Space Sciences Laboratory
University of California
Berkeley, California 94720

GAIN MEASUREMENTS OF STANDARD ELECTROMAGNETIC HORNS IN THE K AND K_a BANDS

by

Gerard Wrixon

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Errata:

TABLE II, Note No. 2, should read:

2. $2a^2/\lambda$ for this frequency = 138.60 cms.

TABLE III, Theoretical Gain Value (Col. 6) at a frequency of 26 GHz should read:

27.080

ABSTRACT

Measurements are described in which the gain of the standard horns in the K and K_a bands were determined at a number of frequencies. The two-antenna method was used. It is believed, after a detailed error analysis, that the gain is known to well within ± 0.1 db at each frequency measured. The gain of the K-band horn was also determined at one frequency by numerically integrating the power pattern. This resulted in a gain value which confirmed that obtained using the two antenna method.

I. INTRODUCTION

At frequencies above 1 GHz pyramidal horn antennas are the most commonly used standard antennas. In addition to the fact that their gains can be easily and fairly accurately calculated, they are widely used because they are rugged, relatively easy to construct, and have wide bandwidth properties. It has been well established[1,2] that the measured gains of these horns agree with the calculated gains to within ± 0.3 db. The main function of standard antennas is their use in relative gain measurements to calibrate other antennas, and for most antenna calibrations the above degree of accuracy is sufficient. However, advancements in radio astronomy, communications, and radar systems have created important demands for improved accuracy. To calibrate antennas for state-of-the-art field strength measurements, standard antennas are needed that have gain values known to within ± 0.1 db. Since this requirement exceeds the verified accuracy of calculated gain values, these standards must be established by making absolute gain measurements.

Recent literature on the subject of absolute gain measurement[3] has tended to be rather cautious in accepting the results of past measurements. This is because of some important experimental details that previously had been overlooked or inadequately discussed. It is concluded that the previous error estimates of less than ± 0.1 db for horn gain measurements have been somewhat optimistic.

In this report measurements of the gains of standard horns in the K (18 GHz — 28 GHz) and K_a (28 GHz — 38 GHz) band are described. The gain of the K-band horn was measured at three different frequencies (18, 22, and 26 GHz) and that of the K_a-band horn at four different frequencies (28, 32, 36, and 38 GHz). It is believed, after having taken all possible known error sources into account, that the gain at each frequency measured is known to well within ± 0.1 db. The two-antenna method [IEEE Standard, no. 149, 1965] was used in these measurements, two identical horns being used in the K_a band and two almost identical horns* in the K band. The immediate motivation for these measurements was to be able to use the standard horns to calbrate, by means of a relative gain measurement, the 20-foot millimeter wave, parabolic radio telescope at the Hat Creek Radio Astronomy Observatory of the University of California. The measurements were carried out during the months of May and June, 1967.

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*See Appendix.

II. DESCRIPTION OF MEASUREMENT TECHNIQUE

The measurement technique used was exactly the same for the K- and K_a-band horns; thus the following descriptions will apply to both. Suppose two identical horn antennas are set up as shown in *Fig. 1*, at a distance R apart and directly facing each other with the centers of their apertures on a straight line perpendicular to the aperture planes. Let P_T be the power delivered to the transmitting horn and P_R be that received at the perfectly matched load of the receiving horn. The gain G can be defined as the ratio of the peak power radiated per unit solid angle to the power per unit solid angle from a hypothetical isotropic radiator radiating the same total power as the horn under consideration. An isotropic antenna of this sort would have, therefore, a power per unit solid angle of $P_T/4\ \pi$. Hence, the peak power radiated per unit solid angle for the horn is $P_T\ G/4\ \pi$. If A_f is the effective area of the receiving horn, the solid angle subtended at the transmitter by that area is A_f/R^2; then the power P_R received by the receiving horn is:

$$P_R = \frac{P_T\ G\ A_f}{4\ \pi\ R^2}$$

However, since the horns are identical, A_f is related to the transmitting gain function of the receiving horn by

$$A_f = \frac{G\ \lambda^2}{4\ \pi}$$

Therefore

$$P_R = \frac{P_T G^2 \lambda^2}{(4 \pi)^2 R^2}$$

and[4]

$$G = \frac{4 \pi R}{\lambda} \sqrt{\frac{P_R}{P_T}}$$

or

$$G \text{ (db)} = 10 \text{ Log}_{10} \frac{4 \pi R}{\lambda} + 1/2 \left(10 \text{ Log}_{10} \frac{P_R}{P_T} \right) \qquad (1)$$

Beatty,[5] taking into account the fact that mismatches can occur in a measurement such as described above proposes the following expression for the gain

$$G \text{ (db)} = 10 \text{ Log}_{10} \frac{4\pi R}{\lambda} + 10 \text{ Log}_{10} \left(\frac{1}{1 - |\Gamma_A|^2} \right) - \frac{\alpha \ell}{2} + 1/2 \left(10 \text{ Log}_{10} \frac{P_R}{P_T} \right) \qquad (2)$$

Where Γ_A is the reflection coefficient of either horn radiating into free space and, α_ℓ is the attenuation of the section of waveguide used to connect the source and the transmitting horn.

Γ_A was measured for each horn and was found to be less than 0.01 [—40 db]. From curves in Beatty's article it is seen that a reflection coefficient of such magnitude contributes an error of less than 0.001 db. This can be neglected and thus, effectively,

$$\Gamma_A = 0 \qquad (3)$$

It will be seen below that, in the experimental setup that was employed,

$$\alpha_\ell = 0 \qquad (4)$$

Introducing (3) and (4), Equation (2) reduces to Equation (1) provided other assumptions that Beatty made in order for (2) to hold are also valid. These are:

(a) The reflection coefficient (Γ_L) at the load of the receiving antenna is zero.

(b) The reflection coefficient (Γ_G) at the junction of the source and the transmitting antenna is zero.

(c) The antennas are separated far enough so that interaction and near-field effects can be safely neglected.

