NASA CONTRACTOR REPORT

NASA CR-1039

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THE ROLE OF ENERGY IN DEFORMATION

by I. J. Gruntfest

Prepared by
GENERAL ELECTRIC CO.
Philadelphia, Pa.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1968

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ABSTRACT

This report is a review of exploratory analytical studies of the influence of energy conservation on the mechanical behavior of a variety of models of materials in diverse test situations. The results are presented and discussed so to display their physical implications rather than the details of their development. The work is generally complementary to the existing body of theory relating to mechanical behavior. However, consideration of the necessary coupling between the thermal and mechanical fields in deforming materials does suggest some new approaches to the problems of fracture in solids and flow stability in fluids. Some insights into the role of microstructure in determining the behavior of solids are also suggested.

The methods, some of which involve the use of electric analogs of the mechanical system, show how necking, yield and plastic flow may arise. In addition, scale effects are introduced which are likely to be of interest in geophysical phenomena.

THE ROLE OF ENERGY IN DEFORMATION

INTRODUCTION

We have been exploring the influence of energy conservation on the mechanical behavior of materials. The approach involves the analysis of the behavior of model systems. Our concern has been the physical understanding and the mathematical description of the responses of materials to stress and strain.

Historically, the energy balance has had an important place in the theory of the mechanical strength of solids in which the work of fracture is compared with the energy of new surface that is formed (1). However, the observed strengths of materials often greatly exceed those computed on this basis. It may be inferred, therefore, that energy consuming deformations generally precede fracture.

Our studies have roots in the discovery made by Copple, Hartree, Porter and Tyson (2) who showed that the failure of a dielectric in an alternating field occurs because the electric loss, which produces heat in the test piece, increases rapidly as the temperature increases. As a result, regenerative thermal feedback, a chain reaction effect, enhances the heating and can lead to thermal instability. We contend that the response of materials to mechanical stress is similarly exothermic and temperature sensitive.

Heat production in deforming materials has often been noted and is sometimes comspicuous (3-7). Furthermore, these thermal effects have been suspected of influencing mechanical behavior for some time (8)(9). In the current work, quantitative relations between the thermal and mechanical variables in model systems have been developed. This has been made possible by the use of modern computing machinery, special analog devices and by the careful selection of problems.

In the text below, the various results (10-18) are reviewed and integrated. These show that the effects of heating can indeed be decisive. Plausible physical rationalizations have been provided for many details of the mechanical behavior of real materials including time and size effects, yield, necking, plastic flow and fracture. Furthermore, these rationalizations are altogether complementary to the main stream theories of the physics of materials and involve the details of the microstructure which are now being widely studied.

While some of these results seem to be directly applicable, emphasis has been given to the physical insights that the analyses provide. These are broadly relevant and could, for example, influence the strategy and tactics of material development and design programs. In addition, they are pertinent to the stability of flows and to large scale events of geophysical interest such as mountain formation and meteorology. Experimental tests of the theory are also discussed.

II. DISCUSSION OF THE MODEL

Every mathematical description of the mechanical behavior of a material involves a commitment to a model. For example, the theory of elasticity implies a model of a solid consisting of elementary masses connected to one another by uniform linear springs. More than a century ago Maxwell pointed out that the simulation of the behavior of real materials could be improved by including viscous elements in the connections between the elementary masses. In this way, the imperfect reversibility of all real experiments could be described.

Visco-elastic models of the type suggested by Maxwell have been found very useful for describing the behavior of bread dough, rubber, synthetic organic polymers and other materials of commercial importance. In some of these cases relationships between the details of the model and the molecular structure of the material have been proposed (20). These applications and correlations have been particularly fruitful when the material under consideration is relatively soft. Application to the harder materials which are more familiar to structural engineers have been less successful.

This is precisely the expected consequence of energy conservation. All of the mechanical work done on a viscous element is converted to heat. If the amount of work involved is low, the process is essentially isothermal. When, on the other hand, the heat production is substantial, the coupling of the temperature field with the mechanical field in the material cannot be neglected.

This coupling is particularly strong in the case of the viscous element because of its temperature dependence. For the purposes of the present analyses the elastic elements are considered to be indifferent to the temperature. This compromise is not essential but it greatly simplifies the computations. Furthermore it does not offend our physical intuition since the relaxation phenomena represented by the viscous elements are biased diffusions which characteristically have an exponential dependence on temperature. On the other hand the reversible phenomena represented by the elastic elements depend on the potentials of intermolecular force which are much less sensitive to the temperature.

As a result of coupling, the temperature depends on the length of time that the heat source, the viscous process, has been operating. Thus, the apparent viscosity can be expected to be time dependent. To the extent that heat conduction occurs, the temperature distribution, and therefore the apparent viscosity, also depends on the thermal boundary conditions and the size of the sample.

In the purely viscous model, the temperature changes can also be expected to influence the stability of laminar flows and the development of cavitation in liquids (10-11). In viscoelastic materials, some of the time and size effects which are often observed in experiments with real materials can be rationalized in a straightforward manner. From this point of view, the equations describing the behavior of a single element are non-linear. It is not necessary, therefore, to assume elaborate spectra of relaxation times to account for the intricate behavior that is often observed in experiments.

Although the approach is in principle more general, the analyses are simplified by focussing attention on one dimensional problems as is the custom in many rheological discussions. Thus, the material is represented by an array of masses connected to one another in series through combinations of elastic and viscous elements. For sufficiently slow deformations, the inertia of the masses may be neglected. However, in this quasi-static situation, the connections between the masses cannot be lumped, as is done in classical rheology, because, even though the stresses are uniform, the temperature in the sample need not be. This simple feature of the model can account for some of the non-uniform strain distributions that are often observed in real deformations. We recall that the necking of a wire or filament is rarely if ever initiated close to the jaws of the testing maching which can function as heat sinks (12)(21).

At higher rates of deformation, or more accurately at higher rates of applied boundary velocity, the inertia of the elementary masses retards the propagation of the disturbance into the material. The earlier appearance of the stress on the connecting elements near the moving boundary produces local heating and enhanced local strain response. This can account for what has been called plastic wave propagation. Obviously, at sufficiently high rates of boundary motion, thermal instability and fracture can occur locally before the stress wave arrives at the more remote stations in the sample at which sensors may be located. This effect can complicate the experimental study of the influence of strain rate on the behavior of materials. These dynamic effects, which can occur in homogeneous materials, are also of interest in heterogeneous systems simulating, for example, imperfect crystals or composites.

III. DISCUSSION OF THE ANALYSIS

The simplest connection between the elementary masses that is of interest in this context is the purely viscous element (10). In the quasi-static deformation of such a model the uniform stress, σ, is related to the local velocity gradient, $\frac{\partial u}{\partial y}$, by the coefficient of viscosity, η, which is assumed to be temperature dependent. A plausible form for the temperature dependence is indicated by

$$\sigma = \eta_0 \frac{\partial u}{\partial y} e^{-a(T-T_0)} \qquad (1)$$

The rate at which heat is produced in this element is given by the product of the stress and the velocity gradient. In an isolated element, the heat production rate can be equated to the product of the heat capacity and the rate of temperature rise.

$$\sigma \frac{\partial u}{\partial y} = c \frac{\partial T}{\partial t} \qquad (2)$$

Combining these equations we have, for example,

$$\frac{\sigma^2}{\eta_0} e^{a(T-T_0)} = c \frac{dT}{dt} \qquad (3)$$

Solution of this equation for a constant applied stress shows that the temperature becomes unbounded in the finite time,

$$t_\infty = \frac{c\eta_0}{a\sigma^2} \qquad (4)$$

which depends on the stress level and the properties of the model. In other words, the typical adiabatic response of a viscous model to a constant stress in unstable.

Notice that the constant strain rate case for which the adiabatic energy balance equation would have the form

$$\eta_0 \left(\frac{\partial u}{\partial y}\right)^2 e^{-a(T-T_0)} = c \frac{\partial T}{\partial t} \qquad (5)$$

has very different solutions. It turns out that in the time t_∞, the stress decays to one half its initial value. While the temperature can become very high it remains finite. The threat of instability and the distinction between the constant stress and constant strain rate cases are among the elementary consequences of energy conservation in imperfectly reversible processes.

