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DYNAMIC STABILITY OF SPACE VEHICLES

VOLUME XIV - TESTING FOR BOOSTER PROPELLANT SLOSHING PARAMETERS

By D. M. Eggleston

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FOREWORD

This report is one of a series in the field of structural dynamics prepared under contract NAS 8-11486. The series of reports is intended to illustrate methods used to determine parameters required for the design and analysis of flight control systems of space vehicles. Below is a complete list of the reports of the series.

Volume I	Lateral Vibration Modes
Volume II	Determination of Longitudinal Vibration Modes
Volume III	Torsional Vibration Modes
Volume IV	Full Scale Testing for Flight Control Parameters
Volume V	Impedence Testing for Flight Control Parameters
Volume VI	Full Scale Dynamic Testing for Mode Determination
Volume VII	The Dynamics of Liquids in Fixed and Moving Containers
Volume VIII	Atmospheric Disturbances that Affect Flight Control Analysis
Volume IX	The Effect of Liftoff Dynamics on Launch Vehicle Stability and Control
Volume X	Exit Stability
Volume XI	Entry Disturbance and Control
Volume XII	Re-entry Vehicle Landing Ability and Control
Volume XIII	Aerodynamic Model Tests for Control Parameters Determination
Volume XIV	Testing for Booster Propellant Sloshing Parameters
Volume XV	Shell Dynamics with Special Applications to Control Problems

The work was conducted under the direction of Clyde D. Baker and George F. McDonough, Aero Astro Dynamics Laboratory, George C. Marshall Space Flight Center. The General Dynamics Convair Program was conducted under the direction of David R. Lukens.

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1/INTRODUCTION

The stability, control, and structural-loading analysis of large liquid-fueled boosters cannot be accomplished without some mathematical representation of the effects of propellant slosh, since propellants constitute by far most of the vehicle weight. Furthermore, the very important tradeoff between vehicle weight and strength cannot be pushed further than the mathematical model will allow.

An accurate propellant slosh mathematical model permits vehicle design close to the best strength-to-weight ratio by eliminating the need for the large safety factors required by lack of understanding of the phenomena involved.

Despite advances in theoretical techniques it is not possible at present to completely predict the parameters of a mathematical model of propellant slosh for a given tank. It is very difficult even to provide instrumentation and telemetry on a vehicle to evaluate propellant slosh forces and moments during flight, because the effects of propellant motion cannot be adequately separated from those resulting from aerodynamics, elastic motion, and other causes. For these reasons propellant slosh testing of reduced or full-scale physical models under controlled laboratory conditions is an attractive alternative, just as wind tunnel testing is important to aerodynamic analysis.

With application of known methods to the design of boosters such as Jupiter, Atlas, Titan, Thor, and Saturn — and upper stages such as Agena, Centaur, and the SIV-B — much has been learned, and a considerable number of slosh model studies have been performed. Slosh model tests have been used to investigate selected phenomena such as spill-over into vents, flow to pump inlets, bubble formation, longitudinal modes, roll axis rotational modes, and damping provided by flexible baffles.

The preferred approach to analysis and simulation of booster propellant slosh begins with use of the linearized inviscid flow theory to obtain parameters for a pendulum or spring-mass model of the basic fluid motion. Digital programs for generating these parameters for body-of-revolution tanks of arbitrary shape are available.[1,2] Such a model is usually quite accurate for small angle slosh in smooth-walled tanks without flow obstructions, where the assumptions of the theory are satisfied. Critical areas are then isolated and a series of slosh tests is defined to give modal parameters for these areas, to evaluate fluid natural damping in the tank, and to give added confidence in the parameters obtained from the linearized, inviscid theory.

Analysis is usually limited to consideration of the fundamental liquid mode, since the forces and moments generated by higher modes are usually quite small in comparison.

The present monograph is intended to provide a concise summary and guide to booster propellant slosh testing, with the following objectives in mind.

a. To summarize the state of the art in booster propellant slosh testing.

b. To organize known data and techniques for convenient use in new situations and for new vehicles.

c. To provide a test guide that will permit rapid, logical, and economical gathering of experimental data by means of propellant slosh model tests.

A literature guide is given, with significant studies closely related to propellant slosh testing arranged under subject headings.

A number of related subjects such as vehicle longitudinal oscillation (POGO effect), flexible baffle damping, and testing at greater or less than one g are beyond the scope of this monograph.

2/STATE OF THE ART

Although renewed interest in fluid motion in containers occurred with the advent of aircraft and liquid-fueled boosters, basic theoretical solutions were published by Lord Rayleigh in 1876[(47)] and Lamb in 1879.[(53)] Experimental slosh tests to compare results with theory were published by Guthrie[(48)] in 1875 and Rayleigh in 1876.

An introduction to theoretical and experimental studies of liquids in moving containers previous to 1960 is contained in the readily available literature survey by Cooper (see Reference 30). For those readers desiring detailed information, the works most important to propellant slosh testing are arranged under subject headings in the references (Section 4 of this monograph).

With regard to slosh testing, the feasibility of the basic technique of dynamic modeling based on similitude theory is well established. It is not always possible to provide full similitude in a reduced scale model tank, however, as will be discussed later.[(64,65)] For vehicle stability and control analysis it is the general practice to use propellant slosh mathematical model parameters obtained in tests on a dynamic model tank. Comparison of flight results with simulations based on such mathematical models has shown satisfactory agreement as regards first-order effects. Smaller effects are continually found which are not predicted by mathematical models presently in use.

In the early days of booster development one program experienced a flight failure that was traced to propellant slosh instability. Means of stabilizing the propellants without gross vehicle redesign or severe weight penalties were sought. Considerable effort was expended in slosh testing to determine methods of increasing the natural damping in propellant tanks by means of baffles.[(9-12)] Damping produced by floating cans was also evaluated.[(66)] The annular ring baffle was found to provide very effective damping[(9,11,18)] when submerged just below the surface. Empirical relationships to describe natural damping and ring baffle damping in circular cylindrical flat bottom tanks were developed.[(3,15)] Experimental data for a large range of tank sizes and fluid properties were obtained, and the effects of draining on slosh frequency and damping were investigated and found to be very small.[(5)] The pressure distribution on tank walls due to liquid slosh[(76)] and the net force on a ring baffle during slosh motion[(4)] were measured. The natural frequencies of various tank shapes were investigated experimentally.[(26-28)] Viscous damping in smooth-walled tanks without baffles has been found to have negligible effect on slosh frequencies.[(64,3)]

Although baffles are usually installed to provide energy dissipation, it is also possible to design baffles which will cause energy exchange between modes. A spiral baffle will couple lateral slosh modes into roll rotational modes. An asymmetrical baffle was tested by Cole and Gambucci[(10)] which transfers energy from the first to the second lateral slosh mode. Baffles for energy exchange between modes have the obvious disadvantage that control system or structural coupling with slosh modes may be aggravated. For example, a strong coupling between lateral and roll rotational modes could cause significant problems in the roll control system.

