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LOW NOISE AND DYNAMIC RANGE IN SYMMETRIC MIXER CIRCUITS*

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This was to have been a paper on the applications of symmetric circuits to increased output power, efficiency and dynamic range in varactor multipliers and mixers. However, D. H. Steinbrecher, my research associate, and I have been embroiled in a continuing controversy on low-noise mixers with others in the field and I felt that this conference might be an appropriate forum for a second round. The first round occurred at the 1967 International Solid-State Circuits Conference and left many questions unanswered. This paper represents an attempt to answer some of the questions and will present a brand-new, at least to us, view of mixer performance, unencumbered by clouds of Bessel functions and unrealizable limits.

There are two shibboleths which need to be disposed of before we begin. The first of these concerns the "magic" noise temperature ratio of 0.5 for high-quality p-n junctions and Schottky-barrier.diodes.[1] There is no question but that under DC drive a high-quality diode will have a noise temperture ratio approaching 0.5; however, the DC figure cannot be used in an RF-drive situation, in which the shot noise will be correlated with the local-oscillator waveform. In fact, high-quality diodes can give mixer conversion losses very close to unity and, since noise figure is defined as the product of available conversion loss and noise temperature ratio, a noise temperature ratio of 0.5 thus implies a noise figure of nearly 0.5. A noise figure below unity for a linear amplifier or frequency converter is patently impossible, so it follows that the effective noise temperature ratio for a diode properly operated in a mixer cannot be significantly less than unity.

The second item concerns the local oscillator waveform and the consequent diode conductance [or resistance, if you belong to that school] waveform. It has been known for years that the "best" conductance waveform for a mixer diode is an impulse train.[2,3,4,5] The reason for this is clear; the impulse train gives best ratio of conversion conductance to average conductance for a single-ended mixer. The resistance-mode mixer is not the same as the reactance-mode mixer; one cannot tune-out an average resistance in the same manner as an average reactance. A balanced mixer is different, however.

In fact, the balanced mixer is fundamentally different from the single diode mixer. It is not a matter of local-oscillator noise cancellation; in fact, the local oscillator's viewpoint gives the first clue. The local oscillator sees two parallel, back-to-back diodes constrained to have zero average voltage upon them. This constitutes a limiter and it is

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hard to come up with any other diode waveform than a square wave from such a device. A square wave would be disastrous in a single-diode mixer but in a balanced mixer it turns out to be ideal. In fact, any deviation from a square wave will degrade the mixer performance.

Under normal drive conditions, a diode pair with no charge storage will have each member "on" for alternate halves of the local-oscillator period and, of course, "off" for the other alternate halves. Each diode, therefore, ideally performs as a square-wave switch. A four-diode balanced mixer, illustrated in Fig. 1, can be represented by an ideal reversing switch, provided we neglect the series loss resistance and the depletion-layer capacitance. Call it the "low-frequency" model if you wish.

The reversing switch carries out the operation $\epsilon(t)$ on the voltages and currents such that

$$V_2(t) = \epsilon(t)\, V_1(t) \tag{1}$$

$$I_2(t) = \epsilon(t)\, I_1(t) \tag{2}$$

where $\epsilon(t)$ is illustrated in Fig. 2. Since

$$\epsilon(t) \cdot \epsilon(t) = 1, \tag{3}$$

we also have

$$V_1(t) = \epsilon(t)\, V_2(t) \tag{4}$$

$$I_1(t) = \epsilon(t)\, I_2(t). \tag{5}$$

Since we have a true reversing switch, a set of open-circuit constraints placed on port 1 will necessitate short-circuit constraints on port 2 and vice versa. That is, if we constrain port 1 so that only a current at the signal frequency ω_s can flow, the out-of-band (other than $\omega_s - \omega_o$) frequencies at port 2 must be short-circuited.

If ω_o is the local oscillator frequency and ω_s the signal frequency, chosen for convenience so that $\omega_s - \omega_o$ (the intermediate frequency) is positive, then the frequencies of interest at $k\omega_o \pm \omega_s$. Those with k even appear at port 1 (since k = 0 is already there); those with k odd appear at port 2.

