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## RADIATION TRANSPORT FOR BLUNT-BODY FLOWS INCLUDING THE EFFECTS OF LINES AND ABLATION LAYER

by Join H. Chin



Prepared by
LOCKHEED MISSILES \& SPACE COMPANY Sunnyvale, California for Ames Research Center

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By Jin H. Chin


#### Abstract

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Prepared under Contract No. NAS 2-4219 by LOCKHEED MISSILES \& SPACE COMPANY Sunnyvale, California
for Ames Research Center
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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## FOREWORD

The work described in this report was completed for the National Aeronautics and Space Administration, Ames Research Center, under Contract No. NAS 2-4219, with N. Vojvodich as Technical Monitor.

The author extends his appreciation to L. F. Hearne and K. H. Wilson for their help and discussion during this study.

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# INCLUDING THE EFFECTS OF LINES AND ABLATION LAYER 

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SUMMARY

Efficient numerical methods are developed for fully radiation-coupled inviscid blunt-body flows. The spectral nature of radiation transport is treated in detail, using as many as 21 spectral regions and accounting for air and ablation vapor continua and 75 discrete atomic lines. A concept of line-group equivalent width and average transmittance is introduced in a finite-difference formulation of radiation transport. This concept is used to account for line overlaps and to formulate the numerical procedures.

The application of the numerical methods is described. The effects of environmental variables, of the numerical methods used, and of the uncertainties in radiative properties are discussed. The results demonstrate that the self-absorption and energy loss effects decrease the sensitivity of the heat fluxes to the changes in environmental variables and to the uncertainties in radiative properties.

## INTRODUCTION

This report presents the analyses and results of a study of radiation transport for blunt vehicles returning from space missions at superorbital velocities. The study is an extension of the author's previous investigation (refs. 1 and 2). In refs. 1 and 2, the effects of non-grey, self-absorption and energy loss for inviscid blunt-body flows were considered. The air continuum spectral absorption coefficients were represented approximately by a 2 -band (level) and a 6 -band model. The contributions of discrete atomic line radiation were not taken into account. The solutions of the flow field for the stagnation and nonstagnation regions were formulated separately. A similarity solution was obtained for the stagnation region and a streamtube method was used for the nonstagnation region. The formulation of the present investigation includes the effects of discrete atomic-line radiation and the presence of an inviscid ablation layer adjacent to the wall. Due to time limitations, however, these effects are incorporated in the numerical calculations for the stagnation region only. A newer set of air continuum absorption coefficients is used for both the stagnation and nonstagnation regions.

The purpose of this investigation is to develop economical numerical methods for fully radiation-coupled, inviscid, blunt-body flows, and to assess the effects of environmental variables, of the numerical procedures used, and of the uncertainties in radiative properties.

- The flow-field analysis is reviewed in Section 2. The flow field is assumed inviscid with an interface between the ablation products and the shock layer air. The stagnation region air layer and ablation layer are analyzed using similarity transformations, with continuity of pressure across the interface. The streamtube formulation for the nonstagnation region is very briefly described, as only an example result (fig. 23) is given.

The detailed analysis of radiation transport is given in Section 3. The divergence of the radiative flux is calculated by one-dimensional approximations. For the integration of the energy equation, the shock layer and ablation layer are divided into a number of sublayers, each with constant properties. The integrals of the radiative divergence over the individual sublayers are used in the finite-difference energy balance. The various terms in the expression of these integrals are of the same basic functional form. Studies of the mathematical properties of this basic functional form lead to a theory of calculating the continuum and discrete (atomic lines) radiation contributions. A concept of line-group equivalent width and average transmittance is used to calculate the contribution of a group of lines, accounting for line overlaps. Readers not interested in the details of radiation transport calculations may skip most of Section 3.

Section 4 describes the numerical methods, including the division of continuum bands and line groups, the scheme of flow field iteration, and the scheme of line calculations. Some of the equations [e.g., Eqs. (129) - 156)] presented are used in the computer program and may be omitted by readers not interested in the numerical details.

The results of calculations are given in Section 5. Except fig. 23 which shows an example of the heat flux distribution for a blunt vehicle, all results are for the stagnation region obtained using the STAGRADS code.

The air and ablation vapor thermodynamic properties, the radiative properties of air and ablation species, and the description of the STAGRADS code are given in the appendix.

## 1 NOMENCLATURE

${ }^{*}$
a

A
$\mathrm{A}_{\mathrm{i}_{1}, \mathrm{i} 2, \mathrm{~m}}$
b
b
$\mathrm{b}_{\mathrm{nn}}{ }^{(\nu)}$
B
c.
$\mathrm{d}_{\mathrm{nn}}{ }^{\prime}$
e
E
$\mathrm{E}_{\mathrm{n}}(\zeta)$
f
$f_{n n^{\prime}}$

F
g
$g_{k}$
$\mathrm{g}_{\mathrm{n}}$
h
H
I
k
K
$\ell_{s}, \ell_{y}$
L
m electron mass

| $\dot{\mathrm{m}}_{\mathrm{w}}$ |
| :---: |
| n |
| $\mathrm{n}_{\mathrm{k}}$ |
| N |
| Ne |
| p |
| q |
| $\overrightarrow{\mathrm{q}}_{\mathrm{r}}$ |
| $q_{r_{s}}, q_{1}$ |
| Q |
| $\mathbf{r}$ |
| $\mathrm{r}_{\mathrm{o}}$. |
| R |
| $\mathrm{R}_{\mathrm{M}}$ |
| $\mathrm{R}_{\mathrm{N}}$ |
| $\mathrm{R}_{0}$ |
| R* |
| s |
| $\overline{\mathbf{s}}$ |
| S |
| T |
| Trans |
| u |
| $u_{r}$ |
| v |
| Width |
| y |
| z |
| Z |
| $\gamma \mathrm{e}$ |
| $\bar{\gamma}_{k}$ |

wall blowing mass flux
number of sublayers; state index
k -th line lower state index, see Section 3.4.3
species particle number density
electron particle number density
static pressure
net radiative power gain per unit volume
radiative flux vector
components of radiative flux in $s$ and $y$ directions
radiative power gain for a finite sublayer; partition function distance from plane or axis of symmetry
value of $r$ at streamline entry point
ratio of equivalent width to line-group width for isolated lines
maximum radius, fig. 22
nose radius
distance from stagnation point to effective axis of symmetry
ratio of equivalent width to line-group width by integration
distance along body from stagnation point
distance along body from point of symmetry, fig. 22
line strength, Eq. (110)
absolute temperature
line-group transmittance
velocity parallel to body
value of ablation layer $u$ at interface, $u_{e}=u_{r}\left(s / R_{N}\right)$
velocity normal to body
line-group equivalent width
normal distance from body surface
normal distance from shock wave
compressibility factor
line effective half-width
k -th line half-width due to 1 electron per unit volume

| $\bar{\gamma}_{n^{\prime}}$ | $\bar{\gamma}$ for $\mathrm{n} \rightarrow \mathrm{n}^{\prime}$ transition |
| :---: | :---: |
| $\delta \nu_{\mathrm{k}}$ | integration interval for k -th line |
| $\Delta$ | total thickness between shock and wall |
| $\Delta H_{v}$ | latent heat of vaporization |
| $\Delta y_{i}$ | $\mathrm{y}_{\mathrm{i}+1}-\mathrm{y}_{\mathrm{i}}$ |
| $\Delta z_{i}$ | $\mathrm{z}_{\mathbf{i}+1}-\mathrm{z}_{\mathbf{i}} ; \Delta \mathrm{z}_{\mathrm{i}}=\Delta y_{i}$ |
| $\Delta \nu_{\mathrm{m}}$ | integration interval for m -th line group |
| $\Delta \tau_{j}$ | $\mu_{j} \Delta z_{j}$ |
| $\zeta$ | transformed normal distance; dummy variable |
| $\eta$ | transformed normal distance, Eq. (34) |
| $\mu$ | linear absorption coefficient |
| $\nu$ | frequency |
| $\nu_{\mathrm{a}}$ | average frequency within line group |
| $\nu_{\mathrm{o}, \mathrm{nn}}{ }^{\prime}$ | frequency corresponding to unperturbed transition $n \rightarrow \mathrm{n}^{\prime}$ |
| $\xi$ | transformed distance from stagnation point, Eq. (33); dummy variable |
| $\rho$ | density |
| $\rho_{\text {SL }}$ | sea-level density |
| $\sigma$ | cross section; Stefan-Boltzmann constant |
| $\tau$ | optical thickness |
| $\phi$ | angle between flight vector and normal |
| $\psi$ | stream function |
| $\Omega$ | solid angle |
| Subscripts |  |
| a | air layer |
| ab | ablation layer; with ablation layer effect |
| c | continuum contribution |
| d | discrete contribution |
| e | at interface between air layer and ablation layer |
| i | species index; dummy index for sublayer |
| $\mathrm{i}_{1}, \mathrm{i}_{2}$ | specific sublayers |

dummy sublayer index; dummy index for frequency integration dummy index for lines
for transition $n \rightarrow n^{\prime}$
immediately behind shock; toward shock
at satellite velocity
at wall conditions
at frequency $\nu$
at ambient conditions

## 2 FLOW FIELD ANALYSIS

### 2.1 Governing Equations

To simplify the formulation of the problem, the flow field is assumed quasisteady; inviscid, non-conducting, and in thermodynamic equilibrium. An interface exists between the ablation products and the shock-layer air. The wall is assumed to be at the equilibrium sublimation temperature corresponding to the surface pressure.

In body-oriented coordinates ( $s, y$ ) shown in fig. 1, the conservation equations for a two-dimensional or axisymmetric, inviscid flow may be written as follows: continuity

$$
\begin{equation*}
\frac{\partial r^{L} \rho u}{\partial s}+\frac{\partial A r^{L} \rho v}{\partial y}=0 \tag{1}
\end{equation*}
$$

s-momentum

$$
\begin{equation*}
u \frac{\partial u}{\partial s}+A v \frac{\partial u}{\partial y}+K u v=-\frac{1}{\rho} \frac{\partial p}{\partial s} \tag{2}
\end{equation*}
$$

y-momentum

$$
\begin{equation*}
u \frac{\partial v}{\partial s}+A v \frac{\partial v}{\partial y}-K u^{2}=-\frac{A}{\rho} \frac{\partial p}{\partial y} \tag{3}
\end{equation*}
$$

energy

$$
\begin{equation*}
u \frac{\partial H}{\partial s}+A v \frac{\partial H}{\partial y}=\frac{A}{\rho} q \tag{4}
\end{equation*}
$$

The net radiative power gain per unit volume is related to the divergence of the radiative flux:

$$
\begin{equation*}
q=-\operatorname{div} \vec{q}_{r}=-\left[\frac{\partial q_{r_{y}}}{\partial y}+\frac{\partial q_{r_{S}}}{A \partial s}\right] \tag{5}
\end{equation*}
$$

The equations-of-state of equilibrium air and equilibrium ablation vapor may be expressed in the following form:

$$
\begin{equation*}
\rho, \mathrm{T}, \mathrm{~N}_{\mathrm{i}}, \ldots=\text { function }(\mathrm{p}, \mathrm{~h}) \tag{6}
\end{equation*}
$$

The boundery conditions are as follows:
At the wall,

$$
\begin{gather*}
u=u_{w}=0  \tag{7}\\
v=v_{w}=\dot{m}_{w} / \rho_{w}=\left(q_{w}-\sigma T_{w}^{4}\right) / \rho_{w} \Delta H_{v}  \tag{8}\\
h=h_{w}=h_{w}\left(p_{w}\right) \tag{9}
\end{gather*}
$$

The wall is assumed to be black.
At the shock wave,

$$
\begin{gather*}
u=u_{S}=u_{\infty}\left[\sin \phi_{S} \cos \left(\phi_{\mathrm{w}}-\phi_{\mathrm{S}}\right)+\epsilon \cos \phi_{\mathrm{S}} \sin \left(\phi_{\mathrm{w}}-\phi_{\mathrm{S}}\right)\right]  \tag{10}\\
\mathrm{v}=\mathrm{v}_{\mathrm{S}}=\mathrm{u}_{\infty}\left[\sin \phi_{\mathrm{S}} \sin \left(\phi_{\mathrm{w}}-\phi_{\mathrm{S}}\right)-\epsilon \cos \phi_{\mathrm{S}} \cos \left(\phi_{\mathrm{w}}-\phi_{\mathrm{S}}\right)\right]  \tag{11}\\
\mathrm{p}=\mathrm{p}_{\mathrm{S}}=\rho_{\infty} u_{\infty}^{2}(1-\epsilon) \cos ^{2} \phi_{\mathrm{S}}+\mathrm{p}_{\infty}  \tag{12}\\
\mathrm{h}=\mathrm{h}_{\mathrm{S}}=\left(\frac{1}{2}\right) \mathrm{u}_{\infty}^{2}\left(1-\epsilon^{2}\right) \cos ^{2} \phi_{\mathrm{S}}+\mathrm{h}_{\infty} \tag{13}
\end{gather*}
$$

At the interface,

$$
\begin{align*}
& \mathrm{p}_{\mathrm{a}, \mathrm{e}}=\mathrm{p}_{\mathrm{ab}, \mathrm{e}}  \tag{14}\\
& \frac{\mathrm{v}_{\mathrm{a}, \mathrm{e}}}{\mathrm{u}_{\mathrm{a}, \mathrm{e}}}=\frac{\mathrm{v}_{\mathrm{ab}, \mathrm{e}}}{u_{\mathrm{ab}, \mathrm{e}}} \tag{15}
\end{align*}
$$

These equations will be approximated for different regions of the flow field.

## 2. 2 Stagnation Flow Field

For the stagnation region, $\mathrm{s} / \mathrm{R}_{\mathrm{N}} \ll 1$, the conservation equations may be simplified. The kinetic energy change is of a smaller order than the enthalpy change and the pressure variation normal to the wall is small. Equations (3) and (4) may then be approximated by Eqs. (16) and (17), respectively.

$$
\begin{equation*}
\frac{\partial p}{\partial y}=0 \quad \text { or } \quad p=p_{S} \tag{16}
\end{equation*}
$$

[^0]\[

$$
\begin{equation*}
u \frac{\partial h}{\partial s}+A v \frac{\partial h}{\partial y}=\frac{A}{\rho} q \tag{17}
\end{equation*}
$$

\]

For this study, the shockwave and the interface are further assumed to be concentric with the body surface so that $\phi_{\mathrm{S}}=\phi_{\mathrm{e}}=\phi_{\mathrm{w}}$.
2.2.1 Air layer. - For the stagnation air layer, Eqs. (10) to (13) reduce to:

$$
\begin{gather*}
u_{s}=u_{\infty} \frac{s}{R_{N}}  \tag{18}\\
p_{s}=\rho_{\infty} u_{\infty}^{2}(1-\epsilon)\left(1-\frac{s^{2}}{R_{N}^{2}}\right)+p_{\infty}  \tag{19}\\
h_{s}=\frac{1}{2} u_{\infty}^{2}\left(1-\epsilon^{2}\right)+h_{\infty} \tag{20}
\end{gather*}
$$

For a concentric interface,

$$
\begin{equation*}
v_{e}=0 \tag{22}
\end{equation*}
$$

For inviscid flow, the interface conditions for $u$ and $h$ cannot be specified; they are part of the solution to be obtained.

Since there are more kıown boundary conditions at the shockwave, it-is more convenient for the air layer solution to use the shockwave as the datum for the normal distance. Using transformations similar to the Lees-Dorodnitsyn transformations for compressible boundary layers and assuming similarity, the following results were obtained in ref. $1^{*}$ :

$$
\begin{align*}
\mathrm{d} \zeta \equiv & \equiv(\mathrm{~L}+1) \frac{\rho}{\rho_{\mathrm{S}}} \frac{\mathrm{dz}}{\mathrm{R}_{\mathrm{N}}}  \tag{23}\\
\mathrm{~F} \equiv & -\frac{\rho \mathrm{v}}{\rho_{\infty} u_{\infty}}=\frac{\rho \mathrm{v}}{\rho_{\mathrm{S}} \mathrm{v}_{\mathrm{S}}}  \tag{24}\\
& -\frac{\mathrm{dF}}{\mathrm{~d} \zeta}=\frac{u}{u_{\mathrm{S}}} \tag{25}
\end{align*}
$$

[^1]\[

$$
\begin{gather*}
g \equiv \frac{h}{h_{s}}  \tag{26}\\
F \frac{d^{2} F}{d \xi^{2}}=\frac{1}{(L+1)}\left[\left(\frac{d F}{d \zeta}\right)^{2}-2 \epsilon(1-\epsilon) \frac{\rho_{s}}{\rho}\right]  \tag{27}\\
F \frac{d g}{d \zeta}=\left[\frac{R_{N}}{(L+1) \rho u_{\infty}}\right] q \tag{28}
\end{gather*}
$$
\]

or

$$
\begin{equation*}
F \mathrm{dg}=\frac{2}{\rho_{\infty} u_{\infty}^{3}}(\mathrm{qdz}) \tag{28a}
\end{equation*}
$$

where z is the distance from the shockwave toward the interface. In Eqs. (23) to (28), the shock layer has been assumed thin compared to the nose radius so that $\mathrm{A} \approx 1$.

The boundary conditions are:
At the shockwave

$$
\begin{equation*}
\zeta=0, \quad F=-\frac{d F}{d \zeta}=g=1 \tag{29a}
\end{equation*}
$$

at the interface,

$$
\begin{equation*}
\zeta=\zeta_{e}, \quad F=0 \tag{29b}
\end{equation*}
$$

where $\zeta_{e}$ is the value of $\zeta$ at the interface. The value of $\zeta_{e}$ must be determined from the solution.

The numerical integration of Eqs. (27) and (28a) is described in Section 4. 2.
2.2.2 Ablation layer. - For the stagnation ablation layer, Eqs. (16) and (20) indicate:

$$
\begin{equation*}
p=p_{e}=\rho_{\infty} u_{\infty}^{2}(1-\epsilon)\left(1-\frac{s^{2}}{R_{N}^{2}}\right)+p_{\infty} \tag{30}
\end{equation*}
$$

Since the layer edge is a streamline and is concentric with the wall, using $u_{e}=u_{r}\left(s / R_{N}\right)$ one obtains:

$$
\begin{equation*}
\frac{d p_{e}}{d s}=-\rho_{e} u_{e} \frac{d u_{e}}{d s}=-\rho_{e} u_{r}^{2} \frac{s}{R_{N}^{2}}=-2 \rho_{\infty} u_{\infty}^{2}(1-\epsilon) \frac{s}{R_{N}^{2}} \tag{31}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\frac{\mathrm{u}_{\mathrm{r}}}{\mathrm{u}_{\infty}}=\left[\frac{2 \rho_{\infty}(1-\epsilon)}{\rho_{\mathrm{e}}}\right]^{1 / 2} \tag{32}
\end{equation*}
$$

where $\rho_{\mathbf{e}}$ is not known before the solution is obtained.
Equations (1), (2), and (17) may be simplified by introducing the following transformations:

$$
\begin{align*}
\xi & =\int_{0}^{s}\left(\frac{r}{A}\right)^{2 L} u_{e} d s  \tag{33}\\
\eta & =\frac{u_{e}}{\sqrt{2 \xi}}\left(\frac{r}{A}\right)^{L} \int_{0}^{y} A^{L} \rho d y  \tag{34}\\
\frac{\partial \psi}{\partial \mathbf{y}} & =-\rho \mathbf{u r}^{L}, \frac{\partial \psi}{\partial s}=A \rho v r^{L}  \tag{35}\\
\mathbf{f}(\xi, \eta) & =\frac{\psi}{\sqrt{2 \xi}}  \tag{36}\\
\mathbf{g}(\xi, \eta) & =\frac{\mathbf{h}}{h_{w}} \tag{37}
\end{align*}
$$

If similarity is assumed so that all dependent variables are functions of $\eta$ only, the transformed equations may be shown as follows:

$$
\begin{align*}
& \mathrm{f} \frac{\mathrm{~d}^{2} \mathrm{f}}{\mathrm{~d} \eta^{2}}=\frac{1}{(\mathrm{~L}+1)}\left\{\left(\frac{\mathrm{df}}{\mathrm{~d} \eta}\right)^{2}-2 \frac{\rho_{\infty} u_{\infty}^{2}}{\rho u_{r}^{2}}-\frac{\mathrm{K}}{\mathrm{~A}^{\mathrm{L}+1} \rho}\left[\frac{(\mathrm{~L}+1) \mathrm{R}_{\mathrm{N}}}{\mathrm{u}_{\mathrm{r}}}\right]^{1 / 2} \mathrm{f} \frac{\mathrm{df}}{\mathrm{~d} \eta}\right\}  \tag{38}\\
& \mathrm{f} \frac{\mathrm{dg}}{\mathrm{~d} \eta}=\frac{\mathrm{AR}}{\mathrm{~N}}  \tag{39}\\
&(\mathrm{~L}+1) \rho \mathrm{u}_{\mathrm{r}} \mathrm{~h}_{\mathrm{w}}
\end{align*} \mathrm{q} .
$$

The transformation also yields the following relations:

$$
\begin{equation*}
f=A^{\mathrm{L}+1}{ }_{\rho \mathrm{v}}\left[\frac{\mathrm{R}_{\mathrm{N}}}{(\mathrm{~L}+1) \mathrm{u}_{\mathrm{r}}}\right]^{1 / 2} \tag{40}
\end{equation*}
$$

$$
\begin{align*}
& \frac{d f}{d \eta}=-\left(\frac{u}{u_{e}}\right)  \tag{41}\\
& \frac{d \eta}{d y}=\left[\frac{(L+1) u_{r}}{R_{N}}\right]^{1 / 2} A^{L} \rho \tag{42}
\end{align*}
$$

Further simplification of Eqs. (38) and (39) is possible by introducing the following:

$$
\begin{align*}
& F=\frac{f}{\rho_{w} v_{w}\left[\frac{R_{N}}{(L+1) u_{r}}\right]^{1 / 2}}=\frac{A^{L+1} \rho v}{\rho_{w} v_{w}}  \tag{43}\\
& \mathrm{~d} \xi=\frac{\left(\frac{2 \rho_{\infty} u_{\infty}^{2}}{\rho_{w} u_{r}^{2}}\right)^{1 / 2}}{\rho_{w} v_{w}\left[\frac{R_{N}}{(L+1) u_{r}}\right]^{1 / 2}} \mathrm{~d} \eta=(L+1)\left(\frac{2 \rho_{\infty} u_{\infty}^{2}}{\rho_{w} v_{w}^{2}}\right)^{1 / 2} \cdot \frac{\rho}{\rho_{w}} A^{L} \frac{d y}{R_{N}} \tag{44}
\end{align*}
$$

Equations (38) and (39) become, respectively,

$$
\begin{align*}
F \frac{d^{2} F}{d \zeta^{2}} & =\frac{1}{(L+1)}\left\{\left(\frac{d F}{d \zeta}\right)^{2}-\frac{\rho_{w}}{\rho}\left[1+\frac{1}{A^{L+1}}\left(\frac{\rho_{w} v_{w}^{2}}{2 \rho_{\infty} u_{\infty}^{2}}\right)^{1 / 2} \mathrm{~F} \frac{d F}{d \zeta}\right]\right\}  \tag{45}\\
F d g & =\frac{1}{\rho_{w} v_{w} h_{w}} A^{L+1} q d y \tag{46}
\end{align*}
$$

For $\Delta_{a b} / R_{N} \ll 1, A \approx 1$.
For conditions with

$$
\rho_{\mathrm{w}} \mathrm{v}_{\mathrm{w}}^{2} / \rho_{\infty} \mathrm{u}_{\infty}^{2} \ll 1
$$

one may neglect the last term in Eq. (45).

$$
\begin{equation*}
\mathrm{F} \frac{\mathrm{~d}^{2} \mathrm{~F}}{\mathrm{~d} \zeta^{2}}=\frac{1}{(\mathrm{~L}+1)}\left[\left(\frac{\mathrm{dF}}{\mathrm{~d} \zeta}\right)^{2}-\frac{\rho_{\mathrm{W}}}{\rho}\right] \tag{47}
\end{equation*}
$$

The boundary conditions are as follows:
at $\zeta=\mathrm{y}=0$

$$
\begin{equation*}
\frac{\mathrm{dF}}{\mathrm{~d} \underline{\zeta}}=-\left(\frac{\mathrm{u}}{u_{e}}\right)\left(\frac{\rho_{\mathrm{w}}}{\rho_{\mathrm{e}}}\right)^{1 / 2}=0 \quad, \quad \mathrm{~F}=1 \quad, \quad \mathrm{~g}=1 \tag{48a}
\end{equation*}
$$

at $\zeta=\zeta_{\mathrm{e}}$ or $\mathrm{y}=\Delta_{\mathrm{ab}}$

$$
\begin{equation*}
\mathrm{F}=0 \quad, \quad \frac{\mathrm{dF}}{\mathrm{~d} \zeta}=-\left(\frac{\rho_{\mathrm{w}}}{\rho_{\mathrm{e}}}\right)^{1 / 2}, \quad \mathrm{p}=\rho_{\infty} \mathrm{u}_{\infty}^{2}(1-\epsilon)+\mathrm{p}_{\infty} \tag{48b}
\end{equation*}
$$

Because of the last term in Eq. (47), the momentum equation is coupled to the energy equation. The surface blowing rate and the layer thickness are not known before the solution is obtained.

