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Study of Limit Cycles and Stability of a Space Vehicle Attitude Control System

by

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I. INTRODUCTION

This memorandum extends the work reported in the Fifteenth Technical Report (NAS8-20104), Determination of the Limit Cycles for a Dual-Channel Missile Yaw-Axis Control System [1], to a three-axis attitude control system for a space vehicle. The orientation of the control thrusters about the center-of-gravity and about the three-axes is shown in Figure 1.

The objective of the study was to determine the limit cycles, the regions of stability, the effects of variations in parameters on the stability, and methods to make the system more stable. Analytical and analog computer simulations were employed.

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[1] D. W. Russell, et al., "Determination of Limit Cycles for a Dual-Channel Missile Yaw-Axis Control System," Auburn University, Fifteenth Technical Report, NAS8-20104, March 31, 1967.

II. DESCRIPTION OF THE THREE-AXIS ATTITUDE CONTROL SYSTEM

In this system, attitude control is achieved by modulating the force (thrust) produced by the four engines or thrusters; these forces are modulated about a value set by the throttle command signal, T_r. The direction of the forces is fixed.

A second-order actuator positions a valve which controls the propellent flow and therefore the force produced by an engine. The actuator is velocity limited, which limits the rate of change of force. The parameters of the system are such that the second-order, velocity-limited actuator can be represented adequately as an ideal relay; this greatly simplifies the study and is used in all the analyses.

Angular position, ϕ, and angular velocity, $\dot{\phi}$, about each axis are fed back to form a three-axis closed-loop control system. The block diagram is given in Figure 2.

A. Torque Equations

The torques (T) about the three axes, in terms of the engine thrusts, F_1, F_2, F_3, F_4, the direction cosines, α, β, γ, and the moment arms, a, b, c, are:

$$\begin{aligned} Tx = {} & c \cos \beta \, (F_1 - F_2 - F_3 + F_4) + \\ & + b \cos \gamma \, (-F_1 + F_2 + F_3 - F_4) \end{aligned} \tag{II-1}$$

$$Ty = c \cos \alpha \, (-F_1 + F_2 - F_3 + F_4) + a \cos \gamma \, (-F_1 + F_2 - F_3 + F_4) \tag{II-2}$$

$$Tz = b \cos \alpha \, (F_1 + F_2 - F_3 - F_4) + a \cos \beta \, (F_1 + F_2 - F_3 - F_4) \quad , \tag{II-3}$$

which simplify to :

$$Tx = (F_1 - F_2 - F_3 + F_4) \, (c \cos \beta - b \cos \gamma) \tag{II-4}$$

$$Ty = (-F_1 + F_2 - F_3 + F_4) \, (c \cos \alpha + a \cos \gamma) \tag{II-5}$$

$$Tz = (F_1 + F_2 - F_3 - F_4) \, (b \cos \alpha + a \cos \beta) \; . \tag{II-6}$$

Thus the angular accelerations about each axis, in terms of the torques and moments of inertia, I, are:

$$\begin{aligned} \ddot{\phi}_x &= \frac{Tx}{Ix} = C_x \, (F_1 - F_2 - F_3 + F_4) \\ \ddot{\phi}_y &= \frac{Ty}{Iy} = Cy \, (-F_1 + F_2 - F_3 + F_4) \quad , \\ \ddot{\phi}_z &= \frac{Tz}{Iz} = C_z \, (F_1 + F_2 - F_3 - F_4) \end{aligned} \tag{II-7}$$

where

$$\begin{aligned} Cx &= \frac{1}{Ix} (c \cos \beta - b \cos \gamma) \\ C_y &= \frac{1}{Iy} (c \cos \alpha + \alpha \cos \gamma) \quad . \\ C_z &= \frac{1}{Iz} (b \cos \alpha + a \cos \beta) \end{aligned} \tag{II-8}$$

