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DESIGN CRITERIA FOR FLIGHT EVALUATION
MONOGRAPH IV — CONTROL SYSTEM EVALUATION

15 MARCH 1968

Prepared for
Marshall Space Flight Center
National Aeronautics and Space Administration
NAS 8-20314

TRW SYSTEMS
AN OPERATING GROUP OF TRW INC.

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APPENDICES

1. INTRODUCTION

This monograph describes the flight evaluation of control systems for multistage launch vehicles. The discussion deals with the powered flight portion of the boost phase, but in many instances the descriptions will also be applicable for coast phases. The vehicle control system executes the steering signals and engine start and cutoff discretes which are outputs of the guidance system. Evaluation of the guidance system is presented in the monograph, "Guidance System Evaluation", (Ref. 34).

Section 2, Statement of the Problem, defines the purpose and importance of control system flight evaluation, parameters which must be evaluated, and the limits and constraints involved in practical flight evaluation programs.

A description of current flight evaluation methods, techniques, etc., are given in Section 3, State-of-the-Art. This includes real-time evaluation, functional analysis, and engineering analysis.

The elements to be considered in planning a control system flight evaluation program are discussed in Section 4, Design Criteria.

Section 5, Recommended Procedures, describes the flow of a comprehensive flight evaluation program from definition of the data requirements through methods and analysis.

The appendices present techniques useful for control system evaluation including: filtering, smoothing, and transforming; deviation of computed sensor output; and simulation technique equations. The techniques defined in the appendices are basic tools of a comprehensive comparative flight evaluation for boost vehicle control systems.

2. STATEMENT OF THE PROBLEM

This section defines the objectives and constraints of control system flight evaluation. A brief description of the indices by which performance is judged and the parameters involved in the flight analysis are given.

2.1 PROBLEM DEFINITION

The flight test of the control system provides experimental assurance and verification on the following major points in the development of a launch vehicle system:

- o System functions properly in the actual flight environment
- o System generates the thrust vector command signals
- o System executes the thrust vector command signals
- o Mathematical models (i.e., autopilot, thrust vector control, vehicle dynamics, etc.) used in simulation programs to perform evaluation analytical studies
- o Analog simulators (i.e., preflight breadboard models) to support launch operations and aid in predicting postflight results.

The general purpose of the postflight evaluation is to verify system integrity and performance and determine the cause of any system malfunction. The evaluation generally consists of a) a real-time evaluation, b) a functional analysis, and c) an engineering analysis. Real-time evaluation establishes that system integrity was maintained through a visual inspection of telemetry data (using envelopes about the nominal that represent the allowable system excursions) during the mission. Functional analysis establishes that the system operated correctly in the actual flight environment and confirms that the system generated and executed the appropriate steering signals and discretes during the operation. Engineering analysis determines that appropriate thrust vector command signals were generated and appropriate thrust vector command signals and engine start and cutoff discretes were executed through a comparison of telemetered and computed control system sensor outputs. The analysis also confirms the adequacy of previously developed simulations and mathematical models used in evaluation and analytical studies.

On a nominal flight, control system activity will occur primarily during major programmed events such as execution of steering maneuvers, maximum dynamic pressure, separation of stages, and jettisoning of interstages. However, an evaluation is required in order to confirm system integrity and performance and to upgrade flight evaluation processes and techniques for subsequent missions.

2.2 IMPORTANCE OF POSTFLIGHT EVALUATION

Flight evaluation is required to provide a basis for the initiation of any changes necessary in design of control system hardware or flight evaluation programs used for real-time analysis, minimizes the impact of any anomalies on launch operations and provides for alternate mission or abort selection during a flight operation.

Real-time analyses performed during the countdown and launch aid in the detection of anomalies or malfunctions as they occur and support alternate or abort decisions as required. Verification of system integrity based on functional analysis (generally performed in the immediate post-flight time period) provides for rapid feedback of information which can affect planning for subsequent tests, and highlights areas which require intensive engineering analysis. Engineering analysis is the long-term detailed analysis in which careful scrutiny is given the data for confirmation of control system characteristics and to uncover subtle anomalies. Engineering analysis results in upgrading system technology and state-of-the-art of control system design criteria and flight evaluation techniques by improving the analytical tools used for analysis.

Preflight and real-time observations of control system performance are used to support launch abort and alternate mission decisions in order to optimize achievement of mission objectives and in some cases enhance astronaut safety. Evaluation of control system performance and the control systems contribution to trajectory or orbital errors are important for verification of mission success. In the event of failure, evaluation determines the nature of malfunctions and permits identification and correction of design problems. Postflight evaluation can also be important for verification of the system dynamic models assumed in design or analysis of control systems.

Postflight evaluation provides important information for the system designer and mission planner, but the extent of the effort will usually be constrained by many factors; such as mission objectives, evaluation time, type and quality of available data, and systems characteristics.

2.3 EVALUATION PARAMETERS

Evaluation parameters for assessing control system performance are in general based on characteristics of the control system sensor output signals. Differences between actual and expected vehicle responses are also commonly used.

The signals that are normally available from flight control system telemetry are the guidance signals (discrete and steering commands), the control system attitude errors from the strapped-down gyros or inertial platform pick-offs, attitude rates from rate sensors, angle-of-attack meter or Q-ball (dynamic pressure sensor) outputs, and accelerometer outputs.

Amplitudes of engine command signals and engine deflection signals are used to evaluate performance of the control system components. Engine command signals from the autopilot are related to the control sensor signals through the autopilot equations or models. Engine deflection signals are related to engine command signals through the thrust vector control system model. Control system sensor output signals are related to the engine deflection signals through the vehicle dynamics model, thus completing the control loop. Differences between the flight output signals and predicted signals are fundamental to evaluation of performance. Model parameter or influence coefficient variations required to match the signals are also valuable performance indicators.

Signals obtained from intermediate measurement points within the auto pilot will allow greater detail in performance evaluation. Additional signals from the thrust vector control system are frequently required since this portion of the control system is subjected to large stresses and therefore more susceptible to damage.

Data which are available from the control electronics include discrete signals and actuator commands. Actuator signals consist of actuator positions (e.g., from potentiometers), actuator rates (e.g., from tachometers), differential pressures, hydraulic supply pressures, and reaction control valve positions. Propellant sloshing information can be obtained from propellant utilization probes, level sensors or cameras within the propellant tanks. Sloshing information can also be obtained from filtering actuator positions data. In some cases, accelerometer and rate gyro sensors judiciously distributed along the vehicle can be used to provide bending information. A series of accelerometers and rate gyros could be used to provide bending mode and bending rate data.

2.4 EVALUATION CONSTRAINTS

2.4.1 Time Constraints

Time constraints arise from the need to provide data to interfacing system evaluations and to designers. This usually results in the evaluation proceeding through discrete stages of increasing depth truncated by the objectives of the evaluation program.

2.4.2 Evaluation Tools

Flight evaluation is constrained by the availability of the tools needed for evaluation. These tools include the instrumentation, telemetry data links and processors, evaluation techniques, and computation facilities to automate the techniques. The number of data channels available for control measurements is limited by the needs and priorities of other subsystems.

Elaborate instrumentation is often not warranted because of cost and difficult interface problems which compromise the integrity of the control system itself.

2.4.3 Instrumentation Accuracy and Coverage

The instrumentation accuracy and coverage of control system and interfacing system data are a constraint to insure that adequate instrumentation channels are available for control system data. A flight program with minimal instrumentation severely limits the control system evaluation.

The range and accuracy of the telemetry channel, the transmission frequency bandwidth, and the bandwidth of recording devices all affect the ultimate usefulness of the data. Calibration errors and linearity of the instrumentation system are also significant data error sources.

2.4.4 Data Acquisition

The problem of data transmission loss due to the limited receiving station range is usually overcome by blending the data if overlapping coverage is available. The boost vehicle engine exhaust may cause RF interference or telemetry dropout problems. Consideration should be given to these problems during flight planning, particularly if major events occur during such periods.

In upper stage boost phases, particularly for flights into an orbit, the problem of maintaining communication with ground stations is greater and restricts the ability to perform flight evaluations.

2.4.5 Data Processing

The form of data available strongly determines the evaluation methods employed. Functional evaluations are generally performed with unfiltered data plots from frequency modulated (FM), analog signals which include FM/FM and FM/FM/FM, or from pulse code modulated (PCM) data points which have been plotted or printed. Pulse-amplitude modulated data (PAM) may also be available for detailed analysis.

Engineering analysis which employs computer programs requires careful consideration of the data processing costs. If analog signals are available and a digital simulation is to be performed, conversion of these signals into a digital form would be required. If PCM data are employed in an analog simulation, the data must first undergo a curve-fit before being inserted into a digital-to-analog converter channel of the analog computer.

Data filtering and data smoothing techniques can often be incorporated in the data conversion process to reduce costs. In the data conditioning process performed on the PAM, PCM, and any other discretely sampled data, digital filters and data editing routines are included. If analog data are employed in analog simulations, the frequency filters can be included in the computer simulation.

3. STATE OF THE ART

Flight evaluations are divided into three areas of analysis which are distinguished by their timing, objectives, methods, and depth. The three areas of analysis are:

- o real-time evaluation
- o functional analysis
- o engineering analysis.

Real-time evaluation and functional analysis are performed for verification of system integrity and adequacy of the control system operation. Engineering analysis provides confirmation of performance characteristics through a comprehensive comparison of measured and computed sensor outputs.

3.1 REAL-TIME EVALUATION

Real-time evaluation involves monitoring and evaluating the control system during flight operation by observation of instrumented signals displayed to support launch abort or alternate mission decisions. Decisions may be made by the range safety officer, the test conductor, and in the case of a manned flight, by the astronaut. This evaluation is keyed to the immediate identification of anomalies and malfunctions, and the prompt recommendation of in-flight trajectory alterations to maximize achievement of test objectives and mission success. The value of a real-time evaluation depends upon the control system analysis support provided to flight operations. Failure analyses and malfunction simulations should be conducted prior to the flight operation to select meaningful abort criteria and performance indicators. Personnel monitoring the flight must be able to interpret telemetered data by comparing it with predicted data to provide timely and accurate recommendations. The monitor must have a comprehensive knowledge of the control system functions.

3.1.1 Data Evaluation Methods

Typical control system signals monitored in real-time evaluations include control system attitude error and rate, hydraulic pressure and temperature, actuator deflection, accelerometer, and angle-of-attack meter outputs. These signals are compared to flight envelopes superimposed on the displays of these signals. The bounds on the parameters are determined prior to the flight from control simulators or models and preflight predictions of performance parameters. However, the limits may be updated based on actual day wind soundings to provide additional data to the test conductor.

Data may be presented as basic parameters which are measured and compared with predictions made prior to the flight. Nominal values are derived from design studies or previous flight operations, or they may be based on actual control system and environmental data processed through a simulator in real-time. The criteria for evaluation of these parameters are established prior to flight. Deviation in parameters from the established limits result in corrective or alternative action.

3.1.2 Real-time Evaluation Limitations

Real-time evaluation requires assessing the control system performance based on a limited number of parameters which are either read directly from the data output stream or by filtering and then comparing with the corresponding resultant values of these parameters predicted prior to the flight or generated in real-time by simulations which account for the simulated environment. The process is limited by time, quality, and quantity of data. Alternative solutions to problems must be defined prior to flight and are therefore limited by the inability of the analyst to predict all possible combinations of events. Further, anomalies in the data stream, e.g., transducer malfunctions, spurious signals, data transmission noise, malfunctions in ground data handling and processing equipment or anomalies in the system software, could result in an inadvertant abort or unnecessary corrective action unless some degree of redundancy is employed.

The time allowed for real-time evaluation or the quantity of available data may preclude selection of an absolute best solution to a problem at the time it develops. The alternative solutions for various combinations of parameter excursions must be defined prior to flight and only minor modifications in planning are possible in the real-time evaluation of a system. Although limited, real-time evaluation is important because it can minimize the cost of an operation by allowing for rapid reaction to unpredictable occurrences.

3.2 FUNCTIONAL ANALYSIS

Functional analyses are conducted subsequent to a flight operation to verify system integrity; to demonstrate the system operating in an actual flight environment; and to verify that commands were generated and that the system executed steering signals and discrete commands, as required. Frequently subtle malfunctions or areas warranting further detailed engineering analysis may be uncovered during this process.

3.2.1 Data Analysis Methods

Functional analysis involves a comparison of telemetered responses to a priori knowledge of these responses and verification of proper execution of the control system functions in response to commands and environmental disturbances.

Data used for functional analysis consist of raw and filtered analog traces, oscillograph records, and tabulation of events. The data is usually processed and edited, and performances of measuring instruments are assessed. If the data is in a digital format, listing of digital words is required. Display of data in a central display room provides the data analyst with ready information. Tabulations and plots of processed data should be distributed immediately.

If an instrument fails, a redundant measurement may be used or significant information may be derived from an alternate source. Simple manipulations of these signals, such as the summing of two or more signals, may be performed to show the effects of filtered signals and drift characteristics. Similar comparisons are made on a subsystem level. In addition, the output to input gain and the phase characteristics of any observable oscillations should be compared with the expected subsystem frequency response.

A comparison is then made of the amplitudes of commanded and executed signals, e.g., commanded TVC position and actuator position. Frequency, damping, and time constants of responses are compared to known values obtained from design studies, ground tests or analog simulation.

Response of the vehicle to significant commands, such as vehicle pitchover, are compared to the predicted vehicle response. If vehicle oscillations result from the commands or if they are expected to occur during any period of the flight, the amplitude and frequency of the observed oscillations are compared to the predicted oscillation curves. These comparisons enable a quick evaluation of system performance and provide an indication of system integrity.

Digital programs are available for determining the power spectral density, auto correlation function, and cross correlation function. An example of this type of program is described in Reference 6. Spectral density analysis of the telemetered data are compared to expected spectrum portraits; their usefulness, however, may be limited due to lack of precise data. In systems where the vehicle dynamic modes are unknown or uncertain, the power spectral density analysis will provide a useful method for exposing these modes.

This type of analysis provides a detailed input for planning engineering analysis, provides for rapid feedback of results to initiate modifications in program planning, and allows for updating analytical data on a timely basis. Further, identification of subtle malfunctions or discovery of anomalous behavior will initiate intensive engineering analysis along specific lines.

3.2.2 Functional Analysis Limitations

Functional analysis allows for an assessment of the data system and the flight operations of the control system. However, this analysis is limited by available data and time allowed for performing the analysis. There are physical limitations on the extent to which the system may be interrogated. The quantity of data depends on the capability and capacity of the telemetry system. Priority of the control system measurements are compared to measurements required for other systems in the accomplishment of mission objectives. Design specifications, manufacturing tolerances, and instrument accuracies will affect and limit the quality and accuracy of the measurements. Furthermore, anomalies and subtle malfunctions may produce performance data which require detailed investigation to produce conclusive results.

If an instrument or a segment of the data system produces questionable information or breaks down completely, this fact can be noted. However, additional tools and time to perform an engineering evaluation may be required to resolve or fully understand such anomalies.

Since functional analysis takes place immediately after the operation, there is no possibility of using the results to upgrade the actual performance of the mission. However, functional analysis permits: a) initiation of further postflight analysis in specific areas of concern, b) upgrading of hardware, c) revision of software such as modifications to simulations to be used in real-time evaluation, d) revision of techniques for preflight and real-time predictions, and e) reevaluation of limits placed on acceptable parameter bands for a real-time operation on future missions. If the lead time for planning future missions is short, functional analysis may be the primary link in the upgrading process. If this is the case, the upgrading process may be more subjective than desirable. In such a case, the limitations described under real-time evaluation also apply to functional analysis.

3.3 ENGINEERING ANALYSIS

Engineering analysis includes a detailed long term control system analysis of a flight test or operation. This evaluation uses refined flight data as inputs to produce highly realistic synthesis of the operation. Data which are particularly useful for component as well as system evaluation involves parameter variations in simulation studies to verify vehicle dynamic modes, to investigate malfunction of a non-obvious nature, and to obtain sensitivity coefficients which are useful in predicting effects of non-nominal behavioral or environmental conditions. Engineering analysis uses ground and laboratory tests to supplement analytical evaluations. Error analysis also provides important results.

3.3.1 Data Analysis Methods

One of the most precise methods of developing and verifying subsystem modes is to use actual flight control system input parameters for the control subsystems to generate predicted outputs for comparison with actual flight data. Computed sensor outputs may be compared with raw or processed (i.e., smooth and/or filtered) telemetry outputs. Computed outputs are generated by simulations of control system laws which predict the behavior of the system using basic simulation techniques such as 6-D rigid body equations, elastic body equations, and control filter equations, or a combination of these simulations. By comparing outputs of these simulations with real data it is possible to verify and improve the simulations which have a basis on state-of-the-art knowledge control laws and control system behavior, as well as affect or improve the capabilities to perform meaningful real-time evaluation and general upgrading of the control system analysis techniques for future operations. Flight data may point out discrepancies or indicate areas where significant improvement in simulations evaluation or techniques are warranted.

Analysis should include the uses of harmonic analysis of the control system error response to a measured input enabling verification of the control system frequency responses which were established in the design studies. The analysis utilizes digital or analog simulations as required, to verify control system parameters and to compare the data with postulated malfunctions or anomalies. Evidence of sustained oscillations should be carefully investigated using linear control system analysis techniques with nonlinear or linearized models of the element or component suspected of causing or sustaining the oscillation. Recently, sophisticated parameter identification programs have been employed to verify statistically inputs to control system performance. The Appendices provide typical comprehensive mathematical representation of control systems simulations including dynamic effects and filtering techniques which are fundamental for engineering evaluations.

The process of constructing or modifying the control system model to match the vehicle flight reaction is perhaps best accomplished through model parameter variations conducted in simulation studies. The range of parameter variations depends upon the degree of certainty in the initial system parameters, that is, precisely known model parameters should not be varied beyond the expected range predicted for malfunction evaluations. Variations in particularly sensitive control system parameters would be appropriate, since small percentage variations in these parameters produce larger effects than a large change in less sensitive parameters. A comprehensive understanding of the control system and its variations and sensitivity is required to conduct this type of evaluation accurately and efficiently. Examples of malfunction simulation studies are given in Reference 7.

3.3.2 Engineering Evaluation Limitations

The engineering evaluation is primarily limited by the constraints of time and resources which are available for the analysis and to the state-of-the-art of the evaluation tools.

Quality and quantity of the data available after a flight from test measurements and computations, or derived from subsequent ground tests, may not be sufficient to answer all questions. The accuracy of available data may be such that anomalies or unpredicted characteristics cannot be attributed to a particular cause and effect relationship. Limitations of this type preclude the modes and simulations which depend on the flight results for refinement from ever becoming absolute in their ability to predict every one of potentially infinite number of subtle variations which may occur. Simulation of every control system component down to the detail of every resistor, wire, and capacitor is prohibitive; the models normally used tend to focus attention primarily on control system laws and control system behavioral characteristics. Evaluation program simulations are themselves limited by real world constraints such as development funds, time, computer capacity, and availability of real data. However, the state-of-the-art is sufficiently refined to allow for prediction and, therefore, evaluation of most control system characteristics which are of interest to the analyst.

The analysis may also be limited by time constraints dictated by the requirements for feedback of the results of the postflight analysis to subsequent flight operations. In many instances, the prime value of the analysis is achieved only if the findings can be fed into the planning of the next launch operation, which may be imminent. For example, if only a few operations of a given type or configuration are planned, the improvement of bending characteristics may only be of academic value unless the resultant improvements can be immediately obtained and the program modified. This can frequently be the case where the nature of the program involves a limited number of flights, such as Saturn or Apollo.

4. DESIGN CRITERIA

This section describes the considerations used in developing an effective flight evaluation program of a control system. Although there is no precise formula for a postflight evaluation procedure, there are major elements which should be defined and considered for development of a successful postflight evaluation procedure. These elements, used as a yardstick for establishing and assessing a program plan, include:

- statement of the objectives
- the evaluation processes
- knowledge of the mechanization of the control system and its characteristics and interfacing evaluation parameters
- understanding the actual test and its support requirements
- the influence of the mission and flight events
- requirements for data handling and processing
- resource requirements.

All the above elements must be defined in an adequate program plan of an evaluation process.

Considerations in preparing a flight evaluation program plan are illustrated in the flow chart of Figure 4-1 and discussed below. Combined in a plan, the objectives and constraints provide the structure for the evaluation of data through the preflight, flight and postflight analysis phases of the evaluation process.

4.1 MISSION OBJECTIVES

The mission and objectives of flight evaluation dictate the planned approach and the commitment of resources. If mission objectives are primarily developmental, analysis of system performance and/or malfunctions and interfacing problems may predominate. If the operation involves astronaut participation, crew safety will become a predominate factor in planning (for example, requirements for redundant real-time evaluations and decision making procedures such as the emergency detection system to be used for Saturn manned launches capable of acting

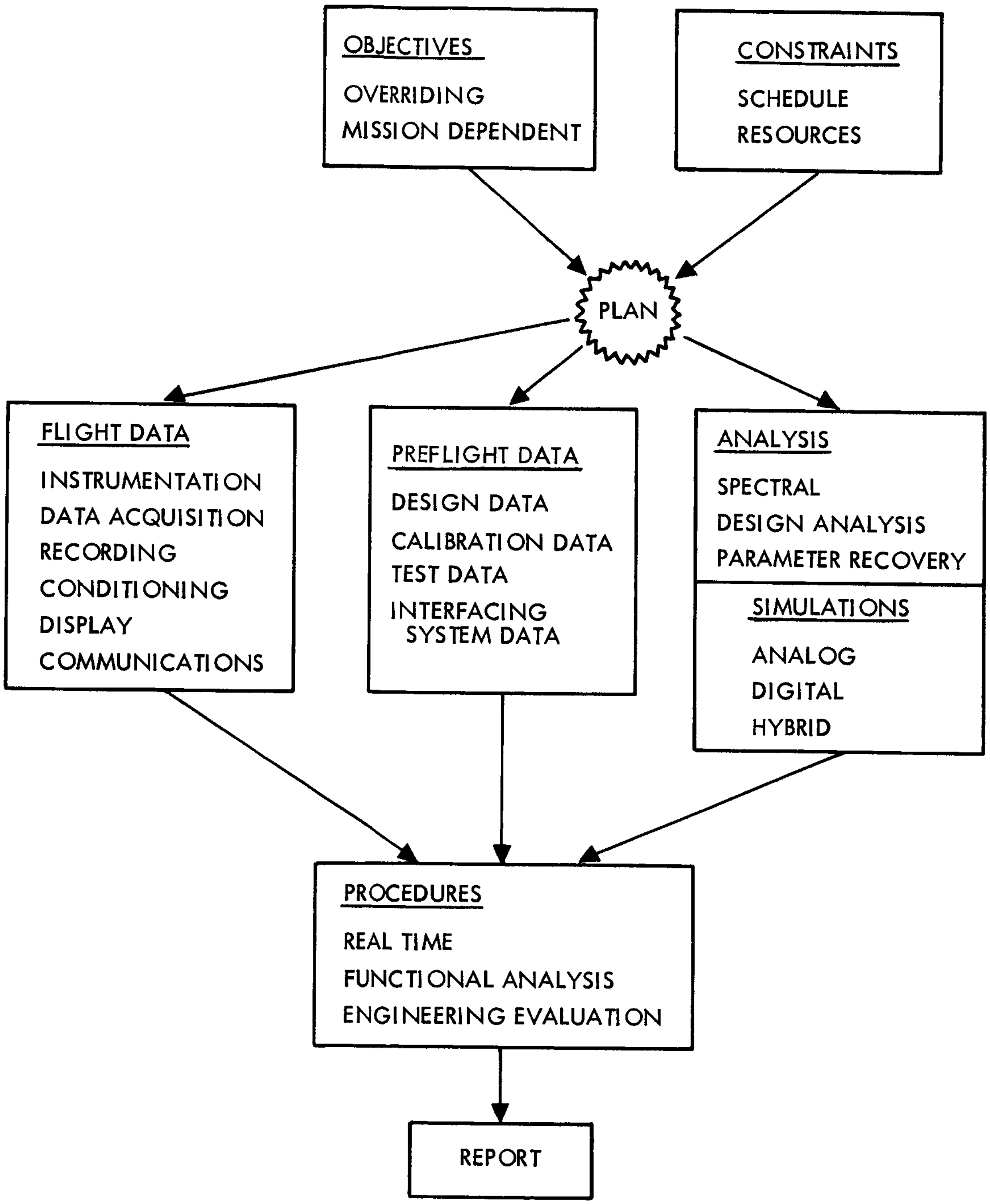

Figure 4.1 Program Plan Preparation

on control, propulsion, or command to enhance real-time mission control). The flight evaluation program plan proceeds from a consideration of the flight objectives and the constraints imposed upon the evaluation program. Some control system evaluation objectives are often mission dependent or closely related to other systems whose performance is being emphasized on a particular test. For example, control system performance verification is usually a priority objective in the early development of a launch vehicle system. Adequacy of the control system must be assured before vehicles and payloads are committed to subsequent flights. In such cases, confidence in the design, analytical prediction techniques, and the magnitude of design margins are the primary reasons for evaluation. Control system flight evaluation objectives common to any flight are the assurance of range and astronaut safety; the analysis of malfunctions, should one occur; and the support of top priority interfacing system evaluations.

When the primary purpose for launching a booster is that of payload delivery, the objectives may include or be oriented toward evaluation or demonstration of systems and subsystems performance, integrity, compatibility and capability. When these are the objectives, the purpose of gathering data is to compare actual to predicted performance, and to determine malfunctions and deviations in performance for refinement of future flights and designs. Evaluation of objectives can lead to uncovering deficiencies in predictions and prediction methods, pinpoint problem areas, or lead to advances in the state-of-the-art in both design and evaluation.

Where the objective of the flight is of a Research and Development nature, the objectives of the control system evaluation will be to determine the adequacy of design, the compatibility of control system with the vehicle and with interfacing systems, and verification of design characteristics, and analytical models. The sensors should be tailored to meet the objectives; ground tests may be required to supplement flight tests; or components may be flown "piggy-back" (open loop) on other flights to obtain component assurance prior to the actual flight of the system.

Where objectives of the flight are operational, the control evaluation verifies system integrity, refinement of prediction techniques, and performance characteristics. Alternate control system sensors may provide redundant sources of data. Where accuracy is an important consideration, redundant components may be utilized to eliminate random errors by correlation of data sources.

The procedures and resources committed for evaluation will be dictated by the mission objectives and the requirements for the evaluation. The degree of emphasis on real-time evaluation, functional and engineering analysis, and provisions for reporting on the accomplishment of objectives will depend on the time urgency of providing feedback of the results.

Data for the evaluation and tools of preflight analysis and simulation to carry out these evaluations are determined by the need to satisfy the objectives and constraints of the mission.

The gathering of preflight or flight data should be geared to the requirements of the evaluation process. It is recognized that in some instances the analyst must use the parameter available due to constraints such as overriding or higher priority requirements. To obtain the necessary flight data, signals must be identified and instrumentation provided. This includes the allocations of tracking and optical devices, as well as transducers. Telemetry links must be provided and allocated for data acquisition. Data display, recording, calibration and linearization, conditioning and handling provisions must be made in order to place needed data before the analysts. Communications must be provided to support real-time evaluation and functional analysis requirements.

Preflight data, control system design and test data, instrumentation calibration data, and flight data on interfacing systems must be transmitted to the control system evaluator.

4.2 CONTROL SYSTEM DESIGN AND CHARACTERISTICS

Ideally the evaluation process should start when the design of the system is begun. Meaningful evaluation is dependent on recognition of the system design objectives as well as the resultant system characteristics.