While by far most experimental tests have used planar translational excitation, several tests involved tank rotation,[5,14,46] and one experimental and theoretical study involved completely coupled pitch rotation and lateral translation with simulated booster engine excitation.[40] This study showed that ring baffle effectiveness extends to greater depth with rotational excitation than with translational excitation, and that the coupled mode frequencies were in good agreement with those analytically predicted from test data on uncoupled modes.

Damping produced by annular ring baffles has been found to be very insensitive to changes in viscosity of various liquid propellants.[9,11]

After basic small-angle effects were determined theoretical and experimental studies of large-angle propellant slosh were made. The phenomenon of unstable rotational modes due to steady sinusoidal excitation was the subject of a number of studies[58-62] and a nonlinear mechanical model has been proposed to represent the observed results.[62]

Recent experimental studies have evaluated the damping provided by flexible ring baffles.[81,82,12] Under certain conditions flexible ring baffles provide more energy dissipation than rigid ring baffles of equal weight.

Experimental studies of longitudinal oscillations of propellants have been performed. These oscillations have caused difficulty in Atlas and Titan boosters and involve structural vibration of the propellant tanks. Dynamic modeling of partially filled flexible tanks has been discussed[70]; however, structural vibration tests are outside the scope of this monograph.

Some experimental studies of propellant slosh response for unusual disturbances are available. Responses of reservoirs to earthquakes[80] have been studied in model tests. Motion, pressures, and net force on a domed propellant tank due to sudden engine cutoff have been reported.[79] The related problem of "zero-g" or low Bond number propellant slosh has been studied extensively using model tests but this work is outside the scope of the present monograph. Propellant slosh oscillations in a spinning tank have been investigated experimentally[55] including both steady rotation and transient effects.

While analysis and previous test results are of considerable help in a new tank design, the determination of slosh mathematical model parameters with adequate accuracy and confidence level usually requires propellant slosh testing.

Mathematical models of propellant slosh effects have an important relation to slosh testing and will be discussed in the following section.

3/RECOMMENDED PROCEDURES

3.1 MATHEMATICAL MODELS

The incorporation of slosh test measurements into a liquid-fueled booster control stability analysis or simulation requires use of a mathematical model of slosh forces and moments.

The slosh mathematical model is defined as that set of differential and functional equations which is used to represent the forces and moments exerted on the vehicle due to the liquid propellants in simulating the motion of the vehicle. For gross effects a simple mathematical model is adequate. For fine detail or special effects a more detailed mathematical model is necessary. The closer to the state of the art is the booster structural and control design, the more important it becomes to have a high accuracy and confidence level associated with the mathematical model and its parameters.

There is no logical necessity to associate a mathematical model with an analogous physical system. Nevertheless such analogies have a long and venerable history. Both Guthrie(48) and Lord Rayleigh(47) computed "equivalent pendulum" lengths to represent observed or calculated frequencies of liquid motion.

More recently, Graham,(45) Lorell,(35) Graham and Rodriguez,(36) Kachigan,(32) Schmitt,(34,35) Abramson,(37) Bauer,(39) Lomen,(51) and Fontenot(52) have discussed sloshing and one or more variants of a mathematical model.

Beginning with the work of Lord Rayleigh the analogy between fluid motion in a tank and motion of a mechanical system has been considered repeatedly. A succession of attempts have been made to provide a rigorous justification for this analogy. One derives equations of motion for a fluid system and for an assumed mechanical system, and then attempts to associate coefficients between the two sets of equations of motion to justify the analogy.

In deriving equations of motion of a heavy liquid in a partially filled, rigid tank having arbitrary motion, assumptions usually made are:

1. Inviscid flow
2. Fluid irrotational with respect to inertial axes
3. Small motions about an equilibrium state.

It is common to restrict the more general problem to planar tank motion either sooner or later in the derivation, leaving two translational and one rotational degree of freedom for the rigid tank. Other simplifying assumptions are made, particularly in linearizing the final equations of motion.

Likewise, in deriving equations of motion of a mechanical system such as a set of pendulums hinged on a rigid vehicle having planar motion, many simplifying assumptions are usually made, particularly in the method of linearizing these equations.

Gradually, over the years, the level of rigor of both fluid motion and mechanical system motion equations have been improved, and some of the simplifying assumptions have been removed.

Each time these improvements are made the comparison of the new fluid equations and the new mechanical equations yields a slightly different relationship in making associations between the two sets of equations. Some analysts even preface their vehicle dynamic analyses with the phrase "... using the latest basic equations of motion..."

Mathematically speaking, there are still many unresolved problems, chief among which is careful treatment of the time-varying aspect of the mathematical models of the fluid and vehicle motion.

However, since most fluid and vehicle parameters change slowly, any new effects that are subsequently uncovered are likely to be quite small. The large, first-order effects are by now well established.

The most popular models for routine engineering control and stability analysis have been the pendulum model and the spring-mass model. For rigid body analysis either pendulum or spring mass models can be constructed to duplicate the equations of motion of the small disturbance, inviscid, irrotational flow solution for the planar case.[51] Models incorporating fluid damping have also been presented by Abramson, et al,[37] Bauer,[39] and others.

A sketch of the configuration of a pendulum model is shown in Figure 1. One pendulum is used for each slosh mode to be represented. In addition, a rigid mass and a moment of inertia are used.

Effects identical to those of the pendulum model can be obtained using a spring-mass representation as in Figure 2. One mass and spring for each mode plus a fixed mass provide suitable equations of motion.

If it is desired to simulate propellant slosh forces and moments using a linear time-varying model, either linearized pendulum equations or linearized spring-mass equations may be used. The linearized pendulum and spring-mass equations of motion given below are derived in References 51 and 54 based on the following assumptions:

a. The vehicle executes small translations from a vertical flight path and small rotations from a vertical vehicle attitude.

b. Pendulum angles (or deflections of slosh masses) and vehicle pitch rotation angles, together with their first time derivatives, are assumed small enough that products of these quantities may be neglected in comparison with the quantities themselves.