The numerical integration of Eqs. (46) and (47) is described in Section 4. 2.

### 2.3 Streamtube Formulation

For the nonstagnation region, the streamtube formulation described in ref. 1 is used. For the inviscid air layer, the conservation equations for an infinitesmal streamtube may be written as follows:
continuity

$$
\begin{equation*}
\rho \operatorname{ur}^{L} \frac{d y}{d s}=\rho_{\infty} u_{\infty} r_{o}^{L} \frac{d r_{o}}{d s} \tag{49}
\end{equation*}
$$

momentum

$$
\begin{equation*}
\rho u \frac{d u}{d s}=-\frac{d p}{d s} \tag{50}
\end{equation*}
$$

energy

$$
\begin{equation*}
\rho u \frac{d\left(h+\frac{u^{2}}{2}\right)}{d s}=q \tag{51}
\end{equation*}
$$

where $r_{o}$ is the radial distance at which the streamtube enters the shock. The entry conditions for the streamtube are given by Eqs. (10) to (13). The application of the streamtube method during this study has been limited to air-layer calculations with continuum radiation only. The integration of Eqs. (49) to (51) follows the procedures described in ref. 1 and will not be discussed further in this report. The streamtube method may also be applied to the ablation-layer calculations.

### 3.1 Basic Radiative Equations

The net radiative power gain per unit volume by the gas in local thermodynamic equilibrium is given by

$$
\begin{equation*}
\mathrm{q}(\mathrm{~s}, \mathrm{y})=\int_{\nu} \mathrm{d} \nu \int_{\Omega=4 \pi} \mu_{\nu}(\mathrm{s}, \mathrm{y})\left[\mathrm{I}_{\nu}(\mathrm{s}, \mathrm{y}, \vec{\Omega})-\mathrm{B}_{\nu}(\mathrm{s}, \mathrm{y})\right] \mathrm{d} \Omega \tag{52}
\end{equation*}
$$

where $I_{\nu}$ is the monochromatic specific intensity (power per unit solid angle per unit area normal to the direction of propagation), $\mathrm{B}_{\nu}$ the Planck distribution function, and $\mu_{\nu}$ is the spectral (monochromatic) absorption coefficient accounting for induced emission. The monochromatic specific intensity is governed by the following equation of radiative transfer:

$$
\begin{equation*}
\left.\operatorname{div}\left[\vec{\Omega} \mathrm{I}_{\nu}(\vec{\Omega})\right] \equiv \vec{\Omega} \cdot \operatorname{grad} \mathrm{I}_{\nu}(\vec{\Omega})=\mu_{\nu} \mid \mathrm{B}_{\nu}-\mathrm{I}_{\nu}(\vec{\Omega})\right] \tag{53}
\end{equation*}
$$

In body-oriented coordinates, Eq. (53) becomes

$$
\begin{equation*}
\ell_{\mathrm{s}} \frac{\partial \mathrm{I}_{\nu}(\vec{\Omega})}{\mathrm{A} \partial \mathrm{~s}}+\ell_{\mathrm{y}} \frac{\partial \mathrm{I}_{\nu}(\vec{\Omega})}{\partial \mathrm{y}}=\mu_{\nu}\left[\mathrm{B}_{\nu}-\mathrm{I}_{\nu}(\vec{\Omega})\right] \tag{54}
\end{equation*}
$$

where $\ell_{S}$ and $\ell_{y}$ are the directional cosines of the vector $\vec{\Omega}$ with respect to the coordinate axes. The flow field is assumed either two-dimensional or axisymmetric so that the gradient of the specific intensity in the circumferential direction vanishes.

The net radiative flux crossing a unit area with unit normal $\vec{\Omega}^{\prime}$ is given by

$$
\begin{equation*}
\mathrm{q}_{\nu}\left(\vec{\Omega}^{\prime}\right)=\int_{\Omega=4 \pi}\left|\vec{\Omega} \mathrm{I}_{\nu}(\vec{\Omega})\right| \cdot \vec{\Omega}^{\prime} \mathrm{d} \vec{\Omega} \tag{55}
\end{equation*}
$$

Then,

$$
\begin{align*}
\mathrm{q}_{\mathrm{r}_{\nu, \mathrm{s}}} & =\int_{\Omega=4 \pi} \ell_{\mathrm{S}} \mathrm{I}_{\nu}(\bar{\Omega}) \mathrm{d} \Omega  \tag{56a}\\
\mathrm{q}_{\mathrm{r}_{\nu, \mathrm{y}}} & =\int_{\Omega=4 \pi} \ell_{\mathrm{y}} \mathrm{I}_{\nu}(\bar{\Omega}) \mathrm{d} \Omega \tag{56b}
\end{align*}
$$

Integrating Eq. (54) over all solid angles and using Eqs. (52), (56a), and (56b), one obtains the monochromatic form of Eq. (5)

$$
\begin{equation*}
\frac{\partial \mathrm{q}_{\nu, \mathrm{s}}}{\mathrm{~A} \partial \mathrm{~s}}+\frac{\partial \mathrm{q}_{\mathbf{r}_{\nu, \mathrm{y}}}}{\partial \mathrm{y}}=\operatorname{div} \overrightarrow{\mathrm{q}}_{\mathrm{r}_{\nu}}=-\mathrm{q}_{\nu}(\mathrm{s}, \mathrm{y}) \tag{57}
\end{equation*}
$$

Under conditions that the temperature gradients in the s-direction are small compared to that in the $y$-direction, one may neglect $\ell_{S}\left[\partial I_{\nu}(\vec{\Omega}) / \mathrm{A} \partial \mathrm{s}\right]$ in Eq. (54) and

$$
\left(\partial q_{r_{\nu, s}} / A^{\prime} \partial s\right)
$$

in Eq. (57) to obtain the "tangent slab" or one-dimensional approximation for radiation transport.

For the environmental condition of interest in this study ( $\mathrm{u}_{\infty}<60,000 \mathrm{ft} / \mathrm{sec}$, $\mathrm{R}_{\mathrm{N}}<10 \mathrm{ft}$ ) the effect of precursor radiation heating of the ambient air ahead of the bow shock is to increase the wall heat flux by the order of 10 percent or less (refs. 3 and 4). Thus, it does not appear that the neglect of precursor heating in the basic radiative calculations contributes significantly to the uncertainty in magnitude of radiation heating. For this investigation, the precursor heating effect and the shock-wave reflectivity are neglected and the body surface is assumed black.

For one-dimensional radiation transfer, the monochromatic net radiative power gain or the radiative flux divergence is given by refs. 1 and 5.

$$
\begin{align*}
-\frac{\partial \mathrm{q}_{\mathrm{r}}, \mathrm{y}}{}= & \mathrm{q}_{\nu}(\mathrm{y}) \\
= & -4 \pi \mu_{\nu} \mathrm{B}_{\nu}+2 \pi \mu_{\nu} \mathrm{B}_{\mathrm{w}, \nu} \mathrm{E}_{2}\left(\int_{0}^{\mathrm{y}} \mu_{\nu}^{\prime} \mathrm{dy}^{\prime}\right) \\
& \quad+2 \pi \mu_{\nu} \int_{\mathrm{o}}^{\Delta} \mu_{\nu}^{\prime} \mathrm{B}_{\nu}^{\prime} \mathrm{E}_{1}\left(\left|\int_{\mathrm{y}^{\prime}}^{\mathrm{y}} \mu_{\nu}^{\prime \prime} \mathrm{d} y^{\prime \prime}\right|\right) \mathrm{dy}^{\prime} \tag{58}
\end{align*}
$$

where $\mathrm{E}_{\mathrm{n}}(\zeta)$ is the $\mathrm{n}^{\text {th }}$ exponential integral and $\Delta$ is the thickness of the "tangent slab."

The succeeding terms on the right-hand side of Eq. (58) can be recognized as (1) radiative loss, (2) absorption of radiation from body surface attenuated by the medium between the body and the local point, and (3) absorption of radiation emitted by medium on both sides of the local point.

- In terms of $z=\Delta-y$, the monochromatic optical thickness is given by

$$
\begin{align*}
\tau_{\nu}(\mathrm{z}) & =\int_{0}^{\mathrm{z}} \mu_{\nu}\left(\mathrm{z}^{\prime}\right) \mathrm{dz}  \tag{59}\\
\tau_{\Delta, \nu} & =\int_{0}^{\Delta} \mu_{\nu}\left(\mathrm{z}^{\prime}\right) \mathrm{dz}^{\prime} \tag{60}
\end{align*}
$$

Equation (58) may then be rewritten as

$$
\begin{equation*}
\mathrm{q}_{\nu}\left(\tau_{\nu}\right)=-4 \pi \mu_{\nu} \mathrm{B}_{\nu}+2 \pi \mu_{\nu} \mathrm{B}_{\mathrm{w}, \nu} \mathrm{E}_{2}\left(\tau_{\Delta, \nu}-\tau_{\nu}\right)+2 \pi \mu_{\nu} \int_{0}^{\tau} \Delta, \nu \mathrm{B}_{\nu}(\mathrm{t}) \mathrm{E}_{1}\left(\left|\tau_{\nu}-\mathrm{t}\right|\right) \mathrm{dt} \tag{61}
\end{equation*}
$$

The monochromatic heat fluxes incident to the wall and leaving the shockwave toward the ambient air are given by Eqs. (62) and (63), respectively,

$$
\begin{gather*}
\mathrm{q}_{\mathrm{W}, \nu}=2 \pi \int_{0}^{\tau} \Delta, \nu  \tag{62}\\
\mathrm{B}_{\nu}(\mathrm{t}) \mathrm{E}_{2}\left(\tau_{\Delta, \nu}-\mathrm{t}\right) \mathrm{dt}  \tag{63}\\
\mathrm{q}_{\mathrm{s}, \nu}=2 \pi \int_{0}^{\tau} \mathrm{B}_{\nu}(\mathrm{t}) \mathrm{E}_{2}(\mathrm{t}) \mathrm{dt}+2 \pi \mathrm{~B}_{\mathrm{w}, \nu} \mathrm{E}_{3}(\tau \Delta, \nu)
\end{gather*}
$$

Integration of Eq. (61) over all frequencies then yields the q required in Eqs. (28), (28a), and (46). Equations (62) and (63) may be integrated over all frequencies to obtain the total heat fluxes.

### 3.2 Equations for a Finite Number of Sublayers

In order to reduce the computation time required for integration of the conservation and radiative transport equations, the shock layer and ablation layer may be divided into a number of sublayers. Across the width of the sublayer, constant properties are assumed, but radiation traversing through is attenuated by each of the differential widths within the sublayer. Equation (61) may be integrated across the width of the ${ }^{\text {ith }}$ sublayer from the shockwave (fig. 2).

$$
\mathrm{Q}_{\mathbf{i}}(\nu) \cong \int_{\mathbf{z}_{\mathbf{i}}}^{\mathbf{z}_{\mathbf{i}+1}} \mathrm{q}_{\nu}(\mathrm{z}) \mathrm{dz}
$$

Carrying the integration, one obtains

$$
\begin{align*}
Q_{i}(\nu)=2 \pi B_{w}\left[E_{3}\left(\tau_{n+1}-\tau_{i+1}\right)-E_{3}\left(\tau_{n+1}-\tau_{i}\right)\right]+2 \pi & \sum_{j=1}^{n} B_{j}\left[E_{3}\left(\left|\tau_{i}-\tau_{j+1}\right|\right)\right. \\
& \left.-E_{3}\left(\left|\tau_{i}-\tau_{j}\right|\right)+E_{3}\left(\left|\tau_{i+1}-\tau_{j}\right|\right)-E_{3}\left(\left|\tau_{i+1}-\tau_{j+1}\right|\right)\right] \tag{65}
\end{align*}
$$

For convenience, the subscript $v$ has been omitted in Eq. (65) and the equations following. Equations relating monochromatic quantities may be recognized by the presence of at least one term with argument $\nu$. The optical thickness for the sublayers are given by

$$
\begin{align*}
\Delta \tau_{i}(\nu) & =\mu_{i} \Delta z_{i}  \tag{66a}\\
\tau_{1}(\nu) & =0  \tag{66b}\\
\tau_{i+1}(\nu) & =\sum_{j=1}^{i} \Delta \tau_{i}, i=1,2, \ldots, n \tag{66c}
\end{align*}
$$

The monochromatic radiative heat flux to the wall, $\mathrm{q}_{\mathrm{w}}(\nu)$, is given by Eq. (67), obtained from Eq. (62).

$$
\begin{equation*}
\mathrm{q}_{\mathrm{w}}(\nu)=2 \pi \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~B}_{\mathrm{i}}\left[\mathrm{E}_{3}\left(\tau_{\mathrm{n}+1}-\tau_{\mathrm{i}+1}\right)-\mathrm{E}_{3}\left(\tau_{\mathrm{n}+1}-\tau_{\mathrm{i}}\right)\right] \tag{67}
\end{equation*}
$$

The monochromatic radiative heat flux leaving the shock front may be similarly derived.

$$
\begin{equation*}
\mathrm{q}_{\mathrm{S}}(\nu)=2 \pi \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~B}_{\mathrm{i}}\left[\mathrm{E}_{3}\left(\tau_{\mathrm{i}}\right)-\mathrm{E}_{3}\left(\tau_{\mathrm{i}+1}\right)\right]+2 \pi \mathrm{~B}_{\mathrm{w}} \mathrm{E}_{3}\left(\tau_{\mathrm{n}+1}\right) \tag{68}
\end{equation*}
$$

The total energy gain or heat fluxes may be obtained by integration of Eqs. (65), (67), and (68) over all frequencies. The spectral integration for the continuum radiation and the line radiation will be discussed separately in the following subsections.

The third exponential integral, $\mathrm{E}_{3}(\zeta)$, may be calculated using numerical correlations and series. However, considerable computation time may be saved by using the "exponential kernal approximation" for the exponential integrals. For instance,

$$
\begin{equation*}
\mathrm{E}_{3}(\zeta) \sim \frac{\mathrm{b}}{\mathrm{a}} \mathrm{e}^{-\mathrm{b} \zeta} \tag{69}
\end{equation*}
$$

Matching the values of the approximate function and its slope at zero argument with the exact values $\mathrm{E}_{3}(0)$ and $\mathrm{E}_{3}^{\prime}(0)=-\mathrm{E}_{2}(0)$, respectively, one obtains $\mathrm{b}=2$ and $\mathrm{a}=4$ so that

$$
\begin{equation*}
\mathrm{E}_{3}(\zeta) \sim 0.5 \mathrm{e}^{-2 \zeta} \tag{70}
\end{equation*}
$$

The individual terms of Eqs. (65), (67), and (68) are of the general form

$$
\begin{align*}
\mathrm{I}_{\mathrm{ijk}}(\nu) & =\mathrm{B}_{\mathrm{k}} \mathrm{E}_{3}\left(\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|\right) \\
& \simeq \frac{\mathrm{b}}{\mathrm{a}} \mathrm{~B}_{\mathrm{k}} \exp |-\mathrm{b}| \tau_{\mathrm{i}}-\tau_{j}| | \tag{71}
\end{align*}
$$

One then can study the spectral integration of Eq. (71) as a basic step for performing the spectral integration of the more complicated Eqs. (65), (67), and (68). It will be shown in the Section 3.4.2 that the exponential kernal approximation enables a convenient formulation of line radiation transport.

### 3.3 Treatment of Continuum Contribution

3.3.1 The continuum absorption coefficient. - The spectral absorption coefficiènts and the optical thicknesses may be separated into two parts - the continuum (subscript c) and the discrete (subscript d).

$$
\begin{align*}
& \mu(\nu)=\mu_{c}+\mu_{d}  \tag{72}\\
& \tau(\nu)=\tau_{c}+\tau_{d} \tag{73}
\end{align*}
$$

The discrete part varies with frequency much more rapidly than the continuum part. For the present study, the discrete part corresponds to the contribution due to the atomic lines. The continuum part corresponds to the contribution of molecular band systems, free-free, and photo-absorption processes.

[^2]3.3.2 The multi-band model. - For the continuum radiation, the multi-band model is used. The continuum spectral absorption coefficient is assumed constant within the individual spectral bands or intervals. The Planck intensity function is integrated using appropriate series expansions. For instance, consider spectral integration of Eq. (71) over a band with $\nu_{1} \leq \nu \leq \nu_{2}$.
\[

$$
\begin{align*}
\int_{\nu_{1}}^{\nu_{2}} \mathrm{I}_{\mathrm{ijk}}(\nu) \mathrm{d} \nu & =\mathrm{E}_{3}\left(\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|{ }_{\mathrm{c}}\right) \int_{\nu_{1}}^{\nu_{2}} \mathrm{~B}_{\mathrm{k}}(\nu) \mathrm{d} \nu \\
& \simeq \frac{\mathrm{~b}}{\mathrm{a}} \exp \left(-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|_{\mathrm{c}}\right) \int_{\nu_{1}}^{\nu_{2}} \mathrm{~B}_{\mathrm{k}}(\nu) \mathrm{d} \nu \tag{7}
\end{align*}
$$
\]

The value of the spectral absorption coefficient for a given band is calculated by appropriate averages. For bands which are expected to be optically thin for most calculations, the partial Planck-mean defined by Eq. (75) may be used.

$$
\begin{equation*}
\mu\left(\mathrm{T}, \nu_{1}, \nu_{2}\right)=\frac{\int_{\nu_{1}}^{\nu_{2}} \mu(\mathrm{~T}, \nu) \mathrm{B}(\mathrm{~T}, \nu) \mathrm{d} \nu}{\int_{\nu_{1}}^{\nu_{2}} \mathrm{~B}(\mathrm{~T}, \nu) \mathrm{d} \nu} \tag{75}
\end{equation*}
$$

### 3.4 Treatment of Line Contributions

3.4.1 The line absorption coefficient. - The absorption cross-section of a line (bound-bound) may be written for the transition $n \rightarrow n^{\prime}$ as follows:

$$
\begin{equation*}
\sigma_{\mathrm{nn}^{\prime}}(\nu)=\left(\frac{\pi \mathrm{e}^{2}}{\mathrm{mc}^{2}}\right) \mathrm{f}_{\mathrm{nn}^{\prime}} \mathrm{b}_{\mathrm{nn}^{\prime}}(\nu)\left(1-\mathrm{e}^{-\mathrm{h} \nu_{\mathrm{nn}}}{ }^{\prime} / \mathrm{kT}\right) \tag{76}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{e}, \mathrm{~m} & =\text { electron charge and mass, respectively } \\
\mathrm{c} & =\text { velocity of light }
\end{aligned}
$$

$$
\begin{align*}
& \mathrm{k}=\text { Boltzmann's constant } \\
& \mathrm{f}_{\mathrm{n} \mathrm{n}^{\prime}}=\text { absorption f-number for transition } \mathrm{n} \rightarrow \mathrm{n}^{\prime} \\
& \nu_{\mathrm{nn}}{ }^{\prime}=\text { line center frequency for transition } \mathrm{n} \rightarrow \mathrm{n}^{\prime} \\
& \mathrm{b}_{\mathrm{nn}^{\prime}}(\nu)=\text { line shape factor normalized according to } \\
& \int_{0}^{\infty} \mathrm{b}_{\mathrm{nn}^{\prime}}(\nu) \mathrm{d} \nu=1 \tag{77}
\end{align*}
$$

The factor ( $1-e^{-\mathrm{h} \nu_{n n^{\prime}} / \mathrm{kT}}$ ) accounts for induced emission.
To obtain the spectral absorption coefficient, the cross section is multiplied by the occupation number, $\mathrm{N}_{\mathrm{n}}$, of state n (the number density of a given species in state n). For a monatomic gas in local thermodynamic equilibrium, the occupation number is given by the Boltzmann formula

$$
\begin{equation*}
N_{n}=\frac{g_{n}}{Q} N e^{-\left(E_{n} / k T\right)} \tag{78}
\end{equation*}
$$

where
$g_{n}=$ statistical weight of the $\mathrm{n}^{\text {th }}$ state
$\mathbf{Q}=$ electronic partition function of monatomic species
$\mathrm{E}_{\mathrm{n}}=$ energy of $\mathrm{n}^{\text {th }}$ electronic state above ground state
$\mathrm{N}=$ total number of particles of the given species per unit volume
The partition function is given by

$$
\begin{equation*}
Q=\sum_{n} g_{n} e^{-\left(E_{n} / k T\right)} \tag{79}
\end{equation*}
$$

The energy levels, partition functions, and statistical weights for nitrogen and oxygen atoms and ions are given in ref. 6.

From Eqs. (76) and (78), the bound-bound absorption coefficient for transition $\mathrm{n} \rightarrow \mathrm{n}^{\prime}$ is then given by

$$
\begin{equation*}
\mu_{n n^{\prime}}=\left(\frac{\pi e^{2}}{m c^{2}}\right) f_{n n^{\prime}} \frac{g_{\mathrm{n}}}{Q} N e^{-\mathrm{E}_{\mathrm{n}} / \mathrm{kT}} \mathrm{~b}_{\mathrm{nn}^{\prime}}(\nu)\left[1-\exp \left(\frac{-\mathrm{h} \nu_{\mathrm{n} n^{\prime}}}{\mathrm{kT}}\right)\right] \tag{80}
\end{equation*}
$$

For electron-impact broadening, the lines follow a Lorentz shape with the halfwidth proportional to the electron particle density:

$$
\begin{align*}
& b_{n n^{\prime}}(\nu)=\frac{1}{\pi} \frac{\gamma_{n n^{\prime}}}{\left[\nu-\left(\nu_{0, n n^{\prime}}-d_{n n^{\prime}}\right)\right]^{2}+\gamma_{n n^{\prime}}^{2}}  \tag{81}\\
& \gamma_{n n^{i}}=\bar{\gamma}_{n n^{\prime}} N e \tag{82}
\end{align*}
$$

where

$$
\begin{aligned}
\nu_{\mathrm{o}, \mathrm{n} n^{\prime}} & =\text { frequency corresponding to unperturbed transition } \mathrm{n} \rightarrow \mathrm{n}^{\prime} \\
\mathrm{d}_{\mathrm{nn}}{ }^{\prime} & =\text { line-center shift } \\
\bar{\gamma}_{\mathrm{nn}} & =\text { half-width due to } 1 \text { electron per unit volume for } \mathrm{n} \rightarrow \mathrm{n}^{\prime} \\
\mathrm{Ne} & =\text { electron particle number density }
\end{aligned}
$$

The normalized half-width $\bar{\gamma}_{n n^{\prime}}$, as calculated by R. Johnston, for carbon, nitrogen, and oxygen atoms and ions are tabulated in ref. 7.

Excited electronic states having the same core configuration, principal and orbital quantum numbers but with different spin multiplicity and total orbital angular momentum for L-S coupling have nearly the same $\mathrm{E}_{\mathrm{n}}$. Therefore, it is convenient to combine these states into a single group so that only the occupation number of this group of states need be calculated. However, the f-numbers of the individual transitions must be multiplied by the ratio of the statistical weight of the individual lower state to that of the group of states. In other words, one may consider in Eq. (80) that $g_{n}^{\prime}$ is the statistical weight of a group of lower states of similar energies and $f_{n n^{\prime}}$ is an effective $f$-number which is equal to the f-number of the particular transition multiplied by a statistical-weight ratio.