Recognition of the evaluation process in the design period allows the analyst to develop appropriate tools of the control system in the operating environment in an effective and efficient manner. The control system analytical tools which are developed for system synthesis are frequently compatible with requirements for evaluation tools. The selection of instrumentation and points of data output for the control system functions and interfacing subsystem functions, which affect the performance and operation of the control system, should be considered during the early period of system development, thus allowing optimum usage of the least time between system concepts and system operations to develop sophisticated evaluation tools. This is especially true in cases where real-time evaluation is used as a means of optimizing the vehicle system operations.

The different methods of mechanizing control system functions dictate the method, procedures, and techniques used for the evaluation of the control system performance in the flight environment. Basic control system design choices are discussed to show the resultant differences in the evaluation process. For the purpose of this discussion, the control system is separated into its two major elements:

- The autopilot, including the control system sensors which measure vehicle motion and provide controlled commands
- The thrust vector control system, including roll jet subsystems for single engine vehicles, which execute the control system command.

4.2.1 Autopilots and Control Laws

The basic function of the autopilot is to obtain information from control sensors and issue thrust vector commands based on the design control law. Numerous autopilot designs are possible, but only the basic categories will be discussed. Control commands for space vehicle applications are commonly derived from one of the following types of basic control systems, i.e., conventional rate feedback, load relief, or adaptive autopilot subsystems. The analysis processes will be geared to the type of control system utilized, the control laws involved, and the hardware components used to mechanize the system selected.

4.2.1.1 Conventional Rate Feedback

The conventional rate feedback control system employs a control law which is essentially a linear function of vehicle attitude error and rate. The thrust vector control command is generated within the control system by operating on the vehicle attitude error, and vehicle attitude rates. The operators are gains which may be varied or changed from time to time to preserve the linearity of the system and filters which are used to shape the parameter or reduce noise. These systems can be mechanized with various analog, digital or hybrid control system elements.

(a) The analog autopilot is a direct mechanical or electrical mechanization of the control function generally employing gyros to sense vehicle position and rate. The steering commands may be preprogrammed or issued by an active guidance system. The operations performed on the sensed or commanded signals are gain multiples and the sum of various signals. Filtering may be employed to smooth the gyro output signals, to increase the system stability, to shape the output of the autopilot, to provide control commands compatible with the mechanization of the thrust vector control. A lead-lag filter could be employed to derive rate signals from the position gyro output, thus eliminating the rate gyro. An integrating circuit may be used in the autopilot network to reduce drifts due to winds.

Evaluation of the analog control system requires data output on the steering commands, sensed position and rates or internally derived quantities such as attitude error, and the results of the control commands such as engine deflections or actuator position. Since it is relatively straightforward to duplicate the functions of filters, gain factors, and other logic operations, the data which can be extracted at any of a number of points in the signal flow network can generally be treated with ease to reconstruct functions of the control system.

(b) The digital autopilot generally derives the control functions based on information sensed within a guidance system. The operations in the signals are performed in the complex logic networks of a digital computer. The feedback and command loops may be largely external to the control system or may be integral parts of other systems, i.e., guidance. Operations on the signals as they pass through the network are more complex and the signal loses its identity with respect to the element sensing or generating the command signals. For example, position and rate signal outputs may become incremental changes of these parameters representing a change of state has occurred within the sensing device rather than the magnitude of the parameter.

More output data is required to reconstruct the functional operations on the control signals as they proceed through logic networks. Filters, operators, and transformations which are performed throughout the signal network can be duplicated in the process of ground handling and analysis of the control system data, but, as in the case of the digital system itself, the data analysis process becomes more complex, and digital programs are required.

(c) Hybrid autopilots combine features of both analog and digital systems if a sophisticated guidance system is available; the designer may use its capability for compilations such as computing attitude information from the guidance system platform gimbal analyses. Attitude error angle commands can be transformed into body error angle commands within the computer, or rate gyros may be employed and the signals integrated in the computer to obtain attitude angles and attitude errors. In either case, the output of the sensing device and the output of the computer should be available for postflight evaluation. The operations made on the signals can be performed or duplicated as part of the analysis process and may provide information on the performance of the computer and the control system.

4.2.1.2 Load Relief

The load relief control system, through the addition of lateral force-dependent terms in the control law, is an extension of the conventional rate feedback control. These terms may be a function of vehicle lateral acceleration, angular acceleration, angle of attack, or a combination of these based on a design logic.

By employing logic equations based on sensed vehicle performance, the gains can be changed to include one or more load relief terms. Each load relief feedback term should contain a filter to exclude undesirable high frequency signals. The mechanization of this type of system may be either digital or hybrid analog.

4.2.1.3 Adaptive System

Adaptive autopilots are generally employed for the stabilization of bending modes in vehicle designs where conventional systems are not acceptable due to large variations in payload or where large uncertainties in the bending data exist due to lack of test data. Basically the adaptive control system identifies the bending oscillations in the control sensor output signals and attenuates these oscillatory signals by performing real-time adjustments in the gains and/or filter configurations. This enables the control system to perform its primary task of rigid body vehicle control within aerodynamic disturbance environments.

Due to its versatility, mechanization of this type of system normally employs a digital computer. However, this does not preclude the use of analog systems for this purpose. Any of the previously described systems may be employed as an adaptive system.

4.2.2 Thrust Vector Control System

The thrust vector control (TVC) system is generally a closed-loop actuation system employing actuator position signals to close the loop. An additional rate feedback loop may sometimes be included. The actuation system may be a hydraulic system or an electromechanical system employed to displace a moveable engine, nozzle, vane, or secondary injection valve. A solenoid valve may be employed to activate a control jet for roll control or for secondary injection control of the thrust vector. Secondary control of the thrust vector is accomplished by injecting a liquid or gas into the main thrust stream causing a separation of the flow from one side of the nozzle and affecting a thrust vector deflection.

The basic components of the hydraulic actuation system include servo amplifiers, servo valves, hydraulic actuators, actuator position and rate feedback transducers, power supplies and voltage regulators, hydraulic supply system (including electrical motor), motor speed regulator, pumps, valves, and accumulators.

In an electromechanical actuation system, the basic components include servo amplifiers, high horsepower electrical motors, electromagnetic clutches, gears, actuator linkages, actuator position and rate feedback transducers, power supplies, and voltage regulators.

For a control jet system, the basic omponents are amplifiers with electrical power switches, solenoid valves, power supplies, and voltage regulators. Solenoid valve actuation indicators are also generally included, although they are not part of the control loop. These indicators may be a solenoid current sensing device or a valve position indicator.

The servo valve controlled flow to the actuator is primarily dependent upon the command current and the actuator load pressure which may be nonlinear. In a precise evaluation of the hydraulic actuation system performance, these nonlinear servo valve characteristics should be modeled.

In the direct drive system, shunt winding motors are usually employed since the motor speed is insensitive to load variations and primarily a function of the control shunt field current. In the clutch driven system, since the clutch absorbs the effects of large load variations, a series-wound motor may be employed. The electromagnetic clutches are normally used in pairs to obtain load velocity direction changes, since the motor is designed to run near constant speed in one direction.

In a solenoid valve system, the most significant problem is to detect whether the valve has been actuated or not. Limit switches which are closed when the valve is fully actuated are not reliable due to contamination or damage from the severe environment of boost flight. Therefore, solenoid voltages and currents are often monitored. The voltage measurement indicates that the servo amplifier is operating properly. The current trace indicates the presence of continuity in the solenoid circuit. A spike in the current trace results from the back electromotive force generated when the solenoid plunger is actuated, indicating its movement. To augment these measurements, pressure transducers may be mounted in the jet nozzles to indicate the presence of jet thrust, since the possibility of propellant blockage exists even if the solenoid valve functions properly.

The thrust vector control system is the power output stage of the control system and is subject to stress and strain and susceptible to malfunctions. Malfunctions which may occur in the TVC system are listed below:

<u>Common Problems</u>

- Actuator lock due to excessive frictional loads, increasing with time due to thermal effects

- Damaged actuators due to high transient loads during engine startup and shutdown
- Feedback transducer damage

Electromechanical System

- Motor overload damage due to heavy duty cycle due in part to thrust vector misalignments
- Clutch overheating damage due to similar heavy duty cycles

Hydraulic System

- Hydraulic leak and pressure loss
- Clogged orifices in servo valves and actuators

Control Jet

- Open solenoid valve coil or short circuit.

4.3 EVALUATION PARAMETERS AND INTERFACES

The parameters which are pertinent to the evaluation of control system performance are inherent in the various subsystem operations. The following is a list of signals organized by subsystem, including related data signals for completeness:

Autopilot, Switching Logic, and Control Subsystem Sensors

Common autopilot parameters:

- Thrust vector deflection commands (β_c) which indicate operation of autopilot
- Stage selector switch indicator showing which boost vehicle stage is receiving thrust vector deflection commands

Analog autopilot parameters:

- Attitude error signal (ϕ_e) which indicates errors in commanded attitudes is typical
- Filter output, including specialized circuits such as a gyro-blender output, indicating performance of the autopilot circuits

Digital autopilot parameters

- Attitude error signal, ϕ_e (Z), which indicates errors in commanded attitudes

- o Digital computer control system equation parameters which indicate detailed executions of control system equations such as outputs from digital filters

Control system sensor parameters:

- o Sensor output which indicates control system performance and sensor operation

- o Wheel speed indicator for gyros which indicates operating speed of gyro wheel

Thrust Vector Control Subsystem Sensors. The thrust vector control subsystems are numerous in design; however, they generally contain basic components such as servo amplifiers, electrical power supplies and voltage regulators, and actuator position and rate sensors. The following list defines the parameters associated with the subsystem and typical performance indications:

Thrust vector control subsystem component parameter:

- o Servo amplifier outputs which indicate operation of power amplifiers and power switches; more often instrumented when driving solenoids or servo valves in an on-off mode in a gas jet, reaction jet, or even secondary injection system.

- o Electrical power supply output (V_S), which indicates voltage and power variations to the subsystem that in turn affects the subsystem performance. It includes power to actuator position and rate sensors.

- o Actuator position and rate sensor outputs (β and $\dot{\beta}$) which indicate if actuators are following commands and the general performance of the servo-actuation system.

Hydraulic actuation system parameter:

- o Actuator load hydraulic pressure (P_L), which indicates inertial and frictional loads on the actuation system as well as thrust vector loads; in particular, flow separation forces during the main engine startup and shutdown transient phases.

- o Actuator supply hydraulic pressure (P_S) which indicates if hydraulic supply system is operating normally and may also indicate flow separation forces during the startup and shutdown transient phases.

Electromechanical actuation system:

- o Electrical motor speed signal (ω_m), which indicates operation of motor and load conditions.
- o Electrical motor current signal (i_m), which indicates load conditions on motor.
- o Electromagnetic clutch current signal (i_c), which indicates that commands are being acted upon and indicates operating state of the clutch.

Secondary injection system parameter:

- o Injectant pressure at outlet (P_i) which indicates thrust deflection variations (may be gas or liquid injection).
- o Servo-actuator hydraulic pressure (P_S), which indicates operating condition of injector actuation system.

Gas jet or reaction jet system parameter:

- o Solenoid current indications showing operation of solenoid jet valves.

Guidance and Program Subsystem Sensor

Common guidance interface parameter:

- o Guidance steering commands (ϕ_c) to the control system.
- o Guidance system errors (ΔV_x, ΔV_y, ΔV_z), which indicates effect of control system performance.
- o Discrete event commands.

Inertial guidance system:

- o Platform gimbal angles, ϕ (Z).
- o Accelerometer or velocity meter signals.

Strapped-down gyro guidance:

- o Attitude command error, ϕ_c.
- o Attitude rate signals, ϕ.

Propulsion Subsystem Sensors

Common propulsion interface parameter:

- o Engine thrust level time history (also given under vehicle parameters-thrust profile).

- Stage separation interstage pressures.

Gimballed engine thrust vector control subsystem:

- Flow separation forces during engine startup and shutdown.
- Viscous and coulomb friction levels.

Gimballed nozzle or movable vane thrust vector control subsystem:

- Nozzle coulomb friction evaluated from actuation system hydraulic pressure data or electromechanical actuation system data.
- Flow separation forces during engine startup and shutdown.

Secondary injection subsystem:

- Injectant supply pressure and temperature at supply tanks and inlet to injector valves.

Gas jet or reaction jet subsystem:

- Propellant temperature and pressure at inlet to jet or outlet of jet.
- Quantity of propellant remaining (confirming control system performance evaluation).

4.4 FLIGHT TEST AND GROUND SUPPORT REQUIREMENTS

The flight sensor data required to evaluate a control system must be telemetered to the ground. The field of telemetry systems and associated ground support equipment is a specialized field of its own, but understanding the general system will help in planning the data requirements for a given telemetry system.

4.4.1 Types and Accuracies of Data Transmission Systems

The three basic types of data transmission systems in frequent use are: a) the FM-FM system, b) the PCM system, and c) the PAM system. A brief description of each system is given below.

4.4.1.1 FM-FM System

In an FM-FM system, the sensor output variation causes a subcarrier oscillator modulating signal to deviate the RF carrier frequency. The FM transmission has the beneficial characteristic of minimizing noise and data loss compared to ordinary RF data transmission.

Selection of the subcarrier frequency determines the frequency response of the transmitted analog signal. The FM-FM transmission system accuracy is in the range of 3 to 5 percent of full scale. Accuracy of the sensor and data display equipment should be added to the transmission system to obtain the overall data accuracy.

4.4.1.2 PCM System

In a PCM system the sensor data are coded to a digital form by an encoder before transmission. The amplitude accuracy is directly related to the analog to digital conversion scheme and is generally a one-bit level. The sensor data are periodically sampled since finite time is required for encoding. The maximum frequency response of the data would depend on the data sampling interval. PCM systems can handle multiple data outputs with a limited frequency response, and the data can be programmed directly into a digital computer.

4.4.1.3 PAM System

In a PAM system the sensor data is sampled and variations in the amplitude of the signal are modulated on amplitude of the subcarrier. The quantity of data or the number of data channels for a given transmitter can be greatly increased by data sampling, if the quasi-static signal vary in amplitude at a rate of less than 1 cps. Commutators used for sampling data may be either mechanical or electronic. The PAM waveform represents the sampling of information taken at discrete intervals. The intervals coincide with the time of occurrence of each individual pulse on the sensor output signal. Therefore, the precise waveform of the signal which is sampled is transmitted during the sampling period. The transmission accuracy is in the range of 3 to 5 percent of the full scale measurement. To assess the overall system, accuracy decommutation as well as sensor and data display accuracies must be considered.

4.4.1.3.1 Frequency Response

Typical rates for Saturn class are 4 or 40 samples per second (sps), which gives 0.8 or 8 cps frequency of response based on criteria of 5 samples to determine a sine wave frequency response = $\frac{\text{No samples/sec}}{\text{5 samples/sec}}$

Typical PAM is 12 or 120 sps which is reduced by linear interpolation to 10 or 100 sps to facilitate evaluation. Multiplying of several sensors on one channel is used to obtain the 12 sps data.

4.4.2 Telemetry Data

All data should be permanently recorded on magnetic tape for later analysis. Individual measurements should be classified according to their importance as follows:

Data Classification

Class I	Mandatory for the accomplishment of mission objectives
Class II	Highly desirable to assess the mission objectives
Class III	Desirable for evaluation of mission objectives

4.4.3 Preflight Instrumentation Calibration

All sensors and the individual telemetry channels should be calibrated in a flight environment. This calibration should be performed periodically, and the last one should be performed as close to the flight date as possible. Confidence in the measurement accuracy is improved by repeated calibration. In-flight calibration is recommended where extreme accuracy is required. The calibration data should include range, linearity, and polarity.

4.4.4 Tracking Data

Tracking data are required during all maneuvers and periods of critical events. These data are correlated with predicted and telemetered flight control events or maneuvers during postflight evaluation. Events such as separation, ignition, burnout, and major maneuvers (programmed pitch and roll and major dogleg maneuvers) could be correlated with flight control data.

4.4.5 Optical Data

Optical data are required during launch and powered flight operations. These data include metric, engineering, sequential and documentary data for cameras and video sources. These data may be used with, or in place of, other tracking data to provide accurate trajectory reconstruction and flight information such as event occurrences which may not be observed in other forms of datum. Responses to some malfunctions and anomalous behavior can only be adequately observed on optical sources. Optical data requires more time for processing than other forms since the raw data requires photo processing for display. Special programs are required for optical metric processing and experienced observers are required for proper review of film data.

TV provides an important data source for real-time analysis since it allows the observer to easily view the overall system performance.

4.4.6 Disturbance Data

Wind sounding data in the launch area before and/or after the vehicle launch or adequate wind models are necessary for control system performance evaluation. Also, nominal or revised values of control system parameters should be obtained and used to update analytical models. These parameters include center-of-mass offsets and thrust vector misalignments as well as values of the control system coefficients.

4.5 TRAJECTORY INFLUENCE/APPLICABLE FLIGHT EVENTS

Two general classes of ascent trajectories are used; a) direct ascent trajectories, and b) multiple burn ascent trajectories. As the name implies, direct ascent is accomplished by nearly continuous propulsive burning from lift-off to the finally desired trajectory or orbit at burnout. Direct ascent trajectories are only possible when the desired final trajectory is a ballistic trajectory, a low altitude (between 80 and 100 nautical miles) circular orbit, or a high altitude elliptical orbit with perigee near the burnout point (Figures 4-2 and 4-3). When other types of high altitude orbits are desired, at least two ascent burn periods are required. A high altitude circular orbit, for example, can be achieved by using the first burn to inject the vehicle into an elliptical orbit whose apogee is at the desired final orbit altitude. After coasting to apogee, a second burn of proper direction and magnitude will put the vehicle into the desired circular orbit (Figure 4-4).

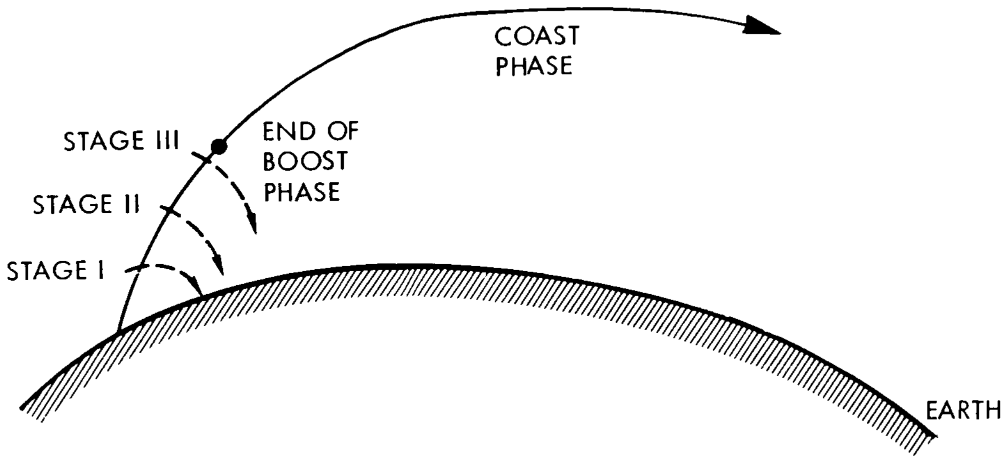

Figure 4-2. Typical "Direct" Ascent Trajectories - Ballistic Coast

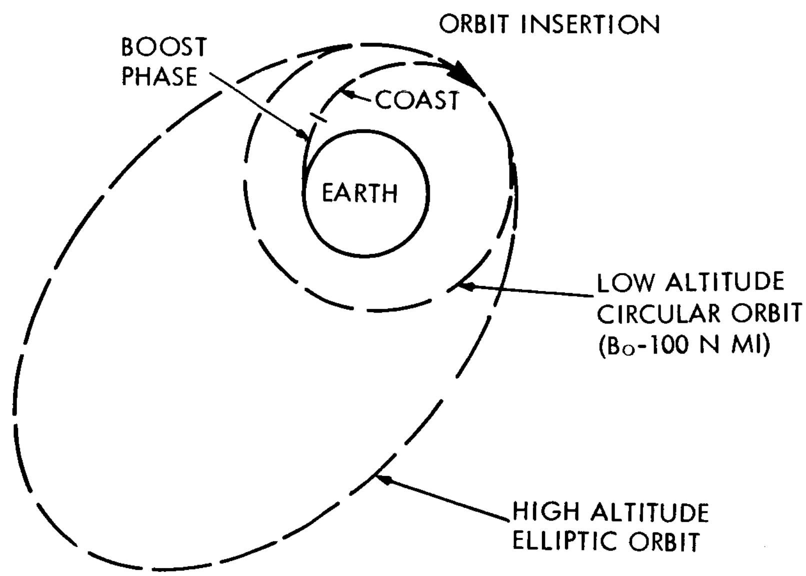

Figure 4-3. Typical Low Altitude Circular or High Altitude Elliptical "Direct" Ascent Trajectory

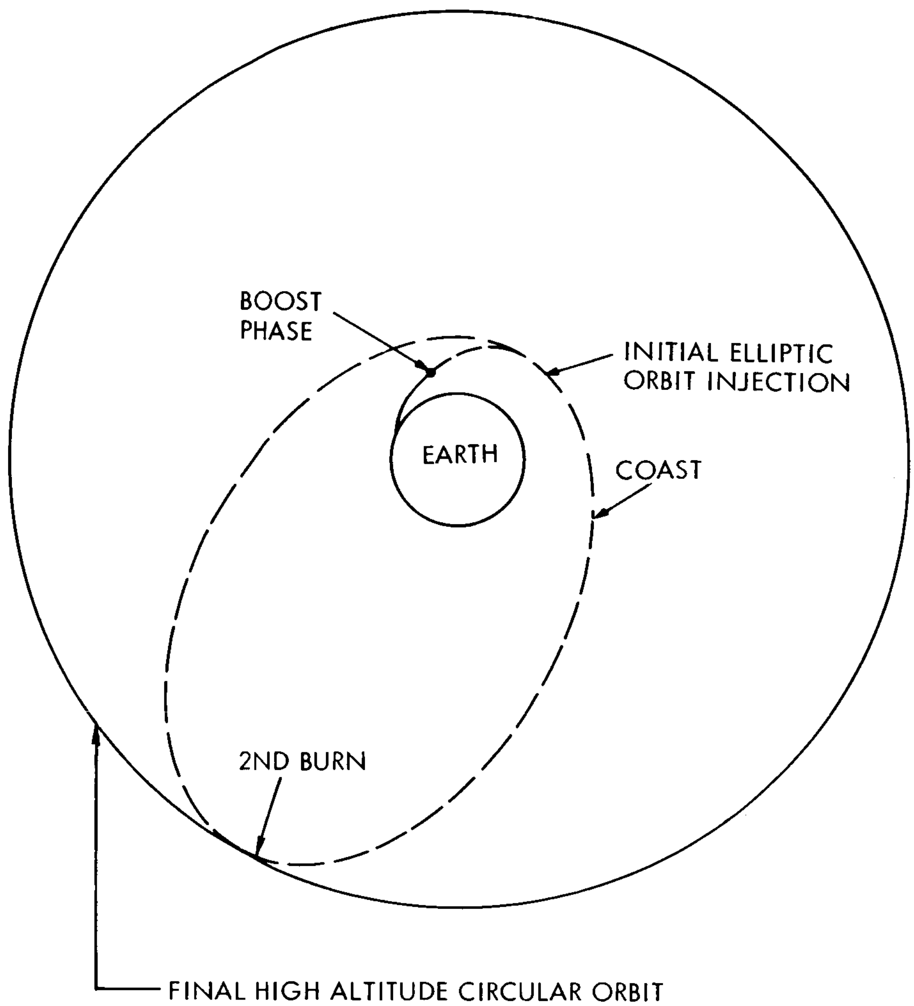

Figure 4-4. Typical "Multiple Burn" Ascent Trajectories

Insofar as control system evaluation is concerned, the important consideration is the number of different vehicle configurations used to achieve the given trajectory. Each stage of the boost vehicle represents a new configuration, and the jettisoning of expendable components such as interstages and payload shrouds produce additional vehicle configurations which must be considered. Boost vehicles may consist of multiple stages, each with a completely separate control system which must be evaluated separately, or each having independent thrust vector control but common autopilots. The typical trajectory dependent events which warrant the attention of the control system analyst are identified below.

4.5.1 Stage I

4.5.1.1 Liftoff

Launch drift of the boost vehicle due to high winds, misalignments in the thrust vector, and c.g. offsets present potential launcher clearance problems. Launch from enclosures such as silos which have higher thermal, acoustical, and gas flow environments present even greater need for detailed launch evaluations.

4.5.1.2 Pitchover or Initiation of Gravity Turn

Pitchover does not typically challenge the control system capabilities but provides an opportunity to evaluate the system response and verify the vehicle model.

4.5.1.3 Maximum Wind and Dynamic Pressure Region

The control system operation may undergo large demands in the region of maximum dynamic pressure or due to the presence of high wind shears in the region of maximum winds. Structural loading and control system aerodynamic stability are important evaluation considerations. The types of guidance equations and steering methods employed have a significant bearing on the severity of the wind shear responses.

4.5.1.4 Gain and Filter Changes

As propellant is expended during the stage operation, changes in the vehicle mass properties require changes to the control system gains and possibly the filter configuration to maintain stability margins. These changes are particularly noticeable if large c. g. offsets, thrust vector offsets, and guidance commands are present. One or more of these changes may be required during the operation of a stage operation and they may also be required during subsequent stages of operation.

4.5.1.5 Thrust Tailoff or Termination

During thrust tailoff or termination the parameters of interest are the vehicle attitude and angular rates existing at the start of the staging sequence due to system limit cycles or staging transients. The magnitudes of these parameters will be dependent upon the frictional level in the thrust vector control system, dead-zones and other nonlinearities in the control system, and the thrust level and vector alignment at the time of staging.

4.5.2 Stage II

4.5.2.1 Staging and Engine Startup

The staging transient is perhaps the dominant control system response during the second stage operation. The magnitude of the response is dependent upon the vehicle condition prior to staging and the changes in guidance commands for the new stage. Aerodynamic torques will also be a significant factor in the system response. The clearance problem between stages requires detailed evaluation, depending upon the method employed. If a "fire-in-the-hole" method is employed, that is, if the upper stage thrust is acquired while the engine is enclosed within the interstage or before the lower stage thrust is terminated, the separation of the stages is accomplished quickly. This method, however, is not without its drawbacks in that large interstage pressures develop which may produce sizeable disturbance torques. The resultant angular rates due to the disturbance represent a potential clearance problem if the torques are unduly high.

If a brief period (a second or two) is allowed between the powered phases, and retrothrusting of the first stage is employed, the interstage pressure problems are alleviated; however, separation of the stages will be slower. Undesirable contact of the stages or binding of separation aids, such as guide rails, may require detailed evaluation.

4.5.2.2 Jettison of Interstages

Depending upon the separation plane between the stages, the interstage, or part of the interstage, may be retained on the second stage. Ejection of this section is desirable to obtain better boost performance of the stage. The effect of jettisoning, due to changes in the vehicle mass properties and aerodynamic parameters, may be evident in the vehicle response.

4.5.2.3 Thrust Tailoff or Termination

During thrust tailoff or termination, control system limit cycle parameters are of primary interest due to staging considerations. The considerations are the same as those described previously for the first-stage thrust tailoff phase.

4.5.3 Stage III and Subsequent Stages

The trajectory events affecting the subsequent stages are essentially similar to the Stage II events. Coast phases may be interspersed between these powered phases with reorientation maneuvers, and propellant settling operations may be employed.