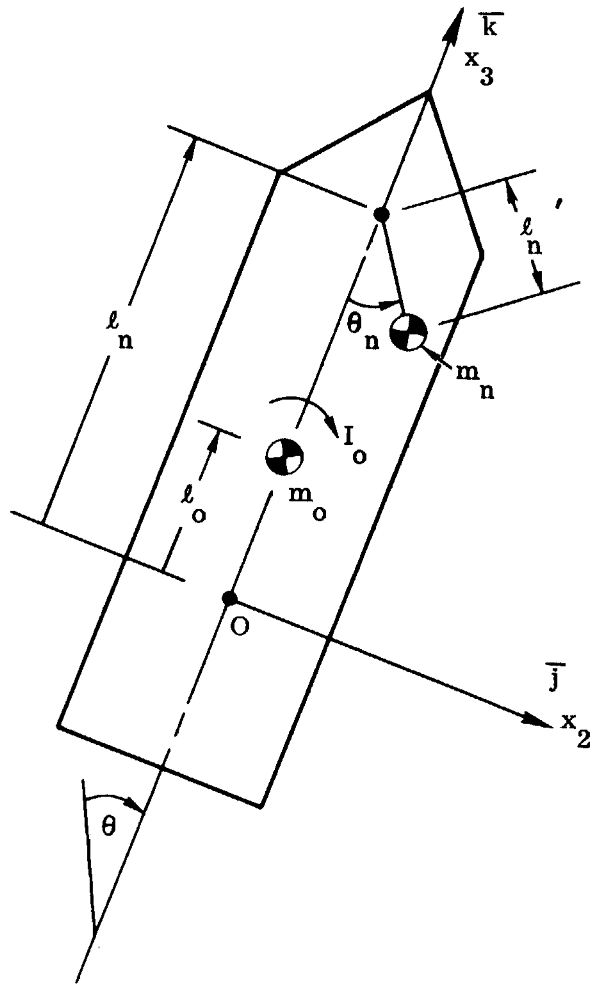


Figure 1. Schematic of Pendulum Model

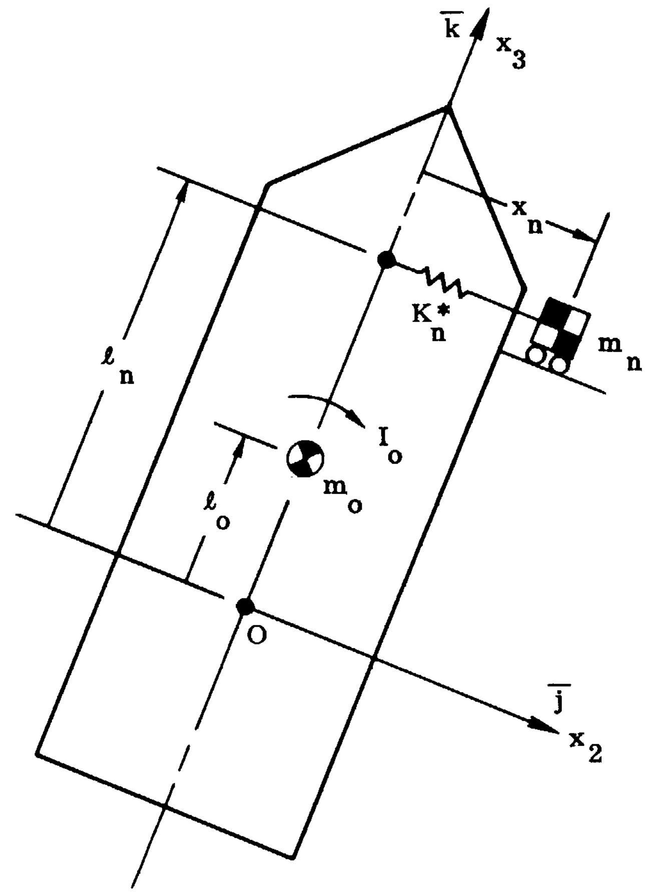


Figure 2. Schematic of Spring-Mass Model

The body-fixed reference axes x_2 and x_3 have their origin fixed to the vehicle at some arbitrary point. Note that this point may or may not be a center of mass for the vehicle structure. These axes are used only to represent the geometry and kinematics in deriving the equations of motion.

Table 1. Nomenclature

General

u_2, u_3		are the components of the inertial velocity of the origin, O, along the body-fixed reference axes
θ	=	vehicle pitch angle
m_o	=	body-fixed mass
ℓ_o	=	distance of m_o forward of origin O of the body-fixed reference axes
I_o	=	moment of inertia in pitch of the body-fixed mass, m_o
x_2, x_3	=	body-fixed reference axes with origin, O
g_2, g_3	=	components of gravity along body axes
$\bar{j}, \bar{k}$	=	unit vectors along x_2, x_3 axes respectively
$\bar{v}$	=	inertial velocity of origin O of the body-fixed reference axes, $\bar{v} = u_2 \bar{j} + u_3 \bar{k}$ along body axes
$\bar{a}$	=	$\frac{d\bar{v}}{dt} - \bar{g}$ = vector thrust acceleration of origin O of the body-fixed reference axes, $\bar{a} = (\dot{u}_2 + \dot{\theta} u_3 - g_2) \bar{j} + (\dot{u}_3 - \dot{\theta} u_2 - g_3) \bar{k}$

Pendulum Equations

m_n	=	mass of pendulum representing n^{th} mode sloshing mass
ℓ_n	=	distance of hinge point of n^{th} mode pendulum forward from origin O of the vehicle reference axes
ℓ_n'	=	length of n^{th} mode pendulum
F_2, F_3	=	sums of external forces exerted by the vehicle on the pendulum + fixed mass system along body-fixed reference axes
M_1	=	total moment exerted by the vehicle on the pendulum + fixed mass system about origin O

Table 1. Nomenclature, Contd

Spring-Mass Equations		
m_n	=	mass representing slosh for the n^{th} mode
K^*_n	=	spring constant of n^{th} mode spring
ℓ_n	=	length of n^{th} mass and spring forward of origin O of the reference axes
F_2, F_3	=	sums of forces exerted by vehicle on spring-mass, fixed mass system along body-fixed reference axes
M_1	=	total moment exerted by vehicle on the spring-mass system about origin O