The total bound-bound absorption coefficient is the sum of all $\mu_{\mathrm{nn}}$ ' due to different lower-state groups of different species.
3.4.2 The line transmittance and equivalent width. - Consider the spectral integration of Eq. (71) over a finite frequency interval $\Delta \nu$ across which the Planck intensity function and the continuum contribution may be approximated by constant values. Then,

$$
\begin{align*}
\mathrm{I}_{\mathrm{ijk}} & \equiv \int_{\Delta \nu} \mathrm{I}_{\mathrm{ijk}}(\nu) \mathrm{d} \nu \simeq \frac{\mathrm{~b}}{\mathrm{a}} \int_{\Delta \nu} \mathrm{B}_{\mathrm{k}} \mathrm{e}^{-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{j}\right|} \mathrm{c}_{\mathrm{e}}^{-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{j}\right|_{d}} \mathrm{~d} \nu  \tag{83}\\
& \simeq \frac{b}{a} \mathrm{~B}_{\mathrm{k}}\left(\nu_{\mathrm{a}}\right) \exp \left[-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|_{\mathrm{c}, \nu_{\mathrm{a}}}\right]_{\Delta \nu} \exp \left[-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|_{\mathrm{d}}\right] \mathrm{d} \nu
\end{align*}
$$

.where $\nu_{\mathrm{a}}$ is an average frequency (e.g., center frequency) within $\Delta \nu$.
Define the average transmittance as follows:

$$
\begin{equation*}
\operatorname{Trans}_{i j} \equiv \frac{1}{\Delta \nu} \int_{\Delta \nu} e^{-b\left|\tau_{i}-\tau_{j}\right|} d_{d \nu} \tag{84}
\end{equation*}
$$

Then,

$$
\begin{equation*}
I_{i j k} \simeq \frac{b}{a} B_{k}\left(\nu_{a}\right) e^{-\mathrm{b}\left|\tau_{i}-\tau_{j}\right|} \mathrm{c}, \nu_{a} \Delta \nu \quad \operatorname{Trans}_{i j} \tag{85}
\end{equation*}
$$

Alternately, one can write Eq. (83) as follows:

$$
\begin{align*}
I_{i j k} & \simeq \frac{b}{a} \int_{\Delta \nu} B_{k} e^{-b\left|\tau_{i}-\tau_{j}\right|_{c}} d \nu-\frac{b}{a} \int_{\Delta \nu} B_{k} e^{-b\left|\tau_{i}-\tau_{j}\right|} c\left(1-e^{-b\left|\tau_{i}-\tau_{j}\right|} d\right) d \nu \\
& \simeq I_{c_{i j k}}-\frac{b}{a} B_{k}\left(\nu_{a}\right) e^{-b\left|\tau_{i}-\tau_{j}\right|} c, \nu{ }_{a} \int_{\Delta \nu}\left(1-e^{-b\left|\tau_{i}-\tau_{j}\right|} d\right) d \nu \tag{86}
\end{align*}
$$

where

$$
\begin{equation*}
I_{c_{i j k}}=\frac{b}{a} \int_{\Delta \nu} B_{k} e^{-b\left|\tau_{i}-\tau_{j}\right|_{c}}{ }_{d \nu} \tag{87}
\end{equation*}
$$

is the continuum contribution.
Define the equivalent width over $\Delta \nu$ as follows:

$$
\begin{equation*}
\text { Width }_{\mathrm{ij}}=\int_{\Delta \nu}\left(1-\mathrm{e}^{-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|}{ }_{\mathrm{d}}\right) \mathrm{d} \nu \tag{88}
\end{equation*}
$$

Then, Eq. (86) may be rewritten as

$$
\begin{equation*}
-\mathrm{b}\left|\tau_{\mathrm{i}}-\tau_{\mathrm{j}}\right|_{\mathrm{c}, \nu_{\mathrm{a}} \text { Width }_{\mathrm{ij}}} \tag{89}
\end{equation*}
$$

The use of Eqs. (87) and (88) permits different spectral divisions for the calculation of the two terms on the right-hand side of Eq. (89).

From Eqs. (84) and (88)

$$
\begin{equation*}
\text { Width }_{\mathrm{ij}}=\Delta \nu\left(1-\text { Trans }_{\mathrm{ij}}\right) \tag{90}
\end{equation*}
$$

The study of line transport then reduces to the calculation of Trans $_{i j}$ or Width ${ }_{i j}$.
3.4.3 Absorption coefficient notation. - In order to avoid using too many indices to describe the variables, the following system of notation will be used:
$\mathrm{i}=$ dummy index for sublayers

- $i_{1}, i_{2}=$ specific sublayers
$k=$ dummy index for lines (except in product $k T$ )
$\mathrm{m}=$ dummy index for line frequency regions
$n_{k}=k^{\text {th }}$ line lower state index (The species index is eliminated and the value of $n_{k}$ specifies the species. The term $N_{n_{k}}$ will be used to denote the total particle number density for the species specified by $\mathrm{n}_{\mathrm{k}}$.)

Thus,

$$
\begin{equation*}
\mu_{i}(\nu)=\left(\frac{\pi e^{2}}{m c^{2}}\right) \sum_{k} f_{k} \frac{g_{n_{k}}}{Q_{i, n_{k}}} N_{i, n_{k}} e^{-E_{n_{k}} / k T_{i}}\left(1-e^{-h \nu_{k} / k T_{i}}\right) b_{i, k}(\nu) \tag{91}
\end{equation*}
$$

represents the spectral absorption coefficient of the $i^{\text {th }}$ sublayer at $\nu$. For many situations, the line-center frequency $\nu_{\mathrm{k}}$ in $\left[1-\exp \left(-\mathrm{h} \nu_{\mathrm{l}} / \mathrm{kT}\right)\right]$ may be approximated by $\nu_{\mathrm{m}}$, an average frequency for the $\mathrm{k}_{\mathrm{m}}$ th frequency region. It should be noted that lines outside the $\mathrm{m}^{\text {th }}$ frequency region may contribute to the value of $\mu_{\mathrm{i}}(\nu)$ at $\nu$ within the $\mathrm{m}^{\text {th }}$ frequency region.

In terms of the new notation, one has the following:

$$
\begin{equation*}
b_{i, k}(\nu)=\frac{1}{\pi} \frac{\gamma_{i, k}}{\left[\nu-\left(v_{k}-d_{i, k}\right)\right]^{2}+\gamma_{i, k}^{2}} \tag{92}
\end{equation*}
$$

$$
\begin{gather*}
\operatorname{Trans}_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}}=\frac{1}{\Delta \nu_{m}} \int_{\Delta \nu_{m}} \exp \left[-\mathrm{b} \sum_{\mathrm{i}}^{\mathrm{i}_{1}, \mathrm{i}_{2}} \mu_{\mathrm{i}}(\nu) \Delta \mathrm{z}_{\mathrm{i}}\right] \mathrm{d} \nu  \tag{93}\\
\text { Width }_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}}=\Delta \nu_{\mathrm{m}}\left(1-\operatorname{Trans}_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}}\right)=\int_{\Delta \nu_{m}}\left\{1-\exp \left[-\mathrm{b} \sum_{\mathrm{i}}^{\mathrm{i}_{1}, \mathrm{i}_{2}} \mu_{\mathrm{i}}(\nu) \Delta \mathrm{z}_{\mathrm{i}}\right]\right\} \mathrm{d} \nu \tag{94}
\end{gather*}
$$

The sum in Eq. (93) represents the following operation:

$$
\sum_{i}^{1_{1}, 1_{2}} \xi_{i}=\left\{\begin{array}{cc}
0 & , i_{2}=i_{1}  \tag{95}\\
\sum_{i=i_{1}}^{i_{2}-1} \xi_{i}, & i_{2}>i_{1} \\
\sum_{i=i_{2}-1} \xi_{i} & , i_{2}<i_{1}
\end{array}\right.
$$

In order to simplify the calculations, the effects of line shift will be neglected. For the conditions of interest, it is reasonable to consider only lines due to neutral nitrogen atoms with an effective density equal to the sum of the actual NI and OI values.* With this approximation, $N_{i}, n_{k}$ may be replaced by $N_{i}$ and $Q_{i}, n_{k}$ by $Q_{i}$. The numerical computation is thus considerably simplified. One may now write

[^3]$$
\text { Width }_{i_{1}, i_{2}, m}=\int_{\Delta \nu_{m}}\left(1-\exp \left\{-b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right]\right.\right.
$$
\[

$$
\begin{equation*}
\left.\left.\Delta z_{i} \sum_{k} \operatorname{gf}_{k} \exp \left(-\frac{\mathrm{E}_{\mathrm{n}_{\mathrm{k}}}}{\mathrm{kT}}\right) \frac{1}{\pi} \frac{\gamma_{\mathrm{i}, \mathrm{k}}}{\left(\nu-\nu_{\mathrm{k}}\right)^{2}+\gamma_{\mathrm{i}, \mathrm{k}}^{2}}\right\}\right) \mathrm{d} \nu \tag{96}
\end{equation*}
$$

\]

where

$$
\begin{aligned}
b^{\prime} & =b\left(\pi e^{2} / m c^{2}\right) \\
{g f_{k}} & =f_{k} g_{n_{k}}, \quad a \text { constant for each line }
\end{aligned}
$$

The expression for the average transmittance now becomes:

$$
\begin{align*}
& \operatorname{Trans}_{i_{1}, i_{2}, m}=\frac{1}{\Delta \nu_{m}} \int_{\Delta \nu_{m}} \prod_{i}^{i_{1}, i_{2}} \exp \left\{-b^{\prime} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{\mathrm{h} \mu_{m}}{k T_{i}}\right)\right] \Delta z_{i}\right. \\
&\left.\sum_{k} \operatorname{gf}_{\mathrm{k}} \exp \left(-\frac{\mathrm{E}_{\mathrm{k}}}{\mathrm{kT}}\right) \frac{1}{\pi} \frac{\gamma_{\mathrm{i}, \mathrm{k}}}{\left(\nu-\nu_{k}\right)^{2}+\gamma_{i, k}^{2}}\right\} \mathrm{d} \nu \tag{97}
\end{align*}
$$

where

$$
\prod_{i} \xi_{i}, i_{2}= \begin{cases}1 & i_{2}=i_{1}  \tag{98}\\ \prod_{2}-1 & \\ \prod_{i} \xi_{i}, & i_{2}>i_{1} \\ i_{1}-1 & \\ \prod_{i} \xi_{i}, & i_{2}<i_{1} \\ i_{2}\end{cases}
$$

In the conditions of interest, it is reasonable to assume that $\bar{\gamma}_{n n^{\prime}}=\bar{\gamma}_{\mathrm{i}, \mathrm{k}}$ is a slow-varying function of temperature. Then, the following approximation may be used:

$$
\begin{equation*}
\gamma_{i, k} \simeq \bar{\gamma}_{k} \mathrm{Ne}_{\mathrm{i}} \tag{99}
\end{equation*}
$$

3.4.4 Line calculation limiting cases. - The study of line transport now reduces to the calculation of Width $i_{1}, i_{2}, m$ or Transin, $i_{2}, m$ according to Eqs. (96) and (97). In general, the integral in Eqs. (96) or (97) can not be expressed in terms of simple combinations of analytic functions. Although numerical integrations can always be used to evaluate these integrals, the computation time required may become excessive. One should then look for appropriate approximations applicable to different ${ }^{\circ}$ asymptotic situations. For situations where numerical integrations must be used, the numerical schemes should be selected with a consideration for the minimization of computation time.

The approximations for different asymptotic situations are discussed below:
(1) Thin-line approximation. -

For $\xi \ll 1, \mathrm{e}^{-\xi} \simeq 1-\xi$. Now,

$$
\begin{aligned}
& b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k}{g f_{k}} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) \frac{1}{\pi} \frac{\gamma_{i, k}}{\left(\nu-\nu_{k}\right)^{2}+\gamma_{i, k}^{2}} \\
& \leq b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{k \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k}{g f_{k}} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) \frac{1}{\pi} \frac{\gamma_{i, k}}{0+\gamma_{i, k}^{2}} \equiv \tau_{i_{1}, i_{2}, m}
\end{aligned}
$$

where the identity defines $\tau_{i_{1}}, \mathrm{i}_{2}, \mathrm{~m}$, an effective optical thickness evaluated at the line centers. Then, for $\tau_{\mathrm{i}_{1}}, \mathrm{i}_{2}, \mathrm{~m} \ll 1$ :
Width $_{i_{1}}, \mathrm{i}_{2}, \mathrm{~m}$

$$
\begin{align*}
& \simeq \int_{\Delta \nu_{m}} b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k} \operatorname{gf} f_{k} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) \frac{1}{\pi} \frac{\gamma_{i, k}}{\left(\nu-\nu_{k}\right)^{2}+\gamma_{i, k}^{2}} d \nu \\
& =b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k} g_{k} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) \int_{\Delta \nu} \frac{1}{\pi} \frac{\gamma_{i, k}}{\left(\nu-\nu_{k}\right)^{2}+\gamma_{i, k}^{2}} d \nu \\
& =b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k} g f_{k} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) G_{\left(\nu_{u_{m}}, \nu_{\ell_{m}}, \nu_{k}, \gamma_{i, k}\right)} \tag{101}
\end{align*}
$$

where

$$
\begin{equation*}
\mathrm{G}\left(\nu_{\mathrm{u}_{\mathrm{m}}}, \nu_{\ell_{\mathrm{m}}}, \nu_{\mathrm{k}}, \gamma_{\mathrm{i}, \mathrm{k}}\right)=\frac{1}{\pi}\left[\tan ^{-1}\left(\frac{\nu_{\mathrm{u}_{\mathrm{m}}}-\nu_{\mathrm{k}}}{\gamma_{\mathrm{i}, \mathrm{k}}}\right)-\tan ^{-1}\left(\frac{\nu_{\ell_{\mathrm{m}}}-\nu_{\mathrm{k}}}{\gamma_{\mathrm{i}, \mathrm{k}}}\right)\right] \tag{102}
\end{equation*}
$$

with $\nu_{u_{m}}$ and $\nu_{l_{m}}$ as the upper and lower limits, respectively, of the $\mathrm{m}^{\text {th }}$ frequency
interval.

When the contributing lines are well within the frequency region (not near the two boundaries), Eq. (102) may be approximated by:

$$
\begin{equation*}
\mathrm{G}\left(\nu_{\mathrm{u}_{\mathrm{m}}}, \nu_{\ell_{\mathrm{m}}}, \nu_{\mathrm{k}}, \gamma_{\mathrm{i}, \mathrm{k}}\right)=1 \tag{103}
\end{equation*}
$$

With the above approximation, the equivalent width then becomes the sum of the socalled strengths* of the contributing lines.

Equation (101) is valid for both isolated and overlapped lines, provided $\tau_{\mathrm{i}_{1}}, \mathrm{i}_{2}, \mathrm{~m} \ll 1$.
(2) Isolated-line approximation

When $\Delta \nu_{\mathrm{m}}$ is large compared with $\gamma_{\mathrm{i}, \mathrm{k}}$ and the contributing lines within the frequency region may be considered isolated, the integral in Eq. (96) may be approximated by the sum of integrals for the individual line. Thus,

$$
\begin{align*}
& \text { Width }_{i_{1}, i_{2}}, m \simeq \sum_{k} \int_{\delta \nu_{k}}\left(1-\exp \left\{-b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta \mathbf{z}_{i}\right.\right. \\
& \left.\left.\operatorname{gf}_{\mathrm{k}} \exp \left(-\frac{\mathrm{E}_{\mathrm{n}_{\mathrm{k}}}}{\mathrm{kT}_{\mathrm{i}}}\right) \frac{1}{\pi} \frac{\gamma_{\mathrm{i}, \mathrm{k}}}{\left(\nu-\nu_{\mathrm{k}}\right)^{2}+\gamma_{\mathrm{i}, \mathrm{k}}^{2}}\right\}\right) \mathrm{d} \nu \tag{104}
\end{align*}
$$

where $\delta \nu_{\mathrm{k}}$ is the integration interval for the $\mathrm{k}^{\text {th }}$ line.
The integral in Eq. (104) for a single value of the index $k$ represents the equivalent width of a single line for a nonisothermal path. The use of an effective half-width (ref. 8) may enable representation of the equivalent width in terms of the LadenburgReiche function (ref. 9).

Consider the following integral:

$$
\begin{align*}
I & =\int_{-\infty}^{\infty}\left[1-\exp \left(-\frac{1}{\pi} \sum_{i} \frac{S_{i} \gamma_{i}}{\xi^{2}+\gamma_{i}^{2}}\right)\right] d \xi \\
& \simeq \int_{-\infty}^{\infty}\left[1-\exp \left(-\frac{1}{\pi} \frac{1}{\xi^{2}+\gamma_{e}} \sum_{i} S_{i} \gamma_{i}\right)\right] d \xi \tag{105}
\end{align*}
$$

[^4]where $\gamma_{\mathrm{e}}$ is an effective half-width. (The determination of $\gamma_{\mathrm{e}}$ will be discussed
later.)
Equation (105) may be integrated to yield:
\[

$$
\begin{equation*}
I \simeq 2 \pi \gamma_{\mathrm{ef}}\left(\frac{\sum_{i} \mathrm{~S}_{\mathrm{i}} \gamma_{\mathrm{i}}}{2 \pi \gamma_{\mathrm{e}}^{2}}\right) \tag{106}
\end{equation*}
$$

\]

where

$$
\begin{equation*}
\mathrm{f}(\xi)=\xi \mathrm{e}^{-\xi}\left[\mathrm{I}_{\mathrm{o}}(\xi)+\mathrm{I}_{1}(\xi)\right] \tag{107}
\end{equation*}
$$

is the Ladenburg-Reiche function with $I_{o}(\xi)$ and $I_{1}(\xi)$ being the modified Bessel
functions.
Two asymptotic expressions of $f(\xi)$ exist:

$$
\mathrm{f}(\xi) \simeq\left\{\begin{array}{cc}
\xi & , \quad \xi \ll 1  \tag{108}\\
\sqrt{\frac{2 \xi}{\pi}} & , \quad \xi \gg 1
\end{array}\right.
$$

at $\xi=\frac{2}{\pi}$, the two asymptotic expressions become equal.
Since the lines are considered isolated, the interval of integration $\delta \mathrm{k}$ may be extended to $(-\infty, \infty)$. Therefore, one may rewrite Eq. (104) as follows:

$$
\begin{equation*}
\text { Width }_{i_{1}, i_{2}, m} \simeq \sum_{k} 2 \pi \gamma_{e_{k}} f\left(\frac{\sum_{i}^{i_{1}} \mathrm{i}_{2} S_{i, k, m} \gamma i, k}{2 \pi \gamma e_{k}^{2}}\right) \tag{109}
\end{equation*}
$$

where

$$
\begin{equation*}
s_{i, k, m}=b^{\prime} \frac{N_{i}}{Q_{i}}\left(1-e^{-\frac{h \nu_{m}}{k T_{i}}}\right) \Delta z_{i} g f_{k} e^{-\frac{E_{n_{k}}}{k T_{i}}} \tag{110}
\end{equation*}
$$

Now, consider the determination of $\gamma \mathrm{e}$. Comparing Eqs. (101) and (109) and using the first of Eq. (108) for isolated thin lines, one obtains:

$$
\begin{equation*}
\gamma_{e_{k}}=\frac{\sum_{i}^{i_{1}, i_{2}} s_{i, k, m} \gamma_{i, k}}{\sum_{i}^{i_{1}, i_{2}} s_{i, k, m}} \tag{111}
\end{equation*}
$$

which is mean weighted according to the line strengths of the individual sub-layers.
Using the second of Eq. (108) for isolated strong lines, one obtains the following equation:

$$
\begin{align*}
\text { Width }_{i_{1}, i_{2}, m} & \simeq \sum_{k} 2 \pi \gamma_{k}\left[\frac{2}{\pi} \frac{\sum_{i}^{i_{1}, i_{2}} s_{i, k, m} \gamma_{i, k}}{2 \pi \gamma_{e_{k}}}\right]^{1 / 2} \\
& =\sum_{k} 2\left(\sum_{i}^{i_{1}, i_{2}} S_{i, k, m} \gamma_{i, k}\right)^{1 / 2}, \quad\binom{\text { isolated strong line }}{\text { square-root approximation }} \tag{112}
\end{align*}
$$

The effective half-width does not appear in Eq. (112). Therefore, it is reasonable to assume that the effective half-width defined by Eq. (111) may be used, as an approximation, in Eq. (109) for situations between the thin and strong limits for isolated. lines.*

For numerical calculation of the Ladenburg-Reiche function, the following approximations are useful:

$$
f(\xi) \simeq \begin{cases}\xi & \xi \leq(0.685)^{4}  \tag{113}\\ 0.685 \xi^{3 / 4} & , \\ \sqrt{\frac{2}{2}} \xi^{1 / 2} & (0.685)^{4}<\xi \leq \frac{4}{\pi^{2}} \frac{1}{(0.685)^{4}} \\ \sqrt{\pi} & \xi>\frac{4}{\pi^{2}} \frac{1}{(0.685)^{4}}\end{cases}
$$

[^5](3) Effective isothermal region. Extending the idea of the effective half-width to non-isolated lines, one may rewrite Eq. (96) as follows:

Width $_{i_{1}, i_{2}, m} \simeq \iint_{\Delta \nu_{m}} 1-\exp \left[-b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left(1-e^{-\frac{h \nu_{m}}{k T_{i}}}\right) \Delta z_{i}\right.$

$$
\begin{equation*}
\left.\left.\sum_{k} \operatorname{gf}_{\mathrm{k}} \mathrm{e}^{-\frac{\mathrm{E}_{\mathrm{n}_{\mathrm{k}}}}{\mathrm{kT}_{\mathrm{i}}}} \frac{1}{\pi} \frac{\nu_{\mathrm{i}, \mathrm{k}}}{\left(\nu-\nu_{\mathrm{k}}\right)^{2}+\gamma_{\mathrm{e}}^{\prime 2}}\right]\right\} \mathrm{d} \nu \tag{114}
\end{equation*}
$$

where $\gamma_{e_{k}}^{\prime}$ is an effective half-width of the $k^{\text {th }}$ line.
Under certain situations, a single lower state or a small number of lower states of nearly the same electronic energy contribute to the $m$ th frequency region. One may then introduce the following approximations:

$$
\begin{gather*}
E_{n_{k}} \simeq E_{m}  \tag{115}\\
\gamma_{i, k} \simeq \bar{\gamma}_{k} \mathrm{Ne}_{i} \tag{116}
\end{gather*}
$$

Equation (114) may then be written as follows:
Width $_{i_{1}, i_{2}, m} \simeq \int_{\Delta \nu_{m}}\left\{1-\exp \left[-b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left(1-e^{-\frac{h \nu}{k T_{i}}}\right) \Delta z_{i} e^{-\frac{E_{m}}{k T_{i}}} N_{i}\right.\right.$

$$
\begin{equation*}
\left.\left.\sum_{\mathrm{k}} \mathrm{gf}_{\mathrm{k}} \frac{1}{\pi} \frac{\bar{\gamma}_{\mathrm{k}}}{\left(\nu-\nu_{\mathrm{k}}\right)^{2}+\gamma_{\mathrm{k}}^{\prime}{ }_{\mathrm{k}}^{2}}\right]\right\} \mathrm{d} \nu \tag{117}
\end{equation*}
$$

The two summations in Eq. (117) may be performed separately, thus saving a considerable amount of numerical calculations. Let

$$
\begin{equation*}
A_{i_{1}, i_{2}}, m=\sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right) N e_{i} \tag{118}
\end{equation*}
$$

Then, Eq. (117) becomes

$$
\text { Width }_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}} \simeq \int_{\Delta \nu_{\mathrm{m}}}\left\{1-\exp \left[-\mathrm{A}_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}} \sum_{\mathrm{k}} \mathrm{~b}^{\prime} \mathrm{gf}_{\mathrm{k}} \frac{1}{\pi} \frac{\bar{\gamma}_{\mathrm{k}}}{\left(\nu-\nu_{\mathrm{k}}\right)^{2}+\gamma_{\mathrm{e}}^{\prime} 2}\right]\right\} \mathrm{d} \nu
$$

The fact that $\mathrm{A}_{\mathrm{i} 1, \mathrm{i} 2}, \mathrm{~m}$ is a sum, independent of the dummy sublayer index i , implies that the equivalent width given by Eq. (119) may be considered to be that due to an effective isothermal region.

Equation (119) may be further simplified, if the thin-line or the isolated strong-line approximation is used.