In general, the connections between the mass elements are not isolated and heat can flow between contiguous elements and to the boundaries of the sample. The quasi-static energy balance condition then becomes a partial differential equation, for example

$$\frac{\sigma^2}{\eta_0} e^{a(T-T_0)} = c\frac{\partial T}{\partial t} - k\frac{\partial^2 T}{\partial y^2} \tag{6}$$

in which y is the single space coordinate and the heat capacity, c, and the thermal conductivity, k, are assumed to be independent of temperature. Closed form solutions of this equation have not been obtained. However, various digital and analog procedures are applicable.

Unlike equation 3, equation 6 has steady solutions when the heat loss by conduction can balance the heat generation rate. This is possible when the thermal relaxation time of the sample is short relative to the adiabatic catastrophe time, t_∞. If the boundaries are isothermal and separated by the distance l, a characteristic thermal relaxation time is given by

$$t_c = \frac{c\ell^2}{k} \tag{7}$$

That is, stability depends on the non-dimensional time ratio

$$\mathcal{S} = \frac{t_c}{t_\infty} = \frac{a\sigma^2\ell^2}{k\eta_0} \tag{8}$$

It is on this basis that the energy conservation condition leads to a dependence of the maximum stress for stability on the size of the sample.

The abbreviated discussion given above is not intended to suggest that the temperature coefficient of viscosity, a, is constant and independent of the temperature. In equation 1 the temperature dependent exponential should be of the form suggested by the concept of the energy of activation for the process. In these terms

$$a \doteq \frac{E_A}{T T_0} \doteq \frac{E_A}{T_0^2} \tag{9}$$

which indicates a strong dependence on the initial temperature. The constant E_A, the ratio of the energy of activation to Boltzmann's constant, is as clearly related to the physics of the material as the viscosity itself. However, the results obtained by the use of "a" are similar to those obtained with E_A and the discussion is then somewhat simpler.

The application of the above analysis to the quasi-static viscoelastic case has also been made (13). Generally, the same type of numerical analysis has been used as for the purely viscous model. However, in the dynamic case, the velocity gradient becomes a function of the space variable so that the computation is more arduous. It is in this connection that the analog methods discussed below are particularly useful. In the static problems the results of the analog method are the same as those of the numerical computation.

IV. DISCUSSION OF THE ELECTRIC ANALOG METHOD

It is well known that the responses of selected electric networks are altogether analogous to those of models of materials of the type described above. In these circuits, if the voltage is the analog of stress, a capacitor is the analog of an elastic element, a resistor is the analog of a viscous element, an inductor is the analog of a mass and the current is the analog of the strain rate. One distinctive feature of our systems is the use of temperature sensitive resistors (thermistors) to simulate the temperature dependent viscosity of the model material. The equations for the thermistor

$$V = I R_0 e^{-a(T-T_0)} \tag{10}$$

$$\frac{V^2}{R_0} e^{a(T-T_0)} = c\frac{\partial T}{\partial t} - k\frac{\partial^2 T}{\partial y^2} \tag{11}$$

are entirely analogous to equations 1 and 6 above. Other non-linear circuit elements are also available and could be used in conjunction with the systems under consideration.

The network shown in figure 1, with the thermistors in thermal contact with one another, simulates the model described in section II for which the connections between the elementary masses are an elastic element in series with a viscous element (Maxwell Model). For the quasi-static case the voltages along the line (numbered stations) are always equal. The shunt capacitors may then be lumped but not the thermistors. A current (strain rate) applied between A and B would charge the lumped capacitor and increase the line voltage until it became high enough so that the sum of the thermistor currents became equal to the total applied. If this current is low enough so that the heating of the thermistors is negligible the network behavior would be analogous to that of a linear Maxwell model and a steady voltage would develop.

At somewhat higher currents the heating of the thermistors would gradually lower their resistance and the line voltage would be reduced. The plot of line voltage against time would have a maximum corresponding to a yield point. If for some reason, for example heat conduction, one of the thermistors were to become hotter than the others, the current would tend to concentrate in that branch corresponding to strain rate concentration or necking. If, then, the energy stored in the capacitor were sufficient, catastrophic local heating could follow yield corresponding to local, perhaps brittle, fracture.

Since in the static case, the voltage drops across the inductors are zero and the capacitors can be lumped, this network is more complicated than in necessary. However, in the dynamic case, there is a voltage drop across the inductors, simulating the inertia of the elementary masses, and the capacitors are not equally charged. That is, the stress level (voltage) depends on the space variable as well as the time.

The network shown in the figure is an example of an electric transmission line. The response of this line to electrical excitation is precisely analogous to the response of the model material to mechanical excitation. The pair of partial differential equations for the voltage and current

$$\frac{\partial V}{\partial y} = L \frac{\partial I}{\partial t} \qquad (12)$$

$$\frac{\partial I}{\partial y} = C \frac{\partial V}{\partial t} + \frac{V}{R} \qquad (13)$$

are to be compared with those for stress and strain rate

$$\frac{\partial \sigma}{\partial y} = \rho \frac{\partial u}{\partial t} \qquad (14)$$

$$\frac{\partial u}{\partial y} = E \frac{\partial \sigma}{\partial t} + \frac{\sigma}{\eta} \qquad (15)$$

While some studies (14) were made using the line as it is shown schematically in figure 1 there are some objections to this system. In the first place an inductor, being typically a coil of wire on an iron core, not only has resistance but tends to have a rating dependent on the current that is being passed. In addition, it is only at considerable expense that well matched capacitors and inductors can be obtained. Furthermore, the time scale of the experiments depends on the ratings of these components and this is not always convenient.

These difficulties were overcome in later work (11)(15)by using electronic integrators to simulate the inductors and capacitors. This simulation depends on the fact that the current through an inductor is proportional to the time integral of the applied voltage and the voltage on a capacitor is proportional to the time integral of the current. Furthermore, the electronic integrators are not only satisfactorily accurate but the proportionality factors can be selected arbitrarily and made to depend on the local values of the temperature, stress or strain. This versatility has not been exploited in the work done so far.

The schematic of the circuit equivalent to that shown in figure 1 is shown in figure 2. The symbols are those used in the literature of analog computation. The power capabilities of the amplifiers and the size of the thermistors that were used were such that the more dramatic non-linearities could not be displayed. However, this is not a necessary objection to the use of the system.

The discussion of the analog so far applies to the responses of homogeneous materials subjected to arbitrary initial and boundary conditions. By connecting two transmission lines, one of which is energized but in equilibrium, the impact of a projectile on a target can be simulated and some of the arbitrariness of the initial conditions could be avoided. Lines representing composite or heterogeneous systems can also be applied. These give some special insights into the importance of microstructure to mechanical behavior.

We note in passing that the typical transmission line problem or the typical deformation problem involves the application of excitations only at the boundaries. In such simple problems as Poiseuille flow, excitation is applied at intermediate points in the material. The analog is well adapted to deal with this situation.

V. DISCUSSION OF RESULTS, QUASI-STATIC PROBLEMS

The behavior of the model may be inferred either from numerical solutions of the non-linear differential equations of from the responses of the analogs. The general implications of thermal feedback are exemplified most simply by the behavior of a single thermistor which represents the quasi-static behavior of a slab of homogeneous viscous material in shear. Figure 3 shows the history of the current in response to various constant applied voltages. When the voltage is low enough so that the heating is negligible this element behaves like an ordinary, constant resistance. At somewhat higher voltages the current becomes time dependent. If the heat losses from the thermisotr can offset the power that is dissipated, steady states are possible. At a critical value of the voltage, the increase of the current and temperature become catastrophic.

In a material in which heat conduction is the only mode of heat transfer, as in solids, the thermal catastrophe can be averted by fracture. If convective heat transport is possible, as in liquids, the response can be a transition to turbulent flow in which the heat transfer rates are enhanced. In a volatile liquid, cavitation is an energy consuming process which also averts the thermal catastrophe.

