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MIE-SCATTERING FUNCTION

by F. Shahrokhi and P. Wolf

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MIE-SCATTERING FUNCTION

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ABSTRACT

For a comparison between experimental results and Mie-scattering theory, a series of calculations was made to find the normalized scattering function $f(\theta)$ from the tables of Legendre-polynomials by G. C. Clark and S. W. Churchill and from tables of scattering coefficients made by C. M. Chu, G. C. Clark, and S. W. Churchill.

This study was carried out as part of a research program being performed under NASA Sustaining Grant NGR-43-001-021.

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Introduction

The angular distribution of the intensity of radiation scattered at spherical particles was worked out by G. Mie (1) in theory. Several tables were prepared for particle materials of real refractive index (absorption by particle material neglected). In these tables coefficients are given which have to be combined with polynomials to give the scattering distribution function for single scattering at spherical particles of defined size in relation to the radiation wave length. The scattering function is usually for an incident beam of unit intensity.

This combination of coefficients and polynomials was done by a small computer program, based on the data provided by the tables for Legendre polynomials by G. C. Clark and S. W. Churchill (2) and by the tables for scattering coefficients by C. M. Chu, G. C. Clark and S. W. Churchill (3).

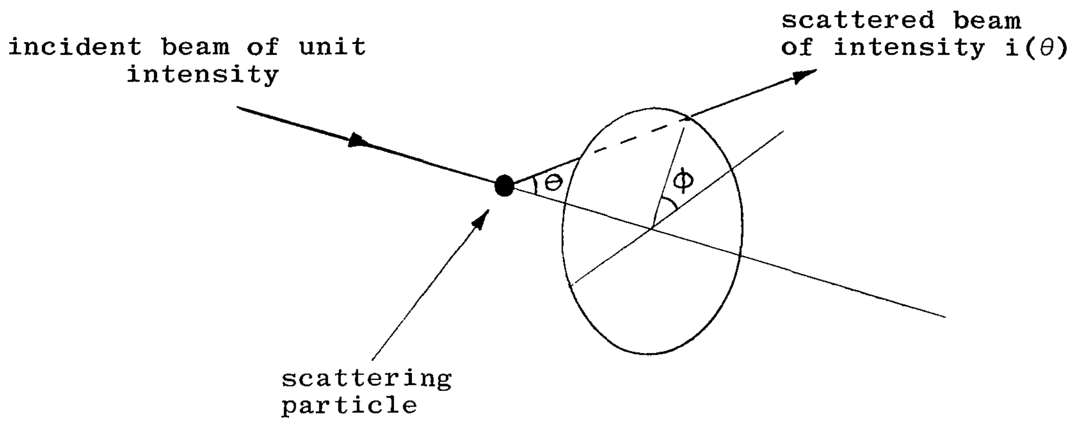
The computations became necessary because of an intention to do a closer comparison of experimental results of scattering in particle clouds with scattering predicted by the Mie-theory for spherical particles suspended in a gas.

Mathematical Definitions and Computation

As Hertel (4) proposed, the normalized scattering function f (θ) may be expressed by

$$f(\theta) = \frac{1}{4\pi} \left(1 + \sum_{M=1}^{\infty} a_n \cdot P_n (\cos \theta) \right)$$

Here is $f(\theta) = \dfrac{i(\theta)}{\frac{\pi D^2}{4}\;K}$ the intensity of a beam scattered in direction θ from an incident beam of unit intensity hitting a spherical particle



The scattered intensity $i(\theta)$ is independent of φ for spherical particles of uniform material and a uniform irradiation.

K is the scattering cross section nondimensionalized with the geometrical cross section $\frac{\pi}{4}\,D^2$ of the spherical particle of diameter D.

The scattering coefficients, a_n, are independent of the angle, θ, but vary with particle size referred to radiation wave length $\alpha = \frac{\pi \cdot D}{\lambda}$ in the medium surrounding the particles and with the particle material of refraction relative to the surrounding medium.

The a_n were computed by authors (3) according to the relations derived by them.

$$a_n = \frac{(2)(-1)^n}{\alpha^2 K(\alpha,\beta)} \sum_{j=1}^{\infty} \sum_{k=1}^{j} \left(\frac{2}{1+\delta_{jk}}\right) \left\{ (2n+1) \left[\frac{j(j+1) + k(k+1) - n(n+1)}{2} \right]^2 w_{jkn} W_{jk} + v_{jkn} V_{jk} \right\}$$

where

$$\delta_{jk} = \begin{cases} 0 \text{ for } j \neq k \\ 1 \text{ for } j = k \end{cases},$$

$$W_{jk} = Re(A_j)Re(A_k) + Im(A_j)Im(A_k) + Re(B_j)Re(B_k) + Im(B_j)Im(B_k),$$

$$V_{jk} = Re(A_j)Re(B_k) + Im(A_j)Im(B_k) + Re(B_j)Re(A_k) + Im(B_j)Im(A_k),$$

$$w_{jkn} = 0, \text{ if } j + k - n \neq 2r, \; r = 0,1,2\ldots k,$$

$$w_{jkn} = \frac{(\angle j + n - k)(\angle k + n - j)(\angle j + k - n)\left(\angle \frac{j + k + n}{2}\right)^2}{(\angle j + k + n + 1)\left[\left(\angle \frac{j + n - k}{2}\right)\left(\angle \frac{k + n - j}{2}\right)\left(\angle \frac{j + k - n}{2}\right)\right]^2}$$

if $j + k - n = 2r$, $r = 0, 1, 2 \ldots k$.

