Faraday Rotation Dispersion Loss at Jicamarca

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In late 1966, a loss mechanism unique to Thompson scatter measurements of Faraday rotation became quite a serious problem at the Jicamarca Radar Observatory (JRO). Dispersion in the total Faraday angle to different parts of the scattering volume causes a loss in S/N if Faraday rotation information is preserved in the experiment. Although Faraday experiments are not done at most other Thompson scatter observatories, they are at the JRO. Faraday rotation provides a very convenient measure of the electron density profile which is not affected by thermal nonequilibrium, as are power profiles. From a simple extension of the Faraday measurement we obtain autocorrelation measurements simultaneously. However, the Faraday dispersion loss increases rapidly with total Faraday angle, or total electron content. At the present (early 1967) level of solar activity, Faraday dispersion sometimes prevents Faraday density measurements above about 600 km and autocorrelation measurements above about 800 km, whereas previously we reached altitudes of about 750 and 1200 km. This loss in range is serious but not fatal, as other techniques now in use at the JRO extend the autocorrelation measurement to beyond the previous maximum height. However, Faraday dispersion loss should be considered in the design of future Thompson scatter experiments making use of the obvious advantages of Faraday rotation.

Before calculating the Faraday dispersion it is desirable to discuss the Faraday experiment. At the JRO, independent but phase coherent halves of the radar operate on the independent magneto-ionic modes. The receiver outputs are complex voltages, say $\underline{A}$ and $\underline{B}$. The product $\underline{A}\underline{B}^* = \underline{C}$ is formed and averaged. $\underline{A}$ and $\underline{B}$ are made up of signal plus noise phasors. Both signal and noise have random phase and amplitude. However, the phase and amplitude of the A and B signal components are correlated, while the phase and amplitude of the A and B noise components are not. Therefore, it is easily shown that the noise components tend to cancel and the signal components tend to add in such a way that $|\underline{C}|$ is proportional to the correlated power and Arg $\underline{C}$ is the total Faraday angle Ω (plus an additive constant). When transmitters A and B operate simultaneously, correlated power is a maximum. When they are separated by a time lag $\Delta\tau$, correlated power is reduced. Thus we have a point on the autocorrelation

$$\rho(\Delta\tau)=\underline{C}(\Delta\tau)/\underline{C}(0). \quad (1)$$

These techniques are discussed in detail in Farley et al[1].

Unfortunately, just as the cosmic and receiver noise always cancel, and as the uncorrelated fraction of the signal cancels in forming $\underline{C}(\Delta\tau)$, so does the fraction of the signal having negatively correlated phase because of differential Faraday rotation in the scattering volume. For a constant electron density a change of Faraday angle of π radians in one radar pulse length in the direction of propagation $\underline{k}$ causes a 2 db loss of correlated signal power. A change of 2π radians in the $\underline{k}$ direction causes a loss of all the correlated power by destructive interference. This type of Faraday dispersion is easy to visualize and easy to eliminate. As it becomes important, the pulse may be shortened. No S/N is lost, because both the dispersion and the signal depend linearly on electron density. In the discussion below, we will assume this kind of dispersion does not exist. Flood et al[2] discuss the problem further.

The really serious type of Faraday dispersion at the JRO is the component from north to south across the scattering volume, perpendicular to $\underline{k}$. The amount of dispersion at any point in the scattering volume depends only on a geometrical constant times Ω. The dispersion loss is a monotonicly increasing function of Ω. Faraday dispersion can be calculated by taking $\underline{C}$ as the vector sum of $\underline{C}(\varphi)$ integrated through the scattering volume. φ is the north-south angle measured from the center of the beam. $\underline{C}(\varphi)$ has the amplitude and phase of the power scattered at a given angle to the magnetic field. The JRO antenna power pattern for Thompson scatter is given by

$$G(\varphi) = \sin^4 x/x^4 , \qquad (2)$$

where

$$x = 44\pi\varphi \qquad (3)$$

approximately[3]. The phase of the scattered power is given by

$$\arg \underline{C}\ (\varphi) = \Omega\,(1 + .8\varphi/\theta_0\ ) \qquad (4)$$

where Ω is the Faraday angle at the center of the beam and θ_0 is $2.9°$, the departure from perpendicularity with the magnetic field at the center of the beam. The factor 0.8 is the result of the curvature of the magnetic field. A one degree shift of the beam changes the magnetic angle by only $0.8°$. Thus we have

$$\underline{C} = \int_{-\pi}^{\pi} \frac{\mathrm{Sin}^4 x}{x^4}\ e^{\,i\Omega(1+x/6.65)}\ dx \qquad (5)$$

By symmetry we see that arg $\underline{C}$ equals Ω , the Faraday angle at the center of the beam, as tacitly assumed above. Tracing the integrand, we see that it forms a straight line for $\Omega = 0$ and an ever tightening spiral for $\Omega > 0$ The Faraday dispersion loss as a function of Ω is shown in Fig.1.