Figure 4 shows the voltage history in response to various constant applied currents. Here, at the first instant, the heat production is proportional to I^2R_o. Later, as the thermistor warms up, the resistance becomes lower so the heat production is reduced. In this analog of the constant strain rate case, the temperature may get high enough to destroy the material but the degenerative nature of the feedback makes it easy to distinguish from the constant stress case.

The static Maxwell model is simulated by a circuit consisting of a thermistor in paralell with a capacitor as shown in figure 5. At low values of the applied current the voltage or stress shows the typical linear response. At higher currents, the steady voltage is reduced because the temperature of the thermistor rises. A voltage maximum can also appear corresponding to a yield point. Notice the low sensitivity of the steady voltage to the current. This is somewhat similar to what is called plastic flow.

As shown in figure 6 relaxation oscillations can also be generated corresponding to stick-slip behavior. Under some conditions the energy stored in the capacitor while the thermistor is heating up is sufficient to catastrophically destroy the thermistor in a manner analogous to brittle fracture.

Figure 7 represents the behavior of an elastic element in paralell with the viscous element (Kelvin-Voigt Model). Figure 8 shows the behavior of a three element model. The shapes of these various curves are familiar to students of the deformation of materials. Similar results were obtained by numerical methods and discussed in reference 13.

In typical quasi-static experiments involving large deformations, the geometry of the test piece is not constant. The simple results discussed above apply to the shear of infinite slabs and do not take this effect into account. The influence of shape changes and heating under quasi-static adiabatic conditions were discussed in two reports. In one of these the responses of a cylinder to axial compression and elongation at constant rate and at constant load were computed (16). A typical result is shown in figure 9. This shows the response of a viscous material to a constant rate of compression. Here the destabilizing effect of the temperature rise can be offset by the increase in the cross section of the test piece. This result is likely to be relevant in material forming processes.

A more complicated, but practically interesting situation is produced by the blow of a hammer on the cylinder. In this case, the duration of the experiment is short in comparison with the time required for the establishement of stress equilibrium, but long in comparison with the thermal relaxation time. The stress depends not only on the instantaneous cross section of the piece, but also on the instantaneous velocity of the hammer which is a variable and becomes zero at the end of the experiment. A typical result is shown in figure 10. This study (17) shows that the response appears to be almost perfectly elastic at low hammer velocities, but almost perfectly viscous for heavier blows. This kind of non-linearity of response is very commonly observed in practice. The reversible part is due to the combined elasticity of the hammer and the test piece, but is dominated by that of the hammer. This variable deformation rate situation is easy to produce in the laboratory and is more likely to occur in nature than the constant deformation rate.

Other static deformations of homogeneous models of materials can be studied by straightforward extensions of the analyses discussed above and some have been mentioned in the referenced articles. Heterogeneity in the model can also be an important consideration in the static case. For example, the relevant characteristic thermal relaxation time of a multilayer configuration in which viscous and elastic materials alternate can depend on the thickness of a single viscous layer and not directly on the gross volume fraction of the viscous material. As a result the static mechanical stability of the system can be expected to depend on what might be called the microstructure (18).

VI. DISCUSSION OF RESULTS, DYNAMIC PROBLEMS

In the dynamic problem the stress depends on the space variable as well as the time. This leads to a more complicated analytical situation. However, the equations for constant values of the model parameters in the one dimensional case have had a great deal of attention in connection with the propagation of electrical disturbance in transmission lines. These analyses are not directly applicable when the coefficients have the complicated dependence on time which arises from the energy condition. However, the idiom developed in the constant coefficient case is useful. For example, in the electrical case one speaks of the attenuation and distortion of signals, the filtering characteristics and Q (the quality factor) of the line. These have precise counterparts in the mechanical system.

Consider first the response of a slab of purely viscous material to the application of a shear velocity at one of its boundaries. This is analogous to the behavior of the network of figure 1 with the storage elements, the capacitors, left out. The relevant constant coefficient equation:

$$L\frac{\partial I}{\partial t} = \frac{\partial}{\partial y} R \frac{\partial I}{\partial y} \qquad (16)$$

is recognized as the diffusion equation and is analogous to

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial y} \eta \frac{\partial u}{\partial y} \qquad (17)$$

The propagation and attenuation in the material depends on the values of the coefficients. When the level of excitation is low and the heating of the thermistors is negligible the linear theory is applicable. At higher levels of excitation the effective viscosity is reduced. The local values of the "velocity" then decrease and the attenuation increases. The nature of the disturbance that reaches an interior station or the far boundary departs from that predicted by linear theory. Examples are discussed in reference (11).

A particularly interesting situation develops if the applied velocity is sinusoidal. This generates a velocity wave with a nominal depth of penetration which depends on the frequency as well as the material properties. As the frequency increases, this depth decreases which increases the local heat production rate. The temperature rise reduces the viscosity which further reduces the depth of penetration. Thus, another regenerative feedback loop develops which must lead to thermal instability. In a real physical system, a reasonable response to this instability would be cavitation which can consume a large amount of energy and avert the thermal catastrophe. A possible explanation of acoustic cavitation thus arises.

In the dynamic Maxwell model, two propagation modes can occur. One of these is the elastic mode which is lossless and characterized by a velocity $(LC)^{-\frac{1}{2}}$. The time and distance are related by

$$t_E = \sqrt{LC} \cdot y \tag{18}$$

The other is the diffusion mode which is lossy and dispersive. In this mode the time and distance are related by

$$t_D = \frac{L}{R} y^2 \tag{19}$$

Now we notice that close to the source of excitation; that is, for y small, t_D is less than t_E. Thus, the initial response is essentially viscous. Farther from the source t_E is less than t_D. Thus a sensor remote from the excitation would show an initially elastic response. At an intermediate station the elastic process would overtake the viscous process. Effects of this sort can be observed in experiments.

One of the reasons for the above development is that it leads to a new non-dimensional time ratio which is a property of the material and its size. We have

$$\Gamma^2 = \left(\frac{t_E}{t_D}\right)^2 = \frac{R^2 C}{L y^2} \doteq \frac{\eta^2}{\rho E y^2} \tag{20}$$

which emerges as a natural property of the material which is related to the Q of the analogous transmission line. This is a vibration damping index. Its value determines whether or to what extent "ringing" occurs in the sample.

A number of "computations" are described in reference 15. One of these relates to more or less the same problem discussed earlier by von Karman and others (21) in terms of plastic wave propagation. Typical results are shown in figure 11. The use of the analog eliminates the necessity for the use of artifical constitutive or state equations for the material. Since authentic physical concepts are involved these methods have some advantages over methods which use the purely mathematical concept of plasticity.

Another feature of the analysis is the development of a dependence of the effective Q of the line on the amplitude of the excitation (14)(15)(17). Some of Mason's experiments with metals (23), show that above a critical amplitude this dependence becomes strong. Further attention is now being given to this point of contact between the analysis and experiment.

The dynamic behavior of the model consisting of alternate elastic and viscous layers has some special interest in connection with the behavior of imperfect crystals. When, in such a model, an instability develops in one of the viscous layers, a stress relief wave propagates from that layer toward the boundaries of the sample. For a time related to the acoustic velocity and the size of the sample, the heat source in the layer is, in effect, turned off. That is, the feedback loop is opened. If during this period the layer can cool, the recovery from the catastrophe can be essentially complete.

The occurrence of the non-lethal catastrophe depends on the relative magnitude of the acoustic transit time in the sample and the thermal relaxation time of the layer in which the instability occurs. The non-dimensional ratio of these times suggests a criterion for ductility in the material. Thin viscous layers, which have short thermal relaxation times are conducive to ductility. This arguement, developed in greater detail in reference (18), leads again to a possible connection between microstructure and mechanical behavior. The fact that dramatic thermal events can be observed to accompany the formation of slip band in metals, tends to confirm the relevance of this analysis (6).

CONCLUDING DISCUSSION

While a few correlations between the results of analyses and experiment have been cited in the text above, many others are possible. For example, heating effects can occur in fatigue tests. These can not only be important by themselves but also can increase the chemical reactivity of the material with its environment to produce secondary effects. To the extent that mechanisms of the failure process are involved, contributions to the theory of reliability are suggested.