It will be seen below that while Γ_L and Γ_G were not precisely zero, their approximate values were known and from the curves given by Beatty their contribution to the error in the final gain value could be found. In no case however did this contribution exceed 0.008 db. As regards antenna separation, at all frequencies the horn apertures were kept apart a distance greater than or equal to four times the Rayleigh distance, i.e.

$$R \geq \frac{2\,a^2}{\lambda}$$

where a is the largest aperture dimension. Even with this separation however a non-negligible proximity error occurs for which correction must be made. This is discussed in the next section.

The experimental setup is shown schematically in *Fig. II.* The measurements were done in the anachoic room of the Electrical Engineering Department at Berkeley; this room is normally used for microwave antenna pattern measurements and the walls have a reflection coefficient, at normal incidence, of less than -20 db across the band.

The horns were mounted on lens holders on a two-meter-long optical bench. That portion of the optical bench between the apertures, and also the table on which the optical bench was placed, was covered with absorbent material, as were any objects, instruments, etc. that were placed in such a way as to have possibly caused reflections. The horns were aligned, at each frequency, at zero separation so that (a) the apertures were flush with each other, (b) the aperture plane was perpendicular to the optical bench, (c) the midpoint of the apertures was directly over the optical bench, as was the midpoint of the end of each horn, and (d) the sides of each horn were perpendicular to the table. The antennas were then fastened securely to the lens holders, which could slide along the optical bench. They were then separated to the distance at which measurements were taken, and points (b) through (d) above were again checked. At each frequency, measurements were taken at ten *randomly spaced* separations (all greater than $2\,a^2/\lambda$), and at each new separation points (b) through (d) above were checked again to make certain that the antennas were still aligned properly. During all the measurements, the horn apertures were vertically polarized.

In this experiment the parameters are λ (the wavelength of the radiation), R (the aperture separation of the antennas), P_T (the power into the transmitting antenna, i.e. the power crossing reference-plane $A' - A'$ in *Fig. II)* and P_R (the power from the receiving antenna, i.e. the power crossing the reference plane $A - A$. λ was measured using the cavity wavemeter, and R was measured using a meter stick. The power sensor was a bolometer, the reflections from which were minimized by incorporating with it an E-H tuner. This was tuned at each frequency, and a reflection coefficient of less than $-$ 33 db was obtained for this matched bolometer. P_R could be measured directly by connecting the matched bolometer to the receiving antenna, however it was inconvenient to interrupt the circuit to measure P_T directly. Using a 20-db directional coupler, a "known fraction" of P_T was tapped off and measured at reference plane $B - B$. This "known-fraction" was determined by accurately calibrating the directional coupler at each frequency. This was done using exactly the same technique as the measurement now being described except that instead of measuring the power at reference planes $A - A$ and $A' - A'$, it was measured at reference planes $A' - A'$ and $B - B$. Ten measurements were taken of the coupling at each frequency, and the average coupling and the standard deviation of the readings for each frequency are shown in Table 1.

From Equation (1) it is seen that to calculate the gain the absolute values of P_R and P_T are not needed; only their db difference is required. This was obtained by means of an audio-substitution measurement. With reference to *Fig. II*, the anode of the Backward Wave Oscillator was modulated with a 1000 Hz sq wave signal. A portion of the modulated r.f. signal was tapped off at the 20-db cross-guide coupler, detected by the bolometer, amplified by the 1000-Hz low-noise amplifier, and fed into the Weinschel Differential Null Detector. This was the reference signal. The matched bolometer was now attached to $A - A$, and a matched load was placed at $B - B$. Using the Attenuator Calibrator the detected signal from $A - A$ was adjusted until a null was obtained with the reference signal in the Differential Null Detector. The value of attenuation shown on the Attenuator Calibrator was noted. Now the matched bolometer and load were exchanged between $A - A$ and $B - B$. The Attenuator Calibrator was adjusted again for a null with the reference signal, which hopefully had not changed in the meantime, and the difference in attenuation as read on the Attenuator Calibrator was the difference in power level from $A - A$ to $B - B$. This procedure was repeated three times at each value of antenna separation, R, and the average taken to be the correct db power difference for the distance involved. To get the difference in power level from $A - A$ to $A' - A'$, which is of course the difference between P_R and P_T, the average coupling for the frequency being used (in db) was just subtracted* from the difference in power level from $A - A$ to $B - B$ (also in db). A typical set of measurements for one frequency are shown in Table II.

From the above description it is seen that any non-zero Γ_G and Γ_L will be due to reflections from the matched bolometer and the matched load. It has been mentioned already that the bolometer could be matched

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*See Note 4 in Table II.

for a reflection coefficient of less than —33 db. From Beatty's article this is seen to lead to an error of less than 0.003 db in the gain. The reflection coefficient of the matched load was measured at each frequency and was found to be less than —30 db; again, from Beatty, this leads to an error in gain of less than 0.005 db. Thus the combined effect was less than 0.008 db.

III. GAIN CALCULATION

Having calculated the the ratio (in db) of P_R/P_T, it can then be substituted into Equation (1) together with the λ and R involved, and the gain can be calculated. An example of the gain calculations for one particular frequency (36 GHz) is shown in Table II. Note that at each frequency ten values of the gain are calculated corresponding to the ten randomly selected values of antenna separation.

Because of the fact that a separation of $2\ a^2/\lambda$ is not sufficient to position the antennas in each other's far fields, it has been recognized that a proximity correction needs to be added to the measured gain values. Chu and Semplak[1] have computed tables of this correction by determining the ratio of the near field to the far field gain. These corrections, which are functions of the horn dimensions as well as the horn separation, were determined for each value of horn separation. They are found in Column 5 of Table II for the frequency 36 GHz, and as can be seen are of substantial magnitude. The corrected gain in Column 6 consists of the measured gain values of Column 4 plus the proximity corrections. The finally adopted gain value was then obtained by averaging the values in Column 6. An exactly similar procedure was followed for all other frequencies. The standard deviation of the corrected gain values, σ_G, was also calculated for each frequency, and its relevance will be dealt with in the next section.

Table III gives the finally adopted measured gain values for the *K*-band horns, and Table IV gives similar data for the K_a band. These tables also give theoretical values for the gain calculated from curves in Schelkunoff and Friis,[6] as well as limiting errors on the measured gain values. These limiting errors are the subject of the next section.