With the nomenclature established, the linearized pendulum equations of motion become:

$$\ddot{\theta}_n + C_n \dot{\theta}_n + \frac{a_3}{\ell'_n} \theta_n = - \frac{a_2}{\ell'_n} - \frac{(\ell_n - \ell'_n)}{\ell'_n} \ddot{\theta} \quad , n=1,\ldots,N. \tag{1}$$

$$(m_o + \sum_{n=1}^{N} m_n) a_3 = F_3 \tag{2}$$

$$(m_o + \sum_{n=1}^{N} m_n) a_2 + [m_o \ell_o + \sum_{n=1}^{N} m_n (\ell_n - \ell'_n)] \ddot{\theta}$$

$$+ \sum_{n=1}^{N} m_n \ell'_n \ddot{\theta}_n = F_2 \tag{3}$$

$$M_1 = - [m_o \ell_o + \sum_{n=1}^{N} m_n (\ell_n - \ell'_n)] a_2 - [m_o \ell_o^2 + I_o$$

$$+ \sum_{n=1}^{N} m_n (\ell_n - \ell'_n)^2] \ddot{\theta} - \sum_{n=1}^{N} m_n (\ell_n - \ell'_n) \ell'_n \ddot{\theta}_n$$

$$+ \sum_{n=1}^{N} m_n \ell'_n \theta_n a_3 \tag{4}$$

A damping term, $C_n \dot{\theta}_n$, was arbitrarily added in Equation (1) to account for energy dissipation within the fluid. In the pendulum model this corresponds to adding viscous friction to the pendulum hinges. The reaction moments on the vehicle that would result from this friction would be very small and have little physical meaning for the fluid system; they are consequently not included.

The corresponding linearized equations of motion for the spring-mass system derived in References 51 and 54, with the addition of a damping term, become:

$$\ddot{x}_n + C_n \dot{x}_n + \frac{K^*_n}{m_n} x_n = -a_2 - \ell_n \ddot{\theta} \quad (n=1,\ldots,N) \tag{5}$$

$$(m_o + \sum_{n=1}^{N} m_n)\, a_3 = F_3 \tag{6}$$

$$(m_o + \sum_{n=1}^{N} m_n) a_2 + (m_o \ell_o + \sum_{n=1}^{N} m_n \ell_n)\ddot{\theta} + \sum_{n=1}^{N} m_n \ddot{x}_n = F_2 \tag{7}$$

$$-(m_o \ell_o + \sum_{n=1}^{N} m_n \ell_n)\, a_2 - (m_o \ell_o^2 + I_o + \sum_{n=1}^{N} m_n \ell_n^2)\,\ddot{\theta}$$

$$- \sum_{n=1}^{N} m_n \ell_n \ddot{x}_n + \sum_{n=1}^{N} m_n x_n a_3 = M_1 \tag{8}$$

As an example of the use of the linearized pendulum equations in slosh testing, assume a simplified case under the following conditions:

$$u_3 = 0,\ \theta = 0,\ u_2 = \dot{x},\ N = 1,\ C_1 << 1.$$

Then $a_3 = g$, $g_2 = 0$, $a_2 = \dot{u}_2 = \ddot{x}$, and the Equations (1), (2), (3), and (4) reduce to:

$$\ddot{\theta}_1 + C_1 \dot{\theta}_1 + \frac{g}{\ell'_1} \theta_1 = - \frac{\ddot{x}}{\ell'_1} \tag{9}$$

$$(m_o + m_1)\, g = F_3 \tag{10}$$

$$(m_o + m_1)\,\ddot{x} + m_1 \ell_1' \ddot{\theta}_1 = F_2 \tag{11}$$

$$-[m_o \ell_o + m_1 (\ell_1 - \ell_1')] \ddot{x} - m_1 \ell_1' (\ell_1 - \ell_1') \ddot{\theta}_1 + m_1 \ell_1' \theta_1 g = M_1 \quad (12)$$

These simple equations are easily verified.

To determine slosh mathematical model parameters one pretends that the test tank contains the mechanical model instead of fluid, and then proceeds to apply any of the test techniques for determining parameters for a damped harmonic oscillator.

Natural frequency and damping may be obtained by exciting the first slosh mode, and recording lateral force versus time for the transient decay with the tank stopped. By plotting lateral force amplitude on a vertical log scale and time on a horizontal linear scale (in these coordinates viscous damping yields a straight line), the data may be smoothed and damping evaluated using the local slope of the curve for any particular amplitude. A procedure similar to this has been worked out which provides a direct measure of fluid damping directly from surface amplitude transducer outputs on the test tank. This device, described in Reference 5, has been named a "dampometer."

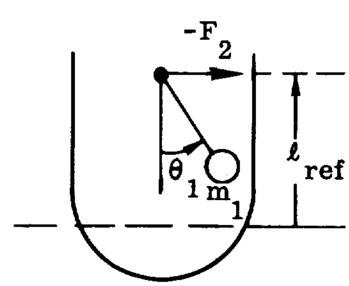
If the test facility is capable of recording lateral force caused by the fluid during tank motion, then the half-bandwidth technique(25) based on the lateral force resonance curve for constant amplitude lateral tank motion yields an alternate method for obtaining equivalent damping coefficients for the fluid. This technique is necessary when fluid damping is large since it is very difficult to evaluate damping from force transient decay in this case.

The pendulum hinge point may be found by recording both lateral force and moment exerted by the fluid on the tank with the tank stopped. Knowing the lateral force and the moment, there is a unique position forward of whatever reference is used for moments in the test rig, at which a pendulum would duplicate both the force and the moment. Combining Equations (9) and (11) with $x \equiv 0$ and $\dot{\theta}_1 = 0$ (which holds at the time of peak force and moment) we have

$$-F_2 = m_1 g \theta_1$$

and the net moment on the tank is given by

$$M = m_1 \ell_{ref} g \theta_1 .$$



Then

$$\ell_{ref} = \frac{M}{-F_2} .$$

and the pendulum hinge point is determined.

This result[43] holds when fluid damping causes no essential force decay over the first half-cycle of fluid motion, which is usually a good assumption. If damping is large the hinge point is given in Reference 24 as

$$\ell_{ref} = \frac{M}{-F_2} + \frac{4\,\xi^2\,\omega_o^2\,g}{\omega_e^2\,[\,\omega_e^2 - \omega_o^2\,(1-4\xi^2)\,]}.$$

Pendulum mass, m_1, may be measured using the following procedure. Let the test tank be subject to a lateral oscillation, $x = x_o \sin \omega_o t$, where ω_o is less than, say, six-tenths of the first-mode natural frequency, ω_e. Wait until a steady oscillation amplitude is established. If the tank is now quick-stopped at a point of zero velocity and the first peak in the lateral force F_2 is measured, the pendulum mass is given by

$$m_1 = \frac{-F_2}{x_o}\left[\frac{1}{\omega_o^2} - \frac{1}{\omega_e^2}\right]$$

where ω_o = excitation frequency and ω_e = first mode natural frequency. This result is derived in Reference 43 for the case when fluid damping is negligible and in Reference 24 for the case when damping is included.