4.6 DATA PROCESSING REQUIREMENTS

The great majority of data processing requirements can be defined prior to flight and the evaluation plan should indicate the type of processing, accuracy, and scheduling requirements for all data so that necessary tools and resources are available in a timely manner. The purpose of data processing is to transform the data (which may be gathered prior to or during the flight) from its raw state, as it comes from the receiver, into a form which can be readily and effectively utilized by the analyst. Data may be presented in a large variety of ways, depending on the needs of the analyst and the analysis techniques used. The data can be presented in analog traces, digital tabulations, oscillograph record, etc., and the data may be presented raw, smoothed, or filtered depending on its use. Annotations for significant events may be added to provide the analyst with a convenient frame of reference. Frequently it is desirable to correct the data for errors in transit time between the transmitter and receiving station to allow for comparison of data from more than one source and to place events accurately within a time frame.

The telemetry data stream can be representative of voltage level variations or can be calibrated by use of instrument or sensor calibrations which can be recorded prior to flight. Calibrations can be applied to the data during processing, or calibration scales may be used in conjunction with uncalibrated traces.

The specific techniques of data processing and presentation are a function of the use for which the information is intended, although most data processing requirements can be established prior to the acquisition of the data. Special processing for specific parameters may be desirable due to observed anomalies or malfunctions. Filtering techniques may be required to reduce effects of noise or component malfunctions. Although it is possible to enhance the use of data by various processing techniques, a reduction of the fidelity of data may result by the elimination of useful data points from the data stream.

Telemetered data must be processed to extract parameters in a form that can be readily analyzed. The data processing requirements will depend on the parameters to be observed and the timeliness of its observation. Some parameters can be measured directly by a sensor, while other parameters can only be obtained indirectly by conditioning the data or by use of evaluation programs. The timeliness of the processed data should be in accord with its classification or priority.

4.7 DATA CONDITIONING AND EVALUATION PROGRAM

Additional operation beyond data processing will be required for detailed analysis of the controls data. In some instances programs developed for analysis of flight data may be applicable to real-time evaluation.

Analytical tools, such as mathematical models and analog simulators, which are available can be adapted to the specific evaluation task. Parameter recovery techniques, simulation for comparative evaluations, analytical parameter identification, and malfunction analysis techniques can also be developed or modified to be consistent with the specific evaluation objectives. Any of the parameters measured may be derived or predicted from simulations based on the initial conditions, the equations of motion, and the specific control laws which are mechanized.

The development of simulation programs for control system evaluation which are tailored to a specific requirement of a mission may require long lead times. Therefore, sufficient time should be allowed to develop such a simulation or to modify an existing tool simulation. The simulation programs used during the design analysis phase are frequently applicable to the flight evaluation. If a heavy launch schedule is anticipated, it may be prudent to establish a redundant simulation input deck reserved for flight evaluation purposes, particularly if analog computers are utilized. In many instances flight data can be fed into digital or analog programs to override computer quantities and thus "drive" the program. Generated outputs can be compared to control system outputs. In the case of digital simulation programs, it may also be desirable to establish a special flight evaluation program since considerably more flexibility in the variation of parameters and malfunction simulation capability would be desired over those employed in the design analysis.

The required flight data obtained from metric sources can be processed and then combined in trajectory reconstruction programs to provide input data for controls analysis. Analysis programs should be capable of handling meteorological data to compute the effect of winds on the vehicle dynamics, in addition to angle-of-attack, Mach number, thrust, drag and other aerodynamic parameters are required as inputs for control system analysis. An aerodynamic evaluation program may provide the capability for loads analysis using environmental pressures.

A control network simulation may be used to compute engine commands and other control parameters based on the established input data which may then be either automatically or visually compared with the sensed values. The equations of motion can be solved for acceleration components which may be compared with measured values or may be filtered or transformed for comparison with the guidance system data.

Stability characteristics, i.e., normal force coefficients and center of pressure, may be computed from acceleration, engine deflection, angles-of-attack, and mass properties data. Control system accelerometer data may be transformed to c.g. coordinates then compared with values of acceleration calculated from moment equations and values of angle-of-attack calculated from the normal accelerations. This data is also compared with outputs of angle-of-attack sensors. Calculated or measured parameters may be input into rigid body simulations for comparison of control parameters or in flexible body models for analysis of the vehicle body dynamics.

Programs such as those mentioned above are extremely important, especially when time constraints or manpower limitations prohibit lengthy, tedious, routine analysis of data. Sophisticated programs can be highly effective tools. The planning and implementation of programs for evaluation purposes should parallel design and evaluation planning and implementation.

4.8 RESOURCES REQUIREMENTS

One of the important elements in the implementation of an effective evaluation are the resources required.

A flight evaluation program plan must specify the resources needed for the evaluation and make provisions for their acquisition, procurement, and application to the flight evaluation program. The resources for the flight evaluation program include facilities, equipment, manpower, training, and funding which are necessary for the performance of tasks defined in the program. Attention should be given to these factors at the beginning of the program since fairly long lead times are associated with providing the necessary resources.

The facilities required for evaluation of a control system include adequate space in the control center for real-time display and analysis, a data display room for viewing the data plots as well as office space for the analysis team during each phase of evaluation. Equipment (such as displays, devices, calculators), the allocation of computer time for data processing, and analysis operations should be considered well in advance of the flight test. Special tools such as hardware models and ground test facilities may be required to determine system characteristics for development of models and for simulation as well as to perform postflight tests to verify conclusions of the analysis or to provide additional control system data for engineering analysis.

Manpower resources are also an important consideration. Sufficient personnel must be available to perform the analysis task in an effective and timely manner. Provisions should be made for the personnel required to monitor the flight in real-time and the types of skills and number of man-hours required to perform the review and detailed analysis of the data. Personnel performing the analysis functions should be skilled in control systems performance theory and should possess a thorough understanding of the performance characteristics of the particular control system which is to be evaluated. Further, it is important that the analyst understand the data operations and the simulations and models and other evaluation tools which will be used for the evaluation of the control system. Time for training personnel and their familiarization with the system are important considerations.

Necessary funds to perform the evaluation can be determined on the basis of manpower, facilities, and equipment required.

All aspects of resource allocation and requirement should be considered during the very early phase of the evaluation process. Acquisition of equipment and facilities can be long lead time items. This is expecially true if development and procurement of special tools are required. Manpower requirements should not be overlooked, for although personnel may be obtained on short order, the training and experience required usually take months to acquire before the analyst can perform his task effectively. It is desirable that personnel involved in the development of the analytical simulations and models also be involved in the actual analysis of the system.

5. RECOMMENDED PROCEDURES

This section provides a description of the integrated flight test evaluation program. It defines the interrelation of the steps of such a process and the important considerations of data extraction and evaluation. It also provides a description of the analyses applicable to each phase of the integrated flight evaluation process. This section then discusses the procedures and techniques of flight evaluation.

5.1 PREFLIGHT AND POSTFLIGHT EVALUATION SEQUENCES

The sequence of control system evaluation activities from control system design description to design verification is illustrated in Figure 5-1. Each of the blocks represent a phase or period in which a number of events in the flow take place. The annotations indicate the relative timing of the evaluation phases which might take place for a launch. It should be noted that although a specific time interval requires adjustment for specific programs, the relative phase relationship presented is valid for most launch vehicles. The analyst should consider each item when planning evaluation of a specific control system since they provide a baseline for planning a comprehensive evaluation.

5.1.1 Initiation of Flight Evaluation

The evaluation process begins as soon as the control system design studies define the control system mechanization and performance characteristics and the flight mission is established. During the initial phase of the evaluation process, preflight data should be compiled and reviewed, emphasis being directed toward the diagnosis of control system functions, sequence of flight operations, vehicle mission and trajectory requirements, stability margins, selection of control system gains, and other compensation parameters required to achieve the performance and stability characteristics of the control system. Preflight data should also include predicted response characteristics of the vehicle control or oscillations which may be inherent in the flight system. Frequency responses of the subsystem or component parts are also important information.

LAUNCH MINUS 2 TO 5 YEARS — LAUNCH MINUS 18 MONTHS — LAUNCH MINUS 6 MONTHS

1. INITIATE FLIGHT EVALUATION

- DESCRIPTIONS OF VEHICLE AND CONTROL SYSTEM DESCRIPTION AND PERFORMANCE
- COMPILE AND REVIEW PRE-FLIGHT TEST DATA
- ESTABLISH INSTRUMENTATION, DATA ACQUISITION AND DATA REDUCTION REQUIREMENTS

2. DEVELOPE EVALUATION PLAN

DEVELOP FLIGHT EVALUATION PLAN AND ESTABLISH EVALUATION PROGRAMS

1. DEVELOP MAG TAPES AND DISPLAYS
2. DEVELOP REAL-TIME SIMULATIONS
3. DEVELOP DATA EDIT AND CONDITION PROGRAMS
4. DEVELOP FUNCTIONAL ANALYSIS PROGRAMS
5. DEVELOP ENGINEERING PROGRAMS

IMPLEM

1. DETA
2. DEVE WITH
3. REVIE REQU
4. INITI REQU

LAUNCH PLUS 4 DAYS

5. FLIGHT TEST SUPPORT

1. PARTICIPATE IN REAL-TIME EVALUATION
2. PARTICIPATE IN FUNCTIONAL EVALUATION
3. OBTAIN FINAL CALIBRATION CURVES
4. VERIFY RESULTS OF DATA, EDITING AND CONDITIONING
5. VERIFY OPERATION OF INSTRUMENTS USED
6. SELECT REDUNDANT INSTRUMENT TO USE FOR FAILED INSTRUMENT
7. INITIATE ANY MINOR MODIFICATION TO PROGRAMS TO OVERCOME UNFORESEEN PROBLEMS

6. INITIAL POST-FLIGHT EVALUATION

1. PERFORM FUNCTIONAL EVALUATI
2. BEGIN ENGINEERING ANALYSIS
3. SET UP SIMPLE PROGRAMS TO EVAL UNEXPECTED PROBLEMS OR ANOM
4. PROVIDE FEEDBACK OF DATA THRU NORMAL CHANNELS
5. PROVIDE FEEDBACK OF RESULTS OF ANALYSIS THRU REVIEW MEETINGS
6. PINPOINT POTENTIAL PROBLEM AR

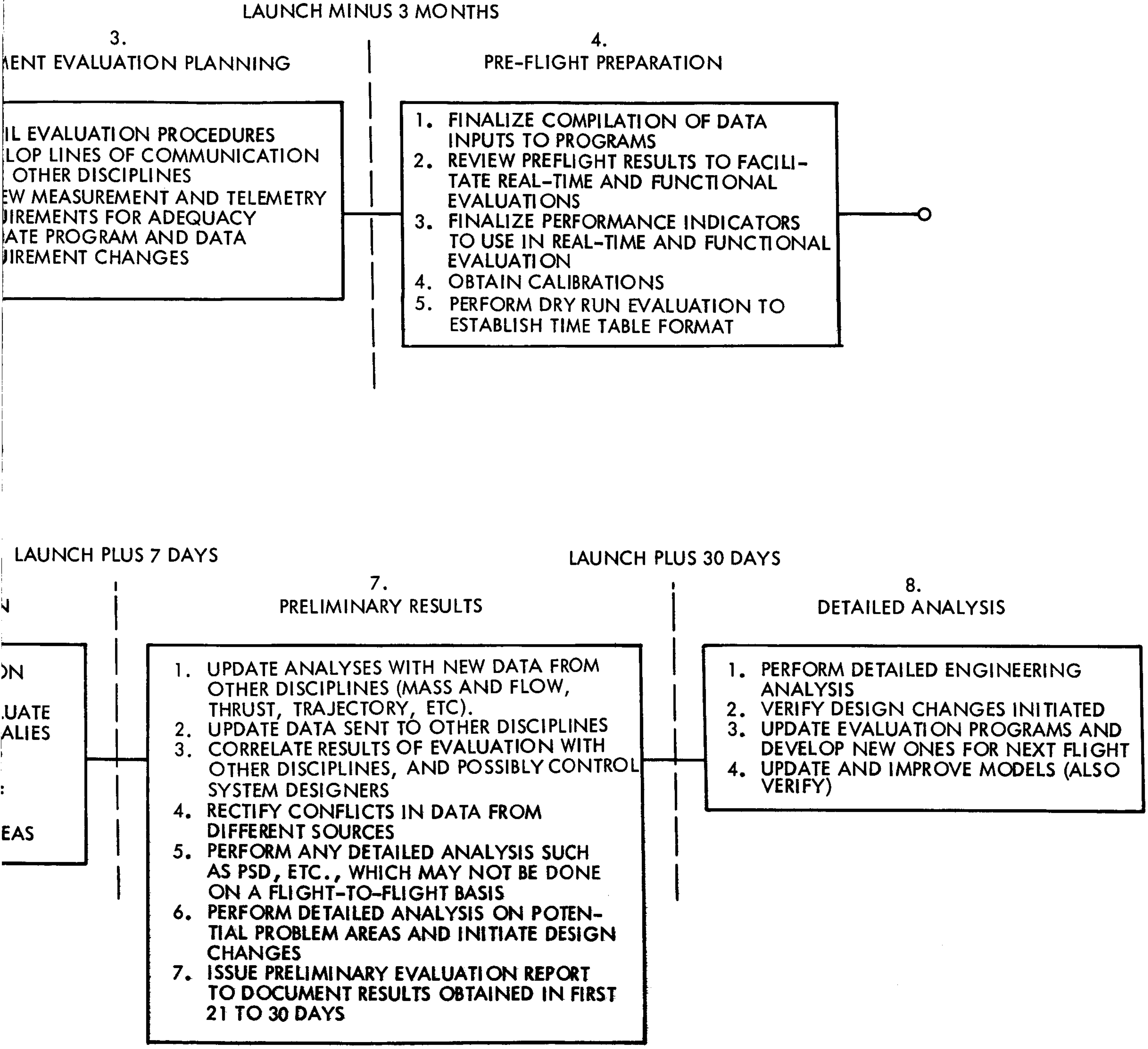

Figure 5-1. Control System Evaluation Sequence

If the control system to be tested is of a known design and has previously undergone flight test operations, the necessary preflight data will be readily available; however, for a new or modified control system or in cases where the mission or flight vehicle is grossly different from previous experience, design analysis may be the only source of preflight data. Frequently in such cases preflight tests are utilized to provide confirmation of design analysis and evaluation models. Data obtained from such tests are significant, and provide insight for planning subsequent preflight tests and prelaunch checkout and design information. Consideration should be given to parameter measurements, data acquisition, data handling, and analysis during control system design since the measurements which can be made, and the instruments and/or points of data extraction, are often established or constrained by the design of the control system and the mission. Usually, compromises must be made in the number of instruments and channel selections transmission requirements and bandwidth of data which are allocated for control system signals. Since evaluation effectiveness is strongly dependent on the adequacy of the data available, preknowledge of the postflight evaluation requirement is necessary during design. Limitations of the instrumentation may result in limitation on the flight evaluation. Instrumentation selection requires an awareness of critical or marginal design areas and potential mission related control system responses.

5.1.2 Development of Evaluation Plan

The next step and possibly the most important phase of a successful evaluation program is the development of the flight evaluation plan and the tools necessary to conduct the desired analysis. Flight evaluation plans provide the framework for the accomplishment of the evaluation. The plan will define the program, including the detailed objectives of the mission and the evaluation, prescribe the data to be obtained from the flight, and establish the requirements of priority, schedule and resources. This phase should begin sufficiently early, as much as a year prior to the intended operation, to allow for the timely development of long lead-time items such as: a) the development of procedures necessary for flight support and postflight analysis (i.e., provisions for data handling, recording and display); b) programs for data editing, conditioning, and presentation; c) simulation for prediction and real-time analysis aids; d) functional and engineering analysis routines and program; and e) the performance preflight tests which may be necessary to define the control system characteristics and performance. Preflight tests may also be required while the system is in a breadboard state or during assembly of the system or vehicle. Significant preflight tests may include polarity tests, state gain checks, determination of subsystem frequency responses and rate limits, calibration of signals to be instrumented, and determination of bias and system errors.

5.1.3 Implementation of Evaluation Plan

After an evaluation plan has been established and the development of the evaluation tools initiated, the next phase of the process is the implementation of the plan which involves the development of procedures and interface relationships. Iterations on planning shall take place as changes are defined.

Detailed step-by-step procedural plans should be formulated. The procedures will define the detailed methods and practices which are necessary to accomplish the objectives of the evaluation. Interface between the control system and other subsystems evaluation functions should be well defined and lines of communication with the other disciplines established. If conflicts exist they should be resolved and any revisions in mission or design should be fed back into the planning requirements, as basic data or analysis tools may change. Telemetry and other measurement requirements should be continuously reviewed for adequacy. Any changes should also be reflected in the development of the analysis tools, since it is important that the analysis techniques are compatible with the system and the data to be analyzed.

5.1.4 Preflight Preparation

During the phase of the evaluation process immediately prior to the actual flight operation, the procedure, methods and techniques for performing evaluation are finalized and preflight data, which are dependent on the final system configuration and on the availability of hardware are acquired. Data from prelaunch checkout, system test, final instrument calibrations, and information on hardware deviations will be reviewed to determine the possible effects on performance indicators. Final preflight data should be used to update the simulations and evaluation program. A careful review of the final preflight data package and analysis tools prior to flight will enhance the effectiveness of the real-time evaluation and functional analysis.

During the preflight period the hardware will undergo final preparations for flight. It is also necessary that final checkout of the system software be accomplished at this time. A dry run of the control system evaluation techniques should be performed to confirm the adequacy of the evaluation process, establish the time table for evaluation tasks, and confirm lines of communication in a near real environment. A test of the complete evaluation process prior to launch will allow for fine tuning of the operation and will establish a sequence for evaluation of flight measurements.

5.1.5 Flight Support

The launch operation period normally begins with the initiation of the countdown and continues until all control or mission functions are complete. During this period the flight evaluation is initiated. Control system analysts and analytical tools are an integral part of the real-time evaluation support of the launch. The real-time observer will monitor performance of the control system, verify the operation of instruments, select redundant instruments if instruments anomalies are noted, and review the data to verify that acquisition, handling, editing, and conditioning provisions are adequate. If unforeseen problems develop, recommendation of mission alternatives or modification to data processing and evaluation programs will be made. Final calibration data and assessment of measurement error should be made during preflight and flight operations.

Functional analysis is also initiated during this period. The observations and data collected in real-time are fed directly into functional analysis. By the end of the launch operation a preliminary or gross assessment of the performance of the control system in its operational environment should be made. The quick-look assessment is normally made the first day.

5.1.6 Initial Postflight Evaluation

The first phase of the postflight evaluation process normally takes place during the first week after flight operation. The detailed functional analysis should be completed and the engineering analysis begun during this time. The functional analysis should emphasize verification of the system performance in its operational environment and determine the results of any observed departures of the control system, its operation performance or environment from the mission plans. Malfunctions, anomalies, and problem areas should be pinpointed. Simple or minor modifications to existing evaluation programs may be initiated for analysis of unexpected problems. If the nature of a problem is subtle, detailed evaluation will be continued in the engineering analysis. Feedback of the results of real-time analysis and functional analysis will provide important data for the evaluation of systems interfacing with the control system. Feedback should occur through the normal channels of communication and early review meetings and preliminary reports. The analysis performed directly after the flight provides insight into the selection of special data processing and conditioning requirements which will expedite the engineering analysis.

5.1.7 Preliminary Results

The next step in the process is concerned with preliminary results and refinement of the evaluation. The control system analysis should be updated based on the additional data acquired by conditioning control system data and the data from interfacing systems and other disciplines such as guidance, propulsion, mass properties, and reconstructed trajectory parameters. As soon as revised or updated control system data becomes available, they will be correlated with the results of analysis of other disciplines and undergo intensive review by control system evaluation and design specialists. Conflicts in data from different sources will be resolved and rectified promptly.

Noted problem areas result in the initiation of design changes in the hardware components or evaluation programs for future missions as soon as the analysis indicates changes are warranted. The results of the preliminary evaluation are normally documented within the first month after the flight.

5.1.8 Detailed Analysis

The final phase of postflight evaluation includes performing comprehensive engineering analysis, modifying control system design characteristics, and updating or improving evaluation programs. Changes in hardware or software result in planning changes for future missions and can feedback into the evaluation process flow at any point. Major changes are normally verified by ground test. Results of the engineering analysis should be documented at specified intervals after launch. However, significant results which may affect design changes or the state-of-the-art should be documented as soon as possible to allow for their expedient and effective use.

5.2 DATA REQUIREMENTS

Data requirements are established during the initial phase of the evaluation process. The requirements for control system data are based on the projected mission, evaluation objectives, and control system characteristics. Control system evaluation parameters will be directly measured or calculated based on direct measurement. These parameters are obtained from interfacing systems as well as the control system. The selection of measurements will determine quality and accuracy of the analysis. Applicable control laws and mechanization will determine the measurements which can be made and the analytical tools required for analysis.

5.2.1 Indices of Performance

In order to perform an effective evaluation of the control system, the control system data must be sufficient to satisfy the objectives of the evaluation. Applicable data from interfacing systems are also required. Interfacing systems, such as the propulsion, structural, or separation systems, are independent of the control system; i.e., provide forcing functions or environmental conditions which affect the performance and operation of the control system. Other interfacing systems such as the electrical, hydraulic, guidance, or navigation systems are interdependent or perform an integral part of the control system operation.

Typical control system performance indicators and evaluation parameters are described in the following paragraphs.

5.2.1.1 Stability

A basic control system performance index is stability. Its criteria are highly dependent upon the particular vehicle dynamic mode of interest. Oscillations observed in vehicle attitude caused by propellant slosh may be considered acceptable if the amplitudes of oscillation are within expected limits (even though they may be a manifestation of an unstable system). On the other hand, excitation of a stable mode (such as a bending mode) may be considered unacceptable if the amplitudes are large, due to structural considerations. Therefore, stability must be viewed in terms of the potentially deleterious effects of an unstable system upon total vehicle performance. Emphasis is normally placed on the evaluation of the evidence of instability rather than on determination of system stability and performance, unless a specific inflight test is performed for these purposes. However, when a control system or launch vehicle configuration change takes place on a flight-to-flight basis, it becomes increasingly important to analyze all performance indicators to verify models and assumptions used in design assurance studies. The reason for this is that such changes may tend to aggravate conditions which would otherwise be marginal.

5.2.1.2 Response

Comparison of actual vehicle response and expected response to commands provides a measure of control system performance. An example of this is the vehicle response after the initiation of a pitch program (or gravity turn phase). Since the pitch program usually begins at low vehicle velocities, aerodynamic effects are small and do not adversely affect control system performance evaluation. If the rigid body response is clearly evident, or if it can be made so by proper filtering of data, the rigid body frequency and damping factors can be computed.

5.2.1.3 Attitude Error

Attitude error is one of the parameters used in evaluation of the general performance of the control system. Small attitude errors (ϕ_e) invariably are indicative of proper performance of the control system. Large discontinuities may point to problems in the guidance commands (ϕ_c) or the attitude feedback signals (ϕ). If the data are oscillatory, the amplitudes of the oscillations may be a factor in launch abort decisions. Attitude error, in conjunction with the vehicle angular rates ($\dot{\phi}$), provides the foundation for evaluation of control system performance, especially where an analog control system is employed.

5.2.1.4 Attitude Rates

In the observation of dynamic mode oscillations (i.e., sloshing, bending) where the frequencies may exceed 1/6 Hertz, the attitude rate signal ($\dot{\phi}$) will provide a better evaluation parameter than the attitude error signal (ϕ_e), since the trace amplitude and the signal-to-noise ratio will be greater. Bending oscillations may be clearly visible on the attitude rate trace, but the bending oscillations may be indistinguishable from the noise levels observed.

If the vehicle rate information is derived in the vehicle computer by differencing attitude angles, quantitization effects will be incorporated in the data; careful filtering of data are required to reconstruct the oscillation amplitudes.

5.2.1.5 Engine Deflection

Data on engine deflection may be obtained either from actuator position measurements or by direct measurement from potentiometers located on the nozzle. Comparing measured engine deflections (β), it is possible to determine the actual control system gains and also provide information on such parameters as stiction or friction in the engine gimbal unit. Sloshing characteristics such as sloshing frequencies may also be observed in the data. By application of filters to the data, the individual contributions of sloshing may be determined. Differential position in coordinate engines or null offset observed in the engine position may be indicative of thrust misalignment.

Engine deflection data also provides a checkpoint for analysis of the hydraulic system; anomalies such as hydraulic pressure transients may not be reflected as engine deflections. Such anomalies could indicate questionable transducer data. A corresponding deviation in nozzle dynamics may be indicative of a hydraulic system problem.

5.2.1.6 Angle-of-Attack

The angle-of-attack of a flight vehicle can be determined either by a direct measurement with an angle-of-attack (β) transducer or through a trajectory reconstruction. Angle-of-attack is used along with dynamic pressure and winds data to determine aerodynamic loading on the vehicle and provides information on forcing functions required for analysis of bending modes and body dynamics.

5.2.1.7 Optimization Criteria

Deviation of the system performance from a given optimum criteria may be employed as a measure of control system performance. For example, the amount of propellant used above the ideal usage in a reaction jet roll control system indicates the performance of the system and the amount of disturbance impulses encountered; or that the integral of the error squared, such as an attitude error, may be employed to give an overall flight performance index. This integral may not be restricted to one parameter, but it may be applied to a combination of several parameters to obtain the performance index.

5.2.1.8 Aerodynamic Response and Stability Derivatives

Verification of the aerodynamic parameters employed in control system analysis insures that the aerodynamic stability margins have not been degraded. The response of the vehicle to known winds (V_W), or to attitude commands (ϕ_C), within the high aerodynamic pressure (Q) phase of flight, enables assessment of the aerodynamic parameters (C_n, X_{cp}). Reconstruction of the aerodynamic pressure (Q), total angle of attack (α_T), and aerodynamic force (F_n) time histories are required to obtain the aerodynamic normal force coefficient (C_n) time history. Use of filtered angular acceleration ($\ddot{\phi}$) information will enable computation of the aerodynamic center of pressure (X_{cp}) time history. Verification of aerodynamic characteristics becomes extremely difficult for regions of very small angles of attack.

5.2.1.9 Thrust Profile

A knowledge of the vehicle thrust (T) time history is a necessity in the evaluation of control system performance. Errors in the determination of thrust levels may produce erroneous conclusions on the performance of the control system and in the evaluation of related parameters. As a part of the reconstruction of the thrust profile from on-board data and radar tracking data, aerodynamic drag coefficients are obtained. Comparison of these coefficients to those employed in trajectory analysis provides verification of the parameters. The differences may reflect the magnitude of error that may be expected in the aerodynamic axial or drag force coefficients.

5.2.1.10 Body-Bending Modes

The existence of unstable body-bending modes or the excitation of stable modes producing vehicles oscillations will be detectable through the higher frequencies in the signals from control system sensor (ϕ, $\dot{\phi}$, $\ddot{\phi}$, Z_{LA}). The oscillations will be more visible from angular rate ($\dot{\phi}$) and acceleration ($\ddot{\phi}$) sensors, if these are located near the nodal points. The oscillations will be more visible from lateral accelerometers ($\ddot{Z}_{LA}$), if they are located near the antinodes. The outputs of these sensors enable evaluation of the severity of the bending oscillation and verification of the bending stability margins when compared to design analysis results.

5.2.1.11 Propellant Sloshing

The existence of unstable or neutrally stable propellant slosh modes or stable modes which undergo excitation is also manifested by vehicle oscillations which are detectable by the control system sensor (ϕ, $\dot{\phi}$, $\ddot{\phi}$). The oscillations will generally be of a rigid body nature and recognizable by its frequency, unless an extremely low frequency bending mode near the slosh frequency is present. Therefore, the location of the control sensors will not generally affect sensing of the oscillations produced by propellant sloshing. The sensor output signals provide a means for verification of control systems analysis and peak-to-peak oscillation amplitudes and their frequencies are employed as evaluation parameters.

