# COMPUTER EVALUATION OF TOPOLOGICAL FORMULAS FOR NETWORK ANALYSIS*

France Rode
Electrical Engineer
Hewlett-Packard Company
Palo Alto, California

and

S. Park Chan
Associate Professor of Electrical Engineering
University of Santa Clara
Santa Clara, California




ABSTRACT

Most of the existing topological formulas developed for network analysis are not well suited for digital computation. Recently Chan and Chan modified Mayeda's formulas

$$Z_{OC} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{V} \begin{bmatrix} W_{1,1'} & W_{12,1'2'} - W_{12',1'2} \\ W_{12,1'2'} - W_{12',1'2} & W_{2,2'} \end{bmatrix} \quad (1)$$

and

$$Y_{SC} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{1}{\Sigma U} \begin{bmatrix} W_{2,2'} & W_{12,1'2'} - W_{12',1'2} \\ W_{12,1'2'} - W_{12',1'2} & W_{1,1'} \end{bmatrix} \quad (2)$$

so that the topological terms can be computed by means of a single tree-finding program.

In this work it is shown that the computational process for evaluating (1) and (2) can be shortened considerably by the use of a new tree-finding program. Specifically it is shown that $\Sigma U$ is equal to the trees of a subnetwork $N_{11',22'}$ of the given network $N$, and that $W_{12,1'2'} - W_{12',1'2}$ can be automatically computed from the set of 2-trees required for the evaluation of $W_{1,1'}$. This represents a significant improvement of the work of Chan and Chan since the tree finding program is applied only four times, instead of nine.

A program for evaluating the various topological terms in (1) and (2) has been written based on these principles using the new tree finding subprogram developed by the authors.


*This work was supported in part by the National Aeronautics and Space Administration, Grant No. NGR-05-017-012.

COMPUTER EVALUATION OF TOPOLOGICAL FORMULAS FOR NETWORK ANALYSIS*

France Rode
Electrical Engineer
Hewlett-Packard Company
Palo Alto, California

and

S. Park Chan
Associate Professor of Electrical Engineering
University of Santa Clara
Santa Clara, California



SUMMARY

## 1. INTRODUCTION

Topological formulas for network analysis have been developed by numerous authors. [1]-[10] Recently Chan and Chan modified the formulas of Mayeda and Seshu [3] for digital computation using nine sets of trees for obtaining all the necessary factors in the expressions. In this study, these topological formulas are further developed so that only four sets of trees are required, using only one tree finding program. In addition, a tree finding algorithm is developed for computing all trees of a network without duplication.

## 2. OPEN-CIRCUIT IMPEDANCE MATRIX $Z_{OC}$

The topological expression for the open-circuit impedance matrix $Z_{OC}$ of a passive two-port N without mutual inductances (Fig. 1), as given by Mayeda and Seshu [1], is:

$$Z_{OC} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{V} \begin{bmatrix} W_{1,1'} & W_{12,1'2'} - W_{12',1'2} \\ W_{12,1'2'} - W_{12',1'2} & W_{1,1'} \end{bmatrix}$$

where V is a sum of all tree-admittance products of the network N, and W indicates the summation of 2-tree admittance products of the same network. The types of 2-trees are defined by the subscripts of each W, namely, the vertices on one side of the comma are in one connected part, and the vertices on the other side in the other connected part of the 2-tree. V can be generated from the graph of N by applying a tree finding program. As known, $W_{i,j}$ of N can be obtained by coalescing vertices i and j of N to form sub-network $N_{ij}$ [11].

*This work was supported in part by the National Acronautics and Space Administration, Grant No. NGR-05-017-012.

$W_{1,1'}$ is therefore the sum of all tree admittance products of $N_{11'}$, and $W_{2,2'}$ is the sum of all tree admittance products of $N_{22'}$.

Here we shall show that $W_{12,1'2'}$ and $W_{12',1'2}$ can be found by selecting the trees of $N_{11'}$. Observe that

$$W_{1,1'} = W_{12,1'} + W_{1,1'2} = W_{122',1'} + W_{12',1'2'} + W_{12',1'2} + W_{1,1'22'}$$

Thus, when a tree of $N_{11'}$ is found, it is also placed in sum $W_{12,1'2'}$ if it has vertices 1 and 2 in one part and vertices 1' and 2' in the other part of the 2-tree of $N$. $W_{12',1'2}$ is generated similarly. Such sorting of trees is much simpler and faster than establishing $W_{12,1'2'}-W_{12',1'2}$ by using any tree finding process four times as done by Chan and Chan [10].