The fixed mass, m_o is determined from the relationship

$$m_o + \sum_{n=1}^{N} m_n = M_{total}$$

where M_{total} is the total mass of fluid in the tank.

Procedures for evaluating model parameters for rotational excitation may be derived in exactly the same way as applied to rotational motion of a tank containing a fixed mass and a pendulum.

Similar procedures may be used to obtain experimentally the parameters of the spring-mass model equations.

While the linearized models discussed above are by far the most widely used models, other, more detailed models have been developed.

The most detailed slosh mathematical model presently available is the nonlinear model proposed by Bauer.[41] This model represents the large-angle effects of frequency decrease and fluid center-of-mass movement as a slosh mass constrained to a paraboloid and restrained by a radial nonlinear spring. Extensive testing would be required to qualify such a nonlinear mathematical model for representation of a tank of complex geometry with high accuracy and confidence levels in the model parameters.

The usual procedure for simulating slosh in booster control and stability analysis is to use equations of motion derived either from a pendulum or a spring-mass system to determine the model parameters as functions of fluid height, incorporating slosh test data, and then to simulate these parameters and their changes with time to obtain booster motions. Although this procedure involves determining a time-varying model from static data, booster propellant flow rates are usually low enough to justify the procedure. Cases where sudden changes of model parameters occur should be given more detailed study.

3.2 SIMILITUDE THEORY

A large number of variables influence fluid slosh phenomena. The relationships which must exist between these variables in the actual vehicle (the "prototype") and in the physical model (the "model") to yield similar fluid phenomena are the subject matter of Similitude Theory.[63-73]

Several types of similarity may be defined. If all corresponding dimensions between model and prototype tanks are in the same ratio, geometric similarity exists between the model and prototype tanks. If the kinematic (i.e., velocity, acceleration, etc.) fields in the liquid are geometrically similar between model and prototype, kinematic similarity of the flows is said to exist. A third type of similarity is called dynamic similarity, where the force distribution between two flows is such that, at corresponding points in the flow, identical types of forces (such as inertial, gravity, viscous, etc.) are parallel and have a ratio which is the same value at all points of correspondence between model and prototype flows.

In booster slosh testing there is no apparent reason to deviate intentionally from geometric similarity.

The displacement, velocity, and acceleration fields of the prototype tank and fluid during flight testing are much too complex to be modeled in a laboratory. Thus in booster slosh testing a very simplified motion of the prototype tank is assumed in order to make kinematic similarity possible. Usually model testing is limited to attempts to simulate vertical flight, vertical flight combined with planar sinusoidal lateral tank motion, or vertical flight combined with sinusoidal planar tank rotation.

Since dynamic similarity is a sufficient condition for kinematic similarity, the usual approach is to try to obtain dynamic similarity.

For dynamic similarity it is sufficient that all dimensionless groups formable from the physical quantities affecting the phenomenon be the same for model and prototype.

As shown by Bridgman[72], fundamental equations can be so arranged that the physical quantities enter the equations through certain combinations that are dimensionless, and the form of such equations is invariant to the size of the units involved in the various terms of the equations. From a mathematical point of view it could be said that the dimensionless groups constitute a mapping from the prototype kinematical system to the model kinematical system which preserves dynamic similarity.

In booster propellant slosh testing some variables which play an important role in slosh phenomena within a rigid tank are shown in Table 2. In certain circumstances other factors, such as surface roughness, bubble formation, etc. can become very important, and the question of what variables may be of significance must be reexamined.

Table 2. Variables of Significance to Propellant Slosh Phenomena

$a = a_3$	=	tank longitudinal thrust acceleration $(\ddot{r} + g)$	LT^{-2}
d	=	diameter	L
p	=	pressure	$MT^{-2}L^{-1}$
Q	=	volumetric flow rate	L^3T^{-1}
τ	=	time	T
β	=	kinematic surface tension = σ/ρ	L^3T^{-2}
μ	=	coefficient of viscosity	$ML^{-1}T^{-1}$
ν	=	kinematic viscosity = μ/ρ	L^2T^{-1}
ρ	=	liquid density	ML^{-3}
σ	=	surface tension	MT^{-2}

Any slosh phenomenon will depend on the physical parameters of the fluid, of the tank, and on the motion of the tank. This dependence can be represented in terms of dimensionless parameters formable from the physical parameters involved, as shown in Table 2. Any fluid flow field will be more sensitive to some of the dimensionless parameters than to the others. For example, the frequency of oscillation of low viscosity fluid in a smooth-walled circular cylindrical tank will be quite insensitive to changes in viscosity, while a change in tank diameter will have a pronounced effect. It often happens that one dimensionless parameter is far more important than the others. Thus, duplicating the Reynolds number of the prototype in the model may provide almost complete dynamic similarity of the fluid motion. In this fortunate circumstance laboratory modeling of the prototype is straightforward.

The ratio of a variable in the model to the same variable in the prototype will be written with a subscript r. Table 3 shows the dimensionless ratios of variables of significance to booster propellant slosh for incompressible fluids of uniform density in rigid tanks.

In applying similitude theory to propellant slosh testing several cases of importance may be distinguished.

CASE 1

Assume a booster which remains vertical and rises straight up. Assume also that the propellant tanks are smooth-walled cylinders free of obstructions to fluid motion, and that small fluid displacements occur due to small initial motions of the propellants. The propellants are assumed to have low viscosity (e.g., liquid oxygen, kerosene, etc.), any compressibility effects are assumed small, and the tanks are assumed large with respect to any capillarity effects at the tank wall.

In this case the major forces on a general fluid element are the inertial, gravitational and pressure forces. Thus, maintaining the Froude number and the Euler number equal in model and prototype will provide dynamic similarity of the basic fluid motion. For equal Froude numbers we must have

$$a_r \tau_r^2 = d_r$$

If the size ratio, d_r, and acceleration ratio, a_r, are fixed, this determines the time scale ratio, τ_r. Equality of Euler numbers, $p_r = \rho_r a_r d_r$ or $F_r = \rho_r a_r d_r^3$, determines the scale ratio for pressures or forces between model and prototype.