From Eq. (100),

$$
\begin{align*}
\tau_{i_{1}, i_{2}}, m & =b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \sum_{k} g f_{k} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) \frac{1}{\pi} \frac{1}{\gamma_{i, k}} . \\
& \simeq b^{\prime} \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right) \sum_{k} g f_{k} \frac{1}{\pi \gamma_{i, k}} \tag{120}
\end{align*}
$$

Then, for $\tau_{\mathrm{i}_{1}}, \mathrm{i}_{2}, \mathrm{~m} \ll 1$, Eq. (101) yields with $\mathrm{G}=1$ :

$$
\begin{equation*}
\text { Width }_{i_{1}, i_{2}, m} \simeq \sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right) \sum_{k} b^{\prime} g_{k} \tag{121}
\end{equation*}
$$

Now, consider Eq. (119) for isolated thin lines, one has in analogy to Eq. (109).

$$
\begin{align*}
\text { Width }_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}} & \simeq \sum_{\mathrm{k}} 2 \pi \gamma_{\mathrm{e}_{\mathrm{k}}}^{\prime}\left[\frac{\mathrm{A}_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}}^{\mathrm{b}^{\prime} \mathrm{gf}_{\mathrm{k}} \bar{\gamma}_{\mathrm{k}}}}{2 \pi \gamma_{\mathrm{e}}^{\prime}{ }_{k}^{2}}\right] \\
& =\mathrm{A}_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}} \sum_{\mathrm{k}} \mathrm{~b}^{\prime} \mathrm{gf}_{\mathrm{k}} \frac{\bar{\gamma}_{\mathrm{k}}}{\bar{\gamma}_{\mathrm{e}_{\mathrm{k}}^{\prime}}} \tag{122}
\end{align*}
$$

Subtracting Eq. (121) from Eq. (122), one obtains the following

$$
\begin{equation*}
\sum_{k} b^{\prime} g f_{k}\left\{A_{i_{1}, i_{2}}, \quad \frac{\bar{\gamma}_{k}}{\gamma_{e_{k}}^{\prime}}-\sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right)\right\}=0 \tag{123}
\end{equation*}
$$

For arbitrary variation of $\mathrm{gf}_{\mathrm{k}}$ and of the number of k -terms, an expression of the effective half-width satisfying Eq. (123) is given below:

$$
\begin{equation*}
\gamma_{e_{k}}^{\prime}=\frac{\left\{\sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right) N e_{i}\right\}}{\left\{\sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right)\right\}} \bar{\gamma}_{k} \tag{124}
\end{equation*}
$$

Equation (124) may also be derived directly from Eq. (111), using Eqs. (115) and (116).

The equivalent width for isolated strong lines for the effective isothermal region, according to the square-root approximation, may be obtained from Eq. (112).

$$
\begin{align*}
\text { Width }_{i_{1}, i_{2}, m} & \simeq \sum_{k} 2\left(\left\{\sum_{i}^{i_{1}, i_{2}} \frac{N_{i}}{Q_{i}}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \Delta z_{i} \exp \left(-\frac{E_{m}}{k T_{i}}\right) N e_{i}\right\}^{\prime}\right. \\
& =2 A_{i_{1}, i_{2}, m}^{1 / 2} \sum_{k}\left(b^{\prime}{g f_{k}}^{\prime} \gamma_{k}\right)^{1 / 2}
\end{align*}
$$

The effective half-width does not appear in Eq. (125). From the above consideration, it is assumed that the effective half-width defined by Eq. (111) (or Eq. (124) as a special case) may be used as an approximation along the whole "curve of growth."
(4) Criteria for line isolation. From the above discussions, it is apparent that the calculations are much simplified if the isolated-line approximation may be made. With the Lorentz line shape, the wings of the lines extend to $+\infty$ and $-\infty$. Therefore, the lines always overlap to a certain degree. One needs then to qualify the description "isolated•lines" and to establish suitable criteria for line isolation for numerical calculations.

Let

$$
\begin{aligned}
\mathrm{Z}= & \text { quantity calculated using the isolated-line approximations } \\
\mathrm{Z}^{*}= & \text { quantity calculated by numerical integration without the isolated-line } \\
& \text { approximations }
\end{aligned}
$$

Then, if

$$
\begin{equation*}
\left|\frac{Z-Z^{*}}{Z}\right| \leq \epsilon \ll 1, \tag{126}
\end{equation*}
$$

the isolated-line approximations may be used.
Now,

$$
\begin{equation*}
\operatorname{Trans}_{i_{1}}, i_{2}, m-\operatorname{Trans}_{i_{1}}^{*}, i_{2}, m=\frac{1}{\Delta \nu_{m}}\left(\text { Width }_{i_{1}}, i_{2}, m-\text { Width }_{i_{1}}^{*}, i_{2}, m\right) \tag{127}
\end{equation*}
$$

The following criterion is more stringent than Eq. (126):

$$
\begin{equation*}
\left|\frac{\operatorname{Trans}_{i_{1}, i_{2}, m}-\text { Trans }_{i_{1}}, \mathrm{i}_{2}, m}{\text { Smaller of }\left(\text { Trans }_{i_{1}, i_{2}, m}, \frac{\text { Width }_{i_{1}, i_{2}, m}}{\Delta \nu_{m}}\right)}\right| \leq \epsilon \ll 1 \tag{128}
\end{equation*}
$$

Unfortunately, the usefulness of the isolated-line approximations is lost if Trans ${ }_{i_{1}}, \mathrm{i}_{2}, \mathrm{~m}$ or (Width* $\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m} / \Delta \nu_{m}$ ) must be calculated for application in Eq. (128). Therefore, other more convenient approaches should be used.

Consider the calculation of the equivalent width of two overlapping lines, as shown in fig. 3a. The values of the integrand in Eq. (96), calculated by summing the two isolated-line contributions exceed that based on two overlapping lines. For the profiles in fig. 3a

$$
\begin{aligned}
\mathrm{R} & \equiv \frac{1}{\Delta \nu_{\mathrm{m}}} \sum_{\mathrm{k}} \text { Width }_{\mathrm{k}, \mathrm{~m}} \sim 0.3^{\dagger} \\
\mathrm{R}^{*} & \equiv \frac{1}{\Delta \nu_{\mathrm{m}}} \text { Width }_{\mathrm{m}}^{*} \\
\mathrm{Z}_{\mathrm{R}} & \equiv \frac{\mathrm{R}-\mathrm{R}^{*}}{\mathrm{R}^{*}} \sim 0.1
\end{aligned}
$$

In fig. 3 b , the line-center separation is three times that in fig. 3 a and the value of $Z_{R}$ is much smaller than 0.1 for the same value of $R$.

Qualitatively, the value of $R$ may indicate the degree of overlapping. As the value of $R$ increases, the value of $R^{*}$ approaches unity, as illustrated in fig. 4. At small values of $R$, the values of $Z_{R}$ are small so that the lines may be considered isolated. For a given group of lines (hence a particular pattern of line spacing) within $\Delta \nu_{m}$, it may be possible to obtain an approximate relation $R^{*}=F(R)$ by correlating the numerical results of some typical distributions of temperature and species concentration (with or without using the approximation of effective isothermal region). This approximate relation may then be used in radiation-coupled flow field calculations. In view of the uncertainties in f-number and half-width values, the above approach appears reasonable.

[^6]
## 4. NUMERICAL.METHODS

The division of the continuum bands and line groups used in the computer code STAGRADS (STAGnation point, RADiation-coupled, with external Source*) is briefly described in Section 4.1. Details of the spectral divisions are given in Appendixes $B$ and $C$. The schemes of flow field iteration and of line calculations are discussed in Sections 4.2 and 4.3, respectively.

### 4.1 Division of Continuum Bands and Line Groups

For the continuum air radiation, eight different models are provided before the lines are incorporated. These models consist of from two to nine spectral bands (see Table 1). The first option (LNE $\varnothing \mathrm{P}=1$ ) of radiation transport calculations accounts for only the air continuum contributions using one of the eight models. For calculations with lines, two options are provided. In one option (LINE $\varnothing \mathrm{P}=2$ ), the air continuum contribution is first calculated using one of the eight models of band divisions. The continuum contribution is then corrected for the presence of the lines [ see Eqs. (86) to (89)]. In another option (LINE $\varnothing \mathrm{P}=3$ ), the spectral division of the continuum bands is made coincident with that of the line groups and the total contribution is calculated according to Eq. (85).

A total of 21 line groups are used (see Appendix C). Three of the groups contain only a single line per group. Three of the groups contain no lines at all so that the last option (LINE $\varnothing \mathrm{P}=3$ ) of transport calculations may be used.

For calculations with ablation layer, only the continuum contributions of the ablation species are included in order to reduce the requirement of computer storage and time. To account for the variation of the ablation species absorption coefficients over a wide spectrum, the division of the continuum bands for these species is made coincident with that of the line groups. For calculations with ablation layer, the last option (LINE $\varnothing \mathrm{P}=3$ ) is used so that the contributions of the air atomic lines are included.

### 4.2 Scheme of Flow Field Iteration

The integration of the conservation equations is for the air layer from the shockwave toward the interface and for the ablation layer from the body surface toward the interface. For the integration of the radiative terms and the energy Eqs. (28a) and (46), the air layer and the ablation layer are divided into a number of sublayers. The air layer is divided in equal increments of the actual distance, but the ablation layer is divided in equal increments of the transformed distance [see Eq. (44)] . At

[^7]present, a maximum of ten air sublayers and ten ablation sublayers is allowed in the computer code in order to reduce the requirement of computer storage and time.

The integration of the momentum Eqs. (27) and (47) requires much less computer time and storage. Consequently, their integration may be performed with finer increments than that for the energy equations. The density ratios in Eqs. (27) and (47) are calculated by interpolations from values obtained using the energy equations.

The flow field calculation begins with the integration of the air layer momentum equation, using an assumed distribution of $\rho_{S} / \rho \simeq \mathrm{h} / \mathrm{h}_{\mathrm{S}}$. The integration is stopped when Eq. (29b) is satisfied at the interface, thus determining the air layer thickness. The air layer is then divided into a number of sublayers and the energy Eq. (28a) integrated. Across a sublayer, an average value of the nondimensional normal velocity $F$ is used, and the net radiative energy gain is given by the spectrally integrated Q [see Eq. (65)] . Iteration is made to obtain a consistent set of velocity and enthalpy distributions.

For calculations with ablation layer, an initial enthalpy distribution, linear in the iransformed normal distance between the wall enthalpy and an enthalpy at the interface estimated from the air layer results, is assumed. The density distribution is calculated using the equations of state of the ablation vapor. The initial blowing rate is estimated using the air layer results, a surface heat balance, and an estimate of the vapor enthalpy rise. The momentum and energy equations are integrated in the same manner as for the air layer. The air layer temperature distribution is maintained unchanged during ablation layer calculations. Iteration on the wall heat flux is also made to obtain a consistent set of wall heat flux, blowing rate, and velocity and enthalpy distributions.

The effect of the presence of the ablation layer on the air layer enthalpy distribution is then checked. If necessary, the air layer calculation is repeated while. the ablation temperature distribution is maintained unchanged. Further repetitions of ablation layer and air layer calculation are then made, if required, to obtain a consistent set of wall heat flux, blowing rate, and velocity and enthalpy distributions for both layers.

At present, the criteria for convergence used in STAGRADS are: enthalpy distribution, 1 percent and heat flux to the wall, 2 percent. The convergence of the blowing rate is governed by that of the wall heat flux. The convergence of the velocity distribution is governed by that of the enthalpy distribution.

### 4.3 Scheme of Line Calculation

When numerical integrations are required to evaluate Width $_{\mathrm{i} 1}, \mathrm{i} 2, \mathrm{~m}$ or Transin, $\mathrm{i}_{2}, \mathrm{~m}$, the selection of the computation schemes should be carefully made to minimize the computer time. For example, the calculation of exponential functions requires much more time than for performing multiplications. Therefore, it may be more economical to calculate

$$
\exp \left(-\sum_{i}^{i_{1}, i_{2}} \quad l l y\right)
$$

according to

$$
\prod_{\mathbf{i}}^{\mathbf{i}_{1}, \mathbf{i}_{2}} \exp \left(\xi_{\mathbf{i}}\right)
$$

if there are many variations of $i_{1}$ and $i_{2}$. Computation time may also be reduced by using common factors and by avoiding calculating the same variables many times (with increasing computer storage requirement, however).

Consider the numerical integration of the integral in Eqs. (96) or (97). The integrand is to be evaluated for many values of $\nu$, say $\nu_{j}$, within $\Delta \nu_{m}$. Let $j$ be the dummy index for $\nu$. In terms of the system of index notation selected; the important variables are summarized as follows: (Note $\Delta y_{i}=\Delta z_{i}$ )

$$
\begin{align*}
& B g f_{k}=b^{\prime} g_{k}  \tag{129}\\
& \operatorname{Dynq}_{i}=\frac{N_{i}}{Q_{i}} \Delta y_{i}  \tag{130}\\
& \text { Dynqe }_{i}=\frac{N_{i}}{Q_{i}} \Delta y_{i} \mathrm{Ne}_{\mathbf{i}}=\operatorname{Dynq}_{i} \mathrm{Ne}_{\mathbf{i}}  \tag{131}\\
& \operatorname{Dynqi}_{i, m}=\frac{\mathbf{N}_{\mathbf{i}}}{Q_{i}} \Delta y_{i}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right]=\operatorname{Dynq}_{i}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right]  \tag{132}\\
& \text { Dynqie }_{i, m}=\frac{N_{i}}{Q_{i}} \Delta y_{i}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \operatorname{Ne}_{i}=\operatorname{Dynqi}_{i, m} N e_{i}  \tag{133}\\
& S_{i, k, m}=\frac{N_{i}}{Q_{i}} \Delta y_{i}\left[1-\exp \left(-\frac{h \nu_{m}}{k T_{i}}\right)\right] \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) b^{\prime} g_{k}=\text { Dynqi }_{i, m} \exp \left(-\frac{E_{n_{k}}}{k T_{i}}\right) B g f_{k}  \tag{134}\\
& S_{i, m}=\sum_{k} S_{i, k, m}=\text { Dynqi }_{i, m} \sum_{k} \exp \left(-\frac{E_{n_{k}}}{k T^{i}}\right) B \operatorname{Bgf}_{k} \tag{135}
\end{align*}
$$

$$
\begin{align*}
& S_{i_{1}, i_{2}, k, m}=\sum_{i}^{i_{1}, i_{2}} S_{i, k, m}=\operatorname{Bgf}_{k} \sum_{i}^{i_{1}, i_{2}} \text { Dynqi }_{i, m} \exp \left(-\frac{E_{k}}{k T_{i}}\right) \\
& S_{i_{1}, i_{2}, m}=\sum_{i}^{i_{1}, i_{2}} S_{i, m}=\sum_{k} S_{i_{1}, i_{2}, k, m}  \tag{137}\\
& \gamma_{i, k}=N e_{i} \bar{\gamma}_{k}  \tag{138}\\
& \tau_{i, k, m, j}=\frac{S_{i, k, m}}{\pi} \frac{\gamma_{i, k}}{\left(\nu_{j}-\nu_{k}\right)^{2}+\gamma_{i, k}^{2}}  \tag{139}\\
& \tau_{i, m, j}=\sum_{k} \tau_{i, k, m, j}  \tag{140}\\
& \tau_{i, k, m}=\frac{S_{i, k, m}}{\pi} \frac{1}{\gamma_{i, k}}  \tag{141}\\
& \tau_{i, m}=\sum_{k} \tau_{i, k, m}  \tag{142}\\
& \tau_{i_{1}, i_{2}, k, m}=\sum_{i}^{i_{1}, i_{2}} \tau_{i, k, m}  \tag{143}\\
& \tau_{i_{1}, i_{2}, m}=\sum_{k} \tau_{i_{1}, i_{2}, k, m}=\sum_{i}^{i_{1}, i_{2}} \tau_{i, m}  \tag{144}\\
& \mathrm{~S} \gamma_{\mathrm{i}, \mathrm{k}, \mathrm{~m}}=\mathrm{S}_{\mathrm{i}, \mathrm{k}, \mathrm{~m}} \gamma_{\mathrm{i}, \mathrm{k}}  \tag{145}\\
& \mathbf{S} \gamma_{i, m}=\sum_{k} \mathbf{S} \gamma_{i, k, m} \tag{146}
\end{align*}
$$

$$
\begin{align*}
& s \gamma_{i_{1}, i_{2}, k, m}=\sum_{i}^{i_{1}, i_{2}} s \gamma_{i, k, m}  \tag{147}\\
& \mathrm{~S} \gamma_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{~m}}=\sum_{\mathrm{k}} \mathrm{~s} \gamma_{\mathrm{i}_{1}, \mathrm{i}_{2}, \mathrm{k}, \mathrm{~m}}  \tag{148}\\
& \gamma_{\mathrm{e}_{1}, i_{2}, k, m}=\frac{\mathrm{S} \gamma_{\mathrm{i}_{1}, i_{2}, k, m}}{\mathrm{~S}_{\mathrm{i}_{1}, \mathrm{i}_{2}, k, m}}  \tag{149}\\
& \xi_{i_{1}, i_{2}, k, m}=\frac{s \gamma_{i_{1}, i_{2}, k, m}}{2 \pi \gamma e_{i_{1}, i_{2}, k, m}^{2}}  \tag{150}\\
& E m \tau_{i, k, m, j}=\exp \left(-\tau_{i, k, m, j}\right)  \tag{151}\\
& \operatorname{Em} \tau_{i, m, j}=\exp \left(-\tau_{i, m, j}\right)  \tag{152}\\
& \mathrm{i}_{1}, \mathrm{i}_{2} \\
& E m \tau_{i_{1}, i_{2}, m, j}=\prod_{i} E m \tau_{i, m, j}  \tag{153}\\
& \text { Width }_{i_{1}, i_{2}}, k, m=2 \pi \gamma_{e_{1}, i_{2}, k, m}{ }^{f}\left(\xi_{i_{1}, i_{2}, k, m}\right)  \tag{154}\\
& \text { Width } \left._{i_{1}, i_{2}, m}=\sum_{k} 2 \pi \gamma_{e_{i_{1}}, i_{2}, k, m}{ }^{f\left(\xi_{i_{1}, i_{2}}, k, m\right.}\right)=\sum_{k} \text { Width }_{i_{1}, i_{2}}, k, m
\end{align*}
$$

[For isolated lines; $f(\xi)$ is given by Eq. (113).]

$$
\begin{equation*}
\text { Width }_{i_{1}}^{*}, i_{2}, m=\sum_{j}\left(1-\operatorname{Em} \tau_{i_{1}, i_{2}}, m, j\right) \delta \nu_{j} \tag{156}
\end{equation*}
$$

[^8]$$
\frac{1}{\Delta \nu_{m}} \operatorname{Width}_{i_{1}}^{*}, i_{2}, m=F\left(\frac{1}{\Delta \nu_{m}} \text { width }_{i_{1}, i_{2}, m}, \text { LINE SPACING PATTERN }\right)(157)
$$
(approximate, empirical-numerical correlation).
The numerical integration of the integral in Eq. (96) then corresponds to the ing to Eq. (153).

In the numerical scheme of line calculations for each line group, the equivalent width is first calculated assuming that the lines are isolated. If this value exceeds 1 percent of the spectral width for the line group, Eq. (156) is then used to calculate the equivalent width with line overlaps. The correlation discussed in connection with fig. 4 may be used for radiation-coupled flow field calculations. However, the computer time for typical flow field cases is found to be of the order of only one minute (UNIVAC 1108). Consequently, efforts in obtaining equivalent-width correlations appear unessential (though desirable) at present.

The numerical calculation of Eq. (156) [or integration of Eq. (96)] is based on a variable interval Simpson's Rule, modified in such a way as to yield results two orders higher in accuracy than the basic Simpson's Rule. Depending upon the pattern of line spacings, the degree of overlap, and the accuracy criterion imposed, the number of evaluations of the integrand [of Eq. (96) or ( $\left.1-\operatorname{Em} \tau_{i_{1}}, \mathrm{i}_{2}, \mathrm{~m}, \mathrm{j}\right)$ ] for a line group may vary over a wide range, say from 20 to 800 for a single equivalent width.

## 5. RESULTS

Results of numerical calculations are presented in this section. Almost all of the results presented are obtained using STAGRADS. Only an example of the heat flux distributions obtained using STRADS is given. Additional results from STRADS may be found in refs. 11 and 12. A reference case $\left(u_{\infty}=15.24 \mathrm{~km} / \mathrm{sec}, \rho_{\infty} / \rho_{\text {SL }}\right.$ $=1.66 \times 10^{-4}, \mathrm{R}_{\mathrm{N}}=256 \mathrm{~cm}$ ) is selected for uncertainty studies.

### 5.1 Effects of Exponential Kernal Approximation

In ref. 1, the exponential integrals used in the radiation transport calculations were calculated using series representation and numerical correlations. In Section 3.2 , the exponential kernal approximation, $\mathrm{E}_{3}(\zeta) \simeq 0.5 \exp (-2 \zeta)$, is introduced. This approximation enables the formulation of the concept of line transmittance and equivalent width, as discussed in Section 3.4.2. The effects of the exponential kernal approximation on the shock layer enthalpy distribution, the radiative flux to the wall and to the shock, and the air layer thickness are shown in fig. 5, for a representative environmental condition (the reference case). The air continuum contributions alone are considered, using band Model No. 4 ( 5 bands) and 10 sublayers. The close agreement of the enthalpy, heat fluxes, and air layer thickness can be seen. The enthalpies calculated with actual $\mathrm{E}_{3}(\zeta)$ are slightly higher than that with approximate $\mathrm{E}_{3}(\zeta)$. However, the optical thicknesses of the four of the five bands with actual $\mathrm{E}_{3}(\zeta)$ are slightly lower than that with approximate $\mathrm{E}_{3}(\zeta)$. The net results are slightly higher heat fluxes with the exponential kernal approximation. The approximation $\mathrm{E}_{3}(\zeta) \simeq 0.5 \exp (-2 \zeta)$ is used in all results presented below.

### 5.2 Effects of Continuum Band Models

The effects of air continuum band models on the radiative flux to the wall can be seen in Table 1. Only air continuum contributions are considered. The results indicate that the two-band model does not account fully for the self-absorption effect in the vacuum ultraviolet. Band Model 4 (Planck-mean for $0.5 \leq \nu \leq 10.95 \mathrm{eV}$ plus 4 vacuum UV bands) provides results sufficiently close to that obtained with more complicated band models and is adopted for "nominal calculations." Figure 6 shows further comparison of results obtained using Models 1 and 4, including line contributions.

### 5.3 Effects of Line Model

Two options (LINEOP $=2$ or 3 ) of line calculations are discussed in Sections 3.4.2 and 4.1. When air layer calculations alone are considered, either of the two options mav be used. When the effects of ablation layer are examined, the option LINEOP $=3$ is used. Figure 7 shows the nondimensional enthalpy distributions, heat fluxes, and air layer thicknesses for the two line calculation options.

The results are for the same environmental condition (the reference case) as for figs. 5 and 6. The two line options yield nearly the same results.

### 5.4 Effects of Ablation Layers

The presence of a relatively high density ablation vapor adjacent to the body surface further attenuates the radiative flux to the wall. In an inviscid analysis, the chemical reaction between the air species and the wall is not considered. The wall is assumed to be at the equilibrium sublimation conditions corresponding to the surface pressure. The surface material considered in this study is 70/30 carbon phenolic (see Appendix A for its thermodynamic properties). A 21-band model of continuum absorption coefficients, with the same spectral divisions as that for the air line groups, is used (see Appendix B. 2).

For the reference case, the air layer temperatures are increased slightly near the interface when the effects of the ablation layer are included, as shown in fig. 8. The heat flux to the shock is increased slightly ( 3 percent) whereas that to the wall is reduced greatly ( 41 percent). The results are for the nominal $f$ numbers for the molecular band systems given in Table.B-2, unless indicated otherwise. Figure 9 shows the temperature and tangential velocity distributions across the air and ablation layers. It is seen that the ablation vapor is heated slowly at first near the wall and very rapidly near the interface. The tangential velocity distribution is replotted in terms of a transformed "mass" distance in fig. 10. The ablation layer is seen to carry about the same mass as the air layer, although it is physically thin compared to the air layer.

The particle number densities in the air and ablation layers are given in fig. 11.* The continuum optical thicknesses are given in fig. 12. The ablation layer is optically thicker than the air layer over all frequencies shown.

Figure 13 shows the spectral heat fluxes. Actually, the heat fluxes should be a series of steps, being constant over a line group frequency region as shown for the optical thickness in fig. 12. In order to show the effects of ablation layer in the same figure, the spectral heat fluxes are "assigned" to a selected point of each line group and straight lines are drawn between adjacent points. The results indicate that the ablation layer effectively blocks all radiation fluxes with frequencies greater than 10.95 eV , due to the photo absorption of C ground and excited states, $\mathrm{C}^{+}$excited states, and H Lyman (see Table B-1).

