University of Arkansas
Grant NGR-04-001-015 by M. K. Testerman

MEASUREMENT OF THE INTENSITY OF TURBULENCE

1. Summary:

An experimental study was made to observe the effect of finite sampling volume on the apparent spectral broadening of the scattered light as observed by the Doppler heterodyne technique. This method, with several necessary modifications, may be suitable for measuring the scattering volume.

An experiment was carried out to plot the velocity profile of laminar flow in a duct of square cross-section. A Doppler shifted back-scattered beam was used in this study. Satisfactory results were obtained which indicated the suitablity of the present setup for initial experimental studies on a turbulent free jet.

2. Work Performed:

A. Measurement of Sampling Volume:

Theoretical consideration: The Doppler shifted signal has frequency spread associated with it and this spread is primarily caused by instrumental broadening.

It has been shown (1) that frequency spread due to finite apertures in the receiving section is eliminated in the symmetrical system. However, there will be broadening due to finite convergency angle of the incident beam.

An approximate calculation of the focal volume (this is the scattering or sampling volume in the present case) can be made from theoretical considerations (2, 3). For simplicity, let us consider that the sampling volume is cylindrical (length $2\Delta z$, diameter D_A), and the focal tolerance Δz, in the direction of the optical axis is

$$\Delta z \approx \pm \tfrac{1}{2}\left(\frac{f}{a}\right)^2 \lambda \qquad (1)$$

$$D_A \approx 2.44 \frac{f}{a} \lambda \qquad (2)$$

where f = focal length of the lens, a = lens diameter and λ = wavelength of incident light. In the present case, f = 40 cm and a = 10 cm. Therefore,

$$f/a = 4,$$

$$\lambda = 6.328 \times 10^{-5} \text{ cm}$$

$$2\Delta z \approx 0.1 \text{ mm} \qquad (3)$$

$$D_A \approx 6.2\mu \qquad (4)$$

These dimensions are comparable to those of the hot wire anemometer (wire diameter $\approx 5\mu$ and length ≈ 0.5 mm) and are frequently used for turbulent intensity measurement (4). Depending on the turbulence scale and particular engineering application, one may find that the use of a larger sampling volume may be advantageous in turbulence studies.

The above consideration of sampling volume is based on a perfect lens. In an actual case the volume may be somewhat larger and the shape may be irregular. One direct method of measuring this sampling volume is by incorporating a velocity gradient in the focal volume and the observation of frequency spread (Δf) around the Doppler shifted signal (f_D) on the spectrum analyzer.

3. Experimental Setup:

Figures 1 and 2 show the schematic of the experimental arrangement. Figure

1 shows that there is a velocity gradient (perpendicular to velocity vectors) along the radial direction, and if a small section of radius D_A is focused, the velocity spread in this section is

$$\Delta v = \frac{dv}{dr} D_A \qquad (5)$$

where v is the velocity component perpendicular to the plane of the paper. Here v depends on r and the speed of the motor. In this geometrical configuration, the Doppler shift (f_D) is given by

$$f_D = \frac{2v}{\lambda} \sin \frac{\theta}{2} \qquad (6)$$

Because of the velocity spread Δv (Equation 5), there will be a frequency spread $df_D^{D_A}$ associated with f_D. By running the motor at high speed, this spread can be observed using the spectrum analyzer.

In the experimental arrangement shown in Figure 2, the velocity gradient is along the optical axis (perpendicular to the focal plane).

The disc was made of clear transparent silicon rubber in which pigment particles were uniformly distributed to form scattering centers within the volume scatterer. The beam entered through and perpendicular to the cylindrical edge of the disc.

The velocity spread (section A, Figure 2) in the direction of the incident beam will be

$$\Delta v = \frac{dv}{dr} 2\Delta z \qquad (7)$$

and the frequency spread $\Delta f_D^{\Delta z}$ will depend on Δv.

It has been stated earlier that primary components of the total spectral broadening (Δf) around f_D are, in the case of the symmetrical system: (1) finite convergence angle and (2) finite scattering volume. The spread due to convergence angle α can be written as

$$\Delta f_D^{\alpha} \simeq \frac{2v}{\lambda} \sin \frac{\alpha}{2} \cos \frac{\theta}{2} \tag{8}$$

If this is large compared to $\Delta f_D^{D_A}$ and $\Delta f_D^{\Delta z}$, the contribution from finite sampling volume will be difficult to measure. In other words, spectral broadening due to finite sampling volume can be considered significant if the total spread (Δf) is experimentally found greater than Δf_D^{α}. It may be pointed out that the spectral broadening Δf will not be the direct algebraic sum of the factors,

$$\Delta f_D^{\alpha},\ \Delta f_D^{D_A},\ \Delta f_D^{\Delta z}$$

and in practice, Δf will be limited by the one which has the largest magnitude.