5.2.1.12 Vibration and Acceleration

In initial flights of a booster vehicle, considerable vibration-measuring strain gauges are attached to the vehicle to determine the stresses applied to critical surfaces. The outputs of these gauges are an aid in determining the noise content detected by the control system sensors. Refined analyses, such as a spectral density analysis of the control sensor outputs, may be performed to determine the frequency spectrum of the noise. If the sensors are located near the main engine, the amount of acoustical transmission through the sensors may also be determined.

5.2.1.13 Injection Accuracy

Since the primary purpose of a booster vehicle is to inject a payload into an orbital or suborbital trajectory, a measure of guidance and control system performance is the injection error developed at the end of the boost phase. Generally, a small error is an indication of good performance by the guidance and control system. However, it is possible that poor performance of the control system can be compensated for by the guidance system since the guidance loop is external to the control system. The reverse is not possible since the control loop does not generate trajectory steering commands. The guidance loop is a low frequency compensator and poor performance of the control system in the form of high frequency oscillations will not be adequately compensated. Hence, significant injection errors due to high frequency control system oscillations may occur.

5.2.2 Measurement and Calculated Data

The measurement requirements must be considered with respect to the mechanization of control laws, the availability or source of signals, and the minimum number of parameters which will allow for the determination of sufficient performance factors to satisfy the objectives of the evaluation. All of the desired data need not be measured directly; some performance parameters can be calculated from combinations of measured and predicted parameters. For example, if an analog control system is to be evaluated, the attitude error, body rates, and gain factors are primary parameters. However, if a lead-lag filter is employed in the design mechanization to provide the rate feedback signals based on attitude position data, there may be no actual rate measurement available. For evaluation purposes, desired rate data can be deduced from postflight calculations using attitude position data or may be obtained from a secondary source such as a guidance system, a forward stage which may employ rate gyros, or other independent monitoring devices.

When specifying the parameters required for evaluation, the analyst should include the anticipated range variations of measurement, accuracy, response frequency, and priority of each measurement.

A typical list of the minimum data required for the control system evaluation of Saturn V boost vehicles is shown in Table 5-1. Table 5-1 includes a list of pertinent parameters, obtained from direct measurement or calculated from measured data, and the priorities of the measurements. The measurement source of data for calculations and the analysis for which the parameter are significant are also shown. Parameters which can be deduced from other data sources or which provide redundant or back-up capability are classed as highly desirable or desirable. Those parameters which are primary measurements for minimum control system evaluations are listed as mandatory measurements.

5.3 RAW DATA PROCESSING

Raw data is usually accompanied by noise, or other distortions in transmission, and some kind of processing is normally required before the data can be used effectively in an evaluation program. The kind of processing depends on use of the data and time factors involved. Considerations of cost also influence the kinds of processing applied. These factors influence data treatment in formatting, display, conversion, recording, collating, and storing.

It is clear that time dictates the type of data treatment feasible. Evaluations in real-time, or near real-time, usually limits the type of processing to simple filtering and smoothing, with display of the data on an oscilloscope or by a plotter. More refined analysis (for example, postflight analysis), which may be delayed days or weeks, can make use of more sophisticated techniques, such as statistical analysis.

Data filtering is discussed in a subsequent paragraph and in Appendix A. It may be noted, as indicative of the time factors involved, that in the Saturn V program, Andrus filtering techniques are applied at any Δt intervals, while smoothing techniques (Ormsby, Graham) are applied at any equidistant Δt intervals.

For refined analysis of data, statistical analysis is the most useful tool. It uses the theory of random sampling (to avoid possible periodic

Parameter	Symbol	Unit	Nominal		Da
Range Time		Sec.	Measured	Computed	
Pitch Attitude Error	ϕ_{ep}	Deg	X		
Yaw Attitude Error	ϕ_{ey}	Deg			
Roll Attitude Error	ϕ_{er}	Deg			
Roll Attitude Error		Volts dc			
Yaw Attitude Error		Volts dc			
Pitch Attitude Error		Volts dc			
Pitch Angular Velocity	$\dot{\psi}_p$	Deg/Sec			
Yaw Angular Velocity	$\dot{\psi}_y$	Deg/Sec			
Roll Angular Velocity	$\dot{\psi}_r$	Deg/Sec			
Pitch Acceleration, I.U.(1)	a_p	G			
Yaw Acceleration, I.U.	a_y	G			
Longitudinal Accel., I.U.	$\ddot{Z}$	G			
Longitudinal Accel., I.U.	$\ddot{Z}$	G			
Longitudinal Accel., S-IC	$\ddot{Z}$	G			
Pitch Acceleration, S-IC	a_p	G			
Yaw Acceleration, S-IC	a_y	G			
Pitch Rate, S-IC	$\dot{\phi}_p$	Deg/Sec			
Yaw Rate, S-IC	$\dot{\phi}_y$	Deg/Sec			
Roll Rate, S-IC	$\dot{\phi}_r$	Deg/Sec			
Pitch Rate EDS(2) Group 1	$\dot{\phi}_p$	Deg/Sec			
Yaw Rate EDS Group 1	$\dot{\phi}_y$	Deg/Sec			
Roll Rate EDS Group 1	$\dot{\phi}_r$	Deg/Sec			
Pitch Rate EDS Group 3	$\dot{\phi}_p$	Deg/Sec			
Yaw Rate EDS Group 3	$\dot{\phi}_y$	Deg/Sec			
Roll Rate EDS Group 3	$\dot{\phi}_r$	Deg/Sec			
Delta Pressure Pitch, Q-Ball	$\Delta P_{a p}$	PSID			
Delta Pressure Yaw, Q-Ball	$\Delta P_{a y}$	PSID			
Vector Sum, Q-Ball	$\Delta P_{a T}$	PSID			
Position Pitch Actr. No. 1	β_{p1}	Deg			
Position Pitch Actr. No. 2	β_{p2}	Deg			
Position Pitch Actr. No. 3	β_{p3}	Deg			
Position Pitch Actr. No. 4	β_{p4}	Deg			
Position Yaw Actr. No. 1	β_{y1}	Deg			
Position Yaw Actr. No. 2	β_{y2}	Deg			
Position Yaw Actr. No. 3	β_{y3}	Deg			
Position Yaw Actr. No. 4	β_{y4}	Deg			

FOLDOUT FRAME 2

ta Source	Priority			Evaluation
	I			Autopilot performance, stability, response
	I			
	I			
		II		
		II		Backup for Angular Error Data
		II		
	I			Stability, response, body dynamics
	I			
	I			
		II		Thrust misalignment, angle of attack, aerodynamics, thrust, dynamics, sloshing
		II		
		II		
		II		Backup, separation dynamics
		II		
		II		
		II		
		II		Autopilot performance, stability, response, sloshing, separation dynamics
		II		
		II		Alternate to angular velocities
			III	Useful as backup information
			III	
			III	
			III	
			III	
			III	
	I			Angle of attack, trajectory and loading analysis
	I			
		II		Check on angle of attack
	I			TVC performance, gains stability, body dynamics, hydraulic
	I			
	I			
	I			
	I			
	I			
	I			
	I			

Table 5-1
Saturn V Control System Data Requirements

FOLDOUT FRAME 1

Parameter	Symbol	Unit	Nominal	
Range Time		Sec.	Measured	Computed
Valve Current Pitch Act. No. 1	i_{p1}	Milliamp	X	
Valve Current Pitch Act. No. 2	i_{p2}	Milliamp	↓	
Valve Current Pitch Act. No. 3	i_{p3}	Milliamp	↓	
Valve Current Pitch Act. No. 4	i_{p4}	Milliamp	↓	
Valve Current Yaw Act. No. 1	i_{y1}	Milliamp	↓	
Valve Current Yaw Act. No. 2	i_{y2}	Milliamp	↓	
Valve Current Yaw Act. No. 3	i_{y3}	Milliamp	↓	
Valve Current Yaw Act. No. 4	i_{y4}	Milliamp	↓	
Mach Number	M	NONE		X
Dynamic Pressure	Q	Newton/ CM^2		↓
Total Pitch Engine Gimbal Angle	β_{PT}	DEG		↓
Total Yaw Engine Gimbal Angle	β_{YT}	DEG		↓
Total Angle of Attack	α_T	DEG		↓
Pitch Angle of Attack	α_p	DEG		↓
Yaw Angle of Attack	α_y	DEG		↓
Pitch Angular Accel.	$\ddot{\psi}_p$	Deg/Sec^2		↓
Yaw Angular Accel.	$\ddot{\psi}_y$	Deg/Sec^2		↓
Roll Angular Accel.	$\ddot{\psi}_r$	Deg/Sec^2		↓
Eng. 1 Pitch Rate	β_{p1}	Deg/Sec		↓
Eng. 2 Pitch Rate	β_{p2}	Deg/Sec		↓
Eng. 3 Pitch Rate	β_{p3}	Deg/Sec		↓
Eng. 4 Pitch Rate	β_{p4}	Deg/Sec		↓
Eng. 1 Yaw Rate	β_{y1}	Deg/Sec		↓
Eng. 2 Yaw Rate	β_{y2}	Deg/Sec		↓
Eng. 3 Yaw Rate	β_{y3}	Deg/Sec		↓
Eng. 4 Yaw Rate	β_{y4}	Deg/sec		↓

(1) I.U. = Inertial Unit

(2) EDS = Emergency Detection System

Data Source	Priority			Evaluation
		II		Component Performance Backup data for Actuator Performance
		II		↓
		II		↓
		II		↓
		II		↓
		II		↓
		II		↓
		II		↓
Best estimate trajectory from optical or tracking				Aerodynamics, trajectory, loads ↓
Actuator Position Data				Thrust misalignment, TVC performance, gains ↓
Q-Ball or acceleration data				Aerodynamics and loads, winds, stability ↓
Angular Velocities Data				Autopilot performance, stability, response, sloshing, dynamics, engine commands ↓
Actuator Position Data				TVC performance, sloshing, response ↓

Table 5-1 (Continued)
Saturn V Control System Data Requirements

bias errors) and sophisticated techniques for estimation of parameter values. Furthermore, statistical analysis may also provide data sampling methods which are optimal. An example of the latter is Kalman filtering, a widely used and successful data analysis tool. Kalman filtering has the advantage of being optimal in the sense that if X_{obs} is a random observation of a parameter x, and $\hat{x}$ is an estimate of the actual value of x, then $(\hat{x}-x)^2$ is minimized which itself is the mathematical model designed to find the optimum estimate x.

Data received is usually subject to certain disturbances, called "noise", which may obscure the signal being transmitted. If the general characteristics of the signal are known, it is often possible to remove unwanted disturbances of the signal by electronic or digital devices called filters, which perform the function of "smoothing" the data. For example, if the data is transmitted within a certain band of frequencies, those frequencies which are significantly higher or lower than the desired frequencies may be eliminated. In general, filters are designed to pass specified frequencies with prescribed gains, reject all other frequencies in a transmitted signal, and thus "clean up" or smooth the transmission so that the desired signal is easily recognized.

Use of filtering will add to the cost of data acquisition and information retrieval. This cost must be weighed against the advantages to be gained. There are no straightforward answers to this question, since no filter will completely accomplish the task desired. Visual inspection of raw data may suffice in making a decision, especially where time is critical.

The basic tool for the design of filters is Fourier analysis. The desired behavior is represented in the frequency domain by a function H (ω), which specifies the gain H as a function of frequency ω. Signal behavior in the time domain is then derived by applying the inverse Fourier transformer to the function H (ω). Details of this design process are described in Appendix A, and only their salient features will be described here.

The basic elementary filter is the low pass filter, which is designed to reject all frequencies except those within a specified range. It transmits the latter with a specified constant gain. The maximum gain error in the low-pass region can be kept as low as 1/2 percent if the total number of data points are properly selected. Since discontinuities are undesirable, the low pass filter adds a roll-off region on each side of the desired frequency interval, producing the appearance shown on Figure 5.2. The choice of roll-off function is at the discretion of the analyst.

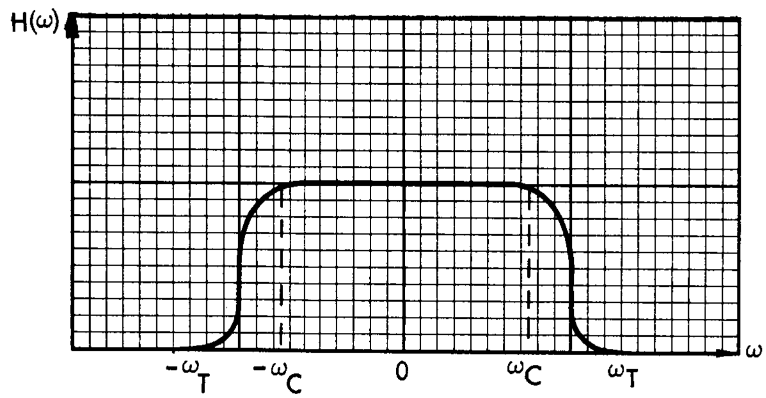

Figure 5-2. Low Pass Filter Gain Vs. Frequency Characteristic

Other filters can be obtained from this basic unit. A high pass filter is the complement of a low-pass filter; i.e., their sum is an all pass filter (one which passes all frequencies). A band pass filter is the difference between two low pass filters. Various combinations of this kind enable the designer to achieve any desired filter.

If data is not transmitted or sampled continuously, but signals are received only at discrete time intervals, the foregoing procedures must be modified. In this case, the integrals involved in the Fourier analysis must be replaced by finite sums which approximate the integrals. Since these sums can use only data values at the given discrete times, a weight must be assigned to each element of the sum. Procedures for doing so are detailed in Appendix A. The resulting filter is called a digital filter, since it cannot be implemented electronically, but requires a digital computer to perform the necessary algebraic operations. Digital filters have the advantages detailed in Appendix A. Among them are higher fidelity, freedom from phase shifts and feedback, and the capability of being changed almost instantly. In the use of digital filters, it is important that data be sampled at a rate at least twice that of the highest expected signal frequency, because of spurious effects introduced at lower rates due to the basic Fourier analysis involved.

Filtering of data may be performed electronically or numerically with a digital computer. Unwanted noise and high frequency oscillations may compromise the preciseness of the evaluations; however, if a low frequency analysis is being performed, visual filtering of the results may be satisfactory. If a comprehensive evaluation of vehicle parameters is desired, the method of filtering must be considered. For sampled or commutated data, there is little choice but to employ numerical filtering. If the data are in a continuous form, electronic filtering is possible and particularly desirable for studying high frequency modes. The phase-shifting effects of an electronic filter must be accounted for by employing the same filter configuration for all signals.

If these measured data signals are compared with simulation or analytically generated parameters, the generated parameter should also be filtered with the same configuration filter to provide consistent results.

Numerical filters, designed to pass or exclude frequency bands contained within a signal, appear to be more useful for control system performance evaluation purposes than the curve-fitting or data-smoothing type of filter. This is primarily because the frequencies of interest are usually known before launch, and oscillations contained in the signals are not random in nature.

5.4 REAL-TIME EVALUATION

Real-time evaluation includes analysis of the control system conducted during the launch operations which involve monitoring of events and performances as they occur.

5.4.1 Objectives

The objectives of real-time evaluation are to verify the integrity of the system by providing a real-time monitoring of the vehicle launch operation which may be required for range and/or astronaut safety and alternate mission decisions and to provide the initial input to the functional and engineering analysis which take place following the launch operation. Due to these factors, it has become increasingly important to link control system analysis to the real-time monitor program. This is especially true for manned space flight.

5.4.2 Analysis Methods and Variations

Real-time evaluation is accomplished by extensive parameter variation analysis in which malfunctions of both low and high criticality are evaluated for their potential effect on the launch operation and their possible impact on crew safety. Since the lives of the astronauts may be at stake and since it is undesirable to inadvertently abort a successful launch due to false flight indications, considerable analyses are required prior to launch to insure that both requirements are met with a high level of confidence.

Each flight and each phase of flight present different constraints on the control system and must be thoroughly analyzed prior to flight to determine meaningful indicators for real-time monitoring. One of the basic problems involved is the time constraint in making proper abort decisions. For instance, if a significant oscillation or divergence ensues in flight, several questions arise in the mind of persons monitoring the launch: What is the nature of the oscillation or divergence? Is it bounded and not harmful? If a subtle malfunction occurs will it endanger the crew or range safety, thereby requiring an abort decision?

The answers to these questions must be resolved almost instantly if an abnormal bending oscillation occurs, within seconds if an abnormal sloshing oscillation occurs, and perhaps longer if a slow divergence is observed. The time constraint is related inversely to the frequency of oscillation or rate of divergence.

Prior to launch the control sensors are monitored to determine status and flight worthiness of the control system. Typical signals monitored during the flight are vehicle attitude and rate signals issued by the control sensors, engine commands, and engine deflection signals. Allowable oscillation amplitudes and boundaries on these signals can be established through design studies or from previous tests. The ability to discern the nature of the oscillations requires knowledge of the control system and will be primarily dependent on launch support crew training with emphasis on the importance of the relationship between real-time monitoring and control system analysis. In Reference 9, the differences between the commanded engine angles and the resulting engine angles are employed as a malfunction indicator. The control system monitor is trained to interpret excessive differences between predicted and sensed parameters, buildup rates or unusual fluctuations in system pressures which may occur. The flight control officer must be alerted in the event than anomalous behavior is indicated.

Other parameters which should be monitored in real-time are the vehicle angle-of-attack which can be measured by an angle-of-attack meter, or special accelerometers which provide a measure of aerodynamic loads on the vehicle structure. Launch abort decision may result if these parameters become excessive. The magnitude of angle-of-attack plus the rate of change of the angle may be employed to obtain additional lead-time for making abort decisions. The rates can be calculated through lead lag filters, or by filtering low frequency body angular rates, which are approximately equal in magnitude to rate of change of angle-of-attack.

The monitoring program is facilitated if the performance of the two basic components of the control system, the autopilot and TVC system, can be evaluated separately. The control sensor signals and guidance commands may be inserted into a computerized set of autopilot equations and the resulting engine deflection commands compared with the telemetered engine commands. The monitor can then base his judgment upon the differences exhibited. Similarly, the telemetered engine commands can be inserted into a computerized set of TVC equations, and the resulting engine deflections compared with the telemetered engine angles. With this implementation, malfunctions within these control components can be rapidly detected, frequently before they are sensed by the vehicle. Problems of telemetry and calibration errors must be considered and accounted for in establishing the allowable margins for comparing differences. The problem of temporary telemetry signal dropout must also be considered. The information obtained shortly after the dropout should be discounted.

To close the control system loop, the telemetered engine angles can be inserted into a computerized set of vehicle rigid-body dynamic equations, and the resulting attitude angles and rate may then be compared with the telemetered data. This would enable detection of control sensor failures or failures external to the control loop, such as breakage of an actuator arm. The existence of aerodynamic effects poses a problem in this instance. Two approaches to overcome this problem are feasible. One method is to reconstruct the angle-of-attack during the flight, based on wind data inserted into the program prior to the flight, and based on presumed velocity data or those reconstructed during the flight. The effects of aerodynamic pressure can then be included in the vehicle dynamic equations, utilizing a priori knowledge of all other aerodynamic parameters. A second method is to include the nominal predicted angle-of-attack, without winds, into the dynamic equations and allowing differences in the attitude angles based on predicted winds. This latter scheme is particularly attractive if manned launches are constrained to relatively low wind conditions.

These implementations would be employed in conjunction with the normal signal monitoring procedures as an aid in making abort decisions in the presence of control system malfunctions.

The extent to which control system evaluation is conducted in real time is dependent on the objectives of the evaluation and the evaluation should be tailored to the needs of the program. Those functions which are not performed in real time should be done in the functional analysis process. For example, TVC, autopilot, rigid-body dynamics are analyzed during functional analysis phase for Saturn launches.

5.5 FUNCTIONAL ANALYSIS

Functional analysis includes the gross analysis of the control system conducted shortly after the operation of the system and involves the review of raw or processed data.

5.5.1 Objectives

The objectives of functional analysis are to verify the integrity of the system and establish how the control system performed all required events and functions. Functional analysis may also uncover subtle malfunctions and performance anomalies, should they occur.

After a launch, a functional analysis is conducted and preliminary conclusions reached on the vehicle and control system performance. This is generally initiated no later than a day after launch. A brief report may be issued giving the flight performance and citing evident problem areas. The data recorded from the real-time monitoring program, including any processed data, are scrutinized, particularly in phases where anomalies may have been noticed during flight. Critical areas and weak points in the control system bear close examination, and a list of these would be an important aid to the evaluation. This list would vary depending upon the mechanization of the control system.

5.5.2 Analysis Methods and Variation

Completion of the functional analysis supports the flight evaluation report issued on a preflight basis, generally within a month after launch. Due to the time constraint, it usually does not include the detailed analysis evaluation, which entails parameter variations in simulation studies. The evaluations included are primarily those which were not performed in the real-time analysis due to time limitations and those which were performed but required a more precise or careful evaluation, particularly if a malfunction occurred. It may be possible, in the functional analysis, to isolate a malfunction in a particular system.

The data used for functional analysis may be raw or edited, filtered, smoothed, or conditioned, depending on the time urgency of the analysis. Oscillograph records with calibration plots are usually available immediately after the data is acquired. Calibrated and annotated data require more time for preparation.

The measured data will be compared with expected values and checked for performance of events and execution of commands and discrete signals.

5.5.2.1 Control System Performance Evaluation

In support of the flight report, it is desirable to plot or record the observations and compare them with expected results. Among these observations may be:

- Vehicle pitch-over response and responses to other distinctive commands.
- Peak-to-peak amplitudes and frequencies of oscillations. In tests where the control system undergoes a programmed excitation, in order to determine its characteristics and

the dynamic model of the vehicle, the amplitudes and frequencies of oscillation are of particular interest and value for comparison.

- Rigid body oscillation. Rigid body oscillations are often observed due to the presence of high coulomb friction in the actuation system. The effects of these oscillations may also be seen in the guidance velocity error traces toward the end of flight. The velocity error components, attributed to the control system, can be discerned directly from these traces, as illustrated in Figure 5-3. In this case, the velocity error attributed to the control system is shown reducing the total velocity error at the end of flight.

- Vehicle c. g. Estimates of vehicle c. g. offsets and engine thrust misalignments that are discernible in the low aerodynamic pressure regions, including regions above the atmosphere.

- Vehicle Response. Response of the vehicle in the high aerodynamic pressure region. In this case, if wind sounding data are available from tests before and after a launch, the launch winds can be interpolated and employed in a digital program to determine the vehicle behavior for comparison purposes.

5.5.2.2 Autopilot Performance

The evaluation of the autopilot performance is a more refined determination of what was briefly observed in the real-time or functional analysis. Digital computer programs would be highly useful in quickly accomplishing this task. The telemetered command error angle and control sensor signals could be inserted into a set of autopilot equations and the resulting engine angle command compared with those obtained in flight. If such programs were not available, manual computations at significant instances during the flight would be a satisfactory compromise. Periods of particular interest would occur during the pitchover phase, in the high aerodynamic force region, and before and after gain changes and stage separations.

If a digital autopilot or a hybrid autopilot is flown, the engine command angles may be quite erratic, due to the discontinuities associated with the quantized digital computer output signals. Visual smoothing of the data to select only low-frequency components should provide a satisfactory means of evaluating the autopilot performance, if only a perfunctory evaluation of an apparently successful flight is required.

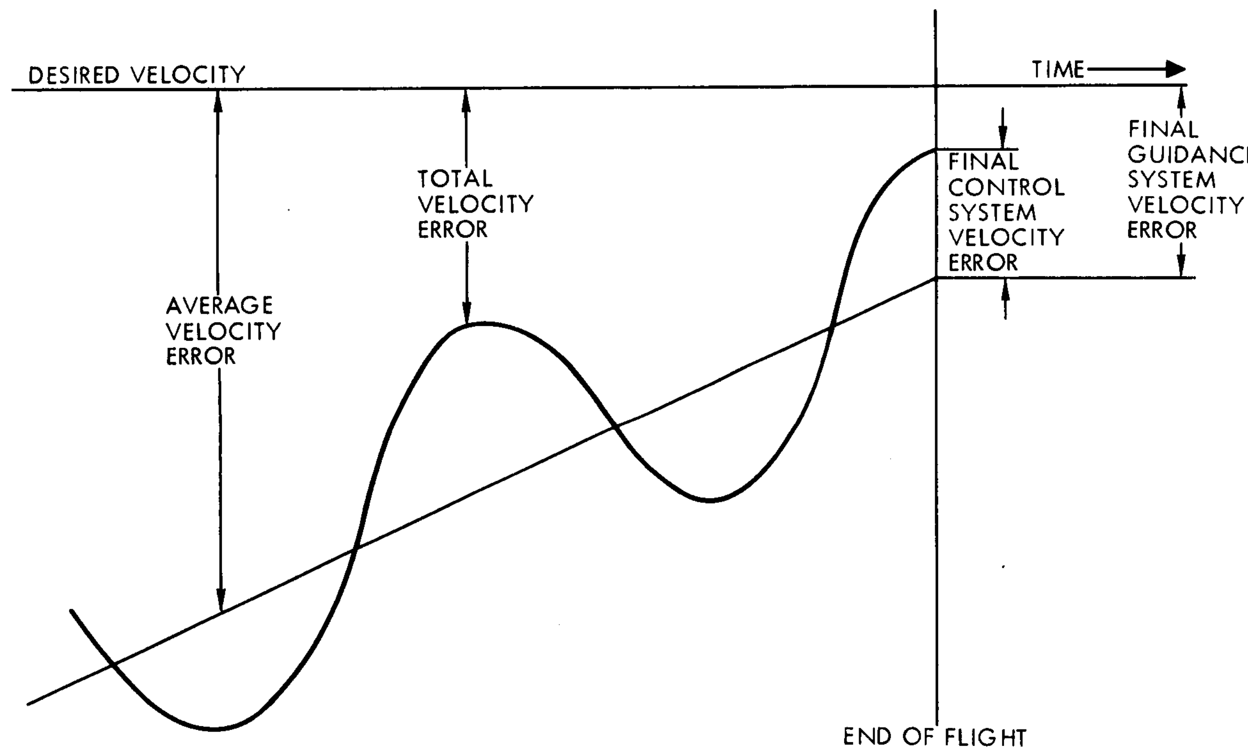

Figure 5-3. Velocity Error Due To Guidance and Control System Performance

In digital autopilots, a malfunction may involve improper operation or computation by the digital computer and its signal converters. Since decoded computer words are available for a functional analysis, the malfunction can be readily identified through a bit-by-bit simulation of the computer operations. This is generally performed by persons responsible for the computer software and hardware operations rather than by control system personnel. If a malfunction has occurred and an analog autopilot is suspect, a simulation study should be initiated. The simulation study would be conducted during engineering analysis. Experience with the autopilot mechanization may enable a rapid deduction of the malfunction and this experience can be gained through malfunction simulation studies performed during the preflight analysis phase.

5.5.2.3 Thrust Vector Control System

The functional analysis will also include review of thrust vector control functions. The engine deflection will be reviewed to determine if the engines follow their commands, and the difference noted. Evaluation of the TVC system can be accomplished similar to the autopilot evaluation, through the use of a digital computer program. By inserting the telemetered engine command angle into the TVC system equations, the output engine deflections and rates can be compared with flight results.

Similarly, the load pressures for the hydraulic actuation system or servo motor current and speed for the electromechanical actuation system can be compared with the telemetered data. It may also be desirable to employ low-pass filters for both simulated and telemetered results, in order to enable comparison of the low-frequency components in the signals.

