IEND = 5
      GO TO 360
  310 CYCLE = NCYCLE
      IF ((STOPC .GT. 0.0) .AND. (CYCLE .GE. STOPC)) GO TO 290
      IF ((WRTPC .GT. 0.0) .AND. (AMOD(CYCLE,WRTPC) .EQ. 0.0)) GO TO 390
      IF ((PRNTC .GT. 0.0) .AND. (AMOD(CYCLE,PRNTC) .EQ. 0.0)) GO TO 350
      IF (TIME .LT. TPRNT) GO TO  10
      TPRNT = TPRNT + DELPR
  350 IEND = 1
  360 CALL CHNXIT
  370 IEND = 4
      GO TO 360
  390 IEND = 3
      GO TO 360
      END
```

```
$IBFTC CMPT10  LIST,REF
      SUBROUTINE COMPUT                                                 CMPT 100
C
C  2-D LAGRANGIAN -- PERFORMS ALL COMPUTATIONS FOR 1 TIME CYCLE.
C
      EQUIVALENCE        (ISPACE(2),LSTN)
      EQUIVALENCE        (ISPACE(4),NSR)
C
C     GET VELOCITY AND POSITION OF EVERY MESH
C
      CALL GETPOS
C
C     START COMPUTATION LOOPS
C
      DMIN = 1.0
      N = 0
      NS = 0
C
      DO 2300  K=1,M
      JMX = JM(K)
      NFRST(K) = 0
      NSLST(K) = 9999
      IENT = 0
      NOIFCE = 0
      IMP = 0
      IDP = 0
C
      DO 2290  J=1,JMX
      NS = NS+1
      NSR = NS
      IMX = IM(K)
      IMP = IMX

      IF (IMX .GT. 0) GO TO 200
      IMX = IM1(J)
      IF (IDP .LT. 0) NSR = NSR+IDP
      IF (J .LT. JMX) GO TO 150
      IMP = 0
      IDP = 0
      GO TO 200
  150 IMP = IM1(J+1)
      IDP = IMP-IMX
C
  200 DO 2280  I=1,IMX
      N = N + 1
      AM = A(N)
      RHOM = RHO(N)
      EM = E(N)
C
C     DEFINE VELOCITIES AND POSITIONS OF THE VERTICIES OF ZONE N.
C                10 = LOWER-LEFT CORNER     (N)
C                11 = LOWER-RIGHT CORNER    (N+1)
C                12 = UPPER-LEFT CORNER     (N+IMX)
C                13 = UPPER RIGHT CORNER    (N+IMX+1)
C
```

```
      UX10  = UX1(N)
      UZ10  = UZ1(N)
      X10   = X1(N)
      Z10   = Z1(N)
      X10M  = X(N)
      Z10M  = Z(N)
      NW = N+1
      UX11  = UX1(NW)
      UZ11  = UZ1(NW)
      X11   = X1(NW)
      Z11   = Z1(NW)
      X11M  = X(NW)
      Z11M  = Z(NW)
      NW = N+IMX
      UX12  = UX1(NW)
      UZ12  = UZ1(NW)
      X12   = X1(NW)
      Z12   = Z1(NW)
      X12M  = X(NW)
      Z12M  = Z(NW)
      NW = NW+1
      UX13  = UX1(NW)
      UZ13  = UZ1(NW)
      X13   = X1(NW)
      Z13   = Z1(NW)
      X13M  = X(NW)
      Z13M  = Z(NW)
C
      IF (J .GE. JMX) GO TO 720
C
C     WE HAVE NOT REACHED THE FINAL ROW OF ZONES OF PLATE K.
      IF (I .LT. IMX) GO TO 500
C
C     WE ARE AT THE LAST ZONE IN THE ROW. RE-DEFINE RIGHT CORNERS.
C
      UX11  = UXS1(NSR)
      UZ11  = UZS1(NSR)
      X11   = XS1(NSR)
      Z11   = ZS1(NSR)
      X11M  = XS(NSR)
      Z11M  = ZS(NSR)
C
  500 IF (I-IMP) 640,800,510
  510 NS = NS+1
      GO TO 740
C
C     COMPUTE AREA AND DENSITY FOR ZONE N.
C
  640 XIBAR = (X11+X12+X10)/3.0
      XOBAR = (X11+X12+X13)/3.0
      AIN1  = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      AON1  = 0.5*((X12-X13)*(Z11-Z13) - (Z12-Z13)*(X11-X13))
      A1(N) = AIN1 + AON1
      TEMP1 = AIN1*XIBAR**(IALF-1) + AON1*XOBAR**(IALF-1)
      RHO1(N) = ZM(N)/TEMP1
      GO TO 2180
```

```
C
C     WE HAVE REACHED THE FINAL ROW OF ZONES FOR PLATE K.
C
  720 NS = NS + 1
      IF (NOSLID .EQ. 1) GO TO 740
      IF (K .LT. M) GO TO 960
C
C     WE ARE WORKING ON THE LAST PLATE. THERE IS NO INTERFACE.
C             RE-DEFINE UPPER CORNERS
C
  740 UX12 = UXS1(NS)
      UZ12 = UZS1(NS)
      X12  = XS1(NS)
      Z12  = ZS1(NS)
      X12M = XS(NS)
      Z12M = ZS(NS)
  800 NW = NS+1
      UX13 = UXS1(NW)
      UZ13 = UZS1(NW)
      X13  = XS1(NW)
      Z13  = ZS1(NW)
      X13M = XS(NW)
      Z13M = ZS(NW)
C
  870 IF ((I .LT. IMX) .OR. (J .LT. JMX)) GO TO 640
C
C     WE ARE AT THE LAST ZONE IN THE PLATE. RE-DEFINE LOWER-RIGHT CORNER

      UX11 = UXS1(NSR)
      UZ11 = UZS1(NSR)
      X11  = XS1(NSR)
      Z11  = ZS1(NSR)
      X11M = XS(NSR)
      Z11M = ZS(NSR)
      NS = NS+1
      GO TO 640
C
C     WE ARE NOT AT THE LAST PLATE. THEREFORE, THERE IS AN INTERFACE.
C
  960 IF (NS .GE. NSLST(K)) GO TO 870
C
C     WE ARE ALONG THE INTERFACE.
C
      NOIFCE = 1
      CALL CMPTIF
C
      IF (NONE .EQ. 2) GO TO 740
C
C     CONTINUE COMPUTATION FOR ZONE N.
C
 2180 TEMP2 = RHO1(N) - RHOM
      IF (ABS(TEMP2/RHOM) .GT. REP    ) GO TO 2190
      TEMP2 = 0.0
      RHO1(N) = RHOM
      A1(N) = AM
      E1(N) = EM
```

```
 2190 RHOBAR = 0.5*(RHO1(N)+RHOM)
      RDOTR = TEMP2 /(DELTA*RHOBAR)
      DRR2   = TEMP2 / RHOBAR**2
      ETA1  = RHO1(N)/RHOZ(K)
      ETA = RHOM/RHOZ(K)
C
C     COMPUTE DEVIATORIC STRESSES, AND EVALUATE ARTIFICIAL VISCOSITIES.
C
      CALL SDST
C
C     DETERMINE STRESSES AND TOTAL ENERGY.
C
      CALL FINDP
      IF (IEND .EQ. 5) RETURN
C
C     COMPUTE NEW DELTA T.
C
      CALL COMPDT
C
C     PERFORM ENERGY CHECKS
C
      CALL'ENRGCK
C
C     END LOOPS

 2280 CONTINUE
      IF (N .GE. LSTN) GO TO 2310
 2290 CONTINUE
 2300 CONTINUE
 2310 RETURN
      END
```

```
$IBFTC GTPS40  LIST,REF
      SUBROUTINE GETPOS
C
C   2-D LAGRANGIAN -- COMPUTES VELOCITIES AND POSITION OF ALL MESHES.


C
      EQUIVALENCE          (ISPACE(1),N3)
      EQUIVALENCE          (ISPACE(2),LSTN)
      EQUIVALENCE          (UXM,SPACE(5)), (UZM,SPACE(6))
      EQUIVALENCE          (ISPACE(4),NSR)
      EQUIVALENCE          (ISPACE(9),N1D)
C
      DSUM = DELTA + DELTAM
      N = 0
      NS = 0
      JZRO = 0
      IND = 0
      IMX = 0
      DO 2080 K=1,M
      JMX = JM(K)
      IENT = 0
      NOIFCE = 0
      TXXSD = 0.0
      TZZSD = 0.0
      TXZSD = 0.0
      IMP = IM(K)
      IMM = 0
      IDM = 0
      IDP = 0
C
C     START PRIMARY COMPUTATION LOOPS.
C
      DO 2075 J=1,JMX
```

```
      NS = NS + 1
      NSR = NS
      IMM = IMX
      IMX = IM(K)
      IF (IMX .GT. 0) GO TO 200
      IMX = IM1(J)
      IF (IDP .LT. 0) NSR = NSR+IDP
      IDM = -IDP
      IF (J .LT. JMX) GO TO 150
      IMP = 0
      IDP = 0
      GO TO 200
  150 IMP = IM1(J+1)
      IDP = IMP-IMX
  200 LMX = IMX
      IFLG = 0
      IZRO = 0
      DO 1670 I=1,IMX
      N = N + 1
C
C     DEFINE POSITION OF VERTICES  OF ZONE N.
C
      X10 = X(N)
      Z10 = Z(N)
      NW = N+1
      X11 = X(NW)
      Z11 = Z(NW)
      NW = N+IMX
      X12 = X(NW)
      Z12 = Z(NW)
      NW = NW+1
      X13 = X(NW)
      Z13 = Z(NW)
C
      IF (J .GT. 1) GO TO 640
C
C     WE ARE WORKING ON ROW 1 OF PLATE K.
C
      IF (K .GT. 1) GO TO 610
C
C     WE ARE IN PLATE 1.
C
  380 NOIFCE = 0
      IFLG = 0
      IF (I .GT. 1) GO TO 440
C
C     WE ARE AT THE LOWER LEFT CORNER.
C
      LL = NBXL(K) + NBZL(K)
      IF (NBZL(K) .EQ. 1) LL=LL-1
      AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      CALL ACRNR
      GO TO 1620
```

```
C
C     WE ARE PROGRESSING ALONG ROW 1.
C
  440 N2 = N-1
      X21 = X12
      Z21 = Z12
      X22 = X(N2)
      Z22 = Z(N2)
      L = 2
      LL = NBXL(K)
      IF (I .LT. IMX) GO TO 570
C
C     WE ARE AT THE LAST ZONE IN ROW 1. RE-DEFINE RIGHT VERTICES.
C
      X11 = XS(NS)
      Z11 = ZS(NS)
      IF (IDP .NE. 0) GO TO 570
      NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
C
  570 AIN2 = 0.5*((X21-X10)*(Z22-Z10) - (Z21-Z10)*(X22-X10))
      AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
  590 CALL ABNDRY
      GO TO 1620
C
C     WE HAVE FINISHED WITH PLATE 1. WE ARE STILL WORKING WITH ROW 1.
C
  610 IF ((N+1 .LE. NFRST(K-1)) .OR.  (N    .GT. NLST(K-1))) GO TO 380
      IF (NOSLID .EQ. 0) GO TO 630
C
C     WE ARE ALONG AN INTERFACE WHICH DOES NOT SLIDE.
C
      IFLG = IFLG+1
      IMX = IMM
      IF (N .NE. NLST(K-1)) GO TO 620
      IF (I .LT. LMX) GO TO 612
C
C     THE RIGHT VERTICES ARE ALONG THE RIGHT BOUNDARY . RE-DEFINE.
C
      X11 = XS(NS)
      Z11 = XS(NS)
      NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
  612 NW = NS-IMX-2
  613 X22 = XS(NW)
      Z22 = ZS(NW)
      N1 = N
      N2 = N-1
      N3 = N2-IMX
      X21 = X(N2)
      Z21 = Z(N2)
```

```
  615 LL = 1
      CALL AANGL
      GO TO 1620
  620 IF ((N .NE. NFRST(K-1)) .OR. (I .EQ. 1)) GO TO 640
      N1 = N-IMX
      N2 = N
      N3 = N-1
      X21 = X12
      Z21 = Z12
      X22 = X(N3)
      Z22 = Z(N3)
      X12 = X11
      Z12 = Z11
      X11 = X(N1)
      Z11 = Z(N1)
      GO TO 615
C
C     WE ARE ALONG A SLIDING INTERFACE.
C
  630 NOIFCE = 1
      CALL AIFACE
      GO TO 1620
C
C     WE HAVE FINISHED WITH ROW1.
C
  640 NOIFCE = 0
      IMX = IMM
      IF (J .GE. JMX) GO TO 830
      IF (I .GT. 1) GO TO 740
C
C     WE ARE ALONG A LEFT BOUNDARY.
C
  660 N2 = N - IMX
      X21 = X(N2)
      Z21 = Z(N2)
      X22 = X11
      Z22 = Z11
      L = 1
      LL = NBZL(K)
      GO TO 570
C
C     WE HAVE A ZONE IN THE INTERIOR.
C
  740 IF (I .LT. LMX) GO TO 750
C
C     THE RIGHT VERTICES ARE ALONG THE RIGHT BOUNDARY . RE-DEFINE.
C
      X11 = XS(NSR)
      Z11 = ZS(NSR)
C
  750 IF (IDP .GT. 0) GO TO 760
      NW = NS+1
      X13 = XS(NW)
```

```
      Z13 = ZS(NW)
      IF (I-IMP-1) 760,752,970
  752 NS = NS+1
      GO TO 1300
  760 IF (IDM) 770,800,800
  770 IF (I-IMM-1) 800,780,440
  780 NW = NS-1
      GO TO 613
  800 AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
  810 CALL AINTR
      GO TO 1620
C
C     WE ARE NOW WORKING ON THE LAST ROW IN PLATE K.
C
  830 IF (K .GE. M) GO TO 970
C
C     WE ARE NOT AT THE FINAL PLATE.
C
      NOIFCF = 1
      NS = NS + 1
      X12 = XS(NS)
      Z12 = ZS(NS)
      NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
C
      IF (I .LT. LMX) GO TO 950
C
C     WE ARE AT A RIGHT BOUNDARY. RE-DEFINE.
C
      NW = NS-IMX
      X11 = XS(NW)
      Z11 = ZS(NW)
      GO TO 800
C
  950 IF (I .EQ. 1) GO TO 660
      GO TO 800
C
C     WE HAVE REACHED THE FINAL PLATE. WE ARE STILL WORKING ON THE LAST
C              ROW.
C
  970 NS = NS+1
C
C     WE WILL FIND THE POSITIONS OF THE BOUNDARY MESH AND FINAL ROW MESH
C              CONCURRENTLY.
C     BOUNDARY MESH.
C
      X11 = X10
      Z11 = Z10
      X10 = XS(NS)
      Z10 = ZS(NS)
      NW = NS+1
      X12 = XS(NW)
```

```
      Z12 = ZS(NW)
      AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
C
      IF (I .GT. 1) GO TO 1210
C
C     WE ARE AT THE UPPER LEFT CORNER
C
      LL = NBXU(K) + NBZL(K)
      IF (NBZL(K) .EQ. 1) LL=LL-1
      CALL ACRNR
      CALL EXTPOS
C
C     SET-UP FOR FINAL ROW MESH. LEFT BOUNDARY.
C
      X10 = X11
      Z10 = Z11
      NW = N+1
      X11 = X(NW)
      Z11 = Z(NW)
      X13 = X12
      Z13 = Z12
      X12 = XS(NS)
      Z12 = ZS(NS)
      GO TO 660
C
C     PROCEED ALONG THE FINAL ROW OF LAST PLATE.
C     BOUNDARY MESH
C
 1210 NW = NS-1
      X21 = XS(NW)
      Z21 = ZS(NW)
      X22 = X11
      Z22 = Z11
      N2 = N-1
      L = 2
      LL = NBXU(K)
      AIN2 = 0.5*((X21-X10)*(Z22-Z10) - (Z21-Z10)*(X22-X10))
      CALL ABNDRY
      CALL EXTPOS
C
 1300 IF (I .GE. LMX) GO TO 1370
      NW = N+1
      X11 = X(NW)
      Z11 = Z(NW)
      GO TO 1530
C
C     WE ARE AT THE UPPER RIGHT CORNER OF PLATE K.
C
 1370 X11 = XS(NS)
      Z11 = ZS(NS)
      NS = NS+1
      X10 = XS(NS)
      Z10 = ZS(NS)
```