The loss reaches 6 db an Ω of 6 π radians, which corresponds to a total electron content of 4×10^{17} electron /m^2. Recently the loss has reached 10 db or more. This can be tolerated in F-region measurements at the JRO, where the normal S/N ratio is 10 or more (with daytime electron densities and 23-35 km pulses). However, a 10 db loss is quite a serious obstacle to autocorrelation measurements above 1000 km, and for this reason we have recently started to use the "double pulse" technique as used at the Arecibo Ionosphere Observatory. This technique is not sensitive to Faraday rotation, of course.

There are two ways of reducing the effect of Faraday dispersion loss. One is to increase the size of the antenna, at least in the north-south direction. The other is to transmit and/or receive the magneto-ionic modes on antennas whose phase centers are separated in the N-S direction by a distance sufficient to just cancel the $\Omega\ (1 + .8\ \varphi/\theta_o)$ Faraday angle dependence at some (large) Ω. Due to the nonlinearity of the Faraday dispersion loss, substantial increases in S/N can be obtained by the first method. Doubling the antenna size in the N-S direction halves the N-S beamwidth and halves the scale of the Ω axis of Figure 1. Thus the S/N ratio at $\Omega = 8\pi$ is increased by 15, a factor of 2 for the increased antenna area and 7.5 for the reduced dispersion.

As an illustration of the second method of reducing Faraday dispersion, we shall consider the effect of transmitting on the full JRO antenna in the normal way, but receiving the "O" and "X" modes on the N-W and S-E halves of the antenna, respectively. The phase centers of the antenna halves are separated by $24\lambda/\sqrt{2}$ in the N-S direction. Thus the Faraday dispersion is zero when

$$48\pi\ \varphi/\sqrt{2} = 0.8\ \varphi\ \Omega/\theta_o \qquad (6)$$
$$\Omega = 2.15\pi$$

The Faraday dispersion relation for this connection is shown as a dashed curve in Figure 1. (The dashed curve is the solid curve moved to the right by 2.15π and multiplied by 0.5 to account for the reduced antenna area.) Note that the dashed curve is above the solid curve for Ω greater than about 5π. The amount of improvement, however, is insignificant. Unfortunately, there is no other convenient connection of the JRO antenna that offers better performance. Therefore, this method has been discarded with hopes that some day the antenna can be enlarged.

Scaling these calculations to apply at other observatories is straightforward. Components of Faraday dispersion both parallel and perpendicular to $\underline{k}$ will be considered. At the JRO, the frequency is 50 MHz and θ_o is 2.9^o. The phase shift between the "O" and

"X" modes is 0.468 radians /10 km (20 km round trip distance) for 10^6 electrons cm^3. In temperate latitudes the magnetic field strength is about 5/3 the equatorial value. Thus the Faraday rotation constant will be about 5/(3 sin 2.9°) or 33 times higher at a given frequency for propagation nearly along the magnetic field. Since the Faraday rotation rate varies as f^{-2}, frequencies in the 200-300 MHz yield rates from nearly equal to about double the JRO value. This is an ideal range, as much higher rates would lead to serious Faraday dispersion in the direction of k in the daytime, when the electron density in high, and much lower rates would yield poor resolution at night.

The component of Faraday dispersion perpendicular to k is a serious problem only when the differential Faraday rotation across the antenna beam reaches several radians. This can happen only when the antenna is directed nearly perpendicular to the magnetic field and when, simultaneously, the total Faraday rotation is fairly large. In other words, it can only happen when the radar parameters are similar to those of the JRO. If a 200 MHz radar, for example, were directed nearly perpendicular to the magnetic field, no Faraday dispersion loss would result, because the total Faraday rotation would be so low (a few radians) that the differential rotation across the beam would be negligible.

