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STUDY TO DETERMINE AN IMPROVED METHOD FOR APOLLO PROPELLANT SYSTEM DECONTAMINATION AND PROPELLANT TANK DRYING

SUMMARY REPORT

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STABILITY OF NONLINEAR PULSE SYSTEMS

[Article by A. Kh. Gelig, Leningrad State University imeni A. A. Zhdanov; Moscow, Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences USSR), Russian, Vol 178, No 4, 1968, pages 793-796]

In his paper at the Third All-Union Conference on Automatic Control [1] E. Jury singled out as one of the urgent problems awaiting a solution the investigation of the dynamics of systems with width and pulse-frequency modulation [2]. Formulated infra are sufficient frequency conditions for the stability and dissipativity of systems of this type.

1. Let us consider a nonlinear pulse system whose mathematical description is reduced to the equations

$$\sigma_i(t) = f_i(t) + f_i^0 - \sum_{j=1}^{l} \int_0^t [\gamma_{ij}(t-\lambda) + \rho_{ij}] \eta_j(\lambda)\, d\lambda; \tag{1}$$

$$\eta_i(t) = \begin{cases} 0, & \text{(1) если } |\sigma_{i,n}| \leqslant \Delta_i, \quad t_{i,n} < t \leqslant t_{i,n+1}, \\ s_{i,n}(t) \operatorname{sign} \sigma_{i,n}, & \text{(1) если } |\sigma_{i,n}| > \Delta_i, \quad t_{i,n} < t \leqslant t_{i,n+1}; \end{cases} \tag{2}$$

$$t_{i,n} = t_{i,n-1} + T_{i,n}, \quad t_{i,0} = 0, \quad n = 1, 2, \ldots \tag{3}$$
$$(i = 1, \ldots, l; \quad \sigma_{i,n} \equiv \sigma_i(t_{i,n} - 0)).$$

Key: 1. if

Here the constants $f_i^{\,0}$ and functions $f_i(t)$ represent natural oscillations of the linear part of the system; $\sigma_i(t)$ is a signal at the input of the i-th pulse element (PE_i) and $\eta_i(t)$ a signal at its output.

The function $s_{i,n}(t)$ describes the shape of the pulse produced by PE_i at moment $t_{i,n}$, and is assumed to be piecewise-continuous nonnegative given $t_{i,n} < t \leqslant t_{i,n+1}$ and equal to zero given $t \leqslant t_{i,n}$, $t > t_{i,n+1}$. The nonnegative constant Δ_i characterizes the value of the insensitivity of PE_i. The quantity $T_{i,n}$ may be a nonlinear function of $|\sigma_{i,n}|$, as well as a functional of $\sigma_i(t)$ [3], the value of $T_{i,n}$ in the latter case depending on the behavior of the function $\sigma_i(t)$ only given $t < t_{i,n}$. It is assumed that there exist positive constants ν_*, ν^* such that

$$1/\nu_* < 1/T_{i,n} < \nu^* \quad (i = 1, \ldots, l;\ n = 1, 2, \ldots), \tag{4}$$

i. e. the pulse frequency has an upper and a lower limit. Formulas (2), (3) encompass all types of modulation with finite pulse duration described in [2]: amplitude, width and time (frequency and phase) modulation of the first and second kind.

Let us define some characteristics of pulse elements. Let $\tau_{i,n}$ be the pulse duration (measure of the set of values $t \in (t_{i,n};\ t_{i,\,n+1}]$, for which $s_{i,n}(t) > 0$),

$$m_{i,n} = \frac{1}{\tau_{i,n}} \int_{t_{i,n}}^{t_{i,n+1}} s_{i,n}(t)\,dt, \quad \alpha_{i,n} = \frac{1}{\tau_{i,n} m_{i,n}} \int_{t_{i,n}}^{t_{i,n+1}} s_{i,n}^2(t)\,dt,$$

$$\varkappa_{i,n} = \frac{2}{m_{i,n}\tau_{i,n}|\sigma_{i,n}|} \int_{t_{i,n}}^{t_{i,n+1}} (t - t_{i,n})\, s_{i,n}(t)\,dt, \quad m_i = \inf_{|\sigma_{i,n}|>\Delta_i} m_{i,n},$$

$$M_i = \sup_{|\sigma_{i,n}|>\Delta_i} \{ \max_{t \in [t_{i,n}, t_{i,n+1}]} |s_{i,n}(t)| \}, \quad \varkappa_i = \sup_{|\sigma_{i,n}|>\Delta_i} \varkappa_{i,n},$$
$$\alpha_i = \sup_{|\sigma_{i,n}|>\Delta_i} \alpha_{i,n}.$$

If PE_i produces square pulses of amplitude a_i, then evidently $m_{i,n} = \alpha_{i,n} = M_i = m_i = \alpha_i = a_i$, while the characteristic $\varkappa_i$ coincides with the "steepness" of the pulse element characteristic introduced in [2].