```
      NW = NSP
      X12 = XS(NW)
      Z12 = ZS(NW)
      AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      LL = NBXU(K) + NBZR(K)
      IF (NBZR(K) .EQ. 1) LL=LL-1
      CALL ACRNR
      CALL EXTPOS
C
C     SET-UP FOR FINAL ROW MESH.
C
      X11 = X12
      Z11 = Z12
      NS = NS-1
 1530 X10 = X(N)
      Z10 = Z(N)
      X12 = XS(NS)
      Z12 = ZS(NS)
      NW = NS+1
      X13 = XS(NW)
      Z13 = ZS(NW)
      GO TO 760
C
C     COMPUTE NEW VELOCITIES AND POSITIONS FOR MESH N.
C
 1620 UXM = UX(N)
      UZM = UZ(N)
      UX1(N) = UX(N) + 0.5*AX*DSUM
      UZ1(N) = UZ(N) + 0.5*AZ*DSUM
      TEMP1 = UEP
      IF (N1D .GT. 0) TEMP1 = 1.0E+10
      IF (ABS(UX1(N) .LT. TEMP1) UX1(N) =0.0
      IF (ABS(UZ1(N)   ) .LT. UEP) UZ1(N)  =0.0
      X1(N)   = X(N) + UX1(N)*DELTA
      Z1(N)   = Z(N) + UZ1(N)*DELTA
      IMX = LMX
C
      IF (NOSLID .EQ. 0) GO TO 1640
      IF (IFLG .EQ. 0) GO TO 1660
C
C     THERE IS AN INTERFACE WHICH DOES NOT SLIDE
C
      NW = NS+IFLG-IMM-2
      UXS1(NW) = UX1(N)
      UZS1(NW) = UZ1(N)
      XS1(NW) = X1(N)
      ZS1(NW) = Z1(N)
      GO TO 1660
C
C     IF THERE IS AN INTERFACE, DETERMINE TANGENT OF LINE THROUGH MESH N.
C
 1640 IF ((J .LT. JMX) .OR.(NOIFCE .EQ. 0)) GO TO 1660
      CALL DETTN
```

```
C
 1660 IF (UX1(N) .NE. 0.0) GO TO 1661
      IF (UZ1(N) .EQ. 0.0) GO TO 1670
 1661 IND  = 1
      IZRO = 1
      JZRO = 0
C
C     END I LOOP
C
 1670 CONTINUE
C
C     TAKE CARE OF RIGHT BOUNDARY MESHES.
C     ROTATE REFERENCE FRAME OF LAST MESH BY 90 DEG.
C
      NSC = NS
      NS = NSR
      X12 = X10
      Z12 = Z10
      X10 = X11
      Z10 = Z11
      X11 = X13
      Z11 = Z13
      IF (J .GT. 1) GO TO 1750
C
C     WE ARE AT THE LOWER RIGHT CORNER MESH.
C
      IF (K .EQ. 1) GO TO 1720
      IF ((NOIFCE .EQ. 0) .OR. (N .GE. NLST(K-1))) GO TO 1720
      I = I+1
      CALL AIFACE
      CALL EXTPOS
      GO TO 2070
C
 1720 IF (N .LT. NLST(K-1)) GO TO 1780
      LL = NBXL(K) + NBZR(K)
      IF (NBZR(K) .EQ. 1) LL=LL-1
 1730 AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      CALL ACRNR
 1735 IF (J .LT. JMX) GO TO 1740
      NSC = NSC+1
      NW = NSC
      GO TO 2040
 1740 CALL EXTPOS
      GO TO 2070
C
C     WE ARE ALONG RIGHT BOUNDARY.
C
 1750 IF (IDM) 1751,1810,1752
 1751 LL = 1
      GO TO 1730
 1752 N1 = N
      N2 = N-IMM
      N3 = N2+1
```

```
      X21 = X(N3)
      Z21 = Z(N3)
      NW = NS+1
      X22 = XS(NW)
      Z22 = ZS(NW)
      LL = 1
      CALL AANGL
      GO TO 1735
 1780 LMX = IMX
      IMX = IMM
 1810 N2 = N-IMX
      X21 = X12
      Z21 = Z12
      L = 1
      LL = NBZP(K)
C
      IF (J .GE. JMX) GO TO 1960
      IF ((J .GT. 1) .OR. (N .GE.NLST(K-1))) GO TO 1860
      IF (IMX .GT.LMX) GO TO 1840
      NW = NS-IMX-2
      GO TO 1870
 1840 NW = N2+1
      X22 = X(NW)
      Z22 = Z(NW)
      GO TO 1890
 1860 NW = NS-1
 1870 X22 = XS(NW)
      Z22 = ZS(NW)
 1890 X10 = XS(NS)
      Z10 = ZS(NS)
      NW = NS+1
      X11 = XS(NW)
      Z11 = ZS(NW)
      NW = NS
      GO TO 2030
C
C     WE ARE NEXT TO THE UPPER RIGHT CORNER.
C
 1960 NSC = NSC+1
      NS = NSC
      NW = NS
      NS = NS-IMX-2
      X22 = XS(NS)
      Z22 = ZS(NS)
      NS = NS+1
      X10 = XS(NS)
      Z10 = ZS(NS)
      X11 = XS(NW)
      Z11 = ZS(NW)
C
 2030 AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      AIN2 = 0.5*((X12-X10)*(Z22-Z10) - (Z12-Z10)*(X22-X10))
      CALL ABNDRY
```

```
 2040 UXM = UXS(NS)
      UZM = UZS(NS)
      CALL EXTPOS
      IF ((J .GT. 1) .OR. (N .GE.NLST(K-1))) GO TO 2050
      N1 = NS-1
      UXS1(N1) = UXS1(NS)
      UZS1(N1) = UZS1(NS)
      XS1(N1) = XS1(NS)
      ZS1(N1) = ZS1(NS)
 2050 NS = NW
      IMX = LMX
C
      IF (NOSLID .EQ. 1) GO TO 2070
      IF ((J .LT. JMX) .OR. (NOIFCE .EQ. 0)) GO TO 2070
      I = I+1
      CALL DETTN
C
 2070 NS = NSC
      IF ((K .EQ. 1) .AND. (M .GT. 1)) GO TO 2075
      IF (NCYCLE .EQ. 1) GO TO 2075
      IF (UXS1(NS) .NE. 0.0) GO TO 2071
      IF (UZS1(NS) .EQ. 0.0) GO TO 2072
 2071 IND  = 1
      IZRO = 1
      JZRO = 0
      GO TO 2075
 2072 IF (IND .EQ. 0) GO TO 2075
      IF (IZRO .NE. 0) GO TO 2075
      JZRO = JZRO+1
      IF (JZRO .LT. 2) GO TO 2075
      GO TO 2100
C
C     END J LOOP
C
 2075 CONTINUE
C
C     END K LOOP
C
 2080 CONTINUE
 2100 LSTN = N
      RETURN
      END
```

```
$IBFTC EXTPOS  LIST,REF
      SUBROUTINE EXTPOS

C  2-D LAGRANGIAN -- COMPUTES VELOCITY AND POSITION OF A BOUNDARY MESH
C
C
      EQUIVALENCE        (ISPACE(9),N1D)
C
      UXS1(NS) = UXS(NS) + 0.5*AX*DSUM
      UZS1(NS) = UZS(NS) + 0.5*AZ*DSUM
      TEMP1 = UEP
      IF (N1D .GT. 0) TEMP1 = 1.0E+10
      IF (ABS(UXS1(NS) .LT. TEMP1) UXS1(NS)=0.0
      IF (ABS(UZS1(NS)) .LT. UEP) UZS1(NS)=0.0
      XS1(NS) = XS(NS) + UXS1(NS)*DELTA
      ZS1(NS) = ZS(NS) + UZS1(NS)*DELTA
      RETURN
      END
```

```
$IBFTC AINTR1  LIST,REF
      SUBROUTINE AINTR                                                  AINT 100
C
C  2-D LAGRANGIAN -- COMPUTES THE ACCELERATION OF AN INTERIOR MESH
C
      N2 = N-1                                                          AINT 110
      X21 = X(N2)                                                       AINT 120
      Z21 = Z(N2)                                                       AINT 130
      N4 = N-IMX                                                        AINT 140
      X22 = X(N4)                                                       AINT 150
      Z22 = Z(N4)                                                       AINT 160
      N3 = N4-1                                                         AINT 170
      AIN2 = 0.5*((X12-X10)*(Z21-Z10) - (Z12-Z10)*(X21-X10))            AINT 180
      AIN3 = 0.5*((X21-X10)*(Z22-Z10) - (Z21-Z10)*(X22-X10))            AINT 190
      AIN4 = 0.5*((X22-X10)*(Z11-Z10) - (Z22-Z10)*(X11-X10))            AINT 200
      TEMP1 = AIN1*RHO(N) + AIN2*RHO(N2) + AIN3*RHO(N3) + AIN4*RHO(N4)  AINT 210
      TEMP2 = 0.25*(ALF-1.0)                                            AINT 220
      IF (TEMP2 .NE. 0.0) GO TO 270                                     AINT 230
      TEMP3 = 0.0                                                       AINT 240
      TEMP4 = 0.0                                                       AINT 250
      GO TO 320                                                         AINT 260
  270 NW  = IALF-1                                                      AINT 270
      TEMP5 = RHO(N )*(0.33333333*(X10+X11+X12))**NW                    AINT 271
      TEMP6 = RHO(N2)*(0.33333333*(X10+X12+X21))**NW                    AINT 272
      TEMP7 = RHO(N3)*(0.33333333*(X10+X21+X22))**NW                    AINT 273
      TEMP8 = RHO(N4)*(0.33333333*(X10+X22+X11))**NW                    AINT 274
      TEMP3 = TEMP2*((TXX(N)-TYY(N))/TEMP5 + (TXX(N2)-TYY(N2))/TEMP6 +  AINT 280
     1(TXX(N3)-TYY(N3))/TEMP7 + (TXX(N4)-TYY(N4))/TEMP8)                AINT 290
      TEMP4 = TEMP2*(TXZ(N)/TEMP5 + TXZ(N2)/TEMP6 + TXZ(N3)/TEMP7 + TXZ(AINT 300
     1N4)/TEMP8)                                                        AINT 310
  320 AZ = -(TXZ(N)*(Z11-Z12) + TXZ(N2)*(Z12-Z21) + TXZ(N3)*(Z21-Z22) + AINT 320
     1TXZ(N4)*(Z22-Z11))/TEMP1 + (TZZ(N)*(X11-X12) + TZZ(N2)*(X12-X21) +AINT 330
     2TZZ(N3)*(X21-X22) + TZZ(N4)*(X22-X11))/TEMP1 + TEMP4              AINT 340
      AX = -(TXX(N)*(Z11-Z12) + TXX(N2)*(Z12-Z21) + TXX(N3)*(Z21-Z22) + AINT 350
     1TXX(N4)*(Z22-Z11))/TEMP1 + (TXZ(N)*(X11-X12) + TXZ(N2)*(X12-X21) +AINT 360
     2TXZ(N3)*(X21-X22) + TXZ(N4)*(X22-X11))/TEMP1 + TEMP3              AINT 370
      RETURN                                                            AINT 380
      END                                                               AINT 390
```

```
$IBFTC ACRNR5  LIST,REF
      SUBROUTINE ACRNR
C
C   2-D LAGRANGIAN -- COMPUTES THE ACCELERATION OF A CORNER MESH
C                     LL = 1 - Z FREE, X FREE
C                        = 2 - Z FREE, X FIXED
C                        = 3 - Z FIXED, X FREE
C                        = 4 - Z FIXED, X FIXED
C
      EQUIVALENCE        (SINB,SPACE(3)),(COSB,SPACE(4))
C
      ZRO = 0.0000005
      ONE = 0.9999995
C
      IF (LL .LT. 4) GO TO 150
  120 AZ = 0.0
      AX = 0.0
      GO TO 290
  150 TEMP1 = AIN1*RHO(N)
      TEMP2 = (ALF-1.0)/(RHO(N)*(0.33333333*(X10+X11+X12))**(IALF-1))
      TEMP3 = X11-X12
      TEMP4 = Z11-Z12
      GO TO (200,210,210,120),LL
  200 AZ = (-TXZ(N)*TEMP4 + TZZ(N)*TEMP3)/TEMP1 + TEMP2*TXZ(N)
      AX = (-TXX(N)*TEMP4 + TXZ(N)*TEMP3)/TEMP1 + TEMP2*(TXX(N)-TYY(N))
      GO TO 290
  210 IF (IALF .LT. 2) GO TO 260
C
C     WHEN CYLINDRICAL SYMMETRY IS REQUESTED, A FIXED BOUNDARY MAY ONLY
C     BE AT 0.0 OR 90.0 DEG.
C
      GO TO (200,242,244,120),LL
  242 IF (J .GT. 1) GO TO 243
      TEMP9 = PSIL(K)
      GO TO 250
  243 TEMP9 = PSIU(K)
      GO TO 250
  244 IF (I .GT. 1) GO TO 245
      TEMP9 = PHIL(K)
      GO TO 250
  245 TEMP9 = PHIR(K)
  250 IF (TEMP9 .GT. 0.7854) GO TO 255
      SINB = 0.0
      COSB = 1.0
      AZ = 0.0
      AX = (-TXX(N)*TEMP4 + TXZ(N)*TEMP3)/TEMP1 +     TEMP2*(TXX(N) - TY
     1Y(N))
      GO TO 290
  255 AZ = (-TXZ(N)*TEMP4 + TZZ(N)*TEMP3)/TEMP1
      AX = 0.0
      SINB = 1.0
      COSB = 0.0
      GO TO 290
```

```
C
C      PLANE GEOMETRY REQUESTED.
C
  260 GO TO (200,262,270,120), LL
  262 IF (J .GT. 1) GO TO 266
      TEMP9 = PSIL(K)+1.5707963
      GO TO 272
  266 TEMP9 = PSIU(K)+1.5707963
      GO TO 272
  270 IF (I .GT. 1) GO TO 271
      TEMP9 = PHIL(K)
      GO TO 272
  271 TEMP9 = PHIR(K)
  272 COSB = COS(TEMP9)
      IF (ABS(COSB) .GT. ONE) COSB = SIGN( 1.0,COSB)
      IF (ABS(COSB) .LT. ZRO) COSB = 0.0
      COSB2 = COSB**2
      SINB = SIN(TEMP9)
      IF (ABS(SINB) .GT. ONE) SINB = SIGN( 1.0,SINB)
      SINB2 = SINB**2
      IF (ABS(SINB) .LT. ZRO) SINB = 0.0
      TIIN = TXX(N)*SINB2+TZZ(N)*COSB2-2.0*TXZ(N)*SINB*COSB
      TJJN = TXX(N)*COSB2+TZZ(N)*SINB2+2.0*TXZ(N)*SINB*COSB
      TIJN = -SINB*COSB*(TZZ(N)-TXX(N))+TXZ(N)*(SINB2-COSB2)
      TEMP4 = (X11-X12)*COSB + (Z11-Z12)*SINB
      TEMP3 = (X11-X12)*SINB - (Z11-Z12)*COSB
      GO TO (200,284,286,120),LL
  284 AW = (-TIIN*TEMP4 + TIJN*TEMP3)/TEMP1
      TEMP10 = SINB
      SINB = -COSB
      COSB = TEMP10
      GO TO 287
  286 AW = (-TIJN*TEMP4 + TJJN*TEMP3)/TEMP1
  287 AZ = AW*SINB
      AX = AW*COSB
  290 RETURN
      END
```

```
$IBFTC ABNDR5  LIST,REF
      SUBROUTINE ABNDRY
C
C   2-D LAGRANGIAN -- COMPUTES THE ACCELERATION OF A BOUNDARY MESH
C                      LL = 1 - DENOTES FREE BOUNDRY
C                         = 2 - DENOTES FIXED BOUNDRY
C
C                       L = 1 - PARALLEL TO Z-AXIS
C                         = 2 - PARALLEL TO X-AXIS

C
      ZRO = 0.0000005
      ONE = 0.9999995
C
      TEMP1  = AIN1*RHO(N) + AIN2*RHO(N2)
      TEMP2  = 0.50*(ALF - 1.0)
      TEMP3  = X11-X12
      TEMP4  = Z11-Z12
      TEMP5  = X21-X22
      TEMP6  = Z21-Z22
      NW = IALF-1
      TEMP7  = 1.0/(RHO(N )*(0.33333333*(X10+X11+X12))**NW)
      TEMP8  = 1.0/(RHO(N2)*(0.33333333*(X10+X22+X21))**NW)
      GO TO (200,240),LL
  200 AZ = -((TXZ(N)*TEMP4 + TXZ(N2)*TEMP6)/TEMP1) + ((TZZ(N)*TEMP3 + TZ
     1Z(N2)*TEMP5)/TEMP1) + TEMP2*(TXZ(N)*TEMP7 + TXZ(N2)*TEMP8)
      AX = -((TXX(N)*TEMP4 + TXX(N2)*TEMP6)/TEMP1) + ((TXZ(N)*TEMP3 + TX
     1Z(N2)*TEMP5)/TEMP1) + TEMP2*((TXX(N)-TYY(N))*TEMP7 + (TXX(N2)-TYY(
     2N2))*TEMP8)
      GO TO 350
  240 IF (IALF .LT. 2) GO TO 260
C
C     WHEN CYLINDRICAL SYMMETRY IS REQUESTED, A FIXED BOUNDARY MAY ONLY
C     BE AT 0.0 OR 90.0 DEG.
C
      GO TO (244,242),L
  242 IF (J .GT. 1) GO TO 243
      TEMP9 = PSIL(K)
      GO TO 250
  243 TEMP9 = PSIU(K)
      GO TO 250
  244 IF (I .GT. 1) GO TO 245
      TEMP9 = PHIL(K)
      GO TO 250
  245 TEMP9 = PHIR(K)
  250 IF (TEMP9 .GT. 0.7854) GO TO 255
C
      AZ = 0.0
      AX = -(((TXX(N)-TXX(N2))*TEMP4)/TEMP1) + ((TXZ(N)*TEMP3 + TXZ(N2)*
     1TEMP5)/TEMP1) +     TEMP2*((TXX(N)-TYY(N))*TEMP7 + (TXX(N2)-TYY(N2
     2))*TEMP8)
      GO TO 350
  255 AZ = -((TXZ(N)*TEMP4 + TXZ(N2)*TEMP6)/TEMP1)+ ((TZZ(N)*TEMP3 +  TZ
     1Z(N2)*TEMP5)/TEMP1)
      AX = 0.0
      GO TO 350
```

