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A PRELIMINARY ANALYSIS OF A THREE-IMPULSE TECHNIQUE FOR TRANSEARTH INJECTION FROM HIGHLY INCLINED ORBITS

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FOREWORD

This report describes work accomplished as a part of the Lunar Orbital Survey Missions Study currently being performed at the Lockheed Missiles and Space Company under Contract NAS 9-5288 for the NASA Manned Spacecraft Center, Houston, Texas. This analysis was performed prior to recent redirection emphasizing flight mechanics and mission analysis. A more thorough analysis of the three-impulse technique is planned as a part of the redirected effort.

SUMMARY

Lunar orbit missions with highly inclined orbits will be useful in accomplishing a complete survey of the lunar surface using Apollo hardware. However, large angles between the orbit plane and transearth trajectory departure asymptote will arise, resulting in prohibitively large ΔV requirements if current single impulse methods are utilized. The two impulse technique will greatly reduce the maximum V required. A further reduction is possible using a three impulse technique, in which highly eccentric elliptical orbits are used to efficiently change the orbit plane.

This report presents a preliminary analysis of the three impulse technique. The ΔV requirements are determined as a function of transearth flight time and the angle between the initial orbit plane and the departure asymptote. Comparisons are also made of payload capabilities for possible missions using two impulses with corresponding missions using three impulses.

A brief summary of the results is given below comparing the payload capabilities of possible missions to polar orbits using Block II vehicle configurations (limited to total mission durations less than 14 days).

Wt in SM Sector 1, Lb.	Translunar Time, Hours	Two Impulse Expmt + Probe Wt, Lb.	Three Impulse Expmt + Probe Wt, Lb.	Difference in Payload Wt, Lb.
2300	60	Negative	9,400	12,200
2300	108	12,300	20,600	8,300
5000	60	Negative	1,900	13,400
5000	72	Negative	12,400	13,900

SUMMARY (Cont.)

For long duration missions to polar orbits the increase in payload capability is estimated to be approximately 9,960 lbs, but in many cases the two-impulse payload capability against which this is compared may be negative.

The preliminary analysis which has been conducted is reported herein in order that the NASA Manned Spacecraft Center might review the results and determine the operational feasibility of this three-impulse technique. In this preliminary analysis certain simplifying assumptions were made, and only six of the most important control parameters were selected for the optimization from the possible 12 parameters. Work is already underway at LMSC on the full optimization of the problem because of the significant results obtained from this preliminary analysis. However, before any great amount of effort is expended it would be desirable to have a response from NASA.

I. INTRODUCTION

After the first lunar landings by Apollo, it is planned to use manned lunar orbit missions as one means of extending our scientific knowledge of the Moon. In order to accomplish a complete survey of the lunar surface, polar orbits will be required. Orbits with high inclinations can result in large angles between the instantaneous departure asymptote and the orbit plane and, consequently, large velocity requirements to perform the transearth injection maneuver.

Various techniques have been investigated for achieving transearth injection when the angle between the orbit plane and the departure asymptote is large (up to 90 degrees). The most efficient technique reported to date involves two impulses, one to leave the lunar orbit and the second near the lunar sphere of influence.

This report considers a three-impulse technique which is significantly more efficient than the two-impulse techniques reported. The first impulse in the three-impulse method primarily injects the vehicle into a highly-eccentric elliptical orbit about the Moon, and the second impulse at apocynthion changes the orbit plane so that it contains the departure asymptote. The third impulse is applied near pericynthion such that the desired departure energy and direction are achieved. Constraints are satisfied which include pericynthion altitudes and time from the first impulse to Earth's atmosphere re-entry.

II. ASSUMPTIONS AND BASIC DATA

This preliminary study of the three-impulse technique for transearth injection is based upon a restricted 2-body, selenocentric analysis. Departure asymptote direction requirements as a function of transearth flight time were taken from Reference (1) assuming average lunar distance. Circular lunar orbits at 43 nm altitude, with a complete range of orbit orientations, were assumed. The requirement for the magnitude of the hyperbolic excess velocity, $|V_\infty|$, as a function of transearth flight time was derived from lunar orbit insertion ΔV data using the vis viva integral. The ΔV data was received from Mr. P. G. Thomas (MSC), and was based on an 80 nm lunar orbit and average lunar distance. The $|V_\infty|$ values thus derived are given below.

Transearth Flight Time, Hours	$\lvert V_\infty \rvert$, Ft/Sec
60	4,222
72	3,328
84	2,937
108	2,681

The lunar constants were based on Reference (2) which is a basic document for the Apollo Navigation Working Group (Reference 3).

Other assumptions are as follows:

- o All translunar and transearth trajectory trip times are between 60 and 108 hours.
- o Maximum elapsed time for anytime abort is 108 hours from first impulse to Earth's atmosphere re-entry.
- o All ΔV's are impulsive.
- o Average Earth-Moon distance.

ASSUMPTIONS AND BASIC DATA (Cont.)

- Circular lunar orbits at 43 nm altitude.
- Saturn V injection weight capability for a 72 hour translunar trajectory is 95,000 Lbs including a 3,800 Lb adapter.
- CSM inert weight is 20,728 Lbs.
- SM impulse propellant is 39,700 Lbs maximum.

III. ONE- AND TWO-IMPULSE TECHNIQUES

The lunar orbit survey missions will utilize high inclination, lunar orbits with the result that large angles may exist between the orbit plane and the departure asymptote for the transearth trajectory. When this "out-of-plane" angle is small, the transearth injection maneuver may be made quite efficiently with a single impulse; when the angle is large, however, the velocity requirement with a single impulse becomes prohibitively great, and more efficient means of attaining a transearth trajectory are desired.

The two-impulse technique provides a great reduction in the maximum velocity required, approximately 5200 Ft/Sec as opposed to approximately 9500 Ft/Sec with a single impulse. A number of investigators have analyzed the two-impulse technique with certain simplifying assumptions, primarily in using only a two-body model (References 4, 5, and 6). However, sophisticated analyses have also been completed with more realistic models, including the trajectory beyond the lunar sphere of influence, and the constraint on transearth flight time. The first such analysis to produce results (to the best of knowledge of this writer) was performed by Mr. P. G. Thomas, NASA, MSC (Reference 7). Results from this analysis are quoted herein for purposes of comparison.

In the two-impulse technique the first impulse injects the vehicle into a hyperbolic trajectory which is followed until the vehicle is a great distance from the Moon. A small amount of plane change is simultaneously accomplished with the first impulse. At a great distance from the Moon (10,000 to 40,000 nm) the second impulse changes the direction and magnitude of the velocity so that the vehicle is on a transearth trajectory which satisfies the vehicle constraints.