It is assumed that the estimates

$$M_i < \infty, \quad m_i > 0 \quad (i = 1, \ldots, l), \tag{5}$$

occur, that functions $f_i(t)$ are absolutely continuous given $t > 0$ and that there is fulfillment of the relations

$$f_i(t) \in L_1[0, \infty), \quad \lim_{t \to \infty} \dot{f}_i(t) = 0 \quad (i = 1, \ldots, l); \tag{6}$$
$$\rho_{ij} = \text{const}, \quad \gamma_{ij}(t) \in L_1[0, \infty) \cap L_2[0, \infty) \quad (i, j = 1, \ldots, l); \tag{7}$$
$$\rho_{ij} + \gamma_{ij}(t) = g_{ij}(t) + \sum_{k=1}^{\infty} c_{i,j}^{(k)} 1(t - \lambda_{i,j}^{(k)}), \tag{8}$$

where the functions $g_{ij}(t)$ are absolutely continuous given $t > 0$, with

$$\dot{g}_{ij}(t) \in L_1[0, \infty), \quad \sum_{k=1}^{\infty} |c_{i,j}^{(k)}| < \infty, \tag{9}$$

$\lambda_{i,j}^{(k)} = \text{const} \geqslant 0, 1(t) = 0$ given $t < 0$, $1(t) = 1$ given $t \geqslant 0$.

The latter requirement is due to the fact that the functions $\gamma_{ij}(t)$ may be discontinuous if the linear part of the control system contains links with distributed parameters.

Let us assume that functions $\chi_{ij}(p) = \int_0^\infty \gamma_{ij}(t) \exp(-pt)dt$ are analytic given $\operatorname{Re} p > 0$, and let us introduce the designations:

$$r_i = \sum_{j=1}^{l} \left(|g_{ij}(+0)| + \int_0^\infty |g_{i,j}(t)|\,dt + \sum_{k=1}^{\infty} |c_{i,j}^{(k)}| \right) M_j, \quad k_i = \frac{r_i \Delta_i \delta_i}{|2a_i|};$$

K and D are diagonal matrices with elements $k_1, \ldots, k_l$ and $d_1, \ldots, d_l$ respectively [See Note]; $\Gamma(p)$ is a square matrix with elements $\chi_{ij}(p)$; R is a square matrix with elements ρ_{ij}.

[Note]: The numbers δ_i and d_i will be described <u>infra</u>.

<u>Theorem 1</u>. Let the following conditions be fulfilled:

1) relations (4)-(9) take place;

2) there exist positive constants δ_i such that $\varkappa_i < 2/r_i - \delta_i$ $(i = 1, \ldots, l)$;

3) if $\Delta_i > 0$, then $\inf\limits_{|\sigma_{i,n}| > \Delta_i} \tau_{i,n} > 0$; if $\Delta_i = 0$, then the dependence of $\tau_{i,n}$ on $\sigma_{i,n}$ is such that $\lim\limits_{\tau_{i,n} \to 0} \sigma_{i,n} = 0$;

4) there exist positive constants $d_1, \ldots, d_l$ such that the matrix DR is symmetric and nonnegative and given all real ω the matrix

$$Q(\omega) = DK + 0{,}5(D\Gamma(i\omega) + \Gamma^*(i\omega)D) \quad (i = \sqrt{-1});$$

5) either det $R \neq 0$ or the constants f_i° are such that the system of equations

$$f_i^0 = \sum_{j=1}^{l} \rho_{ij} u_j \ (i = 1, \ldots, l)$$

is solvable relative to $u_1, \ldots, u_l$.

Then the solution of system (1) possesses the following properties:

$$1) \ \lim_{r_0 \to 0} [\max_i (\sup_{t>0} |\sigma_i(t)|)] = 0, \ \text{где (1)}$$

$$r_0 = \sum_{i=1}^{l} (\sup_{t>0} |f_i(t)| + |f_i^0| + \sup_{t>0} |\dot{f}_i(t)| + \int_0^{\infty} |f_i(t)| \, dt); \qquad (10)$$

$$2) \ \lim_{n \to \infty} \sigma_{i,n} = 0, \ \text{если}^{(2)} \ \Delta_i = 0;$$

Keys:

1. where
2. if

3) if $\Delta_i > 0$, then there will be $T_* > 0$ such that $|\sigma_i(t_{i,n} - 0)| \leqslant \Delta_i$ given all $t_{i,n} > T_*$.