B. The State Equations

The state variables are designated as follows:

$$
\begin{aligned}
x_1 &= F_1 \\
x_2 &= F_2 \\
x_3 &= F_3 \\
x_4 &= F_4 \\
x_5 &= \phi_x \\
x_6 &= \dot{\phi}_x \\
x_7 &= \ddot{\phi}_x \\
x_8 &= \phi_y \\
x_9 &= \dot{\phi}_y \\
x_{10} &= \ddot{\phi}_y \\
x_{11} &= \phi_z \\
x_{12} &= \dot{\phi}_z \\
x_{13} &= \ddot{\phi}_z
\end{aligned}
\qquad . \text{(II-9)}
$$

The state equations are:

$$
\begin{aligned}
\dot{x}_1 &= \dot{F}_L \operatorname{sgn} E_1 \\
\dot{x}_2 &= \dot{F}_L \operatorname{sgn} E_2 \\
\dot{x}_3 &= \dot{F}_L \operatorname{sgn} E_3 \\
\dot{x}_4 &= \dot{F}_L \operatorname{sgn} E_4
\end{aligned}
\qquad . \text{(II-12)}
$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = x_7$$

$$\dot{x}_7 = C_x \dot{F}_L \quad (\text{sgn } E_1 - \text{sgn } E_2 - \text{sgn } E_3 + \text{sgn } E_4) \qquad \text{(II-13)}$$

$$\dot{x}_8 = x_9$$

$$\dot{x}_9 = x_{10}$$

$$\dot{x}_{10} = Cy\dot{F}_L \quad (-\text{sgn } E_1 + \text{sgn } E_2 - \text{sgn } E_3 + \text{sgn } E_4) \qquad \text{(II-14)}$$

$$\dot{x}_{11} = x_{12}$$

$$\dot{x}_{12} = x_{13}$$

$$\dot{x}_{13} = Cz\dot{F}_L \quad (\text{sgn } E_1 + \text{sgn } E_2 - \text{sgn } E_3 - \text{sgn } E_4) \quad , \qquad \text{(II-15)}$$

where

$$E_1 = T_R - X_1 - a_{ox} X_5 - a_{1x} X_6 + a_{oy} X_8 + a_{1y} X_9 - a_{oz} X_{11} - a_{1z} X_{12}$$

$$E_2 = T_R - X_2 + a_{ox} X_5 + a_{1x} X_6 - a_{oy} X_8 - a_{1y} X_9 - a_{oz} X_{11} - a_{1z} X_{12}$$

$$E_3 = T_R - X_3 + a_{ox} X_5 + a_{1x} X_6 + a_{oy} X_8 + a_{1y} X_9 + a_{oz} X_{11} + a_{1z} X_{12}$$

$$E_4 = T_R - X_4 - a_{ox} X_5 - a_{1x} X_6 - a_{oy} X_8 - a_{1y} X_9 + a_{oz} X_{11} + a_{1z} X_{12}$$

(II-16)

III. STUDY OF THE STABILITY BOUNDARY AND LIMIT CYCLES

The three-axis attitude control system is analyzed in a manner similar to that reported in the Fifteenth Technical Report.

A. Stability in the Vicinity of the Origin

For this analysis, the ideal relays are represented as gain terms which, in the limit, approach infinity. Representing the relays as gains, η, then the state equations become:

$$\begin{aligned} \dot{x}_1 &= \eta E_1 \\ \dot{x}_2 &= \eta E_2 \\ \dot{x}_3 &= \eta E_3 \\ \dot{x}_4 &= \eta E_4 \end{aligned} \qquad \text{(III-1)}$$

$$\begin{aligned} \dot{x}_5 &= x_6 \\ \dot{x}_6 &= x_7 \\ \dot{x}_7 &= C_x\eta \ [-x_1 + x_2 + x_3 - x_4 - 4a_{ox}X_5 - 4a_{1x}X_6] \end{aligned} \qquad \text{(III-2)}$$

$$\begin{aligned} \dot{x}_8 &= x_9 \\ \dot{x}_9 &= x_{10} \\ \dot{x}_{10} &= C_y \eta [x_1 - x_2 + x_3 - x_4 - 4a_{oy}X_8 - 4a_{1y}X_9] \end{aligned} \qquad \text{(III-3)}$$

$$\begin{aligned} \dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= x_{13} \\ \dot{x}_{13} &= C_z\eta \ [-x_1 - x_2 + x_3 + x_4 - 4a_{0z}X_{11} - 4a_{1z}X_{12}]. \end{aligned} \qquad \text{(III-4)}$$