```
C
C     PLANE GEOMETRY REQUESTED.
C
  260 GO TO (270,261),L
  261 IF (J .GT. 1) GO TO 264
      TEMP9 = PSIL(K)+1.5707963
      GO TO 272
  264 TEMP9 = PSIU(K)+1.5707963
      GO TO 272
  270 IF (I .GT. 1) GO TO 271
      TEMP9 = PHIL(K)
      GO TO 272
  271 TEMP9 = PHIR(K)
  272 COSB = COS(TEMP9)
      IF (ABS(COSB) .GT. ONE) COSB = SIGN( 1.0,COSB)
      IF (ABS(COSB) .LT. ZRO) COSB = 0.0
      COSB2 = COSB**2
      SINB = SIN(TEMP9)
      IF (ABS(SINB) .LT. ZRO) SINB = 0.0
      IF (ABS(SINB) .GT. ONE) SINB = SIGN( 1.0,SINB)
      SINB2 = SINB**2
      TIIN = TXX(N)*SINB2+TZZ(N)*COSB2-2.0*TXZ(N)*SINB*COSB
      TJJN = TXX(N)*COSB2+TZZ(N)*SINB2+2.0*TXZ(N)*SINB*COSB
      TIJN =-SINB*COSB*(TZZ(N)-TXX(N))+TXZ(N)*(SINB2-COSB2)
      TIIN2 = TXX(N2)*SINB2+TZZ(N2)*COSB2-2.0*TXZ(N2)*SINB*COSB
      TJJN2 = TXX(N2)*COSB2+TZZ(N2)*SINB2+2.0*TXZ(N2)*SINB*COSB
      TIJN2 =-SINB*COSB*(TZZ(N2)-TXX(N2))+TXZ(N2)*(SINB2-COSB2)
      TEMP4 = (X11-X12)*COSB + (Z11-Z12)*SINB
      TEMP6 = (X21-X22)*COSB + (Z21-Z22)*SINB
      TEMP3 = (X11-X12)*SINB - (Z11-Z12)*COSB
      TEMP5 = (X21-X22)*SINB - (Z21-Z22)*COSB
      GO TO (291,310),L

  291 AW = -((TIJN*TEMP4 + TIJN2*TEMP6)/TEMP1) + ((TJJN*TEMP3 + TJJN2*
     1TEMP5)/TEMP1)
      GO TO 330
  310 AW = -(((TIIN-TIIN2)*TEMP4)/TEMP1) + ((TIJN*TEMP3 + TIJN2*TEMP5)/
     1TEMP1)
      TEMP10 = SINB
      SINB = -COSB
      COSB = TEMP10
  330 AZ = AW *SINB
      AX = AW *COSB
  350 RETURN
      END
```

```
$IBFTC AANGL2  LIST,REF
      SUBROUTINE AANGL
C
C  2-D LAGRANGIAN -- COMPUTES THE ACCELERATION OF A MESH AT THE BOUNDARY
C                    OF TWO PLATES OF UNEQUAL LENGTH. THAT IS, THEY FORM
C                    AN ANGLE.
C                    LL = 1 - DENOTES FREE BOUNDARY
C                       = 2 - DENOTES FIXED BOUNDARY
C
      EQUIVALENCE       (ISPACE(1),N3)
C
      GO TO (120,320),LL                                                AANG 110
  120 AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))            AANG 120
      AIN2 = 0.5*((X12-X10)*(Z21-Z10) - (Z12-Z10)*(X21-X10))            AANG 130
      AIN3 = 0.5*((X21-X10)*(Z22-Z10) - (Z21-Z10)*(X22-X10))            AANG 140
      TEMP1 = AIN1*RHO(N1) + AIN2*RHO(N2) + AIN3*RHO(N3)                AANG 150
      TEMP2 = 0.33333333*(ALF-1.0)                                      AANG 160
      IF (TEMP2 .NE. 0.0) GO TO 210                                     AANG 170
      TEMP3 = 0.0                                                       AANG 180
      TEMP4 = 0.0                                                       AANG 190
      GO TO 250                                                         AANG 200
  210 NW = IALF-1                                                       AANG 210
      TEMP5 = RHO(N1)*(0.33333333*(X10+X11+X12))**NW                    AANG 211
      TEMP6 = RHO(N2)*(0.33333333*(X10+X12+X21))**NW                    AANG 212
      TEMP7 = RHO(N3)*(0.33333333*(X10+X21+X22))**NW                    AANG 213
      TEMP3 = TEMP2*((TXX(N1)-TYY(N1))/TEMP5 + (TXX(N2)-TYY(N2))/TEMP6 +AANG 220
     1(TXX(N3)-TYY(N3))/TEMP7)                                          AANG 230
      TEMP4 = TEMP2*(TXZ(N1)/TEMP5 + TXZ(N2)/TEMP6 + TXZ(N3)/TEMP7)     AANG 240
  250 AX = -(TXX(N1)*(Z11-Z12) + TXX(N2)*(Z12-Z21) + TXX(N3)*(Z21-Z22))/AANG 250
     1TEMP1 + (TXZ(N1)*(X11-X12) + TXZ(N2)*(X12-X21) + TXZ(N3)*(X21-X22)AANG 260
     2)/TEMP1 + TEMP3                                                   AANG 270
      AZ = -(TXZ(N1)*(Z11-Z12) + TXZ(N2)*(Z12-Z21) + TXZ(N3)*(Z21-Z22))/AANG 280
     1TEMP1 + (TZZ(N1)*(X11-X12) + TZZ(N2)*(X12-X21) + TZZ(N3)*(X21-X22)AANG 290
     2)/TEMP1 + TEMP4                                                   AANG 300
      GO TO 340                                                         AANG 310
  320 AX = 0.0                                                          AANG 320
      AZ = 0.0                                                          AANG 330
  340 RETURN                                                            AANG 340
      END                                                               AANG 350
```

```
$IBFTC AIFC40  LIST,REF
      SUBROUTINE AIFACE
C
C  2-D LAGRANGIAN -- DETERMINES ACCELERATIONS OF MESHES ON THE INTERFACE
C                         BETWEEN TWO PLATES.
C
C
      EQUIVALENCE        (SINB,SPACE(3)),(COSB,SPACE(4))
C
      KM = K-1
      IF (I .GT. IMX) GO TO 820
C
      IF (I .LT. IMX) GO TO 100
C
      X11 = XS(NS)
      Z11 = ZS(NS)
      NW = NS + 1
      X13 = XS(NW)
      Z13 = ZS(NW)
C
  100 XRT = 0.25*(X11+X13+X12+X10)
      ZRT = 0.25*(Z11+Z13+Z12+Z10)
      TEMP10 = Z11-Z10
      IF (TEMP10 .NE. 0.0) GO TO 120
      TNRT = SIGN(1.0E+8,(-(X11-X10)))
      GO TO 130
  120 TNRT = -(X11-X10)/TEMP10
C
  130 IF (IENT .NE. 0) GO TO 170
C
C     UPON INTIAL ENTRY, SET-UP FLAGS.
C
      LK = 1
```

```
      NL = 1
      TNP = +1.0
C
  170 CALL FDLPTS
      IF (NONE .EQ. 1) GO TO 350
C
C     THE PROPER POINTS COULD NOT BE FOUND.
C
      IF (IM(KM) .GT. 0) GO TO 180
      KM2 = JM(KM)-1
      NW = 0
      DO 175 L=1,KM2
  175 NW = NW + IM1(L)
      IML = IM1(KM2+1)
      GO TO 190
  180 NW = IM(KM)*(JM(KM)-1) + 1
      IML = IM(KM)
  190 IF (KM .LE. 1) GO TO 210
      KM2 = KM-1
      DO 200 L=1,KM2
  200 NW = NW + IM(L)*JM(L)
  210 IF (I .GT. 1) GO TO 250
C
      TXXSD = TXX(NW)
      TZZSD = TZZ(NW)
      TXZSD = TXZ(NW)
      ZMS = ZM(NW)
      IENT = 0
      GO TO 470
C
  250 IF (IENT .NE. 0) GO TO 300
C
      TXXSE = TXX(NW)
      TZZSE = TZZ(NW)
      TXZSE = TXZ(NW)
      GO TO 620
C
  300 NW = NW + IML - 1
      ISR = ISS
      ISS = NW
      TXXSD = TXXSE
      TZZSD = TZZSE
      TXZSD = TXZSE
      IF (N .GE. NLST(KM)) GO TO 320
C
      DS = SQRT((XS(NS-1)-X10    )**2 + (ZS(NS-1)-Z10    )**2)
      RS = SQRT((X10-X11    )**2 + (Z10-Z11    )**2)
      TEMP1 = DS/RS
      IF (TEMP1 .LE. 1.0E-4) TEMP1 = 0.0
      TXXSE = TXX(NW)*TEMP1
      TZZSE = TZZ(NW)*TEMP1
      TXZSE = TXZ(NW)*TEMP1
      GO TO 620
```

```
C
  320 TXXSE = 0.0
      TZZSE = 0.0
      TXZSE = 0.0
      GO TO 620
C
C     POINTS WERE FOUND ON EITHER SIDE OF ZONE N.
C
  350 NSS = NL
      ISS = NH(NSS,KM)
      IF (NL .GT. 1) GO TO 380
      ISR = ISS
      DR = 1.0
      DS = 0.0
      RS = 1.0
      GO TO 450
  380 NSR = NL-1
      ISR = NH(NSR,KM)
      DR = SQRT((XSD-XH(NSR,KM))**2 + (ZSD-ZH(NSR,KM))**2)
      DS = SQRT((XSD-XH(NSS,KM))**2 + (ZSD-ZH(NSS,KM))**2)
      RS = SQRT((XH(NSR,KM)-XH(NSS,KM))**2 + (ZH(NSR,KM)-ZH(NSS,KM))**2)
C
  450 IF (I .GT. 1) GO TO 550
C
      TXXSD = (TXX(ISR)*DS + TXX(ISS)*DR)/RS
      TZZSD = (TZZ(ISR)*DS + TZZ(ISS)*DR)/RS
      TXZSD = (TXZ(ISR)*DS + TXZ(ISS)*DR)/RS
      ZMS = ZM(ISS)
C
  470 LL = 1
      IF (NBZL(K) .EQ. 1) GO TO 500
      LL = 3
      TEMP10 = PHIL(K)
  500 XQ = X11
      ZQ = Z11
      XA = X10
      ZA = Z10
      AIN1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
      CALL ACRNR
      TEMP4 = (X10-X11)*COSB + (Z10-Z11)*SINB
      TEMP3 = (X10-X11)*SINB - (Z10-Z11)*COSB
      AW = AZ/SINB
      AW = AW - (TXZSD*TEMP4 - TZZSD*TEMP3)/TEMP1
      AZ = AW*SINB
      AX = AW*COSB
      GO TO 990
C
  550 IF (I .LE. 2) GO TO 590
      TXXSD = TXXSE
      TZZSD = TZZSE
      TXZSD = TXZSE
C
  590 TXXSE = (TXX(ISR)*DS + TXX(ISS)*DR)/RS
```

```
      TZZSE  =  (TZZ(ISR)*DS  +  TZZ(ISS)*DR)/RS
      TXZSE  =  (TXZ(ISR)*DS  +  TXZ(ISS)*DR)/RS
C
  620 L  = 2
      LL  =  1
      N2  =  N-1
      X22  =  X(N2)
      Z22  =  Z(N2)
      X21  =  X12
      Z21  =  Z12
      XQ  =  X11
      ZQ  =  Z11
      XA  =  X22
      NW  =  NS-IM(K-1)-2+I
      ZA  =  Z22
      IF(ABS((X10-XS(NW))/X10) .GT. 1.0E-4) GO TO 700
      ZMS  =  ZM(ISR)
      GO TO 750
  700 ZMS  =  0.5*(ZM(ISR)+ZM(ISS))
  750 AIN2  =  0.5*((X21-X10)*(Z22-Z10)  -  (Z21-Z10)*(X22-X10))
      AIN1  =  0.5*((X11-X10)*(Z12-Z10)  -  (Z11-Z10)*(X12-X10))
      CALL  ABNDRY
      AX   =  AX -  ((TXXSD*(Z22-Z10)+TXXSE*(Z10-Z11))/TEMP1)  +  ((TXZSD*(X2
     12-X10)+TXZSE*(X10-X11))/TEMP1)
      AZ   =  AZ -  ((TXZSD*(Z22-Z10)+TXZSE*(Z10-Z11))/TEMP1)  +  ((TZZSD*(X2
     12-X10)+TZZSE*(X10-X11))/TEMP1)
      GO  TO  990
C
  820 X10  =  XS(NS)
      Z10  =  ZS(NS)
      NW  =  NS+1
      X11  =  XS(NW)
      Z11  =  ZS(NW)
      X12  =  X(N)
      Z12  =  Z(N)
      XQ  =  X10
      ZQ  =  Z10
      XA  =  X12
      ZA  =  Z12
      ZMS  =  ZM(ISS)
      LL=  1
      IF  (NBZR(K)  .EQ.  1)  GO  TO  940
      LL  =  3
      TEMP10  =  PHIR(K)
  940 AIN1  =  0.5*((X11-X10)*(Z12-Z10)  -  (Z11-Z10)*(X12-X10))
      CALL  ACRNR
      TEMP4  =  (X12-X10)*COSB  +  (Z12-Z10)*SINB
      TEMP3  =  (X12-X10)*SINB  -  (Z12-Z10)*COSB
      AW  =  AZ/SINB
      AW  =  AW  -  (TXZSE*TEMP4  -  TZZSE*TEMP3)/TEMP1
      AZ  =  AW*SINB
      AX  =  AW*COSB
C
```

```
 990 IF (LL .NE. 3) GO TO 1000
     SINAP = SIN(TEMP10-1.5707963)
     GO TO 1060
1000 TEMP4 = XQ-XA
     TEMP5 = ZQ-ZA
     IF (ABS(TEMP5/TEMP4) .GE. 1.0E-4) GO TO 1040
1020 SINAP = 0.0
     GO TO 1060
1040 TEMP6 = SQRT(TEMP4**2 + TEMP5**2)
     SINAP = TEMP5/TEMP6
1060 COSAP = SQRT(1.0 - SINAP**2)
     AXW = AX*COSAP + AZ*SINAP
     AZW =  ZM(N)/(ZM(N)+ZMS)*(-AX*SINAP + AZ*COSAP)
     AX = AXW*COSAP - AZW*SINAP
     AZ = AXW*SINAP + AZW*COSAP
     RETURN
     END
```

```
$IBFTC DTTN10  LIST,REF
      SUBROUTINE DETTN
C
C  2-D LAGRANGIAN -- DETERMINES THE TANGENT OF THE LINE THROUGH MESH  N.
C
      EQUIVALENCE        (ISPACE(4),NSR)
C
      IF (I .GT. IMX) GO TO 1940
      AIN1 = A(N) - 0.5*((X13-X11)*(Z10-Z11) - (Z13-Z11)*(X10-X11))
      IF (I .GT. 1) GO TO 1750
C
C     WE ARE AT THE UPPER LEFT CORNER ZONE OF PLATE K.
C
      X11 = X10
      Z11 = Z10
      X12 = X13
      Z12 = Z13
      IF (NBZL(K) .EQ. 1) GO TO 1670
      LL = 3
      GO TO 1680
 1670 LL = 1
 1680 CALL ACRNR
      TEMP1 = X1(N)
      TEMP2 = Z1(N)
      CALL GTTN
 1700 X11 = X(N+1)
      Z11 = Z(N+1)
      X12 = XS(NS)
      Z12 = ZS(NS)
      GO TO 2170
C
C     WE ARE ALONG THE UPPER BOUNDARY OF PLATE K.
C
 1750 X11 = X10
      Z11 = Z10
      X12 = X13
      Z12 = Z13
      NW = NS-1
      X21 = XS(NW)
      Z21 = ZS(NW)
      X22 = X11
      Z22 = Z11
      N2 = N-1
      L = 2
      LL = 1
      AIN2 = A(N2) - 0.5*((X22-X(N2))*(Z21-Z(N2)) - (Z22-Z(N2))*(X21-X(N
     12)))
      CALL ABNDRY
      TEMP1 = X1(N)
      TEMP2 = Z1(N)
      CALL GTTN
      GO TO 1700
C
C     WE ARE AT THE LAST MESH IN THE PLATE.
```

```
C
1940 X11 = XS(NS-1)
     Z11 = ZS(NS-1)
     X12 = X10
     Z12 = Z10
     AIN1 = A(N) - 0.5*((X12-X(N))*(Z11-Z(N)) - (Z12-Z(N))*(X11-X(N)))
     IF (NBZR(K) .EQ. 1) GO TO 2020
     LL = 3
     GO TO 2030
2020 LL = 1
2030 CALL ACRNR
     TEMP1 = XS1(NSR)
     TEMP2 = ZS1(NSR)
     CALL GTTN
2170 RETURN
     END
```