### 5.5 Effects of the Number of Sublayers

In order to test the accuracy of the finite-difference method of integrating the radiation transport and energy equations, the reference case is recalculated with fewer sublayers, from $10+10$ ( 10 subdivisions in both the air and ablation layers) to $6+6$. The results are compared in fig. 14. It is seen that the effects on the heat fluxes and total layer thickness are small. The surface reradiation is of the same order as the heat flux to the wall, causing a significant deviation in the nondimensional surface blowing rate.
*Only species used in radiative property calculations are plotted (see Appendix B).

For the study of the effects of environmental variables, the combination of 6 air sublayers and 6 ablation sublayers will be considered adequate.

### 5.6 Effects of Environmental Variables

The effects of nose radius variation on the radiative fluxes, for constant velocity and altitude, are shown in fig. 15. Increasing the nose radius from 1 to $8.4 \mathrm{ft}(30.48$ to 256 cm ) only increases the heat flux to the wall by 68 percent without ablation layer effects and 24 percent with ablation layer effects. Increasing the nose radius increases the heat loss to the ambient air more rapidly.

Further results for different environmental variables are given in Table 2 and figs. 16,17 , and 18 . At the highest altitude ( 250 kft ) considered, the surface reradiation exceeds the heat flux to the wall. The assumption that the surface is at the equilibruim sublimation temperature is not physically meaningful. However, the results may still be presented with the understanding that zero ablation is considered. The number of calculated points are insufficient to construct figs. 16, 17, and 18 accurately. Some of the points are obtained by "interpolation" of data points in other figures.

The results indicate that the heat flux to the wall is changed according to the following:
(1) It increases as the velocity increases; the ablation layer tends to decrease the velocity effect.
(2) It increases only slowly as the nose radius increases; the ablation layer reduces the nose-radius effect further.
(3) It decreases due to the presence of the ablation layer, more for lower altitudes, larger nose radii, and higher velocity.

In general, it can be stated that the ablation layer becomes more efficient in reducing the zero-ablation radiative flux to the wall as the latter increases. The analogy between the ablation layer effectiveness in reducing radiative heat flux to the wall and the convective blowing effectiveness is noted.

### 5.7 Effects of Perturbation of Radiative Properties

The effects of uncertainties of the radiative properties are illustrated in figs. 19 and 20. In fig. 19, curve 3 corresponds to the reference case when nominal ( $N$ ) values of the radiative properties are used. Curves $1,2,4$, and 5 represent results without the ablation layer. Curves 1 and 2 are results of calculation ( $\operatorname{LINE} \varnothing P=1$ ) with air continuum contributions only, with nominal cross sections for curve 1 and $2 \times$ the first-band cross sections ( $\nu<10.95 \mathrm{eV}$ ) for curve 2. The effects of air lines may then be seen by comparing curves 1 and 3 and the corresponding heat fluxes. Doubling the first-band cross sections alone is not sufficient to "match" the line effects.

For curve 4 , the line half widths used are twice the nominal value, while for curve 5 the line f-numbers are doubled. The effects are to increase both the heat fluxes to the wall and to the shock, but by less than 10 percent.

For curve 6, the line half-widths, line f-numbers, and the ablation species molecular band system f-numbers are twice the nominal values. Again the heat flux increase is relatively small, being less than 17 percent.

The effects of increasing the ablation species molecular band system f-numbers alone by a factor of two may be seen in fig. 20. The heat flux to the wall is only slightly reduced. Attempts were made to study the effects of increasing the f-numbers by a factor of the order of 10 . However, because of the rapid temperature rise near the wall, convergence difficulties were encountered in iteration of the ablation layer enthalpy distribution. Approximate energy conservation calculations for the enthalpy rise for a one-dimensional ablation vapor flow indicates, however, the enthalpy rise is nearly the maximum. Consequently, increasing the f-numbers further may not change the heat flux to the wall appreciably, although the enthalpy distribution may change significantly.

### 5.8 Effects of Precursor Heating

No detailed analysis of the precursor heating effects is performed in this study. However, using the results of spectral heat fluxes to the shock obtained from STAGRADS and the results of refs. 3,4 , and 13 , a first order estimate of the preheating effect on the radiative flux to the wall may be made. For instance, using fig. 11 of ref. 13 and assuming all radiation leaving the shock with $\nu>12.15 \mathrm{eV}^{*}$ (see Table 2) is absorbed by the cold air near the shockwave, for $u_{\infty}=15.24 \mathrm{~km} / \mathrm{sec}$ and $\rho_{\infty} / \rho_{\mathrm{SL}}=1.66 \times 10^{-4}$, the increase in radiative flux to the wall may be estimated to be in the order of 5 percent for the nose radii considered in Table 2. For velocities higher than $60 \mathrm{kft} / \mathrm{sec}$ the preheating effects may become more significant, however.

### 5.9 Equivalent Width Results

Figure 21 illustrates the results of equivalent widths of the line groups, for isothermal air layers of different thickness. The temperature and density correspond to that immediately behind the normal shock for the reference case. As discussed in Section 4.3 , the equivalent width is first calculated assuming that the lines are isolated. If this value exceeds 1 percent of the spectral width of the line group, the equivalent width is recalculated by numerical calculation of Eq. (156) to account for line overlaps. As the layer thickness increases, the equivalent widths of line groups with strong line overlaps approach the group width asymptotically, for instance line groups 19 and 15 in fig. 21. When the ratio of equivalent width to line-group width is much less than unity, the slope of the line may indicate whether the lines are thin or within the strong-line, square-root regime (see Eq. (113)). For instance, the lines within group 1 are thin whereas that within group 13 are within the square-root regime, for most of the values of $\Delta$ in fig. 21.

[^9]The results of fig. 21 indicate that line overlaps must be taken into account for blunt-body radiation transport calculations.

### 5.10 STRADS Results

Downstream of the stagnation region, the stream-tube formulation is used to calculate the flow field and radiative fluxes. In this study, the computer code STRADS developed in ref. 1 is modified to incorporate the air continuum band models used in STAGRADS. The effects of ablation layer are not included. The air line contributions are "simulated" approximately by multiplying the first-band cross sections with an appropriate factor.

The configuration considered is a blunt vehicle flying at a 33 deg angle-of-attack, as shown in fig. 22. The formulation of STRADS is based on two-dimensional and axisymmetric configurations. In order to simulate the actual attitude, the effective axis of symmetry is displaced at a distance $R_{o}$ from the stagnation point, as shown in fig. 22.

The radiation heating distributions, normalized with respect to the stagnation radiative flux to the wall calculated by STAGRADS, are given in fig. 23. The dimensional symbols are defined in fig. 22. Varying the value of the first-band cross section multipliers from 2 to 4 produces no significant changes in the normalized heating distributions. The radiation heating distribution is seen to exhibit a strong velocity dependency. Further results from STRADS may be found in refs. 11 and 12.

## 6. CONCLUDING REMARKS

An efficient numerical procedure for fully radiation-coupled blunt-body flows has been formulated in this study. The effects of air continuum, atomic line, and ablation layer radiations are taken into account for determining the stagnation point velocity and temperature fields, the heat fluxes to the wall and to the shock, and the surface blowing rate. The efficiency of the numerical methods is due to the finite-difference integration of the radiation-transport terms and the energy equation, to the formulation of the concept of line-group equivalent width and average transmittance, and to the procedure of calculating the equivalent width with line overlaps.

Using the computer code developed, the effects of environmental variables, of the numerical methods used, and of the uncertainties in radiative properties have been delineated. The results indicate the following conclusions:
(1) The exponential kernal approximation of the exponential integral yields results for enthalpy distributions and heat fluxes very nearly the same as that with actual exponential integrals.
(2) The two-band model of air continuum radiation underpredicts the self-absorption effects in the vacuum ultraviolet. Increasing the vacuum UV band numbers from 1 to 4 (Model 4) provides results sufficiently close to that obtained with more complicated band models.
(3) The two options of line calculations, LINEOP $=2$ or 3, yield nearly the same results.
(4) The ablation layer is very effective in reducing the heat flux to the wall and thus the surface ablation rate. It practically blocks all radiative fluxes with frequency greater than 10.95 eV . The ablation layer becomes more efficient in reducing the zero-ablation radiative flux to the wall as the latter increases.
(5) Reducing the number of sublayers from $10+10$ to $6+6$ still yields results adequate for the study of the effects of environmental variables.
(6) The self-absorption and energy loss effects decrease the sensitivity of the heat fluxes to the changes in environmental variables and to the uncertainties in radiative properties. An order of magnitude change of the nose radius (from 1 to 10 ft ) only increases the heat flux to the wall by 30 percent. Considering the change of convective heat rates with nose radius, blunt configurations may prove to be more effective than slender configurations in reducing the total wall heat fluxes.
(7) For velocities less than $18 \mathrm{~km} / \mathrm{sec}$ ( $\sim 60 \mathrm{kft} / \mathrm{sec}$ ), the effect of precursor heating on the radiative flux to the wall is small.

In future work, the following studies are recommended:
(1) The formulation of radiation transport may be extended to include the precursor radiation and the spectral emissivity of the body surface.
(2) The contributions of additional radiative systems, such as the lines due to nitrogen ions, and oxygen and carbon atoms and ions, may be included.
(3) The formulation of the flow field may be extended to include the precursor heating effect and the viscous effect (including conduction and diffusion).
(4) The radiation transport formulation developed may be used for calculations around the body.
(5) The accuracy of the one-dimensional radiation transport may be investigated, particularly when the temperature gradient in the direction parallel to the body is large.
(6) The numerical procedures used in STAGRADS may be further improved to increase the accuracy and to reduce the computation time.

## APPENDIX A <br> THERMODYNAMIC PROPERTIES OF AIR AND ABLATION VAPOR

For a given composition of elementary species, two state variables are sufficient to specify the thermodynamic properties of a gaseous mixture in thermodynamic equilibrium. The state properties may be put in a tabular form so that property calculations may be made by table look-up and interpolation. Alternately, curve fits of one state variable versus a second at several levels of a third may be used. The latter method is particularly convenient when many calculations are made at a fixed level of a certain state variable. For instance, for a blunt body in hypersonic flow, the shock layer normal pressure gradient may be neglected. Curve fits or correlations of temperature and other state properties versus enthalpy at several constant levels of pressure will be very convenient for calculations along a body normal. Correlations of this kind for air may be found in refs. 15,16 , and 17.

For air calculations in the present study, correlations of refs. 15 to 17 may be used. However, for the purpose of reducing the computer time required, new correlations with a more suitable set of units are generated. The division of enthalpy ranges is also selected in such a manner that-interpolations of the coefficients in the correlations between adjacent pressure levels may be more conveniently made.

The units selected for correlation purposes are:

> P , pressure in atm
> T , temperature in $\mathrm{eV}, l \mathrm{eV}=11605.7^{\circ} \mathrm{K}$
> H , enthalpy ratio, $\mathrm{h} / \mathrm{h}_{\text {Satellite }}, \mathrm{h}_{\text {Satellite }}=12,484 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}$
> $\mathrm{PV} / \mathrm{V}_{\mathrm{SL}}$, in atm, $\mathrm{V}_{\mathrm{SL}}=$ sea-level $\left(1\right.$ atm, $\left.288.16^{\circ} \mathrm{K}\right)$ air specific volume

The correlation formula coefficients for air are given in Table A-1.
With the values of the compressibility factor available, the concentrations of air species may be readily calculated if Hansen's (ref. 18) simplified model of air chemistry is used. However, in this study, the particle densities (their logarithm) of neutral nitrogen and oxygen atoms and elections are calculated by double interpolation with respect to temperature and the logarithm of density using the results of Gilmore (ref. 19) and Moeckel and Weston (ref. 20).

The surface material considered in ablation layer calculations is 70/30 carbonphenolic. The 70/30 carbon-phenolic is assumed to be at the equilibrium sublimation conditions corresponding to the surface pressure. The sublimation latent heat is 4,$270 ; 4,220 ; 3,900 \mathrm{cal} / \mathrm{gm}$ at $0.1,1$, and 10 atm , respectively. If the material sensible heat is included in the energy balance, the effective heat of sublimation will be higher.

The thermodynamic properties (temperature, enthalpy, density, species particle concentrations, and pressures) of 70/30 carbon-phenolic vapor in thermodynamic equilibrium were calculated by a free-energy minimization program, and are put in a table form for double interpolation with respect to pressure and enthalpy. Between two consecutive table values, the temperature and the logarithm of specific volume and of species concentrations are assumed to be linear in enthalpy and in the logarithm of pressure.

## APPENDIX B

## RADIATIVE PROPERTIES OF AIR AND ABLATION SPECIES

The absorption coefficient of a given species is the product of its particle number density and absorption cross sections. For a given mixture of species, the absorption coefficient is the sum of these products. The cross sections of air species and ablation vapor species are discussed below.

## B. 1 Absorption Cross Sections of Air Species

B.1.1 Continuum absorption cross sections. - The effective cross sections of neutral nitrogen (NI) and neutral oxygen (OI) given in Tables 4-3 and 4-5 of ref. 7 are used. These cross sections include the bound-free and free-free photon absorption contributions and are per particle of NI or OI. For the conditions of interest, the relative contribution due to NII is small. The incorporation of absorption contributions due to NII and $\mathrm{N}^{-}$continuum has not been made.

For the eight continuum band models described in Section 4.1, partial Planck mean [see Eq. (75)] cross sections are used for bands with frequency less than 10.95 eV . The NI cross sections increase in steps for frequency above 10.95 eV and average values are used for each of the steps. The OI cross sections do not change appreciably between 4.0 and 13.61 eV but increase in steps above 13.61 eV . For OI, partial Planck means are used up to 13.61 eV and average values are used for each of the steps above 13.61 eV . The selection of the band boundaries is based on the locations of the step changes in cross sections. The band boundaries for the eight models are given in Table 1. For frequencies above 10.95 eV , the cross sections for the highestfrequency band correspond to that of the lowest-frequency step within the band. For example, in Model 1, the average cross sections between 10.95 and 12.15 eV are used for the second band with boundaries at 10.95 and 30.0 eV .

For calculations with lines and ablation layer, the frequency regions are relatively narrow. The value of the cross section at a specified point within the frequency region is used. The logarithms of the cross sections are assumed to be linear in frequency and temperature between tabulated values.
B.1.2 Line absorption cross sections. - Only the contribution of NI lines are included in the present STAGRADS code. The effective NI particle density for line calculations is taken as the sum of the actual NI and OI particle densities. A total of 75 NI lines are considered. Some of the lines are "effective" lines formed by treating two or more closely situated lines as single lines. The f numbers and the half-width (at $\mathrm{T}=10,000^{\circ} \mathrm{K}$ ) of the NI lines given in Tables $4-9$ and $4-15$, respectively, of ref. 7 are used. The line shifts are neglected and the half-widths are considered independent of temperature [see Eq. (99)]. See Appendix C. 3 for details of line data used.

## B. 2 Absorption Cross Sections of Ablation Species

Only the continuum contributions of ablation species are included in the present STAGRADS code. The radiative systems considered include the Swan, Fox-Herzberg, Deslandres-D'Azambuja, Freymark, and Mulliken band systems of $\mathrm{C}_{2}$, the CO4+ system, and the continuum processes due to the $\mathrm{C}^{+}$free-free contribution and the photo-absorption of CO, H Balmer, H.Lyman, C ground and excited states, and $\mathrm{C}^{+}$ excited states.

As discussed in Section 4.1, the division of the continuum bands for the ablation species is made coincident with that of the line groups.

The spectral ranges of the 21 bands and the average cross sections of the absorption processes are given in Table B-1. The cross section $\sigma_{0}$ of the molecular band systems were obtained by wavelength averaging the results obtained by John Weisner (ref. 21) and by normalizing to unity f-number. The temperature dependence is determined from the cross-section values at two temperatures $\left(3,000^{\circ} \mathrm{K}\right.$ and $9,000^{\circ} \mathrm{K}$ or $10,000^{\circ} \mathrm{K}$ ) and is very approximate. The cross-sections of the continuum processes are given by the same simple correlation, with $f$ taken as a unity. No perturbation of the continuum cross-sections is considered in this study. The f-numbers of the band systems are given in Table B-2.

The $\mathrm{C}_{2}$ Swan systems are the only band contributors considered within the first five bands. Preliminary results of calculation indicated that the optical thickness for these bands ( $>0.62 \mu$ ) was too low. As the absorption in the IR is generally strong due to the presence of the polyatomic molecules adjacent to the wall and due to systems not included (e.g., C $C_{2}$ Phillips and Ballik-Ramsey Band systems and H Paschen and Brackett continua), the $\mathrm{C}_{2}$ Swan contribution within Band 1 is raised by a factor of 10 to compensate for the neglected absorbing systems.

## APPENDIX C

## THE STAGRADS CODE

The listing of the STAGRADS code, for calculation in UNIVAC 1108 at LMSC, is given after the figures near the end of the report. The input data required for the reference case ( $u_{\infty}=15.24 \mathrm{~km} / \mathrm{sec}, \rho_{\infty} / \rho_{\mathrm{SL}}=0.166 \times 10^{-3}, \mathrm{R}_{\mathrm{N}}=256 \mathrm{~cm}$ ) are then listed. Brief explanations are given for the code and the input. The program is very complex. The purpose of presenting the listing is for the benefit of those who have the desire to modify the program and to make use of some of the subroutines for other related problems.

Representative output data, including details of the line data used, are also given at the end of the report with brief explanations.

## C. 1 STAGRADS Listing (See page 99.)

The listing of STAGRADS given is in FORTRAN IV. STAGRADS consists of a main program and a number of subroutines and functions. In the order of appearance in the listing, they will be explained briefly below.

## C.1.1 Main Program. The main program serves the following functions:

(1) Read all input data.
(2) Write a majority of the output results.
(3) Integrate the energy equation and iterate the enthalpy distribution for the air layer.
(4) Call the appropriate subprograms (i.e., Subroutine VEL for integration of the momentum equation to obtain the velocity distribution).

The built-in data TC (I, J) are used only in Subroutine ABSORP. They are repeated here for the purpose of checking (see listing near statement no. 72).
C. 1. 2 Subroutine STAGAB, STAGAB (STAGrads ABlation) is for calculation of stagnation ablation layer. It serves the following functions:
(1) Integrate the energy equation and iterate the enthalpy distribution, wall heat flux and surface blowing rate for the ablation layer.
(2) Write output of ablation layer results.
(3) Call the appropriate subprograms (i.e., Subroutine VEL for calculation of the velocity distribution).
C.1.3 Subroutine NGGBSP. NGGBSP (Non-Grey Gas) is used for radiation transport calculations involving finite bands of the air continuum contributions. As listed, the exponential kernal approximation is used for $\mathrm{E}_{3}(\xi)$. However, the actual $\mathrm{E}_{3}(\zeta)$ may be used if only the air continua are considered.
C.1.4 Subroutine CRLINE. CRLINE (CoRrection for LINE) is used for radiation transport calculations when the contributions of the air lines and ablation species continua are included. It is used for either of the two line-calculation options discussed in Section 4.1.
C. 1.5 Subroutine ABSORC. ABSORC (ABSORption for Continua) calculates the air continuum absorption coefficients (NI and OI) for a specified frequency within the individual line groups.
C.1.6 Subroutine LINE. LNNE calculates the equivalent widths or transmittance for the line groups. It writes out the calculated results if requested by appropriate controls.
C.1.7 Function FLR. FLR calculates the Ladenburg-Reiche function according to the approximations given by Eq. (113).
C.1.8 Subroutine INTEG. INTEG (INTEGration) performs integration to obtain the equivalent width. It is based on a modified Simpson's rule. It writes out the smallest integration step and other information if requested by the control.
C. 1.9 Subroutine INTRND. INTRND (INTegRaND) calculates the integrands required for equivalent width calculations.
C.1.10 Subroutine DOUBL. DOUBL (DOUBLe interpolation) is used to calculate particle concentrations of NI, OI and e by double interpolation with respect to temperature and the logarithm of specific volume.
C. 1. 11 Subroutine PROPT. PROPT (PROPerty) calculates air temperature, PV product, and compressibility factor for given pressure and enthalpy, using the correlation formulas given in Table A-1.
C.1.12 Subroutine ABSORP. ABSORP (ABSORPtion) calculates the air continuum absorption coefficients (NI and OI) for the individual bands for the eight band models.
C. 1.13 Subroutine SPABCO. SPABCO (SPectral ABsorption COefficient) calculates the ablation species continuum absorption coefficients for the 21 spectral regions.
C. 1.14 Subroutine PROT. PROT (PROperTy) calculates for the ablation vapor the specific volume, temperature, and particle densities of $\mathrm{C}_{2}, \mathrm{C}, \mathrm{CO}, \mathrm{H}$, and $\mathrm{C}^{+}$by double interpolation with respect to enthalpy and the logarithm of pressure. The property tables are input in the main program.
C. 1.15 Subroutine SESB. SESB (State Equation for Surface Blowing) calculates the surface enthalpy for 70/30 carbon phenolic in sublimation equilibrium at the surface pressure. The surface specific volume and temperature calculated in SESB are actually not used in STAGAB; they are recalculated by PROT for consistency with the vapor properties.
C. 1.16 Subroutine PLANCK. PLANCK calculates the fraction of black-body powers within two frequency limits and for a given temperature.
C.1.17 Function EXPF. EXPF limits the absolute value of the argument for EXP to 85 to avoid numerical overflow.
C. 1.18 Subroutine VEL. VEL (VELocity) integrates the momentum equation for the air layer and ablation layer.
C.1.19 Subroutine TBLP4. TBLP4 is used for interpolation of variables such as specific volume ratio, transformed distances and velocities in VEL.
C.1.20 Subroutine MODL1, MODL2, ... MODL8. These subroutines specify the frequency limits for the 8 air continuum band models. They also select the proper cross section indices for calculations in ABSORP.
C.1.21 Function E2F. (listed after input data) E2F calculates the second exponential integral by series and numerical correlations. It may be used in conjunction with E3F in NGGBSP and CRLINE, when the actual exponential integral instead of the exponential kernal approximation is employed.

## C. 2 Input Data Listing (See page 155.)

The input data for an example case (the reference case) are listed immediately following the STAGRADS listing. All input data are read in within the main program.

The input line and line group data are first read. The 10 read statements after the comment "Input Line and Line-Group Data" read in the values of the following variables:

NHV Total number of line groups
NLST Total number of lower states
GEE(I) Statistical weight of I-th lower state
$\operatorname{EPS}(\mathrm{I}) \quad$ Electronic energy of I-th lower state, eV
FHVM(I) Lower frequency limit of I-th line group, eV
FHVP(I) Upper frequency limit of I-th line group, eV
FHV(I) A selected (e.g., central) frequency within I-th line group, eV
ISOE(I) $\quad>0$ if the approximation of effective isothermal region may be used for the I-th line group; otherwise $\leqq 0$.
NUMINT(I) $>0$ if numerical integration is to be used (when required) to calculate the equivalent width of the I-th line group; otherwise $\leqq 0$.
NU(I) Number of lines in I-th line group

| NLINE | Total number of lines, calculated by summing all NU |
| :--- | :--- |
| ND(I) | Lower state index for I-th line |
| HVL(I) | Center frequency of I-th line, eV |
| FF(I) | Effective f-number of I-th line (f-number of the particular transition <br> multiplied by a statistical-weight ratio, see Section 3.4.1) |
| GAMBA(I) | Half-width (eV) of I-th line for 1 electron $/ \mathrm{cm}^{3}$. |

Next, some of the 4 input tables are read, between Statements 8 and 99 in the listing. Each of the 4 tables is headed by a leader card.

| Table 01 | (Leader Card with value of NC $=01$; near statement 10) |
| :---: | :---: |
| L | 0 for two-dimensional and 1 for axisymmetric |
| CASE | Case identification |
| PA | Ambient pressure, atm |
| RHOA | $\stackrel{\rho}{\rho}_{\infty} / \rho_{\text {SL }}$ |
| UA | $u_{\infty}, \mathrm{km} / \mathrm{sec}$ |
| RN | $\mathrm{R}_{\mathrm{N}}, \mathrm{cm}$ |
| NS | $\leqq 10$, number of air sublayers |
| ES | $\rho_{\infty} / \rho_{s}$ |
| HA | $\mathrm{h}_{\infty} / \mathrm{h}_{\text {sat }}$ |
| TW | $\mathrm{T}_{\mathrm{W}}$, eV (if $<0.02$, reset to 0.02 ; recalculated as equilibrium surface temperature if ablation layer effects are considered) |
| MODEL | Air continuum band model number, $=1,2, \ldots, 8$ |
| LINEOP | $\left\{\begin{array}{l}1, \text { only air continua with one of the eight band models } \\ 2, \text { calculate air continua with one of the eight band models and then } \\ \text { correct for air lines } \\ 3, \\ \text { 21 spectral regions with air continua and lines, with or without } \\ \quad \text { ablation continua }\end{array}\right.$ |

FCONTN Multiplying factor for air continuum cross sections ( $\nu<10.95 \mathrm{eV}$ ) for the 8 band models
FDHAB Multiplying factor for surface material latent heat of vaporization
NRITE1 > 0 print nondimensional enthalpy distributions during iteration; $\overline{\overline{<}} 0$ no print
NRITE2 $>0$ print equivalent-width information, final result only; $<0$ no print
NRITE3 $>0$ print equivalent-width information during iteration; $=0$ no print
NRITE4 $>0$ print equivalent-width integration information during iteration; $=0$ final result only; $<0$ no print

NRITE5 $\quad>0$ print line strength of sublayer, etc. during iteration; $=0$ final result only; < 0 no print
NRITE6 $>0$ print line strength of air-layer regions, etc. during iteration; $=0$ final result only; $<0$ no print
NRITE7 $>0$ print nondimensional enthalpy distributions, QWT, etc., when either (but not both) the air or ablation layer iteration converges; $\leqq 0$ no print
CMAG $\quad 0.0001$ times absolute value of the average value of integrand
ERR
Integration accuracy control constant
With err $=0.1$ and 0.01 , the result is said to be accurate to $4-6$ places and $5-7$ places, respectively. Decreasing the value of err will increase the computing time required.