## 3. SHORT-CIRCUIT ADMITTANCE MATRIX $Y_{SC}$

Assuming that $Z_{OC}$ is already known, the only term to be determined in the formula for $Y_{SC}$ [11]

$$Y_{SC} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{1}{\Sigma U} \begin{bmatrix} W_{2,2'} & W_{12,1'2'}-W_{12',1'2} \\ W_{12,1'2'}-W_{12',1'2} & W_{1,1'} \end{bmatrix}.$$

is $\Sigma U$, where

$$\Sigma U = U_{12',2,1'} + U_{1,2,1'2'} + U_{12,2',1'} + U_{1,2',1'2'} \quad . \tag{3}$$

The symbol $U$ indicates the summation of 3-tree admittance products where groups of its subscripts indicate the relationship of four vertices (1,1',2,2').

Here we shall state and prove the following theorem.

<u>Theorem 1</u>. The expression, $\Sigma U$, is equal to the sum of all tree-admittance products of the subnetwork $N_{11',22'}$ (Fig. 2).

<u>Proof</u>. By combining vertices w and 2' into a single vertex 2" a subnetwork $N_{22'}$ results (Fig. 3). Similarly, as shown earlier for $W_{i,j}$, the sum of 2-tree admittance products $K_{1,1'}$ of the network $N_{22'}$ is equal to the sum of all tree admittance products of $N_{11',22'}$. (Here K, instead of W, is used to emphasize that a different subnetwork is used.)

We shall show that $K_{1,1'}$ is equal to $\Sigma U$ as follows. Observe that

$$K_{1,1'} = K_{12'',1'} + K_{1,1'2''} , \tag{4}$$

and that vertex 2" represents the combination of vertices 2 and 2' into one vertex in forming $N_{22'}$. Thus, (4) can be rewritten as

$$K_{1,1'} = K_{1(2,2'),1'} + K_{1,1'(2,2')}$$
$$= K_{12,2',1'} + K_{2,12',1'} + K_{1,1'2,2'} + K_{1,2,1'2'}, \quad (5)$$

K being merely a symbol of summation of the k-tree admittance products. Consequently, if K is replaced by U, with the terms rearranged (5) becomes

$$K_{1,1'} = U_{12',2,1'} + U_{1,2,1'2'} + U_{12,2',1'} + U_{1,2',1'2'} \quad (6)$$

Comparing (3) and (6), we conclude that

$$\Sigma U = K_{1,1'}$$

and the proof is complete.

From the above theorem, it is evident that only one set of trees (of $N_{11',22'}$) is required in obtaining $\Sigma U$.

## 4. EXAMPLE

The following example will illustrate that the set of all trees of $N_{11',22'}$ corresponds to the set of all terms in $\Sigma U$. Also, it will show that the terms involved in $W_{12,1'2'}$ can be extracted from the expression for $W_{1,1'}$.

Example 1. Find all the terms for $Z_{OC}$ and $Y_{SC}$ for two-port N of Fig. 4. (This example is from Chan and Chan [2] in order to make a comparison). Establish also the sum of all admittance products; each admittance was given the value of unity. The result is printed as the output of the computer program.

## 5. TREE FINDING ALGORITHM

This algorithm is developed to compute the terms in the topological formulas presented in preceding sections. In this summary only a brief description of the basic principle involved and some of the advantages of the algorithm are included. The algorithm is based on the following definition of a tree:

> A tree of a connected graph G with v vertices and e edges is a connected subgraph $G_i$ of G with v vertices and v-1 edges [11].

Basically, this algorithm consists of two steps:

Step 1. Form all subgraphs of $G_1$ of G with v-1 edges and v vertices (isolated vertices are not permitted).

Step 2. Select the connected subgraphs.

Some of the advantages of this algorithm are:

a) The determination of each tree is done independently so that the trees need not be stored but can be printed immediately upon being found--this being an advantage over the algorithm developed by Mayeda and Seshu [12].

b) No extra storage is needed for subgraphs as required in Minty's algorithm [13].

c) The selection of subgraphs is not based on circuit comparison, and hence it is not necessary to generate (and hence to store) any circuits in the entire process.