However,

$$-x_1 + x_2 + x_3 - x_4 = -x_7/C_x$$
$$x_1 - x_2 + x_3 - x_4 = -x_{10}/C_y$$
$$-x_1 - x_2 + x_3 + x_4 = -x_{13}/C_z$$

Therefore, (III-2), (III-3), and (III-4) become

$$\begin{aligned} \dot{x}_5 &= x_6 \\ \dot{x}_6 &= x_7 \\ \dot{x}_7 &= -\eta x_7 - 4a_{ox}C_x\eta x_5 - 4a_{1x}C_x\eta x_6 \end{aligned} \tag{III-5}$$

$$\begin{aligned} \dot{x}_8 &= x_9 \\ \dot{x}_9 &= x_{10} \\ \dot{x}_{10} &= -\eta x_{10} - 4a_{oy}C_y\eta x_8 - 4a_{1y}C_y\eta x_9 \end{aligned} \tag{III-6}$$

$$\begin{aligned} \dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= x_{13} \\ \dot{x}_{13} &= -\eta x_{13} - 4a_{oz}C_z x_{11} - 4a_{1z}C_z\eta x_{12} \end{aligned} \tag{III-7}$$

Since the equations for the three axes are not coupled, the conditions for stability in the vicinity of the origin can be determined, for each axis, in the same manner as was done in the Fifteenth Technical Report. Thus, the system is stable in the vicinity of the origin if all the parameters, C_x, a_{ox}, a_{1x}, C_y, a_{oy}, a_{1y}, C_z, a_{oz}, a_{1z} are greater than zero.

B. Limit Cycles

Three limit cycles can be calculated by the periodicity of the phase space trajectory for one axis, with the other two axes in a quiescent state. Therefore, the results of the Fifteenth Technical Report can be applied to calculate a limit cycle for each axis. Since the system is stable at the origin, these are unstable limit cycles.

The possibility of other limit cycles (such as two axes or all three axes being in a limit cycle at one time) was studied. To illustrate, suppose all three axes are in the same limit cycle. The relay switching sequence is shown in Figure 3. At any instant of switching, $E_1 = E_2 = E_3 = E_4 = 0$. Therefore, at that instant, with the δ's as defined in Figure 2,

$$
\begin{aligned}
-\delta_x + \delta_y - \delta_z &= x_1 \\
\delta_x - \delta_y - \delta_z &= x_2 \\
\delta_x + \delta_y + \delta_z &= x_3 \\
-\delta_x - \delta_y + \delta_z &= x_4
\end{aligned}
$$

Adding the above equations,

$$x_1 + x_2 + x_3 + x_4 = 0, \tag{III-8}$$

which must hold for all switching instants. However, reference to Figure 3 reveals that, for any arbitrary set of d-c levels for the variables x_1, x_2, x_3, and x_4, (III-8) does not hold at all switching instants. Therefore, the assumed limit cycles cannot exist in all three channels simultaneously. A similar line of reasoning can be applied to show that a limit cycle with two axes in phase and one axis 180° out-of-phase cannot occur.

The general case of two or three limit cycles of arbitrary frequency occurring simultaneously has not been thoroughly studied. Further work needs to be done in this area.

C. Stability Boundary

The stability boundary was obtained by trial-and-error using the analog computer simulation shown in Figure 4. The stability boundary for initial conditions of the three velocity terms (x_6, x_9, x_{12}), the other state variables having a zero initial value, is shown in Figure 5. It can be visualized as being similar to two four-sided pyramids whose bases are joined. The vertices are points on the limit cycle trajectories.

This work can be extended to establish the boundary for initial conditions on the position and acceleration terms.

IV. CONCLUSIONS AND RECOMMENDATIONS

It was determined that the three-axis control system can possess at least three unstable limit cycles: a limit cycle about each axis, conditions about the other two axis being quiescent. Points on these limit cycle trajectories can be calculated using the results of the Fifteenth Technical Report. Additional analytical and analog computer studies are needed to determine if other limit cycles are possible.