Figs. III and *IV* show the measured and theoretical gains of the horns plotted as a function of frequency for the *K*- and K_a-band horns respectively.

Table V lists, for each frequency, the value of $2\ a^2/\lambda$, the spread of horn separations used in the measurements, the *average* value of the proximity corrections, and the standard deviation in the corrected gain values.

IV. ERROR ANALYSIS

Bowman,[3] in a thorough examination of the errors involved in a measurement of this kind has pointed out the importance of taking into account multipath scattering and interhorn scattering. This scattering results in a periodic variation of the received power with horn separation. Rather than attempt to calculate the magnitude of this variation from the geometrical and physical properties of the experimental setup, it was felt that designing the measurement so that it would be possible to observe this variation would be simpler and more reliable. It is for this reason that, at each frequency, the gain was measured at each of ten randomly chosen spacings. Since the wave-length of this periodic variation is less than or equal to the wavelength of the radiation, and since from Table V it is seen that the measurement interval was at least eight wavelengths long at all frequencies, it was felt that the proximity corrected gain values at the different distances represent a true random sample of the periodically varying gain. Thus averaging these gain values would average out the periodic variations, and the standard deviation of the gain values σ_G would contain the average R.M.S. value of the variation amplitude.

σ_G actually has a number of other contributions; these are variations that might have occurred (a) due to random departures from the true alignment as the horns were separated at the different distances, (b) due to instability of the equipment and repeatability of electrical connectors, and (c) due to misalignment of the waveguide flanges as the bolometer was alternatively removed and reconnected. If $\sigma_{(a)}$, $\sigma_{(b)}$, and $\sigma_{(c)}$ represent the standard deviations due to effects (a), (b), and (c) and $\sigma_{\mathrm{R.M.S.}}$ represents the average R.M.S. value of the periodic variation amplitude, then, because all these effects are independent, we have:

$$\sigma_G = \sqrt{\sigma_{\mathrm{R.M.S.}}^2 + \sigma_{(a)}^2 + \sigma_{(b)}^2 + \sigma_{(c)}^2} \tag{5}$$

$\sqrt{\sigma_{(b)}^2 + \sigma_{(c)}^2}$ can be evaluated separately by computing half the standard deviation of the actual audio substitution measurements and can thus be separated from σ_G. The reason for using half rather than the whole standard deviation is explained in *A* later in this section.

To find the limiting errors on the values of measured gain it is necessary to examine all possible sources of error in the experiment and assign limiting errors to each. In cases where a source of error is directly observable as a non-zero standard deviation of a group of readings (as for instance $\sqrt{\sigma_{(b)}^2 + \sigma_{(c)}^2}$ above) the limiting error is taken to be the limits of the 99% confidence interval for the mean of the group. These limits are $\pm \frac{x\,\sigma}{\sqrt{n}}$, where σ is the standard deviation of the readings, n is the number of readings, and x is a factor obtained from the t-distribution and depends on the confidence interval and the number of readings. For the cases treated here x is usually equal to about 3. Use of the t-distribution assumes the samples are taken from a normal population and, since the errors examined seem randomly distributed, this assumption is presumed correct.

A breakdown of the limiting errors on the gain value for each frequency is shown in Table VI.

A. STABILITY OF EQUIPMENT, REPEATABILITY OF ELECTRICAL CONNECTORS, AND VARIATION IN WAVEGUIDE FLANGE ALIGNMENT

Since these effects are directly responsible for the spread in the actual audio substitution measurements, observation of half the standard deviation in these readings and the use of the number of measurements (30 at each frequency) leads, via the *t*-distribution as described above to the limiting error. The reason that half, rather than the whole, standard deviation is used is that since these random errors occur in the transmission between both horns, the contributions to the random error in the gain of one horn is halved. This remark applies also to *C* and *E* below.

B. MULTIPATH PROPAGATIONS, INTERHORN SCATTERING, AND ALIGNMENT VARIANCE

The standard deviation in gain values due to these effects, $\sqrt{\sigma_{\mathrm{R.M.S.}}{}^2 + \sigma_{(a)}{}^2}$, can be obtained using Equation (5) and a limiting error can be obtained as described above. In this case $n = 10$.

C. DIRECTIONAL COUPLER

This error occurs due to the uncertainly in the value of the coupling constant. Ten measurements were taken of the coupling constant at each frequency, and half[*] the standard deviation in the measurements was used (as described above) to find a limiting error.

D. ACCURACY OF PROXIMITY CORRECTIONS

Bowman[3] proposed that ± 10 % of the decibel value of the proximity corrections is a reasonable limiting error estimate of their accuracy. We adopt Bowman's criterion.

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[*] See *A* above.

E. PRECISION ATTENUATOR ACCURACY AND BOLOMETER LINEARITY*

The maximum error in the Weinschel attenuator measuring system was guaranteed by the manufacturers to be less than 0.005 db over the range in which the horn measurements were made. This was checked using a precision auto-transformer, and the maximum error was found to be less than 0.004 db. When insertion loss measurements are made with the Weinschel system, which lead to a one or greater order of magnitude difference in the power incident on the bolometer (i.e., when the attenuation being measured is 10 db or more), account must be taken of the non-linearity of the bolometer characteristic. This situation did not occur in the horn measurements where the insertion loss typically was 5 db, however it was present in the measurements of the 20-db directional coupler. The manufacturers guarantee that any error arising from such non-linearity will be less than 0.01 db if the power incident on the bolometer is less than 200 microwatts peak. Since this limiting error presumably applies to measurements utilizing the full 200-microwatt range (up to 52 db), it can be used with confidence as a limiting error for this effect in the directional coupler measurement. It is appropriate to mention here that during both the horn and coupler measurements, a power meter was used to ensure that the incident bolometer power never exceeded 200 microwatts.