$$v_{jkn} = 0, \quad \text{if } j + k - n \neq 2r + 1, \; r = 0, 1, 2 \ldots k,$$

and

$$v_{jkn} = \frac{(2n+1)(j+k-n)(j+n-k+1)(k+n-j+1)(\angle j+n-k+1)(\angle k+n-j+1)(\angle j+k-n-1)\left(\angle \frac{j+k+n+1}{2}\right)^2}{4(\angle j+k+n+1)\left[\left(\angle \frac{j+n-k+1}{2}\right)\left(\angle \frac{k+n-j+1}{2}\right)\left(\angle \frac{j+k-n-1}{2}\right)\right]^2}$$

if $j + k - n = 2r + 1$, $r = 0,1,2\ldots k$

where Im = Imaginary part,

Re = Real part,

$\angle$ = factorial.

The Legendre polynomials $P_n(\cos\theta)$ were computed by authors (2) up to the high necessary order n by a recurrence relation

$$(n + 1) P_{n+1}(x) - (2n + 1) x P_n(x) + n P_{n-1}(x) = 0$$

starting with $P_o(x) = 1$

$P_1(x) = x$

$$P_n(-x) = (-1)^n \cdot P_n(x)$$

For the wanted scattering function

$$4\pi \cdot f(\theta) = 1 + \sum_{n=1}^{\infty} a_n \cdot P_n (\cos \theta)$$

$$\cos \theta = x$$

only the sum of the products $a_n \cdot P_n$ has to be formed.

Since the input of the data a_n and P_n was the only time-consuming work, the symmetry relation

$$P_n (-x) = (-1)^n \ P_n (x)$$

was used to necessitate the input of P_n for the range of $0° \leqslant \theta \leqslant 90°$ only and let the computer fill in the P_n for $90° \leqslant \theta \leqslant 180°$ by use of this symmetry relation.

The $f(\theta)$ were computed for scattering angles $\theta = 2°$, 5°, 10°, 15°, 20°, 25°, . . . 160°, 165°, 170°, 175°, 178°, adjusted to the range of possible scattering measurements, the particle size parameter α took values $\alpha = 1$, 2, 3, 4, 5, 6, 8, 10, 15, 20, 25, 30, according to the values for which the coefficients a_n had been computed in (3). The real index of refraction m covered the range m = 1.20, 1.25, 1.30, 1.33, 1.44, 1.50, 1.55, 1.60, 2.0, ∞ for aerosols.

The number of angles θ turned out to be chosen too small in order to represent $f(\theta)$ exactly enough for higher α (15 to 30), because $f(\theta)$ goes through many maxima and minima when scanning $0° \leqslant \theta \leqslant 180°$. Since an experimental investigation by Love (5) has shown that $f(\theta)$ probably cannot be truly represented in all maxima and minima (no unique -α mixtures of particles finite beams of radiation), the calculations have not been extended over more intermediate θ - values.

Tables and Graphs

The computer printout gives $4\pi \cdot f(\theta)$ values written in sequence according to the 37 θ - values5

$\theta_1 = 2°$,	$\theta_2 = 5°$,	$\theta_3 = 10°$,	$\theta_4 = 15°$,	$\theta_5 = 20°$
$\theta_6 = 25°$	"	"	"	$\theta_{10} = 45°$
$\theta_{11} = 50°$	"	"	"	$\theta_{15} = 70°$
$\theta_{16} = 75°$	"	"	"	$\theta_{20} = 95°$
$\theta_{21} = 100°$	"	"	"	$\theta_{25} = 120°$
$\theta_{26} = 125°$	"	"	"	$\theta_{30} = 145°$
$\theta_{31} = 150°$	"	"	"	$\theta_{35} = 170°$
$\theta_{36} = 175°$,	$\theta_{37} = 178°$			

The $4\pi \cdot f(\theta)$ - values are given to seven decimal places; however 5 are only useful because of the limitations of the tables (2), (3). The last two places may be used for round off.

A limited number of $f(\theta)$ - values was graphed to have an immediate picture of the function. The ordinate in the graphs is signed $f(\theta)$, although the factor 4π is still connected to $f(\theta)$.

The graphs 4π $f(\theta)$ for α - values 15, 20, 25, 30 do not truly represent the scattering function because the θ - steps are too wide to give enough information over the many maxima and minima of $f(\theta)$, which often extend over a range $\Delta\theta = 2$ to $5°$ only.

Computer - Program for

$$4\pi\ f(\theta) = 1 + \sum_{n=1}^{N} a_n \cdot P_n\ (\cos\ \theta)$$

```
/INPUT
/JOB GO
      DIMENSION P(70,37),A(70),F(37)
      READ (5,11) ((P(1,J),I=1,70),J=1,19)
11    FORMAT(10F8.0)
      DO 111 J=1, 18
      DO 111 I=1,69,2
      K=J+19
      L=-J+19
      P(I,K)=-P(I,L)
111   P(I+1,K)=P(I+1,L)
12    READ (5,13)N
13    FORMAT (14)
      IF(N) 14,20,14
14    READ (5,10) (A(I),I=1,N)
10    FORMAT(7F11.0)
      DO 15 J=1,37
15    F(J)=1.0
      DO 16 K=1,37
      DO 16 J=1,N
16    F(K) =F(K)+A(J)*P(J,K)
      WRITE (6,17) F
17    FORMAT (1P5E16.7)
      GO TO 12
20    STOP
      END
/DATA
```

P(I,J) = Legendre polynomials $P_n(\cos\ \theta)$ for 37 θ - values and orders n up to N = 70.