Now, if we put a load, R_L, at port 2 at $\omega_s - \omega_o$, make $V_2 = 0$ for all other k odd, and let a current at ω_s alone flow into port 1, a simple analysis shows that the input impedance at port 1 and ω_s is

$$Z_{in} = \frac{4}{\pi^2} R_L. \tag{6}$$

If we reverse the constraints

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$$Z_{in} = \frac{\pi^2}{4} R_L. \tag{7}$$

In either case the conversion loss is unity.

The results are interesting because Eq. (6) represents the IF output impedance of a mixer with a short-circuit image constraint and Eq. (7) is the IF output impedance of a mixer with an open-circuit image constraint. A 3-GHz balanced mixer operated in our laboratory with a 100-ohm source impedance gave 40-ohm and 250-ohm IF output impedances under short- and open-circuit image constraints, remarkably close to the theoretical values.[7] The same mixer, using good GaAs Schottky-barrier diodes, gave a conversion loss under open-circuit image conditions of 1.2 db.

Generally, the minimum conversion loss of a diode mixer is greater than unity. My associate, D. H. Steinbrecher, has demonstrated, however, that a low-frequency mixer can be constructed, according to the theory represented here, with a conversion loss close enough to unity so that the extra loss is unmeasurable. Nevertheless, at high frequencies the diode junction parasitics must limit the performance. That is, we have a bulk series loss resistance, R_s, and a minimum reverse-bias capacitance, C_{min}. The capacitance, shown in Fig. 3, is not the minimum attainable at the breakdown voltage, but is the value attained during the off cycle. The value is $\sim C(0)/2$.

Note that R_s can be removed as a series element at each port and C_{min} can be removed as a shunt element at each port. The equivalence is exact and yields the final model with the $\epsilon(t)$ reversing switch imbedded as shown in Fig. 3. It is now obvious how R_s and C_{min} limit the performance of a diode mixer. In fact, just as the cutoff frequency of a varactor limits the performance of varactor circuits so a similar cutoff frequency is the major limit on performance of p-n junction mixers.

The only other quantity of immediate importance in good mixer diodes is carrier lifetime. Any charge storage will force nonuniform half-periods in the switching waveform and consequent degradation in performance. Some nonzero voltage biasing of the diode can serve to rebalance the switching waveform, and this accounts for the improvement in balanced-mixer performance with bias for some diodes. Schottky-barrier diodes, with essentially zero minority-carrier storage, show little or no improvement with bias (except, of course, for a reduction in required local-oscillator power).

A preliminary and rather algebraically involved analysis of the mixer represented in Fig. 3 yields the following results, based on the circuit of Fig. 4.

The normalized IF output impedance is

$$\mathrm{Re}\,\{z_{out}\} = \frac{20}{9} + \frac{\pi^2}{4} \cdot \frac{1 + r_g}{\left|1 + j(1+z_g)\,\dfrac{\omega_s}{\omega_c}\right|^2} \tag{8}$$

with an available conversion loss of

$$L = 1 + \frac{160}{9\pi^2}\left(\frac{\omega_s}{\omega_c}\right)^2 + \frac{80}{9\pi^2}\left(\frac{\omega_s}{\omega_c}\right)^2 \cdot r_g + \frac{1 + \frac{80}{9\pi^2}\left[\left(\frac{\omega_s}{\omega_c}\right)^2 + \left(1 - x_g\frac{\omega_s}{\omega_c}\right)^2\right]}{r_g} \qquad (9)$$

where

$$\omega_c = [R_s C_{min}]^{-1}$$

$$z_{out} = Z_{out}/R_s$$

$$r_g = R_g/R_s$$

$$x_g = X_g/R_s$$

and it assumed that ω_s, $\omega_s - \omega_o$, $\omega_s + \omega_o$ and $3\,\omega_o \pm \omega_s$ are all less than 0.1 ω_c, and the image is effectively open-circuited.