## 6. CONCLUSIONS

The example illustrates that $Z_{OC}$ and $Y_{SC}$ can be evaluated by generating only four sets of trees. The same tree finding algorithm is used with the following networks:

1. Network N for computing V ,

2. Network $N_{11'}$ (obtained from N with vertices 1 and 1' short circuited) for computing $W_{1,1'}$, $W_{12,1'2'}$, and $W_{12',1'2}$ ,

3. Network $N_{22'}$ (obtained from N with vertices 2 and 2' short circuited) for computing $W_{2,2'}$ ,

4. Network $N_{11',22'}$ (obtained from N with vertices 1 and 1' short circuited together) for computing $\Sigma U$ .

Although the example is simple, this program can be used for a two-port network with up to 20 elements and fifteen vertices. It can be easily expanded if the speed of the computer permits so.

It should also be pointed out that the same principles can be extended to the analysis of active networks with mutual inductances.

# REFERENCES


[1] W.S. Percival, "The Graph of Active Networks," Proc. IEEE, vol. 102, pt C, pp 270-278; Sept. 1955.

[2] S.J. Mason, "Topological Analysis of Linear Nonreciprocal Networks," Proc. IRE, vol 45, pp 829-838; June 1957.

[3] W. Mayeda and S. Seshu, "Topological Formulas for Network Functions," Univ. of Ill. Bul., vol 55, No. 23, Urbana, Ill., Nov. 1957.

[4] W. Mayeda, "Topological Formulas for Active Networks," Int. Tech Rpt., No. 8, U.S. Army Contract No. DA-11-022-ORD-1983, Univ. of Ill., Urbana, Ill., Jan. 1958.

[5] C.L. Coates, "General Topological Formulas for Linear Network Functions," IRE Trans. on Circuit Theory, CT-5, pp 30-42; March 1958.

[6] W.K. Chen, "Topological Analysis for Active Networks," IEEE Trans. on Circuit Theory, CT-12, pp 85-91; March 1965.

[7] A. Talbot, "Topological Analysis of General Linear Networks," IEEE Trans. On Circuit Theory, CT-12, pp 170-180; June 1965.

[8] D.A. Calahan, "Linear Network Analysis and Realization Digital Computer Programs: An Instruction Manual," Univ. of Ill. Bul., Vol. 62, No. 58, Urbana, Ill., Feb. 1965.

[9] F.F. Kuo, "Network Analysis by Digital Computer," Proc. IEEE, vol. 54, No. 6, pp 820-829; June 1966.

[10] S.P. Chan and S.G. Chan, "Topological Formulas for Digital Computation," Proc. 1st Annual Princeton Conf. on Information Sciences and Systems, pp 319-323, March 1967.

[11] S.P. Chan, Introductory Topological Analysis of Electrical Networks, to be published by Holt, Rinehart and Winston, in 1968.

[12] W. Mayeda and S. Seshu, "Generation of Trees without Duplication," IEEE Transactions on Circuit Theory, vol CT-12, No. 2, June 1965.

[13] G.J. Minty, "A Simple Algorithm for Listing All the Trees of a Graph," IEEE Transactions on Circuit Theory, March 1965, p. 120.

09/26/67

FIND ZOC AND YSC OF NETWORK N WITH 6 VERT.,6 EDG.,AND

INCIDENCE METRIX A

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 |
| 3 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 1 | 1 |
| 2' | 0 | 0 | 0 | 1 | 1 | 0 |
| 1' | 0 | 1 | 0 | 1 | 0 | 0 |

F O U R   S E T S   O F   T R E E S

| TREES OF N | | TREES OF $N_{11'}$ | | TREES OF $N_{22'}$ | | TREES OF $N_{11',22'}$ | |
|---|---|---|---|---|---|---|---|
| 1 | 1 2 3 4 6 | 1 | 1 3 4 6 | 1 | 1 2 3 4 | 1 | 1 3 5 |
| 2 | 1 2 3 5 6 | 2 | 1 3 5 6 | 2 | 1 2 4 5 | 2 | 1 3 6 |
| 3 | 1 3 4 5 6 | 3 | 1 4 5 6 | 3 | 1 2 4 6 | 3 | 2 3 5 |
| 4 | 1 2 4 5 6 | 4 | 2 3 4 6 | 4 | 1 2 3 5 | 4 | 2 3 6 |
| | | 5 | 2 3 5 6 | 5 | 1 3 4 5 | 5 | 1 3 4 |
| | | 6 | 2 4 5 6 | 6 | 1 2 3 6 | 6 | 1 4 5 |
| | | 7 | 3 4 5 6 | 7 | 1 3 4 6 | 7 | 1 4 6 |
| | | | | | | 8 | 2 3 4 |
| | | | | | | 9 | 2 4 5 |
| | | | | | | 10 | 2 4 6 |
| | | | | | | 11 | 3 4 5 |
| | | | | | | 12 | 3 4 6 |