A(I) = Scattering coefficients a_n

F(K) = $4\pi f(\theta)$

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1. Mie, G., "Beitrage zur Optik truber Medien", Ann. Physik, 25, 377-445 (1908).

2. Clark, G. C., S. W. Churchill, "Tables of Legendre Polynomials", Univ. of Michigan Press, Engineering Research Inst. Publications, 1957.

3. Chu, C. M., G. C. Clark, S. W. Churchill, "Angular Distribution Coefficients for Radiation Scattered by a Spherical Particle", Journ. Optic. Soc. America, 47, (1957), or in Univ. of Michigan Press, Engineering Research Instit. Publications.

4. Hertel, W., "Zur Theorie der Licht streuung durch trube Schichten", Das Licht, 40, 141-43, 165, 190-91, 214-15, 232-34, (1940)

5. Love, T. J. and R. A. Wheasler, "An Experimental Study of Infra-Red Scattering by Clouds of Particles", Aerospace Research Laboratories, ARL 64-109, June 1964.

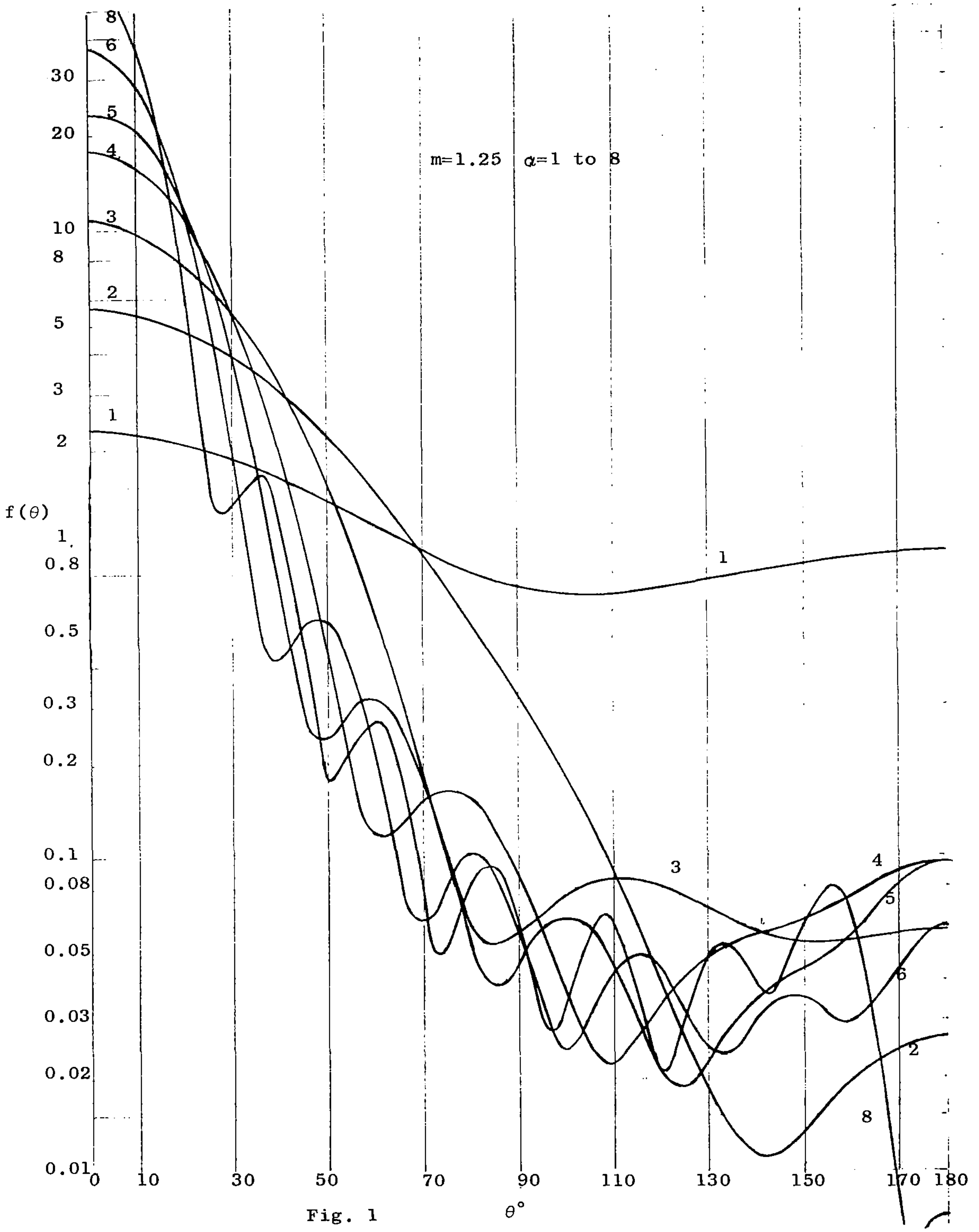
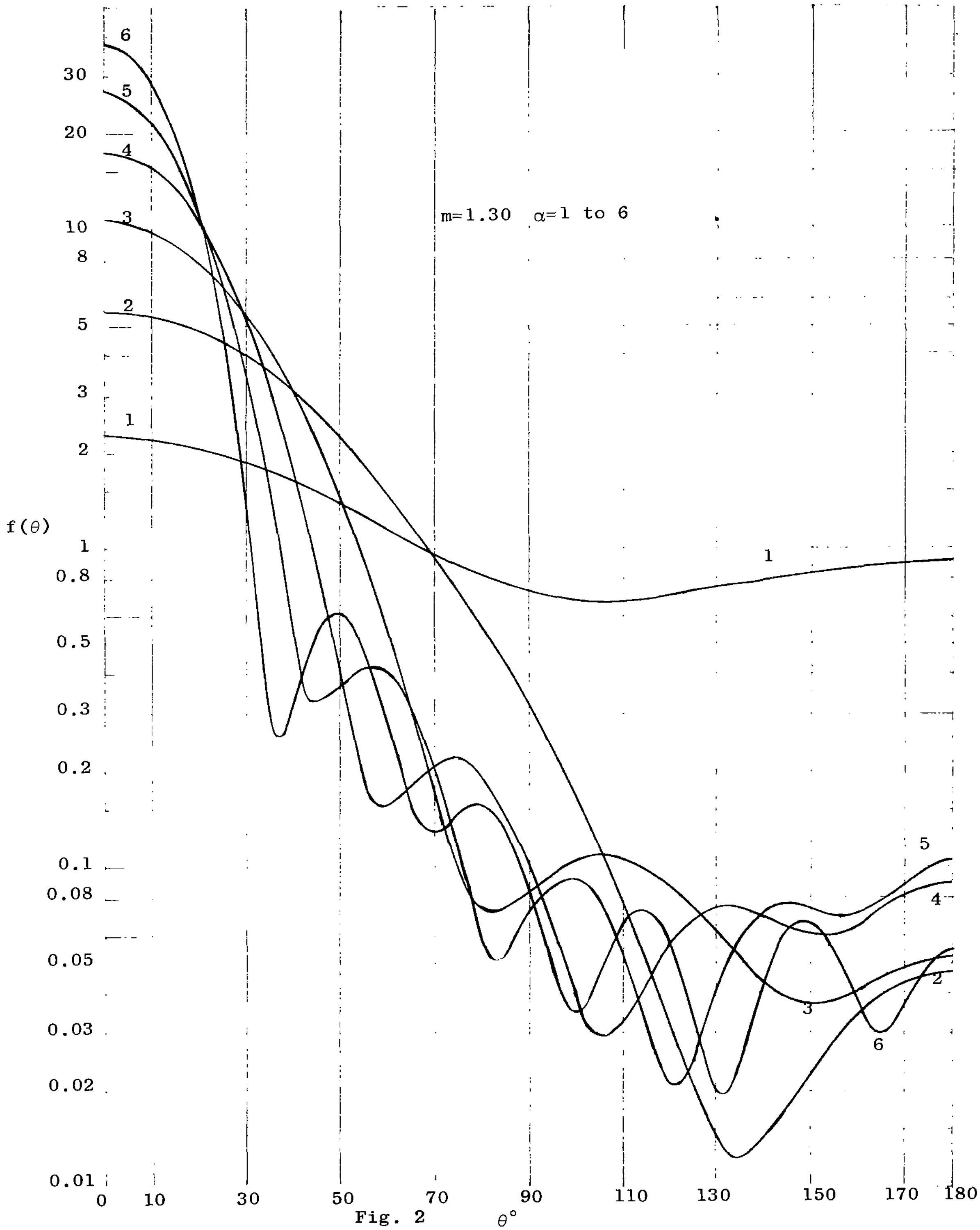