IV. THREE-IMPULSE TECHNIQUE

The three-impulse geometry is illustrated in Figure 1. The first impulse in the three-impulse method primarily injects the vehicle into a highly-eccentric elliptical orbit about the Moon, and the second impulse at apocynthion changes the orbit plane so that it contains the departure asymptote. The third impulse is applied near pericynthion such that the desired departure energy and direction are achieved.

The efficiency of the three-impulse technique has its basis in two astrodynamic principles:

1) Direction of motion can be changed most efficiently where the velocity is smallest.
2) Energy level can be changed most efficiently where the velocity is largest.

The two-impulse technique utilizes the first principle by going a great distance from the Moon to change the direction, but in so-doing, the achievement of the departure energy tends to be penalized. Therefore, it is reasonable to expect some advantage by using a three-impulse technique. First, the velocity at the apocynthion of a highly eccentric orbit is very low, permitting a plane change with a small impulse. Secondly, all the velocity added to achieve the required departure energy is added near the Moon where the velocity is large.

Factors which tend to decrease the efficiency of the three-impulse technique are as follows:

1. The time spent in the elliptical orbit must be compensated for by using a faster transearth trajectory, requiring greater velocity at transearth injection.

2. Ideally the third impulse should be applied at or near the ellipse pericynthion, but with the constraints used in this study along with the V_∞ direction requirements, the position for the third impulse is compromised.

First Impulse

The magnitude of the first impulse is selected to produce an ellipse having a period of 24 hours. This is selected somewhat arbitrarily, since from a performance standpoint, the greater the period and apocynthion distance, the lower the velocity required for the plane change; but also the greater the time which must be compensated for in the transearth trajectory. Also, problems due to Earth and Sun perturbations increase with increasing period. The apocynthion altitude with a 24 hour period is 8,610 nm. It appears to be a reasonable assumption at this time that if the desired orbit orientation would result in undesirable perturbations (for example, lowered pericynthion altitude), then either small impulse corrections may be applied or a compromise orbit orientation can be utilized without significant increase in velocity requirements.

The first impulse is assumed to be applied horizontally; any change in flight path angle would result in a pericynthion altitude lower than the lunar circular orbit altitude. Even though the vehicle does not normally return to pericynthion, it seems undesirable to enter such an orbit. A constraint applied therefore was that the three conics involved in this technique would have pericynthion altitudes equal to or greater than the 43 nm initial altitude.

First Impulse (Cont.)

A small yaw component is optimally contained in the first maneuver. The corresponding velocity savings with respect to turning the total angle at apocynthion is only 6 to 13 Ft/Sec for a 24 hour ellipse. Only the first two impulses contain out-of-plane components in this preliminary analysis, for the sake of simplicity and because the third impulse is not at a point favorable to efficiently changing the plane.

The position in the lunar orbit of the first maneuver is the only control variable for orientation of the line of apsides of the first conic, since a change of flight path angle is ruled out by the constraint discussed above. The line of apsides is oriented such that the total velocity of the three impulses is minimized for the control variables considered.

Second Impulse

The second impulse, applied at apocynthion, has its magnitude and direction specified by the requirements that it change the plane to contain the departure asymptote without changing the pericynthion altitude. The second maneuver is also assumed to be applied in the horizontal direction.

Third Impulse

The optimum position for the third impulse is found by searching for the position which requires the smallest impulse to satisfy the hyperbolic excess velocity (V_∞) magnitude and direction requirements. If the third impulse is applied at the pericynthion, the orbit velocity would be greatest but the change in flight path angle would be large, requiring in general, an excessively large impulse. The optimum position for the third impulse then exists as a compromise between maximizing the orbit velocity and minimizing the path angle change.

V. METHOD OF OPTIMIZATION

A simplified approach to optimization would be to first orient the line of apsides so that the minimum angle is turned through along the ellipse line of apsides ($\Theta = \Theta_{min}$), and then determine the optimum point of the third maneuver, ΔV_3, to transfer from the ellipse established by the first two impulses (ΔV_1 and ΔV_2) to the required departure conditions. This approach would minimize ($\Delta V_1 + \Delta V_2$) and ΔV_3 separately but would not minimize the total ($\Delta V_1 + \Delta V_2 + \Delta V_3$).

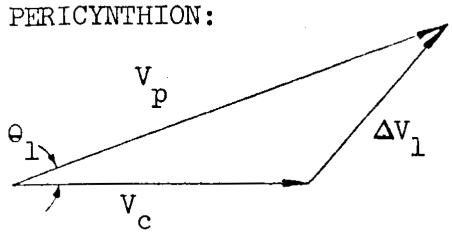
Figure 2 shows that if the ellipse line of apsides is oriented so that the angle Θ is minimized, the angle between the ellipse line of apsides and the departure asymptote, W, is always equal to 90 degrees regardless of the value of the angle between the initial orbit plane and the departure asymptote, α (also called "out-of-plane angle"). Figure 3 illustrates that if W can be made to be less than 90 degrees, the third maneuver can be made at a lower altitude and greater velocity, making it more efficient. Figure 2 shows that W can have values less than 90 degrees at the expense of increasing the angle Θ. The optimization procedure used in this study is based upon such an approach, and is described in the remainder of this section.

The minimization of the total velocity was accomplished in two parts: (1) the optimum distribution of plane change between the first and second impulses, and (2) the optimum orientation of the ellipse line of apsides considering the effect upon the total plane change angle.

The optimum distribution of the plane change angle between the first and second impulses was found by a closed-form solution.* The associated geometry is illustrated as follows:

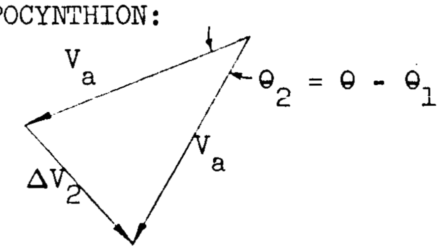
* Performed by K. F. Johansen, LMSC ANVO, dated 29 March 1966.