If malfunctions within the TVC system are suspected and the TVC equations contain models for the components within the actuation loop, variations in these models may perhaps produce a fortunate match with flight results. The more obvious types of malfunctions, such as an amplifier saturation or loss of output, could be successfully duplicated. Less obvious malfunctions, such as servovalve damage or magnetic clutch damage may be extremely difficult to duplicate and may not be resolved in a timely manner to support the scheduled flight report.

5.6 ENGINEERING EVALUATION

The flight report, issued generally 1 month after launch, does not allow enough time for parameter evaluation of the control system. Hence, these reports are augmented by reports on special studies performed in areas where it is deemed necessary, and by final report of engineering evaluation.

5.6.1 Objectives

The detailed engineering analysis is quite costly, and hence, may not be performed on a preflight basis. Engineering analyses that are mandatory pertain to nonobvious malfunctions in which the control system is suspect, or to control system malfunctions in which the exact nature of the failure is not clear. Less critical evaluations would pertain to analyses of design weak points, predominant oscillations, and transients during the flight, the latter being performed essentially to verify the analytic model of the vehicle dynamics employed in design studies. Many times control system evaluation support is required to pinpoint the cause of an oscillation not necessarily attributed to the control system, such as the study detailed in Reference 10. In this case, a longitudinal elastic mode, coupled with a propellant feed system resonance, produced a sustained oscillation which could be detected by the vehicle sensors.

5.6.2 Analysis Methods and Variations

The detailed evaluation of the total control system consists of inserting the flight guidance steering commands and reconstructed trajectory data into a closed-loop simulation, as illustrated in Figure 5-4. The resulting outputs of vehicle attitude, angular rates, and acceleration are compared with the corresponding flight results. If the results are well-matched, this is often the extent of the evaluation desired. If some parameter adjustments appear necessary, comparison of the intermediate output points (engine command, β_C, and engine angle, β) with flight results may give some insight as to which parameters would be likely candidates for adjustment.

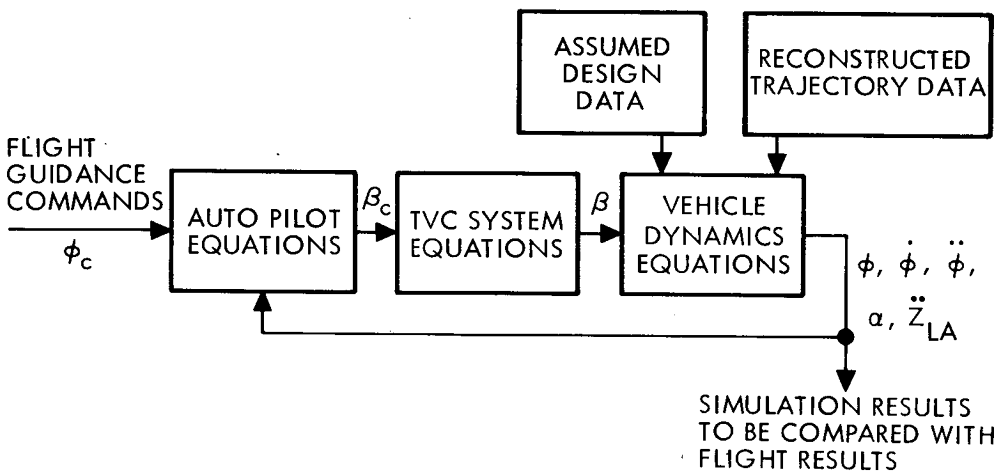

Figure 5.4. Closed Loop Simulation Method

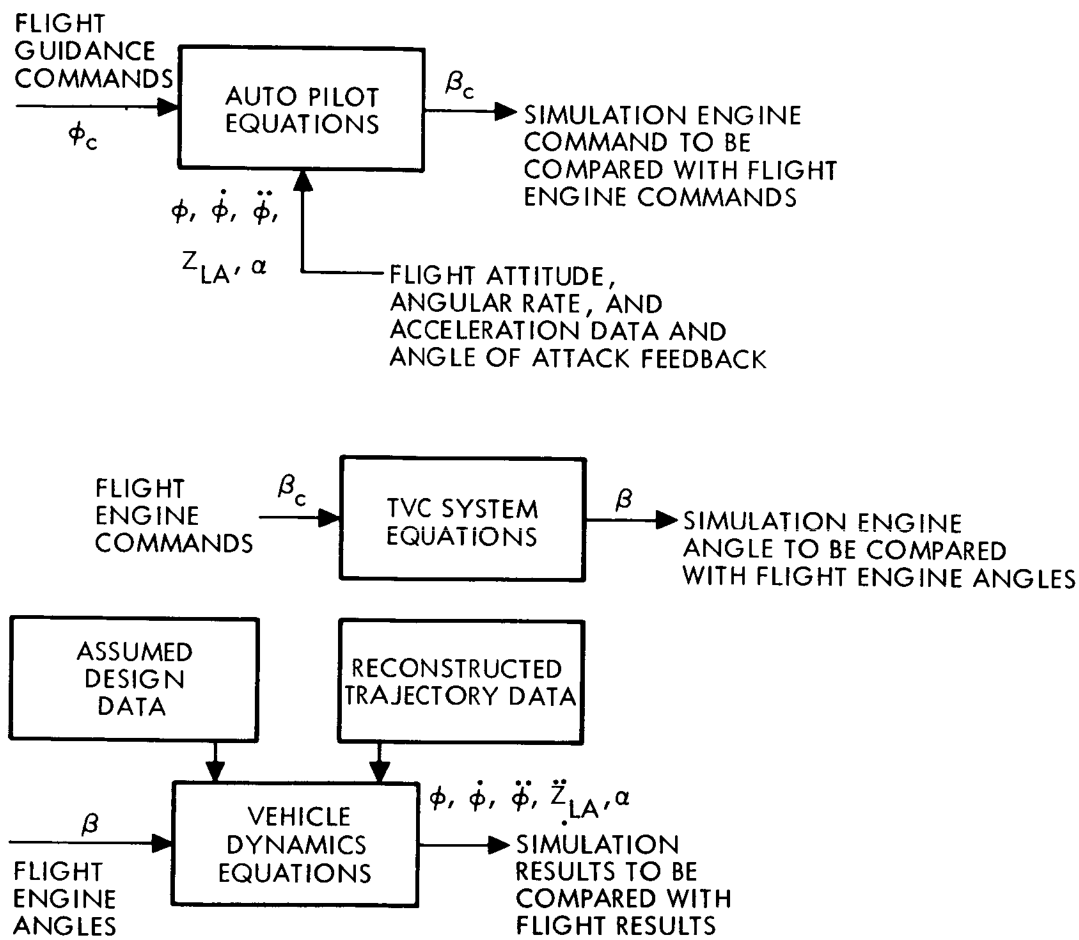

Figure 5.5. Open Loop Simulation Method

5.6.2.1 Simulation Methods

A typical flight evaluation program based on the flight mechanics for Saturn V is included in Appendix B. This program compares a six degree of freedom figid body simulation with a flight mechanics program, including filtering and processing of data, separation analysis, and an assessment of body bending and propellant sloshing effects. The results are displayed for easy comparison on a data plotter.

If a malfunction has occurred and is not subtle, the model of the suspected system can be altered to simulate the flight results with a high degree of success. If the malfunction is less obvious, an open-loop analysis of the component system, as shown in Figure 5-5, would be more desirable, since the effects of the interfacing system equations can be removed. The open-loop analysis is more precise for this reason and, hence, more suitable for a fine grain evaluation of vehicle dynamics parameters, such as the aerodynamics, bending, and sloshing parameters. Likewise, the evaluation of the autopilot performance and TVC systems performance can be accomplished in a more precise manner.

Performance of a vehicle may be judged in terms of its departure from an intended trajectory. However, the cause of such departures may depend on any of several components, and there may not be a priori criteria for the selection of one component or another. For this reason, simulation of vehicle performance, offering a capability of studying the interactions between system parameters is desirable, both for assessing failures and for monitoring nominal performance.

Simulation techniques are principally of two types: a) analog; and b) digital. Analog techniques simulate vehicle performance in terms of a system of electrical networks and servomotors. Digital simulation involves the use of a mathematical model of the system which is solved on a digital computer. The choice of method depends on the purpose of the simulation. Analog simulation has the advantage of immediate assessment of the effect of parameter variation; e.g., by a display on an oscilloscope. Depending on the sophistication of the model, however, it may lack the precision available from computed results obtained from a digital simulation. Both methods are discussed in greater detail in Appendix C.

The simulations may be augmented by adding routines to the basic models which compensate for nonideal conditions which exist during the flight. A discussion of analytical methods for determination and reconstruction of the effects of engine misalignment, total angle-of-attack, propellant sloshing, body bending, and aerodynamic moment parameters is given below. Detail analytical techniques for calculations of performance parameters under non-ideal conditions are included in Appendix C.

(a) Reconstruction of the Effective Engine Misalighment Angle. An effective engine misalignment angle, whose components are engine misalignments and offsets, and center of gravity offsets, can be estimated from flight test results over the low aerodynamic pressure phases both before and after the high aerodynamic pressure phase. With aid from the a priori knowledge of the c. g. offset changes with time, the effective engine misalignment angle over the high aerodynamic pressure phase can be iterated from these results. This is required for determining the aerodynamic parameters, since these effects may falsely appear to be attributed to aerodynamic pressure.

Filtering of the data to remove noise and body-bending effects is desirable, and if propellant slosh effects are dominant, notch filtering of these frequencies may also be desirable. Normally, effects of c. g. offsets and thrust vector misalignments with time are slow and similar to effects of aerodynamic variations or frequencies, excluding regions where jettisoning or staging takes place.

It is also feasible, with appropriate input data, to separate the effects of thrust misalignment angles from the c. g. offset values. These angles would be of particular usefulness to the propulsion analysis area.

(b) Reconstruction of the Total Angle of Attack. The reconstruction of the total angle of attack (α) is required since the vehicle experiences aerodynamic forces from both the angle of attack due to wind (α_W) and the angle of attack due to angular differences between vehicle centerline and velocity vector (α_V).

The wind velocity data used in the simulation are acquired from wind soundings at the launch site and nearby sites or by analytical techniques using previously evaluated statistical data. The tests are generally made just prior to and after a launch, thereby enabling interpolation of launch winds. This can be extended even further if necessary, by reconstructing the wind velocity history through curve fitting of data from several wind sounding tests. Such a necessity may arise if rapidly changing wind speed and direction occurs. However, in most applications, wind velocity models are included in the analytical models used for flight evaluation and the parameters of the wind model are estimated along with the other parameters of the system.

(c) Propellant Slosh Detailed Evaluation. Comparisons between flight test results and expected propellant slosh oscillation amplitudes and frequencies, can be observed in the attitude rate, slosh profiles and level sensor traces. Simulation studies will provide more detailed analysis. One method is to conduct the closed-loop control system simulation studies with the inclusion of the reconstructed thrust, thrust misalignment angles, winds, and trajectory. The propellant slosh parameters may then be varied until the best match with the flight test results are obtained.

A second and more precise method is to utilize just the vehicle dynamics portion of this simulation. By employing the flight test engine gimbal angle as an input to the vehicle dynamics equations and discarding the autopilot and thrust vector control system (TVC) equations, the effects of the actual autopilot and TVC system variations would be included.

The engine angle data is first filtered with a low-pass filter to exclude body-bending effects. The resulting attitude, angular rate, and acceleration signals could be compared with corresponding traces from the flight, which also have been filtered with the low-pass filter. There is no particular sequence in which to vary the slosh parameters to match flight results and thus, it is a matter of trial and observation. However, the most likely candidates for parametric variations are the propellant slosh damping and frequency of the mode under study.

(d) Body Bending Detailed Evaluation. If a dominant bending oscillation is prevalent in the flight results, vehicle body-bending parameters can be obtained through simulation studies and compared with design values. If dynamics effects, in addition to the one being sought, are apparent in the data, data filtering may be necessary. Such effects as propellant sloshing and higher bending mode oscillations can be removed by employing a bandpass filter, allowing only bending mode frequency to pass.

If these frequencies are known to vary considerably over the duration of the flight, the evaluation can be performed over smaller phases with different bandpass filters employed. Usually the visible bending oscillations are of short duration and do not require this consideration.

A simplified bending mode parameter evaluation method employs bandpass filtered engine angle data. By utilizing the actual engine data, the autopilot and TVC system high-frequency dynamics uncertainties are bypassed. The lateral and angular accelerometer outputs and position and rate gyro outputs can be compared with the corresponding flight data which have also been bandpass filtered. Adjustments can be made to the bending mode frequency or slopes and deflections to achieve the desired match.

If slosh frequencies or other bending modes exist in the proximity of the mode under study, a multiple mode analysis may be required, in which the adjustments of coupling terms is necessary. The possibility of satisfactorily matching the data would be diminished in this instance, due to the added complexity of the task.

(e) Aerodynamic Moment Parameters. Engine thrust vector misalignments and offsets, and vehicle center of gravity offsets, produce turning moments on the vehicle which may appear to be attributed to aerodynamics. These misalignment and offset moments generally vary slowly and are predictable, once they have been calibrated. Such calibrations can be performed over periods of flight where aerodynamic pressure is negligible. The presence of noise and high-frequency dynamic effects (vehicle bending) in the data can be filtered from the data quite successfully.

5.6.2.2 Evaluation of Control System Sensor Performance

Since the static and dynamic characteristics of control system sensors are accurately determined in laboratory tests, verification of these characteristics is not under consideration in flight evaluation. Usually, a cursory look at the flight test sensor outputs signals will determine if the sensors were operating satisfactorily. If the telemetry channel for a signal is lost, evaluation of the autopilot performance will indicate if the control sensor was operative or not. Malfunctions in gyroscopic sensors, such as loss of excitation or overheating of the spin motor windings, may be difficult to determine unless indications of the spin motor speed are telemetered. The gyroscope will continue to operate in a continually degrading fashion and simulation studies to match vehicle performance may be necessary, such as described in Reference 12. To determine whether the malfunction can be attributed to an open-spin motor winding or a high torque gain, simulation studies to match trajectory characteristics obtained from radar tracking data will be required. In the event of an instrument malfunction or questionable performance, an alternate source may be selected or the parameter may be calculated from measurements by application of the simulations.

5.6.2.3 Autopilot Performance

The detailed evaluation of the autopilot performance may entail performing the open loop simulation described in the functional analysis evaluation because of the time constraint for analysis or perhaps because a malfunction has occurred and cannot be readily reconciled or because the autopilot is complex and deserves a detailed analysis such as with an adaptive autopilot. Rather than to just compare the simulation outputs, manipulation of the autopilot parameters may be accomplished to obtain a match in these outputs.

In the case of an adaptive autopilot, performance measures can be obtained in the detailed analysis evaluation. For example, with a tracking notch filter autopilot, the identified frequencies and the frequency content of the engine commands can be compared to those for an ideal adaptive autopilot.

In the case of load relief autopilots, the change in autopilot gains due to sensed vehicle performance can be compared with the ideal autopilot performance.

5.6.2.4 Thrust Vector Control System Performance

An open loop simulation study of the thrust vector control system will enable a precise determination of the performance of components within the system and often the exact nature of the malfunction. If the malfunction is obvious, such as a feedback transducer failure or a servo amplifier failure, it would have been identified in the functional analysis and a simulation study would be a means of verifying the conclusions. If a malfunction is subtle, for example, performance degradation of a component rather than complete failure, the problem becomes considerably more complex and the open loop simulation becomes an invaluable tool. The degraded performance of the component can be hypothesized and tested in the simulation in an attempt to match flight results. In a highly non-linear component such as a hydraulic servovalve, blockage of one of its numerous fluid passages or orifices may result in a servovalve performance which is entirely different from its nominal behavior. In this instance, actual hardware tests may be required to duplicate the failure characteristics. Although the possibility exists that the numerous stages of the servovalve including the valve spool dynamics and fluid flow rates can be simulated in detail to reconstruct the servovalve malfunction, the hardware tests may be required to obtain conclusive evidence as to the exact nature of the failure due to the complexity and, hence, uncertainty of such simulation results.

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APPENDIX A

FILTERING SMOOTHING AND TRANSFORMS

INTRODUCTION

Associated with the process of data signals are various editing techniques which are designed to eliminate unwanted signals and to reconstruct the desired signal. There is no universal method for accomplishing this end, since retrieval of information depends on our knowledge of the general character of the information transmitted, or some assumptions about the nature of that information, and also on the means at hand for processing the data. Techniques described in this appendix are limited to a type of data processing which applies certain linear operators to the raw data to force the signals to conform to a proposed shape; i.e., to conform to frequency patterns within which the desired information is known, or assumed, to lie. The success of the methods described depends on the premise, to a large extent applicable to telemetry data, that desired and unwanted frequences are non-overlapping. This assumption places the smoothing, or filtering operation within the classical pass-type frequency filter designs which must be reckoned with when reasonably sharp cut-off and finite time spans are proposed.

When data is not continuously monitored, but is sampled only at discrete time intervals, design of filters requires the development of a system of weights which replace the integrals of the continuous process by finite weighted sums. The discrete process cannot be realized by electronic circuits and requires a digital computer for its implementation. For this reason, filters in this class are usually called digital filters.

The treatment of the filtering process in this appendix is not intended to be exhaustive, but its purpose is to exhibit certain classes of filtering operations which have been successful in applications, and to indicate the general nature of the techniques applicable to the filtering process.

I. A Class of Filters

The general form of the linear operators used in filtering, which relate the input (S(t) to the output S*(t) are of the form

$$S^*(t) = \int_{-T_L}^{T_u} h(\tau)\ S(t - \tau)d\tau \tag{1}$$

For discrete time sampled data the integral is replaced by a finite sum

$$S_m^* = \sum_{n=L}^{V} h_n\ S_{n+m} \tag{2}$$

With infinite time limits, the frequency functions (H(ω) together with the continuous, or discrete, weighting functions form Fourier transform pairs. With finite time limits, we seek a finite set of weights so that the frequency function is a least square fit to the proposed frequency function. Application of the inverse Fourier transform then determines the filter behavior in the time domain (weighting function). Various weighting functions are obtained depending on the shape of the frequency cut-off behavior desired. Suppose, for example, we wish to cut off all frequencies for which $|\omega| > \omega_c$. Since sharp cut-off cannot be achieved because of the discontinuity involved, a filter is designed to pass frequencies in a range $-\omega_T \leq \omega \leq \omega_T$ (ω_T is called the terminal frequency as opposed to the cut-off frequency ω_c) with a gain function H(ω), normalized to unity, represented by the function

$$
\begin{aligned}
H(\omega) &= 0, && |\omega| > \omega_T, \\
H(\omega) &= 1, && |\omega| \leq \omega_c, \\
H(\omega) &= f(\omega), && \omega_c \leq \omega \leq \omega_T, \text{ where } f(\omega_c) = 1,\ f(\omega_T) = 0, \\
H(\omega) &= f(-\omega), && -\omega_T \leq \omega \leq -\omega_c.
\end{aligned}
$$

The function has the following appearance

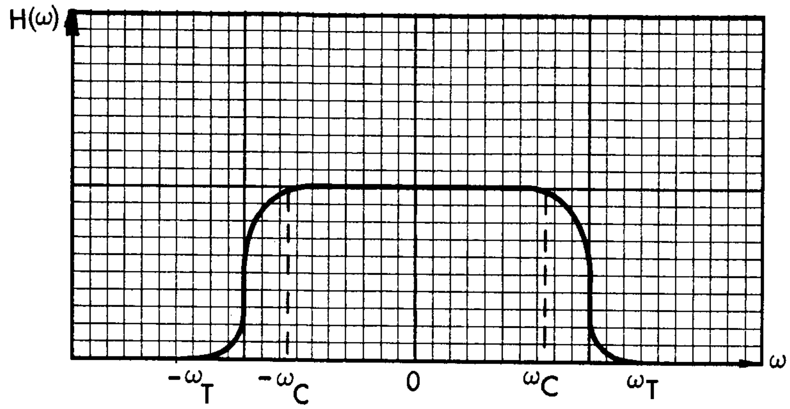

Figure A-1

The nature of the function $H(\omega)$ in the intervals $(-\omega_T, -\omega_c)$ and (ω_c, ω_T) are at the designer's discretion, and these regions are called the roll-off regions.

Example 1

One class of such filters is given by

$$H(\omega) = \begin{cases} 0, & |\omega| > \omega \\ 1, & |\omega| \le \omega \\ \dfrac{1}{\omega_T - \omega_c}^{\,p} (\omega_T + \omega)^p, & -\omega_T \le \omega \le -\omega_c \\ \dfrac{1}{\omega_T - \omega_c}^{\,p} (\omega_T - \omega)^p, & \omega_c \le \omega \le \omega_T. \end{cases}$$

The time domain function (weighting function) is given by the inverse Fourier transform

$$h(t) = \int_{-\infty}^{\infty} e^{i\omega t} H(\omega) \frac{d\omega}{2\pi} \qquad (3)$$

This function is evaluated in Reference 13 and leads to a complicated expression for h(t) for a general value of p. The expression becomes simple if p = 1, and is given by

$$h(t) = \frac{\cos \omega_c t - \cos \omega_T t}{\pi t^2 (\omega_T - \omega_c)} .$$

This function will be used later in a discussion of weights for the discrete-time case.

The type of filter just described in called a low-pass filter. As another example of such a filter (using a different roll-off function), consider the following:

<u>Example 2</u>

$$H(\omega) = \begin{cases} 0, & |\omega| > \omega_T \\ 1, & |\omega| \leq \omega_c \\ \frac{1}{2} \cos \frac{\pi(\omega_c + \omega)}{T - c} + \frac{1}{2}, & -\omega_T \leq \omega \leq -\omega_c \\ \frac{1}{2} \cos \frac{\pi(\omega - \omega_c)}{\omega_T - \omega_c} + \frac{1}{2}, & \omega_c \leq \omega \leq \omega_T \end{cases}$$

For this filter the time function, given by the inverse Fourier transform is

$$h(t) = \frac{\pi}{2t} \frac{\sin \omega_T t + \sin \omega_c t}{\pi^2 - (\omega_t - \omega_c)^2 t^2} .$$

The derivation of h(t) is easily obtained from equation (3), using standard tables of integrals. A detailed derivation is given in Reference 11.

The preceding examples are typical low-pass filters, which are the basic entity in filter design. From the low-pass filter, most filters in common use can be easily derived. Some of them are the following:

(a) High-pass filter. The complement of the low-pass filter, i.e., their sum is an all-pass filter (one which passes all frequencies without change).

(b) Band-pass filter. The difference between two low-pass filters.

(c) Band-reject filter, or notch filter. The difference between an all-pass filter and a band-pass filter.

The differences in the foregoing classification are to be interpreted in the sense of differences in the appropriate weights. Therefore, a wide variety of filters is obtained by taking linear combinations of appropriate weights.

II. Digital Filter Weights

When data transmissions are sampled at discrete time intervals, the integrals of the previous section are replaced by finite sums of the form

$$h(t) = \sum h_n \, \Delta t_n.$$

When the time intervals Δt_n are equal ($\Delta t_n = \Delta t$ = constant) the weights can be obtained directly from $h(t_n)\Delta t$. If this is not the case, the solution for the weights requires solving a set of simultaneous equations, which may not be done directly, but incorporated as a sub-routine in a digital program. We consider only the former case, and refer the reader to the references cited for other applications.

In the case of sampled data, it is important that the sampling rate be at least twice the highest frequency rate expected in the data transmission. The reason for this requirement is associated with the Gibbs phenomenon of Fourier analysis. If a signal is sampled at a frequency rate f_s, any signals having a frequency greater than $1/2\ f_s$ are reflected into the range $(0,\ 1/2\ f_s)$, a phenomenon sometimes called frequency aliasing, and is a consequence of the fact that Fourier analysis deals with expansions in terms of periodic functions.

Returning to Example (1), we compute a finite set of weights corresponding to the function

$$h(t) = \frac{\cos \omega_c t - \cos \omega_T t}{\pi t^2 (\omega_T - \omega_c)}.$$

Since this function is symmetric $h(t) = h(-t)$, we can take the upper and lower limits of the sum equal to each other; i.e., $h_n = h_{-n}$, and therefore have an odd number, $2N + 1$, of weights. If ω_s is the effective sampling angular frequency, we normalize with respect to this frequency. Thus we introduce variables

$$\lambda_c = \frac{\omega_c}{\omega_s}, \quad \lambda_R = \frac{\omega_T - \omega_c}{\omega_s}, \quad t_n = n\Delta t, \quad \Delta t = \frac{1}{f_s},$$

we find that the weights $h_n = h(t_n)\Delta t$ are given by the formula

$$h_n = \frac{\cos 2\pi n\lambda c - \cos 2\pi n\lambda T}{2\lambda_R(\pi n)^2}, \quad n = 0, \pm 1, \pm 2, \ldots \pm N$$

where $\lambda_T = \lambda_c + \lambda_R$.

Other examples of weight computations are given in References 11 and 13.

III. Error Analysis

Since digital filters replace integrals (which are exact) by finite sum approximations, an evaluation of the error involved is necessary in the design of a digital filter. In general, the error will be a function of the number of points selected and of the frequencies which occur.

To derive an analytic expression for the error, we first note that $H(\omega)$ can be expressed as

$$H(\omega) = \sum_{n=-\infty}^{\infty} h_n e^{i\omega n/f_s}.$$

Denote by $\hat{H}(\omega)$ the finite approximation using 2N + 1 points,

$$\hat{H}(\omega) = \sum_{n=-N}^{n=N} h_n e^{i\omega n/f_s},$$

where

$$h_n = \frac{1}{2\pi f_s} \int_{-\pi f_s}^{\pi f_s} H(\omega) e^{-i\omega n/f_s} d\omega,$$

and the effective signal frequency range is $-\omega_{s/2} \leq \omega \leq \omega_{s/2}$. We define the error $\varepsilon(\omega, N)$ to be

$$\varepsilon(\omega, N) = \hat{H}(\omega) - H(\omega).$$

Interchanging summation and integration we can write

$$\hat{H}(\omega, N) = \frac{1}{2\pi f_s} \int_{-\pi f_s}^{\pi f_s} H(\xi) \sum_{n=-N}^{N} e^{i(\omega-\xi)n/f_s} d\xi.$$

Introducing $\lambda = \frac{\omega}{\omega_s}$, $\rho = \frac{\xi}{\omega_s}$, this becomes

$$\hat{H}(\lambda, N) = \int_{-0.5}^{0.5} H(\rho) \sum_{n=-N}^{N} e^{2\pi i(\lambda-\rho)n} d\rho.$$

Now, since

$$\sum_{n=-N}^{N} e^{2\pi i(\lambda-\rho)n} = \frac{\sin (2N + 1)\pi (\lambda - \rho)}{\sin \pi(\lambda - \rho)} \text{ , we find}$$

$$\varepsilon(\lambda, N) = \int_{-0.5}^{0.5} H(\rho) \frac{\sin (2N + 1)\pi(\lambda - \rho)}{\sin \pi(\lambda - \rho)} d\rho - H(\lambda).$$

A calculation of $\varepsilon(\lambda, N)$ can be made digitally for any λ and N, the number of of data points, can be determined in accordance with design requirements.

In addition, the accuracy of a filter is dependent on the sharpness of the foll-off, particularly if the roll-off function produces a discontinuity in slope at the cut-off frequency. Thus, this factor should be evaluated (together with the number of data points) in an evaluation of accuracy.