If energy dissipation is to be modeled for this case, equality of Reynolds numbers would also be required so that $d_r^3 = \nu_r \tau_r$. Solving this simultaneously with the Froude number equation we obtain

$$d_r = \nu_r^{2/3} a_r^{-1/3}$$

This relationship means that if a_r and d_r are fixed, the kinematic viscosity ratio, ν_r, is also fixed. In practice one determines the approximate kinematic viscosity ratio required and then adjusts the scale ratio, d_r, so that this equation is satisfied for the real fluid having the closest kinematic viscosity to that required.

Table 3. Propellant Slosh Dimensionless Parameters for Incompressible Fluids of Uniform Density in Rigid Tanks

Froude number	$= \dfrac{a}{d/\tau^2}$	$\dfrac{\text{inertial force}}{\text{gravitational force}}$
Euler number	$= \dfrac{p}{\rho\, ad}$ or $\dfrac{F}{\rho ad^3}$	$\dfrac{\text{pressure force}}{\text{inertial force}}$
Reynolds number	$= \dfrac{\rho d\,(d/\tau)}{\mu} = \dfrac{d^2}{\nu\tau}$	$\dfrac{\text{inertial force}}{\text{frictional force}}$
Weber number	$= \dfrac{\rho d^3}{\sigma\tau^2}$	$\dfrac{\text{inertial force}}{\text{surface tension force}}$
Bond number	$= \dfrac{\rho ad^2}{\sigma}$	$\dfrac{\text{gravitational force}}{\text{surface tension force}}$

If modeling of a very low kinematic viscosity propellant in a small model is attempted for a fixed acceleration ratio it often happens that a model fluid with extremely low kinematic viscosity would be required. A table of representative fluid properties is shown in Table 4. Here it is seen that methylene bromide has a very low kinematic viscosity, and might be suitable for model tests. However, methylene bromide is somewhat toxic, very flammable, very expensive, and is a solvent for many plastics. These disadvantages increase the cost of model tests and may force consideration of alternate methods of modeling. An approach often used in practice is then to relax the requirement to model Reynolds number. One then attempts to correct the resulting test data for this discrepancy, as will be discussed later.

CASE 2

Let conditions be the same as those of Case 1 except let the tank contain obstructions to fluid flow in the form of fixed annular ring baffles. This type of baffle induces high-velocity gradients with resulting energy dissipation due to fluid viscosity. Dynamic similarity of fluid motion would then require equal Froude, Euler, and Reynolds numbers.

It is interesting to note that experimental results[9,20] obtained for this case have shown that the resulting energy dissipation in the form of damping is quite insensitive to changes in fluid viscosity. Miles[16] attributes this to the fact that in booster propellant slosh, Reynolds numbers above 10^3 are obtained even for small oscillations (2-inch wave height at wall in 10-foot diameter tank with a ring area of 1 percent of the section area).

Table 4. Fluid Properties for Booster Slosh Testing

FLUID	SPECIFIC GRAVITY	VISCOSITY (Centipoise)	KINEMATIC VISCOSITY (Centistokes)	SURFACE TENSION (Dynes/cm)	TEMPERATURE
Liquid oxygen	1.14	0.189[2]	0.166	13.2	-183°C
RP-1	0.802	9.0	11.2	--	-30° F
RP-1			16.5[8]		-34.40° F
Kerosene	0.83	2.4[7]	2.89	23 - 32	70°F
		7.5 - 8.0[7]	--	--	-30°F
Acetone	0.792	0.316[1]	0.399	--	25°C
				23.7	20°C
Water (H_2O)	1.0	1.0[1]	1.0	73	20°C
Carbon tetrachloride	1.59	0.969[1]		26.95[1]	20°C
		0.952	0.598	26.8	20°C
Methylene chloride (CH_2Cl_2)	1.34	0.435[3]			20°C
		0.449[1]	0.34	26.5	15°C
Methylene bromide (CH_2Br_2)	2.495	0.92[1]	--	41.7 ± 2[4]	30° C[1]
		1.09[1]			15° C[1]
Liquid hydrogen	0.0703	0.011[1]	0.157	2.31	-255°C

[1] Handbook of Chemistry and Physics, 47th Ed., 1966-1967.

[2] Liquid Propellants Handbook, Oxygen - Transport Properties - Battelle Memorial Institute, 1958.

[3] Hatschek, E., "The Viscosity of Liquids", Van Nostrand, 1928.

[4] Determined experimentally by author.

[5] This is incorrectly given in Note 6. (See Reference 64)

[6] Handbook of Chemistry and Physics, Chem. Rubber Pub. Co., 34th ed., 1952.

[7] Perry, Chemical Engineers Handbook, 4th Ed., 1963.

[8] AF RPL. Rep. TR66-4, Propellant Handbook, Jan 1966, p 2.7.

Extreme care must be exercised in verifying this insensitivity to Reynolds number in cases where odd-shaped obstructions are present. To say that damping is independent of viscosity is false, since a fluid of zero viscosity would flow past gross obstructions without any energy dissipation whatsoever.

Other special situations require different necessary conditions for similarity. For very low thrust accelerations the body forces on a fluid element become small, and surface tension forces become dominant. Basic fluid phenomena then require equality of Bond numbers between model and prototype.

The importance of each dimensionless group in a given phenomenon may be established from previous studies, from analytical considerations, or by careful laboratory determination of the effect of varying each variable separately. Results based on model testing cannot be considered complete until the sensitivities of the phenomenon studied to variations in all dimensionless parameters that were not modeled are established and appropriate corrections applied.

Similitude theory is used in propellant slosh testing for at least three different purposes. Firstly, similitude relations are used to establish the model tank size, fluid, and other parameters. Secondly, the theory is used in correcting test results for dissimilarity when some dimensionless groups have not been modeled. Thirdly, the final data is correlated and compared with the prototype by means of the dimensionless parameters.

Finally, it should be noted that the most faithful model of the prototype is the prototype itself, and possibilities for measuring slosh parameters using a prototype, if one exists, should not be overlooked.

3.3 TEST OBJECTIVES AND PLANNING

Although there are many reasons why booster propellant slosh testing may be required, it will be assumed here that the primary purpose of testing is to determine the parameters of a mathematical model to represent propellant slosh forces as part of a booster control and stability simulation.

After a mathematical model has been chosen and the need for accurate estimates of the mathematical model parameters is apparent, the objectives of a series of slosh model tests may be stated and the strategy for achieving these objects defined. Following this the many details of the strategy can be planned, including budget, schedule, construction of apparatus, and so on.