PERICYNTHION:



$$\Delta V_1 = (V_p^2 + V_c^2 - 2V_pV_c\cos\theta_1)^{\frac{1}{2}}$$

APOCYNTHION:



$$\Delta V_2 = 2V_a\sin\tfrac{1}{2}\,\theta_2$$

Where V_c = Circular orbit velocity

V_p = Periapsis velocity of intermediate ellipse

V_a = Apoapsis velocity of intermediate ellipse

θ = Total plane change angle

θ_1 = Plane change made at injection into intermediate ellipse

ΔV_1 = Magnitude of first burn

ΔV_2 = Magnitude of second burn

The total cost of the maneuver is given by:

$$\Delta V_T = \Delta V_1 + \Delta V_2 - (V_p - V_c) \qquad (1)$$

For the given orbit parameters and total angle θ this expression is a function of the plane change θ_1 made at periapsis. To find the value of θ_1 which minimizes ΔV_T, equation (1) was differentiated and set equal to zero which leads to the following equation:

$$(V_p^2 + V_c^2 - 2V_pV_c\cos\theta_1)^{-\frac{1}{2}}\,(V_pV_c\sin\theta_1) - V_a\cos\tfrac{1}{2}(\theta - \theta_1) = 0 \qquad (2)$$

This equation was converted into a fourth degree polynomial in cos Θ_1 which was solved numerically. The resulting angles are plotted versus total angle for several elliptical orbit periods in Figure 4. For small total angles the cost is minimized by making the entire plane change at periapsis. The total cost of the maneuver (excluding $V_p - V_c$) and the savings relative to making the entire plane change at apoapsis are plotted in Figures 5 and 6. Note that the velocity savings, for a 24 hour elliptical orbit, range between only 6 and 13 Ft/Sec.

The optimization of the ellipse line of apsides orientation was accomplished utilizing numerical techniques. The angle W and the magnitude of the hyperbolic excess velocity were varied parametrically as inputs to a computer program which calculated conditions of the third impulse based on solutions to conic equations. For each combination of W and $|V_\infty|$, the magnitude of ΔV_3 was minimized by using a numerical search scheme to vary the hyperbola pericynthion altitude. Assuming values for α, the values of Θ corresponding to each of the selected values of W may be obtained from spherical trigonometry (Figure 2) by

$$\sin \Theta = \frac{\sin \alpha}{\sin W} \tag{3}$$

The second impulse velocity requirement, assuming all plane change at apocynthion, may be computed from

$$\Delta V_2 = 2V_a \sin \tfrac{1}{2} \Theta \tag{4}$$

The total ΔV requirement may be computed for each combination of W, α and Θ by adding to the above ΔV_2 and ΔV_3, the ΔV_1 for injecting into a 24 hour ellipse, 1868 Ft/Sec, and subtracting the savings possible by making a portion of the plane change at pericynthion (Figure 6) for the values of Θ computed using equation (3). Results may be plotted as a function of Θ, as shown in Figure 7, to obtain the minimum ΔV for a given value of α.

VI. RESULTS

The velocity requirement for the simplified optimization, described at the beginning of the previous section, is shown in Figure 8 for an 84 hour transearth flight time as a function of α, the angle between the orbit plane and the departure asymptote. The total ΔV for transearth injection in the simplified case is

$$\Delta V_{TEI} = 3375 \text{ Ft/Sec} + 2V_a \sin \tfrac{1}{2}\alpha \ .$$

It should be noted that the simplified case ΔV requirement approaches 3375 Ft/Sec as α approaches zero, which is 597 Ft/Sec greater than the single impulse requirements. This indicates the size of penalty (for small values of α) associated with having the ellipse line of apsides oriented for minimizing the plane change angle, Θ, rather than minimizing the total velocity. It should also be noted that for $\alpha = 90^\circ$ the total velocity is 4430 Ft/Sec; and this does not include a similar penalty, since the velocity requirement is insensitive to the line of apsides orientation for $\alpha = 90^\circ$.

For comparison with the simplified case curve, Figure 8 also shows the ΔV requirement for the case where the ellipse line of apsides orientation is optimized.

The third impulse ΔV requirement is given in Table I as a function of the angle W and the transearth flight time. These data were obtained from solutions to conic equations and serve as a basis for the simplified optimization results above, and for the six-parameter optimization results given in subsequent paragraphs.

Figure 9 presents the three-impulse transearth injection ΔV requirements as a function of the out-of-plane angle and the transearth trajectory flight time, based on the six-parameter optimization. The transearth trajectory flight times indicated do not include the 24 hours spent in the intermediate ellipses.

Figure 10 presents a comparison between the two- and three-impulse ΔV requirements for departure from lunar polar orbits. One-impulse coplanar data is also shown. The two-impulse data (Reference 7) is based on slightly different ground rules than those used for the one- and three-impulse data. The two-impulse data is based upon a maximum earth-moon distance and an orbital altitude of 80 nm. The three-impulse curve shown is for a transearth flight time of 84 hours so that with the 24 hours spent in intermediate ellipses it is comparable to the two-impulse data at 108 hours. The figure gives the transearth injection ΔV requirements as a function of the longitude of the node and as a function of time in lunar orbit. The declination of the departure asymptote, δ, is assumed to be zero. This assumption is conservative with respect to the peak ΔV values, since the maximum out-of-plane angle possible with a polar orbit is

$$\alpha_{max} = 90^{\circ} - \delta .$$

The most critical time in a polar mission of long duration ($>$10d) occurs when the out-of-plane angle is near a maximum for departure on an 84 hour transearth trajectory. The maximum value that this angle can be is 90°, therefore the maximum ΔV of 4424 Ft/Sec in the 84-hour curve of Figure 10 is approximately the maximum abort ΔV for any mission, assuming that 550 Ft/Sec reserves is adequate to cover Earth-Moon distance variations. A breakdown of the maximum ΔV is given below.

ΔV_1	=	1874 Ft/Sec
ΔV_2	=	1043
ΔV_3	=	1507
ΔV_{TEI}	=	4424
$\Delta V_{reserve}$	=	550
Total ΔV_{max}	=	4974 Ft/Sec

A comparison of the two- and three-impulse ΔV requirements shows that the three-impulse technique provides a 750 Ft/Sec reduction to the maximum requirement for anytime abort on long-duration missions. For short-duration missions the maximum reduction might be in the velocity to abort immediately after lunar orbit insertion. The savings here are 1050 Ft/Sec, 985 Ft/Sec and 870 Ft/Sec for coplanar lunar orbit insertion with 60 hr, 72 hr, and 84 hr translunar flight times respectively. These savings will be evaluated in terms of payload in subsequent paragraphs.

One characteristic of the three-impulse curve which might be of interest is the relatively steep slopes before and after the maximum. If the anytime abort constraint is relaxed at some time in the future, the three-impulse technique will provide a greater increase in payload capability for a given period without abort capability than will the two-impulse technique. For example, postponing abort up to 2 days on a long mission will increase the payload capability 3000 Lbs with the three-impulse technique compared with 910 Lbs for the two-impulse technique.

Figure 11 presents the three-impulse transearth injection ΔV requirements for polar orbits as a function of the longitude of the node and the transearth trajectory flight time. A time-in-orbit scale is also shown, based on a 72-hour translunar trajectory. A curve for the one-impulse coplanar injection is also given. Again, the transearth flight times shown do not include the 24 hours spent in the intermediate ellipses. The minimum and maximum points in the curves represent plane change angles of zero and 90 degrees respectively. Notice that the zero-plane-change points do not lie on the single-impulse curve, but are displaced 13.176 degrees to the left (or 24 hours in time) due to the time spent in the intermediate ellipses.