F. MATCHING

It was seen in Section II that the combined effect of Γ_G and Γ_L was to produce a gain error of less than 0.008 db. Even though both horns were perfectly matched to free space it may have been possible that in the

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*By linearity here is meant voltage output proportional to power input.

experimental setup a mismatch occurred. If this was the case, then it cannot have been very serious because of the relatively large horn separation and the fact that the horns were placed at least three aperture heights above the optical bench. Any variations in this mismatch would show up as part of σ_G. However, if we take a conservative estimate and assume a mismatch of -25 db (i.e. Reflection Coefficient $= 0.056$), then from the curves by Beatty this will lead to a limiting error of about 0.017 db, which together with an error of 0.008 db contributed by Γ_G and Γ_L comes to 0.025 db at most. Since errors in the gain value due to mismatching are always of the same sign, i.e. negative (in other words a mismatch always results in a degradation of the gain value), this limiting error of 0.025 db only contributes to the upper bound in the final limiting error and not to the lower bound.

The only other error source that comes to mind is one due to a basic alignment error, as opposed to variations around the true alignment, which were discussed above. If such a basic error did occur it could be observed as a steady decrease in gain value with distance. Such an effect was not observed at any frequency and so the possibility of such an error having occurred was discounted.

The total limiting error in the measurement can be obtained, assuming all limiting errors given in sections A to E above can be treated as independent random variables, from

$$\text{Total limiting error} = +\sqrt{(A)^2 + (B)^2 + (C)^2 + (D)^2 + (E)^2 + (F)^2}$$

$$-\sqrt{(A)^2 + (B)^2 + (C)^2 + (D)^2 + (E)^2}$$

This is done in Table VI, and the result is interpreted as the limiting error of the measurement.

V. INDEPENDENT GAIN DETERMINATION AT 20 GHz

Another way of determining the gain of a horn is to measure its radiated power pattern. The directivity can then be calculated from

$$\text{Directivity} = \frac{4\,\pi}{\int_0^{2\pi} \int_0^{\pi} [U(\theta,\ \phi)]^2 \sin\theta \, d\theta \, d\phi} \tag{6}$$

where $U[\theta,\ \phi]^2$ is the radiated power pattern (normalized so that $U[0,\ 0] = 1$) and θ and ϕ are polar coordinates centered on the horn axis. The gain can now be calculated, assuming the attenuation loss through the horn is known, by subtracting this loss from the directivity.

The radiation pattern was measured for the *K*-band horn at a frequency of 20 GHz over a dynamic range of about 55 db. The central portion of the pattern lying within ± 40° in the *E*-plane and ± 21° in the *H*-plane was found accurately by repeated *E*-plane scans spaced 1.5° apart in the *H*-plane. Outside of this central region the pattern was recorded at 5° intervals in the *H*-plane with *E*-plane scans of ± 180°. At the boundaries of the central region the pattern was down 30 db in the *H*-plane, and 29 db in the *E*-plane, from its maximum value.

The integration in the denominator of Equation 6 was divided into two parts. The central portion was integrated numerically on a computer using a grid size of 1.5° by 1°, and the contribution of the remainder of the pattern was estimated by hand calculation. From these integrations it was seen that the central portion contained 98% of the pattern, the back 2 π steradians 0.2 %, and the remainder of the front 2 π steradians 1.8%. Using Equation 6, the directivity was then found to be 24.362 db.

The loss in the horn was measured as follows: each end of the horn was fitted with a plate so that it became a resonant cavity. A detector was coupled through a small hole in the larger plate, a swept frequency oscillator was coupled through a small hole in the smaller plate, and the Q of the cavity mode associated with the normal field distribution in the horn was measured. This procedure was repeated a number of times, decreasing the coupling hole size each time, until a constant value of Q was arrived at. From this a transmission loss of 0.030 ± 0.003 db was derived for the horn. Upon subtracting this from the value of the directivity, we arrive at a value of 24.332 db for the gain of the horn at 20 GHz. It is estimated that the accuracy of the integration technique was about 2.5%, giving a limiting error of ± 0.115 db on the gain value.

From *Fig. III* it is seen that the measured gain value from the two-antenna method is 24.415 db. To this must be added the loss in the two-inch waveguide piece connected to the horn during the measurement,* which was ascertained to be 0.050 db at 20 GHz. Also, since the shorter of the two K-band horns* was used in the pattern measurement, 0.007 db must be subtracted from the measured value. With these two corrections the measured gain of the shorter K-band horn is 24.458 db. The limiting error on this value, obtained by averaging the errors obtained at 18 and 22 GHz is $^{+\,0.060}_{-\,0.055}$ db.

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*See Appendix.

VI. DISCUSSION

To summarize the results quoted in the last section: the measured gain of the shorter *K*-band horn at 20 GHz as interpolated from *Fig. III* is 24.458, $^{+\,0.060}_{-\,0.055}$ db, while the gain obtained by integrating the pattern is 24.332 ± 0.115 db. The closeness of these results and the fact that their error bars overlap would seem to indicate that a reasonably high degree of confidence can be placed in the methods used, the precautions taken, and, consequently, the results obtained in the measurements described in this report.

Jull[8] in a recent note commenting upon the accuracy of Schelkunoff's expression notes a systematic discrepancy of about 0.3 db between the measured and the calculated values of a high gain horn in the range 13 — 14 GHz and suggest that less reliance can be placed on Schelkunoff's expression when it is applied to small horns at high frequency. While the horns used in measurement cannot be considered small, having aperture dimensions of about 6 λ at the low frequency end of each band, it is interesting to see how their measured gain values compare with these calculated from Schelkunoff. It is seen from *Fig. IV* that the agreement in the K_a band is remarkably good, the measured values all lying approximately 0.1 db below the theoretical values. When it is remembered that the loss in the K_a-band horn is about 0.05 db* the remaining discrepancy of 0.05 db, presumably due to edge diffraction, is exceedingly small indeed. In the *K* band however the agreement is not as good. When

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*See Appendix.

allowance is made for attenuation in the horn (0.03 db) and in the two-inch waveguide piece attached to the horn* (0.05 db) it is seen from *Fig. III* that differences exist between the measured and the calculated values ranging from about 0.47 db at the low frequency end of the band to about 0.17 db at the high frequency end. The average difference over the band is 0.27 db and is within what are usually assumed to be the error limits of theoretical gain expressions. However, the fact that such a large discrepancy exists between the measured and theoretical values at the low frequency end of the band is indicative of how important it is to actually measure the gain in order to obtain a reliable value for it. That this discrepancy, whose magnitude is 50% greater than is normally assumed, really exists is brought out by the fact that the independent gain measurement was made in the same frequency region.