V = Y1Y2Y3Y4Y6+Y1Y2Y3Y5Y6+Y1Y3Y4Y5Y6+Y1Y2Y4Y5Y6

W(1,1') = Y1Y3Y4Y6+Y1Y3Y5Y6+Y1Y4Y5Y6+Y2Y3Y4Y6+Y2Y3Y5Y6+
+Y2Y4Y5Y6+Y3Y4Y5Y6

W(12,1'2')-W(12',1'2) = Y1Y3Y4Y6

W(2,2') = Y1Y2Y3Y4+Y1Y2Y4Y5+Y1Y2Y4Y6+Y1Y2Y3Y5+Y1Y3Y4Y5+
+Y1Y2Y3Y6+Y1Y3Y4Y6

SUM U = Y1Y3Y5+Y1Y3Y6+Y2Y3Y5+Y2Y3Y6+Y1Y3Y4+Y1Y4Y5+
+Y1Y4Y6+Y2Y3Y4+Y2Y4Y5+Y2Y4Y6+Y3Y4Y5+Y3Y4Y6

IF Y1=1OHM Y2=1F Y3=1OHM Y4=1OHM Y5=1F Y6=1OHM

$$Z11 = \frac{2.S**2 + 4.S + 1}{2.S**2 + 2.S} \qquad Y11 = \frac{2.S**2 + 4.S + 1}{2.S**2 + 6.S + 4}$$

$$Z12=Z21 = \frac{1}{2.S**2 + 2.S} \qquad Y12=Y21 = \frac{1}{2.S**2 + 6.S + 4}$$

$$Z22 = \frac{2.S**2 + 4.S + 1}{2.S**2 + 2.S} \qquad Y22 = \frac{2.S**2 + 4.S + 1}{2.S**2 + 6.S + 4}$$

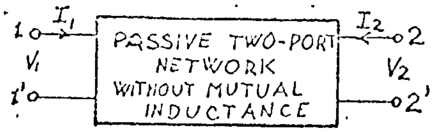


FIG. 1. PASSIVE NETWORK N

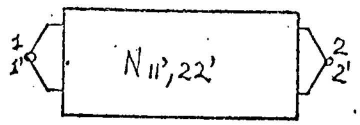


FIG. 2. MODIFIED NETWORK

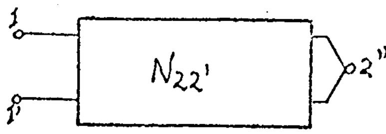


FIG. 3. MODIFIED NETWORK

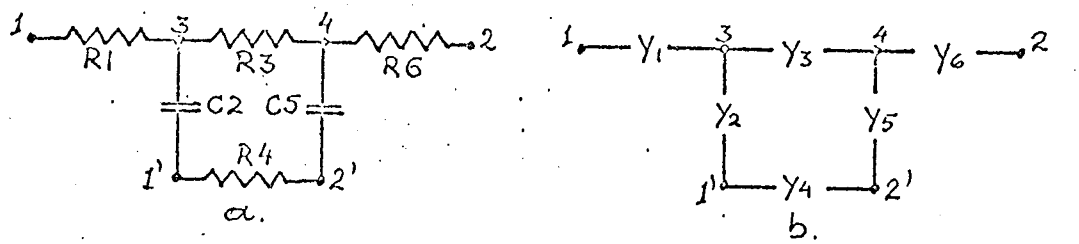


FIG. 4. NETWORK N OF THE EXAMPLE a.; AND IT'S GRAPH G b.