The conditions for stability about the origin were found to be the same as those for a single-axis system: namely, all the system gains must be positive. The stability boundary for initial conditions in velocity only (see Figure 5) was determined by trial-and-error using an analog computer simulation. Additional analog computer studies would yield information about the stability boundary for other initial conditions and the effects on the stability boundary of variations in system parameters.

It would be useful to extend the conclusions reached in the Fifteenth Technical Report for a single-axis system to this three axis system. Here again, additional analog computer simulations are suggested.

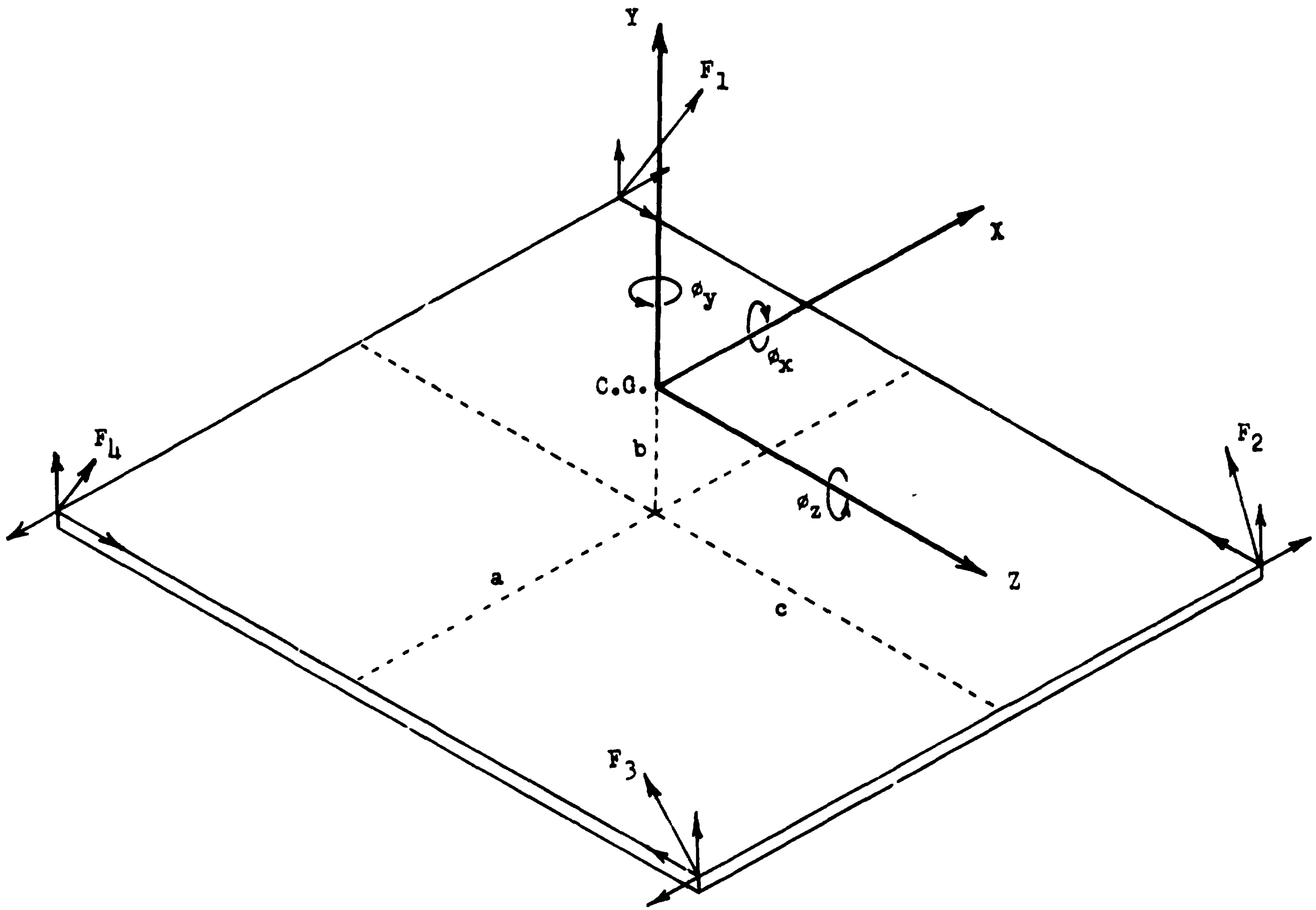

Figure 1. Orientation of Control Engines.