From the results of equivalent-width calculations for a variety of conditions, a value of err $=0.1$ is found adequate for most situations. Under certain situations when the lines are very narrow and with a small degree of overlapping, it is found that a value of err in the order of 0.01 is required for line groups 4 and 5 in order to obtain accurate results in equivalent widths by numerical integration. However, a value of err in the order of 0.1 is adequate for flow field and total heat flux calculations.

FRAC First integration step size divided by total integration interval NSAB $\leqq 10$, number of ablation sublayers

Table 01 must be input for each run of flow field calculations. For flow field calculations without the ablation layer, Table 01 ends with a card with -1 (value of NC) in the first two columns. For flow fields calculations with ablation layer, Tables 03 and 04 are also required; the " -1 " card is then put at the end of Table 04.

For flow field calculations, each case must end with a "-1" card. For subsequent cases of the same computer run, Tables 03 and 04 may be omitted if they are identical with the corresponding ones in the preceding case.

Table 02 (Leader card with value of $\mathrm{NC}=02$; near Statement 80)
Table 02 is input for equivalent width calculations with given layer thickness, number of sublayers, and distributions of temperature and specific volume. No "-1" card is required to end each case.

N Total number of sublayers
DELTA Total thickness of layer, cm
$T(I) \quad$ Temperature of I-th sublayer, eV
V (I) $\quad \rho_{\mathrm{SL}} / \rho$ of I-th sublayer
CMAG, ERR, FRAC
NRITE1 $\rightarrow$ NRITE6 see Table 01

Table 03 (Leader card with value of NC $=03$; near Statement 7010)
Table 03 inputs the thermodynamic properties of the ablation layer vapor.

| IP | Total number of pressure values in table (3 allowed in present program) |
| :---: | :---: |
| IT | Total number of temperature values in table ( 20 allowed in present program) |
| PRTAB(I) | The i-th pressure value, in $\log _{10}(\mathrm{p}$, atm) |
| TTAB(I, J) | $J$-th temperature value for I-pressure value, ${ }^{\circ} \mathrm{K}$ |
| ENTAB(I, J) | Enthalpy for I-th pressure and J-th temperature, cal/gm |
| RHOTAB( I , J) | Density for I-th pressure and J-th temperature, gm $/ \mathrm{cm}^{3}$ |
| C2TAB(I, J) | Particles/ $\mathrm{cm}^{3}$ for I-th pressure and J-th temperature, for $\mathrm{C}_{2}$ |
| C1TAB(I, J) | For $\mathrm{C}_{1}$ |
| COTAB(I, J) | For CO |
| HTAB (I, J) | For H |
| CPTAB(I, J) | For $\mathrm{C}^{+}$ |
| EMTAB(I, J) | For e |

Table 04 (Leader card with value of $N C=04$; near Statement 7020)
Table 04 inputs the f-numbers and continuum cross section multipliers for ablation layer radiating systems.

ICOSR Total number of input COSR values
$\operatorname{COSR}(\mathrm{I}) \quad \mathrm{f}$-numbers for band systems. For continua, $\operatorname{COSR}=1.0$ if the standard values of the cross sections are used. The order of $\operatorname{COSR}(\mathrm{I})$ is given in Table C-1.
C. 3 Output Data Listing, Including Details of Line Data Used (See page 163.)

The data for the 21 line groups are first listed. The headings of the data are:

| HV | Frequency within a line group for calculating Planck intensity function <br> and continuum cross section, eV |
| :--- | :--- |
| HV + | Upper frequency limit of line group, eV |
| HV- | Lower frequency limit of line group, eV |
| N | Number of lines within line group |

TABLE C-1
$\operatorname{COSR}(\mathrm{I})$ VALUES

I
1

LINE
NLST
HV (I)
$F(\mathrm{I})$
GAM(1)

Absorption Processes
$C_{2}$ Swan
$C_{2}$ F-H
$C_{2}$ D-D
$C_{2}$ Frey
$C_{2}$ Mull
Unassigned
Unassigned
Unassigned
CO $4+$
CO Photo
Unassigned
Unassigned
Unassigned
H Balmer
H Lyman
Unassigned
Unassigned
C Photo
Unassigned
Unassigned
$\mathrm{C}^{+}$Photo
Unassigned
$\mathrm{C}^{+}+\mathrm{e} \rightleftharpoons \mathrm{C}+\mathrm{h} \nu,\left(\mathrm{C}^{+}\right.$free-free $)$

Remarks
Use f value
Use f value
Use f value
Use f value
Use f value
Use 0 value
Use 0 value
Use 0 value
Use $f$ value
Use 1 for standard value
Use 0 value
Use 0 value
Use 0 value
Use 1 for standard value
Use 1 for standard value
Use 0 value
Use 0 value
Use 1 for standard value
Use 0 value
Use 0 value
Use 1 for standard value
Use 0 value
Use 1 for standard value

BFG(I) b'gf , Eq. (129) eV-cm ${ }^{2} /$ particle
ISOE $>0$, if effective isothermal region may be assumed; $\overline{\overline{<}} 0$ otherwise
NUMINT $>0$, if equivalent width is calculated by integration when its value according to isolated lines exceeds 1 percent of line-group width; $\overline{<} 0$ if calculated according to isolated lines only
GEE(I) Statistical weight of I-th lower state
EPS(I) Electron energy of I-th lower state, eV
The next group of output data is the air continuum cross sections TC(I, J) used in ABSORP. In ABSORP, TC( $1 \rightarrow 14,1$ ) are temperatures in eV and $\mathrm{TC}(1 \rightarrow 14$, $2 \rightarrow 27$ ) are values of $38+\log _{10}$ (cross-section, $\mathrm{cm}^{2}$ ). These values have been converted to ${ }^{\circ} \mathrm{K}$ and $\mathrm{cm}^{2}$, respectively, in the main program before being printed out. Each index $J(2 \rightarrow 27)$ is for a particular frequency interval (either a partial Planck mean or a representative average).

| J: | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Species: | NI | NI | NI | NI | NI | NI | NI | NI |  |
| $\nu_{1}, \mathrm{eV}:$ | 0.5 | 2.0 | 4.0 | 6.0 | 9.50 | 10.95 | 12.15 | 13.61 |  |
| $\nu_{\mathrm{u}}, \mathrm{eV}:$ | 2.0 | 4.0 | 6.0 | 9.5 | 10.95 | 12.15 | 13.61 | 14.50 |  |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |  |
| NI | OI | OI | OI | OI | NI | OI | NI | OI |  |
| 14.50 | 0.5 | 2.0 | 4.0 | 13.61 | 0.50 | 0.50 | 2.00 | 2.00 |  |
| 30.00 | 2.0 | 4.0 | 10.95 | 30.00 | 10.95 | 10.95 | 10.95 | 10.95 |  |
|  |  |  |  |  |  |  |  |  | . |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |  |
| NI | NI | NI | NI | NI | OI | OI | OI | OI |  |
| 0.5 | 1.0 | 2.0 | 3.0 | 4.00 | 0.5 | 1.0 | 2.0 | 3.0 |  |
| 1.0 | 2.0 | 3.0 | 4.0 | 10.95 | 1.0 | 2.0 | 3.0 | 4.0 |  |

The next group of output consists of data of particle per air atom for equilibrium air. Following a column of 14 temperatures ( $3,000 \rightarrow 24,000^{\circ} \mathrm{K}$ ), the particles per air atom for NI, OI, and e ( 3 column as a group for one density) for $\log _{10}\left(\rho_{\mathrm{SI}} / \rho\right)=$ $1.97634,2.97634$, and 3.97634 are given. The values of SPTABO (I, J, K) with the main program are in $-\log _{10}$ (particles/air atom). These values have been converted to particles/air atom before being printed out.

The next group of output corresponds to Table 01 of input. The units of the variables and the definitions of the symbols are given there.

If Table 03 is input, the data will be printed out. The data printed for the reference case are for three pressures $\left(\log _{10} \mathrm{p}\right.$, atm $\left.=-1.0,0.0,1.0\right)$. Columns 1 to 9 are respectively temperature ( ${ }^{\circ} \mathrm{K}$ ), enthalpy (cal/gm), density ( $\mathrm{gm} / \mathrm{cm}^{3}$ ), and particles $/ \mathrm{cm}^{3}$ for $\mathrm{C}_{2}, \mathrm{C}_{1}, \mathrm{CO}, \mathrm{H}, \mathrm{C}^{+}$, and e .

If Table 04 is input, the values of $\operatorname{COSR}(\mathrm{I})$ are printed out. See Table C-1 for identification of the index.

The next group of output repeats the information for the line groups including the selected frequency and the lower and upper line indices.

The symbols appearing in the output are defined below:

| NCOUNT | Number of iterations for enthalpy distribution, one counter |
| :---: | :---: |
| DELTA | for the air layer and another counter for the ablation layer Layer thickness for air or ablation layer, cm |
| PO | Air and ablation layer pressure, atm |
| HO | $\mathrm{h}_{\mathrm{s}} / \mathrm{h}_{\text {sat }}$ |
| TO | $\mathrm{T}_{\mathrm{s}}$, eV |
| QWT | Total heat flux to wall, watts $/ \mathrm{cm}^{2}$ |
| QSHOKT | Total heat flux to shock, watts/ $\mathrm{cm}^{2}$ |
| I | Sublayer index from shock |
| YC | $z / \Delta$ for air layer; $y / \Delta_{a b}$ for ablation layer (measured from center of sublayers) |
| UC | Normalized tangential velocity |
| VC | Normalized normal velocity |
| HC | $\mathrm{h} / \mathrm{h}_{\mathrm{s}}$ for air layer; $\mathrm{h} / \mathrm{h}_{\mathrm{w}}$ for ablation layer |
| T | Temperature, eV |
| V | $\rho_{\text {SL }} / \rho$ |
| Z | Compressibility |
| HV | Selected (e.g., central) frequency of line group, eV |
| HV+ | Upper limit frequency of line group, eV |
| HV- | Lower limit frequency of line group, eV |
| NLINE | Number of lines in line group, eV |
| TAUC | Continuum optical thickness for air layer |
| TAUCT | Continuum optical thickness from shock to wall |
| LINEOP | Line-calculation option (See Section 4.1) |
| EMD | $\rho_{\mathrm{w}} \mathrm{v}_{\mathrm{w}} / \rho_{\infty} \mathrm{u}_{\infty}$ |

NQWT

NCONH

QWTA1
QWTA2
QWTC1
QWTC2
QWT1
HW
TW
VW
VCW
UR

RMOM
URMAS

IAB
NC1, NC2, NCO $\mathrm{NH}, \mathrm{NC}^{+}$

M
IP1, I
ISO . WIDTH
ACT . WIDTH
DNU* TRN
ISO-ACT WIDTH
WIDTH (IP1, I)
DELTAB
DELTAT
US

ZETA

NCNTR1

Number of iterations for heat flux to the wall for each ablation-layer calculation
Counter for enthalpy distribution iteration for each assumed wall heat flux

Previously assumed wall heat flux, watts $/ \mathrm{cm}^{2}$
Previously calculated wall heat flux, watts $/ \mathrm{cm}^{2}$
New assumed wall heat flux, watts $/ \mathrm{cm}^{2}$
$h_{w} / h_{\text {sat }}$
$\mathrm{T}_{\mathrm{w}}$, eV
$\rho_{\mathrm{SL}} / \rho_{\mathrm{w}}$
$v_{w} / u_{\infty}$
$u_{r} / u_{\infty}$ calculated based on interface streamline momentum consideration
$\rho_{\mathrm{w}} \mathrm{v}_{\mathrm{w}}^{2} / \rho_{\infty} \mathrm{u}_{\infty}^{2}$
$u_{r} / u_{\infty}$ calculated based on mass balance for a finite number of sublayers
Ablation sublayer index from wall
Particle $/ \mathrm{cm}^{3}$ for $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{CO}, \mathrm{H}$, and $\mathrm{C}^{+}$, respectively
Line group index
Upper and lower index for region with indices IP1 and I
Equivalent width assuming isolated lines, eV
Equivalent width with overlap, eV
$\Delta \nu$ • TRANS , eV
ISO • WIDTH - ACT • WIDTH, eV
Equivalent width, eV
$\Delta_{a b}$, ablation layer thickness, cm
$\Delta$, thickness between shock and wall, cm

$$
\begin{aligned}
& \int_{\mathrm{o}}^{\mathrm{u}_{\mathrm{s}}} \\
& \mathrm{z}_{\mathrm{s}} \\
& \rho \\
& \rho_{\mathrm{s}} \\
& \frac{\mathrm{dz}}{\Delta} .
\end{aligned}
$$

Counter for number of air layer enthalpy iteration convergence

| NCNTR2 | Counter for number of ablation layer calculation convergence |
| :---: | :---: |
| NCNTR3 | Counter for number of either (but not both) air layer or ablation layer convergence |
| FNUL(KK) | Lower frequency limit of KK-th air continuum band, eV |
| FNUH(KK) (FNUU) | Upper frequency limit of KK-th air continuum band, eV |
| FNU | Central frequency of air continuum band, eV |
| AB | 1.0, signifies with self-absorption |
| QW (KK) | Continuum heat flux to wall for KK-th band, watts/ $\mathrm{cm}^{2}$ |
| QWTC | Total continuum heat flux to wall, watts $/ \mathrm{cm}^{2}$ |
| QDEL(KK) | Continuum heat flux to shock for KK-th band, watts/ $\mathrm{cm}^{2}$ |
| QDELT | Total continuum heat flux to shock, watts $/ \mathrm{cm}^{2}$ |
| SQW (KK) | Spectral continuum flux to wall for KK-th band, watts $/ \mathrm{cm}^{2} \mathrm{eV}$ |
| SQDEL(KK) | Spectral continuum flux to shock for KK-th band, watts $/ \mathrm{cm}^{2} \mathrm{eV}$ |
| TAU(I, KK) | Continuum optical thickness for KK-th band from I-th boundary to shock ( $\mathrm{I}=1$ ) |
| K, KM | Line index and line index within line group, respectively |
| JD | Lower-state index |
| I | Sublayers index from shock |
| IP1 , J | Boundary upper and lower indices |
| GAM(I) | Line half-width, eV |
| SS(I) | Line strength, Eq. (110), eV |
| $\mathbf{S S T}(\mathrm{I}+1)$ | $\sum_{J=1}^{\mathrm{I}} \mathrm{SS}(\mathrm{~J}), \mathrm{eV}$ |
| SGAM(I, KM ) | $\mathrm{SS}(\mathrm{I}) * \mathrm{GAM}(\mathrm{I}),(\mathrm{eV})^{2}$ |
| $\operatorname{SGAMT}(\mathrm{I}+1)$ | $\sum_{J=1}^{I} \operatorname{SGAM}(J, K M),(e V)^{2}$ |
| TAUD(I) | Line-center optical thickness, SS(I)/ $\pi$ GAM(I) |
| TAUDT( $\mathrm{I}+1$ ) | $\sum_{\mathrm{J}=1}^{\mathrm{I}} \operatorname{TAUD}(\mathrm{~J})$ |
| SS12 | Line strength between boundaries IP1 and J, eV |

SGAM12
GAME12
ZETA12

WID12K

WID12(IP1 , J)

TAU12

NCOUNT, NREJE

Value of SGAM between boundaries IP1 and J, (eV $)^{2}$ Effective half-width, SGAM12/SS12, eV
Argument of the Ladenburg-Reiche function, Eqs. (106) and (107)

K-th line contribution to isolated line equivalent width for region between boundaries IP1 and J, eV
Equivalent width according to isolated lines for region between boundaries IP1 and $\mathrm{J}, \mathrm{e} \overline{\mathrm{V}}$
Line-center optical thickness for region between boundaries IP1 and J

Counter for advancing to next integration interval or for rejecting existing integration step size, respectively. In Subroutine INTEG. Approximate number of evaluations of integrand for each region between two boundaries $=$ (NCOUNT $\div$ NREJE) $\times 4$
DNU
DXX(I)
Normalized Smallest Step $=$ DXX/DNU

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TABLE 1
EFFECT OF CONTINUUM BAND MODEL ON STAGNATION RADIATIVE FLUX (Calculation Performed Before Line Contributions are Incorporated)

| Model No. | No. of Bands | Spectral Range in eV |  |  |  |  |  |  |  |  |  |  | Wall Radiative Flux watts/cm ${ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} 1 \\ 0.5- \\ 1.0 \end{gathered}$ | $\begin{gathered} 2 \\ 1.0- \\ 2.0 \end{gathered}$ | $\begin{gathered} 3 \\ 2.0- \\ 3.0 \end{gathered}$ | $\begin{gathered} 4 \\ 3.0- \\ 4.0 \end{gathered}$ | $\begin{gathered} 5 \\ 4.0- \\ 6.0 \end{gathered}$ | $\begin{gathered} 6 \\ 6.0- \\ 9.5 \end{gathered}$ | $\begin{gathered} 7 \\ 9.5- \\ 10.95 \end{gathered}$ | $\begin{gathered} 8 \\ 10.95- \\ 12.15 \end{gathered}$ | $\begin{gathered} 9 \\ 12.15- \\ 13.61 \end{gathered}$ | $\begin{gathered} 10 \\ 13.61- \\ 14.5 \end{gathered}$ | $\begin{gathered} 11 \\ 14.5- \\ 30.0 \end{gathered}$ |  |
| 1 | 2 |  |  |  |  |  |  |  |  | - |  | $\rightarrow$ | 2,603 |
| 2 | 3 |  |  |  |  |  |  |  | - 2 | - |  |  | 2,563 |
| 3 | 4 |  |  |  |  |  |  |  | 2 | $3 \rightarrow$ | - | - | 2,475 |
| 4* | 5 |  |  |  |  |  |  |  | - 2 | $3 \rightarrow$ | $-4=$ | 5 | 2,433 |
| 5 | 4 |  | $\rightarrow$ |  |  | $2$ |  | $\rightarrow$ | $-3$ |  |  |  | 2,563 |
| 6 | 9 |  | $\rightarrow 1$ |  |  | $3-1$ | $141$ | 5 | $-6=$ | $7 \rightarrow$ | $1-7$ | $-9$ | 2,414 |
| 7 | 9 | $1=$ | $2=$ | $3$ | $4=$ |  | $5$ |  |  | $7 \rightarrow$ | $-8$ | $9$ | 2,434 |
| 8 | 5 | — |  |  |  |  | $3-$ | $\rightarrow+$ | $-4=$ |  |  | $\longrightarrow$ | 2,560 |

*Band model 4 is adopted for "nominal calculation."
TABLE 2

| $\mathrm{R}_{\mathrm{N}}$ | $\mathbf{u}_{\infty}$ | $\rho_{\infty} / \rho_{\text {SL }}$ | $\mathrm{q}_{s}$ | $q_{s, a b}$ | $\mathrm{q}_{\mathrm{w}}$ | $q_{\text {w, ab }}$ | $\sigma \mathrm{T}_{\mathrm{w}}^{4}$ | $\left(\frac{\rho_{w} \mathrm{~V}_{w}}{\rho_{\infty} \mathrm{u}_{\infty}}\right)$ | $\left(\frac{\rho_{w} V_{w}}{\left.\rho_{\infty}{ }^{u}\right)_{\infty}}\right)_{\text {ab }}$ | $\Delta_{\text {a }}$ | $\Delta_{\text {ab }}$ | $\mathrm{p}_{s}$ | $\mathrm{T}_{\mathrm{s}}$ | $\begin{gathered} \text { Precu } \\ \% \end{gathered}$ | sor Heatin $\text { of } q_{s} \text { (or } q_{s}$ | Effect ab) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cm | km/sec |  | $\rightarrow$ watts $/ \mathrm{cm}^{2} \longrightarrow$ |  |  |  |  |  |  | cm | cm | atm | eV | $\nu>14.5$ | $\nu>12.15$ | $\nu>10.95$ |
| 30.48 | 12.20 | $1.66(-4)$ | 1,488 | $\begin{aligned} & 3,529 \\ & 6,449 \end{aligned}$ | 385 | $\begin{aligned} & 1,324 \\ & 1,626 \end{aligned}$ | 1,127 | 0 | 0 | 1.295 | 0 | 0.282 | 1.045 | 3.0 | 12.0 | 17.2 |
|  | 15.24 |  | 3,452 |  | 1,806 |  | 1,236 | 0.103 | 0.017 | 1.142 | 0.082 | 0.447 | 1.245 | 8.8 | 29.6 | 42.1 |
|  | 17.38 | $\begin{aligned} & 1.66(-4) \\ & 1.66(-4) \end{aligned}$ | 6,070 |  | 3,513 |  | 1,304 | 0.352 | 0.054 | 1.048 | 0.218 | 0.575 | 1.416 | 11.7 | 37.7 | 53.0 |
|  | 15.24 | 3.26(-5) | 1,537 |  | 170 |  | 1,435* | 0 | 0 | 1.130 | 0 | 0.087 | 1.122 | 1.7 | 5.7 | 7.7 |
|  | 15.24 | 5. 94(-4) | 14,227 | 14,278 | 8,086 | 3,602 | 1,606 | 0.336 | 0.102 | 1.155 | 0.327 | 1.573 | 1.352 | 9.2 | 31.3 | 47.8 |
| 122 | 15.24 | 1.66(-4) | 5,360 | 5,565 | 2,591 | 1,479 | 1,236 | 0.245 | 0.046 | 4.250 | 0.671 | 0.447 | 1.245 | 7.8 | 29.0 | 44.0 |
| 256 | 12.20 | 3.26(-5) | 1,404 |  | 106 |  | 1, 361* | 0 | 0 | 9.910 | 0 | 0.056 | 0.955 | 0.8 | 3.8 | 5.2 |
|  | 12.20 | 5.94(-4) | 6,375 | 6,625 | 3,482 | 2,396 | 1,458 | 0.130 | 0.062 | 10.450 | 1.338 | 1.006 | 1.120 | 2.0 | 10.4 | 21.5 |
|  | 15.24 | 1.66(-4) | 6,518 | 6,569 | 3, 030 | 1,642 | 1,236 | 0.324 | 0.076 | 8.501 | 1.999 | 0.447 | 1. 24.5 | 6.8 | 26.4 | 41.8 |
|  | 17.38 | $3.26(-5)$ | 2,288 |  | 921 |  | 928 | 0 | 0 | 7.447 | 0 | 0.113 | 1.274 | 10.1 | 31.4 | 41.8 |
|  | 17.38 | 5.94(-4) | 50,650 | 52,919 | 22, 826 | 6,120 | 1,696 | 0.966 | 0.201 | 7.247 | 6.083 | 2.048 | 1.545 | 7.3 | 22.8 | 36.9 |

*Value too high due to pressure below input ablation property table pressure values and no extrapolation allowed.
$q_{s}, q_{w}$ radiative flux toward shock and wall, respectively, without ablation layer effect.
$q_{s, a b}, q_{w, a b}$ with ablation layer effect. $\quad \mathrm{T} 1 \mathrm{eV}=11605.7^{\circ} \mathrm{K}$
Carbon-phenolic surface, 6 air sublayers +6 ablation sublayers.