Condition 2) of Theorem 1 cannot be essentially relaxed since there exists an example for which $\varkappa_i = 2/r_i$ and all the rest of the conditions of Theorem 1 are satisfied, but its conclusions are not fulfilled.

Theorem 1 encompasses critical cases where the characteristic equation of the linear part of the system has not more than l zero roots. Let us now consider a noncritical case. In this case in equations (1) $f_i^{\circ} = \rho_{ij} = 0$ (i, j = 1, ..., l) and the following assertion occurs.

<u>Theorem 2</u>. Let in equations (1) and expressions (8), (10) $f_i^{\circ} = \rho_{ij} = 0$ (i, j = 1, ..., l) conditions 1) and 2) of Theorem 1 be fulfilled and let there exist positive con-

stants $d_1, \ldots, d_l$ such that the matrix $Q(\omega)$ is nonnegative given all real .

Then $\lim\limits_{t\to\infty} \sigma_i(t) = 0$ ($i = 1, \ldots, l$) and

$$\lim_{r_j\to 0} [\max_i (\sup_{t>0} |\sigma_i(t)|)] = 0.$$

2. Now let us consider the system of equations

$$\sigma_i(t) = f_i(t) + f_i^0 - \int_0^t [\gamma_{ij}(t-\lambda) + \rho_{ij}] \eta_j(\lambda)\, d\lambda + \psi_i(t); \tag{11}$$

$$\eta_i(t) = \sum_{n=1}^{\infty} \lambda_{i,n} \delta(t - t_{i,n}); \tag{12}$$

$$\lambda_{i,n} = \begin{cases} 0, & \overset{(1)}{\text{если}}\ \varphi_i(t_{i,n}-0) = 0, \\ \operatorname{sign}\varphi_i(t_{i,n}-0), & \overset{(1)}{\text{если}}\ \varphi_i(t_{i,n}-0) \neq 0 \end{cases} \tag{13}$$
$$(i = 1, \ldots, l).$$

Key: 1. if

Here $\sigma_i(t)$, $f_i(t)$, $f_i^0 =$ const, $\gamma_{ij}(t)$, $\rho_{ij} =$ const, $t_{i,n}$ have the same sense as in equations (1); $\psi_i(t)$ are constantly operating perturbations; $\delta(t)$ is the Dirac delta function [4] and the moments $t_{i,n}$ are defined according to formula (3), with $T_{i,n} > 1/\nu^*$.

Each function $\varphi_i(t)$ is piecewise continuous and is the value on $\sigma_i(t)$ of some nonlinear operator A_i. Operators A_i are defined on piecewise continuous functions and satisfy the following conditions:

1) $\varphi_i(t-0)$ depends on the values of function $\sigma_i(\tau)$ only given $0 \leqslant \tau \leqslant t-0$;

2) $\varphi_i(t-0)\sigma_i(t-0) > 0$ for those moments t given which $\sigma_i(t-0) \notin [-\Delta_i'', \Delta_i']$, where Δ_i'', Δ_i' are nonnegative constants.

Equations (11)-(13) describe a nonlinear pulse system with l pulse elements, which effect frequency modulation by instantaneous pulses [5].

It is assumed that the functions $\gamma_{ij}(t)$ are continuous given $t > 0$, belong to $L_2[0, \infty)$, and their Laplace transforms $\chi_{ij}(p)$ are analytic given $\operatorname{Re} p > 0$. With respect to the functions $f_i(t)$ and $\psi_i(t)$ we shall assume that they are continuous given $t > 0$ and, whatever may be the sequence $t_1, t_2, \ldots$, which satisfies the condition $t_{n+1} - t_n > 1/\nu^*$, there is fulfillment of the relations

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} |f_i(t_k)| = 0, \quad \overline{\lim_{n\to\infty}} \frac{1}{n}\sum_{k=1}^{n} |\psi_i(t_k)| < \infty \quad (i = 1, \ldots, l).$$

It can be seen from equations (11) that at moment $t_{j,k}$ the functions $\sigma_i(t)$ undergo a jump by the quantity $-(\gamma_{ij}(+0) + \rho_{ij})\lambda_{j,k}$, which, generally speaking, does not tend to zero given $k \to \infty$, even if $\Delta_i'' = \Delta_i' = \psi_i(t) \equiv 0$ $(i = 1, \ldots, l)$. Therefore instead of the problem of stability as a whole, let us study the dissipativity of system (11)-(13).