```
$IBFTC GTTN5   LIST,REF
      SUBROUTINE GTTN
C
C   2-D LAGRANGIAN -- COMPUTES TANGENT OF MESH BOUNDARIES AT INTERFACE
C
C
      EQUIVALENCE       (UXM,SPACE(5)), (UZM,SPACE(6))
C
      COSAB = SQRT(1.0-SNAB(NS)**2)
      AIN   = AX*COSAB + AZ*SNAB(NS)
      TEMP8      =   UXM   + 0.5*AIN*DSUM*COSAB
      TEMP9      =   UZM   + 0.5*AIN*DSUM*SNAB(NS)
      IF (ABS(TEMP8    ) .LT. UEP) TEMP8    =0.0
      IF (ABS(TEMP9    ) .LT. UEP) TEMP9    =0.0
      XT = XS(NS) + TEMP8    *DELTA
      ZT = ZS(NS) + TEMP9    *DELTA
      TN(NS) = ATAN2(ZT-TEMP2,XT-TEMP1)
  220 IF ((I .GT. 1) .AND. (I .LE. IMX)) GO TO 400
      IF (I .GT. 1) GO TO 300
      TEMP9 = PHIL(K)
      IF (TN(NS).GT. TEMP9) GO TO 320
      GO TO 400
  300 TEMP9 = PHIR(K)
      IF (TN(NS).GT. TEMP9) GO TO 400
  320 TN(NS) = TEMP9
  400 RETURN
      END
```

```
$IBFTC FDLP10  LIST,REF
      SUBROUTINE FDLPTS                                               LPT  100
C
C  2-D LAGRANGIAN -- FIND POINTS ON LOWER BOUNDARY OF INTERFACE
C
      EQUIVALENCE        (ISPACE(4),NSR)
C
      NONE = 1
      L = K-1
      IMM = IM(L)
      IF (IM(L) .GT. 0) GO TO 120
      JMM = JM(L)
      IMM = IM1(JMM)
  120 TEMP10 = XH(NL,L)-XRT
      TEMP9 = ZH(NL,L)-ZRT
      IF (TEMP10 .NE. 0.0) GO TO 130
      TNR = SIGN(1.0E+8, TEMP9)
      GO TO 140
  130 TNR = TEMP9/TEMP10
  140 IF (ABS(TNRT) .GE. 1.0E+4) GO TO 190
      IF (ABS(TNR)  .GE. 1.0E+4) GO TO 170
      TEMP8 = TNRT-TNR
      TNR =    TEMP8    /(1.0 + TNRT*TNR)
      IF (ABS(     TNR      ) .LT. 1.0E-4) GO TO 220
      GO TO 230
  170 TNR = -1.0/TNRT
      GO TO 230
  190 IF (ABS(TNR)  .GE. 1.0E+4) GO TO 220
      TNR = +1.0/TNR
      GO TO 230
  220 TNR = 0.0
      XSD = XH(NL,L)
      ZSD = ZH(NL,L)
      GO TO 350
  230 IF ((TNR*TNP) .LE. 0.0) GO TO 320
  270 TNP = TNR
      NL = NL + 1
      IF (NL .LE. IMM) GO TO 120
      NL = IMM
      NONE = 2
      GO TO 360
  320 IF (NL .LE. 1) GO TO 350
      TEMP1 = (ZH(NL-1,L)-ZH(NL,L))/(XH(NL-1,L)-XH(NL,L))
      IF ((ABS(TEMP1).GT.1.0E-4) .OR. (ABS(TNRT).LT.1.0E+4)) GO TO 330
      XSD = XRT
      ZSD = ZH(NL,L)
      GO TO 350
  330 TEMP2 = TNRT - TEMP1
      XSD = (ZH(NL-1,L)-ZRT+TNRT*XRT-TEMP1*XH(NL-1,L))/TEMP2
      ZSD = (TNRT*(ZH(NL-1,L)-TEMP1*(XH(NL-1,L)-XRT))-TEMP1*ZRT)/TEMP2
  350 IENT = 1
      TNP = +1.0
  360 RETURN
      END
```

```
$IBFTC CMPI20  LIST,REF
      SUBROUTINE CMPTIF
C
C  2-D LAGRANGIAN -- COMPUTES AREA AND DENSITY OF ZONES ALONG LOWER
C                           EDGE OF INTERFACE.
C
      EQUIVALENCE          (XW,TEMP4),       (ZW,TEMP5),       (UXRT,SPACE(1)),
     1   (UZRT,SPACE(2))
      EQUIVALENCE          (ISPACE(4),NSR)
C
      IF (IENT .NE. 0) GO TO 1170
C
C     THIS IS THE FIRST ZONE ON THE INTERFACE. INITIALIZE FLAGS.
C
      LK = 1
      IJK = 1
      NL = 1
      DO 990 L=1,K
      IF (IM(L) .GT. 0) GO TO 980
      NW = JM(L)
      DO 970 LL=1,NW
  970 NL = NL+IM1(LL)
      GO TO 990
  980 NL = NL + IM(L)*JM(L)
  990 CONTINUE
      XRT = X1(NL)
      ZRT = Z1(NL)
      UXRT = UX1(NL)
      UZRT = UZ1(NL)
      NFN = NL-1
      IENT = 1
C
```

```
 1100 TNNS = TN(NS)
C
C      SEARCH J=1 MESH OF PLATE K+1  TO FIND POINTS ON EACH SIDE OF N.
C
       XW = X1(N)
       ZW = Z1(N)
       CALL FDUPTS
       IF (NONE .EQ. 1) GO TO 1160
C
C      NO PAIR OF POINTS WERE FOUND.
C
       IENT = 0
       NOIFCE = 0
       GO TO 1600
C
C      A PAIR OF POINTS WERE FOUND. IF THIS WAS THE FIRST PAIR, SET FLAG.
C
 1160  IF (NFRST(K) .GT. 0) GO TO 1170
       NFRST(K) = NL-1
       IF (NL-1 .LT. NFN+1) NFRST(K) = NFN+1
 1170  X12 = XS1(NS)
       Z12 = ZS1(NS)
       X12M = XS(NS)
       Z12M = ZS(NS)
       UX12 = UXS1(NS)
       UZ12 = UZS1(NS)
       N1 = NL
       NS = NS + 1
C
C      CONTINUE SEARCH FOR POINTS ON EACH SIDE OF N+1. IF NO POINTS FOUND.
C              SET FLAG.
C
       TNNS = TN(NS)
       XW = X1(N+1)
       ZW = Z1(N+1)
       IF (N+1 .LE. NFN) GO TO 1220
       XW = XS1(NSR)
       ZW = ZS1(NSR)
 1220  CALL FDUPTS
       IF (NONE .EQ. 1) GO TO 1250
       NSLST(K) = NS
       WRITE (6,1240) K,I,J,N,NS,NL,LK,IJK,NFN,TN(NS),TNR,XRT,ZRT,XW,ZW
 1240  FORMAT (1H0,45HCAUTION - INTERFACE POINT OUTSIDE UPPER PLATE /1H0,
     1 9I6 / 1H0,6E16.8 )
 1250  X13 = XS1(NS)
       Z13 = ZS1(NS)
       X13M = XS(NS)
       Z13M = ZS(NS)
       UX13 = UXS1(NS)
       UZ13 = UZS1(NS)
       N5 = NL
       NS = NS-1
       IF (I .LT. IMX) GO TO 1280
```

```
C
C      WE ARE AT THE LAST ZONE IN THE PLATE. RE-DEFINE LOWER-RIGHT CORNER
C              AND SET A FLAG
C
       NLST(K) = N5
       NW = NSR
       X11 = XS1(NW)
       Z11 = ZS1(NW)
       X11M = XS(NW)
       Z11M = ZS(NW)
       UX11 = UXS1(NW)
       UZ11 = UZS1(NW)
       NS = NS+1
C
C      COMPUTE AREA AND DENSITY FOR ZONE N.
C
 1280  AI1 = 0.5*((X11-X10)*(Z12-Z10) - (Z11-Z10)*(X12-X10))
       IF (N1 .LE. NFN+IM(K+1)) GO TO 1290
       XINT = (X10+X12+X13)/3.0
       A1(N) = 0.5*((X13-X10)*(Z12-Z10) - (Z13-Z10)*(X12-X10))
       RDEN = A1(N)*XINT**(IALF-1)
       GO TO 1470
 1290  XINT = (X10+X12+X1(N1))/3.0
       AIN  = 0.5*((X1(N1)-X10)*(Z12-Z10) - (Z1(N1)-Z10)*(X12-X10))
       IF (XINT .NE. 0.0) GO TO 1310
       RDEN = 0.0
       GO TO 1320
 1310  RDEN = AIN *XINT**(IALF-1)
 1320  A1(N)= AIN
       N3 = N5-1
       IF (N3-N1) 1470,1430,1350
 1350  N3 = N3-1
       DO 1420 N4=N1,N3
       XINT = (X10+X1(N4)+X1(N4+1))/3.0
       AIN  = 0.5*((X1(N4+1)-X10)*(Z1(N4)-Z10) - (Z1(N4+1)-Z10)*(X1(N4)-
      1X10))
       RDEN = RDEN + AIN *XINT**(IALF-1)
       A1(N) = A1(N) + AIN
 1420  CONTINUE
 1430  XINT = (X10+X1(N5-1)+X13)/3.0
       AIN  = 0.5*((X13-X10)*(Z1(N5-1)-Z10) - (Z13-Z10)*(X1(N5-1)-X10))
       RDEN = RDEN + AIN *XINT**(IALF-1)
       A1(N) = A1(N) + AIN
 1470  AIN1 = A1(N)
       XINT = (X11+X10+X13)/3.0
       AIN  = 0.5*((X11-X10)*(Z13-Z10) - (Z11-Z10)*(X13-X10))
       RDEN = RDEN + AIN *XINT**(IALF-1)
       A1(N) = A1(N) + AIN
       RHO1(N) = ZM(N)/RDEN
C
       NH(I,K) = N
       XH(I,K) = 0.25*(X10+X11+X12+X13)
       ZH(I,K) = 0.25*(Z10+Z11+Z12+Z13)

 1600  RETURN
       END
```

```
$IBFTC FDUP80  LIST,REF
      SUBROUTINE FDUPTS                                                 UPT  100
C
C  2-D LAGRANGIAN -- FIND POINTS ON UPPER BOUNDARY OF INTERFACE
C                         THIS VERSION IS FOR THE COUPLED CAMEO-2-D SERIES.
C
      EQUIVALENCE         (XW,TEMP4),      (ZW,TEMP5),      (UXRT,SPACE(1)),
     1   (UZRT,SPACE(2))
      EQUIVALENCE         (ISPACE(4),NSR)
C
      NONE = 1
  120 TEMP10 = XRT-XW
      TEMP9 = ZRT-ZW
      TNR = ATAN2(TEMP9,TEMP10)
      IF (ABS(TNR-TNNS) .LT. 1.0E-4) GO TO 220
      TEMP7 = SIN(TNNS)
      TEMP8 = COS(TNNS)
      TNR = TEMP10*TEMP7 - TEMP9*TEMP8
      GO TO 230
  220 TNR = 0.0
      XS1(NS) = XRT
      ZS1(NS) = ZRT
      UXS1(NS) = UXRT
      UZS1(NS) = UZRT
      IF (LK .GT. 1) GO TO 400
      LK = 2
      TNP = -1.0
      XLFT = XRT
      ZLFT = ZRT
      NL = NL+1
      XRT =  X1(NL)
      ZRT = Z1(NL)
```

```
      GO TO 400
  230 IF (LK .NE. 1) GO TO 260
      LK = 2
      TNP = -1.0
      GO TO 270
  260 IF ((TNR*TNP) .LE. 0.0) GO TO 320
      TNP = TNR
  270 IF (IJK .GE. 2) GO TO 310
      XLFT = XRT
      ZLFT = ZRT
      NL = NL + 1
      XRT = X1(NL)
      ZRT = Z1(NL)
      UXRT = UX1(NL)
      UZRT = UZ1(NL)
      IF (NL .LE. NFN+IM(K+1)) GO TO 120
      NW = 1
      DO 300 L=1,K
      IF (IM(L) .GT. 0) GO TO 290
      N3 = JM(L)
      DO 280 LL = 1,N3
  280 NW = NW+IM1(LL)
      NW = NW+1
      GO TO 300
  290 NW = NW+JM(L)+IM(L)+1
  300 CONTINUE
      XRT = XS1(NW)
      ZRT = ZS1(NW)
      UXRT = UXS1(NW)
      UZRT = UZS1(NW)
      IJK = IJK+1
      GO TO 120
  310 NONE = 2
  320 TEMP1 = (ZLFT-ZRT)/(XLFT-XRT)
      IF (ABS(TNNS-1.5707963) .LE. 0.3E-7) GO TO 340
      TNNS = TAN(TNNS)
      IF (ABS(TNNS) .LE. 1.0E+9) GO TO 350
  340 TNNS = SIGN(1.0E+9,TNNS)
  350 TEMP2 = TNNS - TEMP1
      XS1(NS) = (ZLFT-ZW +TNNS*XW-TEMP1*XLFT)/TEMP2
      ZS1(NS) = (TNNS*(ZLFT-TEMP1*(XLFT-XW))-TEMP1*ZW)/TEMP2
      TEMP1 = XS1(NS)-XS(NS)
      TEMP2 = ZS1(NS)-ZS(NS)
      UXS1(NS) = TEMP1/DELTA
      UZS1(NS) = TEMP2/DELTA
      IF (ABS(UXS1(NS)) .LT. UEP) UXS1(NS)=0.0
      IF (ABS(UZS1(NS)) .LT. UEP) UZS1(NS)=0.0
  400 TEMP1 = XRT-XLFT
      TEMP2 = ZRT-ZLFT
  420 IF (ABS(TEMP2/TEMP1) .GE. 1.0E-4) GO TO 490
  430 SNAB(NS) = 0.0
      GO TO 500
  490 TEMP1 = SQRT(TEMP1**2 + TEMP2**2)
      SNAB(NS) = TEMP2/TEMP1
  500 RETURN
      END
```

```
$IBFTC SDST    LIST,REF
      SUBROUTINE SDST                                                  SDST 100
C
C  2-D LAGRANGIAN -- DETERMINES DEVIATORIC STRESSES
C
C     DDD IS DEVIATORIC STRESS FUNCTION
C
      DDD(TEMP1,TEMP2,TEMP3) = (DELTA/    TEMP4     )*(TEMP1*(2.0*DXX+DZ
     1Z) + TEMP2*(2.0*DZZ+DXX) + TEMP3*(2.0*DXZ))
C
      TEMP1 = RDOTR/3.0
      TEMP2 = 2.0*(A1(N)+AM)
      TEMP3 = UX10-UX13
      TEMP4 = UX12-UX11
      TEMP5 = UZ10-UZ13
      TEMP6 = UZ12-UZ11
      TEMP7 = Z12+Z12M-Z11-Z11M
      TEMP8 = Z10+Z10M-Z13-Z13M
      TEMP9 = X12+X12M-X11-X11M
      TEMP10= X10+X10M-X13-X13M
      DXX = -(TEMP3*TEMP7 - TEMP4*TEMP8)/TEMP2 + TEMP1
      DZZ =   (TEMP5*TEMP9 - TEMP6*TEMP10)/TEMP2 + TEMP1
      DYY = -(DXX+DZZ)
      DXZ =   (TEMP3*TEMP9 - TEMP4*TEMP10 - TEMP5*TEMP7 + TEMP6*TEMP8)/
     1(2.0*TEMP2)
      WXZ =   (TEMP3*TEMP9 - TEMP4*TEMP10 + TEMP5*TEMP7 - TEMP6*TEMP8)/
     1(2.0*TEMP2)
      TEMP1 = 0.5*(RHO1(N)+RHOM)/RHOZ(K)-1.0
      G = GO(K)*(1.0+TEMP1*(G1(K)+TEMP1*(G2(K)+TEMP1*(G3(K)))))
      TEMP2 = DELTA*WXZ
      TEMP3 = G*DELTA
      TEMP4 = 2.0*TEMP2*(DTXZ(N)+TEMP3*DXZ)
      DETXX = DTXX(N) + 2.0*TEMP3*DXX + TEMP4
      DETYY = DTYY(N) + 2.0*TEMP3*DYY
```