The optimum conversion loss is

$$L_{opt} = 1 + \frac{160}{9\pi^2}\left(\frac{\omega_s}{\omega_c}\right)^2\left[1 + \sqrt{1 + \frac{9\pi^2}{80}\left(\frac{\omega_c}{\omega_s}\right)^2}\right] \qquad (10)$$

for

$$r_{g,opt} = \sqrt{1 + \frac{9\pi^2}{80}\left(\frac{\omega_c}{\omega_s}\right)^2} \qquad (11)$$

$$x_{g,opt} = \frac{\omega_c}{\omega_s}$$

which also gives

$$\mathrm{Re}\,\{z_{out}\}_{opt} = \frac{20}{9} + \frac{\pi^2}{4}\left(\frac{\omega_c}{\omega_s}\right)^2 \cdot \frac{1}{1 + \sqrt{1 + \frac{9\pi^2}{80}\left(\frac{\omega_c}{\omega_s}\right)^2}} \qquad (12)$$

For $\omega_s/\omega_c = 10^{-2}$ we have $L_{opt} = 1.02$, $r_{g,opt} = 105$, $x_{g,opt} = 100$, and $\mathrm{Re}\,\{z_{out}\}_{opt} = 234$. Note that for a nominal R_s of 10 ohms the values of $R_g = 1050$ ohms and $\mathrm{Re}\,\{Z_{out}\} = 2340$ ohms are far from those normally designed for. Now it is clear why no one observes such low conversion losses. It is also clear why smaller junction areas are better at higher frequencies, even if the diode has the same cutoff frequency as a larger-area unit; Most mixer circuits are usually designed around RF impedance levels of 50-200 ohms, regardless of frequency.

The shot-noise contribution in a good quality, square-wave-driven mixer diode is minimal. We can invoke correlation as an argument or realize that the shot current only flows when the diode junction is essentially a short circuit; consequently, no shot noise. The internal noise sources are primarily thermal, uncorrelated, and due to R_s. An analysis of the low-frequency noise figure gives

$$(F-1)_{opt} = \frac{88}{9\pi^2}\left(\frac{\omega_s}{\omega_c}\right)^2\left[1 + \sqrt{1 + \frac{9\pi^2}{44}\left(\frac{\omega_c}{\omega_s}\right)^2}\right] \tag{13}$$

for

$$r_{g,opt} = \sqrt{1 + \frac{9\pi^2}{44}\left(\frac{\omega_c}{\omega_s}\right)^2} \tag{14}$$

and

$$x_{g,opt} = \frac{\omega_c}{\omega_s}. \tag{15}$$

It is no surprise that the noise figure and conversion loss do not optimize for the same source impedance. In fact, for $\omega_s/\omega_c = 10^{-2}$ we have $(F-1)_{opt} = 0.014$ and $r_{g,opt} = 142$, with $x_{g,opt} = 100$. Comparing these figures with those for the conversion loss, we see that the noise temperature ratio is slightly less than unity. Only for very large r_g, and consequently very poor noise figures, can the noise temperature ratio become as low as 11/20. The number, however, has as much physical significance as the noise temperature ratio of a varactor upper-sideband upconverter.

The large dynamic range of such balanced mixers is obvious. The local-oscillator current can be very large and little or no perturbation of the switching waveform can occur for moderate level signals.[8] The larger the junction area is, the larger the dynamic range. In fact, large-area, GaAs Schottky-barrier diodes should make excellent low-impedance, high dynamic range mixers.

I should like to credit the reversing-switch model for the mixer to Donald H. Steinbrecher and thank him for the many stimulating discussions we have had over the past few years.

REFERENCES

1. Uhlir, Arthur, Jr. "p-n junction frequency converters: considerations of gain and noise," BTL Monograph 3113, 1958; and Bell System Tech. J., Vol. 37, Part I, pp. 951-1008, July 1958.
2. Torrey, H. C. and C. A. Whitmer, "Crystal Rectifiers," Radiation Laboratory Series, Vol. 15, McGraw-Hill, New York, 1948.
3. Uhlir, A. A., Jr., "Frequency Conversion in PN Junction Devices," BTL Second Interim Report on Crystal Rectifiers, 1954.
4. Bakanowski, A. E., "Frequency Conversion by a Nonlinear Resistor," BTL Second Interim Report on Crystal Rectifiers, 1954.
5. Messenger, G. C. and C. T. McCoy, "Theory and Operation of Crystal Diodes as Mixers," Proc. IRE, Vol. 45, No. 9, September 1957.
6. O'Neill, H. J., "Image Frequency Effects in a Microwave Crystal Mixer," Proc. IEEE, Vol. 112, No. 11, November 1965.
7. Vugrinec, J., "The Effect of Image-Frequency Termination in Microwave Mixers Using Schottky-Barrier Diodes," S. M. Thesis, M. I. T., August, 1966.
8. Thompson, K. D., "The Use of Schottky-Barrier Diodes in VHF/UHF Mixer Applications," S. M. Thesis, M. I. T., May, 1966.