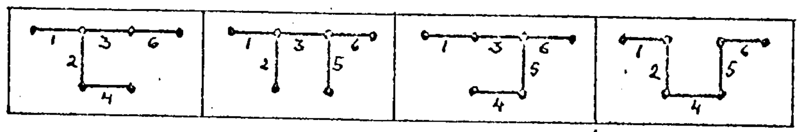

FIG. 5. THE TREES OF NETWORK N

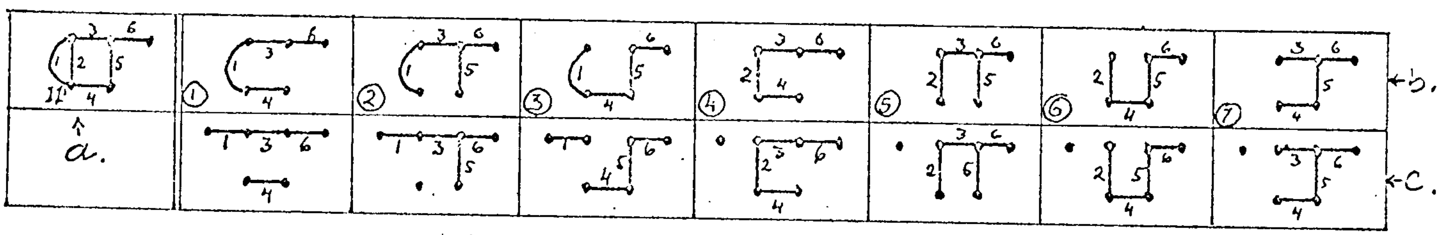


FIG. 6. a. NETWORK $N_{11'}$ b. TREES OF $N_{11'}$ c. 2-TREES

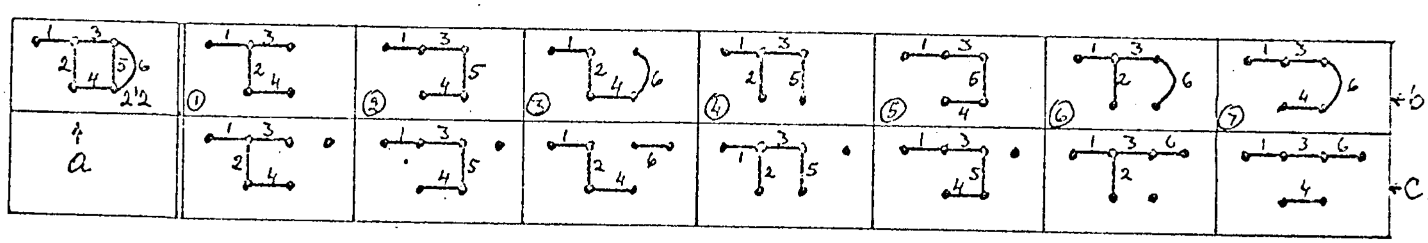


FIG. 7. a. NETWORK $N_{22'}$ b. TREES OF $N_{22'}$ c. 2-TREES

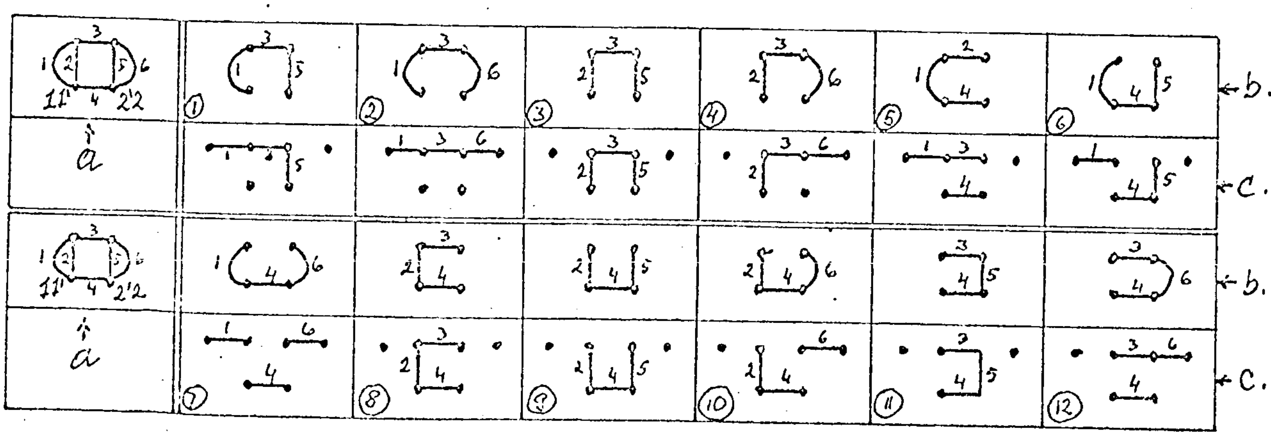


FIG. 8. a. NETWORK $N_{11',22'}$ b. TREES OF $N_{11',22'}$ c. 3-TREES