```
      DETZZ = DTZZ(N) + 2.0*TEMP3*DZZ - TEMP4
      DETXZ = DTXZ(N) + 2.0*TEMP3*DXZ - TEMP2*(DTXX(N)+TEMP3*DXX-DTZZ(N)
     1-TEMP3*DZZ)
      BOX   = DETXX**2 +DETYY**2 +DETZZ**2 +2.0*DETXZ**2
      TEMP6 = 0.66666667*CAPY(K)**2
      DTXXM = DTXX(N)
      DTYYM = DTYY(N)
      DTZZM = DTZZ(N)
      DTXZM = DTXZ(N)
      IF (BOX   .GT. TEMP6) GO TO 500
      DTXX(N) = DETXX
      DTYY(N) = DETYY
      DTZZ(N) = DETZZ
      DTXZ(N) = DETXZ
      GO TO 550
  500 TEMP7 = SQRT(TEMP6/BOX   )
      DTXX(N) = DETXX*TEMP7
      DTYY(N) = DETYY*TEMP7
      DTZZ(N) = DETZZ*TEMP7
      DTXZ(N) = DETXZ*TEMP7
C
C     EVALUATE ARTIFICIAL VISCOSITY.
C
  550 QXXM = QXX1(N)
      QZZM = QZZ1(N)
      QXZM = QXZ1(N)
      CALL EVALQ
C
      TEMP4 = RHO1(N)+RHOM
      TEMP1 = DTXX(N)+DTXXM
      TEMP2 = DTZZ(N)+DTZZM
      TEMP3 = DTXZ(N)+DTXZM
      DEED = DDD(TEMP1,TEMP2,TEMP3)
      TEMP1 = QXX1(N)+QXXM
      TEMP2 = QZZ1(N)+QZZM
      TEMP3 = QXZ1(N)+QXZM
      DEQD = DDD(TEMP1,TEMP2,TEMP3)
      DED = DEED + DEQD
      RETURN
      END
```

```
$IBFTC EVALQ2  LIST,REF
      SUBROUTINE EVALQ                                                  EVLQ 100
C
      EQUIVALENCE        (SPACE(7),QM)
C
      PM = -TXX(N)+DTXXM-Q(N)-QXX1(N)                                   EVLQ 110
      QM = Q(N)                                                         EVLQ 111
      P1 = PM                                                           EVLQ 120
      CALL GETCN                                                        EVLQ 121
      IF (NBQ .GT. 0) GO TO 160                                         EVLQ 122
      IF (RDOTR .GT. 0.0) GO TO 160                                     EVLQ 130
      Q1(N) = 0.0                                                       EVLQ 140
      GO TO 200                                                         EVLQ 150
  160 TEMP1 = 0.5*(RHO1(N)+RHOM)                                        EVLQ 170
      TEMP2 = 0.5*(A1(N)+AM)                                            EVLQ 180
      Q1(N) = CZ**2*TEMP1*TEMP2*RDOTR**2 + CL*TEMP1*CN*RDOTR*SQRT(TEMP2)EVLQ 190
  200 TEMP3 = C3*TEMP1*TEMP2*CN                                         EVLQ 200
      QXX1(N) = TEMP3*DXX                                               EVLQ 210
      QZZ1(N) = TEMP3*DZZ                                               EVLQ 220
      QXZ1(N) = TEMP3*DXZ                                               EVLQ 230
      RETURN                                                            EVLQ 240
      END                                                               EVLQ 250
```

```
$IBFTC FDP53   LIST,REF
      SUBROUTINE FINDP
C
C   2-D LAGRANGIAN -- DETERMINES E AND P, ACCORDING TO E.O.S. SELECTED.
C                       IF MEQ(K) = 1   -  TILLOTSON METALLIC E.O.S.
C                                 = 2   -  POLYNOMIAL E.O.S.
C                                 = 3 - OTHER

      EQUIVALENCE        (SPACE(7),QM)
C
      IF (MEQ(K) .LE. 0) MEQ(K)=1
      TEMP1 = ETA1-1.0
      IF (MEQ(K) .GT. 1) GO TO 500
C
C     TILLOTSON METALLIC EQUATION OF STATE
C   2-D LAGRANGIAN -- DETERMINES E AND P BY SOLVING A QUADRATIC. USES
C                       MOST POSITIVE VALUE.
C
      IND = 0
      SA1 = 0.0
      IF (DRR2 .EQ. 0.0) GO TO 120
      SA1 = 2.0/DRR2
  120 SA2 = -(SA1*E(N)+PM+ QM+Q1(N)+ SA1*DED)
      SB1 = SA(K)*RHO1(N)
      SB2 = SB(K)*RHO1(N)
      SB3 = 1.0/(EZERO(K)*ETA1**2)
      SB4 = TEMP1*(CAPA(K) + TEMP1*CAPB(K))
      TEMP2 = 1.0/ETA1 - 1.0
      TEMP3 = EXP(-ALPHA(K)*TEMP2**2)
      SC2 = SB(K)*RHO1(N)*TEMP3
      SC3 = CAPA(K)*TEMP1*TEMP3*EXP(-BETA(K)*TEMP2)
      IF (DRR2 .NE. 0.0) GO TO 230
      E1(N) = E(N)
      IF (TEMP1 .GE. 0.0) GO TO 210
      IF ((ETA1 .GE. RHOS(K)) .AND. (E1(N) .LE.EPRS(K))) GO TO 210
      SB2 = SC2
      SB4 = SC3
  210 P1 = E1(N)*(SB1+SB2/(SB3*E1(N) +1.0)) + SB4
      GO TO 460
  230 IF (SC2 .NE. 0.0) GO TO 270
      IF (SC3 .NE. 0.0) GO TO 270
      E1(N) = SA2/(SB1-SA1)
      GO TO 450
  270 IF (TEMP1 .LT. 0.0) GO TO 360
  320 BA = SB3*(SA1-SB1)
      BB = SA1-SB1+SB3*(SA2-SB4)-SB2
      BC = SA2-SB4
      GO TO 400
  360 IF (ETA1 .GE. RHOS(K)) IND=1
      BA = SB3*(SA1-SB1)
      BB = SA1-SB1+SB3*(SA2-SC3)-SC2
      BC = SA2-SC3
```

```
  400 CALL QUAD(BA,BB,BC)
      IF (BC .NE.(-999.)) GO TO 410
      WRITE (6,403) N
  403 FORMAT (1H0,32HERROR - ROOTS IMAGINARY. ZONE = ,I4)
      IEND = 5
      RETURN
  410 E1(N) = BA
      IF (IND .NE. 1) GO TO 450
      IND = 0
      IF (E1(N) .LE. EPRS(K)) GO TO 320
  450 P1 = SA1*E1(N)+SA2
      IF (E1(N) .LT.EEP    ) E1(N)=0.0
  460 IF (ABS(P1) .LT. PEP    ) P1=0.0
      GO TO 800
C
C     POLYNOMIAL EQUATION OF STATE
C
  500 CONTINUE
      CAPF = 1.0
      TEMP2 = FK0(K)*TEMP1*(1.0+TEMP1*(FK1(K)+TEMP1*(FK2(K)+TEMP1*(FK3(K
     1)))))
      TEMP3 = H0(K)*(1.0+TEMP1*(H1(K)+TEMP1*(H2(K)+TEMP1*(H3(K)))))
  700 E1(N) = (EM + (0.5*(TEMP2*CAPF+PM)+Q1(N))*DRR2 + DED)/(1.0-0.5*TEM
     1P3*CAPF*DRR2)
      P1 = (TEMP2 + TEMP3*E1(N))*CAPF
      IF (E1(N) .LT. EEP) E1(N)=0.0
      IF (ABS(P1) .LT. PEP) P1=0.0
C
  800 TEMP1 = -(P1+Q1(N))
      TXX1(N) = TEMP1+DTXX(N)+QXX1(N)
      TYY1(N) = TEMP1+DTYY(N)-(QXX1(N)+QZZ1(N))
      TZZ1(N) = TEMP1+DTZZ(N)+QZZ1(N)
      TXZ1(N) =       DTXZ(N)+QXZ1(N)
C
C     COMPUTE PRINCIPAL STRESSES, AND DIRECTION OF STRESSES.
C     LARGEST POSITIVE VALUE IS COMPARED WITH ULTIMATE TENSILE STRENGTH
C     (UTS) OF MATERIAL. IF THE STRESS EXCEEDS THE UTS, A FRACTURE WILL
C     OCCUR.
C
      TEMP1 = 0.5*(TXX1(N)+TZZ1(N)+SQRT((TXX1(N)-TZZ1(N))**2+(2.0*TXZ1(N
     1))**2))
      TM    = AMAX1(TEMP1,TYY1(N))
      IF (TM   .LT. UTS(K)) GO TO 950
      PHITM = 0.5*ATAN2(2.0*TXZ1(N),TXX1(N)-TZZ1(N))*57.2957795
      IEND = 6
      WRITE (6,920) N,K,TM  ,PHITM
  920 FORMAT (1H0,30HFRACTURE HAS OCCURRED AT MESH ,I4, 8H, PLATE ,I2, 1
     1H. / 11X,18HVALUE OF STRESS = ,E25.8, 8H D/SQCM. / 11X,21HDIRECTIO
     2N OF CRACK = ,F6.1, 5H DEG.  )
C
  950 RETURN
      END
```

```
$IBFTC QUAD2D  LIST
      SUBROUTINE QUAD (SA,SB,SC)
C
C  DETERMINES THE ROOTS OF A QUADRATIC EQUATION -  A*X**2 + B*X + C = 0
C     UPON RETURNING, A = THE MOST POSITIVE ROOT
C                     B = THE REMAINING ROOT
C
      DOUBLE PRECISION A,B,C,DISC,ALPHA,BETA
      A = SA
      B = SB
      C = SC
      DISC = B**2 - 4.0*A*C
      IF (DISC)  50, 70,100
   50 SC = -999.0
      RETURN
   70 SA = -B/(2.0*A)
      SB = SA
      RETURN
  100 IF (C) 103,101,103
  101 DISC = DABS(B)
      GO TO 104
  103 DISC = DSQRT(DISC)
  104 IF (B) 110,130,150
  110 ALPHA = (-B + DISC )/(2.0*A)
      GO TO 160
  130 ALPHA = DSQRT(-C/A)
      GO TO 160
  150 ALPHA = (-B - DISC )/(2.0*A)
  160 BETA  = C/(A*ALPHA)
      IF (ALPHA-BETA) 180,210,240
  180 SA = BETA
      SB = ALPHA
      RETURN
  210 SA = ALPHA
      SB = -SA
      RETURN
  240 SA = ALPHA
      SB = BETA
      RETURN
      END
```

```
$IBFTC COMPDT  LIST,REF
      SUBROUTINE COMPDT                                                 CDT  100
C
C  2-D LAGRANGIAN -- DETERMINES MINIMUM DT FROM DT OF EACH MESH

C
      CALL GETCN                                                        CDT  110
C
      TEMP1  = SQRT((Z10-Z13)**2 + (X10-X13)**2)                        CDT  120
      TEMP2  = SQRT((Z11-Z12)**2 + (X11-X12)**2)                        CDT  130
      TEMP3  = AMAX1(TEMP1,TEMP2)                                       CDT  140
      DL = A1(N)/TEMP3                                                  CDT  150
      IF (DL .LE. 0.0) WRITE (6,300) N,DL,A1(N)
  300 FORMAT (1H0,I4,2E16.8)
      IF (RDOTR .LE. 0.0) GO TO 190                                     CDT  160
      TEMP4  = 2.0*CZ*DL*RDOTR                                          CDT  170
      GO TO 200                                                         CDT  180
  190 TEMP4  = 0.0                                                      CDT  190
  200 TEMP5  = DL/SQRT(CN**2 + TEMP4**2)                                CDT  200
      DMIN  = AMIN1(DMIN,TEMP5)                                         CDT  210
      RETURN                                                            CDT  220
      END                                                               CDT  230
```

```
$IBFTC GETCN1  LIST,REF
      SUBROUTINE GETCN
C
C  2-D LAGRANGIAN -- DETERMINES SPEED OF SOUND IN MESH
C
C
      TEMP9 = 0.0                                                       GTCN 100
      IF (GO(K) .EQ. 0.0) GO TO 140                                     GTCN 110
      TEMP10 = 1.226                                                    GTCN 120
      GO TO 150                                                         GTCN 130
  140 TEMP10 = 1.0                                                      GTCN 140
  150 CN = CN0(K)                                                       GTCN 150
      IF (P1 .LE. 0.0) GO TO 180                                        GTCN 160
      IF (MEQ(K) .GT. 1) GO TO 170                                      GTCN 161
      IF (ETA1 .GE. RHOS(K)) GO TO 170                                  GTCN 162
      TEMP9 = SQRT(SA(K)*E1(N)*(1.0+SA(K)))                             GTCN 163
  170 CN = CN + (CNA(K)*(P1*1.0E-12)**CNEXP(K))*1.0E+5                  GTCN 170
      CN = AMAX1(CN,TEMP9)                                              GTCN 171
  180 CN = CN * TEMP10                                                  GTCN 180
      RETURN                                                            GTCN 190
      END                                                               GTCN 200
```

```
$IBFTC ENRGCK  LIST,REF
      SUBROUTINE ENRGCK
C
C  2-D LAGRANGIAN -- COMPUTES TOTAL ENERGY AND ENERGY DISTRIBUTION
C                   XP      = TOTAL RADIAL MOMENTUM FOR EACH CYCLE
C                   ZP      = TOTAL AXIAL MOMENTUM FOR EACH CYCLE
C                   ZK      = TOTAL AXIAL KINETIC ENERGY
C                   XK      = TOTAL RADIAL KINETIC ENERGY
C                   TIE     = TOTAL INTERNAL ENERGY
C                   TE      = TOTAL ENERGY FOR EACH CYCLE
C                   TDEE    = TOTAL SPHERICAL STRESS WORK
C                   TDEQ    = TOTAL SPHERICAL VISCOUS STRESS WORK
C                   TDEED   = TOTAL DEVIATORIC STRESS WORK
C                   TDEQD   = TOTAL DEVIATORIC VISCOUS STRESS WORK
C                   TDEPD   = TOTAL PLASTIC WORK
C                   TDEQPD  = TOTAL ENERGY
C
      EQUIVALENCE         (SPACE(7),QM)
C
      TEMP1  =  (6.2831853)**(IALF-1)*ZM(N)
      TEMP2  =  0.25*(UX10+UX11+UX12+UX13)
      TEMP3  =  0.25*(UZ10+UZ11+UZ12+UZ13)
      XP  =  XP  +  TEMP1*TEMP2
      ZP  =  ZP  +  TEMP1*TEMP3
      TEMP4  =     0.5*TEMP1*TEMP2**2
      TEMP5  =     0.5*TEMP1*TEMP3**2
      TEMP6  =       0.5*TEMP1*(E1(N)+EM)
      TE    =  TE  +  TEMP4+TEMP5+TEMP6
      XK  =  XK  +  TEMP4
      ZK  =  ZK  +  TEMP5
      TIE  =  TIE  +  TEMP6
      TEMP2  =  0.5*TEMP1*(P1+PM)*DRR2
      TEMP3  =  TEMP1*DRR2*0.5*(Q1(N)+QM)
      TEMP4  =  TEMP1*DEED
      TEMP5  =  TEMP1*DEQD
      TEMP7  =  0.0
      IF  (G  .EQ.  0.0)  GO  TO  300
      TEMP7  =-TEMP1*(CAPY(K)*(SQRT(0.66666667*BOX)-0.66666667*CAPY(K)))
     1/(G*(RHO1(N)+RHOM))
  300 TDEQPD  =  TDEQPD  +  TEMP2+TEMP3+TEMP4+TEMP5+TEMP7
      TDEE  =  TDEE  +  TEMP2
      TDEQ  =  TDEQ  +  TEMP3
      TDEED  =  TDEED  +  TEMP4
      TDEQD  =  TDEQD  +  TEMP5
      TDEPD  =  TDEPD  +  TEMP7
      RETURN
      END
```

```
$IBFTC LNK320  LIST,REF
C   2-D LAGRANGIAN -- LINK3 FOR IBM 7040 CHAIN
C
      DIMENSION           ZZ(15778)
C
      EQUIVALENCE         (ZZ(1),NCYCLE)
C
C     REWIND CHAIN TAPE (TAPE 0)
C
      CALL RWNDO
C
C     CHECK ERROR INDICATOR, IEND, AND WRITE MESSAGE -
C                  =1 - CONTINUE COMPUTING AFTER OUTPUT
C                  =2 - PLOT ONLY AND CONTINUE COMPUTING
C                  =3 - UPDATE RESTART TAPE AND OUTPUT
C                  =4 - ERROR STOP - STRESS VALUES EXCEED LIMIT
C                  =5 - NORMAL STOP AFTER OUTPUT
C                  =6 - ERROR STOP, PRINT ONLY.
C
      GO TO (200,210,130,110,130,200),IEND
  110 WRITE (6,120) N
  120 FORMAT (1H0,38HERROR - STRESS LIMIT EXCEEDED AT ZONE ,I4 )
      GO TO 200
C
C     UPDATE RESTART TAPE
C
  130 WRITE (6,140) NCYCLE
  140 FORMAT (1H0,30HRESTART TAPE UPDATED AT CYCLE ,I6)
      REWIND 4
      WRITE (4) ZZ
C
C     PRINT OUTPUT, PLOT OUTPUT
C
  200 CALL PRNT
  210 CALL GRAPH
C
      CALL CHNXIT
      END
```