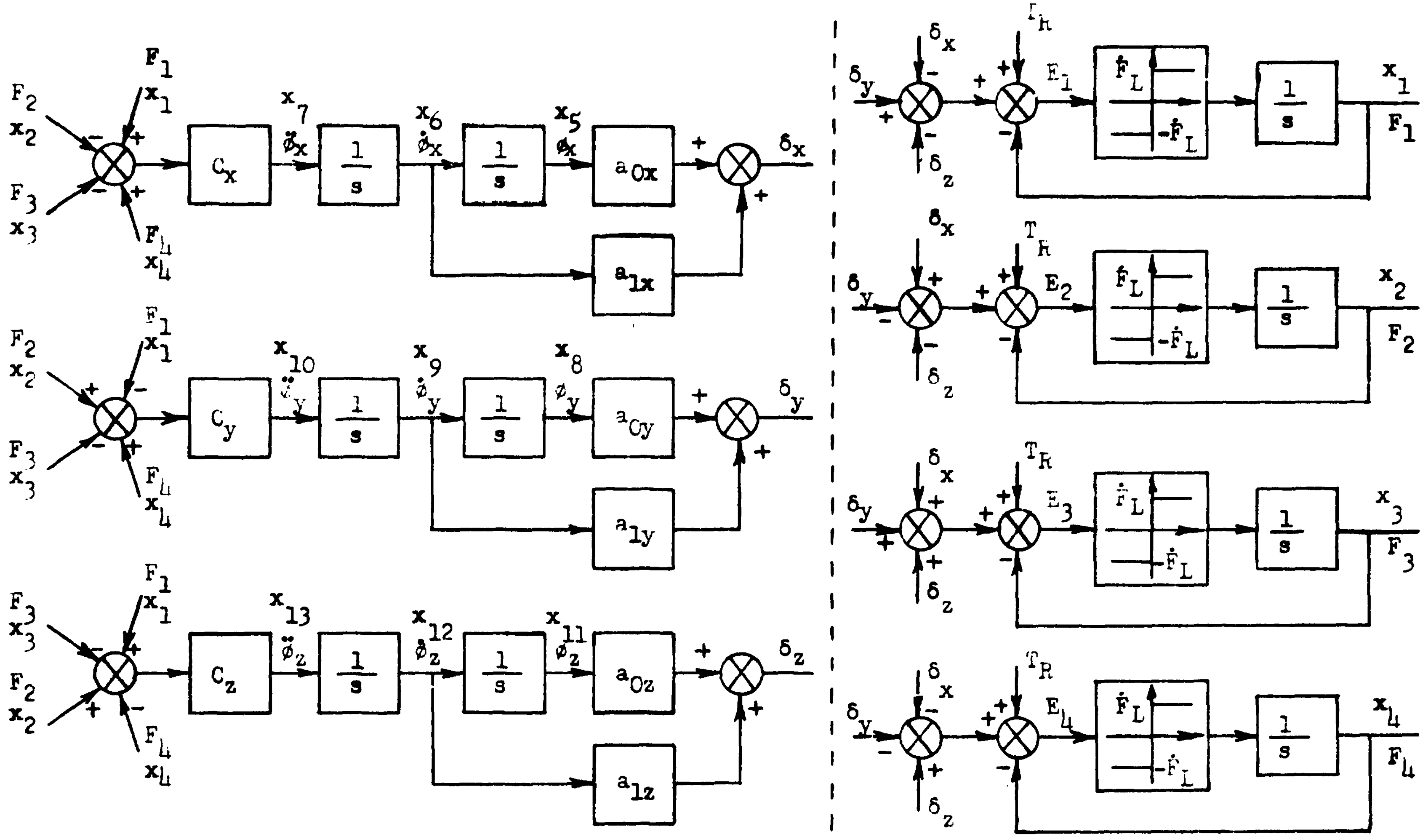

Figure 2. Simplified Block Diagram Of The Attitude Control System.