4. Experimental Results: In the experiments, the motor speed was adjusted to have $v \simeq 1920$ cm/sec. In a typical experiment, the Doppler shift was $f_D = 4.55$ MHz, $\Delta f = 270$ KHz, motor speed = 4350 rpm and r = 4.45 cm. From Equation 8,

$$\Delta f_D^{\alpha} = 280 \text{ KHz}$$

which shows that total spread was caused by the finite convergence angle alone. The maximum velocity gradient used in these experiments was limited by the maximum allowable speed of the motor, which was severely limited while using a silicon rubber disc. Thus, in these experiments with the velocity gradients used, the contribution from the finite scattering volume was found insignificant in the spectral broadening.

Figure 3 shows the frequency versus velocity and intensity versus distance plots as the rotating silicon rubber disc was moved along the optical axis (Figure 2).

Intensity is a maximum when the beam is focused at the edge of the disc (air-silicon rubber interface). This is shown as $\Delta x = 0$ point. When the disc is moved away from this point the intensity falls off sharply, as shown on the right side of the $\Delta x = 0$ line, i.e. when Δx is positive. The curve is less steep than expected because the outer surface of the disc is not perfectly spherical and concentric. When the disc is moved toward the lens (Δx is negative), i.e. with the scattering volume inside the silicon rubber disc, the intensity falls off again because of the absorption of the light in the medium. In this case, however, frequency f_D decreased linearly with Δx which is expected because, as r decreased, v also decreased linearly.

5. Measurement of Laminar Gas Flow in a Duct:

A duct of square cross-section (2 cm x 2 cm) was used in which dry air was allowed to flow at a known rate. Talc particules (2.5μMMD) were used as aerosol. Experimental data obtained show good agreement with the expected values of velocity. Back-scattered beams were used in the symmetrical system.

It is planned to measure turbulent intensity of a free air jet. The above experiment shows that the system parameters are suitable for initial studies on a known turbulent system.

REFERENCES

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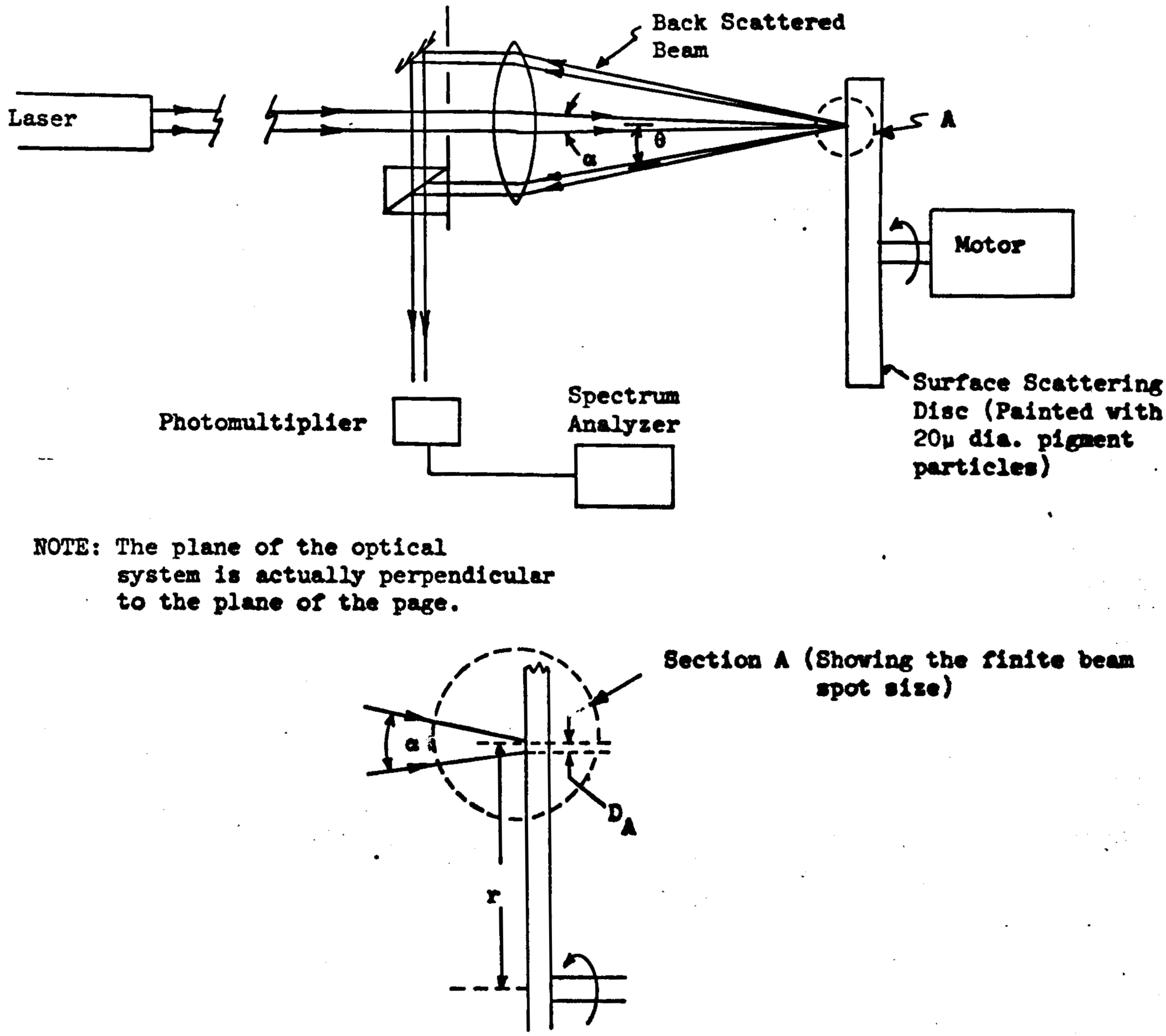