```
$IBFTC PRNT10  LIST,REF
      SUBROUTINE PRNT
C
C  2-D LAGRANGIAN --
C
      EQUIVALENCE        (ISPACE(2),LSTN)
      EQUIVALENCE        (ISPACE(4),NSR)
C
      WRITE (6,120) (TITLE(L),L=1,12)
  120 FORMAT (1H1,29X,12A6/ )
      WRITE (6,140) NCYCLE,TIME,DELTA
  140 FORMAT (1H0,6H CYCLE, 5X, 8HTIME-SEC, 6X,11HDELTA T-SEC /1H ,I6,
     12E16.8/ )
      TEMP1 = TDEQ + TDEQD
      TEMP2 = XK + ZK
      WRITE (6,153) TIE,TEMP2,TE,XP,ZP,TEMP1,TDEPD
  153 FORMAT (1H0,35H  PER CYCLE ENERGIES - INTERNAL-ERG, 4X,11HKINETIC-
     1ERG, 6X, 9HTOTAL-ERG, 9X,20HRADIAL MOM.-G CM/SEC, 1X,19HAXIAL MOM.
     2-G CM/SEC  /1H ,20X,3E16.8, 4X,2E20.8 /
     *                  1H0,35HACCUMULATED ENERGIES - VISC.ST.-ERG, 4X,
     311HPLASTIC-ERG /1H ,20X,2E16.8 / )
      WRITE (6,170)
  170 FORMAT (1H , 5HPLATE, 1X, 1HZ, 4X, 1HX, 6X,13HSIGMA X-D/CSQ, 3X,13
     1HSIGMA Y-D/CSQ, 3X,13HSIGMA Z-D/CSQ, 3X,14HSIGMA XZ-D/CSQ, 5X, 7HQ
     2-D/CSQ, 9X, 8HRHO-G/CC, 5X,13HENERGY-ERG/GM     / 6X,3HST., 2X,3HS
     3T., 5X,13HX VEL.-CM/SEC, 3X,13HZ VEL.-CM/SEC, 8X, 4HX-CM,12X, 4HZ-
     4CM, 9X, 9HQ X-D/CSQ, 8X, 9HQ Z-D/CSQ, 5X,10HQ XZ-D/CSQ /)
      LCM = 53
      LC  = 14
      NS = 0
      N = 0
      DO 780 K=1,M
      KJ = 1
      JMX = JM(K)
      IMX = 0
      IDM = 0
      DO 771 J=1,JMX
      LL = 0
      IMM = IMX
      IMX = IM(K)
      IF (IMX .GT. 0) GO TO 300
      IMX = IM1(J)
      IDM = IMM - IMX
  300 NS = NS+1
      DO 470 I=1,IMX
      N = N + 1
      IF (LC .LE. LCM) GO TO 380
      WRITE (6,340)
  340 FORMAT (1H1)
      WRITE (6,170)
      LC = 1
      KJ = 1
  380 GO TO (390,430), KJ
```

```
390 WRITE (6,410) K,J,I,TXX1(N),TYY1(N),TZZ1(N),TXZ1(N),Q1(N),RHO1(N),
   1E1(N),UX1(N),UZ1(N),X1(N),Z1(N),QXX1(N),QZZ1(N),QXZ1(N)
410 FORMAT (1H0, 1X,I2,I5,I5, 3X, 7E16.8/17X,7E16.8)
    KJ = 2
    GO TO 460
430 WRITE (6,450)  J, I,TXX1(N),TYY1(N),TZZ1(N),TXZ1(N),Q1(N),RHO1(N),
   1E1(N),UX1(N),UZ1(N),X1(N),Z1(N),QXX1(N),QZZ1(N),QXZ1(N)
450 FORMAT (1H0, 3X,I5,I5, 3X, 7E16.8/17X, 7E16.8)
460 LC = LC + 3
470 CONTINUE
480 IF (LC .LE. LCM) GO TO 550
    WRITE (6,340)
    WRITE (6,170)
    LC = 1
    KJ = 1
550 GO TO (560,590), KJ
560 WRITE (6,570) K, J, I,UXS1(NS),UZS1(NS),XS1(NS),ZS1(NS)
570 FORMAT ( 1H0,  1X,I2,I5,I5,3X, 7E16.8 )
    KJ = 2
    GO TO 610
590 WRITE (6,600)    J, I,UXS1(NS),UZS1(NS),XS1(NS),ZS1(NS)
600 FORMAT (1H0, 3X,I5,I5, 3X, 7E16.8)
610 LC = LC + 2
    IF (IDM .LE. 0) GO TO 620
    IF (LL .GE. IDM) GO TO 620
    LL = LL+1
    NS = NS+1
    GO TO 480
620 IF (J .NE. JMX) GO TO 770
    J = J + 1
    DO 750 I=1,IMX
    IF (LC .LE. LCM) GO TO 700
    WRITE (6,340)
    WRITE (6,170)
    LC = 1
    KJ = 1
700 GO TO (710,730), KJ
710 WRITE (6,570) K,J,I,UXS1(NS),UZS1(NS),XS1(NS),ZS1(NS)
    KJ = 2
    GO TO 740
730 WRITE (6,600)  J, I,UXS1(NS),UZS1(NS),XS1(NS),ZS1(NS)
740 LC = LC + 2
750 CONTINUE
    NS = NS + 1
    GO TO 480
770 IF (N .GE. LSTN) GO TO 790
771 CONTINUE
780 CONTINUE
790 RETURN
    END
```

```
$IBFTC GRAPH1  LIST,REF
      SUBROUTINE GRAPH                                                  GRAF 100
C
C  2-D LAGRANGIAN --
C
      DIMENSION DATA(100)
C
      CALL PLOTS (DATA,100)
C
      IF (NPLOT .GT.  5) GO TO 300                                      GRAF 120
      IF (NPLOT .GT.  0) GO TO 150                                      GRAF 130
      CALL GRP1                                                         GRAF 140
      GO TO 300                                                         GRAF 141
  150 IPLOT = NPLOT                                                     GRAF 150
      IF (NPLOT .LT.  5) GO TO 180                                      GRAF 160
      IPLOT = 1                                                         GRAF 170
  180 GO TO (190,210,230,250),IPLOT                                     GRAF 180
  190 CALL GRP2 (TXX1, 8HSIGMA X )                                      GRAF 190
      GO TO 260                                                         GRAF 200
  210 CALL GRP2 (TYY1, 8HSIGMA Y )                                      GRAF 210
      GO TO 260                                                         GRAF 220
  230 CALL GRP2 (TZZ1, 8HSIGMA Z )                                      GRAF 230
      GO TO 260                                                         GRAF 240
  250 CALL GRP2 (TXZ1, 8HSIGMA XZ)                                      GRAF 250
  260 IF (NPLOT .LT. 5) GO TO 300                                       GRAF 260
      IF (IPLOT .GE. 4) GO TO 300                                       GRAF 270
      IPLOT = IPLOT + 1                                                 GRAF 280
      GO TO 180                                                         GRAF 290
  300 RETURN                                                            GRAF 300
      END                                                               GRAF 310
```

```
$IBFTC GRP101  LIST,REF
      SUBROUTINE GRP1
C
C  2-D LAGRANGIAN -- DRAWS PICTURE OF MESHES - X VS. Z
C
      EQUIVALENCE       (ISPACE(4),NSR)
      EQUIVALENCE       (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
      EQUIVALENCE       (TEMP1,XMN),     (TEMP2,ZMN)
C
      DIMENSION XT(2),ZT(2)
C
C     DETERMINE RANGE OF VALUES OF X AND Z
C
      XMIN = X1(1)
      XMAX = XMIN
      ZMIN = Z1(1)
      ZMAX = ZMIN
      DO 200 N=2,NMAX
      IF (X1(N) .LT. XMIN) XMIN = X1(N)
      IF (X1(N) .GT. XMAX) XMAX = X1(N)
      IF (Z1(N) .LT. ZMIN) ZMIN = Z1(N)
      IF (Z1(N) .GT. ZMAX) ZMAX = Z1(N)
  200 CONTINUE
      DO 260 NS=1,NSMAX
      IF (XS1(NS) .LT. XMIN) XMIN = XS1(NS)
      IF (XS1(NS) .GT. XMAX) XMAX = XS1(NS)
      IF (ZS1(NS) .LT. ZMIN) ZMIN = ZS1(NS)
      IF (ZS1(NS) .GT. ZMAX) ZMAX = ZS1(NS)
  260 CONTINUE
C
C     SCALE SO THAT Z DOES NOT EXCEED 10 IN.,DRAW AXIS, LABEL
C
      XT(1) = XMIN
```

```
      XT(2) = XMAX
      ZT(1) = ZMIN
      ZT(2) = ZMAX
      CALL SCALE (ZT,2,10.0,ZMN,DZZ, 1)
      ZLNGTH = AINT(ZT(2)+0.99)
      XMN    = AINT(XT(1)/DZZ)
      XLNGTH= AINT((XT(2)-XMN   )/DZZ+ 0.99)
      IF (XLNGTH .LE. 10.0) GO TO 340
      ZMN = ZMN*DZZ
      CALL SCALE (XT,2,10.0,XMN,DZZ,1)
      XLNGTH = AINT(XT(2)+0.99)
      ZLNGTH = AINT((ZMAX-ZMN)/DZZ+0.99)
      ZMN = ZMN/DZZ
  340 PLN = AMIN1(XLNGTH,2.0)
      CALL AX2  (0.0,0.0,13HRADIAL (X)-CM,-13,XLNGTH,  0.0,XMN,DZZ, PLN)
      PLN = AMIN1(ZLNGTH,2.0)
      CALL AX2  (0.0,0.0, 12HAXIAL (Z)-CM, 12,ZLNGTH, 90.0,ZMN,DZZ, PLN)
      CALL SYMBOL (0.5,10.0,0.14,TITLE,0.0, 72)
      CALL SYMBOL (0.5, 9.75,0.14,5HTIME=,0.0,5)
      T = TIME*1.0E+6
      CALL NUMBER (1.10,9.75,0.14,T     ,0.0, 6)
      CALL PMU (2.18, 9.75,0.14)
      CALL SYMBOL (2.30, 9.75, 0.14,4HSEC.,0.0, 4)
C
C     START PLOT OF X VS. Z
C
      NS = 0
      N = 0
      N1 = 0
      DO 900 K=1,M
      IF (IM(K) .GT. 0) GO TO 440
C
C     THIS PLATE IS A DEBRIS CLOUD.
C
      CALL GRP4
      GO TO 900
C
C     FIRST DRAW LINES OF CONSTANT  J
C
  440 JMX = JM(K)
      DO 580 J=1,JMX
      NS = NS + 1
      IMX = IM(K)
      IC = 3
      DO 540 I=1,IMX
      N = N + 1
      XPLT = (X1(N)-XMN)/DZZ
      YPLT = (Z1(N)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  540 CONTINUE
      XPLT = (XS1(NS)-XMN)/DZZ
      YPLT = (ZS1(NS)-ZMN)/DZZ
```

```
      CALL PLOT (XPLT,YPLT,IC)
  580 CONTINUE
C
C     NOW DRAW LINES OF CONSTANT  I
C
      K1 = K-1
      IF (K1 .LE. 0) GO TO 630
      IF (IM(K1) .GT. 0) GO TO 620
      N1 = NMAX1+N1
      GO TO 630
  620 N1 =  IM(K1)*JM(K1)+N1
  630 DO 760 I=1,IMX
      IC = 3
      NS = NS + 1
      DO 720 J=1,JMX
      N2 = N1 + (J-1)*IMX + I
      XPLT = (X1(N2)-XMN)/DZZ
      YPLT = (Z1(N2)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  720 CONTINUE
      XPLT = (XS1(NS)-XMN)/DZZ
      YPLT = (ZS1(NS)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
  760 CONTINUE
      IC = 3
      N2 = NS - IMX - JMX
      DO 850 J=1,JMX
      N2 = N2 + 1
      XPLT = (XS1(N2)-XMN)/DZZ
      YPLT = (ZS1(N2)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  850 CONTINUE
      NS = N2 + IMX + 1
      XPLT = (XS1(NS)-XMN)/DZZ
      YPLT = (ZS1(NS)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
  900 CONTINUE
      DO 1000 I=1,IMX
      NS = NS-1
      XPLT = (XS1(NS)-XMN)/DZZ
      YPLT = (ZS1(NS)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
 1000 CONTINUE
      CALL PLOT (XLNGTH+2.0, 0.0,-3)
      RETURN
      END
```

```
$IBFTC GRP2E2  LIST,REF
      SUBROUTINE GRP2 ( S1,LABL)
C
C 2-D LAGRANGIAN -- PLOTS SIGMA VS. X AND Z
C
      EQUIVALENCE       (ISPACE(5),IMXM),(ISPACE(6),NMAX1)
C
      DIMENSION          XT(2),ZT(2),ST(2),S1( 600),LABL(2)
C
      THETA = -30.0
      TH = THETA*0.017453294
      SNTH = SIN(TH)
      CSTH = COS(TH)
      GAMMA = +30.0
      GM = GAMMA*0.017453294
      SNGM = SIN(GM)
      CSGM = COS(GM)
      YOFF = 3.5
C
      NSV = 0
      NSSV = 0
      N1SV = 0
      NSTRT = 0
      NLAST = 0
      NSSTRT = 0
      NSLAST = 0
C
      DO 1000 K=1,M
C
      IF (K .LE. 1) GO TO 100
      THETA = -150.0
      TH = THETA*0.017453294
      SNTH = SIN(TH)
```

```
      CSTH = COS(TH)
      GAMMA = -30.0
      GM = GAMMA*0.017453294
      SNGM = SIN(GM)
      CSGM = COS(GM)
      YOFF = 5.5
C
  100 NSTRT = NLAST+1
      NSSTRT = NSLAST+1
      N1 = IM(K)*JM(K)
      N2 = IM(K)+JM(K)+1
      IF (N1 .GT. 0) GO TO 130
      N1 = NMAX1
      N2 = JM(K)+IMXM+1
  130 NLAST = NLAST+N1
      NSLAST = NSLAST+N2
C
C     DETERMINE RANGE OF VALUES OF X AND Z
C
      XMIN = X1(NSTRT)
      XMAX = XMIN
      ZMIN = Z1(NSTRT)
      ZMAX = ZMIN
      SMIN = 0.0
      SMAX = ABS(S1(NSTRT))
      DO 200 N=NSTRT,NLAST
      IF (X1(N) .LT. XMIN) XMIN = X1(N)
      IF (X1(N) .GT. XMAX) XMAX = X1(N)
      IF (Z1(N) .LT. ZMIN) ZMIN = Z1(N)
      IF (Z1(N) .GT. ZMAX) ZMAX = Z1(N)
      IF (ABS(S1(N)) .GT. SMAX) SMAX = ABS(S1(N))
  200 CONTINUE
      DO 260 NS=NSSTRT,NSLAST
      IF (XS1(NS) .LT. XMIN) XMIN = XS1(NS)
      IF (XS1(NS) .GT. XMAX) XMAX = XS1(NS)
      IF (ZS1(NS) .LT. ZMIN) ZMIN = ZS1(NS)
      IF (ZS1(NS) .GT. ZMAX) ZMAX = ZS1(NS)
  260 CONTINUE
      XT(1) = XMIN
      XT(2) = XMAX
      ZT(1) = ZMIN
      ZT(2) = ZMAX
      ST(1) = SMIN
      ST(2) = SMAX
C
C     SCALE SO THAT X DOES NOT EXCEED  5 IN.,DRAW AXIS, LABEL
C
      CALL SCALE (ST,2, 5.0,SMN,DSS,1)
      SLNGTH = AINT(ST(2)+0.99)
      CALL SCALE (XT,2, 5.0,XMN,DXX,1)
      XLNGTH = AINT(XT(2)+0.99)
      DZZ  = DXX
      IF (K .GE.2) DZZ  = DXX/10.0
```

```
      ZMN = AINT(ZT(1))
      ZLNGTH = AINT((ZT(2)-ZMN)/DZZ+0.99)
      IF (ZLNGTH .LE. 5.0) GO TO 380
      CALL SCALE (ZT,2, 5.0,ZMN,DZZ,1)
      ZLNGTH = AINT(ZT(2)+0.99)
C
  380 CALL PLOT ( 4.0,YOFF,-3)
      CALL AX2 (0.0,0.0,LABL(1), 12,SLNGTH, 90.0,SMN,DSS,-2.0)
      CALL SYMBOL (0.5,4.85,0.14,TITLE,0.0, 72)
      CALL SYMBOL (0.5, 4.60,0.14,5HTIME=,0.0,5)
      T = TIME*1.0E+6
      CALL NUMBER (1.10,4.60,0.14,T     ,0.0, 6)
      CALL PMU (2.18, 4.60,0.14)
      CALL SYMBOL (2.30, 4.60, 0.14,4HSEC.,0.0, 4)
      THR = 180.0+THETA
      CALL AX2 (XLNGTH*CSTH,XLNGTH*SNTH,1H , 1,-XLNGTH,THR   ,XLNGTH* DXX
     1,-DXX, XLNGTH)
      CALL SYMBOL ((XLNGTH+2.0)*CSTH,(XLNGTH+2.0)*SNTH,0.14,13HRADIAL (X
     1)-CM,THR   , 13)
      CALL AX2 ( 0.0, 0.0, 1H , 1,ZLNGTH,GAMMA,ZMN,DZZ,-ZLNGTH)
C
      ISWTCH = 2
  500 NS = NSSV
      N = NSV
      N1 = N1SV
C
C     IF ISWTCH = 1, PLOT AT CONSTANT Z
C     IF ISWTCH = 2, PLOT AT CONSTANT X
C
      GO TO (520,720),ISWTCH
  520 JMX = JM(K)
      DO 680 J=1,JMX
      NS = NS+1
      IC = 3
      IMXT = IM(K)
      IMX = IM(K)
      IF (IMX .GT. 0) GO TO 580
      IMX = IM1(J)
      IMXT = IMXM
  580 DO 640 I=1,IMX
      N = N+1
      TEMP1 = X1(N)-XMN
      TEMP2 = Z1(N)-ZMN
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT = -(S1(N)-SMN)/DSS + (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  640 CONTINUE
C
      IF (K .LE. M) GO TO 680
C
  650 TEMP1 = XS1(NS)-XMN
      TEMP2 = ZS1(NS)-ZMN
```