| Property y | $\mathrm{H}_{\text {lower }}$ | $\mathrm{H}_{\text {upper }}$ | $\mathbf{P}$ (atm) | ${ }^{2} 0$ | 21 | ${ }_{2}$ | 3 | ${ }_{4}$ | 3 | ${ }^{2} 6$ | Accuracy <br> (b) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T, ev | 0 | 0.0785 | .01-10 | $T=(0.264+2.642 \mathrm{x})^{1 / 2}-0.5138$ |  |  |  |  |  |  |  |
|  | 0.0785 | 1.50 | 0.01 | 1.1835678-1 | 6.5812617-1 | 9.3439634-1 | -3. 053483 | 2.4147548 | -5.8669396-1 |  | 6.4 |
|  |  |  | 0.1 | $1.0487607-1$ | 9.5327098-1 | 3. 2467991-3 | -1.5121987 | 1.3027109 | -3. 0545366-1 |  | 6.4 |
|  |  |  | 1.0 | 9.6979514-2 | 1.1433893 | -3.5657184-1 | -1.0256385 | 1. 0029850 | -2.4502322-1 |  | 6.2 |
|  |  |  | 10.0 | 1.1613734-1 | 1.0843619 | 1.1034320-1 | -1.6526663 | 1.3729724 | -3.3199664-1 |  | 3.9 |
|  | 1.50 | 5.50 | 0.01 | -1.5819349 | 3.3455871 | -1.8624081 | 5. $2266841-1$ | -7.2395315-2 | 3. 9710259-3 |  | 1.0 |
|  |  |  | 0.1 | -1.9378666 | 3.8547943 | -2. 0904657 | 5.7519457-1 | -7.8194046-2 | 4. $2031910-3$ |  | 1.0 |
|  |  |  | 1.0 | -1.8226109 | 3.4993767 | -1. 7229472 | 4.3881665-1 | -5. 5808967-2 | 2.8371443-3 |  | 1.0 |
|  |  |  | 10.0 | -1. 2891001 | 2.5154559 | -1.0005790 | 2.1112489-1 | -2. 2369219-2 | 9.6330532-4 |  | 1.3 |
|  | 5.50 | 11.0 | 0.01 | 1.9205406 | -5.1167326 | 2. 2507384 | -3.7221565-1 | 2.7302427-2 | -7.470516-4 |  | 2.2 |
|  | 5.50 | 9.9 | 0.10 | $1.9295706+2$ | -1.3236946+2 | $3.5826685+1$ | -4. 7588249 | 3.1126795-1 | -8.0365421-3 |  | 1.0 |
|  | 5.50 | 7.43 | 1.0 | 2.8774962+1 | -1.3395786+1 | 2.1445183 | -1.1088207-1 |  |  |  | 1.0 |
|  | 5.50 | 6.78 | 10.0 | -7.3068604-2 | 9.3749066-1 | -1.9204991-1 | 1.5081737-2 |  |  |  | 1.0 |
| T (alternate) | 0.0361 | 0.36 | 0.01 | -9.5098987-3 | 2.7438727 | $7.6381683-1$ | -1.2213621+2 | 5.5486666+2 | -7.1270856+2 |  | 1.3 |
|  |  |  | 0.10 | 3.5647251-2 | 2.4819323-1 | $4.7476579+1$ | -5.0754271+2 | 2.1990801+3 | -4.2729834+3 | 3.0866908+3 | 1.0 |
|  |  |  | 1.0 | -1.9368604-3 | 2.3758528 | 3.8650062 | -9.1807305+1 | 3.2603521+2 | -3.4814242+2 |  | 1.3 |
|  |  |  | 10.0 | 1.7905563-2 | 1.15133458 | .1.4744230+1 | -1.3762337+2 | $4.0607364+2$ | -3.9827684+2 |  | 1.0 |
|  | 0.36 | 1.50 | 0.01 | -9.1095707-1 | 1.0041215+1 | -3.1187126+1 | $5.0529627+1$ | -4.4270302+1 | $1.9837832+1$ | -3. 5439965 | 1.0 |
|  |  |  | 0.10 | -1.1822706-1 | 2.8525348 | -5. 5224045 | 5.6209047 | -2.9035087 | 6.1978861-1 |  | 1.0 |
|  |  |  | 1.0 | 6.5851264-2 | 1.5634838 | -1.7494294 | 8.5083235-1 | -1.1486050-1 |  |  | 1.0 |
|  |  |  | 10.0 | -9.2105463-2 | 2.4273938 | -3.0304437 | 1.7403332 | -3.4802097-1 |  |  | 1.0 |
|  | 5.50 | 11.0 | 0.01 | 3.5245559+2 | -2.8068595+2 | $9.1419875+1$ | -1.5572593+1 | 1.4670453 | -7. 2599443-2 | 1.4764338-3 | 1.4 |
| PV/V SL $^{(a t m)}$ | 0 | 0.0785 | .01-10 |  | $\mathrm{PV} / \mathrm{v}^{\text {s}}$ | = 40.27 | < 40.27 [ 0.264 | + $2.642 x)^{1 / 2}-$ | .5138] |  |  |
|  | 0.0785 | 1.50 | 0.01 | 4.1466486 | 3.1607714+1 | $7.0980384+1$ | -1.7031562+2 | $1.2768093+2$ | -2.9994045+1 |  | 5.7 |
|  |  |  | 0.10 | 3.4124600 | 4.5715653+1 | 2. $2226337+1$ | -8.3560087+1 | $6.3467061+1$ | -1.3608968+1 |  | 5.4 |
|  |  |  | 1.0 | 3.1948184 | 5.0919529+1 | $1.6382194+1$ | -7.3823582+1 | $5.7745495+1$ | -1.2956107+1 |  | 4.0 |
|  |  |  | 10.0 | 3.7750871 | $4.8311173+1$ | $3.6479840+1$ | $-9.8644062+1$ | $7.2579584+1$ | -1.6722375+1 |  | 2.6 |
|  | 1.50 | 5.50 | 0.01 | -1.2724814+2 | $2.5536017+2$ | -1.3807613+2 | $4.0678168+1$ | -5.8880258 | 3.3852882-1 |  | 1.0 |
|  |  |  | 0.10 | -1.4284936+2 | $2.7319929+2$ | -1.4179943+2 | $4.0480512+1$ | -5.6722231 | 3.1539598-1 |  | 1.0 |
|  |  |  | 1.0 | -1.3549011+2 | $2.488272+2$ | -1.1659636+2 | $3.1199285+1$. | -4.1153845 | 2.173742-1 |  | 1.0 |
|  |  |  | 10.0 | -7. $3263048+1$ | 1.3473468+2 | -3.6334966+1 | 6.1591101 | -4.2335399-1 | 9.2513935-3 |  | 1.3 |
|  | 5.50 | 11.0 | 0.01 | -6.3540412+2 | -1.7310820+2 | 1.8382531+2 | -3.6014227+1 | 2.8527773 | -8.1544340-2 |  | 1.9 |
|  | 5.50 | 9.9 | 0.10 | $2.5708608+4$ | -1.7743649+4 | $4.8244025+3$ | -6.4277878+2 | $4.2158885+1$ | -1.0911416 |  | 1.0 |
|  | 5.50 | 7.43 | 1.0 | $3.6719114+3$ | -1.7467843+3 | $2.8518962+2$ | -1.4827085+1 |  |  |  | 1.0 |
|  | 5.50 | 6.78 | 10.0 | 1.1714926+2 | $1.7900484+1$ | -2.9186889 | 7.5179851-1 |  |  |  | 1.0 |
| $\mathrm{PV} / \mathrm{V}_{\mathrm{SL}} \text { (alternate) }$ | 0.0361 | 0.36 | 0.01 | -2.9150224-1 | $1.0816331+2$ | 1.8632507 | -4.0867975+3 | $1.9336595+4$ | -2.5392596+4 |  | 1.1 |
|  |  |  | 0.10 | -3.9293492-1 | 1.1212841+2 | -9.4952171+1 | -2.5456987+3 | 1. $2088268+4$ | -1.5053815+4 |  | 1.5 |
|  |  |  | 1.0 | -7.9702854-2 | 9.8552205+1 | 4.7099724+1 | -2.5586988+3 | 9.8867489+3 | -1.1004637+4 |  | 1.1 |
|  |  |  | 10.0 | 5.5855155-1 | 7.0821747+1 | 3. $9679145+2$ | -4.0339294+3 | 1. $2496950+4$ | -1.2649937+4 |  | 1.0 |
|  | 0.36 | 1.5 | 0.01 | 6.9132261 | $2.5195433+1$ | 5.9373260+1 | -1.3021310+2 | 9.3944820+1 | -2.0986612+1 |  | 1.6 |
|  |  |  | 0.10 | 1.3690118 | $7.0848172+1$ | -5.9618294+1 | 2.5006220+1 |  |  |  | 1.0 |
|  |  |  | 1.0 | 2.1078820 | 6.9536693+1 | -5.0323748+1 | 2. $0157349+1$ |  |  |  | 1.0 |
|  |  |  | 10.0 | -4.1878757 | 1.0303833+2 | -9.9332450+1 | 5.6323946+1 | -1.0105138+1 |  |  | 1.0 |
|  | 5.5 | 11.0 | 0.01 | $4.6079518+4$ | -3.6897522+4 | $1.2067170+4$ | -2.0617303+3 | $1.9472369+2$ | -9.6571314 | 1.9676061-1 | 1.3 |

$\mathrm{p}=\ldots$
$\rho$

| ${ }^{0}$ | 1 | 2 | 3 | $a_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $z=1.0$ |  |  |  |  |
| 0.90204210 | 1.3765963 | -1.5847662 | 1.5546703 | -0.51615931 |
| 0.89964231 | 1.3061862 | -1.4626525 | 1.4298675 | -0.46987220 |
| 0.90237909 | 1.1686618 | -1.1587565 | 1.1151654 | -0.36066937 |
| 0.88730931 | 1.1296640 | -1.0581270 | 0.95701218 | -0.29414222 |
| 1.2736413 | 0.50509824 | -. 54156490-2 |  |  |
| 1.3301822 | 0.44379041 | 0.97737477-3 |  |  |
| 1.3685294 | 0.39338470 | 0.48567735-2 |  |  |
| 1.4055444 | 0.33971451 | 0.82709458-2 |  |  |
| 2.8602478 | 0.16921459 | . 25806779-2 |  |  |
| 2.8614600 | 0.15494900 | . $32019459-2$ |  |  |
| -. 13604584 | 1.0478795 | -. 6454679-1 |  |  |
| -. 64086440 | 1.1224999 | -.66709173-1 |  |  |

 $H_{\text {upper }}$
0.0785
1.50

5.50

11.0
9.9
7.43
6.78


[^10]
## TABLE B-1

SPECTRAL RANGES AND AVERAGE CROSS SECTIONS FOR 70/30 CARBON-PHENOLIC "21 BAND" MODEL


TABLE B-2
ABLATION SPECIES BAND SYSTEM f NUMBERS

| System | f-Number |
| :--- | :--- |
| $\mathrm{C}_{2}$ Swan | 0.0059 |
| $\mathrm{C}_{2}$ Fox-Herzberg | 0.02 |
| $\mathrm{C}_{2}$ Deslandres- |  |
| $\mathrm{D}^{\prime}$ Azambuja |  |
| $\mathrm{C}_{2}$ Freymark | 0.006 |
| $\mathrm{C}_{2}$ Mulliken | 0.002 |
| $\mathrm{CO} 4+$ | 0.02 |

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Fig. 1 Geometry of Inviscid Flow Field


Fig. 2 Geometry for a Finite Number of Sublayers



Fig. 4 Qualitative Variation of $R^{*}$ and $R$


Fig. 5 Effects of Exponential Kernal Approximation


Fig. 6 Effects of Air Continuum Band Models


Fig. 7 Effects of Line-Calculation Options


Fig. 8 Effects of Ablation Layer


Fig. 9 Temperature and Tangential Velocity Distribution in Physical Normal Distance


Fig. 10 Tangential Velocity Distribution in Transformed Mass Normal Distance


Fig. 11 Particle Density Distributions


Fig. 12 Continuum Optical Thicknesses


Fig. 13 Spectral Heat Flux to Wall and Shock


Fig. 14 Effects of Number of Sublayers


Fig. 15 Effects of Nose Radius, Constant Velocity and Altitude


Fig. 16 Effects of Velocity, Constant Nose Radius


Fig. 17. Effects of Nose Radius, Constant Altitude


Fig. 18 Effects of Nose Radius, Constant Velocity


Fig. 19 Effects of Perturbation of Radiative Properties


Fig. 20 Effects on Ablation Layer Enthalpy Distribution and Heat Flux to Wall by Perturbing Ablation Molecular Species f-numbers


Fig. 21 Equivalent Widths for Isothermal Layers of Different Thickness


Fig. 22 Blunt Vehicle Configuration for STRADS Calculations


Fig. 23 Effect of Velocity on Radiation Heating Distribution, Blunt Vehicle

## C. 1 Listing of STAGRADS and Input Data

TEM
$\underset{~ Z}{Z}$
suniy vale, calif
0.0
100
1010




[^11]

$\begin{array}{ll}0 & -1 \\ 0 & 0 \\ -1 & -1 \\ 0 & 0\end{array}$
5
0
0
9102 FORMAT(IIgI 1X,3E12•2)


迹安妾 トャトロート $\stackrel{5}{\square}$
$\underset{5}{2}$



7913
7901

$=$ = E12.5.



## MP206

[^12]DO $203 \quad I=2 \boldsymbol{N} 5$
$Y E R=Y E R+D Y E R / G H(I-1)$
SIT(I) $=$ SIT $(I-1)+L Y Y$
$099) 204 \cdot 203 \cdot 203$
\[

$$
\begin{aligned}
& 115 \\
& 05
\end{aligned}
$$
\]

GU(I) $=(1.0+$ GAMA $5 *((1.0-Y E R) * *(-0.8)-1.0)) *(-0.25)$
$=(\operatorname{GH}(I)+G \operatorname{Hi}(I+1)) / 2 \cdot 0$.
NUE

$$
\begin{aligned}
& +1)=2 \cdot 0 \% 3 I T(N S) \\
& +1 .=V I S(N 3) \\
& E 0 .=0.3) 60 \text { TO } 104 \\
& (6,901 \mathrm{~J}) \\
& (6.9012) \\
& \mathrm{J}=1 . \mathrm{KN}
\end{aligned}
$$


$N$
$N$

$$
203
$$

$\pm 10$
0
No
0
$\wedge$
$\therefore 0$
$\therefore \sim$
100

## 

 1FIUSG:FこいUN

CALL VEL (I,SIT,VIS,-1.O,DYY2,AE, CLI,N, ANS, DUVI,DUM,DY,FVA,FUA:

$$
\begin{aligned}
& \text { COITIUE } \\
& \text { URITE } G \text { ' } \\
& \text { NAS } \\
& \text { CALL VEL } \\
& \text { IUYER2) }
\end{aligned}
$$


Y
$(N+1)$ D $I F(N T \cdot G T O 2 O) C A L L E X I T$
$N T P I=N T+I$
$D O 3 O 2 I=1, N$
$H(I)=G H(I) * H O$
$C A L L$ PROPT(PO,H(I),V(I):T(I),Z(I),NOI)
$V(I)=V(I) / P O$
$C A L L ~ D O U B L(A L O G I O(V(I)), V T A B O, T(I), T T A B O, X N, S P T A B O, I, 3, I 4,3)$忈 DO 207 I

$$
\begin{aligned}
& I F(N A \cdot G T \circ I \cdot A N D \circ I \cdot E Q \cdot 1) N A=2 \\
& \text { IF }(N A \cdot G T \circ I \cdot A N D \cdot I \cdot G T \cdot 1) N A=3
\end{aligned}
$$

$$
I F(\because A A \circ G T \bullet I \cdot A N D \circ I \cdot E Q \cdot I) N A A=2
$$

$$
\text { IF (NAAOGTOI AND.I.GTOI INAA }=
$$

$$
\begin{aligned}
& I F(N A A \cdot G T \cdot I \cdot A N D \circ I \cdot E Q \cdot I) N A A=2 \\
& I F(N A A \circ G T \odot I \cdot A N D \cdot I \cdot G T \odot I) N A A=3
\end{aligned}
$$

$$
\text { GO TO }(910: 910,911), \text { LINEO }
$$

## 912

 $G O T O \quad 912$CALL CRLIHE(CQ)
OGH=SIG\%CW/FVA(I)
$G H C(I+I)=G H C(I)+U G H$
$G H A(I)=(G H C(I)+G H C(I+I)) / 2 \bullet 0$
$I F(A B S(G H A(I)=G H(I)) / G H(I)-D E L H) 205,305,304$
$A Q=1$

$$
\begin{aligned}
& I F(G H A(I) \cdot G T \cdot I \cdot 2) G H A(I)=I \cdot 2 \\
& I F(G H A(I) * H O \cdot L T \cdot 0 \cdot 3) G H A(I)=0 \cdot 3 / H O \\
& I F(N C O U T T-1) 305 I \cdot 3053,3053 \\
& G A Z(I)=G H(I) \\
& G C 2(I)=G H A(I) \\
& G A I(I)=G A Z(I)+G C 2(I)
\end{aligned}
$$

$H(I)=(G H A(I)+G H(I)) / 2.0$
$N$
$N$
$N$
N
O
$\stackrel{\underset{\sim}{\mathrm{O}}}{\stackrel{-}{+}}$
$H N$
$H \mathrm{~N}$
H C
304
$n$
n
n
in

| $\sim$ |
| :--- |
| 0 |
| 0 |

$G A I(I)=G A 2(I)$
$G C I(I)=G C 2(I)$

Gi2(I)=GH(I)

IF(LINEO.EQ.I)QWT=QWTC
IF (LINEO.EQ.I)QSHOK=QWST
IF(LINEO.EQ.I)GO TO 331
CALL CRLINE(QUTL)
QUT=QUTC-QWTL
QSHOK=QWST-QUTS

931
331
7030
$m$
$m$
1034
I二(SHCHC)GOTO
NCNTR2=RCITR3-1
IF (VRITETOLEOTGOTOTO31
0
$1-$
0
2
2
2
2

$V C=F V A(I) \cdots V(T) / V O$

$Y C=(Y(I)+D Y(I) / 2 \cdot O) / D E L T A$
$U C=-F Y A(I)$
$V C=F V A(I) \neq V(I) / V O$
$H C=G H(I)$


$$
00735 \quad I=1 \sin
$$

$$
\mathrm{DO} 734 \quad \mathrm{~J}=\mathrm{I} \cdot 3
$$

$n$
$m$
$m$
IF (NSAB.GTOO)TS=T(N)
$\because R I T E(6,9021)$
DO $735 I=1, N$
$D O 734 J=1.3$
$\operatorname{PART}(J)=10.0 * *(-X N(I, J)) \div 5.07 E+19 / V(I)$
IF (NSAB.GTOO)TS=T(N)
$\because R I T E(6,9021)$
DO $735 I=1, N$
$D O 734 J=1.3$
$\operatorname{PART}(J)=10.0 * *(-X N(I, J)) \div 5.07 E+19 / V(I)$
$734 \operatorname{PART}(J)=10.0 * \%(-X N(I, J)) \div 5 \cdot 07 E+19 / V(I)$


$00750 \mathrm{~J}=1 \mathrm{ok}$ SQ: (J) = OW(J)/ SOHS $J$ ) $=0: 1 / 5(J) / \operatorname{DNU}(J)$ $\xrightarrow[3]{2}$
$R$
$R$
$U C=-F \cup A(I)$

$$
\text { RITE( } 6,7907) I, Y C, \cup C, T(I), Z E T A
$$

7041

$$
\text { GO TO } 7045
$$

$$
\begin{aligned}
& G O T O 7045 \\
& \mathrm{IAB}=\mathrm{NTPI-I}
\end{aligned}
$$

DZETA $=\operatorname{DYAB}(I A B) /(V I W(I A B+1) * V W O D)$
ZETA=ZETA + DZETA




AEYZ＝AEE＊EHD
AEYZ＝AEEFEND
IF（NGUTOEQ．U）GO TO 206
NGUT＝O
GO TO 220
DO 209 I＝ISNS
SIT $I+I)=S I T(I)+D S I$
CONTINUE
$N=N S$
IT $(N+2)=S I T(N+1)+D S I / 2 \cdot 0$ $S I T(N+3)=2 \cdot 0 * S I T(N+2)$
$\stackrel{8}{\circ}$
0
0
$\sim$
$\sim$
O
N
in＝int 2
いu 10
$F(: 1 H-1) 300,800,221$
（FLOAT（I）－0．5）／FLOAT（N）） $V(I)=G H(I) * * I .50 \% V \%$
$V I!(I+I)=V(I) / V W$
N
+
$\pm$
$\vdots$
$\vdots$
$V \operatorname{In}(N+3)=V \operatorname{In}(N+2)$
1F（Mn•OT．0） 0 （
IF（NRITEI。GT•O）WRITE（6，3）BL $\because H=: H+1$
$S U R=0.0$
CALL VEL $2, S I T, V I W, 0.0, D Z E T A, 2.0, C L 1, N, A N S, F U E, A E Y Z, D Y, F V A, F U A$, 1DU日）

$\begin{array}{cc}\text { द8月 } & 0 \\ 0 & 0\end{array}$
N
$\sim$
$\stackrel{N}{N}$




00
0
5
00
$y+4$

$Q U T I=(Q U T A 2+Q: T C Z) / 2 \cdot 0$
$\begin{array}{ll}N & N \\ 0 & \boxed{O} \\ \forall & \ddots\end{array}$
こV1:OO=TV1•O $\because 9+$

CONTINUE
NCOTH $=0$
$\begin{array}{cccccc}\text { in } & 0 & 0 & 0 & H & \rightarrow \\ 0 & 0 & 1 & 0 & 0 & N \\ + & j & + & 0 & 0 & +\end{array}$

$\because \times 1=(G T C$
IF ( $Q$ NTCI-QWTAI) $\because(Q W T C 2-Q \omega T A 2)) 460,462,462$

IF (NRITEI•LE O)GO TO 471
$\because R I T E(O, 9 O S U) N Q U T, N C O N H: N C O U N T ~$
NCOUNT =, 1.10 )
$=, E 18 \cdot 5 \cdot 8 H$ QWTC2 2 (
EルH=Co* (QWTI-1.03E+5*TW**4) (1.386-1.13*TW)

UR=-SQRT(2.O*ESI*RHOA*VH)*FUE
URMAS =EGD*RHOA*RB*FUE/(CLI*SUM)
IF(NRITET.LEOOIRETURN
$E A D=E M D / F D H A O$
$A E Y Z=A E E * E M D$
$G O T O 220$
$V C H=E G D * R H O A * V W$
$R M O M=E M D H V C W$

$$
6,90131
$$

6.9014)
$I=1 \cdot N$
V1730/10.2/1I) 1010


$H C=E H(I)$
$H(I)=H C * H W$
$n$
$n$
$n$
$\square$
(i)
(1)


| Nmon | $\rightarrow$ | ᄂ 0 | ホ | $\pm 0$ |
| :---: | :---: | :---: | :---: | :---: |
| Tr- | $\cdots$ |  | - | (1) $m$ |
| 0009 | 9 | 90 | 0 | 00 |
| $\geq \geq \geq 2$ | $\geq$ | z < | z | $\geq$ z |

IF $(S \because C H C) F U A(I)=U C W U R M A S$


VIT FOK HOGuSP

COMiON/BIGGCR/DY(10),T(10),V(10), XN(10,4):TWONONPI,I
SUBROUTINE NGGBSP(CQ)




| 0 |
| :--- |
|  |
|  |

$N m$
00
00
20


[^13]
COMON/RCRLA/FHVP(24), FHVB(24), FHVP:(24)sFHV(24), ISOE(24),NU(24), iN


$E 3 F(A)=0.50 \% E \times P F(-200 \% A)$
SIF (FRE, TT) =FRE**3/(EXPF(FRE/TT)=1.0)*0.153990 IF (NCQAB.LE.0)60 TO 1001
GO TO (1002,1006,60), NCQAB 1001 CCNTINUE

## GO TO $(2,20,51,60)$, NAA

 GO TO $(3,4,5)$,LINEOP$$
N(i) J
$$

VUE

$$
\rightarrow
$$ FORMAT(17H (KKoKL1,KL2,FHV):31100E20.6) 10 CONTINUE $\therefore A A=i N A+1$

continue
 1(-2. $384 / T(J))+60 * \operatorname{EXP}(-3 \cdot 576 / T(J))))$

1002
20
1010 FFF(J)=0。
25 COMTNUE
CCC*FHVP:I(KK)