The important ideas of work hardening and work softening are given a richer physical interpretation. In this connection, the remarkable durability of rubber in tires may depend to some extent on the fact that the restoring forces in this material increase as work is done and the temperature increases.

The analyses also suggest a revised attitude toward what are called properties of materials. The non-dimensional parameters or similarity criteria which arise are of particular interest in this connection. Even in conventional terms, what may be called the viscosity of solids is given an important role.

Given the model, there can be no question about the validity of the analyses. The problem is the identification of the model with the diverse realities of the situation. As the model becomes more realistic, and therefore more complex, the analyses become more difficult. Then the physical insights that are developed by examining the "stripped down" models may still be useful.

While in the discussion, the versatility of the approach is emphasized, its application is not without hazard. For example, one of the key ideas in the development is that of regeneration. Regenerative processes entered the theory of material failure many years ago in connection with the direct current, electrical breakdown of gases (24). There, the regenerative process and instability involves the development of avalanches of electrons. In that case the above analysis cannot be applied with confidence because of the difficulity of defining a temperature in mixtures of electrons and atoms. This complication does not arise in the low voltage experiments of Copple et al (2) or in the mechanical experiments which are considered here.

VIII. ACKNOWLEDGEMENTS

Special thanks are due Mr. M. Rosche' and Dr. Howard Wolko of the office of Advanced Research and Technology of the National Aeronautics and Space Administration, who, in their role as monitors of this program, were a constant source of encouragement and guidance. Our colleagues Dr. J. Stewart and Mr. T. Bechtel of the General Electric Company also provided important support and counsel.

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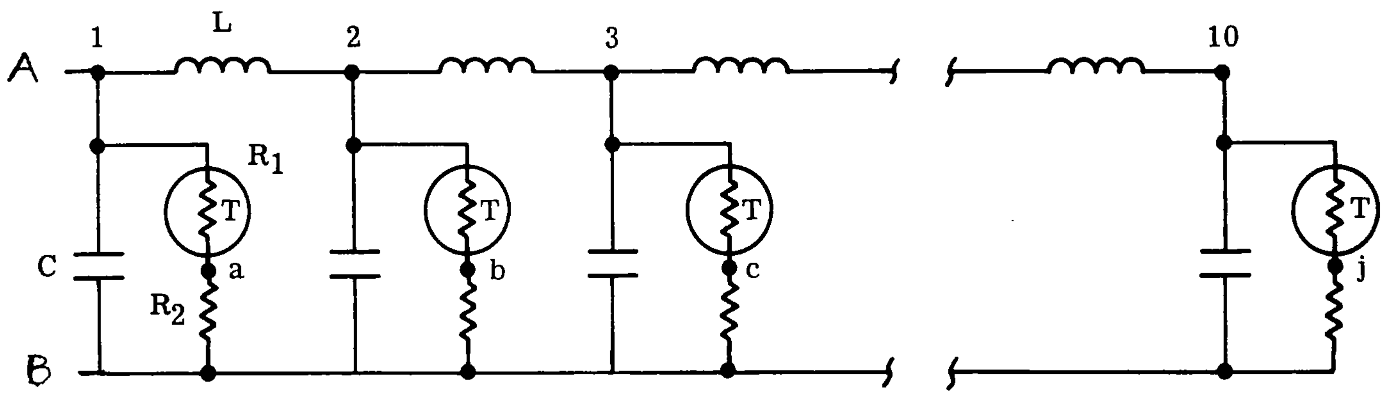

Figure 1.- Electric circuit analog of homogeneous, one dimensional Maxwell Model Material taken from reference 14. The configuration represents a slab excited at the left hand boundary and held fixed at the right.

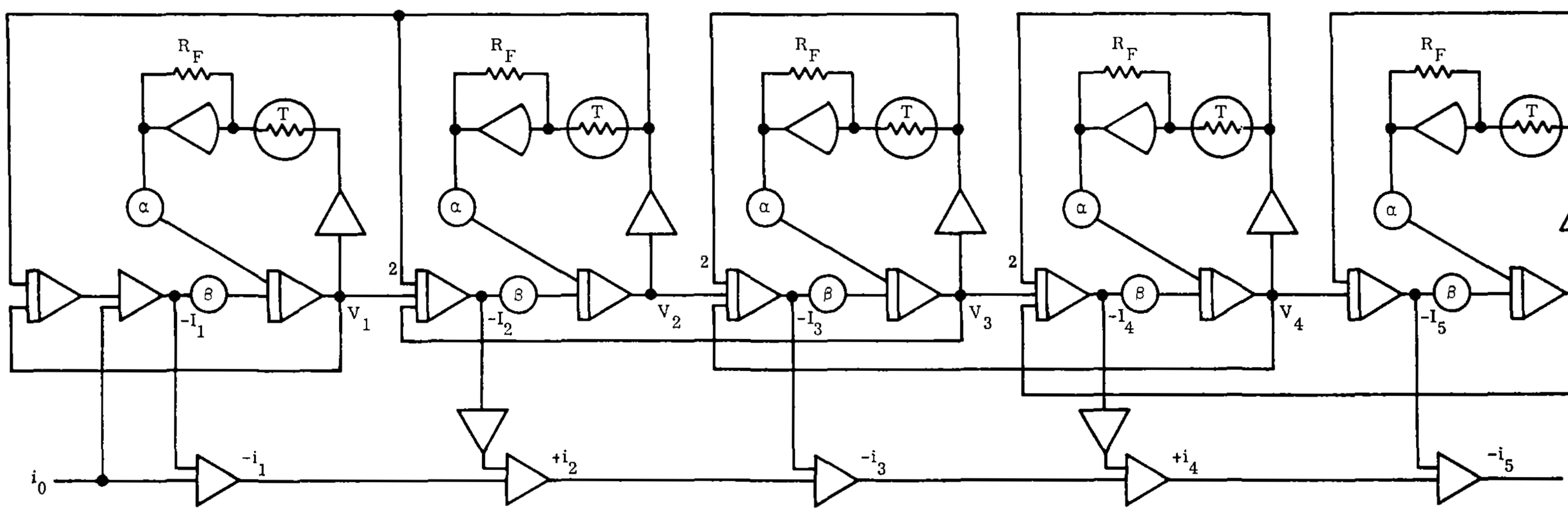

Figure 2.- Simulation of the circuit of figure 1 taken from reference 15.