```
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT =                    (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
  680 CONTINUE
      IF (K .LT. M) GO TO 700
      IC = 3
      DO 690 I=1,IMX
      NS = NS+1
      TEMP1 = XS1(NS)-XMN
      TEMP2 = ZS1(NS)-ZMN
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT =                    (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  690 CONTINUE
      NS = NS+1
      GO TO 960
  700 NS = NS+IMXT+1
      GO TO 960
C
  720 N1 = N
      IMX = IM(K)
      IF (IMX .GT. 0) GO TO 740
      IMX = IMXM
  740 DO 850 I=1,IMX
      IC = 3
      JMX = JM(K)
      MXT = 0
      IF (NCRVS .EQ. 0) GO TO 780
      DO 773 L=1,NCRVS
      IF (I .EQ. JCRVS(L)) GO TO 780
  773 CONTINUE
      GO TO 850
  780 DO 840 J=1,JMX
      IMXT = (J-1)*IMX
      IF (IM(K) .GT. 0) GO TO 800
      IF (J .LE. 1) GO TO 790
      MXT = MXT+IM1(J-1)
  790 IMXT = MXT
  800 N = N1+IMXT+I
      IF ((I .LE. IM1(J)) .OR.  (IM(K) .NE. 0)) GO TO 820
      IC = 3
      GO TO 840
  820 TEMP1 = X1(N)-XMN
      TEMP2 = Z1(N)-ZMN
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT = -(S1(N)-SMN)/DSS + (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  840 CONTINUE
      IF (K .LT. M) GO TO 850
      L = JMX+I+NS
      TEMP1 = XS1(L)-XMN
```

```
      TEMP2 = ZS1(L)-ZMN
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT =                    (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
  850 CONTINUE
C
      IF (K .LE. M) GO TO 950
C
  860 IC = 3
      DO 930 J=1,JMX
      NS = NS+1
      TEMP1 = XS1(NS)-XMN
      TEMP2 = ZS1(NS)-ZMN
      XPLT = (TEMP2*CSGM)/DZZ + (TEMP1*CSTH)/DXX
      YPLT =                    (TEMP2*SNGM)/DZZ + (TEMP1*SNTH)/DXX
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  930 CONTINUE
      NS = NS+IMX+1
  950 CONTINUE
      ISWTCH = 1
      GO TO 500
C
  960 CALL SYMBOL((ZLNGTH+0.75)*CSGM,(ZLNGTH+0.75)*SNGM,0.14,12HAXIAL (Z
     1)-CM, GAMMA, 12)
      CALL PLOT (ZLNGTH+4.50,-YOFF,-3)
C
      NSSV = NS
      NSV = N
      N1SV = N1
C
 1000 CONTINUE
      RETURN
      END
```

```
$IBFTC GRP400  LIST,REF
      SUBROUTINE GRP4
C
C  2-D LAGRANGIAN -- DRAWS MESHES FOR A DEBRIS CLOUD
C
      EQUIVALENCE         (ISPACE(4),NSR)
      EQUIVALENCE         (TEMP1,XMN),      (TEMP2,ZMN)
C
      JMX = JM(K)
      IMX = 0
      IDP = 0
      DO 600 J=1,JMX
      IMM = IMX
      IMX = IM1(J)
      IDM = -IDP
      IF (J .LT. JMX) GO TO 150
      IMP = 0
      IDP = 0
      GO TO 200
  150 IMP = IM1(J+1)
      IDP = IMP-IMX
  200 NS = NS+1
      NSR = NS
      IF (IDM .GT. 0) NSR = NSR-IDM
      IC = 3
      DO 400 I=1,IMX
      N = N+1
      XPLT = (X1(N)-XMN)/DZZ
      YPLT = (Z1(N)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
      IF (I .LE. IMP) GO TO 320
      NS = NS+1
```

```
      XPLTW= (XS1(NS)-XMN)/DZZ
      YPLTW= (ZS1(NS)-ZMN)/DZZ
      GO TO 350
  320 NW = N+IMX
      XPLTW= (X1(NW)-XMN)/DZZ
      YPLTW= (Z1(NW)-ZMN)/DZZ
  350 CALL PLOT (XPLTW,YPLTW,IC)
      IC = 3
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
  400 CONTINUE
      XPLT = (XS1(NSR)-XMN)/DZZ
      YPLT = (ZS1(NSR)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
      IF (IDP .GT. 0) GO TO 440
      NW = NS+1
      XPLTW= (XS1(NW)-XMN)/DZZ
      YPLTW= (ZS1(NW)-ZMN)/DZZ
      GO TO 470
  440 NW = NW+1
      XPLTW = (X1(NW)-XMN)/DZZ
      YPLTW = (Z1(NW)-ZMN)/DZZ
  470 CALL PLOT (XPLTW,YPLTW,IC)
      IC = 3
      CALL PLOT (XPLT,YPLT,IC)
      IC = 2
      IF (IDM .LE. 0) GO TO 600
      NW = NSR
      DO 550 LL=1,IDM
      NW = NW+1
      XPLT = (XS1(NW)-XMN)/DZZ
      YPLT = (ZS1(NW)-ZMN)/DZZ
      CALL PLOT (XPLT,YPLT,IC)
  550 CONTINUE
C
  600 CONTINUE
C
      NS = NS+1
C
      RETURN
      END
```

```
$IBFTC PMU       LIST,REF
      SUBROUTINE PMU (X,Y,H)
C
      HU = H*0.715
      CALL SYMBOL (X,Y,HU,1HU, 0.0,1)
      RETURN
      END
```

```
$IBFTC AX2M    LIST
C      SUBROUTINE AX2 - MODIFIED AXIS TO ALLOW AXIS ANNOTATION AT OTHER  AX200010
C                        THAN 1.0 INCH INCREMENTS.  Z IS THE NUMBER OF   AX200020
C                        INCHES BETWEEN AXIS ANNOTATIONS.                AX200030
      SUBROUTINE AX2 (X,Y,BCD,NC,SIZE,THETA,YMIN,ZDY,Z )                 AX200040
      CALL OVERFL (KOV)                                                  AXS40031
      SIG  = 1.0                                                         AXIS2050
      IF(NC) 1,2,2                                                       AXIS2060
    1 SIG=-1.0                                                           AXIS2070
    2 NAC=IABS(NC)                                                       AXIS2080
      TH=THETA*0.017453294                                               AXIS2081
      ZSIG = Z                                                           AX2M 090
      Z = ABS(Z)                                                         AX2M 091
      SSIG = SIZE                                                        AX2M 092
      SIZE = ABS(SIZE)                                                   AX2M 093
      CTH = COS  (TH)                                                    AXIS2100
      STH = SIN  (TH)                                                    AXIS2110
      DY = Z *ZDY                                                        AX200120
      SIZE = SIZE + 0.0001                                               AX200121
      N  = SIZE / Z                                                      AX200130
      WN = N                                                             AX200140
      XB = X                                                             AXIS2143
      YB = Y                                                             AXIS2144
      XA = X - 0.1 * SIG  * STH                                          AXIS2150
      YA = Y + 0.1 * SIG  * CTH                                          AXIS2160
      CALL PLOT (XA,YA,3)                                                AXIS2170
      IC = 2                                                             AX2M 171
      IF (ZSIG .LT. 0.0) IC = 3                                          AX2M 172
      DO 20  I =1,N                                                      AXIS2180
      CALL PLOT (XB,YB,IC)                                               AX2M 190
      IC = 2                                                             AX2M 191
      XC = XB + CTH * Z                                                  AXIS2200
      YC = YB + STH * Z                                                  AXIS2210
      CALL PLOT (XC,YC,2)                                                AXIS2220
      XA = XA + CTH * Z                                                  AXIS2230
      YA = YA + STH *Z                                                   AXIS2240
      IF ((SSIG .LT. 0.0) .AND. (I .EQ. N)) GO TO 226                    AX2M2241
      CALL PLOT (XA,YA,2)                                                AXIS2250
  226 XB = XC                                                            AXIS2260
   20 YB = YC                                                            AXIS2270
      KEXP = 0                                                           AXS40271
      IF ( SIZE - ABS(WN * Z ))    28, 28,271                            AX200270
  271 CALL PLOT (XB,YB, 2)                                               AX200271
      XC = CTH * SIZE + X                                                AX200272
      YC = STH * SIZE + Y                                                AX200273
      CALL PLOT (XC,YC, 2)                                               AX100274
      XA = XC - 0.1 * SIG * STH                                          AX200275
      YA = YC + 0.1 * SIG * CTH                                          AX200276
      CALL PLOT (XA,YA, 2)                                               AX100277
   28 CHAR = ABS (YMIN)                                                  AXS40280
      ABSV = ABS (YMIN + WN * DY)                                        AXS40290
      CALL OVERFL (KOV)                                                  AXS40291
      GO TO (293, 13,295), KOV                                           AXS40292
```

```
293 KEXP = KEXP + 38                                                    AXS40293
    GO TO 296                                                           AXS40294
295 KEXP = KEXP - 38                                                    AXS40295
296 CHAR =  ABS (YMIN/ 10.0 ** KEXP)                                    AXS40296
    ABSV =  ABS (YMIN/(10.0 ** KEXP)+ (WN * DY) /(10.0 ** KEXP))        AXS40297
 13 IF (ABSV .GE. CHAR)  GO TO  32                                      AXS40300
    ABSV = CHAR                                                         AXS40310
 32 DO   35   J = 1,40                                                  AXS40320
    K = J - 1                                                           AXS40330
    IF (ABSV .LT. (10.0 ** K))  GO TO  39                               AXS40340
 35 CONTINUE                                                            AXS40350
 36 WRITE (6,37)                                                        AXS40360
 37 FORMAT (1H0,24HERROR IN SUBROUTINE AXIS)                            AXS40370
 39 IF (K)  402,391, 41                                                 AXS40390
391 DO   40   J = 1,40                                                  AXS40391
    K = -J                                                              AXS40392
    IF (ABSV .GT. (10.0 **   K ))  GO TO 402                            AXS40393
 40 CONTINUE                                                            AXS40400
    GO TO 36                                                            AXS40401
402 IEXP =  K + 1                                                       AXS40402
    KM = 5                                                              AXS40403
404 ABSV = ABSV /(10.0 ** (IEXP - 3))                                   AXS40404
    GO TO   422                                                         AXS40405
 41 IF (K .LT. 4)  GO TO  42                                            AXS40410
    KM = 4                                                              AXS40411
    IEXP = K - 1                                                        AXS40412
    GO TO  404                                                          AXS40413
 42 KM = 5 - K                                                          AXS40420
    IEXP = 0                                                            AXS40421
422 NSIG = 3                                                            AXS40422
423 NT    = ABSV                                                        AXS40423
    CHAR = NT                                                           AXS40430
    IF (ABSV .EQ. CHAR) GO TO  93                                       AXS40431
    NSIG = NSIG + 1                                                     AXS40432
    IF (NSIG .LT. KM) GO TO  423                                        AXS40433
    NSIG = KM                                                           AXS40434
 93 IEXP = IEXP + KEXP                                                  AXS40435
    ADY = DY * 10.0 ** (-IEXP)                                          AX200446
    ABSV= YMIN * 10.0 ** (-IEXP) + WN * ADY                             AX200450
    XA = XB - (.20 * SIG  - .05) *STH - .0857 * CTH                     AXIS2460
    YA = YB + (.20 * SIG  - .05) * CTH - .0857 * STH                    AXIS2470
    N = N + 1                                                           AXIS2480
    DO 30 I = 1,N                                                       AXIS2490
    IF ((ZSIG .LT. 0.0) .AND. (I .EQ. N)) GO TO 51                      AX2M 491
    IF (SSIG .LT. 0.0) GO TO 51                                         AX2M 492
    CALL NUMBER (XA,YA,0.1,ABSV,THETA,NSIG)                             AX200500
 51 ABSV = ABSV - ADY                                                   AXIS2510
    SSIG = ABS(SSIG)                                                    AX2M 511
    XA = XA - CTH * Z                                                   AXIS2520
 30 YA = YA - STH * Z                                                   AXIS2530
    TNC = NAC + 7                                                       AXIS2540
    XA = X + (SIZE / 2.0 -.06 * TNC)*CTH - (-.07 + SIG *.36)* STH       AXIS2550
    YA = Y + (SIZE / 2.0 -.06 * TNC)*STH + (-.07 + SIG *.36)* CTH       AXIS2560
```

```
      CALL SYMBOL (XA,YA,0.14,BCD,THETA,NAC)                            AXIS2570
      XA = XA +((TNC -6.0) * 0.12)* CTH                                 AXIS2580
      YA = YA +((TNC -6.0) * 0.12)* STH                                 AXIS2590
      IF(IEXP) 61,50,61                                                 AXS40600
   61 CALL SYMBOL (XA,YA,0.14,7H(X10   ) ,THETA,7)                      AXS40610
      XA = XA + .48 * CTH  -.07 * STH                                   AXS40620
      YA = YA + .48 * STH  +.07 * CTH                                   AXS40630
      IF(IEXP) 65,50,65                                                 AXS40640
   65 EXP = IEXP                                                        AXS40641
      CALL NUMBER (XA,YA,0.10,EXP,THETA,-1)                             AXS40650
   50 RETURN                                                            AXIS2640
      END
```

```
$IBMAP RWND2    LIST                                                  RWD2   10
*                              CALL RWND2                             RWD2   20
        ENTRY   RWND2                                                 RWD2   30
RWND2   SAVE    1,2,4                                                 RWD2   40
        TSX     S.IOOP,4       REWIND TAPE 2                          RWD2   50
        IOSKP   RWND             WITH NO END-OF-FILE MARK             RWD2   60
        PZE     S.SU02                                                RWD2   70
        RETURN  RWND2                                                 RWD2   80
RWND    PZE     0,,-1                                                 RWD2   90
        END
```

```
$IBMAP RWND0    LIST                                                  RWD0   10
*                              CALL RWND0                             RWD0   20
        ENTRY   RWND0                                                 RWD0   30
RWND0   SAVE    1,2,4                                                 RWD0   40
        TSX     S.IOOP,4       REWIND TAPE 0.                         RWD0   50
        IOSKP   RWND             WITH NO END-OF-FILE MARK             RWD0   60
        PZE     S.SU00                                                RWD0   70
        RETURN  RWND0                                                 RWD0   80
RWND    PZE     0,,-1                                                 RWD0   90
        END
```

```
$IBMAP FPTMOD  LIST,REF
       PMC     ON
DPOPT  SET     1                      NOW SET TO GIVE D P SIMULATION
*DPOPT SET     0                      USED TO REMOVE D P SIMULATION
*                                     ROUTINE FOR DP INSTRUCTIONS
*                                     EXECUTED ON SP MACHINES.
FPTLIM EQU     1000                   FLOATING POINT TRAP LIMIT
*
       ENTRY   SETFP.
       ENTRY   OVERF.
       ENTRY   FPT               CALL FPT(SILNCE,I)
       EXTERN  FEXEM.,EXIT,FIL06.,HNLIO.,FILIO.,SLOIO.,STHIO.,IOHEF.,
       ETC     IOHHC.,IOHOC.,IOHAC.,S.SLOC,S.PLOC,S.FBOU
*   *    *    *    *    *    *    *    *    *    *    *    *    *    *
*
* ENTRY FROM MAIN PROGRAM TO SET UP SPECIAL ENTRY FOR FLOATING PT. TRAPS.
SETFP. TRA     **
       CAL     FLPTRP            SET RETURN TO THIS ROUTINE IN
       SLW     8                     CASE OF FLT-PT TRAPS
       CAL     INTTIM           IGNORE INTERVAL TIMER RESET TRAP, CON-
       SLW     31                    TINUE WITH EXECUTION.
       TRA     SETFP.            RETURN TO CALLING PROGRAM
INTTIM TRA*    30
*   *    *    *    *    *    *    *    *    *    *    *    *    *    *
*
* ENTRY FROM FORTRAN PROGRAMS TO SET UP SPECIAL HANDLING OF FLT-PT TRAPS.
MINUS  STA     FPTLMM            SET NEW LIMIT
       MSM     LIM               SET LIMIT SW ON
XR4    AXT     **,4              RESTORE INDEX 4
FPT    TRA     **                CALL FPT (SILNCE,I)
*   IF SILNCE=.TRUE.= -,THEN DONT WRITE OUT MESSAGE WHEN UNDERFLOW OR
*   D.P. TRAPS OCCUR.
*   IF I IS + THEN EXECUTE I UPON LEAVING FLPTRP ON OVERFLOW, NO LIMIT
*   TO UNDERFLOWS OR D.P. TRAPS, CONTINUE PROGRAM EXECUTION.
*   IF I IS - THEN MAG(I) = FPTLIM
*   IF I IS 0 THEN TREAT OVERFLOWS THE SAME AS UNDERFLOW AND D.P. TRAPS
*   AND ALLOW AN INFINITE NUMBER OF TRAPS.
       SXA     XR4,4             SAVE INDEX 4
       LAC     FPT,4
       MSP     SILNCE
       PLT*    2,4               IF 1ST ARGUMENT IS TRUE, THEN
       MSM     SILNCE                DON-T WRITE MESSAGE
AR2    CLA*    3,4
       TMI     MINUS
       TZE     ZERO
       STA     IRT               I=+, SO SET OVERFL EXIT'
       MSP     IGNORE
NOLIM  MSP     LIM
       TRA     XR4               RETURN
ZERO   MSM     IGNORE
       TRA     NOLIM
*   *    *    *    *    *    *    *    *    *    *    *    *    *    *
*
```