It should be noted that at certain times the velocity requirement to use a 72 hour transearth trajectory is less than it is to use an 84 hour trajectory. This is true because the out-of-plane angle is less with the 72 hour than with the 84 hour trajectory, and the resulting savings exceeds the expense of the faster transfer. The increase in ΔV necessary to use a 60 hour trajectory is not great in some regions because of a similar savings. Also, advantage may be taken of the crossing of the curves to slightly reduce the maximum ΔV required for anytime abort.

The payload (lab + experiments) capability as a function of transearth ΔV is given in Figures 12 through 15, based upon 2300, 1500, 5000 and 0 Lbs in Sector 1 of the Service Module respectively. These correspond to configurations defined for the LOSM Study by the Manned Spacecraft Center in January 1966 as Cases I-A and I-B, Cases II-A and II-B, Case III-A, and Case III-B. In addition to assumptions and basic data stated earlier, the performance is based upon Block II Command and Service Module weights received from MSC and shown in Tables II and III. To the ΔV requirements obtained from Figures 9 through 11, the velocity reserves of 550 Ft/Sec (for the transearth midcourses and lunar distance and other variations) must be added to enter Figures 12 through 15.

The effect on payload capability of using the three-impulse technique on possible Block II (short duration) polar orbit missions may be determined by using Figures 10 through 15. Table IV gives some examples of advantageous use of the three-impulse technique.

The Block II configurations are designed for total mission durations of 14 days or less, therefore the effect of utilizing the three-impulse technique on the payload capability for long duration missions cannot be determined from figures presented in this report. However, the effect can be estimated using a tradeoff of payload with velocity of approximately -13 Lb/Ft/Sec. On this basis the reduction of the maximum ΔV from 5190 Ft/Sec to 4424 Ft/Sec will provide an increase in the payload capability of approximately 9,960 Lb for long duration missions with the anytime abort capability. The two-impulse payload capability against which this is compared may be negative, as was found in some Block II polar orbit missions.

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TABLE I

THIRD IMPULSE ΔV REQUIREMENTS AS A FUNCTION OF W AND TRANSEARTH FLIGHT TIME

W, Deg.	THIRD IMPULSE ΔV REQUIREMENTS			
	Transearth Trajectory Flight Time, Hours			
	60	72	84	108
90	1889.0	1605.7	1507.1	1450.5
85	1806.7	1531.5	1437.6	1384.5
80	1728.4	1456.9	1367.0	1317.0
75	1654.5	1382.6	1295.4	1248.0
74			1281.0	
72			1252.3	
70			1223.5	
68			1194.8	
66			1166.3	

TABLE II

COMMAND MODULE WEIGHT SUMMARY

	Block II 11/1/65
Weight Empty	(9318)
Structure	5374
S&C	188
G&N	366
Crew Systems	85
Environmental Control	424
Earth Landing	620
Instrumentation	38
Electrical Power	1243
Reaction Control	300
Communications	334
Controls and Displays	346
Useful Load Expendables	(1282)
Crew Systems	911
Environmental Control	101
Reaction Control	270
Total CM	(10600)

TABLE III

SERVICE MODULE WEIGHT SUMMARY

	Block II 11/1/65
Weight Empty	(7787)
Structure	4302
E. C. S.	137
Instrumentation	56
Electrical Power	1667
Main Propulsion	1208
RCS	307
Communications	110
Useful Load	(2341)
E. C. S.	150
E. P. S.	503
Main Prop.	850
R. C. S.	838
Total	(10128)