Fig. 1



m=1.30 α=1 to 6
f(θ)
θ°
30
20
10
8
5
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180
1
2
3
4
5
6

Fig. 2

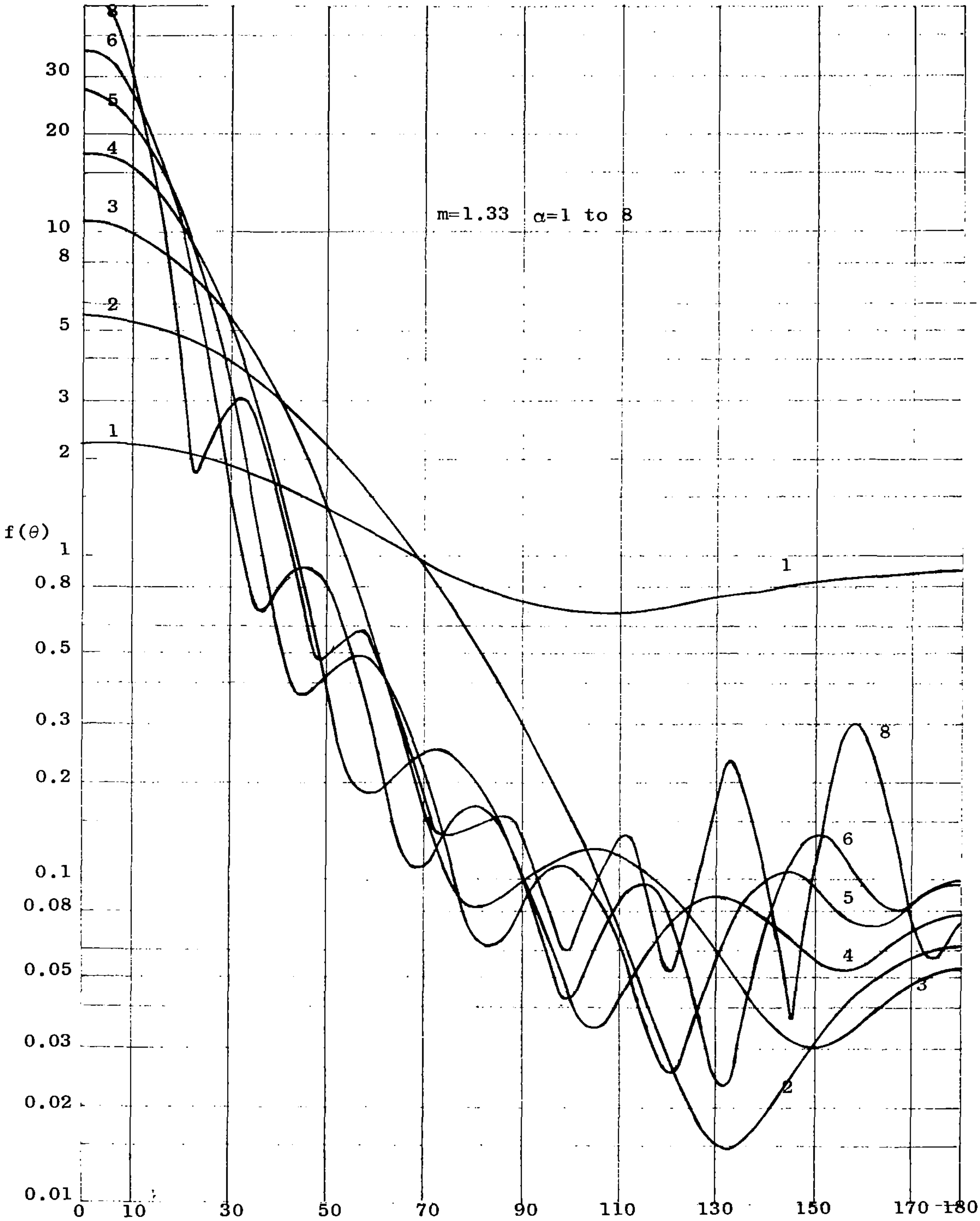

m=1.33 α=1 to 8
f(θ)
30
20
10
8
5
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180
1
2
3
4
5
6
8