```
* SPECIAL ENTRY FOR FLT-PT TRAPS OCCURRING IN FORTRAN PROGRAMS.
FLPTRP TRA    *+1
       SXA    FPTX,4              SAVE XR4
       STO    SAC                 SAVE AC
       STQ    SMQ                   AND MQ
       CLA    0                   SAVE CONTENTS OF TRAP CELL
       SLW    OVERF.                FOR OVERF. ROUTINE
       IFT    DPOPT=1
       TMI    DPFLPT                   DP INSTRUCTION ON SP MACHINE
       IFF    DPOPT=1
       TMI    SETAC                    IGNORE THIS TRAP
       ANA    =O000040000000      WAS THIS A DOUBLE PRECISION ERROR
       TZE    CAL                 NO
       CLA    D                   YES, WRITE COMMENT, TEST LIMIT, CONTINUE
SILNCE TXL    TESTLM,0,           IF SILNCE=.TRUE. DONT WRITE MESSAGE
       TSL    WRITE
       TRA    TESTLM
CAL    CAL    OVERF.
       ANA    =O000004000000      WAS TRAP CONDITION OVERFLOW
       TZE    UNFL                NO, MUST BE UNDERFLOW
       CLA    O                   YES, WRITE COMMENT
       PLT    IGNORE
       MIT    SILNCE
       TSL    WRITE
       MSM    OVERF.              SET OVERFLOW SW. ON
LIM    TXL    FPTLMM,0,           TXL A,0 OR TXH A,0 =BRN A =BRANCH TO A IF
*                                 THIS CELL IS NEGATIVE
       TRA    CAL1
UNFL   CLA    U                   WRITE UNDERFL COMMENT
       XEC    SILNCE
       TSL    WRITE
TESTLM MIT    LIM                 IS THERE A LIMIT
       TRA    CAL1                NO
FPTLMM AXT    FPTLIM,4            HAS TRAP LIMIT BEEN REACHED
       TNX    FPTOUT,4,1          YES, THEN RESTORE TRAPS, CALL FEXEM.
       SXA    *-2,4               NO, REDUCE COUNTER
CAL1   CAL    OVERF.
       ANA    =O000002000000      WAS CONDITION IN AC
       TZE    MQDB                NO, ONLY IN MQ OR DOUBLE PRECISION
       MSP    ACMQSW              INITIALIZE BYPASS SW
       CAL    OVERF.
       ANA    =O000001000000      WAS CONDITION IN BOTH AC AND MQ
       TNZ    *+2                 YES
       MSM    ACMQSW              NO, ONLY IN AC, SET BYPASS SW ON
       ZAC                        UNDERFL, MAKE AC ALL ZEROS
       PLT    OVERF.
       COM                        OVERFLOW, MAKE AC ALL ONES
       PLT    SAC                 RESTORE AC SIGN
       CHS
       STO    SAC                 SAVE AC SETTING
ACMQSW BRN    SETAC               TRA IF CONDITION ONLY IN AC
SETMQ  LDQ    SMQ
       LLS    0                   RESTORE MQ SIGN
```

```
        STO     SMQ               SAVE MQ SETTING
SETAC   CLA     SAC               RESTORE AC
        LDQ     SMQ                AND MQ SETTINGS
FPTX    AXT     **,4              RESTORE XR4,
        MIT     LIM
        MIT     OVERF.
        TRT*    OVERF.            RETURN TO TRAPPED LOCATION +1
IGNORE  BRN     *-1               IGNORE PRESET EXIT
TRT     TRT     **                GO TO PRESET EXIT SET BY CALL TO FPT
*
MODB    CAL     OVERF.
        ANA     =000004000000O    WAS THIS A DOUBLE PRECISION ERROR
        TNZ     SETAC
        PLT     OVERF.            AC =0 NOW
        COM                       OVERFLOW, MAKE AC ALL ONES
        TRA     SETMQ
FPTOUT  TRT     *+1
        TSX     S.PLOC,4          PURGE OUTPUT BUFFER
        PZE     S.FBOU,,149
        TSL     FFXEM.            ERROR TRAP LIMIT REACHED
        PZE     EXIT,,31          NO OPTIONAL RETURN
*
SAC     PZE
SMQ     PZE
OVERF.  PZE                       INDICATORS FROM LAST TRAP
D       PZE     DOUB,,2
O       PZE     OV,,2
U       PZE     UN,,2
DOUB    BCI     2, D.P. ADRESS
OV      BCI     2,   OVERFLOW
UN      BCI     2,   UNDERFLOW
*
X2      AXT     **,2              RESTORE XR2
WRITE   TRA     **
        SXA     X2,2              SAVE XR2
        STO     TYPE
        LXA     S.SLOC,4          FIND NAME OF ROUTINE IN
        TXL     M1,4,0             WHICH TRAP OCCURED, NAME
        CLA     0,4                OF CALLING ROUTINE AND
        PAC     ,2                 CALLING STATEMENT
        CLA     -6,2
        STO     CULPRT
        CLA     2,4
        STO     CALLS
        CLA     -3,2
        PAX     ,4
        TXL     M2,4,0
        CLA     0,4
        PAC     ,2
        CLA     -6,2
        STO     CALLR
        TRA     STH
M1      STZ     CULPRT            S.SLOC=0 WHEN CONTROL IS IN
```

```
        STZ     CALLS                MAIN PROGRAM
M2      STZ     CALLR
*   WRITE (6,FMT) TYPE,CULPRT,CALLS,CALLR,CELL 0
STH     TSX     STHIO.,4
        PZE     FIL06.
        PZE     FMT
        TSX     SLOIO.,4
TYPE    PZE     **,,**
        CLA     0
        TSL     HNLIO.
        TSX     FILIO.,4
        TRA     X2
*
FMT     AXT     2,2                 WRITE TYPE
        TSX     IOHAC.,4
        PZE     6
        TSX     IOHHC.,4
        PZE     63
        BCI     2,,     TRAP IN
CULPRT  PZE
        BCI     2,, CALLED BY
CALLS   PZE
        BCI     2, IN ROUTINE
CALLR   PZE
        BCI     2,, CELL 0=
        TSX     IOHOC.,4
        PZE     12
        TRA     IOHEF.
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *
*
*
        TTL     SIMULATE D P HARDWARE
*               SET DPOPT TO 1 IN ORDER TO USE THE ROUTINE
DPTRAN  MACRO
DPFLPT  CLA     OVERF.
        STO     THEND               SAVE RETURN ADDRESS
        SUB     =1
        STA     DPINS
        CLA     SAC                 PROTECT ARGUMENTS FROM POSSIBLE
        STO     AC                  FLOAT PT TRAP WHILE SIMULATING
        CLA     SMQ
        STO     MQ
        SXA     THER,4              SACE INDEX IN CASE OF FPT
DPINS   CAL     **                  PICKUP DP INSTRUCTION
        ANA     =O000060000000
        SUB     =O000060000000
        MSM     INDRK               INITIALIZE SWITCH
        TNZ     *+2
        MSP     INDRK               SET INDIRECT ADDRESS SWITCH ON
        XEC     DPINS
        ANA     =O000060777777
        ORA     CLA
        STO     *+1
```

```
ADD1     CLA      **                  CLA Y / CLA Y,T / CLA* Y / CLA* Y,T
         STO      ARG1                STASH Y
INDRK    BRA      ADD2                BRANCH IF ADDRESS IS DIRECT
         XEC      DPINS
         ANA      =0000000777777
         ORA      CLA
         STO      *+1
         CLA      **                  CLA Y / CLA Y,T
         ANA      =0000000777777
         ORA      CLA                 CLA* Y / CLA* Y,T
         TRA      *+2
ADD2     CLA      ADD1
         ADD      =1
         STO      *+1
         CLA      **                  CLA Y+1 / CLA Y+1,T
         STO      ARG2                STASH Y+1
         XEC      DPINS
         ANA      =0777700000000      GET OP CODE
         LAS      DFDP
         TRA      SETAC
         TRA      DPDVP               -0241 (DP DIVIDE)
         LAS      DFSB
         TRA      SETAC
         TRA      DPSUB               +0303 (DP SUBTRACT)
         LAS      DFAD
         TRA      SETAC
         TRA      DPADD               +0301 (DP ADD)
         TRA      DPMPY               +0234 (DP MULTIPLY)
*        DFSB     SUBTRACT            SAME AS DFAD WITH SIGN CHANGE
*
DPSUB    CLA      ARG1
         CHS
         STO      ARG1
         CLA      ARG2
         CHS
         STO      ARG2
*        DFAD     ADD                 (A+B)+(C+D)=B+D+(A+C)LOW+A+C HIGH
DPADD    CLA      AC                  FORM A+C HIGH AND LOW
         FAD      ARG1
         STO      TEMP                A+C HIGH
         STQ      SMCHAR              A+C LOW
         CLA      MQ                  FORM B+C
         FAD      ARG2                LOSE MQ LOW ORDER
         FAD      SMCHAR              A+C LOW
         FAD      TEMP                A+C HIGH ANSWER NOW IN AC MQ
         TRA      THER                EXIT WITH ANSWER
*DOUBLE PRECISION MULTIPLY            (A + B) * (C + D) = A*C + B*C +A*D
DPMPY    LDQ      AC                  A*C
         FMP      ARG1
         STO      MARG1
         STQ      MARG2
         LDQ      MQ                  B*C
         FMP      ARG1
```

```
         STO      MQ                 B*C HIGH ORDER B*C NOT SIGNIFICANT
         LDQ      AC                 A*D
         FMP      ARG2
         FAD      MQ                 ADD IN B*C HIGH
         FAD      MARG2              ADD IN A*C LOW
         FAD      MARG1              ADD IN A*C HIGH
         TRA      THER               RETURN
*DOUBLE PRECISION DIVIDE             (A + B) / (C + D) = Q1 + Q2
DPDVP    CLA      AC                 BEGIN DIVIDE SIMULATION
         FDP      ARG1               (A + B) / C = Q1 + R1
         STO      Q1
         STO      R1
         FMP      ARG2               Q1 * D
         CHS
         FAD      R1                 R1 - Q1 * D
         FAD      MQ                 ADD IN B TO BE DIVIDED
         FDP      ARG1
         STQ      TEMP               (R1 - Q1 * D) / C = Q2 + R2
         CLA      Q1
         FAD      TEMP
THER     AXT      **,4               RESTORE INDEX ON EXIT
         TRT*     THEND
*
AC       PZE      0
MQ       PZE      0
THEND    PZE      0
CLA      CLA      **
ARG1     PZE      0                  HIGH-ORDER OPERATOR
ARG2     PZE      0                  LOW-ORDER OPERATOR
DFDP     DFDP     **
DFSB     DFSB     **
DFAD     DFAD     **
SMCHAR   PZE      0                  SMALLER CHARACTERISTIC
TEMP     PZE      0                  TEMPORARY STORAGE
MARG1    EQU      SMCHAR
MARG2    EQU      TEMP
Q1       EQU      SMCHAR
R1       EQU      TEMP
         ENDM     DPTRAN
         IFT      DPOPT=1
         DPTRAN
         END
```

The term (ρA) appearing in these equations is required at 0, where ρ and A are quantities averaged over the meshes 1, 2, 3 and 4. This term is well represented by the expression

$$(A\rho)_o = A_{I1}\rho_1 + A_{I2}\rho_2 + A_{I3}\rho_3 + A_{I4}\rho_4$$

where A_{I1} is the area of triangle AOD, A_{I2} of AOB, A_{I3} of BOC, and A_{I4} of COD.

The last term on the right of the momentum equation is written

$$\left(\frac{t^{xx} - t^{yy}}{\rho x}\right)_o = \frac{1}{4}\left\{\frac{\left[(t^{xx})_1 - (t^{yy})_1\right]}{\rho_1 \bar{x}_{I1}} + \frac{\left[(t^{xx})_2 - (t^{yy})_2\right]}{\rho_2 \bar{x}_{I2}} + \frac{\left[(t^{xx})_3 - (t^{yy})_3\right]}{\rho_3 \bar{x}_{I3}} + \frac{\left[(t^{xx})_4 - (t^{yy})_4\right]}{\rho_4 \bar{x}_{I4}}\right\}$$

where

$$\bar{x}_{I1} = \frac{1}{3}(x_O + x_D + x_A)$$

$$\bar{x}_{I2} = \frac{1}{3}(x_O + x_A + x_B)$$

$$\bar{x}_{I3} = \frac{1}{3}(x_O + x_B + x_C)$$

$$\bar{x}_{I4} = \frac{1}{3}(x_O + x_C + x_D)$$

Similarly for the second momentum equation

$$\left(\frac{1}{\rho}\frac{\partial t^{xz}}{\partial x}\right)_o = \frac{-1}{(\rho A)_o}\left\{(t^{xz})_1(z_D - z_A) + (t^{xz})_2(z_A - z_B) + (t^{xz})_3(z_B - z_C) + (t^{xz})_4(z_C - z_D)\right\}$$

$$\left(\frac{1}{\rho}\frac{\partial t^{zz}}{\partial Z}\right)_o = \frac{1}{(\rho A)_o}\left\{(t^{zz})_1(x_D - x_A) + (t^{zz})_2(x_A - x_B) + (t^{zz})_3(x_B - x_C) + (t^{zz})_4(x_C - x_D)\right\}$$

and

$$\left(\frac{t^{zz}}{\rho x}\right)_o = \frac{1}{4}\left\{\frac{(t^{xz})_1}{\rho_1 \bar{x}_{I1}} + \frac{(t^{xz})_3}{\rho_2 \bar{x}_{I2}} + \frac{(t^{xz})_3}{\rho_3 \bar{x}_{I3}} + \frac{(t^{xz})_4}{\rho_4 \bar{x}_{I4}}\right\}$$

Substituting these finite difference quantities in the momentum equation enables us to evaluate the accelerations a^x and a^z:

$$(a^x)^n_o = -\frac{(t^{xx})^n_1 (z^n_D - z^n_A) + (t^{xx})^n_2 (z^n_A - z^n_B) + (t^{xx})^n_3 (z^n_B - z^n_C) + (t^{xx})^n_4 (z^n_C - z^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(t^{xz})^n_1 (x^n_D - x^n_A) + (t^{xz})^n_2 (x^n_A - x^n_B) + (t^{xz})^n_3 (x^n_B - x^n_C) + (t^{xz})^n_4 (x^n_C - x^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(\alpha-1)}{4}\left\{\frac{\left[(t^{xx})^n_1 - (t^{yy})^n_1\right]}{\rho^n_1 \bar{x}^n_{I1}} + \frac{\left[(t^{xx})^n_2 - (t^{yy})^n_2\right]}{\rho^n_2 \bar{x}^n_{I2}} + \frac{\left[(t^{xx})^n_3 - (t^{yy})^n_3\right]}{\rho^n_3 \bar{x}^n_{I3}}\right.$$

$$\left. + \frac{\left[(t^{xx})^n_4 - (t^{yy})^n_4\right]}{\rho^n_4 \bar{x}^n_{I4}}\right\}$$

$$(a^z)^n_o = -\frac{(t^{xz})^n_1 (z^n_D - z^n_A) + (t^{xz})^n_2 (z^n_A - z^n_B) + (t^{xz})^n_3 (z^n_B - z^n_C) + (t^{xz})^n_4 (z^n_C - z^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(t^{zz})^n_1 (x^n_D - x^n_A) + (t^{zz})^n_2 (x^n_A - x^n_B) + (t^{zz})^n_3 (x^n_B - x^n_C) + (t^{zz})^n_4 (x^n_C - x^n_D)}{A^n_{I1}\rho^n_1 + A^n_{I2}\rho^n_2 + A^n_{I3}\rho^n_3 + A^n_{I4}\rho^n_4}$$

$$+ \frac{(\alpha-1)}{4}\left\{\frac{(t^{xz})^n_1}{\rho^n_1 \bar{x}^n_{I1}} + \frac{(t^{xz})^n_2}{\rho^n_2 \bar{x}^n_{I2}} + \frac{(t^{xz})^n_3}{\rho^n_3 \bar{x}^n_{I3}} + \frac{(t^{xz})^n_4}{\rho^n_4 \bar{x}^n_{I4}}\right\}$$