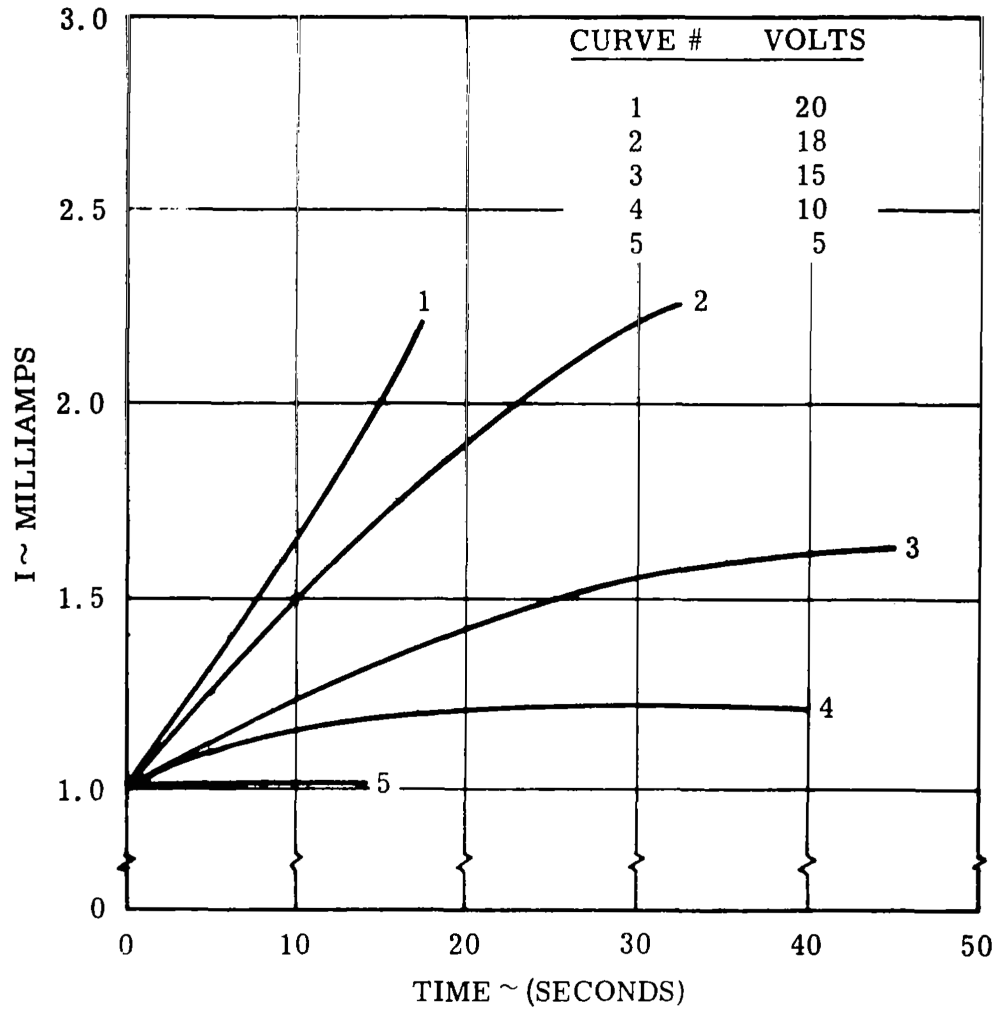

Figure 3.- Time dependent current through a thermistor at various constant voltages showing the effect of regenerative feedback.

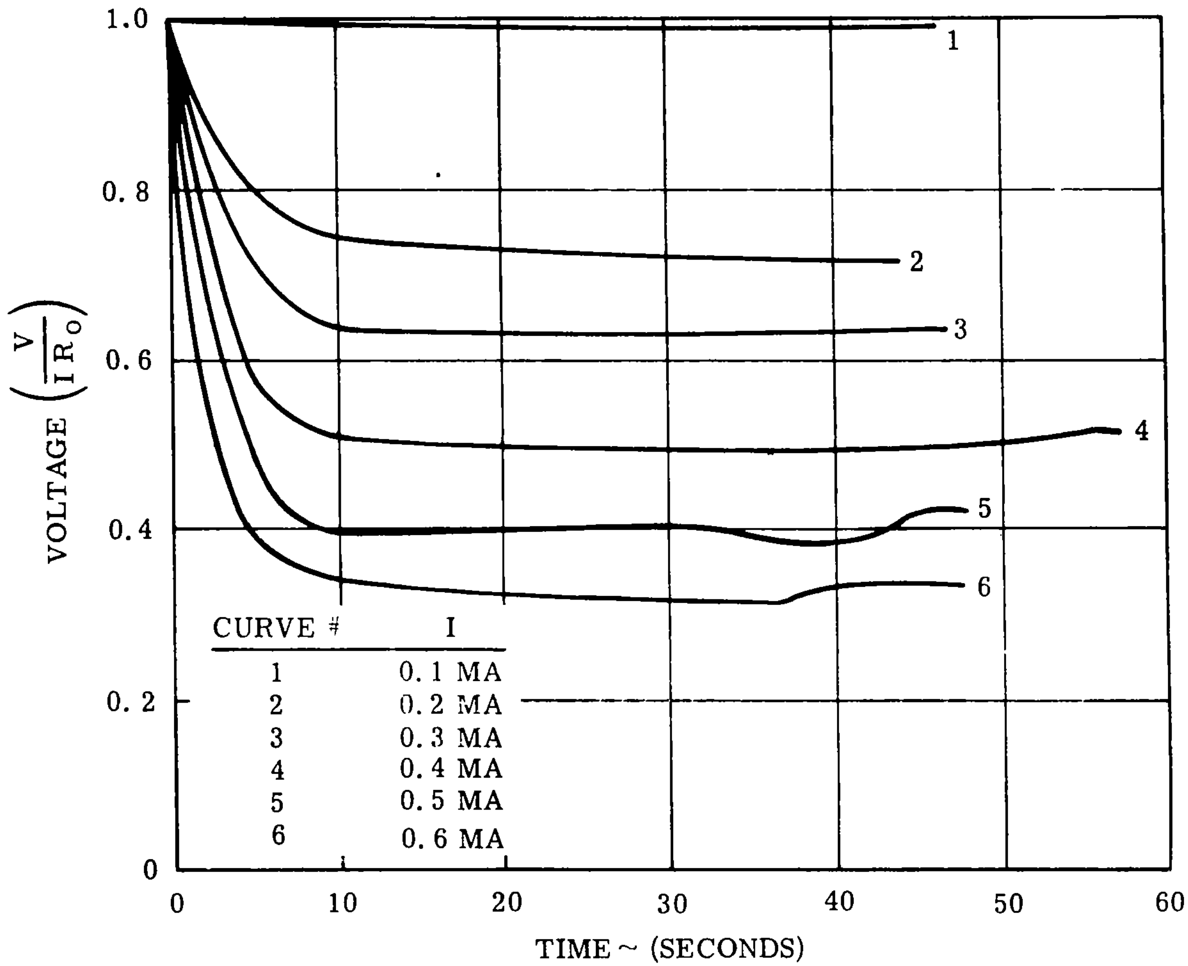

Figure 4.- Time dependent voltage across a Thermistor at various constant currents.

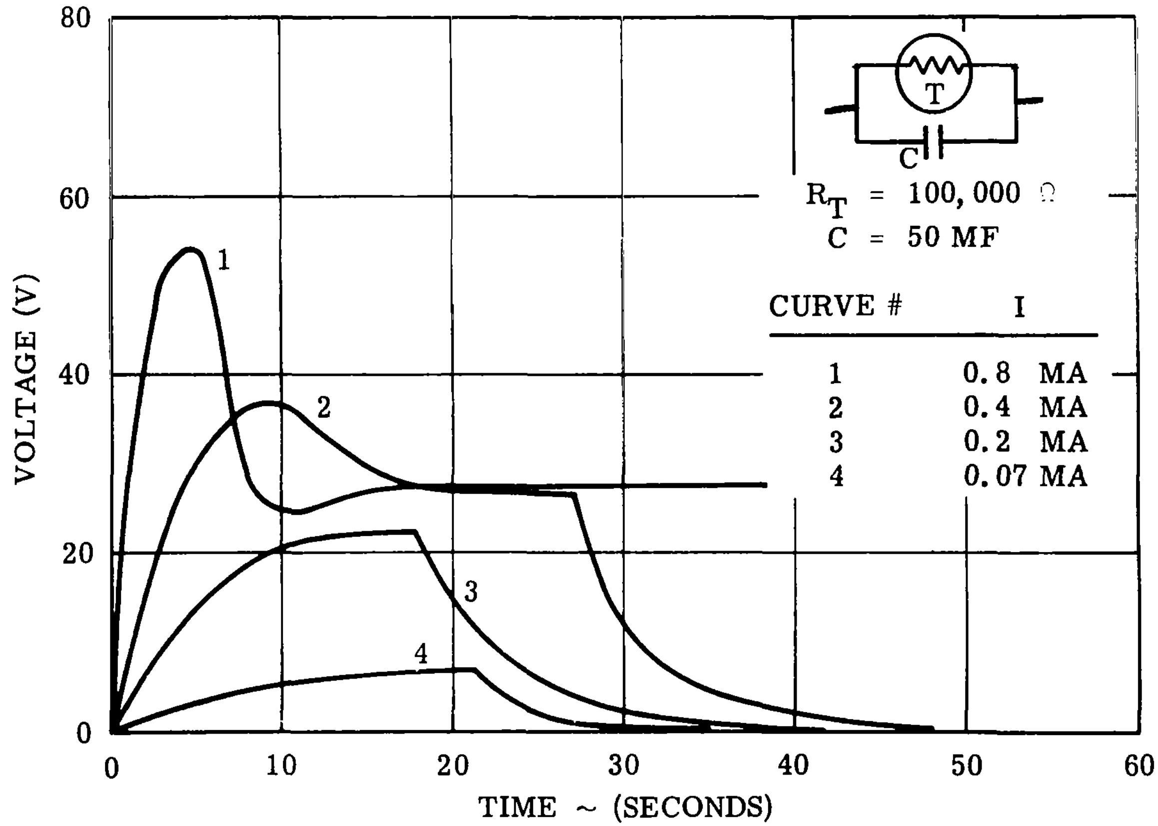

Figure 5.- Behavior of a thermistor analog of a Maxwell Model Material at various constant currents showing phenomena resembling yield and plastic flow.