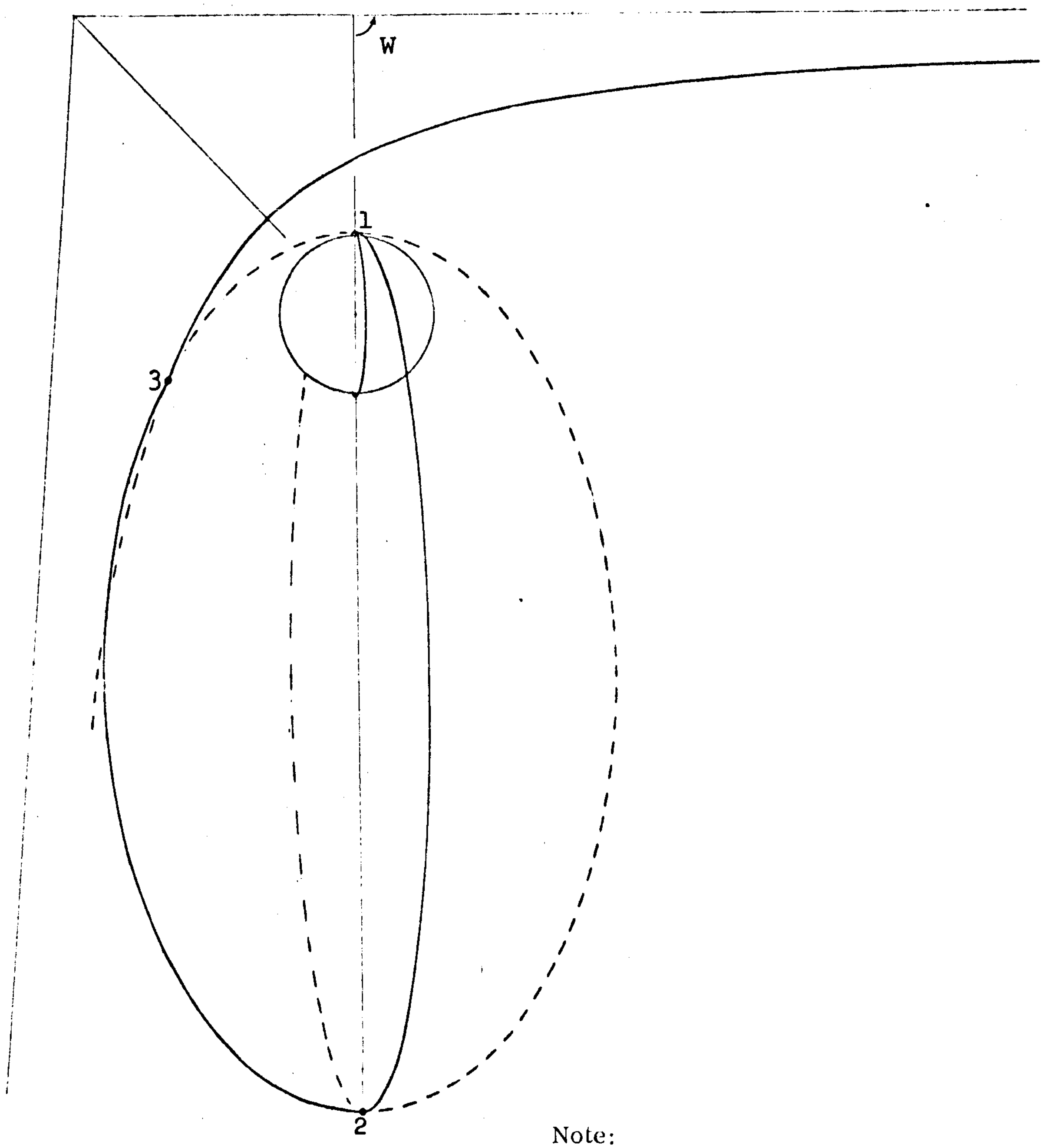
TABLE IV

PAYLOAD CAPABILITY COMPARISON

Config. Case No.	TL Time, Hr	Time in Orbit at TEI, Days	TWO-IMPULSE Total ΔV, Ft/Sec	TWO-IMPULSE Lab. + Expmt.* Wt, Lb	TWO-IMPULSE Expmt. Wt,** Lb	THREE-IMPULSE Total ΔV, Ft/Sec	THREE-IMPULSE Lab. + Expmt.* Wt, Lb	THREE-IMPULSE Expmt. Wt,** Lb	Difference in Payload Wt, Lb
I-A or -B	60	0	5595	8,700	Negative	4545	20,900	9,400	12,200
I-A or -B	72	0	5320	19,100	7,600	4335	29,250	17,750	10,150
I-A or -B	84	0	5010	24,800	13,300	4140	30,500	19,000	5,600
I-A or -B	84	7	4845	28,000	16,500	4230	30,400	18,900	
I-A or -B	108	7	5340	23,800	12,300	4560	32,100	20,600	8,300
III-A	60	0	5595	0	Negative	4545	13,400	1,900	13,400
III-A	72	0	5320	10,000	Negative	4335	23,900	12,400	13,900

*Experiments, probes, or combination experiments and probes
**Lab weight assumed to be 11,500 lb

THREE-IMPULSE GEOMETRY

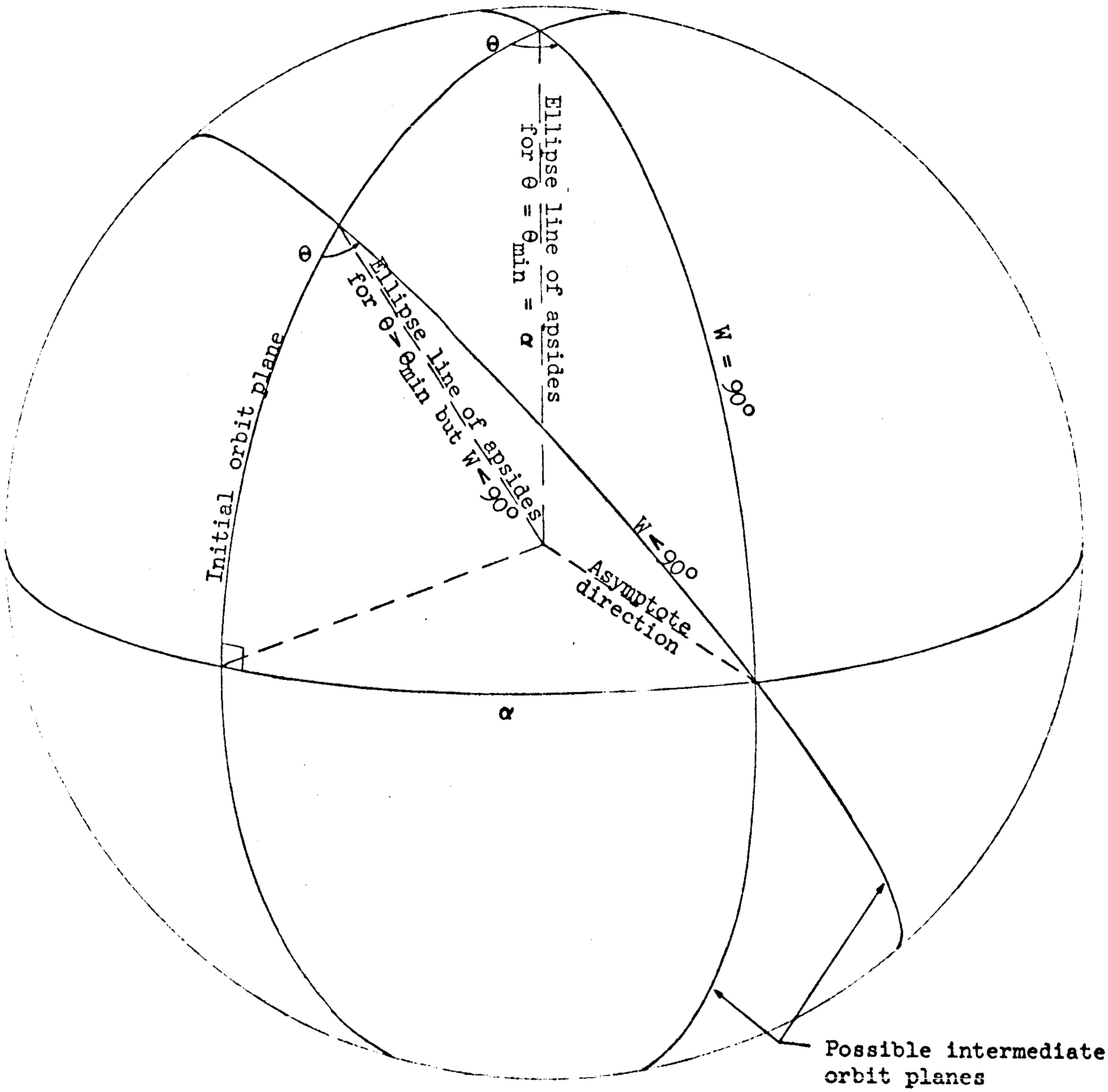


Note:

To scale for 24 hr ellipse, 84 hr TE trajectory, and ΔV_3 minimized for W = 90°

Figure 1

RELATIONSHIPS OF ELLIPSE ORIENTATION



$$\sin \theta = \frac{\sin \alpha}{\sin W}$$

Figure 2

ELLIPSE ORIENTATION WITH RESPECT TO DEPARTURE HYPERBOLA

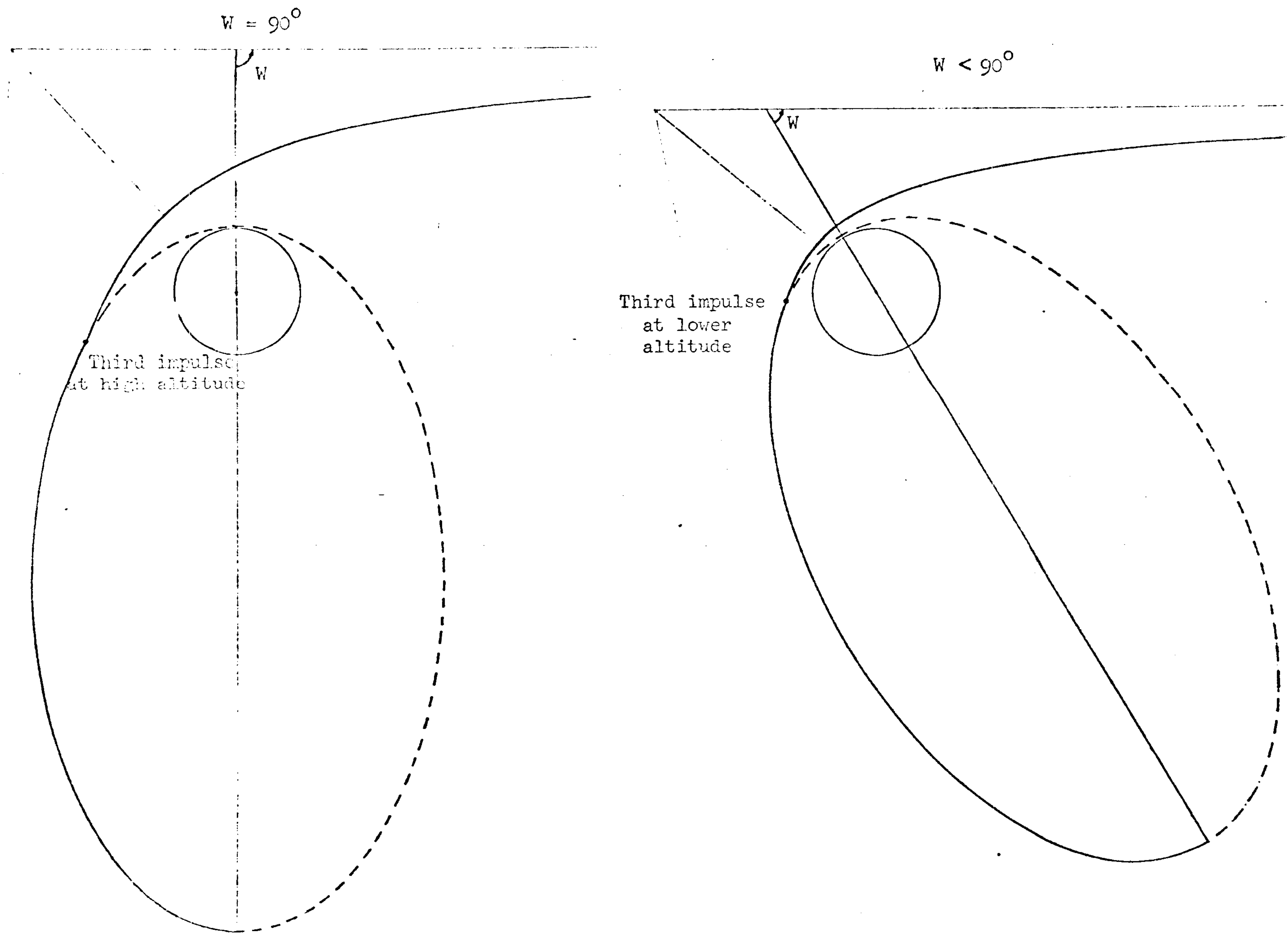