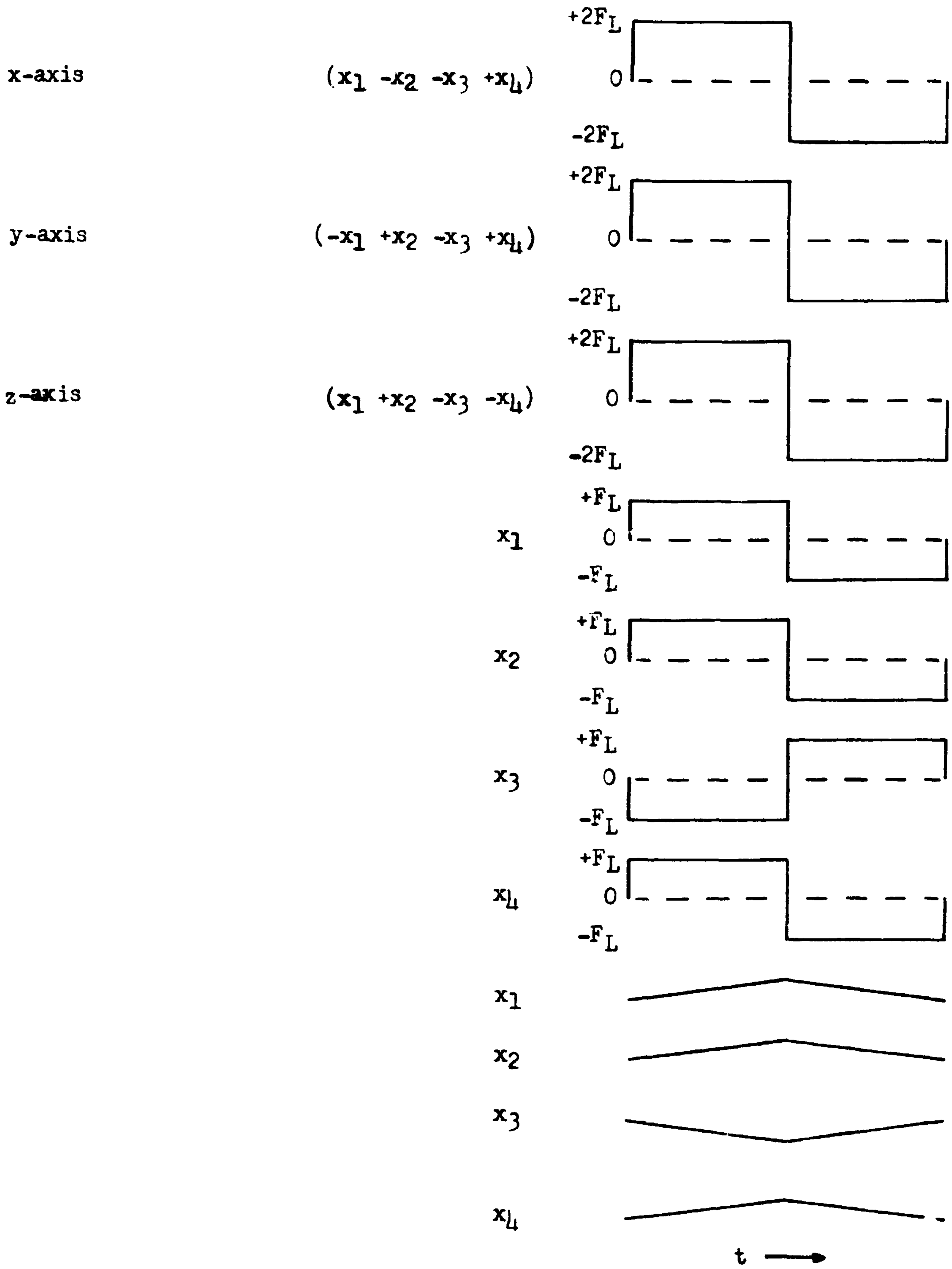

Figure 3. Switching Sequences for Possible Concurrent Limit Cycles.