Fig. 3 θ°

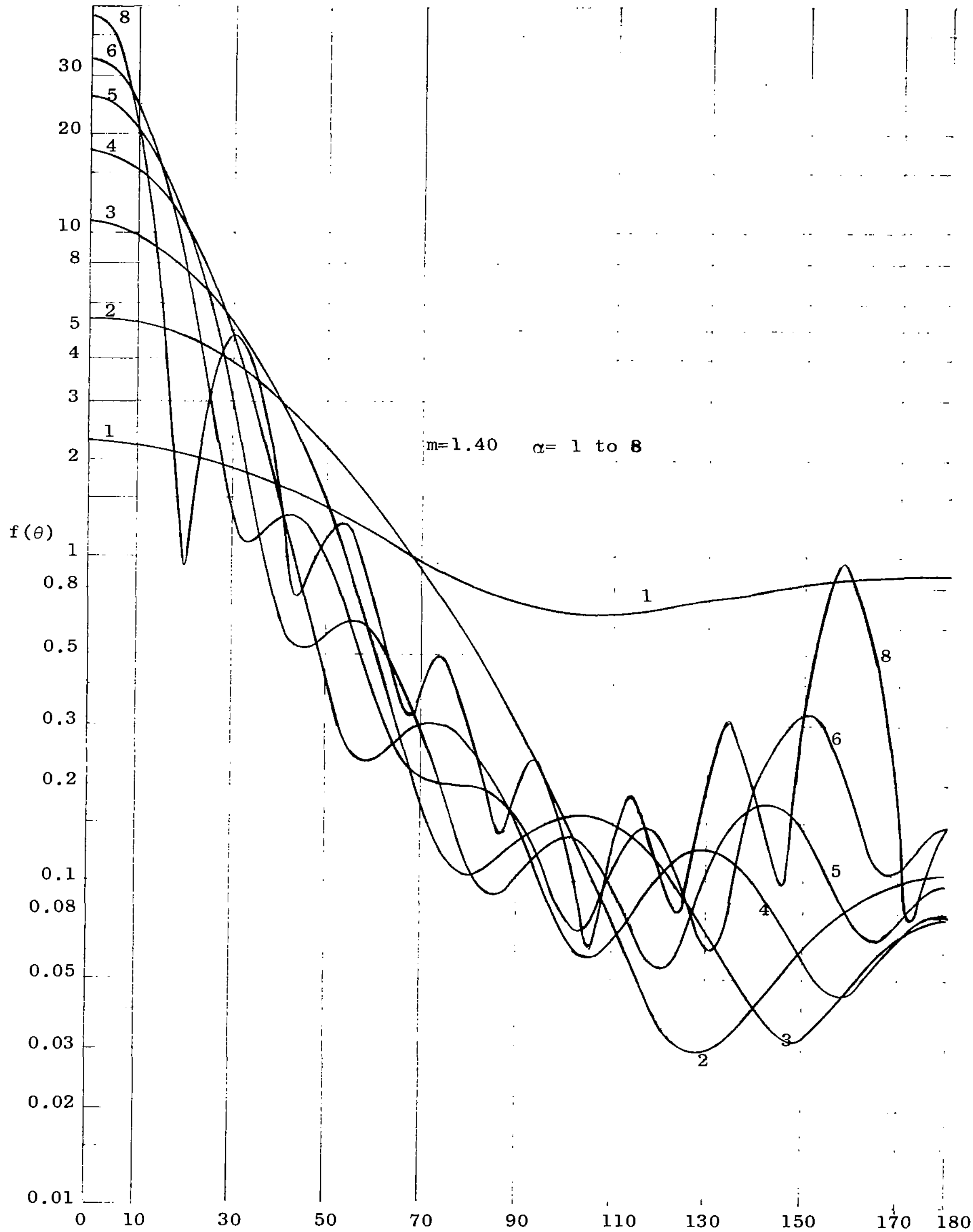
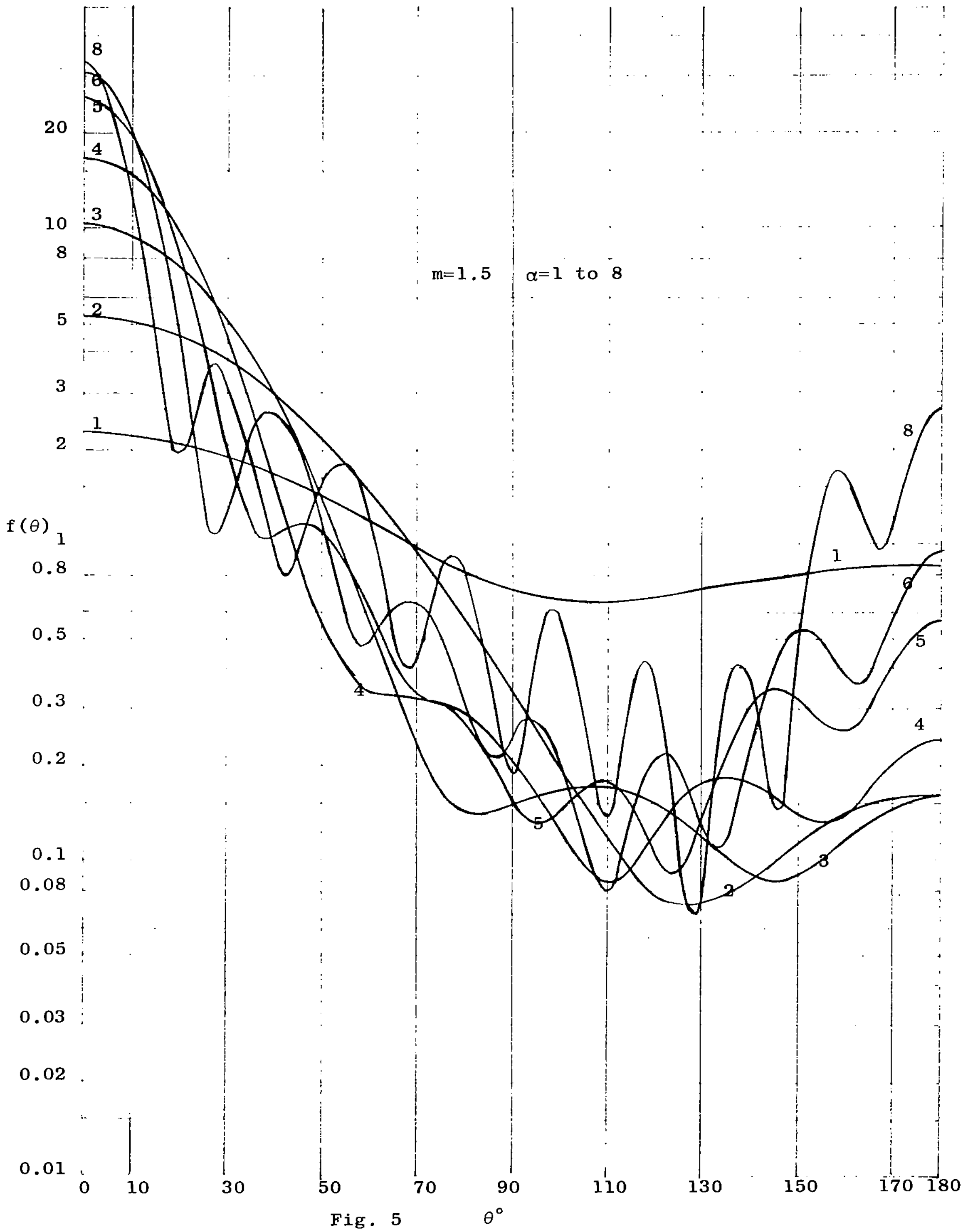

f(θ)
30
20
10
8
5
4
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180
m=1.40 α= 1 to 8
1
2
3
4
5
6
8

Fig. 4 $\theta°$


m=1.5 α=1 to 8
f(θ)
θ°
20
10
8
5
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180
Fig. 5