Figure 3

PLANE CHANGE AT PERIAPSIS VS TOTAL PLANE CHANGE ANGLE TO MAXIMIZE VELOCITY SAVINGS

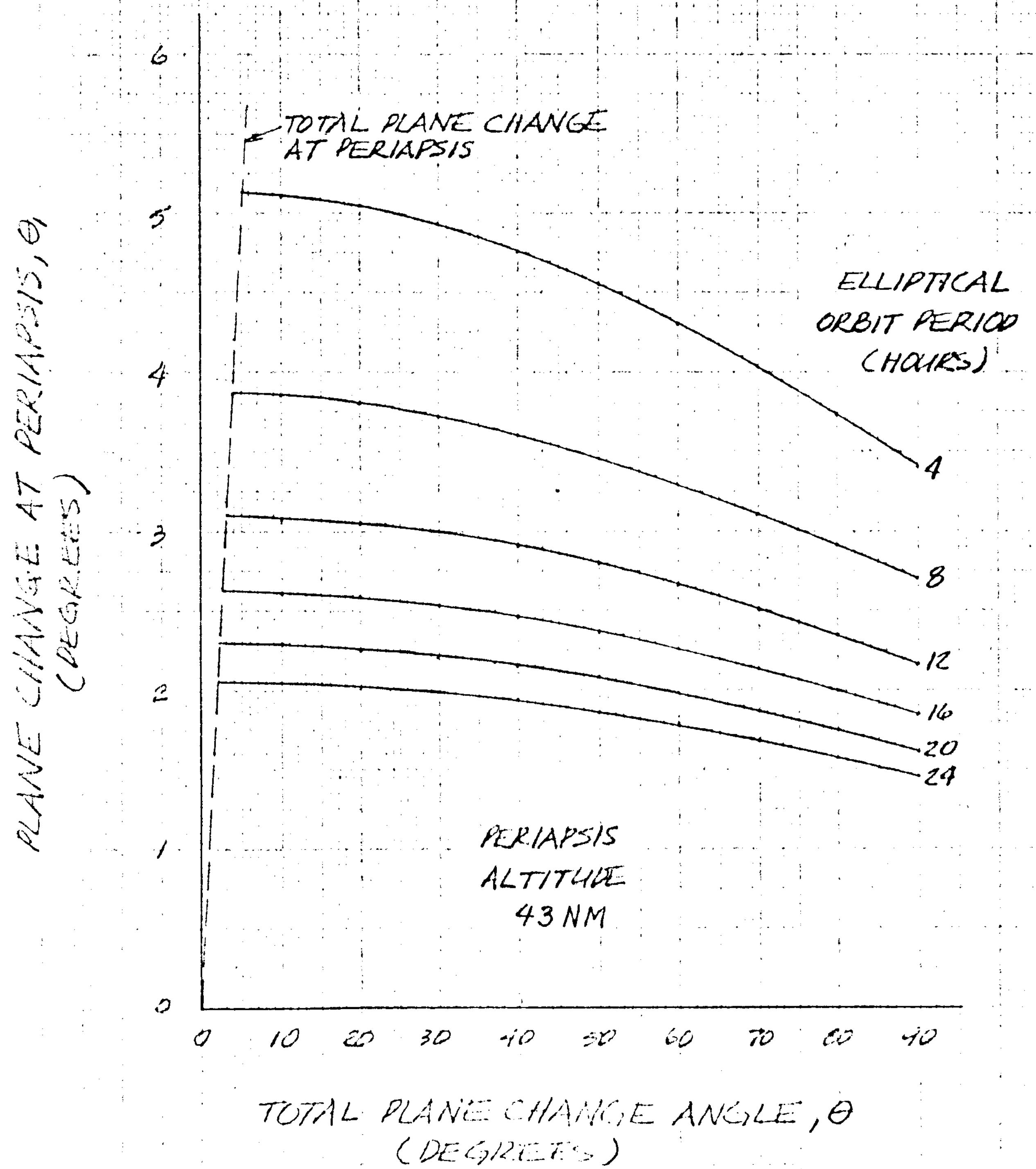


Figure 4

TOTAL VELOCITY TO CHANGE PLANES UTILIZING OPTIMUM PLANE CHANGE AT PERIAPSIS VS TOTAL PLANE-CHANGE ANGLE

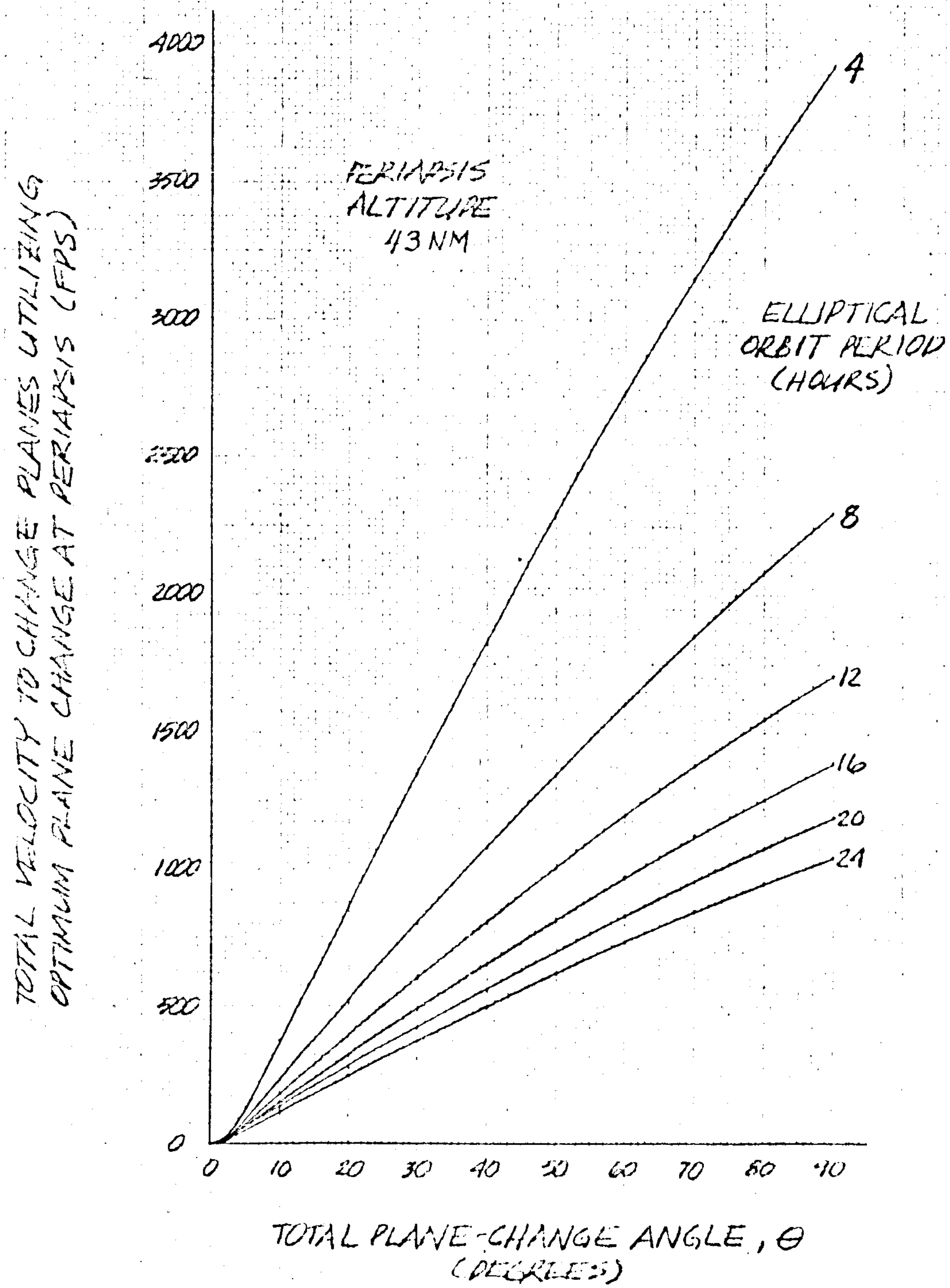