Consider the filter of Example (1), assuming $\lambda_R = \frac{\omega_T - \omega_c}{\omega_s} = 0.02$ and that N = 50, $\lambda_c = 0.10$. We get a maximum pass band error of less than 0.25 percent up to $\lambda = 0.081$ and a maximum rejection band error less than 0.25 percent for $\lambda > 0.139$. The effective λ_R with these errors is 0.058. For maximum pass and rejection band errors of less than 0.5 percent, we get $\lambda_R = .038$; that is, a pass band up to $\lambda = 0.091$ and a rejection band after $\lambda = 0.129$. A plot of this design is given in the accompanying Figure A-2.

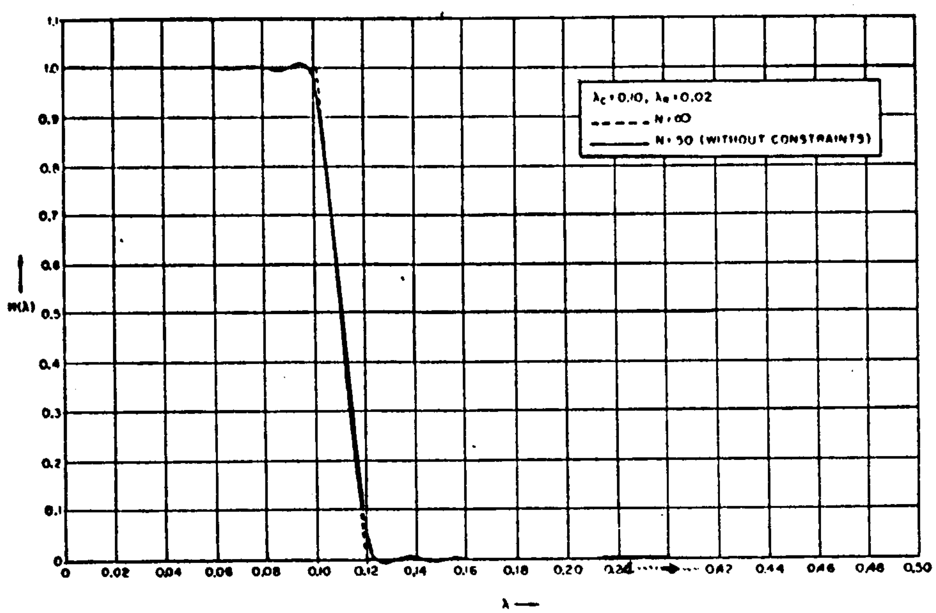

Figure A-2. Characteristic Error for Example 1

IV. Constraints

Certain trends given as general polynomial time forms and considered as desirable data require constraints on the basic weights in order to pass without error. To prevent this type of error one should have:

$$\sum_{n=-N}^{n=N} h_n N^k = \frac{d^k H(\omega)}{d\omega_k} = 0 \Big|_{\omega=0}; \quad k \geq 1$$

$$= 1 \Big|_{\omega=0}; \quad k = 0$$

Practical considerations restrict k to 3, and $\frac{d_k H(\omega)}{d\omega_k} = 0 \,|_{\omega=0}$ is automatically satisfied if k is odd. The conditions become more acute as $\omega_c \to 0$, and the weights must be constrained to satisfy this condition.

Satisfaction of the constraint $H(\omega) = 1|_{\omega=0}$ gives new weights h_n' as follows:

$$h_n' = h_n + \delta$$

$$\delta = \frac{1 - H(o)}{2N + 1}$$

Satisfaction of the constraint $\frac{d^2 H(\omega)}{d\omega 2} = 0|_{\omega=0}$ introduces a new δ which depends on N in a non-simple manner. The final results are:

$$h_o' = h_o + \frac{\sigma_2 \Delta_1 + 2\sigma_1 \Delta_2}{\sigma_3}$$

$$h_n' = h_n + \frac{\sigma_2 \Delta_1 + 2\sigma_1 \Delta_2 - n^2[\sigma_1 \Delta_1 + (2N + 1)\Delta_2]}{\sigma_3}, \quad n \geq 1,$$

where

$$\Delta_1 = 1 - h_o - 2 \sum_{n=1}^{N} h_n; \quad \Delta_2 = \sum_{n=1}^{N} n^2 h_n; \quad \sigma_1 = \sum_{n=1}^{N} n^2;$$

$$\sigma_2 = \sum_{n=1}^{N} n^4 ; \quad \delta_o = \frac{\sigma_2 \Delta_1 + 2\sigma_1 \Delta_2}{(2N + 1)\sigma_3 - 2\sigma_1^{\,2}} .$$

Details of the derivation of these equations can be found in References 13 and 15.

V. Other Types of Filters

The filters described thus far have all been in-phase filters, involving no shifts in frequency. It is sometimes desirable to introduce filters having a $+90°$, or a $-90°$ shift in phast (so-called quadrature filters).

The usefulness of phase shift filters can be illustrated by the comparison of two different designs for a band-pass filter. The first of these is the one which has already been discussed; namely the difference between two low-pass filters having different cut-off frequencies. The objection to this type of filter is that the error involved can be twice the error of either of the component filters, due to addition of errors.

Another band-pass filter, generally preferred, is obtained by frequency shifting, as follows. Given a low-pass filter with response $H(\lambda)$ and weights h_n, define $H_B(\lambda) = H(\lambda - \lambda_o) + H(\lambda + \lambda_o)$, where λ_o is the center of the desired pass region. Here $H(\lambda)$ is given by

$$H(\lambda) = h_o + 2 \sum_{n=1}^{N} h_n \cos n\lambda \quad .$$

We then obtain

$$H_B(\lambda) = 2h_o + 2 \sum_{n=1}^{N} (2h_n \cos 2\pi n\lambda_o) \cos 2\pi n\lambda \quad .$$

The weights for $H_B(\lambda)$ are

$$h_{Bn} = 2h_n \cos 2\pi n\lambda_o \quad .$$

Comparison of the two types of band-pass design, we find that using two low-pass filters results in positive and negative errors across the entire pass band, while the frequency shift filter emphasizes errors near the edges of the pass band. The comparison is illustrated graphically in the accompanying Figure A-3.

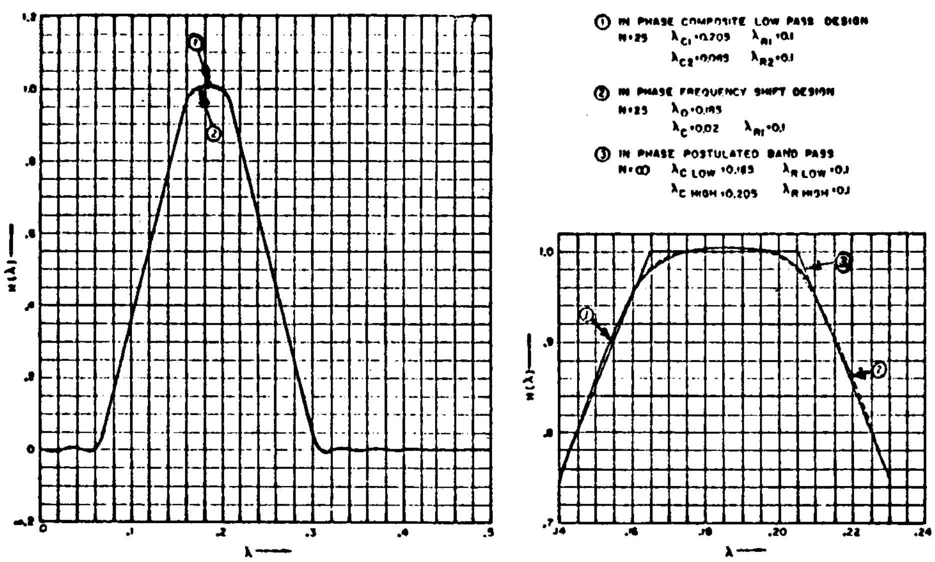

Figure A-3. H(λ) versus λ for Bandpass

An additional variant to the designs which have been discussed is a class of filters using the first derivative for smoothing of the raw data. We refer the reader to Reference 13 for a detailed treatment of filters in this category.

VI. A Closed Form Solution for Control System Equations

The foregoing sections have discussed salient mathematical considerations underlying the design of filters, having as their purpose the extraction of viable information from signal transmissions accompanied by various forms of "noise". The actual physical devices which accomplish the intended function are usually electronic or digital networks which are described by systems of differential equations. The solution of these equations relates the output of a filter to its input. If the filter can be realized by an analog device, which could display the output on, say, an oscilloscope, the relation of output to input is easily realized. However, in most cases the solution of the system of equations requires the use of digital computation. For use in control systems it is almost always desirable to accomplish the solution in real time, or something approaching real time, in order that data transmissions can be promptly translated into control commands. In most systems this objective cannot be realized by direct integration of the system of differential equations, even with the largest and most sophisticated computer systems presently available.

The present section presents a solution of a system of linear differential equations in closed form which is applicable to a large class of control system equations, and which comes close to realizing the real time relationship of input, in the form of electrical signals to output in the form of electromotive forces actuating the physical controls. The method is due to J. F. Andrus.

1. Statement of the Problem

The problem under consideration is the following:
Given a system of differential equations

$$\dot{\bar{q}} = A\bar{q} + E_{in}\,\bar{b}$$

and a relation

$$E_{out} = \bar{u}^T\bar{q} \ ,$$

where $\overline{q}$, $\overline{b}$ and $\overline{u}$ are n-dimensional column vectors, $\overline{u}^T$ is the transpose of $\overline{u}$, A is an n-dimensional square matrix, E_{in} is the input function, and E_{out} the output, find a solution of the system in closed form at time $T + \Delta t$ in terms of conditions at time T.

In order to accomplish the solution desired, we assume that E_{in} can be represented as a polynomial

$$E_{in} = \sum_{k=0}^{m} r_k t^k$$

on the interval $(T, T + \Delta t)$. This form of input could be achieved, say, by a least-square fit. It is also assumed that the matrix A is similar to a diagonal matrix; i.e., there exists a non-singular matrix P such that

$$P^{-1}AP = D, \text{ where}$$

D has non-vanishing elements only on its main diagonal. This is not a severe restriction, since it is true for any matrix whose eigen values are all distinct, as well as certain other matrices.

The first step in the solution is to make a transformation of variables

$$\overline{p} = P^{-1}\,\overline{q}, \text{ resulting in}$$

$$P\dot{\overline{p}} = A(P\overline{p}) + E_{in}\,\overline{b}$$

$$E_{out} = \overline{u}^T(P\overline{p}).$$

These equations can be written as

$$\dot{\overline{p}} = D\overline{p} + E_{in}\,\overline{c}$$

$$E_{out} = \overline{v}^T\,\overline{p}, \quad \text{where} \quad D = P^{-1}AP, \quad \overline{c} = P^{-1}\,\overline{b}, \text{ and } \quad \overline{V} = P^T\overline{u}$$

For the i-th component, we have

$$p_i = \lambda_i p_i + c_i \, E_{in}$$

$$E_{out} = \sum_{i=1}^{n} v_i p_i ,$$

since we have assumed that D is a diagonal matrix. Letting $h_i = v_i p_i$ and $\gamma_i = v_i c_i = (\bar{u}^T \bar{x}_i)(\bar{y}_i^T \bar{b})$, we get

$$\dot{h}_i = \gamma_i h_i + \gamma_i \, E_{in} .$$

This scalar equation has a standard solution

$$h_i = e^{\Lambda i} \int e^{-\Lambda i} \, \gamma_i \, E_{in} dt + c_i \, e^{\Lambda i}$$

where $\frac{d \Lambda_i}{dt} = \lambda_i$, and c_i are constants of integration.

This equation yields, for sufficiently small Δt (i.e., when such approximations as

$$\int_T^{T+\Delta t} \lambda_i dt = \lambda_i \Delta t$$ are valid) the following expression:

$$h_i(T + \Delta t) = e^{\lambda_i \Delta t} \left[\gamma_i \int_T^{T+\Delta t} e^{\lambda_i (t-T)} \, E_{in} \, (t) \, dt + h_i(T) \right] .$$

Now, if $E_{in}(t)$ has the polynomial form $E_{in} = \sum_{k=0}^{m} r_k (t - T)^k$,

the foregoing equation may be integrated to yield

$$\int_T^{T+\Delta t} e^{-\lambda_i (t-T)} \, E_{in} dt = \sum_{k=0}^{m} r_k \int_T^{T+\Delta t} (t - T)^k \, e^{-\lambda_i (t-T)} dt$$

$$= \sum_{k=0}^{m} r_k \int_0^{\Delta t} \tau^k \, e^{-\lambda_i \tau} d\tau .$$

Repeated integrations by parts give us

$$\int \tau^{k} e^{-\lambda_i \tau} dt = -\frac{e^{-\lambda_i t}}{\lambda_i^{k+1}} \sum_{s=0}^{k} \frac{k!}{S!} (\lambda_i \tau)^s$$

Hence

$$h_i(T + \Delta t) = h_i(T)\, e^{\lambda_i \Delta t} + \gamma_i \sum_{k=0}^{m} \frac{k!}{k + 1} \left[e^{\lambda_i \Delta t} - \sum_{s=0}^{k} \frac{(\lambda_i \Delta t)^s}{S!} \right] r^k .$$

Now, if $\lambda_i \Delta t$ is small, the quantity

$$\frac{k!}{\lambda_i^{k+1}} \left[e^{\lambda_i \Delta t} - \sum_{s=0}^{k} \frac{(\lambda_i \Delta t)^s}{s!} \right]$$

can be computed by means of the series

$$K!\ (\Delta t)^{k+1} \sum_{s=0}^{\infty} \frac{(\lambda_i \Delta t)^s}{(S + k + 1)!}$$

This computation has the advantage of avoiding the subtraction of two quantities which may be very nearly equal, and could lead to the loss of several significant figures. The series can be safely truncated after several terms.

Summary

An expression for $h_i(T + \Delta t)$ in terms of $h_i(T)$ has been obtained in the form

$$h_i(T + \Delta t) = h_i(t)\, e^{\lambda_i \Delta t} + \sum_{k=0}^{m} f_{ik}\, r_k$$

$$E_{out}(T + \Delta t) = \sum_{i=1}^{n} h_i(T + \Delta t)$$

where

$$f_{ik} = \gamma_i \frac{k!}{\lambda_i^{\,k+1}} \left[e^{\lambda_i \Delta t} - \sum_{s=0}^{k} \frac{(\lambda_i \Delta t)^s}{s!} \right], \quad \lambda_i \neq 0,$$

$$f_{ik} = \gamma_i \frac{(\Delta t)^{k+1}}{k+1}, \quad \lambda_i = 0,$$

$$\gamma_i = (\bar{u}^T \bar{x}_i)(\bar{y}_i^T \bar{b}).$$

$\bar{x}$ and $\bar{y}$ are respectively right and left eigen vectors of the matrix A corresponding to $\lambda = \lambda_i$, scales so that $\bar{y}_i^T \bar{x}_i = 1$.

These equations are the desired closed form solution for $E_{out}(T + \Delta t)$ in terms of conditions at time T.

It should be remarked that if the value of Δt is changed during the integration, the quantities $e^{\lambda_i \Delta t}$ and f_{ik} must be recomputed. Furthermore, if A, $\bar{b}$ and $\bar{u}$ vary with time, the λ_i and γ_i will also vary with time. However, the solution presented here assumed the λ_i and γ_i remained essentially constant over the interval T to T + Δt. One should also observe that any of the quantities λ_i, γ_i, h_i and f_{ik} might be complex, in which case one must use either complex arithmetic, or real arithmetic by using appropriate pairs of real numbers.

APPENDIX B

DERIVATION OF COMPUTED SENSOR OUTPUTS

INTRODUCTION

A complete flight evaluation program consists of a comparison between a precomputed vehicle trajectory and the actual observed trajectory, as modified by flight commands transmitted in response to telemetered data observed during the flight. The technique for evaluation is illustrated by the accompanying flow chart, which is illustrative of a typical procedure.

One may note the following typical features of such evaluations. First, one observes telemetered flight data, subject to certain processing techniques. The data thus treated is used as an input, or inputs to a flight mechanics program. The flight mechanics program may be typified by programs such as that presented in this appendix. The data processing techniques illustrated included in the diagram include the Andrus technique for treating control equations as a direct input to the flight mechanics program, at .01 second intervals, and compared after smoothing by techniques such as those of Graham and Ormsby, with data obtained at 0.1 second.

The outputs thus obtained are compared with those of a six degree of freedom simulation, using a plotter for direct visual observation.

The equations governing the six degree of freedom simulation are typified by the following flight mechanics for Saturn V. Based on observations of attitude errors, and attitude rates, the following control commands are generated.

$$\beta_{pc} = A_{op} \; \overline{X}_p + A_{1p} \; \overline{\dot{\varphi}}_{pc}$$

$$\beta_{yc} = A_{oy} \; \overline{X}_y + A_{1y} \; \overline{\dot{\varphi}}_{yc}$$

$$\beta_{rc} = A_{or} \; \overline{X}_r + A_{1r} \; \overline{\dot{\varphi}}_{rc}$$

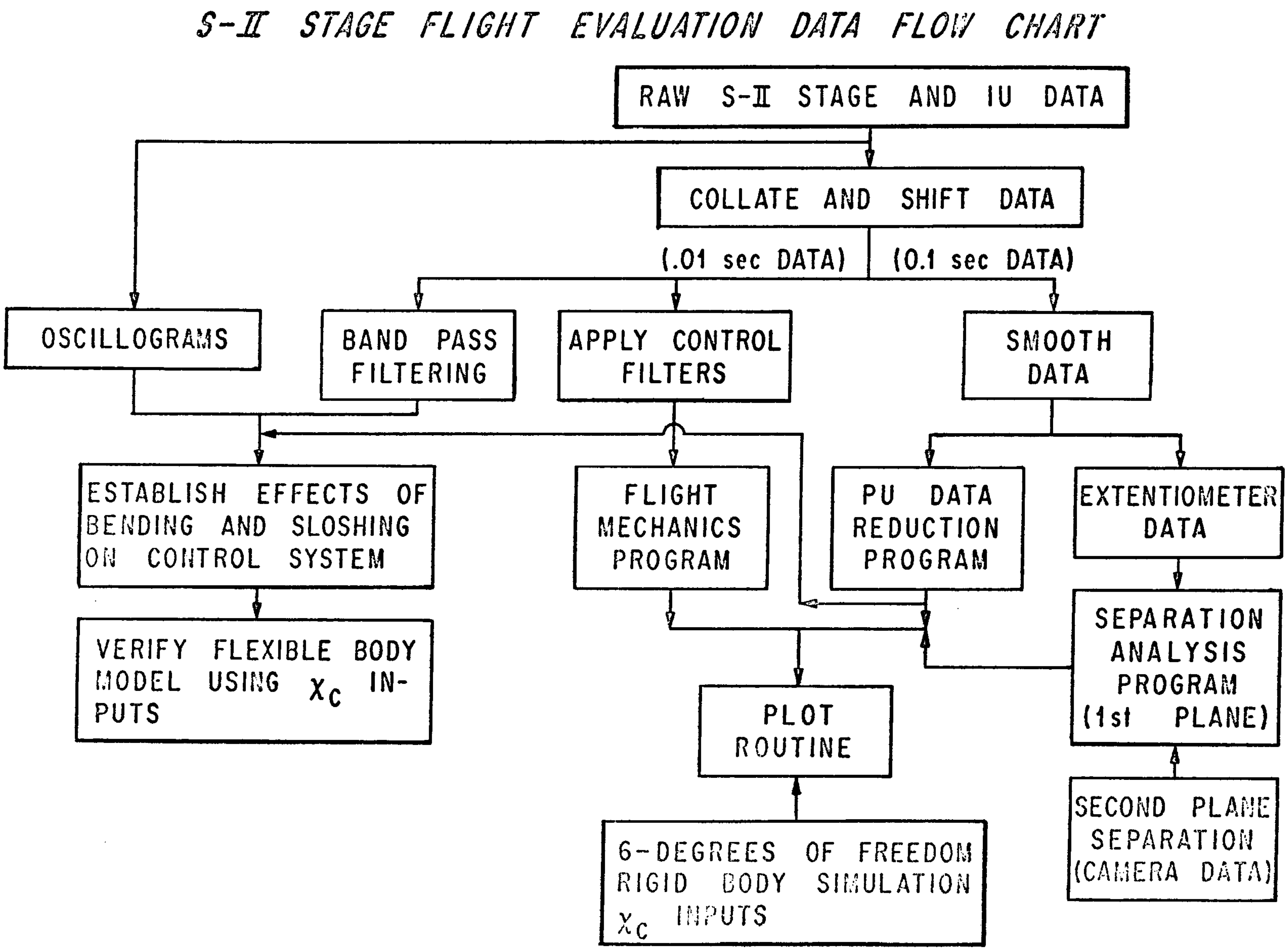

Figure B-1. Saturn SII Stage Flight Evaluation Data Flow Chart

In these equations, β_{pc}, β_{yc}, and β_{rc} are control commands in pitch, roll, and yaw - $\overline{X}_p$, $\overline{X}_y$, and $\overline{X}_r$ are the corresponding attitude error vectors, and $\overline{\dot{\varphi}}_{pc}$, $\overline{\dot{\varphi}}_{yc}$, and $\overline{\dot{\varphi}}_{rc}$ are the corresponding attitude rates. A_{op}, A_{oq}, A_{or}, A_{1p}, A_{1y}, A_{1r} are coefficient matrices derived from appropriate equations describing the flight mechanics.

Because commands may be given to certain combinations of engines, rather than to individual engines, we consider the following commands.

$$\beta_{1pc} = \beta_{pc} - \beta_{rc}/\sqrt{2}$$
$$\beta_{2pc} = \beta_{pc} - \beta_{rc}/\sqrt{2}$$
$$\beta_{3rp} = \beta_{pc} + \beta_{rc}/\sqrt{2}$$
$$\beta_{4pc} = \beta_{pc} + \beta_{rc}/\sqrt{2}$$
$$\beta_{1yc} = \beta_{yc} + \beta_{rc}/\sqrt{2}$$
$$\beta_{2yc} = \beta_{yc} - \beta_{rc}/\sqrt{2}$$
$$\beta_{33yc} = \beta_{yc} - \beta_{rc}/\sqrt{2}$$
$$\beta_{4yc} = \beta_{yc} + \beta_{rc}/\sqrt{2}$$

$$\beta_{pT} = \tfrac{1}{4}\left(\beta_{1p} + \beta_{2p} + \beta_{3p} + \beta_{4p}\right)$$
$$\beta_{yT} = \tfrac{1}{4}\left(\beta_{1y} + \beta_{2y} + \beta_{3y} + \beta_{4y}\right)$$
$$\beta_{rT} = \frac{\sqrt{2}}{8}\left(\beta_{1y} - \beta_{1p} - \beta_{2y} - \beta_{2p} - \beta_{3y} - \beta_{3p} + \beta_{4y} - \beta_{4p}\right)$$
$$\beta_m = \sqrt{(\beta_{pT})^2 + (\beta_{yT})^2}$$

$$\begin{cases} \Delta\beta_p = \beta_{pc} - \beta_{pT} \\ \Delta\beta_y = \beta_{yc} - \beta_{yT} \\ \Delta\beta_r = \beta_{rc} - \beta_{rT} \end{cases}$$

The latter three quantities are ideally zero, and departures from zero are an indicator of control effectiveness.

Other useful quantities used in the evaluation are

χ_p = predicted pitch tilt

χ_r = predicted roll program

$$\varphi_p = 90° - \bar{\chi}_p - \chi_p$$

$$\varphi_R = \chi_R + \bar{\chi}_R$$

In the program, angular accelerations in pitch, roll and yaw, as well as pitch rates and yaw rates for individual engines are computed in terms of observed engine gimbal angles and angles of attack. Attitude errors in pitch, roll and yaw, as well as corresponding angular velocities and accelerations are sensed from telemetered data.

During first stage action, a polynomial approximation to the pitch profile is used for guidance. Saturn V, for example, requires a body rotation to an angle of 72° from true north, starting at time T + 10.

Saturn V Dynamics

I. Thrust

$$F_i = CFV_i\,(PC_i)(AT_i) - AE_i\,(PA) + FE_i$$

where F_i = thrust of i-th engine

CFV_i = coefficient of vacuum thrust for i-th engine

PC_i = combustion chamber pressure for i-th engine

AT_i = throat area of i-th engine

AE_i = exit area of i-th engine

PA = ambient pressure at engine bell

FE_i = turbine exhaust thrust of i-th engine

$$F_S = \frac{1}{5} \sum_{i=1}^{5} F_i$$

II. Angle of Attack

The angle of attack may be determined from two different approaches and differences, if any, may be used in the flight evaluation program.

1. Angle of attack from Q - Ball measurements

$$PAPQD = \frac{\Delta \overline{PAP}}{1.45038\ (q)\ Cp\alpha} \qquad \text{(degrees pitch)}$$

$$PAyQD = \frac{\Delta \overline{PAy}}{1.45038\ (q)\ Cp\alpha} \qquad \text{(degrees yaw)}$$

$$\Delta PRESS = \sqrt{\overline{\Delta PAP}^2 + \overline{\Delta PAy}^2}$$

$$\alpha TOTAL = \frac{\Delta PRESS}{1.45038\ (q)\ (C_{p\alpha})}$$

In these equations ΔPAP and ΔPAy are obtained as pressure differentials in pitch and yaw from Q-ball measurements. Definition of other symbols are:

a = aerodynamic pressure

C_{pa} (function of Mach number) = aerodynamical normal force coefficient for angle of attack a

1.45038 is a conversion factor from Newtons/cm^2 to lbs/in^2

2. Angle of attack from filtered accelerations

$$\alpha_{pA} = \frac{\ddot{\alpha}\ pF}{(N'/M)pitch} - \frac{4\ Fs\ \sin\beta_{pT}}{(N'/M)\ pitch} \qquad \text{(deg pitch)}$$

$$\alpha_{yA} = \frac{\ddot{\alpha}\ \ yF}{(N'/M)\ yaw} - \frac{4\ FS\ \sin\beta_{yT}}{(N'/M)\ yaw} \qquad \text{(deg yaw)}$$

Symbols not previously defined are:

$\ddot{\alpha}_{pF}$ = pitch acceleration from filtered data

$\ddot{\alpha}_{yF}$ = yaw acceleration from filtered data

$$\left(\frac{N'}{M}\right) \text{pitch} = 17.797405\left(\frac{C_{z_p} qs}{M}\right), \quad \frac{m/sec^2}{degree}$$

$$\left(\frac{N'}{M}\right) \text{yaw} = 17.797405\left(\frac{C_{z_y} qs}{M}\right), \quad \frac{m/sec^2}{degree}$$

C_{zp} = aerodynamic normal force coefficients (functions of C_{zy}, Mach number and angle of attack), $\frac{1}{rad}$

S = reference area of vehicle, m^2

M = mass of vehicle, $\frac{Kg\ sec^2}{m}$

Comparison of the two methods for computing angle of attack, one from Q-ball measurements and the other from filtered acceleration data, provides another measure for evaluation of the control system.

Transformations of the pitch and yaw angles of attack into the flight azimuth, so that winds may be compared with rawinsonde data is accomplished by the simple transformations.

$$P\alpha PQFT = PAPQD \cos \varphi_R + PAyQD \text{ sim } \varphi_R$$

$$P\alpha yQFT = PAPQD \sin \varphi_R + PAyQD \cos \varphi_R$$

III. External Moments

$$M\ pitch = -\left[C_{1p}\ (PAPQD) + C_{2p}\ (\beta_{pT}) + I_p\ \ddot{\varphi}_{pF}/57.2957\right]$$

$$M\ yaw = -\left[C_{1y}\ (PAPQD) + C_{2y}\ (\beta_{yT}) + I_y\ \ddot{\varphi}_{yT}/57.2957\right]$$

where

$$C_{1p} = \frac{C_{zp}\ (q)(s)(CG - CP)}{57.2957795} \qquad \left(\frac{Kg\text{-}m}{deg\ \alpha_p}\right)$$

$$C_{1y} = \frac{C_{zy}\ (q)(s)(CG - CP)}{57.2957795}$$

$$C_{2p} = 4F_s\ (CG) = C_{2y}$$

CG = distance from center of gravity to gimbal plane

CP = distance from center of pressure to gimbal plane

As previously remarked in another formulation, M_{pitch} and M_{yaw} should ideally be zero; deviations provide a measure of control evaluation.