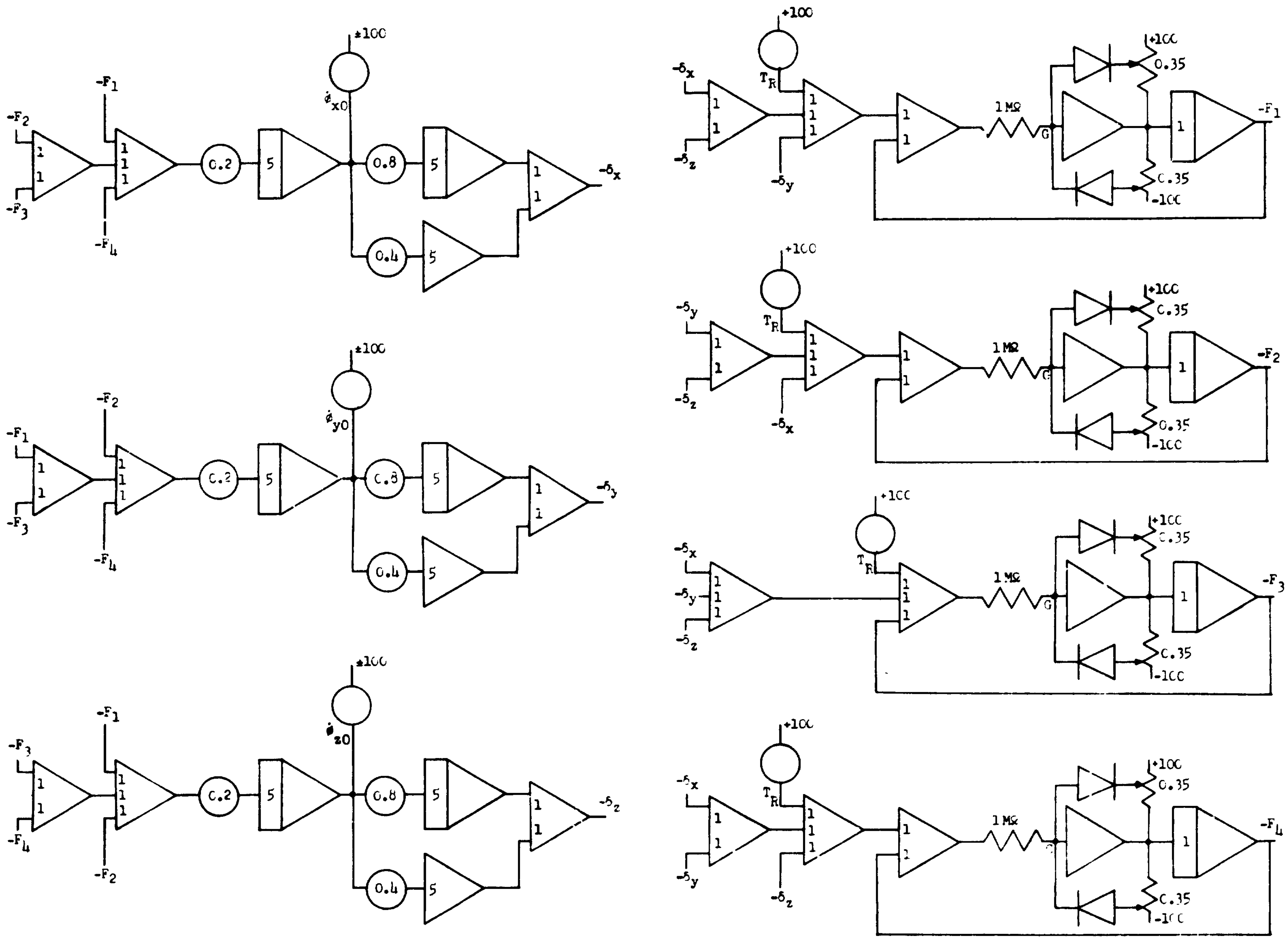

Figure 4. Analog Computer Diagram for Determining the Stability Boundary.