References

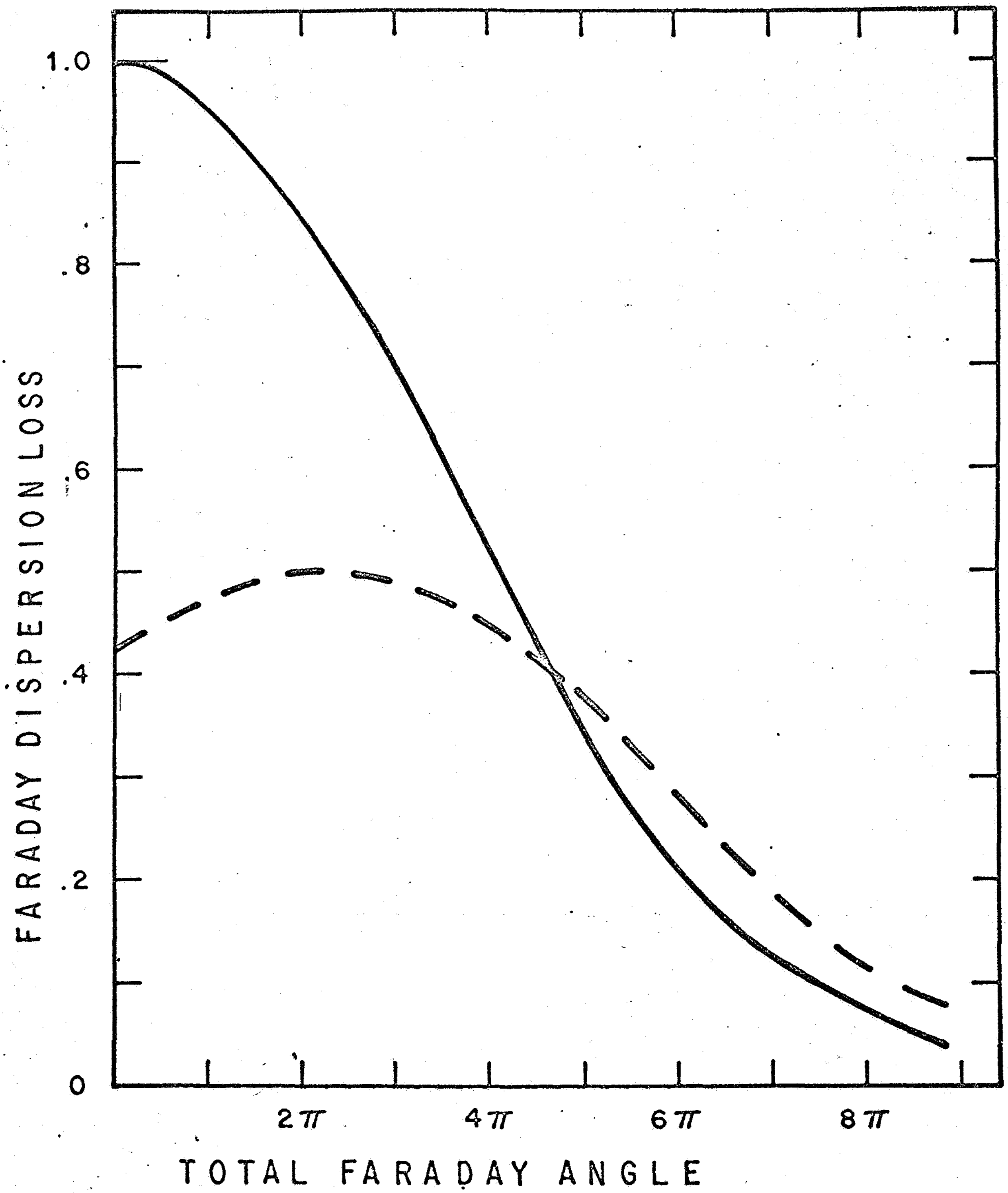
(1) Farley, D. T., J. P. McClure, D. L. Sterling and J. L. Green, in press, 1967.

(2) Flood, W. A. and H. G. Camnitz, J. Geophys. Res., 70, 4391, 1965.

(3) Bowles, K. L., G. R. Ochs and J. L. Green, J. Res. Natl. Bur. Std. D66, 395, 1965.

Figure Captions

Fig.1. Faraday Dispersion Loss. The solid curve shows the result of destructive interference due to differential Faraday rotation in the scattering volume at the Jicamarca Radar Observatory. Only Faraday dispersion in the direction perpendicular to k is considered. Dispersion in the direction parallel to k is not a serious problem at Jicamarca (though it may be at other locations). The dashed curve applies to a special case discussed in the text. Note that the above loss applies only to Thompson scatter Faraday rotation experiments.



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