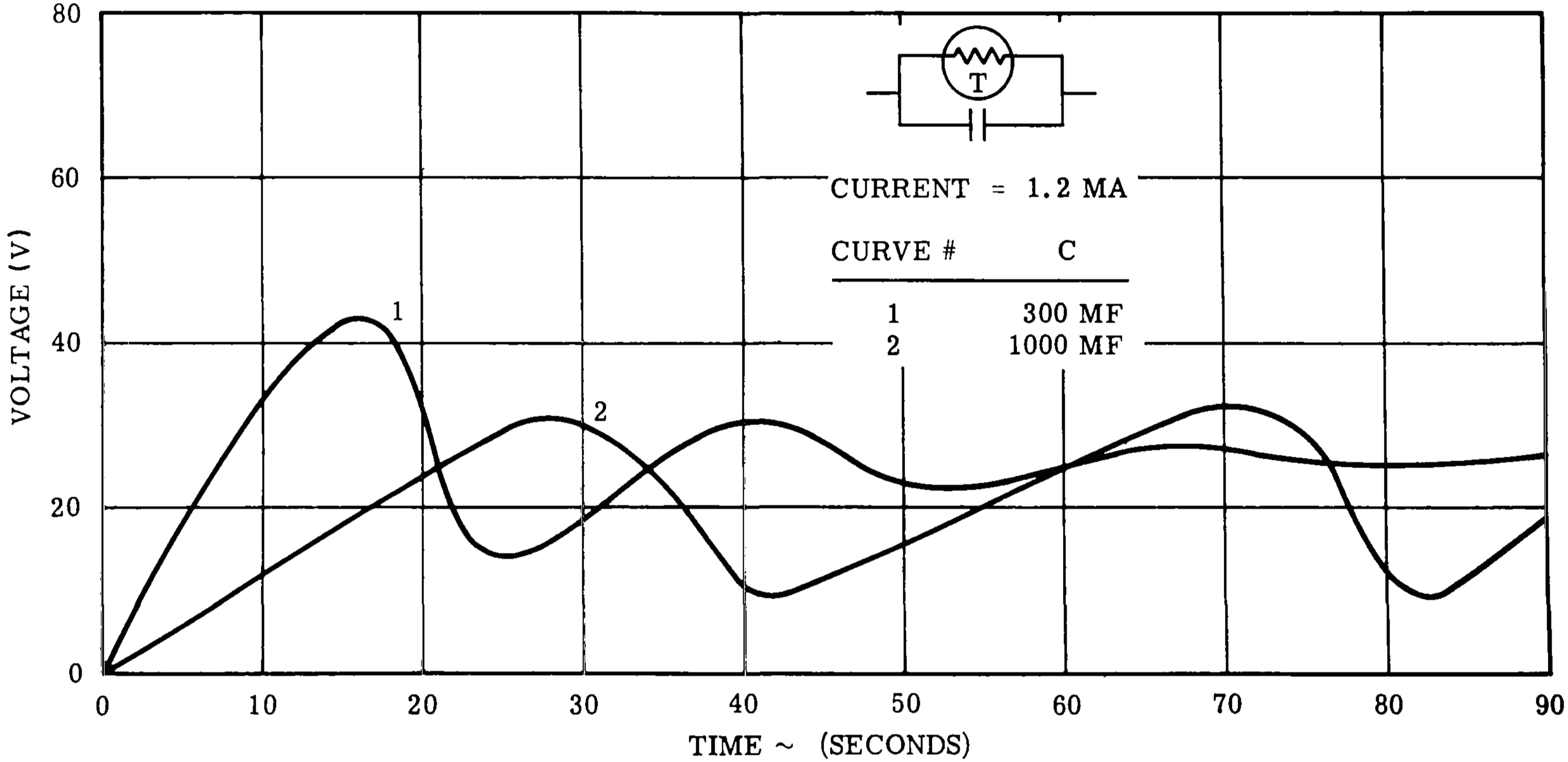

Figure 6.- Same as figure 5 showing relaxation oscillations.

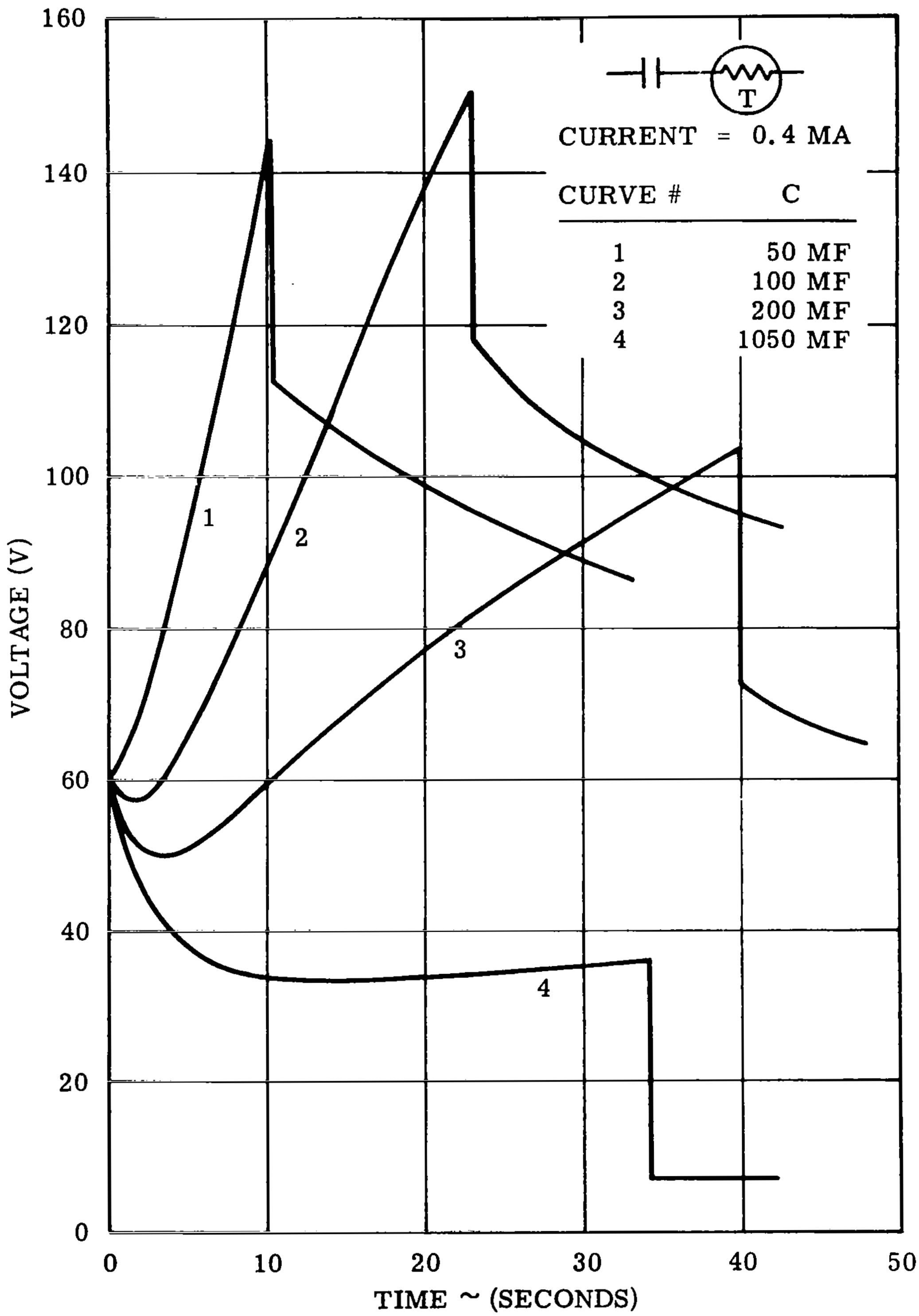

Figure 7.- Time dependent voltage across a thermistor analog of a Voigt model.