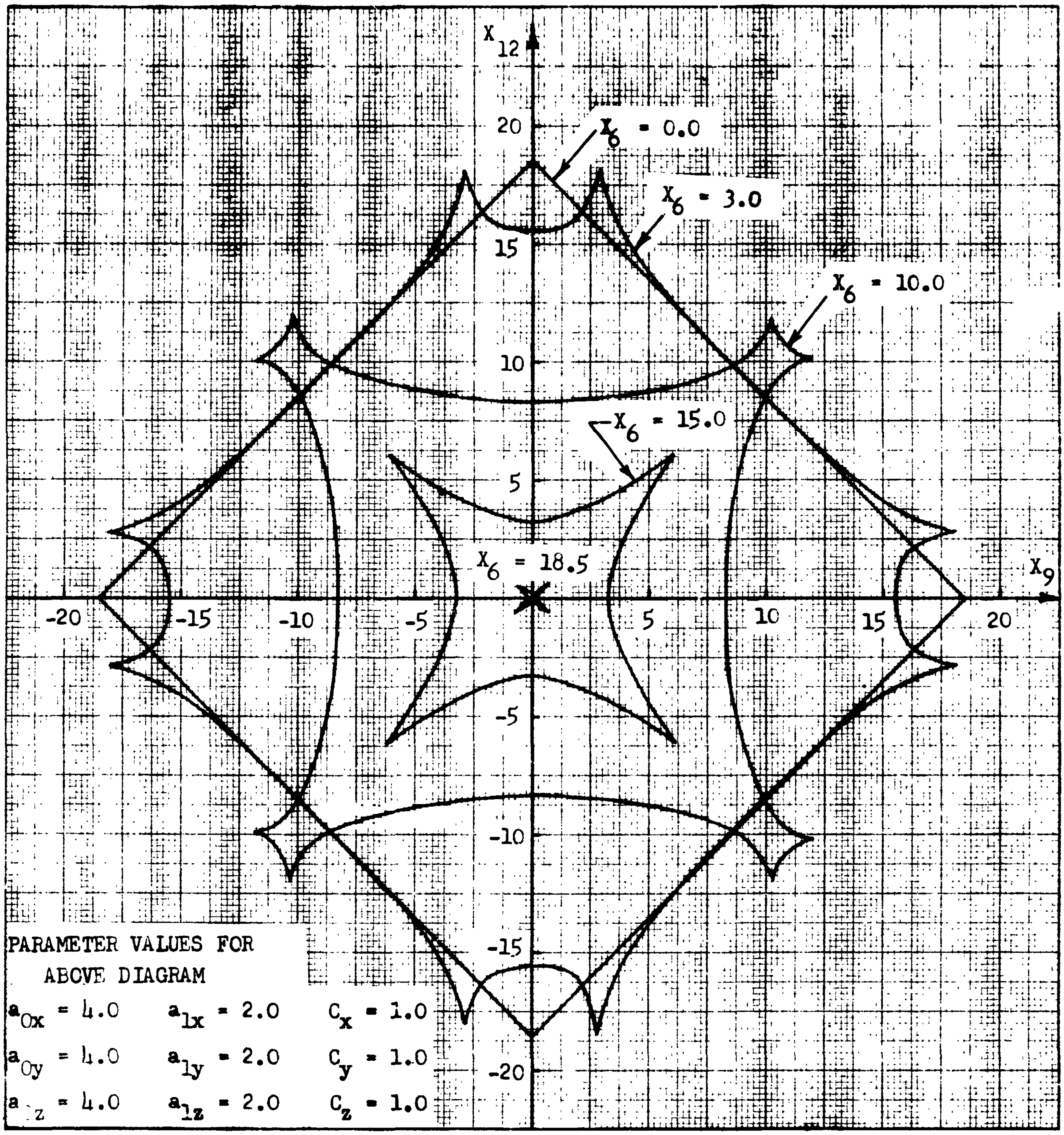

Figure 5. Stability Boundary Projected onto the X_9-X_{12} Plane for Various Values of X_6. All Other Variables Are Assumed To Be Initially Zero.