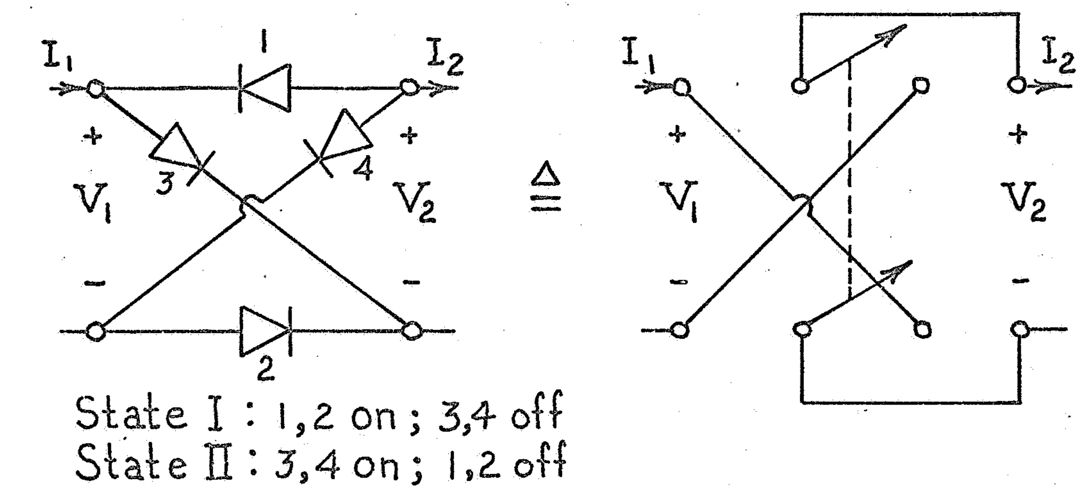

Figure 1: Four-diode balanced mixer and its reversing-switch model.

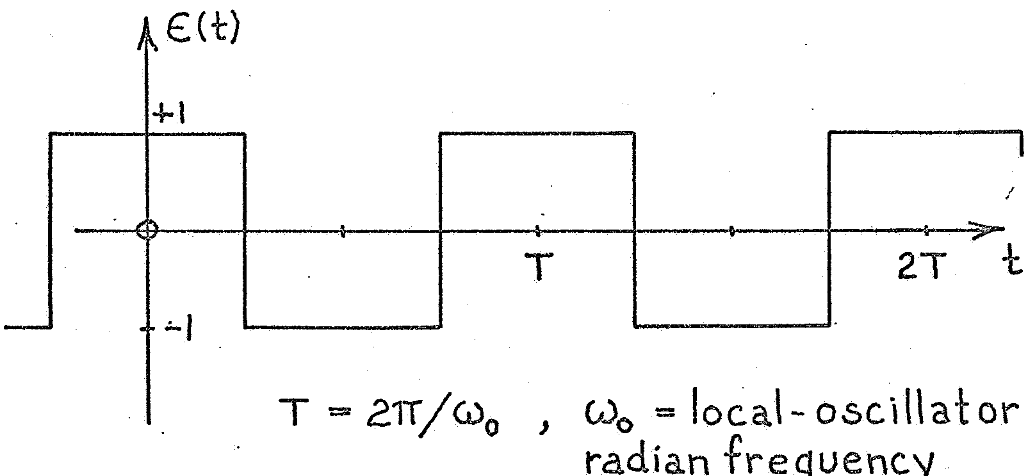

Figure 2: Square-wave switching waveform produced by the local oscillator.