The completion of any physical experiments — to give useful results within budget and on schedule — requires careful attention to a number of precepts of an elementary nature, many of which are easily forgotten in the heat of battle with the real world.

Although a complete discussion of these matters is beyond the scope of this monograph, they are treated in a book by E. B. Wilson, Jr., Reference 29.

An important decision is whether or not tests using tank (pitch plane) rotation are necessary. Certain parameters such as effective pitch plane inertia of the fluid cannot be obtained in tests limited to tank translation.

Fundamental to successful testing are the following requirements:

1. Statement of test objectives
2. Test strategy to satisfy the objectives
3. Work schedule
4. Budget
5. Facilities and instrumentation criteria
6. Model tank design and construction criteria
7. Test data processing procedure
8. Documentation procedure

Although the details will vary, the following set of questions will serve as a check list in planning the tests.

a. <u>Statement of Test Objectives</u>

1. Why should the tests be performed?
2. What is the intended use of the results?
3. What test objectives are most important and which one could be relaxed?
4. Is it feasible to try to satisfy these test objectives using dynamic model testing?
5. Where is data needed most?
6. What accuracy would be required for the data to be of use?
7. Would translational planar tests be adequate, or would more complex tank motions be required?
8. Can the data needed be measured in a test?

b. <u>Test strategy to satisfy objectives</u>

1. Will rotational, as well as translational, tests be required?
2. How will dynamic similarity requirements be met?
3. Will several tank sizes and several test fluids be required?

4. Can test fluid properties be varied satisfactorily by heating?

5. What special precautions are required by the test fluid because of fire hazard, toxicity, reactivity, solubility of tank material or seals, etc.?

6. Will measurements of forces and moments during tank motion be required?

7. If force measurements during tank motion are required, how will apparatus inertial forces be subtracted?

8. Will visual observation of the slosh motion be mandatory?

9. Will scaling problems due to lack of dynamic similarity be so difficult to resolve that full-size tests will be required to provide results of sufficient accuracy and confidence level?

10. If high slosh amplitudes are to be studied, how will rotational instability be prevented or allowed for?

11. How might changes in strategy affect the schedule?

12. What funding is available, and what funding would permit accomplishment of the objectives?

13. Can data on sensitive parameters be obtained in several different ways as a check?

14. What ranges of variables will be required in the test, e.g., amplitudes, frequencies, etc.

15. Would two-dimensional model tests be useful for study of basic fluid motion, flow visualization etc., in conjunction with three-dimensional tests?

c. Work Schedule

A work schedule is necessary and should include adequate allowances for manufacturing, transportation, test equipment setup and calibration, unforeseen delays, time to conduct the tests, data reduction, data analysis, and documentation.

d. Budget

The budget is usually allocated simultaneously with decisions on test objectives, strategy, work schedule, facilities, and data processing, following a preliminary estimate which essentially determines the level of effort.

e. Facilities and instrumentation criteria

1. How will the model tank be supported?

2. What forces and moments must be measured, and how will they be measured?

3. Will force transducers have adequate sensitivity to low forces and adequate capacity for high-amplitude measurements under full-depth conditions?

4. Will the test rig be rigid enough that structural resonances are well outside the frequency ranges to be used in the tests?

5. How will quiescent fluid height be measured?

6. If fluid forces during tank motion are to be measured, how will structural inertial forces be subtracted?

7. What ranges of excitation amplitude and frequency are required?

8. What variables must be recorded?

9. Is data recording on magnetic tape necessary? (This is convenient for selective filtering and scaling on playback.)

10. What other instrumentation is necessary?

11. Are still or motion pictures required? When should they be scheduled?

12. If the tank is to be quick-stopped at a point of zero velocity, what method will be used?

f. Model tank design and construction criteria

1. How large must the tank be to provide the necessary dynamic similarity?

2. What visual observation is necessary? Can the tank be of transparent plastic?

3. Will fluids which dissolve plastic or seal materials be necessary for dynamic similarity?

4. What tank strength will be required at maximum depth and the highest amplitude and frequency to be used?

5. How will the model tank be manufactured?

6. What deviations from geometric similarity are permissible in the model tank?

7. Should the model tank be in several sections or just one piece?

8. If rotational tests are required, how will the tank be supported?

9. How will the tank be filled and drained?

10. Are heating coils necessary to vary fluid properties?

11. Will the tank be strong enough so that load deflections do not cause excessive leakage?

12. What measurements of the model tank are necessary to verify geometric similarity?

g. Test data processing

1. What manual data recording is necessary?
2. How can the tests be arranged to yield the maximum amount of useable data per run?
3. How can a digital computer be most profitably used to alleviate tedious data reduction and scaling?
4. How will recorder charts be processed?
5. What methods of data control can be used during the tests?
6. Can magnetic tape data recording be used?
7. How will extraneous effects, such as out-of-plane slosh, be detected in final data?
8. How will data be corrected for deviations in important parameters, such as geometric dissimilarity, lack of dynamic similarity, etc?

h. Documentation procedure

If test data is necessary, it is usually of interest to a large number of people engaged in vehicle design and analysis. Communication of test results requires documentation. In many cases the results will be useful for a number of years hence, so that documentation should be permanent, clear, and in sufficient detail to be understood by persons who had no connection whatever with the original tests. Test data about which there are essential doubts is useless and may as well not have been taken.

Test documentation should be planned from the beginning and relevant analysis, design, and results should be written up immediately after they are performed in preliminary form. Still pictures and motion pictures may form part of the permanent documentation.

3.4 FACILITY DESIGN AND INSTRUMENTATION

Of prime importance in performing propellant slosh testing is the test facility to be used. Since several good slosh model test facilities have been built and are still in existence, the performance, capacity, convenience, capability, availability, etc., of these existing test setups should be examined before considering design and construction of a new facility. If an existing facility can be used, then reports of all previous work performed on the facility should be consulted for detailed information on performance and limitations. If a new facility is required, then characteristics of existing test setups should be carefully analyzed prior to design and preparation of specifications.

Fairly detailed description of slosh model test facilities are contained in References 4, 6, 8, 64, 10, 11, and 26. Very little slosh test data obtained in full-size, 1-g tests is available in the general literature, although some data is discussed in Reference 64, and correspondingly little has been published to document full-size test facilities used for booster propellant slosh tests.