Because of the precautions taken to avoid errors and, where unavoidable, the effort made to determine their limiting magnitude it is felt that the quoted values of limiting error are both reasonable and realistic.

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*See Appendix.

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TABLE I

CALIBRATION OF K- AND K_a-BAND DIRECTIONAL COUPLERS*

K-Band Coupler

Frequency (GHz)	Coupling (db)	σ (db)
18	18.676	0.015
22	19.948	0.009
26	19.659	0.011

K_a-Band Coupler

Frequency (GHz)	Coupling (db)	σ (db)
28	20.040	0.028
32	20.491	0.016
36	20.343	0.014
38	20.134	0.016

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*Each coupling value is the average of 10 readings and σ is the standard deviation of these readings.

TABLE II

Frequency
(36 GHz)

Antenna Separation R (cms)	10 Log₁₀ P_{AA}/P_{BB}	10 Log₁₀ $P_{AA}/P_{A'A'}$	Gain (db)	Proximity Correction (db)	Corrected Gain (db)
138.65	8.300	− 12.043	27.182	0.338	27.520
139.35	8.419	− 11.924	27.263	0.332	27.595
140.60	8.380	− 11.963	27.282	0.328	27.610
142.20	8.349	− 11.994	27.316	0.324	27.640
144.10	8.055	− 12.288	27.227	0.318	27.545
145.72	8.072	− 12.271	27.284	0.312	27.596
148.12	7.920	− 12.423	27.279	0.306	27.585
149.70	7.848	− 12.495	27.289	0.301	27.590
151.50	7.756	− 12.587	27.295	0.298	27.593
152.68	7.729	− 12.614	27.315	0.296	27.611

Average Corrected Gain = 27.589

Standard Deviation in Corrected Gain Values $[\sigma_G]$ = 00.030

Notes

1. The distances are randomly spaced.
2. $2a^2/\lambda$ for this frequency = 140.00 cms.
3. Each entry in the second column above (10 Log₁₀ P_{AA}/P_{BB}) represents the average of three readings taken at that particular antenna separation.
4. 10 Log₁₀ $P_{AA}/P_{A'A'}$ = 10 Log₁₀ P_{AA}/P_{BB} − 10 Log₁₀ $P_{A'A'}/P_{BB}$ = 10 Log₁₀ P_{AA}/P_{BB} − 20.343. 20.343 is the average coupling of the K_a-band 20 db directional coupler at 36 GHz. Also, of course, $P_{AA} = P_R$ and $P_{A'A'} = P_T$.
5. Gain (db), from Equation (1) = 10 Log₁₀ 4 π R/λ + 1/2[10 Log₁₀ P_R/P_T]. Notice that the gain is calculated for each distance.
6. For proximity correction see Section III.
7. Corrected gain = gain + proximity correction.

TABLE III
Final Results for *K*-Band Horns

Frequency (GHZ)	Measured Gain Value (db)	Gain of Longer Horn (db)	Gain of Shorter Horn (db)	Limiting Error (db)	Theoretical Gain Value
18	23.481	23.487	23.475	+ 0.062 — 0.057	24.050
22	25.301	25.308	25.294	+ 0.058 — 0.052	25.610
26	26.803	26.811	26.795	+ 0.051 — 0.044	26.290

Notes

1. The measured gain value is the average of the gain of the longer and shorter horns and includes the effect of the 2" waveguide piece (see Appendix).
2. The values of limiting error are the same for both the longer and shorter horns.
3. The theoretical gain values are calculated from curves in reference 6 for a horn of length equal to the average of the longer and shorter horns and are thus directly analogous to the measured gain values.

TABLE IV

Final Results for K_a-Band Horn

Frequency (GHz)	Measured Gain Value (db)	Limiting Error (db)	Theoretical Gain Value (db)
28	25.674	+ 0.062 − 0.057	25.750
32	26.723	+ 0.055 − 0.048	26.790
36	27.589	+ 0.052 − 0.045	27.720
38	28.022	+ 0.061 − 0.056	28.120

TABLE V

	Frequency (GHz)	$2\frac{a^2}{\lambda}$ (cms)	Spread of Distances (cms)	Average Value of Proximity Correction (db)	Standard Deviation in Corrected Gain Values σ_G (db)
K-Band Horn	18	109.00	142.25 — 158.90	0.11	0.053
	22	133.00	136.02 — 157.50	0.26	0.044
	26	157.00	144.18 — 157.75	0.34	0.026
K_a-Band Horn	28	108.00	146.50 — 161.82	0.18	0.041
	32	123.00	140.55 — 162.48	0.25	0.051
	36	138.60	138.65 — 152.68	0.31	0.030
	38	146.20	153.55 — 168.16	0.30	0.045

TABLE VI

ERROR ANALYSIS OF MEASUREMENTS*

		K-Band			K_a-Band			
Frequency (GHz)		18	22	26	28	32	36	38
Limiting Errors (db)								
(A) Stability of equipment, repeatability of electrical connectors, and variation in flange alignment	±	0.006	0.006	0.006	0.005	0.007	0.004	0.004
(B) Multipath propagation, interhorn scattering, and alignment variance	±	0.053	0.044	0.025	0.041	0.050	0.030	0.045
(C) Directional coupler	±	0.008	0.005	0.006	0.014	0.008	0.007	0.008
(D) Accuracy of proximity corrections	±	0.018	0.027	0.035	0.019	0.025	0.032	0.031
(E) Precision attenuator accuracy and bolometer linearity	±	0.006	0.006	0.006	0.006	0.006	0.006	0.006
(F) Matching	+	0.025	0.025	0.025	0.025	0.025	0.025	0.025
Total Limiting Error	+	0.062	0.058	0.051	0.062	0.055	0.052	0.061
	−	0.057	0.052	0.044	0.057	0.048	0.045	0.056

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*See Section IV.