<u>Theorem 3</u>. Let system (11)-(13) satisfy the assumptions made in this section and let conditions 4) (given $K = 0$) and 5) of Theorem 1 be fulfilled. Then for the solution of system (11)-(13) the estimate

$$\overline{\lim_{T\to\infty}} \|\sigma\|_T \leq 2\Delta + \gamma + \overline{\lim_{T\to\infty}} \|\psi\|_T, \qquad (14)$$

is valid, where

$$\Delta = \sum_{i=1}^{l} d_i \max(\Delta_i', \Delta_i''), \quad \gamma = l \sum_{i=1}^{l} d_i \sum_{j=1}^{l} (|\gamma_{ij}(+0)| + |\rho_{ij}|),$$

$$\|\sigma\|_T = \frac{1}{N} \sum_{i=1}^{l} d_i \sum_{0 < t_{i,k} < T} |\sigma_i(t_{i,k} - 0)|, \quad N = \max_{\substack{t_{i,k} < T \\ i=1,\ldots,l}} k,$$

$$\|\psi\|_T = \frac{1}{N} \sum_{i=1}^{l} d_i \sum_{0 < t_{i,k} < T} |\psi_i(t_{i,k})|.$$

There exists an example satisfying the conditions of Theorem 3 for which an equality is realized in (14).

Let us assume that nonlinear operators A_i satisfy the supplementary condition [See Note]: $\varphi_i(t) \equiv 0$ for those t given which $\sigma_i(t) \in [-\Delta_i'', \Delta_i']$. Then Δ can be put on the right-hand side of inequality (14) instead of 2Δ.

[Note]: This case takes place when insensitivity is present in pulse elements.

The proof of the above-formulated theorems is based on the method of a priori integral estimates, which is used for estimating a functional which is quadratic relative to $\sigma_1, \ldots, \sigma_l, \eta_1, \ldots, \eta_l$. In the proof of Theorem 3, in addition, the delta function is preliminarily replaced by a delta-shaped sequence (del'taobraznaya posledovatel'nost'), and at the end of the calculations a transition is made to the limit.

It is interesting to note that the results of this work are finding use in the investigation of the dynamics of mathematical models of neuron networks.