In the foregoing equations, $\Delta\overline{PAP}$, $\Delta\overline{PAY}$, ΔPRESS are measured from telemetered Q-ball, or accelerometer data. Mass characteristics, thrust and control parameters, and trajectory parameters are computed taped inputs to the program. Wind data, if available, may be a measured input, or wind deflections may be sensed from differential pressures during flight. Attitude errors, angular velocities in pitch, yaw, and roll, and angular accelerations in pitch, roll, and yaw are sensed from telemetered data.

The analysis presented is to be regarded as a typical program for evaluation studies. Any other formulations of flight dynamics and formats for comparing performance with preflight computations is acceptable, provided that it permits a reasonably comprehensive basis for comparison between expected performance and actual achievement in flight.

APPENDIX C

SIMULATION TECHNIQUE EQUATIONS

1. Introduction

Evaluation of system performance is normally concerned with a comparison of actual performance, obtained from flight data, with anticipated performance predicted by a system model. Models normally consist of systems of differential equations which theoretically describe the system being considered. The system of equations may be solved by digital computation, as may be simulated by electrical and servo-mechanical networks. In the former case, one deals with digital simulation of the systems;in the latter case, analog simulation. For a given analytical model, digital simulation generally has the advantage of greater precision and accuracy. However, analog devices may provide adequate data in a shorter time and at smaller expense. Which type should be used depends on evaluation of such factors as time requirements, the adequacy of the mathematical model, expense, and required accuracy. It should be remarked that the actual physical control system must itself be an analog system, consisting of relays, servomotors, filters, etc.

2. Analog Simulations

The detailed evaluation of the total control system consists of inserting the flight guidance steering commands and reconstructed trajectory data into a closed loop simulation. Such a simulation is best illustrated by a block diagram such as that shown in Figure C-1.

The resulting outputs of vehicle attitude, angular rates, and acceleration are compared with the flight results. If the results are well-matched, this is often the extent of the needed evaluation. If some parameter adjustment is indicated, comparison of intermediate outputs (e. g., engine command β_c and engine angle β) with flight results may give some insight as to which parameters are contributing to the discrepancies. If a malfunction has occurred which is not obvious from such evaluations, an open loop parameter evaluation is more precise for a fine-grained evaluation and therefore more desirable.

Inputs at various points of the analog simulation may, or may not be filtered. If the system is "noisy", varying amounts of data filtering may be desirable. (See Appendix A.)

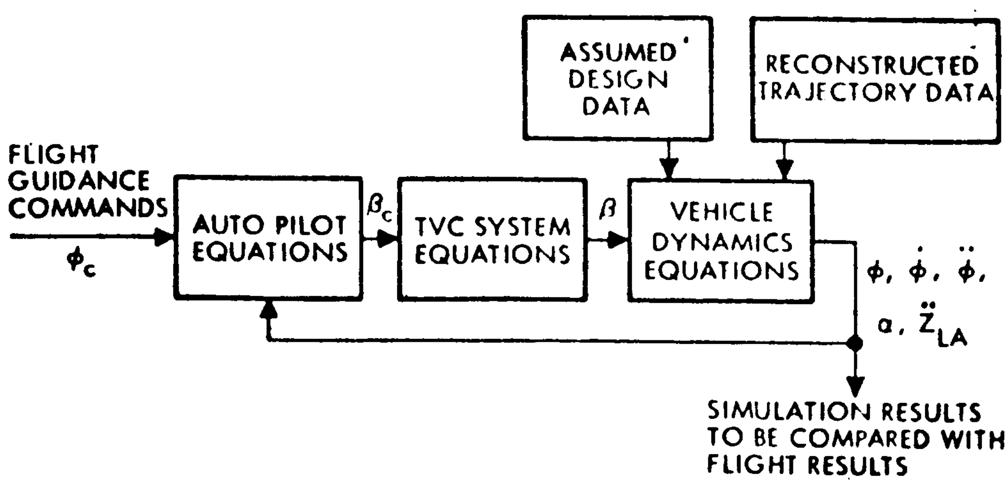

Figure C-1. Closed Loop Simulation Method

If the vehicle is considered to be a rigid body, detailed analysis of the system may be carried out using the following data parameters and equations:

Data for simulation methods (Figure C-1) are as follows:

Assumed Design Data

M = vehicle mass

I = vehicle inertia

ℓ_x = distance between cg and engine gimbal point

G = gravity

Reconstructed Trajectory Data

T = vehicle thrust

a = total angle of attack

Q = aerodynamic pressure

V = vehicle velocity

γ = angle of velocity vector from the local vertical

M_H = Mach number

β_{eo} = effective thrust misalignment angle

β_o = thrust misalignment angle

Z_{cg} = effective cg offset including thrust vector offsets

If the aerodynamic pressures are negligible, the vehicle angular acceleration and sensed lateral accelerations after filtering are given by:

$$\ddot{\varphi} = \mu_c (\beta - \beta_o) - \frac{T}{I} Z_{cg}$$

$$\ddot{Z}_{LA} = -a_\beta(\beta - \beta_o) + \ell a \ddot{\varphi}$$

where

$\ddot{\varphi}$	=	vehicle angular acceleration
μ_c	=	$\frac{T\ell_x}{I}$ = control moment coefficient
β	=	engine deflection angle
β_o	=	engine misalignment angle
T	=	vehicle thrust
ℓ_x	=	distance between vehicle c. g. and engine gimbal point
I	=	vehicle inertia
Zcg	=	lateral c. g. offset and thrust vector offset
$\ddot{Z}_{LA}$	=	sensed vehicle lateral acceleration
a_β	=	control thrust acceleration
ℓ_a	=	distance between lateral accelerometer and the vehicle c. g.

The effective engine misalignment angle, β_{eo}, is given by

$$\beta_{eo} = \beta_o + \frac{Zcg}{\ell_x} = \beta - \frac{\ddot{\theta}}{\mu_c}$$

The thrust misalignment angle, β_o, is obtained from:

$$\beta_o = \beta + \frac{\ddot{Z}_{LA} - \ell_a \ddot{\theta}}{a_\beta}$$

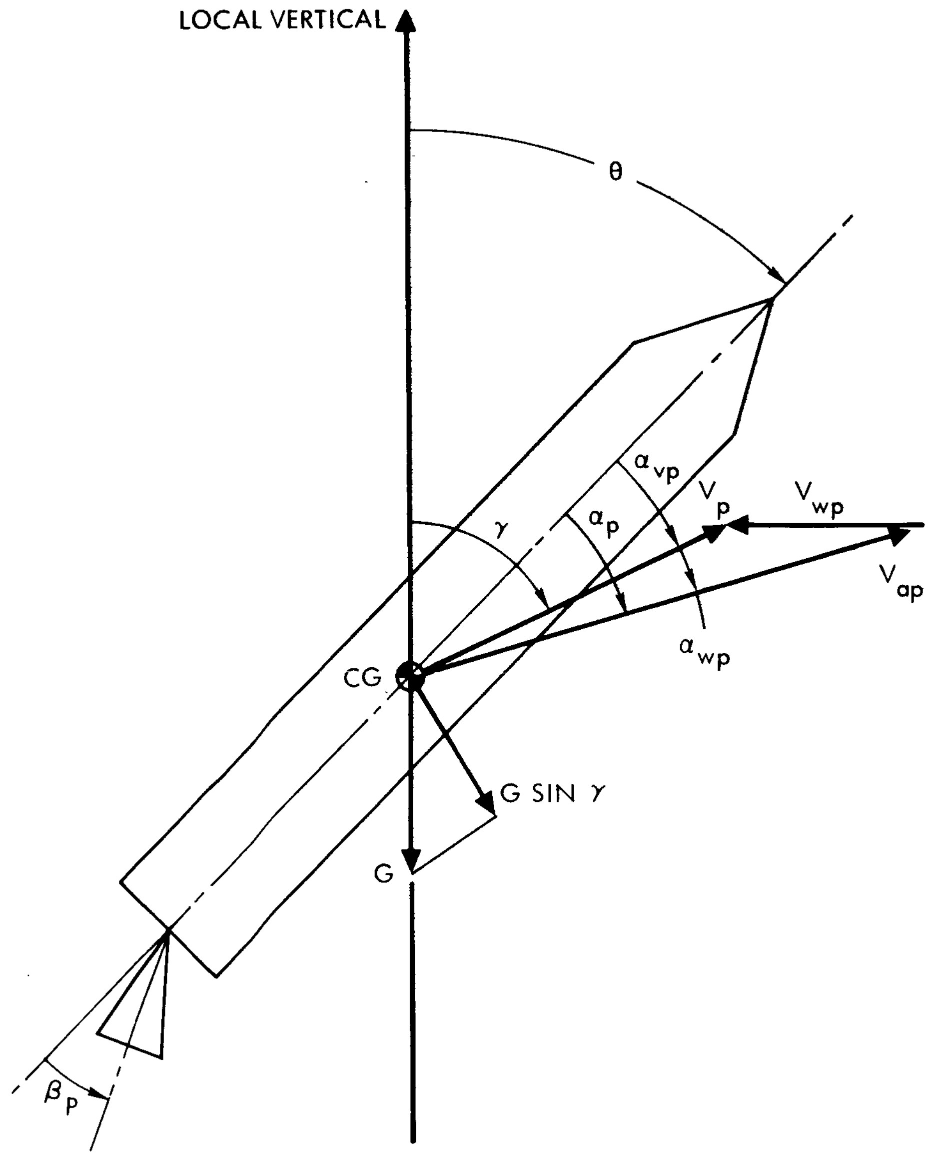

Figure C-2. Vehicle Pitch Plane Angles

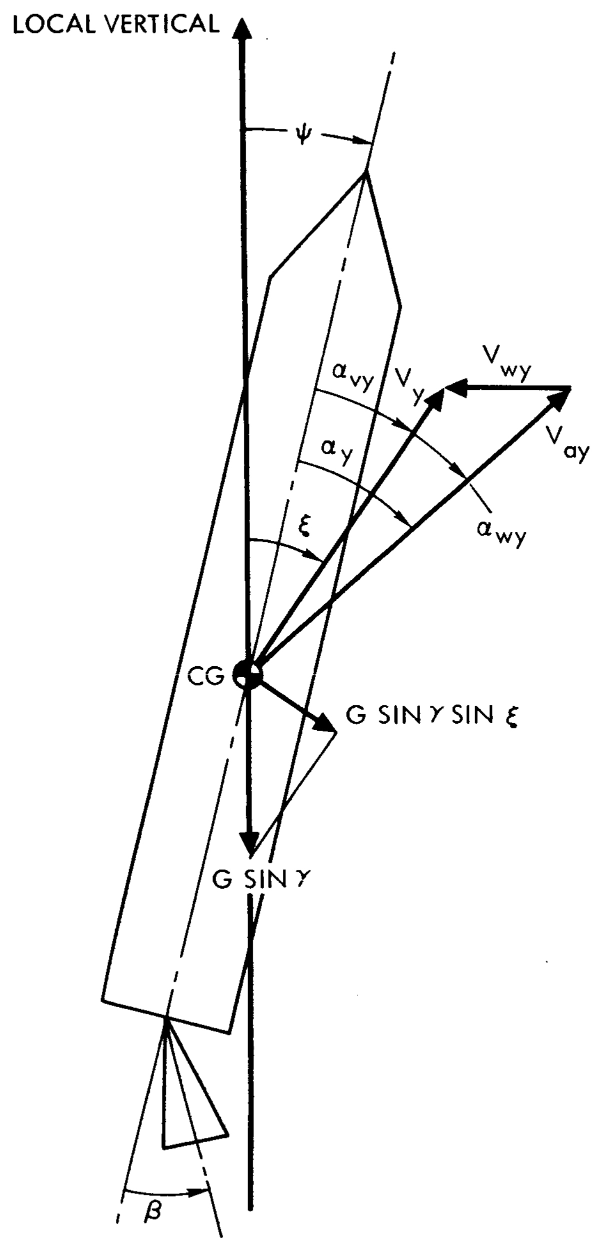

Figure C-3. Vehicle Yaw Plane Angles

The lateral offset value is obtained from:

$$Zcg = \frac{I}{a_\beta T} \left[\ddot{\theta} \left(\mu_c \;\; a - a_\beta \right) - \mu_c \ddot{Z}_{LA} \right]$$

If the angular acceleration, $\ddot{0}$, is negligible, $\beta_{eo} = \beta$ and

$$Zcg = - \frac{\ddot{Z}_{LA\ x}}{a\beta}$$

2.1 Reconstruction of the Total Angle of Attack

The reconstruction of the total angle of attack (α) is required since the vehicle experiences aerodynamic forces from both the angle of attack due to wind (α_w), and the angle of attack due to angular differences between vehicle centerline and velocity vector (α_v). The total angle of attack, illustrated in Figures C-2 and C-3, is given by

$$\alpha = \alpha_v + \alpha_w$$

Using the subscript "p" to denote the pitch plane, this equation becomes

$$\alpha_p = \alpha_{vp} + \alpha_{wp}$$

$$\alpha_{wp} = \tan^{-1} \frac{V_{wp} \cos \gamma}{V_p}$$

Similarly using 'y' in the yaw plane, the equation becomes

$$\alpha_y = \alpha_{vy} + \alpha_{wy}$$

$$\alpha_{wy} = \frac{\tan^{-1} V_{wy} \cos t}{V_y}$$

where V_{wp} and V_{wy} are the wind velocities in the pitch and yaw planes and V_p and V_y are the component vehicle velocities.

The wind velocity data are obtained from wind sounding tests at the launch site and nearby sites or by analytical techniques using previously evaluated statistical data. The tests are generally made within an hour before and after a launch, thereby enabling interpolation of launch winds. This can be extended even further, if necessary, by reconstructing the wind velocity history through curve fitting of data from several wind sounding tests. Such a necessity may arise if rapidly changing wind speed and direction occurs. However, in most applications, wind velocity models are included in the analytical models used for flight evaluation and the parameters of the wind model are estimated reversively along with the other parameters of the system.

2.2 Bending and Sloshing

The principal factors which will modify the foregoing rigid-body analysis are the effects of bending due to aerodynamic and control moments acting on the vehicle, and the effect of sloshing in propellant tanks. A quick-analysis evaluation of these effects may be made by a comparison between the flight test results and the expected amplitudes and frequencies of oscillation due to bending and sloshing as shown in the attitude rate traces. If such evaluation is not conclusive, there is little recourse but to include an analysis of these effects in the full-scale simulation.

One method is to perform the closed-loop control system simulation studies used in the design analysis with the additional inclusion of reconstructed thrust, thrust misalignment angles, winds and trajectory, and then varying propellant slosh parameters until a best match with flight data is obtained.

A more precise method is to utilize just the vehicle dynamics portion of the simulation. By employing the flight test engine angle as an input to these equations, and discarding the autopilot and TVC equations, the effect of the autopilot and TVC system variations could be assessed. This method is shown in Figure C-4 as an open-loop simulation method. The propellant slosh evaluation is performed over a time duration in which aerodynamic effects are negligible, and therefore, the requirements for the reconstructed trajectory data are reduced

ASSUMED DATA: M, I, ℓ_x

RECONSTRUCTED TRAJECTORY DATA: a_T, β_{eo}

LOW PASS FILTERED FLIGHT DATA: β^{LF}

PARAMETERS TO BE VARIED: ω_o, ω_f, ζ_o, ζ_f, ℓ_o, ℓ_f, m_o, m_f

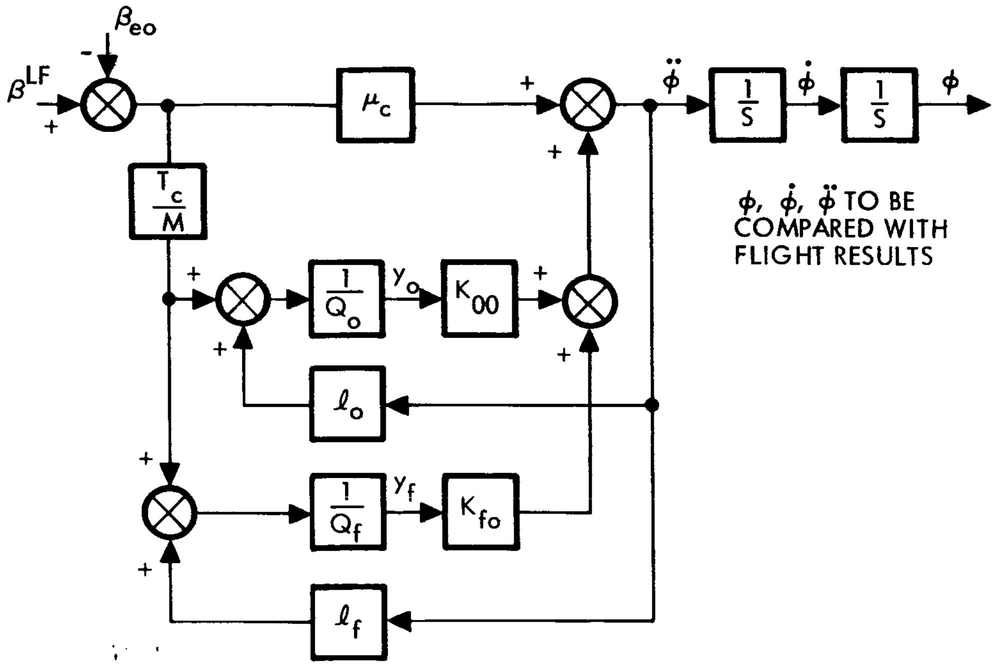

Figure C-4. Open Loop Propellant Slosh Simulation (Approximation) to thrust profile and engine misalignment angles. The engine angle data is first filtered through a low-pass filter to eliminate the effects of body bending.

Propellant damping equations are generally known in design analysis, and variations in these models may be accomplished through variations in multiplying factors. In some cases, it may be necessary to include such effects as viscosity and propellant consumption rate.

An open loop simulation technique for sloshing effects is shown in Figure C-3.

An approximate system of equations for the effects of sloshing is the following, including cylindrical link sloshing both with and without the effects of ring damping.

Approximated Propellant Slosh Equations

$$\ddot{\varphi} = \mu_c (\beta^{LF} - \beta_{eo}) + y_o K_{oo} + y_f K_{fo}$$

$$y_o = \frac{1}{Q_o} (\ell_o \ddot{\varphi} + a_T \beta)$$

$$y_f = \frac{1}{Q_f} (\ell_f \ddot{\varphi} + a_T \beta)$$

where the transfer functions are defined as:

$$Q_o = s^2 + 2 \zeta_o \omega_o s + \omega_o^2 \quad 1 + \frac{m_o}{M}$$

$$Q_f = s^2 + 2\zeta_f \omega_f s + \omega_f^2 \quad 1 + \frac{m_f}{M}$$

$$K_{oo} = \frac{m_o}{I} (\ell_o \omega_o^2 - a_T)$$

$$K_{fo} = \frac{m_f}{I} (\ell_f \omega_f^2 - a_T)$$

$$\mu_c = \frac{T_c \ell_x}{I}$$

where

s = Laplace transform operation

y_o, y_f = oxidizer and fuel slosh mass displacement

M_o, m_f = oxidizer and fuel slosh mass

ℓ_o, ℓ_f = oxidizer and fuel slosh mass moment arm

ω_o, ω_f = oxidizer and fuel slosh mass frequency

ζ_o, ζ_f = oxidizer and fuel slosh mass damping

M = vehicle mass excluding slosh masses

T_c = control thrust

$\ddot{\varphi}$ = vehicle angular acceleration

β = engine deflection angle

a_T = total (thrust minus drag) vehicle acceleration

β^{LF} = low pass filter estimate of engine angle

I = vehicle inertia

ℓ_x = control moment error

Cylindrical Tank Smooth Wall Damping

$$\zeta = \frac{0.886\nu^{1/2}}{a_T^{1/2}\ R^{3/4}\ (30.48\frac{cm}{ft})} \qquad \zeta < 0.05, \quad \frac{h_o}{R} > 0.1$$

ν = kinematic viscosity, cm^2/sec

a_T = axial acceleration, ft/sec^2

R = tank radius, ft

h_o = propellant wave height, ft

Cylindrical Tank Ring Baffle Damping

$$\zeta = 4.5\, k_r^{3/2} \left(\frac{h_o}{R}\right)^{1/2} \exp\left(\frac{-4.6 d_r}{R}\right) \qquad H \ \ R \qquad \frac{h_o}{w_R} < 3$$

H	=	propellant level above tank bottom, ft
w_R	=	ring baffle width, ft
d_r	=	distance of propellant level above baffle, ft
A_r	=	tank area
$k_r A_r$	=	ring baffle area = $2\pi R w_R$

Maximum Force on Ring

$$F_r = 8.25\, h_o^{3/2} \exp\left(\frac{-2.76\, d_r}{R}\right) k_r A \rho G$$

ρ	=	propellant density
G	=	gravity

2.2 Body Bending

If a dominant bending oscillation is prevalent in the flight results, vehicle body-bending parameters can be obtained through simulation studies and compared with design values. If dynamics effects, in addition to the one being sought, are apparent in the data, data filtering may be necessary. Such effects as propellant sloshing and higher bending mode oscillations can be removed by employing a bandpass filter, allowing only bending mode frequency to pass.

If these frequencies are known to vary considerably over the duration of the flight, the evaluation can be performed over smaller phases with different bandpass filters employed. Usually the visible bending oscillations are of short duration and do not require this consideration.

The simplified bending mode parameter evaluation method shown in Figure C-5 employs the bandpass filtered engine angle data. By utilizing the actual engine data, the autopilot and TVC system high-frequency dynamics uncertainties are bypassed. The lateral and angular accelerometer outputs and position and rate gyro outputs can be compared with the corresponding flight data which have also been bandpass filtered. Adjustments can be made to the bending mode frequency or slopes and deflections to achieve the desired match.

If slosh frequencies or other bending modes exist in the proximity of the mode under study, a multiple mode analysis may be required, in which the adjustment of coupling terms is necessary. The possibility of satisfactorily matching the data would be diminished in this instance, due to the added complexity of the task.

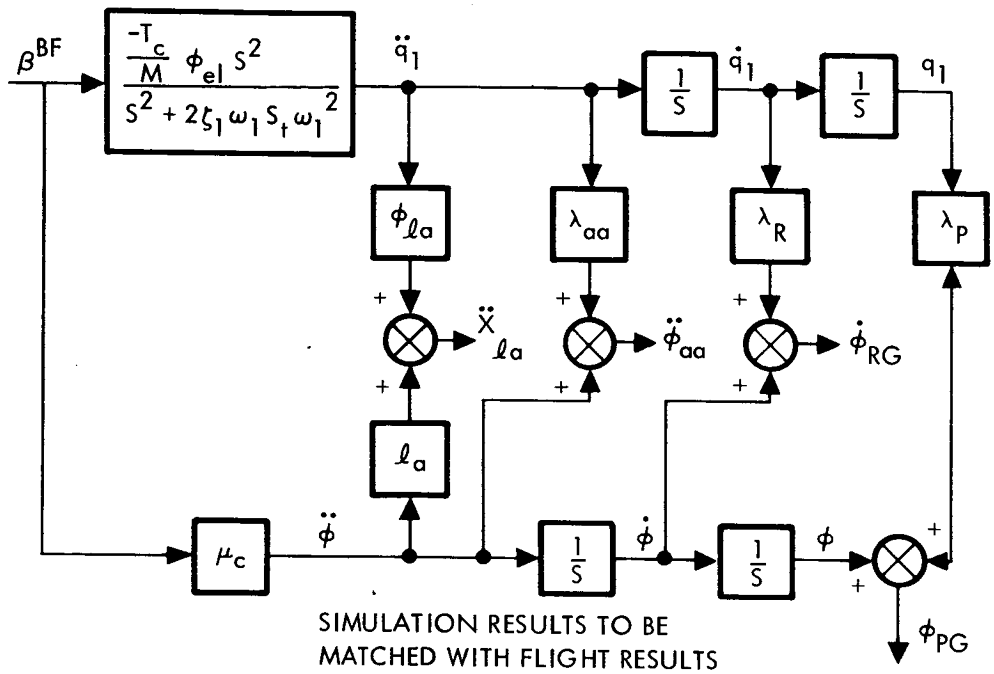

Figure C-5. Simplified Bending Mode Parameter Evaluation

Identification of symbols for Figure C-5 is as follows:

βBF = band pass filtered flight engine angle data

φ_{e1} = engine gimbal station bending deflection

$\varphi_{\ell a}$ = lateral accelerometer station bending deflection

λ_{aa} = angular accelerometer station bending slope

λ_R = rate gyro station bending slope

λ_p = position gyro station bending

ℓ_a = distance between lateral accelerometer and vehicle c. g.

T_c = control thrust

M = vehicle mass

μ_c = control moment coefficient

ω_1 = bending mode frequency

ζ_1 = bending mode damping

S = LaPlace operator

q_1 = normalized bending mode amplitude

$\ddot{X}_{\ell a}$ = lateral accelerometer output from the simulation

$\ddot{\varphi}_{aa}$ = angular accelerometer output from the simulation

$\dot{\varphi}_{RG}$ = rate gyro output from the simulation

φ_{PG} = position gyro output from the simulation

3. Digital Simulation

The digital simulation of a control system will be illustrated by a method developed for application to the Saturn V control system. This system uses phase shaping filter networks, which are analyzed by a closed form input-output relation in the electric circuitry, which has the advantage of giving a true response and is not as restrictive as other methods on the form of the input function, and has the advantage of using integration time steps which are compatible with the rest of the system.

The method presented has provided a complete representation of the Saturn control system and improves the integration time steps by a factor of 100 over other methods, such as that of Runge-Kutta.

3.1 Description of the Control System

The control system for the Saturn V vehicle is based on sensing the pitch, yaw, and roll attitude errors and the pitch and yaw translational accelerations. The attitude errors are obtained from a space-fixed platform on board the vehicle. The pitch and yaw acceleration signals are obtained from body-fixed accelerometers. These attitude error and acceleration signals are transformed through electrical phase shaping networks to pitch and yaw actuator commands for each of the four gimballed engines.

The following diagram shows the sign conventions and the body-fixed orientation of the vehicle used in this report.