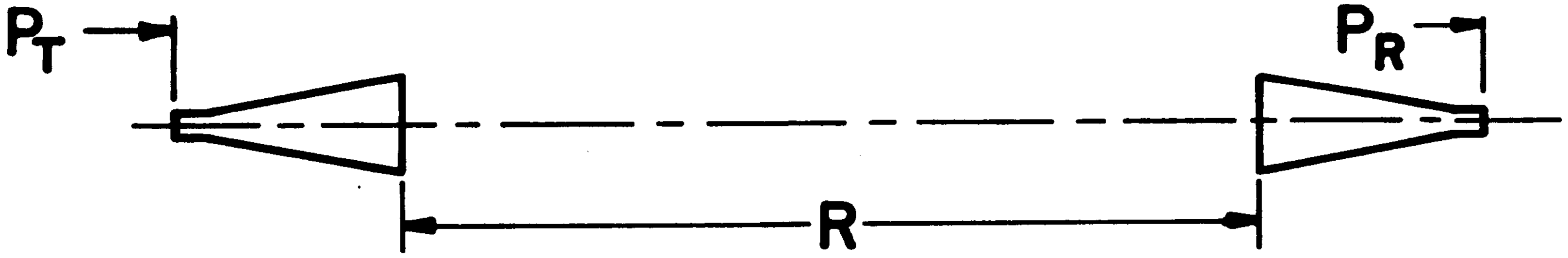

FIG. I

POSITION OF HORNS FOR GAIN DETERMINATION

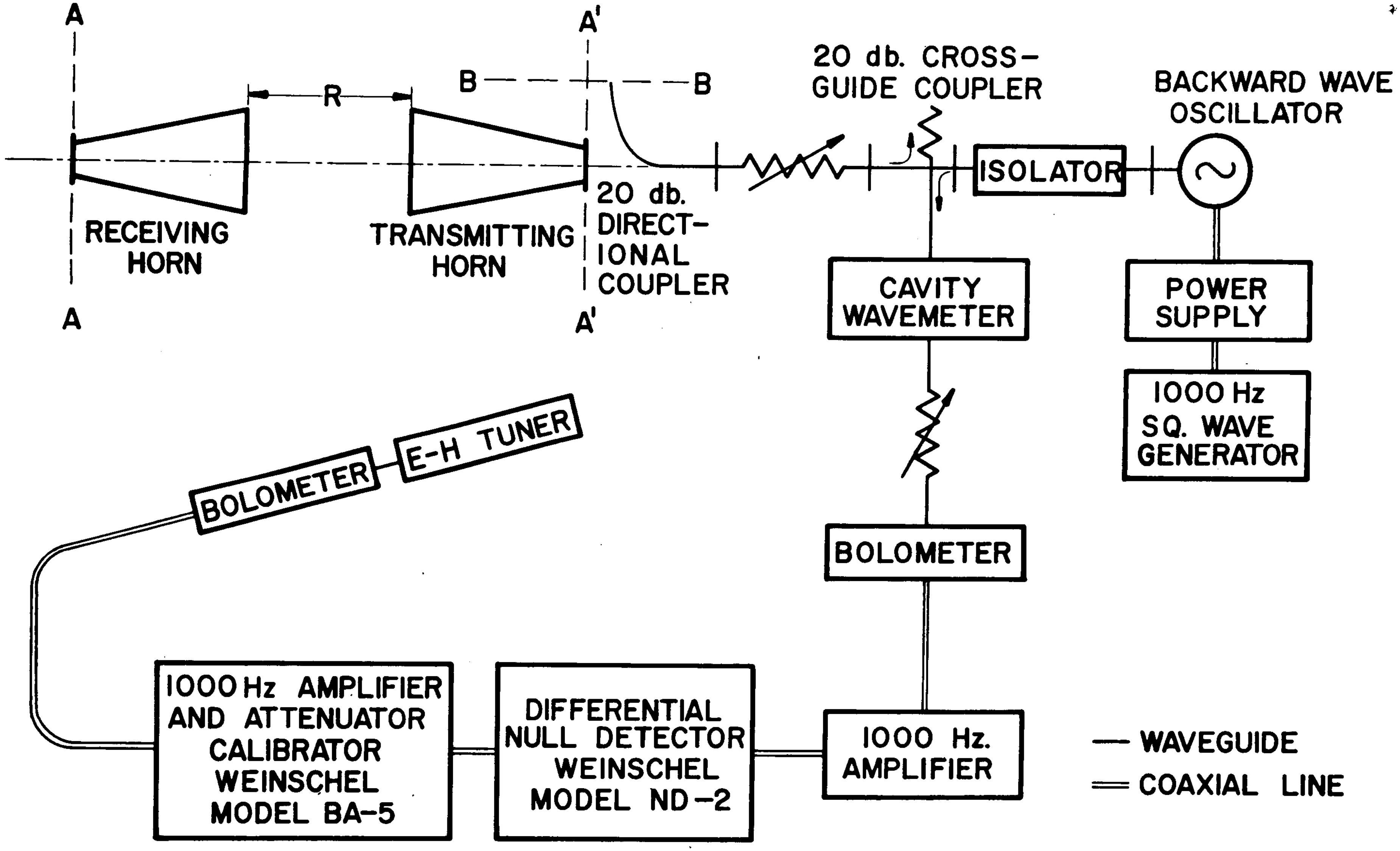

FIG. II

BLOCK DIAGRAM OF MEASURING EQUIPMENT

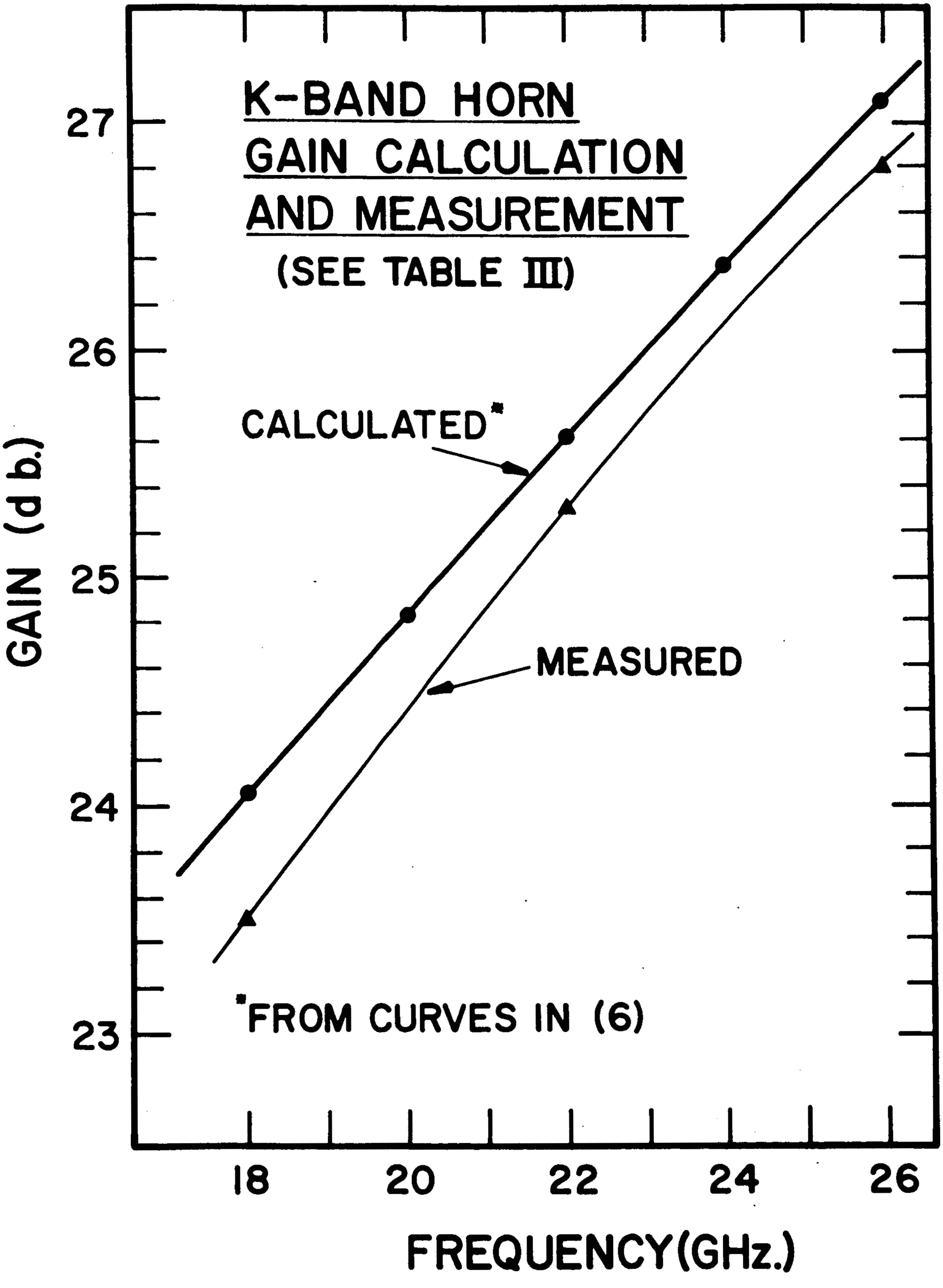
K-BAND HORN
GAIN CALCULATION
AND MEASUREMENT
(SEE TABLE III)
CALCULATED*
MEASURED