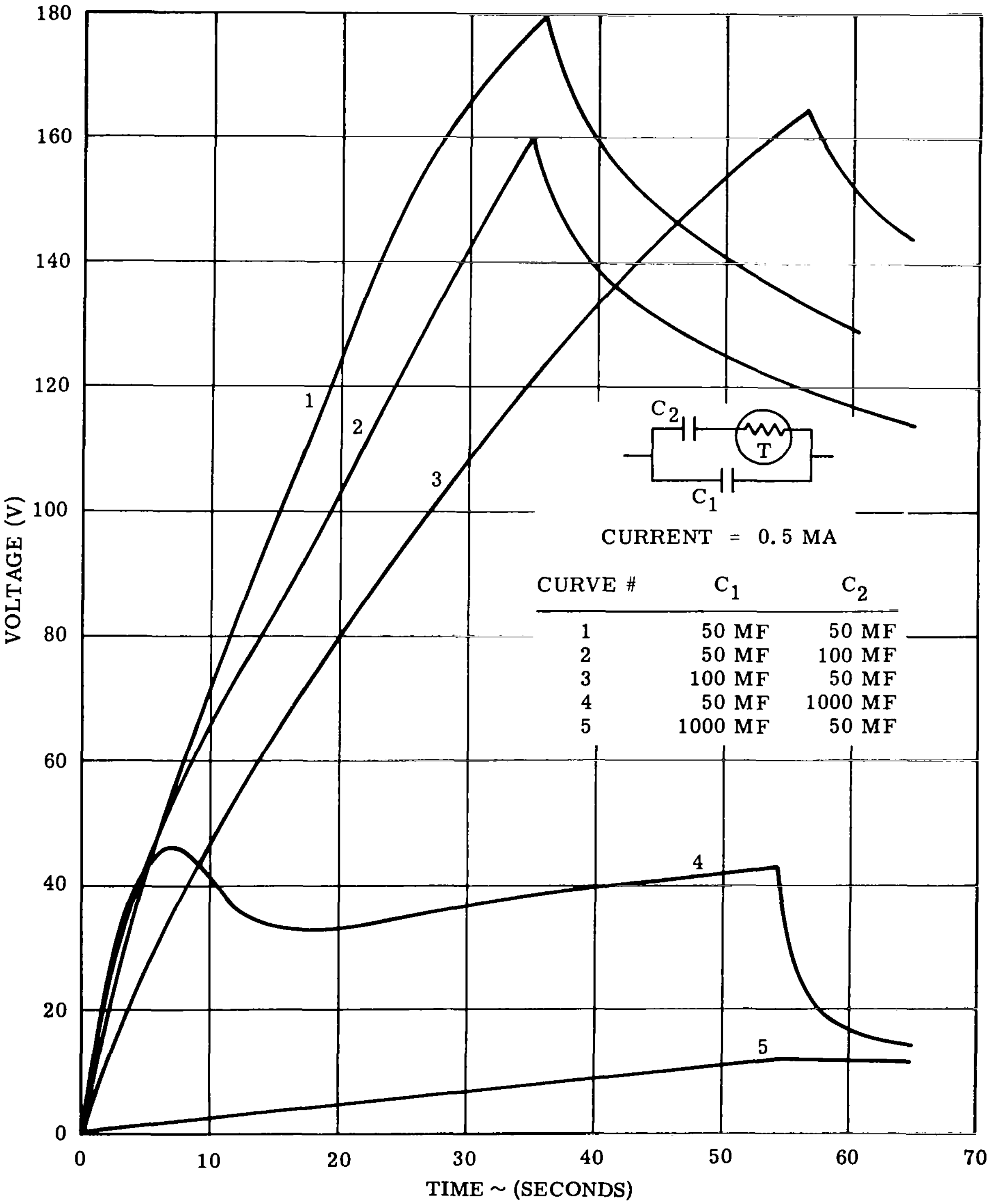

Figure 8.- Time dependent voltage across a thermistor analog of a three element model.

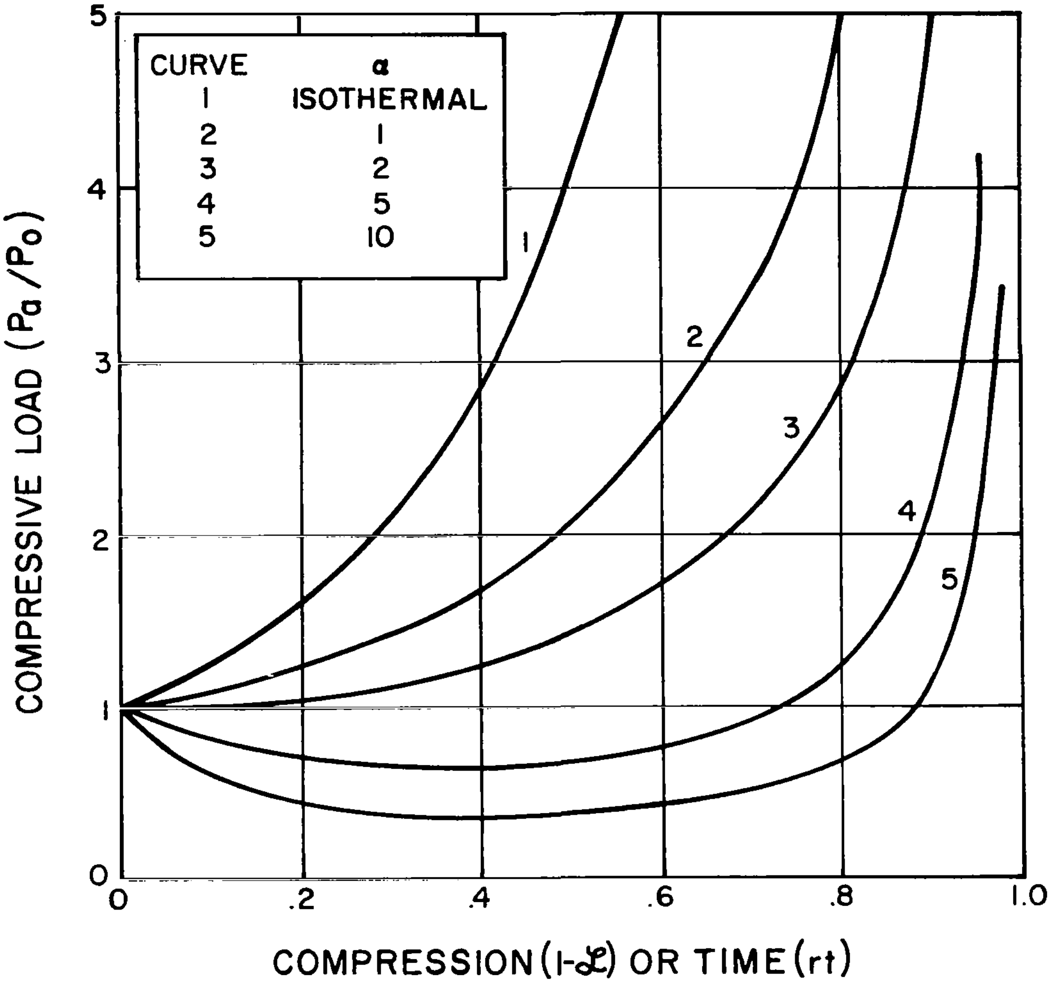

Figure 9.- Behavior of a viscous cylinder subjected to various constant rates of axial compression showing how the force decrease due to thermal softening at high strain rates can be offset by the increase in the cross section (from ref. 16).