A booster propellant slosh test facility should meet suitable requirements in the following areas:

1. Size and capacity — adequacy for desired test
2. Measuring capability — for forces, moments, pressures
3. Structural rigidity — structural resonances should be well outside the frequency band to be used
4. Provision for fluid excitation — hydraulic servo, offset cam arrangements, and paddles have been used.
5. Provision for required degrees of freedom — translation, rotation, or both.
6. Provision for minimization of sensor noise pickup
7. Provision for recording — both for data control and for permanent recordings

In addition, safety of operation, sensitivity to local disturbances such as building vibration, provision for handling exotic model propellants with regard to flammability, toxicity, etc., must also be considered.

Detailed information on transducers, recorders, and other required equipment may be obtained from manufacturers and will not be discussed here.

Two methods have been proposed and used for measurement of fluid forces and moments during tank motion. In the balance tank method a second test tank duplicating the motion and the mass distribution of the actual test tank is used. Force transducers on the balance tank record tank inertial forces while similar transducers on the actual test tank record the sum of fluid forces and tank inertial forces, so that subtraction yields the fluid forces alone (see Reference 64). A second method, the accelerometer method, is to mount a suitable accelerometer on the test tank and subtract inertial forces from tank plus fluid forces. This method is illustrated schematically in Figure 3. Apparently the balance tank method provides better inertial force cancellation and is more easily adjusted.

3.5 TEST OPERATIONS AND DISTURBANCE EFFECTS

During testing operations meticulous care in recording test conditions is necessary. Conditions should be recorded in a permanent notebook and may also be written

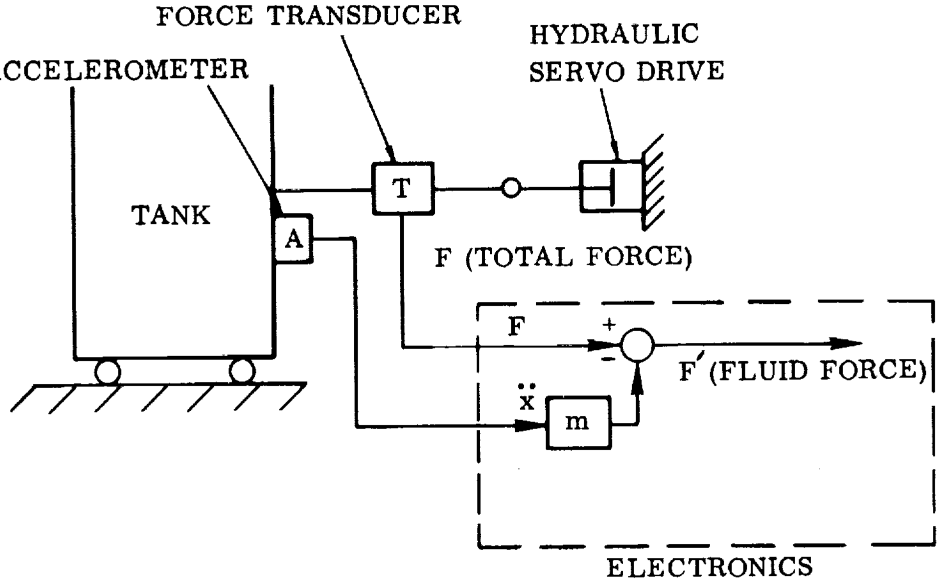


Figure 3. Inertial Force Subtraction Using an Accelerometer

directly on data traces. Notation on any special or peculiar phenomena observed, or erratic instrumentation noticed should be made after each run.

It is always something of a shock to embark upon a physical test equipped with a presupposed simplified mathematical model. In propellant slosh testing many complex effects are observed. A fluid is a continuous medium or a collection of a very large number of particles. Even the solution of the linearized, inviscid, irrotational equations of motion has an infinite set of natural frequencies and corresponding modes. In the motion of a real fluid one observes surface waves, vortex motion, splashing at high amplitudes, and fluid motion due to draining or filling. Essentially nonlinear effects occur, such as generation of roll mode fluid rotation[61] from small amplitude lateral tank translation, and amplitude-dependence of frequency and damping. Motion of cryogenic propellants is more complex still, since heat transfer, evaporation, bubble formation, and layering all occur; thus tests using an essentially incompressible fluid such as water are already a simplification of the real phenomena.

Nonlinear effects observed during model tests may be of second order as regards net effect on a similar tank in flight and yet cause significant difficulties in reduction and evaluation of experimental data. Perhaps the most difficult task is to obtain data on mathematical model parameters for high slosh amplitudes, where out-of-plane rotational modes are likely to occur. One approach (Reference 17) is to artificially eliminate rotational modes using a thin vertical baffle centered in the tank in the plane of tank excitation.

It is important to be able to detect out-of-plane forces and moments when conducting planar tests; recorder traces should be carefully examined for signs of out-of-plane motion.

Besides these difficulties other problems always develop which were not foreseen.

Finally, during test operations one must constantly push for fulfilling test objectives, maintaining data control, and completion of each test phase according to plan — on schedule and within budget.

3.6 DATA REDUCTION AND SCALING

For complete fluid slosh tests a large amount of data can be generated, and data reduction can be a long and tedious job. In this situation it is worth the effort to generate a digital computer program to perform the routine calculations. Program output can be arranged to give well-indexed, convenient, and permanent data records. Some computer installations have provision for automatic curve plotting, which can be a great convenience.

Recorder force amplitude traces are usually the main source of information. Considerable care must be exercised in evaluating force amplitudes, eliminating faulty data, plotting, and checking. Plotted data must be checked for general behavior and should be compared with results of other related tests or theories to establish the confidence level.

Final data must be scaled to account for any significant deviations from dynamic similarity in model tests. For example, if the Reynolds number was not modeled but could cause differences, the sensitivity of each output data parameter to Reynolds number should be evaluated and appropriate corrections added.

The sensitivity of a given output parameter to changes in a particular dimensionless group may be established from previous studies, from analytical considerations, or by careful laboratory determination of the effect of varying each variable separately.

One must be quite careful to avoid biasing the data in the direction of theory. Any corrections or adjustments to the data should be based on standard procedures and properly applied statistical analysis. Because unexpected difficulties often arise in the analysis of experimental data, it is well to perform data analysis as soon as possible, preferably while it is still possible to perform further tests if required. A detailed discussion of the analysis of experimental data is contained in Reference 29, Chapter 8.

It is always necessary to make some estimates of the possible errors associated with data. Standard techniques for error analysis are discussed in Reference 29, Chapter 9.

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