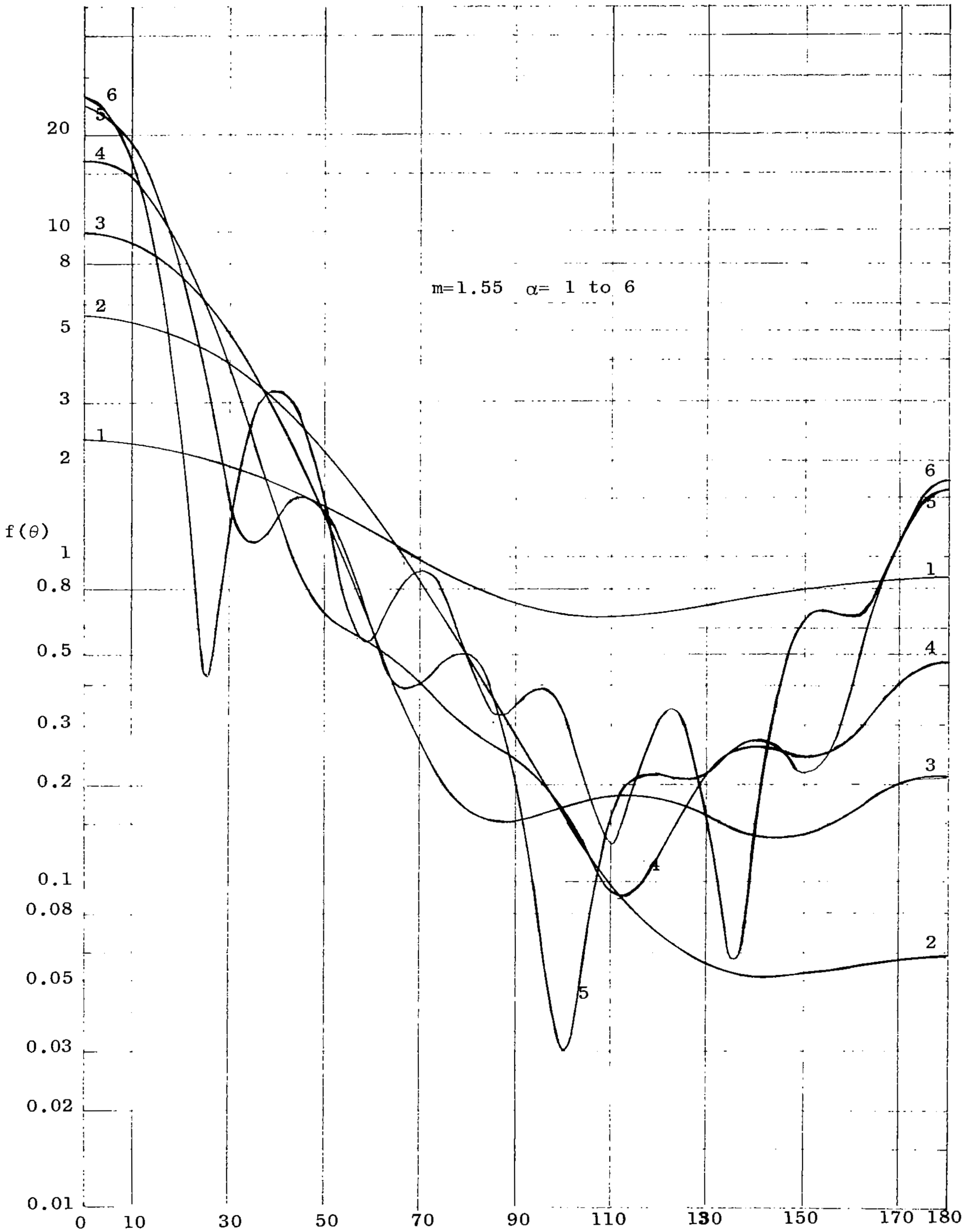
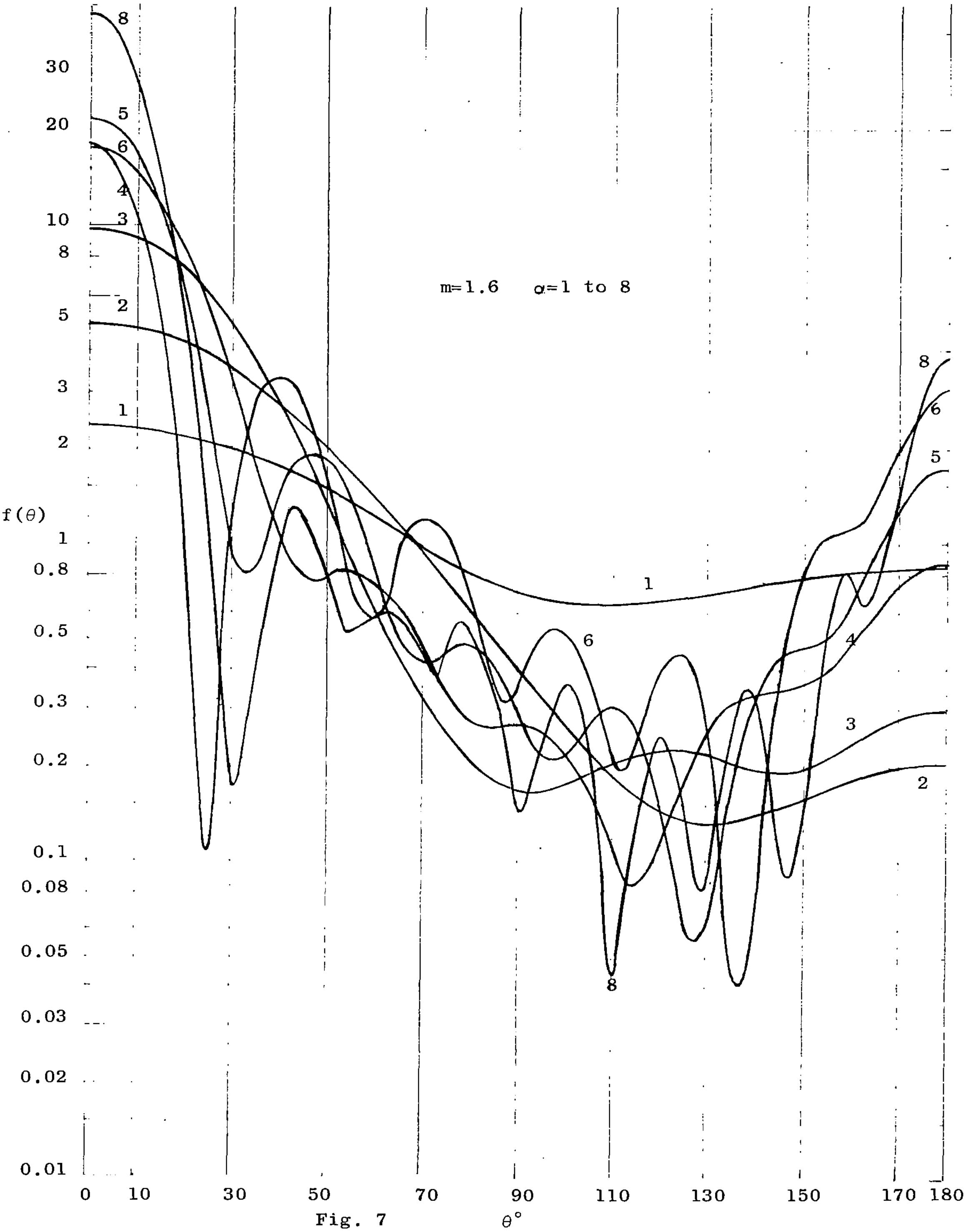