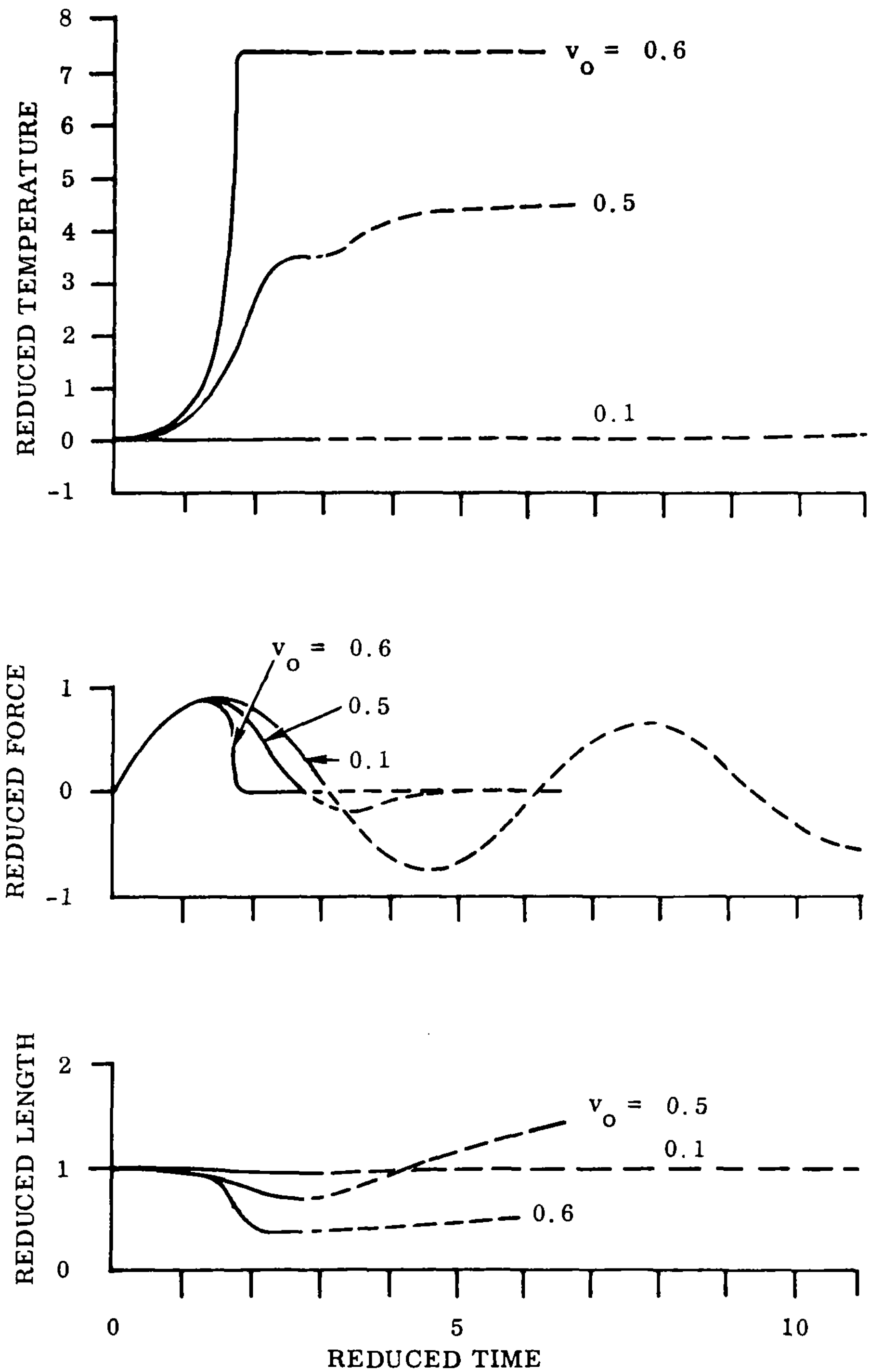

Figure 10.- Plots of temperature, force and length as functions of time for a viscoelastic cylinder subjected to hammer blows of various severities. While the material shows isothermal, essentially elastic behavior at low levels, damping is conspicious at the higher levels (from ref. 17).

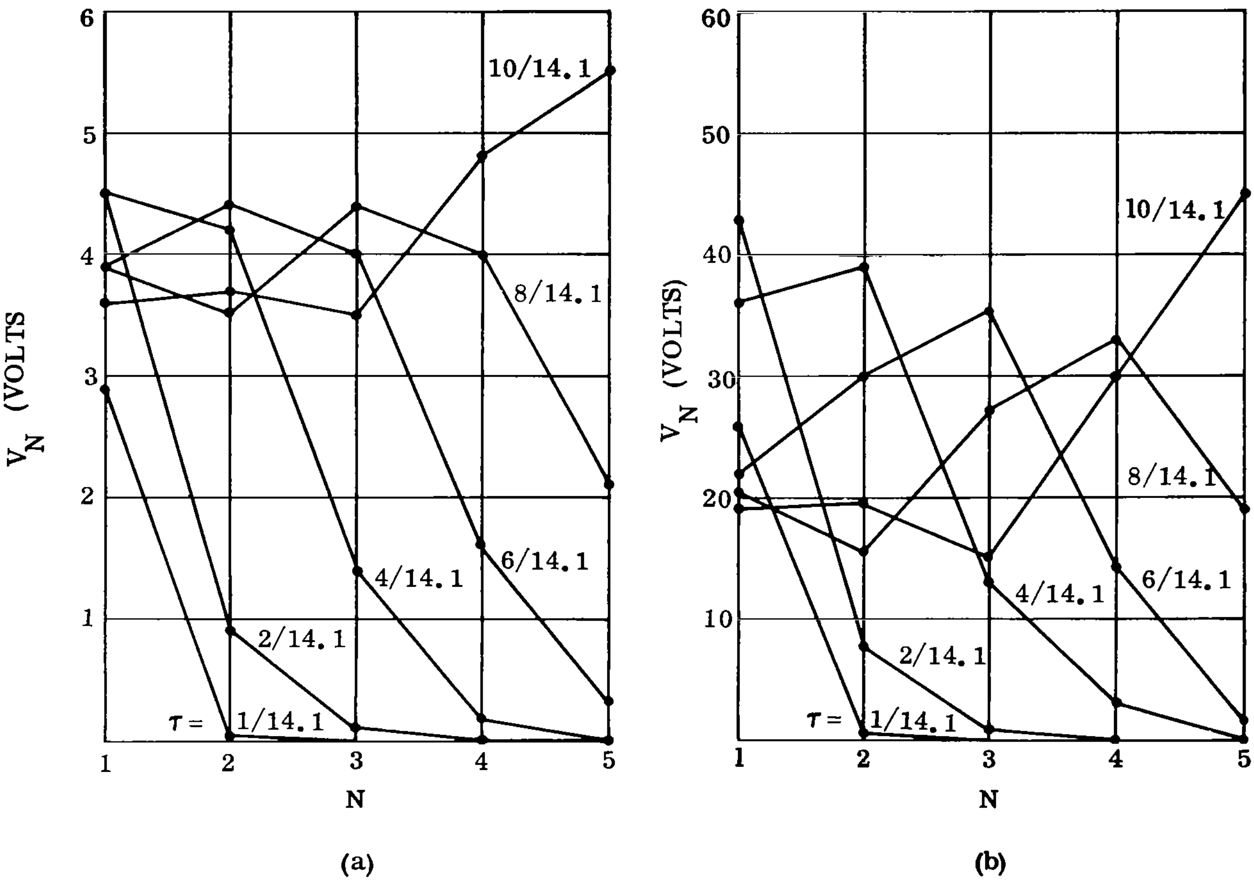

Figure 11.- Plots of the stress distribution in a slab of viscoelastic material at various times after the application of a velocity step at one boundary. The left hand curves show the essentially linear evolution of the wave at low levels of excitation. The other shows the stress relief at the input end due to heating when the level is higher (from ref. 15).