*FROM CURVES IN (6)
GAIN (d b.)
23
24
25
26
27
18
20
22
24
26
FREQUENCY(GHz.)

FIG. III

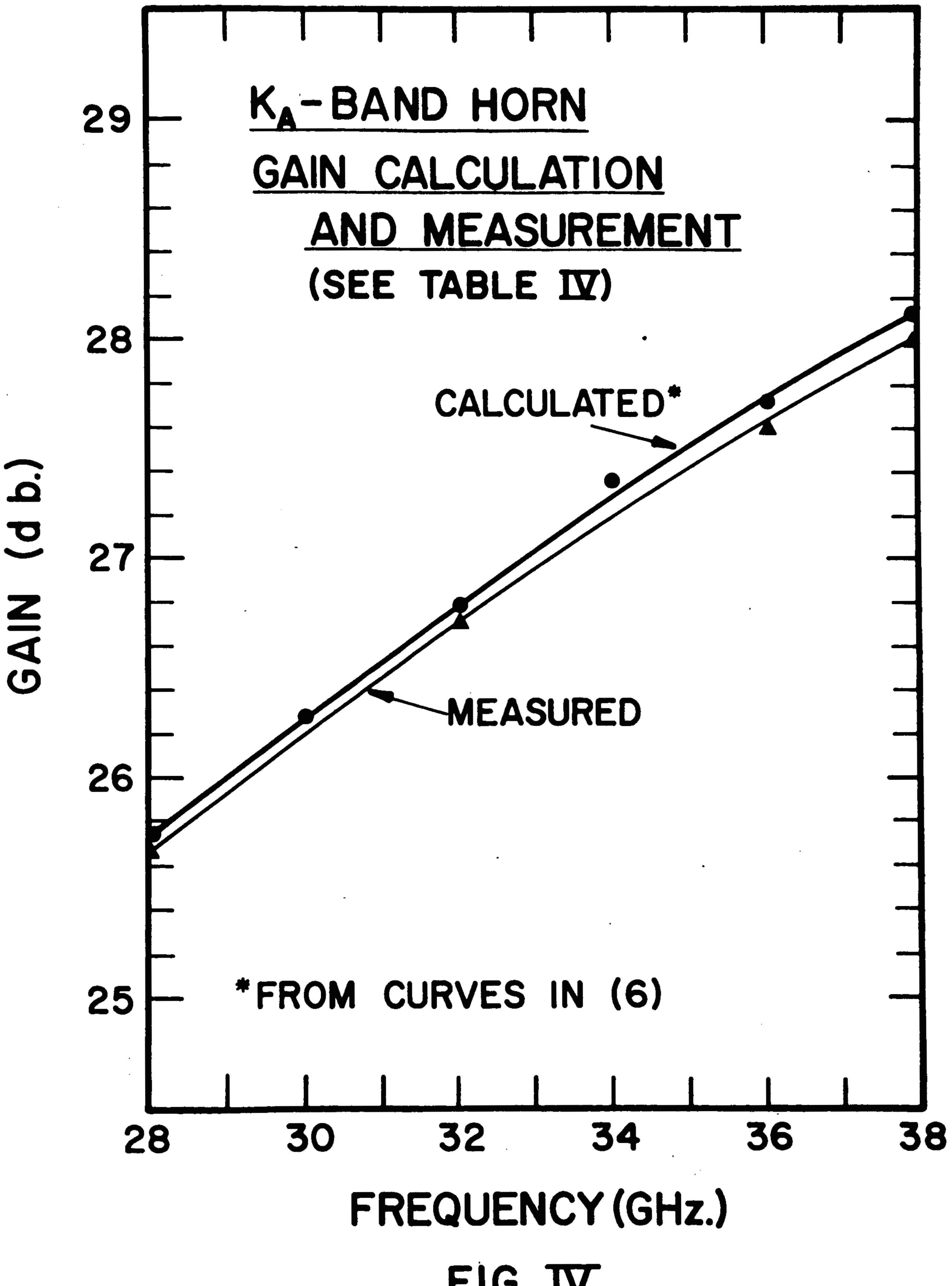
K_A-BAND HORN
GAIN CALCULATION
AND MEASUREMENT
(SEE TABLE IV)
CALCULATED*
MEASURED
*FROM CURVES IN (6)
GAIN (d b.)
29
28
27
26
25
28
30
32
34
36
38
FREQUENCY (GHz.)