m=1.55 α= 1 to 6
f(θ)
θ°
20
10
8
5
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180
1
2
3
4
5
6

Fig. 6



m=1.6 α=1 to 8
f(θ)
θ°
30
20
10
8
5
3
2
1
0.8
0.5
0.3
0.2
0.1
0.08
0.05
0.03
0.02
0.01
0
10
30
50
70
90
110
130
150
170
180

Fig. 7

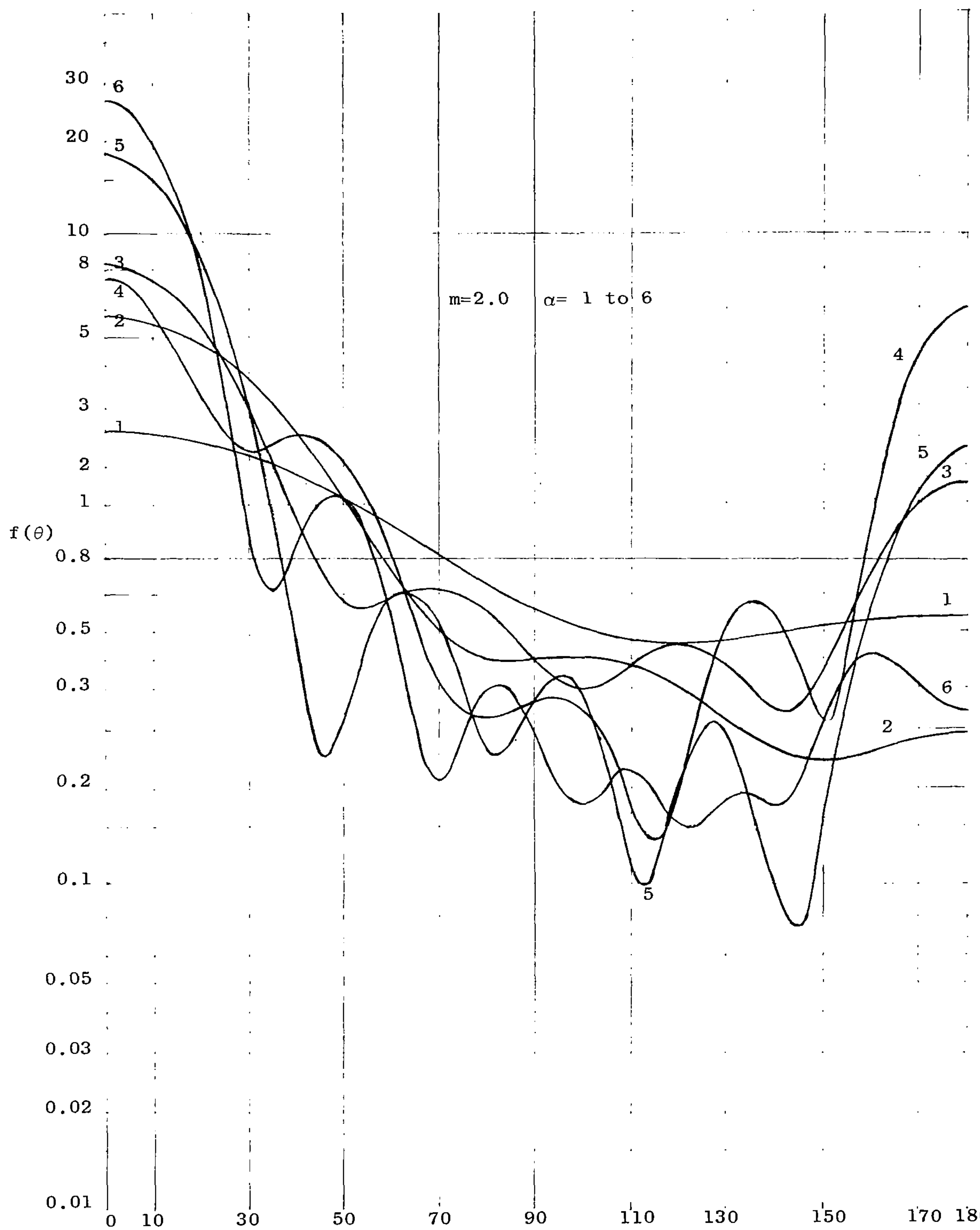


Fig. 8 $\theta°$

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