Figure 1

EXPERIMENTAL ARRANGEMENT WITH THE VELOCITY GRADIENT PERPENDICULAR TO THE VELOCITY VECTORS

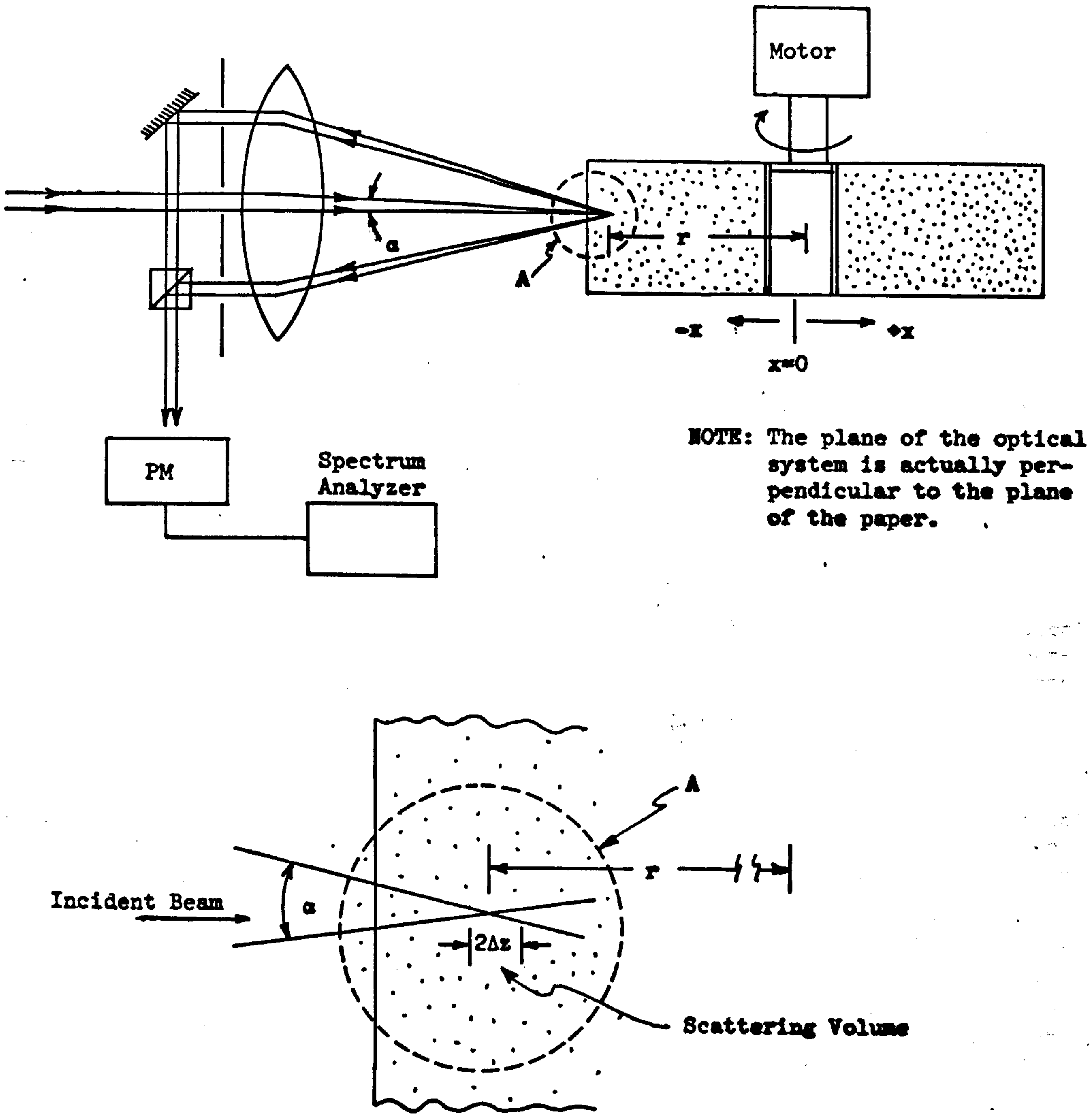

Figure 2