FIG. IV

HORN DIMENSIONS

K-Band Horn

a = 9.52 cms

b = 6.99 cms

R_E = 68 cms (shorter horn); 71 cms (longer horn)

R_M = 68 cms (shorter horn); 71 cms (longer horn)

K_a-Band Horn

a = 7.60 cms

b = 5.55 cms

R_E = 60 cms

R_M = 62 cms

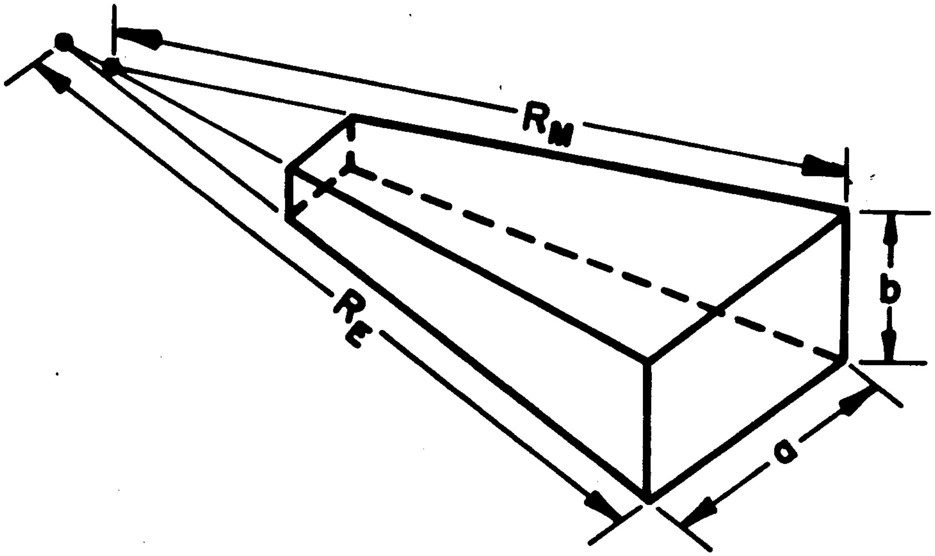

FIG. V

R_M is the length of the dihedral horn that would be formed by extending those faces of the given horn that are normal to the magnetic plane until they meet. R_E is the length of the dihedral horn that would be formed by extending the top and bottom faces, which are perpendicular to the electric plane. The aperture dimensions are a and b.

APPENDIX

PHYSICAL PROPERTIES OF THE HORNS

Both sets of horns were manufactured in the machine shop of the Space Sciences Laboratory, University of California, Berkeley. Their dimensions are shown in *Fig. V.*

A. *K*-BAND HORNS

These horns were electroformed and are identical in aperture size but are different in length by exactly one inch. This leads to differences in R_e and R_m as shown in *Fig. V.* Since the horns are already quite long, approximately 50 wavelengths in the center of the band, an additional two wavelengths does not radically change the phase distribution across the aperture; it is changed slightly, however, and the resulting gain difference in the horns can be roughly estimated from curves in Jasik[7] to within about 10%. These gain differences turn out to be 0.02 db at 18 GHz, 0.014 db at 22 GHz, and 0.016 at 26 GHz.

Since these differences are very slight, being less than 0.3% of the total gain at all frequencies, the two antenna method can still be used, and the result, to a first approximation, will be an average gain. Thus, to find the gain of the longer horn at a particular frequency, half of the above difference should be added to the measured gain and for the gain of the shorter horn it should be subtracted. This is shown in Table III. The limiting error in the value of these corrections is very small (less than 0.001 db at all frequencies) and is neglected in the error analysis.

Since these horns were electroformed, a process resulting in extremely uniform surfaces, it was felt that the loss through each horn would be the same thus obviating the need to take further inhomogeneities into consideration.

For ease of connection, two-inch pieces of waveguide were fastened to the horns and the measurements taken with the waveguide connected. Since it is intended that these pieces remain fastened to the horns their attenuation effects were not added to the gain.

B. K_a-BAND HORNS

These horns have identical dimensions and are labeled *A* and *B* for identification purposes. They were first soldered up on brass and then silver-plated. Since this manufacturing process does not give quite as uniform a surface finish as electroforming, it was considered necessary to know the loss through each horn separately. This was done by measuring the *Q* of the resonant cavities formed from the horns, as described in Section V. This procedure was followed at a number of frequencies randomly spaced throughout the band. It was found that while the average loss at the center of the band was 0.046 db, there existed a slight non-zero difference in loss value at some frequencies. The loss through horn *B* was always greater than or equal to that through *A* and the maximum value of the difference in loss was 0.010 db. As was mentioned above, the measured gain to a first approximation is an average of the gain of both horns, so that the maximum error in the measured value due to this effect is 0.005 db and contributes to the upper bound only of the total limiting error for horn *A* and to the lower bound only for horn *B*. The effect of these bound changes on the total limiting errors given in Table VI is so negligible, however, that no significant advantage would be gained by calculating separate total limiting errors for the two horns.