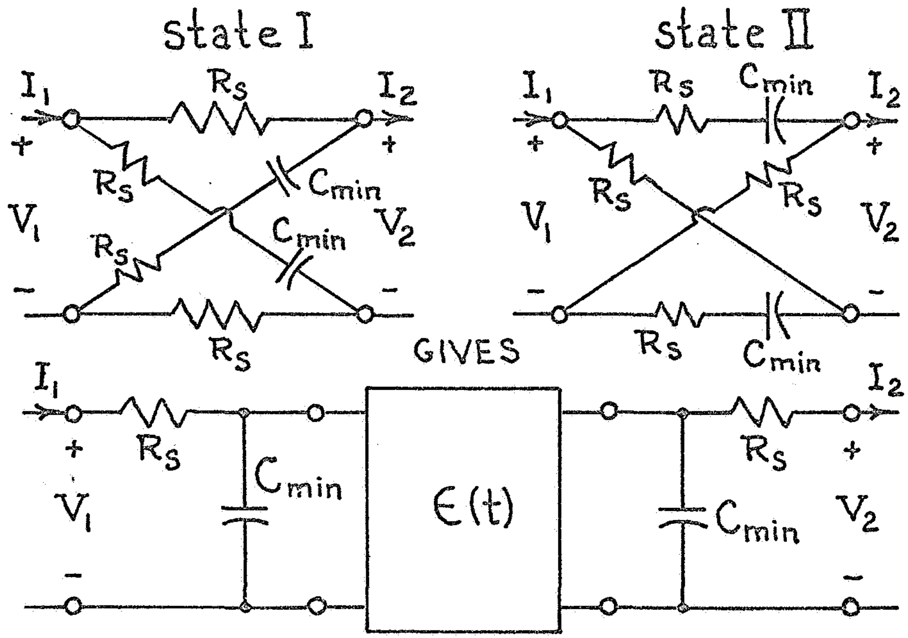

Figure 3: Development of a model for a four-diode balanced mixer in terms of the diode series loss resistance and the minimum, as pumped, depletion-layer capacitance.

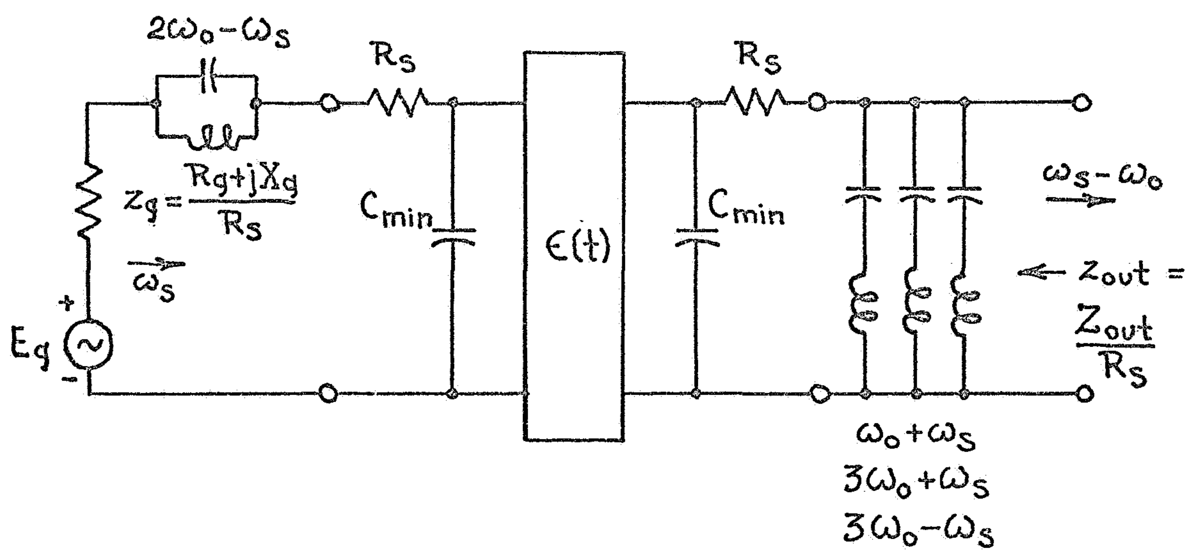

Figure 4: Operational mixer with open-circuited image.