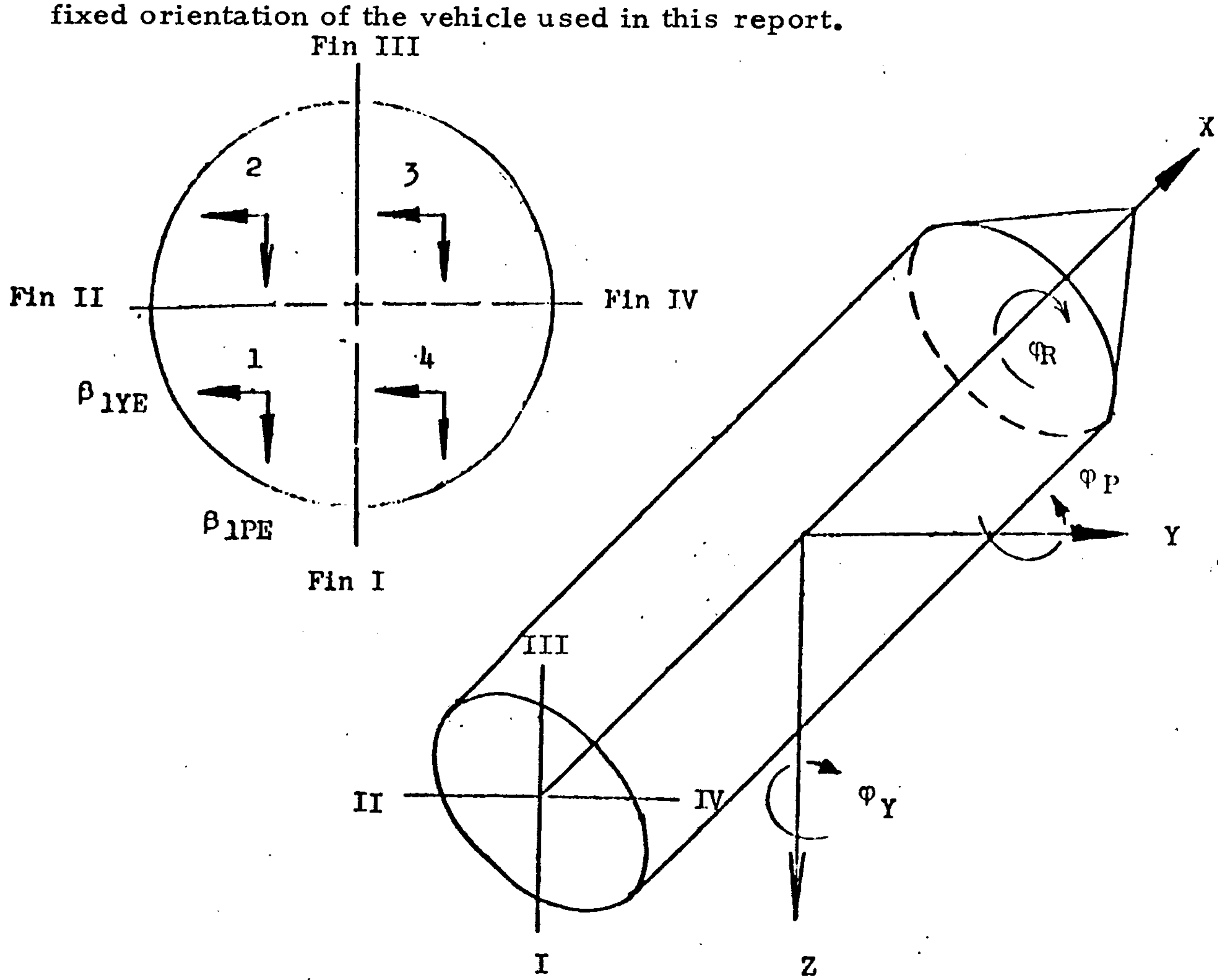

Figure C-6. Sign Convention

$+\varphi_R$ roll about X-axis, clockwise from rear

$+\varphi_P$ pitch about Y-axis, nose up

$+\varphi_Y$ yaw about Z-axis, nose right

The sign conventions are such that a positive pitch engine deflection (β_P) gives a restoring force to correct for a positive pitch attitude error (φ_P) and a negative pitch normal acceleration ($\ddot{\gamma}_P$). A positive yaw control deflection (β_Y) corrects for a positive yaw attitude error (φ_Y) and a positive yaw normal acceleration ($\ddot{\gamma}_Y$). The positive roll engine deflection (β_R) gives a counterclockwise (looking from rear) restoring moment which corrects for a positive roll attitude error (φ_R).

The components of the control system considered in this note are shown in the block diagram shown in Figure 2 which is drawn for a single engine. The gain and filter networks are identical for pitch and yaw due to body symmetry. The gain coefficients, a_o and g_2, are based on rigid body stability analyses and are given as a function of flight time. The desired attitude and accelerometer transfer functions $A_{\varphi}(S)$ and $A_{\ddot{\gamma}}(S)$ are established by the electric phase shaping networks which may vary with flight time. The transfer functions representing the actuator and the engine dynamics are considered as a single transfer function throughout flight for this analysis.

The equations relating the pitch, yaw and roll commands to the attitude errors and accelerations are:

$$\beta_{PC}(S) = a_{OP}\, A_{\varphi P}(S)\, \varphi_P(S) - g_{2P} A_{\ddot{\gamma} P}(S) \ddot{\gamma}_P(S)$$

$$\beta_{YC}(S) = a_{OY}\, A_{\varphi Y}(S) \varphi_Y(S) + g_{2Y} A_{\ddot{\gamma} Y}(S) \ddot{\gamma}_Y(S)$$

$$\beta_{RC}(S) = a_{OR}\, A_{\varphi R}(S) \varphi_R(S).$$

The individual engine deflection commands are:

$$\beta_{1PC} = \beta_{PC} - \beta_{RC}$$

$$\beta_{1YC} = \beta_{YC} + \beta_{RC}$$

$$\beta_{2PC} = \beta_{PC} - \beta_{RC}$$

$$\beta_{2YC} = \beta_{YC} - \beta_{RC}$$

$$\beta_{3PC} = \beta_{PC} + \beta_{RC}$$

$$\beta_{3YC} = \beta_{YC} - \beta_{RC}$$

$$\beta_{4PC} = \beta_{PC} + \beta_{RC}$$

$$\beta_{4YC} = \beta_{YC} + \beta_{RC} .$$

The actuator - engine equations are

$$\beta_{1PE}(S) = A_{AE}(S) \quad \beta_{1PC}(S)$$

$$\beta_{1YE}(S) = A_{AE}(S) \quad \beta_{1YC}(S). \tag{5}$$

The engine deflections β_{1PE} and β_{1YE} in equation (5) are the actual engine deflections of engine 1 which are telemetered from the flight vehicle. The average actual engine deflections become:

$$\beta_{PE} = \frac{\beta_{1PE} + \beta_{2PE} + \beta_{3PE} + \beta_{4PE}}{4}$$

$$\beta_{YE} = \frac{\beta_{1YE} + \beta_{2YE} + \beta_{3YE} + \beta_{4YE}}{4} \tag{6}$$

$$\beta_{RE} = \frac{\beta_{1YE} - \beta_{1PE} - \beta_{2PE} - \beta_{2YE} + \beta_{3PE} - \beta_{3YE} + \beta_{4PE} + \beta_{4YE}}{8}$$

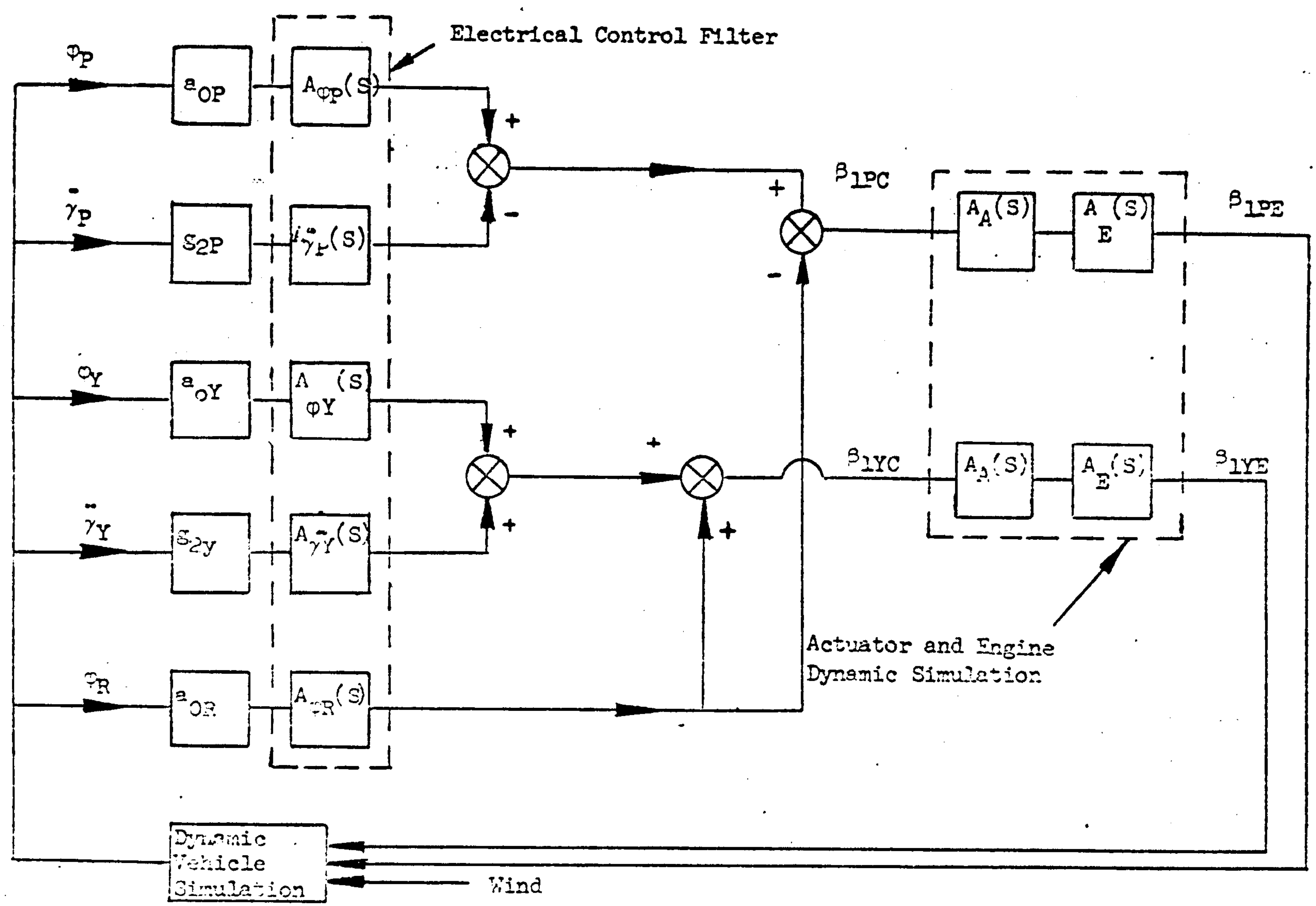

Figure C-7. Control System Components (Engine 1)

3.2 Differential Equation Representation of Filter Networks

The electrical control filters are used to delay or modify the control sensor signals. These control filter networks are designed by analyzing the body bending, propellant sloshing, and engine dynamic data so that a stable control system will be maintained. The control transfer functions which are indicated as $A_\phi(S)$ and $A_{\dot{\gamma}}(S)$ in Figure C-7, are determined by analyzing the electrical network schematics.

A typical electric filter is shown schematically in Figure C-8. This will be used as an example to indicate the first step in developing the required equations for the digital simulation. A single input voltage E_{in} is assumed and a system of differential equations is written using Kirchoff's voltage law. To avoid a system of integro-differential equations, the electronic charge, q, is selected as the dependent variable. The voltage drops across the individual electrical elements (inductors, resistors and capacitors) are determined from the relationships:

$$
\begin{aligned}
E_{inductor} &= L_i \ddot{q}_k \\
E_{resistor} &= R_i \dot{q}_k \\
E_{capacitor} &= \frac{1}{C_i} q_k
\end{aligned}
\qquad (7)
$$

Where q is the time integral of current

$$q_k = \int_0^t i_k dt \qquad (8)$$

In the example shown on Figure C-8, there are three independent loop networks. Therefore, three equations are required to evaluate the dependent variables q_1, q_2 and q_3. The system of equations obtained by setting the voltage drop around each loop to zero are:

$$L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{1}{C_1} q_1 - \frac{1}{C_1} q_3 = E_{in} \qquad (9\text{-}1)$$

$$R_2 \dot{q}_2 + \frac{1}{C_2} q_2 - R_2 \dot{q}_3 = 0 \qquad (9\text{-}2)$$

$$-\frac{1}{C_1} q_1 - R_2 \dot{q}_2 + (R_2 + R_3 + R_4) \dot{q}_3 + \frac{1}{C_1} q_3 = 0 \qquad (9\text{-}3)$$

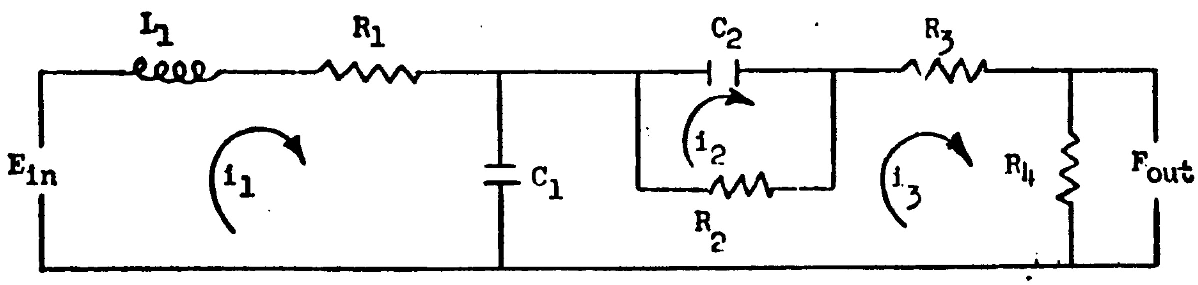

Figure C-8. Typical Network Schematic

By Laplace transforming equation (9), using the initial conditions $q_k(0) = \dot{q}_k(0) = 0$, a system of algabraic equations, linear in $L(q_k)$, is formed as follows:

$$\begin{bmatrix} (L_1S^2 + R_1S + \frac{1}{C_1}) & 0 & -\frac{1}{C_1} \\ 0 & (R_2S + \frac{1}{C_2}) & -R_2S \\ -\frac{1}{C_1} & -R_2S & (R_5S + \frac{1}{C_1}) \end{bmatrix} \cdot \begin{bmatrix} L(q_1) \\ L(q_2) \\ L(q_3) \end{bmatrix} = \begin{bmatrix} L(E_{in}) \\ 0 \\ 0 \end{bmatrix} \qquad (10)$$

where $R_5 = R_2 + R_3 + R_4$.

S is the Laplace operator and L denotes the transformation process.

The transfer function for the electrical filter is

$$A(S) = \frac{L(E_{out})}{L(E_{in})} \qquad (11)$$

For the network shown in Figure C-8, the output voltage is equal to $R_4 \dot{q}_3$. Therefore,

$$A(S) = \frac{L(R_4\dot{q}_3)}{L(E_{in})} = \frac{R_4 S\ L(q_3)}{L\ (E_{in})}. \qquad (12)$$

A general solution for equation (12) can be developed by generalizing equation (10) to k independent loops.

$$\begin{bmatrix} f_{11}(S) & f_{12}(S) & \ldots f_{1k}(S) \\ f_{21}(S) & f_{22}(S) & \\ \vdots & \vdots & \\ f_{k1}(S) & f_{k2}(S) & \ldots f_{kk}(S) \end{bmatrix} \cdot \begin{bmatrix} L(q_1) \\ L(q_2) \\ \vdots \\ L(q_k) \end{bmatrix} = \begin{bmatrix} L(E_{in}) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (13)$$

Solution of $L(q_k)$ from equation (13) using Cramer's rule gives

$$L(q_k) = \frac{\begin{vmatrix} f_{11}(S) & \ldots f_{1,k-1}(S) & L(E_{in}) \\ \ldots & \ldots & \ldots \\ f_{k,1}(S) & \ldots f_{k,k-1}(S) & 0 \end{vmatrix}}{\begin{vmatrix} f_{11}(S) & \ldots f_{1,k}(S) \\ \ldots & \ldots \\ f_{k,1}(S) & \ldots f_{k,k}(S) \end{vmatrix}} \tag{14}$$

which reduces to

$$L(q_k) = (-1)^{1+k} \frac{L(E_{in})}{D(S)} \begin{vmatrix} f_{21}(S) & \ldots & f_{2,k-1}(S) \\ \ldots & \ldots & \ldots \\ f_{k1}(S) & \ldots & f_{k,k-1}(S) \end{vmatrix} \tag{15}$$

$$= (-1)^{1+k} \frac{L(E_{in})\, N(S)}{D(S)}$$

where D(S) is the determinant of coefficients in equation (13). Substituting equation (15) in equation (12) yields,

$$A(S) = (-1)^{k+1} \frac{P_4 S N(S)}{D(S)} \tag{16}$$

where N(S) and D(S) are polynomials in S.

The frequency response of the transfer function A(S) can be found by substituting $(j\omega)$ for S in both the numerator and denominator polynomials. For practical purposes the transfer function is multiplied by a constant (K) so that the amplitude gain at D.C. (zero frequency) is one. The normalized phase and amplitude gains are calculated by the following expression utilizing the rules of complex algebra.

$$A(S) = \frac{K\left[N_m (j\omega)^m + N_{m-1} (j\omega)^{m-1} + \ldots N_o\right]}{D_n (j\omega)^n + D_{n-1} (j\omega)^{n-1} + \ldots D_o} = \frac{K\left[R_N + jI_N\right]}{R_D + jI_D} \tag{17}$$

where n and m are positive integers with the condition m_n. The transfer characteristics can be determined at any particular frequency from the real and imaginary terms of equation (17).

$$\text{Amplitude Gain} = \frac{K\left[R_N^2 + I_N^2\right]}{R_D^2 + I_D^2} \qquad (18)$$

$$\text{Phase Angle} = \tan^{-1}\left(\frac{I_N}{R_N}\right) - \tan^{-1}\left(\frac{I_D}{R_D}\right) \qquad (19)$$

The output voltage signal (E_{out}) from each control filter can be solved numerically from the differential loop equation (9). But in order to utilize the closed form solution outlined in this report, it is necessary to transform this system of equations into a set of linear first order differential equations.

Sometimes a direct transformation is not obvious, but with a combination of rearranging the loops and manipulating the loop equations, a transformation can be obtained. To illustrate, Figure C-9 can be redrawn as follows:

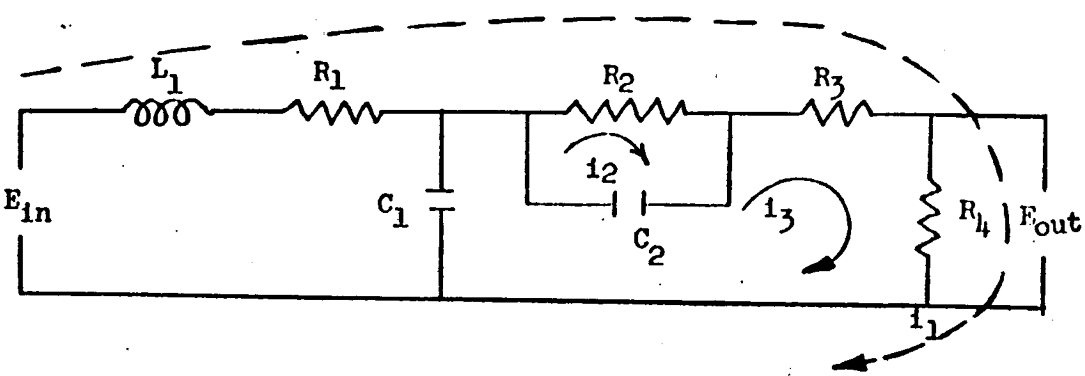

Figure C-9. Rearranged Figure C-8 Network

The loop equations are

$$L_1\ddot{q}_1 + (R_1 + R_2 + R_3 + R_4)\,\dot{q}_1 + R_2\dot{q}_2 + (R_3 + R_4)\,\dot{q}_3 = E_{in} \qquad (20\text{-}1)$$

$$R_2\dot{q}_1 + R_2\dot{q}_2 + \frac{1}{C_2}\,q_2 - \frac{1}{C_2}\,q_3 = 0 \qquad (20\text{-}2)$$

$$(R_3 + R_4)\,\dot{q}_1 - \frac{1}{C_2}\,q_2 + (R_3 + R_4)\,\dot{q}_3 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)q_3 = 0 \qquad (20\text{-}3)$$

$$E_{out} = R_4\,(\dot{q}_1 + \dot{q}_3). \qquad (20\text{-}4)$$

Now let us define a set of transformation equations.

$$Q_1 = q_1$$
$$Q_2 = q_2 \qquad (20\text{-}5)$$
$$Q_3 = q_3$$

Then

$$E_{out} = R_4 (Q_1 + \dot{Q}_3). \qquad (20\text{-}6)$$

A first order system can now be written.

$$L_1 \dot{Q}_1 + R_6 Q_1 + R_2 \dot{Q}_2 + R_7 \dot{Q}_3 = E_{in} \qquad (21\text{-}1)$$

$$R_2 Q_1 + R_2 \dot{Q}_2 + \frac{1}{C_2} Q_2 - \frac{1}{C_2} Q_3 = 0 \qquad (21\text{-}2)$$

$$R_7 Q_1 - \frac{1}{C_2} Q_2 + R_7 \dot{Q}_3 + C_3 Q_3 = 0 \qquad (21\text{-}3)$$

where

$$R_6 = R_1 + R_2 + R_3 + R_4$$
$$R_7 = R_3 + R_4$$
$$C_3 = (\frac{1}{C_1} + \frac{1}{C_2})$$

or in Matrix form

$$\begin{bmatrix} L_1 & R_2 & R_7 \\ 0 & R_2 & 0 \\ 0 & 0 & R_7 \end{bmatrix} \cdot \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \end{bmatrix} = - \begin{bmatrix} R_6 & 0 & 0 \\ R_2 & \frac{1}{C_2} & -\frac{1}{C_2} \\ R_7 & -\frac{1}{C_2} & C_3 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} + \begin{bmatrix} E_{in} \\ 0 \\ 0 \end{bmatrix} \qquad (21\text{-}4)$$

To relate (21-4) to Figure C-7, it is necessary to multiply equation (21-4) by the inverse of the coefficient matrix for the Q's. This can be shown here by diagonalizing this coefficient matrix with a few simple manipulations as follows:

(a) Subtract the second row of equations (21-4) from the first.

(b) Subtract the last row from the resultant of (a) and use this result as the first row.

$$\begin{bmatrix} L_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_7 \end{bmatrix} \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \end{bmatrix} = - \begin{bmatrix} (R_6 - R_2 - R_7) & 0 & (\frac{1}{C_2} - C_3) \\ R_2 & \frac{1}{C_2} & -\frac{1}{C_2} \\ R_7 & -\frac{1}{C_2} & C_3 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

$$+ \begin{bmatrix} E_{in} \\ 0 \\ 0 \end{bmatrix} \qquad (21\text{-}5)$$

Now the coefficients A and $\overline{b}$ used in equation (A-1) can be determined.

$$A = - \begin{bmatrix} L_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_7 \end{bmatrix}^{-1} \begin{bmatrix} (R_6 - R_2 - R_7) & 0 & (\frac{1}{C_2} - C_3) \\ R_2 & \frac{1}{C_2} & -\frac{1}{C_2} \\ R_7 & -\frac{1}{C_2} & C_3 \end{bmatrix}$$

or

$$A = - \begin{bmatrix} \frac{(R_6 - R_2 - R_7)}{L_1} & 0 & \frac{(\frac{1}{C_2} - C_3)}{L_1} \\ 1 & \frac{1}{R_2 C_2} & \frac{-1}{R_2 C_2} \\ 1 & \frac{-1}{R_7 C_2} & \frac{C_3}{R_7} \end{bmatrix} \qquad (21\text{-}5a)$$

$$\bar{b} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_7 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \qquad (21\text{-}5b)$$

The output signal (E_{out}) from this network can now be written from the above equation as,

$$\begin{aligned} E_{out} &= R_4 (Q_1 + \dot{Q}_3) \\ &= R_4 \left\{ Q_1 + \left[A_{31}, A_{32}, A_{33} \right] \bar{Q} \right\} \\ &= R_4 \left\{ \left[0, \frac{1}{R_7 C_2}, - \frac{C_3}{R_7} \right] \bar{Q} \right\} . \end{aligned} \qquad (22)$$

This can be reduced to a form similar to (A-2)

$$E_{out} = \bar{u}^T \bar{Q} \qquad (22\text{-}a)$$

where the transpose of the vector $\bar{u}$ is

$$\bar{u}^T = R_4 \left[0, \frac{1}{R_7 C_2}, - \frac{C_3}{R_7} \right] . \qquad (23)$$

Summarizing, a set of first order linear differential equations can be obtained for an electrical network which has the form:

$$\dot{\bar{Q}} = [A] \bar{Q} + E_{in} \bar{b} \qquad (24)$$

$$E_{out} = \bar{u}^T \bar{Q} . \qquad (25)$$

As is seen in the preceding section, the control filters each can be represented as a system of linear first order differential equations. The solutions to these equations are now obtained in closed form by the method of Andrus, which has been described in Appendix A-VI.

3.3 Treatment of Aerodynamics Moment Parameters

The following procedures are usually adequate to evaluate the anomalous effects due to engine misalignments and offsets, body bending effects, and other slowly varying effects which might be spuriously considered as aerodynamic effects.

The steps given below successfully solve or avoid these problems.

(a) Filter the angular acceleration trace, $\ddot{\phi}$, the lateral acceleration trace, $\ddot{X}_{LA}$, the engine angle trace, β, the reconstructed angle-of-attack trace, α, with the same low pass filter to reduce noise and vehicle high frequency dynamics, while maintaining equal filter effects on these traces.

(b) Compute the estimated value of the normal force, $F_{N\alpha}\alpha$, (the parameters being functions of time from the normal force equation):

$$\left(F_{N\alpha}\right)\alpha = (\ddot{X}_{LA}\left[- L_A\ddot{\phi}) - T_c\beta\right]$$

where $\ddot{X}_{LA}$ is the lateral accelerometer trace, L_A is the expected distance between the vehicle c.g. and the lateral accelerometer, M is the vehicle mass, and T_c is the vehicle control thrust (gimbaled engines only).

(c) Compute the expected value of the normal force $\overline{F_{N\alpha}\alpha}$, employing the reconstructed total angle of attach, α, and the reconstructed mach number trace, M_H:

$$\overline{F_{N\alpha}\alpha} = C_{N\alpha}AQ\alpha$$

where A is the reference area and Q is the reconstructed aerodynamic pressure and where $C_{N\alpha}$ is the normal force coefficient, obtained from tables, as a function of total angle-of-attack and mach number.

(d) Plot and compare the expected value of the normal force with the estimated normal force value and determine if it is within an allowable difference, ΔF_n:

$$F_{N\alpha}\alpha - \overline{F_{N\alpha}\alpha} < \Delta F_n$$

(e) Compute the expected value of the aerodynamic moment,

$$\overline{F_{N\alpha}\alpha \ell p} = C_{N\alpha} AQ \ell p \alpha$$

where ℓp is the expected value of the aerodynamic moment arm obtained from the difference between the expected c. g. location and the expected center of pressure; the latter is also a function of the total angle of attack and mach number.

(f) Compute the estimated value of the aerodynamic moment, $F_{N\alpha}\alpha \ell p$, from the moment equation:

$$F_{N\alpha}\alpha \ell p = I \ddot{\phi} - T_c \ell_x (\beta - \beta_{eo})$$

where ℓ_x is the control moment arm and β_{eo} is the effective engine misalignment angle.

(g) Plot and compare the expected value of the aerodynamic moment with the estimated value and determine if it is within an allowable difference, ΔM_N:

$$F_{N\alpha}\alpha \ell p - \overline{F_{N\alpha}\alpha \ell_p} < \Delta M_N$$

(h) Passage of these comparison tests constitutes verification of the design aerodynamic moment parameters.

(i) An extension of this computation would be to obtain $F_{N\alpha}$ by dividing a conditioned α. This conditioning would entail setting a lower limit on α to avoid dividing by zero. The normal force coefficient $C_{N\alpha}$ could then be obtained and compared directly. Similarly, a lower limit may be placed on $F_{N\alpha}\alpha$ to obtain the aerodynamic moment arm. A similar procedure should be developed for the axial force setting in the launch vehicle.