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END

FIGURE 5

BLOCK DIAGRAM OF TEST UNIT

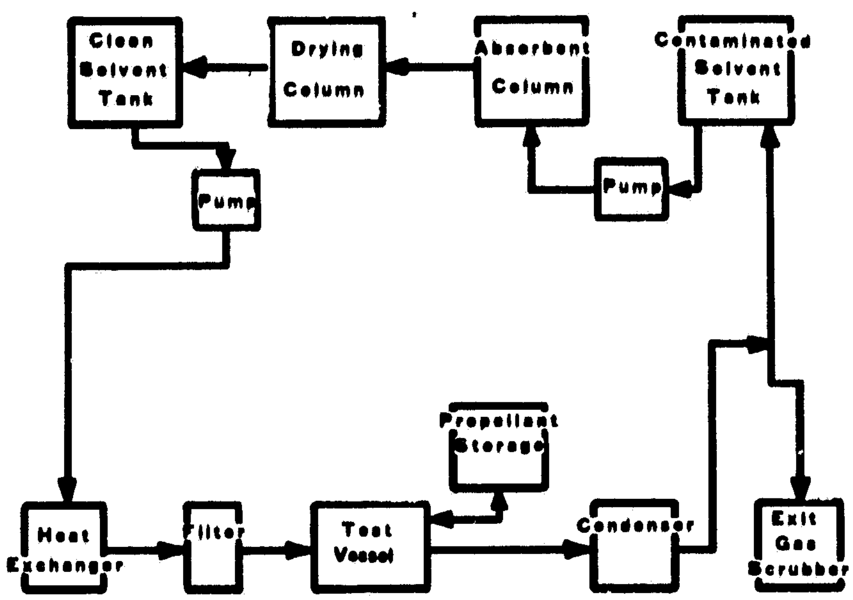

FIGURE 6

REMOVAL OF AEROZINE 50 FROM TEST VESSEL

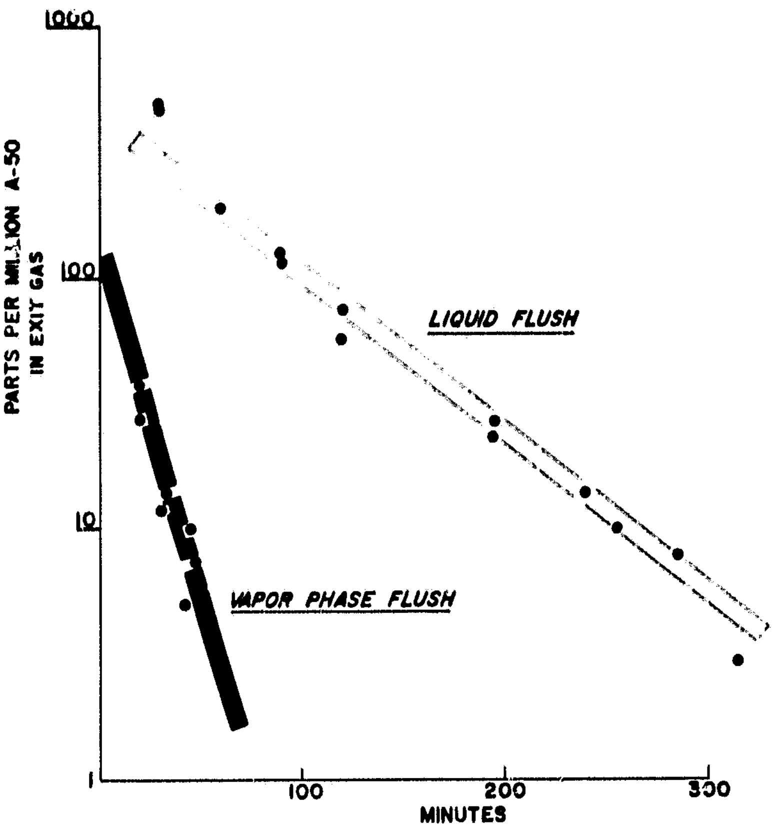

FIGURE 7

REMOVAL OF METHANOL FROM TEST VESSEL

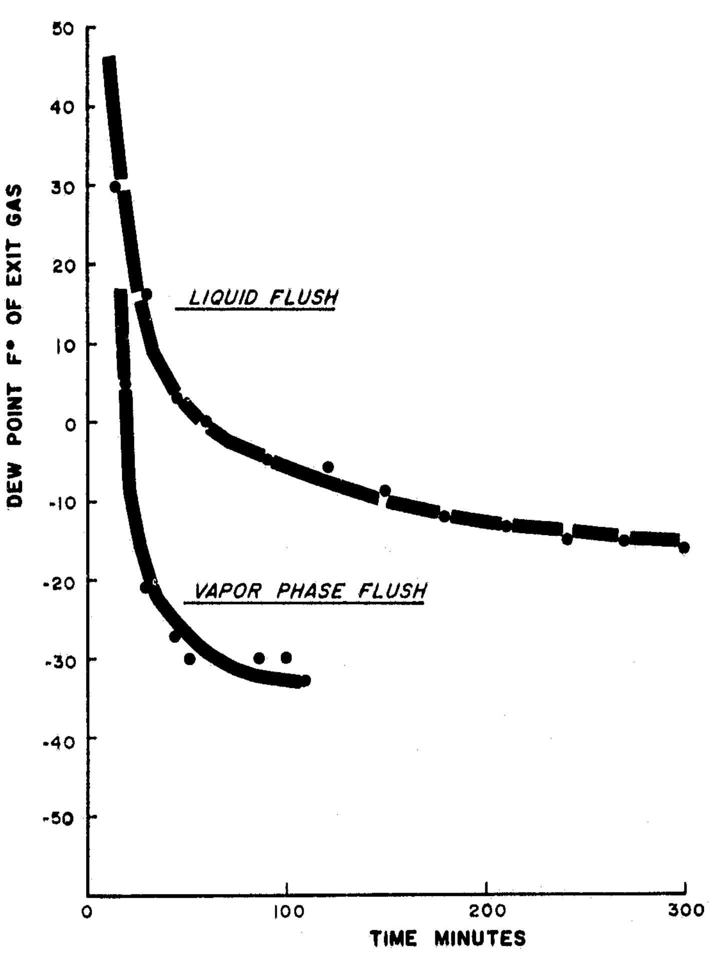

FIGURE 8

REMOVAL OF N_2O_4 FROM TEST VESSEL USING FREON® MF