EXPERIMENTAL ARRANGEMENT WITH THE VELOCITY GRADIENT PERPENDICULAR TO THE FOCAL PLANE

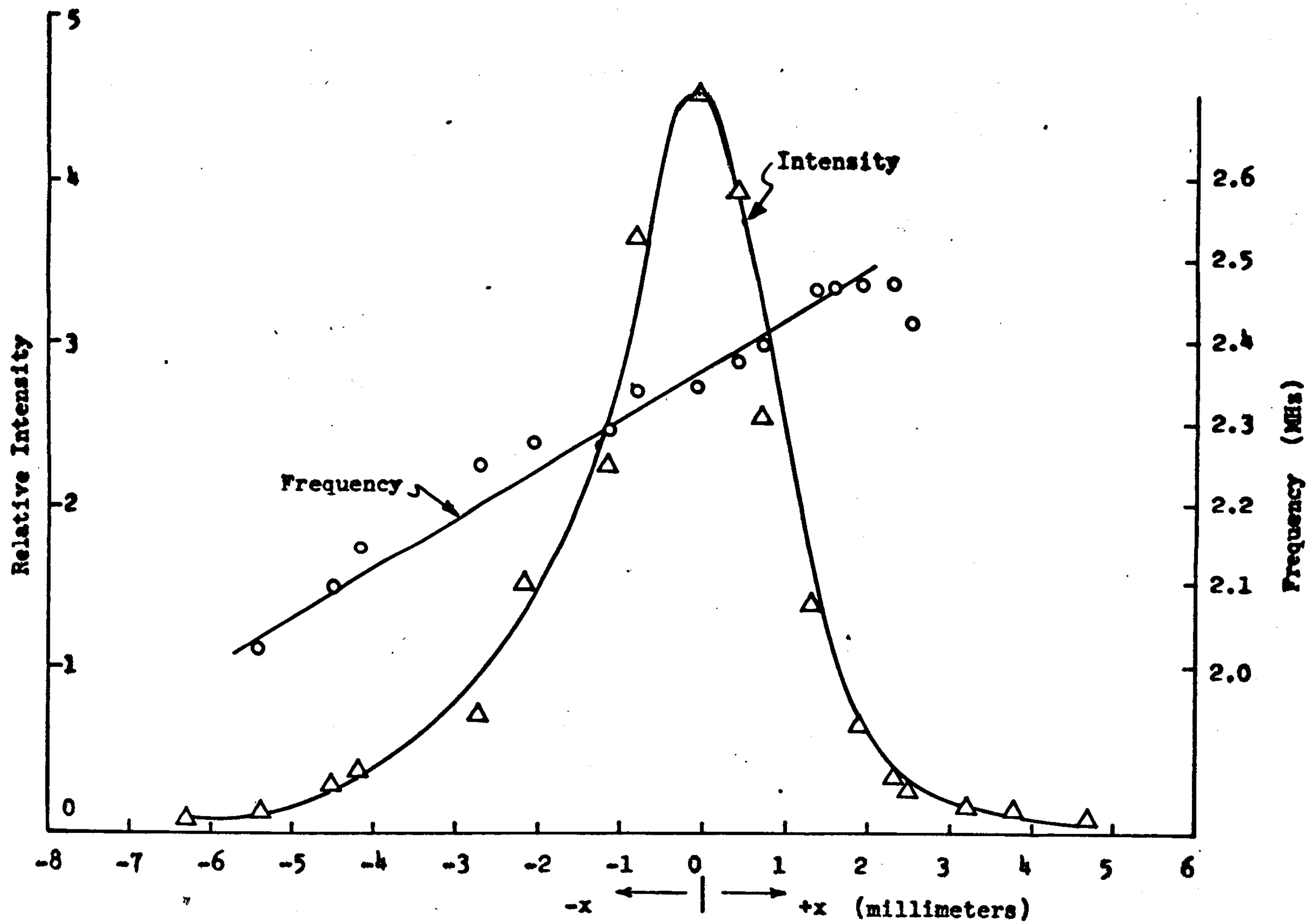

Figure 3

FREQUENCY VERSUS VELOCITY AND INTENSITY VERSUS DISTANCE PLOTS