EN: STORE (K, KK)

-JPI $=T E_{M}$


### 11102.1105 .1105


14
2
$0045 \mathrm{~J}=2$, NT
$043 \quad I J=1, \because T$
$F F=E I(I J G K K) *(E T A U 12(J, I J+1)-E T A U 12(J, I J)+E T A U 12(J P I, I J)-E T A U 12($

$$
\text { 玉jPI I I } \downarrow 111
$$

FFF $(J)=F F F(J)+F F$
FFF $(J)=F F F(J)+U I W(K K) * G(J, K K)$
CONTINUE

$$
\begin{aligned}
& \text { GUS(KK)=ETAU12(NTP1,I) } \\
& 0 \text { CONTINUE }
\end{aligned}
$$

> $I F(N C Q A B \cdot G T \cdot O) N C Q A B=N C Q A E+1$
IF (NCQABOGT•O)GO TO 1006
IF (NCWABOLE.O) GO TO 51
$I=N T P I-I A B$
$C_{G}=F F F(I)$
1005
1202
1105
1100
1930
1042
1042

45


075 NM=
$\therefore 0(K K)=0$
$\frac{0}{11}$





| $\begin{aligned} & \infty \\ & \underset{\sim}{m} \\ & + \\ & \vdots \\ & \mathbf{~} \\ & \underset{\sim}{3} \end{aligned}$ |
| :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



$$
1 I
$$

GAMi I $=$ GAMDA（K）＊PARTE（I）
IF（NUMINT．EQ。O）GO TO 21

 GAㄹ（I，K（i）$=$ GAM（I）$\% * 2$
1F（NELOMO20100

IF（NUMINT。GT。O）SGAVP（I．KM）＝SGAM（I，KM） 20.31830989
 TAULT（IPI）＝TAUDT（I）＋TAUD（I）


 a IF（KA•GT•IのAND。ISOE。GT•O）GOTO 22
DYGI（I）＝DYNOI（I）KEXP（－EPS（JD）／T（I））
GG（I）＝DYGI（I）＊OGF（K） 527 ＊GAM（I））
[P1)
$N$
$N$
6イガ（1）S
โT•Xて＇（1）

## anNILMOD IO8





[^14]
$X 13=0.33333333 * 0 X$
$\times 23=0.66666657 * 0 X$

$O F X$
1405 NG END, ADJUST
$-X A$,

$3 * 0 x$ 10 m
$m$
$m$
$m$
$0 \times$




XU
VE

$$
0.25
$$

$$
\begin{aligned}
& 0 m \\
& 0 \\
& 0 \\
& 0
\end{aligned} \mathrm{~m}+\mathrm{m}++
$$

UA



$$
11111 \frac{x}{S}
$$ ,NL, NLP 1,K1,K2,HVL,GAM2,SGAMP $\frac{1}{0}$

$$
D x
$$ §。

$$
\begin{array}{ccccc}
n & 11 \\
n & n & 11 & 1 & 11
\end{array} 11
$$ F (NCOUNTOET

$D 0700 I=I$ ONL
$I P I=I+I$

$$
\begin{aligned}
& \begin{array}{l}
\text { NTRND (X2, T2 NL, NLPI,KI,K2,HVL,GAM2, SGAMP) } \\
\text { NTRND }(X 3, T 3, N L, N L P I, K 1, K 2, H V L, G A M 2, S G A M P)
\end{array} \\
& \text { a }
\end{aligned}
$$

404
$C \quad$
405
901

## 

## 

300

$00000 \mathrm{~J}=1: 1$



$\% \underset{\sim}{\sim}$
$\stackrel{\sim}{\circ}$
anyovan
any
IF (ANS (I +1, J).GT•XULA)ANS $(I+1, J)=X U L A * 0.99999990$
CONTINUE GO TO
TA(IPl, J) $=$ TB(IP1, J)
GO TO 401

| $\infty$ |
| :--- |
| $\stackrel{\circ}{+} \quad \circ$ |

$\because$
0
$E R R=E 12.5$, STEP $=, E 12.51$ $\stackrel{a}{4}$ in

FORFAT ( 8 H NREJE = I I 12)

101
102 FORHAT(20H DXX(I) VALUES BELOW)
102 FORMAT(12E11.3)
RETURN
END
 1PTIJ(10), HVLNU(12) $\because 21=k i-1$
$0020 K=$
$K \because=K-K I I$
HVLNU(KM) $=(X X-H V L(K)) *$ 关2
-
HXR=HX/XULA
$H X R=H X / X U L A$
URITE(G,IOO)NCOUNT,HX,

DO $408 \quad I=1$,NL
WRITE $(6,101)$ NREJE
IF (NCOUNT.GT. 50 )NC
IF (NRITE4-LE•OIRETURN
URITE 6,100$)$ NCOUNT,HX,ERR, XULA,HXR
WRITE 6,101$)$ NREJE
IF (NCOUNT•GT•50) NCOUNT $=50$
$\because R I T E(6,102)$
$\because R I T E(6,103)(0 \times X(I), I=1, N C O$


- TIKJ=SGAMP(I,Kin)/(HVLNU(KHi)+GAM2(I)Kivi)
VIT
$3 C T I J(I)=T I J(I)+T i K J$

| $30$ | TIJ(I)=TIJ(I)+TIKJ |
| :---: | :---: |
|  | $E \times P T I J(I)=E \times P F(-T I J(I))$ |
|  | $[P]=T \div 1$ |
|  | Friov (TPI*IPI) =1。 |
|  | $0040 \mathrm{~L}=1.1$ |
|  | $I J=I P I-J$ |
| 40 |  |
|  | i) $45 \quad \downarrow=1, I$ |
| 45 | PRODK (IPI, J)=I.O-PRODK(IPI, J) |
| 5 | CONTINUE |
|  | RETURIN |
|  | END |

## VITFOR $\mathcal{F O U N L}$

©I FUBROUTIAE DOUBL (X,XT,Y,YT,Z,ZT,IL,NX, NV,NZ)





| $F 4(J)=E 4(I, J)$ |  |
| :---: | :---: |
|  |  |
|  |  |
| 1 1) $=$ P1 (T, |  |
| $C 2(J)=\mathrm{C} 2(I) J)$ |  |
|  |  |
| $C<(J)=G 4(I) J)$ |  |
| $C E(J)=E 5(I, J)$ |  |
| $D 0(J)=A 0(I) J)$ |  |
| $D I(J)=A 1(I, J)$ |  |
| $\bigcirc 2(J)=A 2(I: J)$ |  |
| b3 (J) $=$ A $3(I, J)$ |  |
| $34(J)=A 4$ (I.J) |  |
| $5(J)=A 5(I, ~ J) ~$ |  |
|  |  |
|  |  |



- NiN





$$
\begin{aligned}
& I F(K \cdot G I \cdot I) G O \text { TO } 20 \\
& I=I \\
& I F(T C(I \cdot I)-T) I, 2: 3 \\
& I=I+I \\
& I F(I \cdot E Q \cdot I 4) G O T O 8 \\
& G O T O 4 \\
& R=I \cdot O
\end{aligned}
$$

|  | GOTO 20 |
| :---: | :---: |
| 3 | $I F(I \circ E Q \circ 1) I=2$ |
| 8 | $R=(T-T C(I-1,1)) /(T C(I, 1)=T C(I-1,1))$ |
| 20 | 5 Su＝0．0 |
|  | RTEG（IL）$=$ R |
|  | IRTEN（IL）＝I |
|  | DO $30 \mathrm{~N}=1.2$ |
|  | $J A=I \therefore T(N 力 K)$ |
|  | SIGA＝R关（TC（I，JA）－TC（I－I，JA））＋TC（I－IのJA） |
|  |  |
|  | $A E=10 \cdot 0 * * A B$ |
|  |  |
| 30 | SiUU $=$ SHU 4 AB |
|  | SPMU＝SMU／V |
|  | RETURiN |
|  | END |




| N0N | Nintino | $r \infty$ | OnNMす | ¢ |
| :---: | :---: | :---: | :---: | :---: |
| 000 | $\bigcirc 0 \bigcirc 0$ | 00 | Frimern |  |
| $N \sim \sim$ | $N N N N$ | ONN | $N N N N$ | N |
| \& mu |  |  |  |  |
| $\theta 64$ | 67696960 | 6) ts 6 | 6769606 | 47 |

SUMUC=SPMS

$\stackrel{-}{C}$

VIT FOR PROT $\quad$| SUZKOUTINE PRUTIXX,XT,YY,YT,ZP,ZT,AA,AT,BE,BT,CC,CT,DD,DT,EE,ET,F |
| :--- | FT NL M



$Z=A L O G 1 C(Z P)$
IF (ZZ-ZT(1).LT.O.O.AND.XX-XT(1.1).GT.O.O)GOTO 902
GOTO 906
IF (ZZ-ZT(1).LT.O.O.AND.XX-XT(1.1).GT.O.O)GOTO 902
GOTO 906
$=(X X-X T(1,1)) 802,803,803$
II -10
60
00
CEC32

NNHNN NHN
（）约 4

Qn
$N N$
$6) 6$

Nmさに NNNN NNNN




 －j」 j」 j」

 $\hat{Y} T U R$
$Y Y=$ Y Y
$A A=$
IF (XX-XT(L,J)) 911,912,913
912 COTINUE
तill
1
906
 $\quad \begin{aligned} & i=J \\ & i=J-1\end{aligned}$ $L L_{i=J-1}$
$I F(X X-X T(L L, J)) 2001,2002,2000$空 $\left.\begin{array}{l}\text { GO TO } 906 \\ Y \\ Y \\ =Y T(L L E L M\end{array}\right)+R A T I P *(C I-Y T(L L, L M))$ $Y Y=Y T(L L=L M)+R A T I P *(C I-Y T(L L, L G))$ $A A=A T(L L O M)+R A T I P *(C \angle-A T(L L O L M))$ $C C=C T(L L, L i n)+R A T I P *(C 4-C T(L L, L i j))$



$0 D \mathrm{H}$
Nmm
a. $\frac{1}{a} \frac{1}{a}$

|  |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |



TBLP4（ZETA（J），SIT，VIJ（J），VI，DLVioFV，DUF，FU，1） 10，20）：LAOB

F（J－20）6204，6205，6205

$$
\begin{aligned}
& \text { DFV }=F U(J) * D X \\
& \text { DFU= } \\
& \text { FU(J) }
\end{aligned}
$$

GO TO（10，20）：LAOB
UX＝DYERZ／VIJ（J）

$$
Z E T A(J+1)
$$

IF (J.GE.4O)CAL

$$
\begin{aligned}
& \text { IAT/ANS } \\
& 35 \cdot 301: L A O B
\end{aligned}
$$

$$
\begin{aligned}
& \text { AT/ANS } \\
& 5.30)=L A O B
\end{aligned}
$$

$$
j+1)
$$


0
$r$
$\begin{array}{ll}-1 & N \\ 0 & 8 \\ 0 & 0\end{array}$
8
0
0
$\begin{array}{cc}9 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0\end{array}$
亿
呙
相
0
0 m

$$
\angle \approx 0 S I
$$

EXIT

$$
\begin{aligned}
& \text { O) 6204,620 } \\
& \text { T(J) } 125 \cdot 0 \\
& \text { C*DZETA } \\
& \text { C*DYER2 }
\end{aligned}
$$

$$
\begin{aligned}
& I=1, N \\
& =1
\end{aligned}
$$

（2「「IN•
RETURN
END $\nabla I T F O R$ TBLP4

[^15]$\mathrm{B}=\mathrm{R} * \mathrm{BT}(I)+R R * B T(I-1)$
IF (JF。EQ.IIRETURN
$C=R * C T(I)+R R * C T(I-1)$
$\cdots$

## SUEROUTINE

DINENSION AT (20), BT (20):CT(20), DT(20)
SULROUT INE TBLP4(A,AT, B, BT,C,CT,D,DT, JF)
$I F(A T(I)-A) I, 2,3$
$=I \div 1$
GO TO 4

$\stackrel{1}{\square}$

$N N$

$m m m$



o


NTS

## FivUUU（I）＝FNUU（I） $0010 \mathrm{~J}=1,2$ <br> IKTTT（J，I）$=$ IKT（J，I） 20 continue <br> ETURN <br> VLX XGT MAIN


in
$\uparrow$

|  | N |  |  |
| :---: | :---: | :---: | :---: |
|  | N－900 | $\bigcirc 80 \mathrm{~m}$ | ONJ |
|  | $\bigcirc$ | $\bigcirc 0$ | $\pm$ \％ |
| 0 | 細号が品 | $\cdots \stackrel{\square}{\square} \stackrel{\square}{\sim}$ | － |

in


$$
\begin{aligned}
& 98 \circ 2 T \\
& 05^{\circ} I t
\end{aligned}
$$

$$
\begin{aligned}
& 002 \cdot c \\
& 050 \cdot \tau \\
& \hline 00007
\end{aligned}
$$

$$
\begin{aligned}
& 00 \cdot 02 \\
& G \tau \cdot Z T
\end{aligned}
$$

$$
002^{\circ} \mathrm{I}
$$

$$
0 S \circ g
$$

SヨMI7 2IHOLV とO」 SOUV）U1YO






columin numiber 0


$$
4.100
$$

$$
56^{\circ} 01
$$

$$
3.200
$$



$$
\begin{array}{ll}
.657 & 1.02 \\
.674 & 2.08
\end{array}
$$



















$$
\begin{aligned}
& \text { OQ心の }
\end{aligned}
$$

N
+1
+1 $\cdots$ $\cdots \infty$ $\therefore$ $+4$ 10 r
+4
+4 $\pm i$ N $\infty$ +
+

+ in
$+4$ $\dot{m} 0$ $+6$ $\stackrel{+}{6}$ +4
+1 $+1$ \& $0+6$


$$
\begin{aligned}
& 1900+03+5343+05+3569-04+2218+14+3356+10+7515+12+1661+18+1307+19+1660+19 \\
& 2000+05+5716+05+3262-04+1106+14+2216+18+2616+12+1070+18+1279+19+1657+19 \\
& 2200+05+6294+05+2327-04+3275+13+1023+18+1809+11+4411+17+1193+19+1592+19 \\
& 2500+05+7004+05+2395-04+3163+12+3701+17+8544+09+1298+17+1032+19+1458+19
\end{aligned}
$$

[^16]

## OUTPUT DATA LISTING

INCLUDING DETAILS OF
LINE DATA USED



| 12 | －60003＋05 | －دBどうー3t | ．71945－19 | －275－2－19 | ．19099－19 | ．42170－19 | ． $79616-19$ | －¢P445－17 | ．39537－17 | ．76208－17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | －くiouutuj | －67453－1t | ．12：34－10 | －45769－19 | －3nA32－19 | ．63241－19 | ． $90353-10$ | －26977－17 | ．37154－17 | ．6668I－17 |
| 14 | － $23945+05$ | ．1010＜゙ー1\％ | －177i3－10 | ．05917－19 | －43053－19 | －57794－19 | －76208－13 | － $24155-17$ | －32961－17 | $.56234-17$ |
| 1 | －3000u＋ct | －316c3－37 | －316ç－37 | －31623－37 | －40272－17 | －31623－37 | －31623－37 | －31623－77 | ．31623－37 |  |
| 2 | － $46000+6+$ | ． $14521-36$ | － $594 i v-31$ | ．1745，8－29 | －4 $41179-17$ | －82945－33 | －126．77－3： | －6035n－34 | －6f6？ |  |
| 3 |  | －363しか－2c | ． 7375 －28 | －0．35こ3－27 | －． $39094-17$ | －22961－26 | ．5741：－ |  |  |  |
| 4 5 | －OVOOL＋ 104 |  | －－2？${ }^{\text {a }}$ | － $110 \times 0-25$ | ． $3^{0191}$ |  |  |  |  |  |
| 5 | ． $76006+04$ | ．17539－24 | ＋Tヶこ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | ．615395－21 | ．36983－21 |
|  |  |  |  |  |  |  | － 1 | ． $73790-20$ | ．33497－20 | ． $19320-20$ |
|  |  |  |  |  |  | －u5v－2u | －21846－19 | －27990－19 | ．12050－19 | －66989－20 |
|  |  |  |  |  | －23121－19 | －12077－19 | ．26540－15 | ． $84918-19$ | ．34119－19 | －18323－19 |
|  |  |  | M，＋－10 | ．10375－19 | ． $48417-19$ | ．24546－29 | ．63090－18 | ．19498－18 | ．75162－19 | ．39537－19 |
|  |  | － $11140-17$ | － $2110=-15$ | －17ÊS0－18 | －6．2234－10 | －40365－17 | ．12303－17 | ．36983－1R | ．13836－1A | ．71450－19 |
| 14 | $.23795+05$ | ．25こ63－17 | ．73730－15 | －2fose－is |  | ．56494－19 | －20277－17 | －59429－18 | －21777－18 | －11117－18 |
| 1 | －うしつうutう4 | －1ヵ281－ら4 | ．0123j－01 | － $26558-07$ | －5021u－04 | ．13335－00 | －11298－06 | －18．365－c．3 | ．19143－00 | －24099－06 |
| $<$ | －400ustu． | ．20277－0 | ．13830́－0 1 | ． $37154-05$ | －64121－0？ | ．20512－00 | － 088 cos－05 | － $20184-n 1$ | －27893－00 | －12399－04 |
| 3 | －Suvoutist | ．33260－41 | ． $23404-\mathrm{Cu}$ | ．41733－04 | － $13040+00$ | － $2 \cos 3-00$ | ．73790－04 | －27470－n0 | －20989－00 | －12589－n3 |
| 4 | ． $06000+04$ | －1946－Cl | ． 205 こナ－2し | －20791－n3 | ．45709－00 | ．20941－00 | ． $30933-63$ | －70469－n0 | －21038－00 | － 23176003 |
| 5 | －7リワリレ＋ 1 － | ．50916－ut | ．23895－30 | ．758，3－03 | ． $72611-20$ | －En¢89－00 | ．200．15－02 | －773ロ9－ก0 | －27941－00 | －61660－02 |
| 0 | －0u0uutit | $\because 71121-0$（ | ． $239+1-30$ | －3n26y－02． | ．76736－20 | －20693－00 | －9397e－02 | ．75．454－rc | －27512－00 | －29309－01 |
| 7 | －Yu0jutor |  | －20341－30 | ．35701－0？ | ． $74817-01$ | －20701－0」 | －30339－81 | －63973－n0 | －19187－00 | －10471＋00 |
| 3 | －1100utuj |  | ． $2046+00$ | ．25119－01 | －72946－ju | ．19568－00 | －329a5－01 | －40＋31－nu | －19982－00 | －27479－00 |
| 9 | －1くらすい +05 | ．05234－cs | ．19320－ju | －1174．940n | － $51240-00$ | ．15922－00 | －32434－03 | －22＇439－n0 | ．34140－01 | ．69024－00 |
| 10 | ．1400u＋25 | －+8750 －心 | ．15171－05 | －359\％ 5 －0．1 | － $24401-00$ | －95940－01 | －659M8－0C | －68365－01 | －25002－01 | －8R920－00 |
| 11 | －16のJコt55 | ． 29942000 | .11137540 | ． 583384000 | － 4 （6900－ワ1 | －41976－01 | －B6ィ96－no | －1495？－n1 | ．59979－02 | ．97724－00 |
| 12 | －1 h00utíj | ． $10612-00$ | ．05013－01 | －762ワ3－00 | －25A23－01 | －14791－01 | ．96333－00 | －27353－n2 | －12190－02 | $\cdot 10000+01$ |
| 13 | －2じからい＋いう | $.05901-41$ | $.3597 j-92$ | － $87902-10$ | －10230－01 | －50933－02 | －98855－00 | －10100－02 | ．29854－03 | －10000＋01 |
| $1+$ | $.2397 y+65$ | ．19949－01 | ．75913－02 | －78175－90 | －20701－02 | － $44723-63$ | $.10814+01$ | －1636A－n3 | －94125－04 | －13996＋01 |


$\begin{array}{ll}\text { FCNiNI．} & = \\ \text { rijhas } & = \\ & .1 J 000+51 \\ \end{array}$

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| 18 18 | 11 11 | $10^{9}$ | $.34013 y 90-00$ $.22871784-00$ | -0141140ij-10 <br> $.26371534-00$ <br> .192512?4-00 | $\begin{array}{r} 18588132=00 \\ .23128456-00 \\ .30748770=00 \end{array}$ | $\begin{array}{r} .11777019+00 \\ .79424568-01 \end{array}$ $.36205591-01$ |
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| 19 | 3 | 2 | . $43258344-00$ | . 3268541 )3-00 | , 3P114606-nn | -19699087-00 |
| 19 | 4 | 1 | .70907:11-00 | .47373n ${ }^{\text {a }}$ |  |  |

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23 / 3 & I & I \\
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[^0]:    *In superorbital velocities, $\mathrm{p}_{\infty}$ and $\mathrm{h}_{\infty}$ are negligible compared to $\mathrm{p}_{\mathrm{S}}$ and $\mathrm{h}_{\mathrm{S}}$, respectively. Precursor heating may increase $h_{\infty}$. However, for the velocities of interest in this study, the precursor heating is small, as will be discussed in Section 3. 1.

[^1]:    *The analysis of the stagnation flow field for the ablation layer is similar to that for the air layer. Consequently, only the analysis for the ablation layer is presented in detail in the following subsection.

[^2]:    *With $b=1$ and $a=1$, Eq. (71) and many of the following equations may be used for radiation transport calculations for a pencil of radiation.

[^3]:    *One may also neglect the contribution of the OI vacuum ultraviolet lines.

[^4]:    *The strength of a single line over an isothermal path ( $L$ ) may be defined as $\left(N_{i} / Q_{i}\right)\left[1-\exp \left(-h \nu_{k} / k T_{i}\right)\right] L g f_{k} \exp \left(-E_{n_{k}} / k T_{i}\right)$

[^5]:    *This may be compared with the Curtis-Godson approximation (ref. 10).

[^6]:    $\dagger_{\text {The value of } R}$ is inversely proportional to $\Delta \nu_{m}$.

[^7]:    *In ref. 1, the radiation transport formulation includes the presence of external radiation sources. For this study, no external radiation sources are considered.

[^8]:    (according to appropriate numerical integration scheme)

[^9]:    *Cold $\mathrm{N}_{2}$ has a multitude of narrow molecular bands starting at 12.4 eV , but strong absorption may not begin until the $\mathrm{N}_{2}$ photo-ionization continua are reached at about 15.5 eV (ref. 14).

[^10]:    $x=H=\mathrm{h} / \mathrm{h}_{\text {satellite }}, \mathrm{h}_{\text {satellite }}=12484 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}, \mathrm{V}_{\text {SL }}$ for alr at 1 atm $\% \mathbf{2 8 8 . 1 0 ^ { \circ } \mathrm { K }}$
    (1) Hansen, C.F. NACA TR R-50 for $1,000 \leq T^{\bullet} K \leq 12,000$ (2) LMSC Report 4-74-64-1, for $12,000 \leq T^{\bullet} K \leq 25,000$

[^11]:    OOOOOOOOOOOOOO
    
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[^12]:    $02=(T)$
    CAL DOUEL(ALOG10(VO):VTABO,TO:TTABO,XN:SPTABO.1.3,14.3)
    
    
    GOTO 030 ORTM,
    CALL HOOLZ (FNUL FNUU, IKT, KK)
    CALL $\because O D L 3(F N U L, F N U U, I K T, K K)$
    GOTC 8こ0
    CALL $\because O D L 4$ (FNUL,FNUU IKT,KK)
    GO TO 330
    L $\because O D L 5(F N U L, F N U U$ IKT, NK)
    GO 830 (FNUL FNUU IKT,KK)
    

    830
    $O D L$
    (FNUL,FNUU。IKTOKK) 830
    ,OOLO (FNUL, FNUU,IKT,KK)
    

[^13]:    VIT FOR CRLINE

[^14]:    - IT FOR INTLG

    GULROUTINE INTEG(XLA,XUA,NL,NLPIのANS,KI,K2,HVL,GAM2,SGAMP,NRITE + ) I, I1), GAM2 10,12$), \operatorname{SGAMP}(10,12)$, $\operatorname{HVL}(80)$
    DIHENSION DXX(50)

    COWAON/AIHTEG/CMAG,ERRGFRAC

[^15]:    $R=(A-A T(I-1)) /(A T(I)-A T(I-1))$
    $R R=1 。 C-R$

[^16]:    EXPONTIAL KERNAL APPROXIMATION IS USED
    CAlculation without line and ablationi,
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    열
    

[^17]:    URMAS
    $35951-00$
    
    
    

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