Figure 5

MAXIMUM VELOCITY SAVINGS RESULTING FROM PARTIAL PLANE CHANGE AT PERIAPSIS VS TOTAL PLANE CHANGE ANGLE

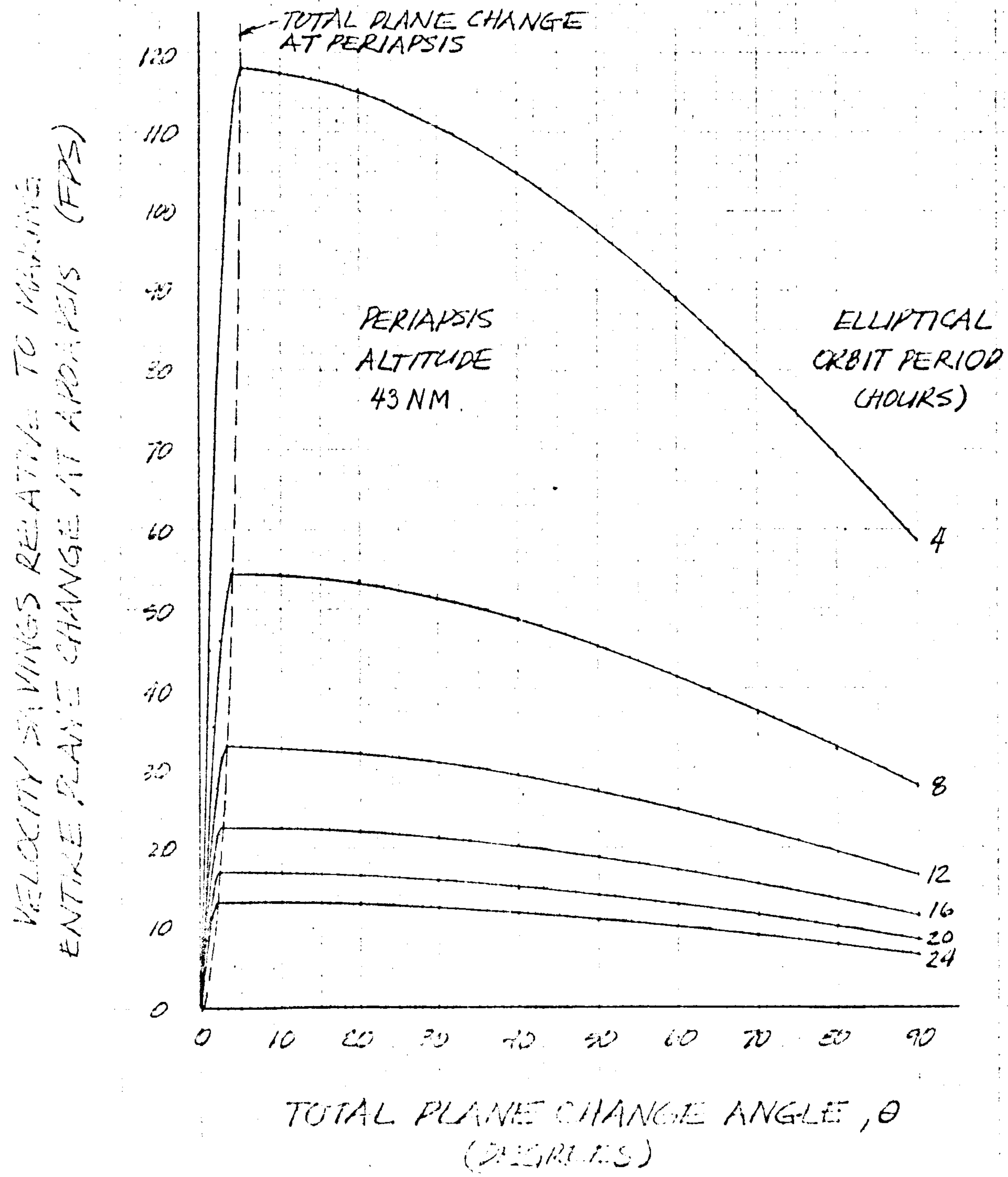


Figure 6

OPTIMIZATION OF THE ANGLE TURNED ALONG THE ELLIPSE LINE OF APSIDES

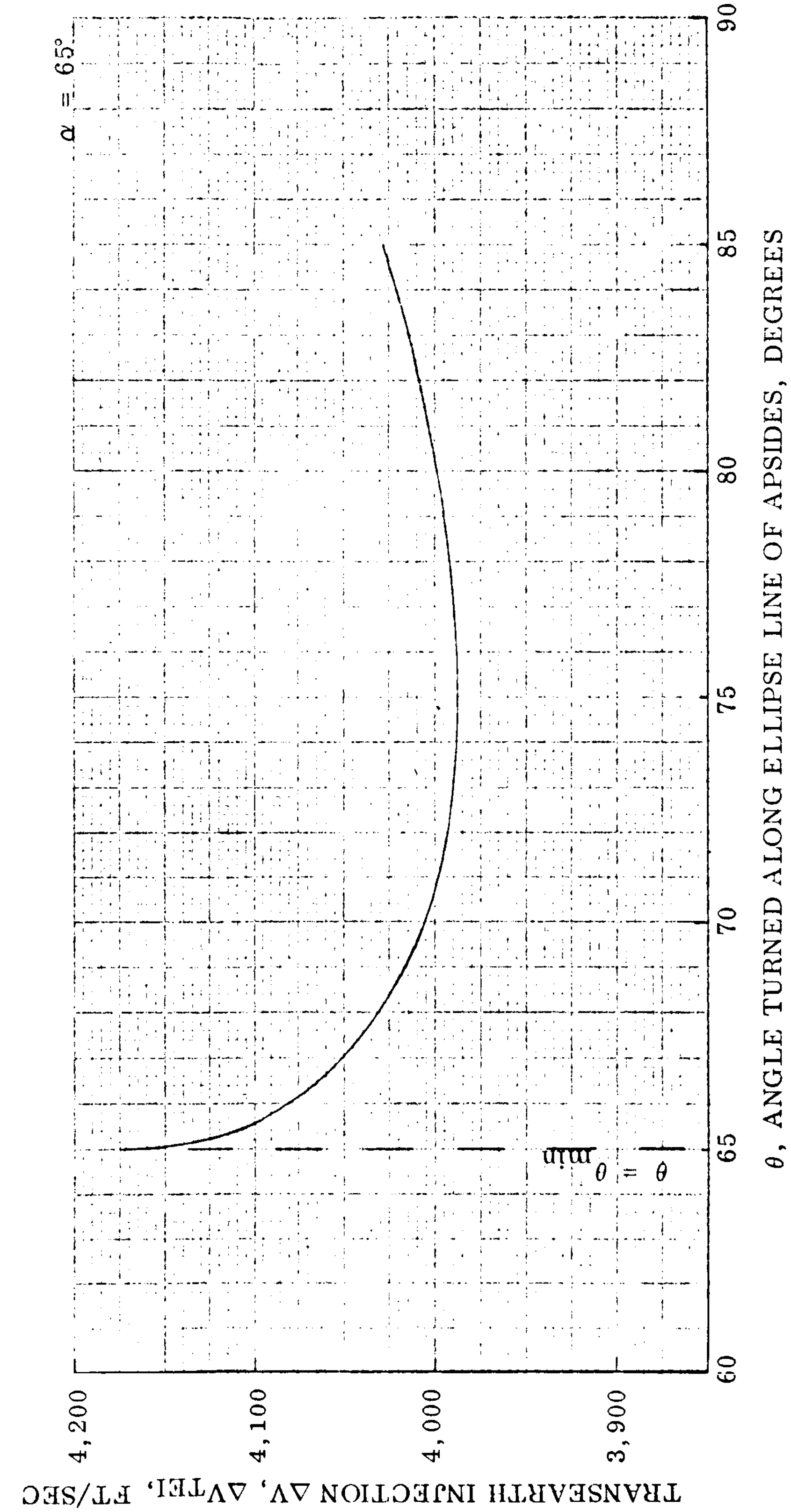


Figure 7

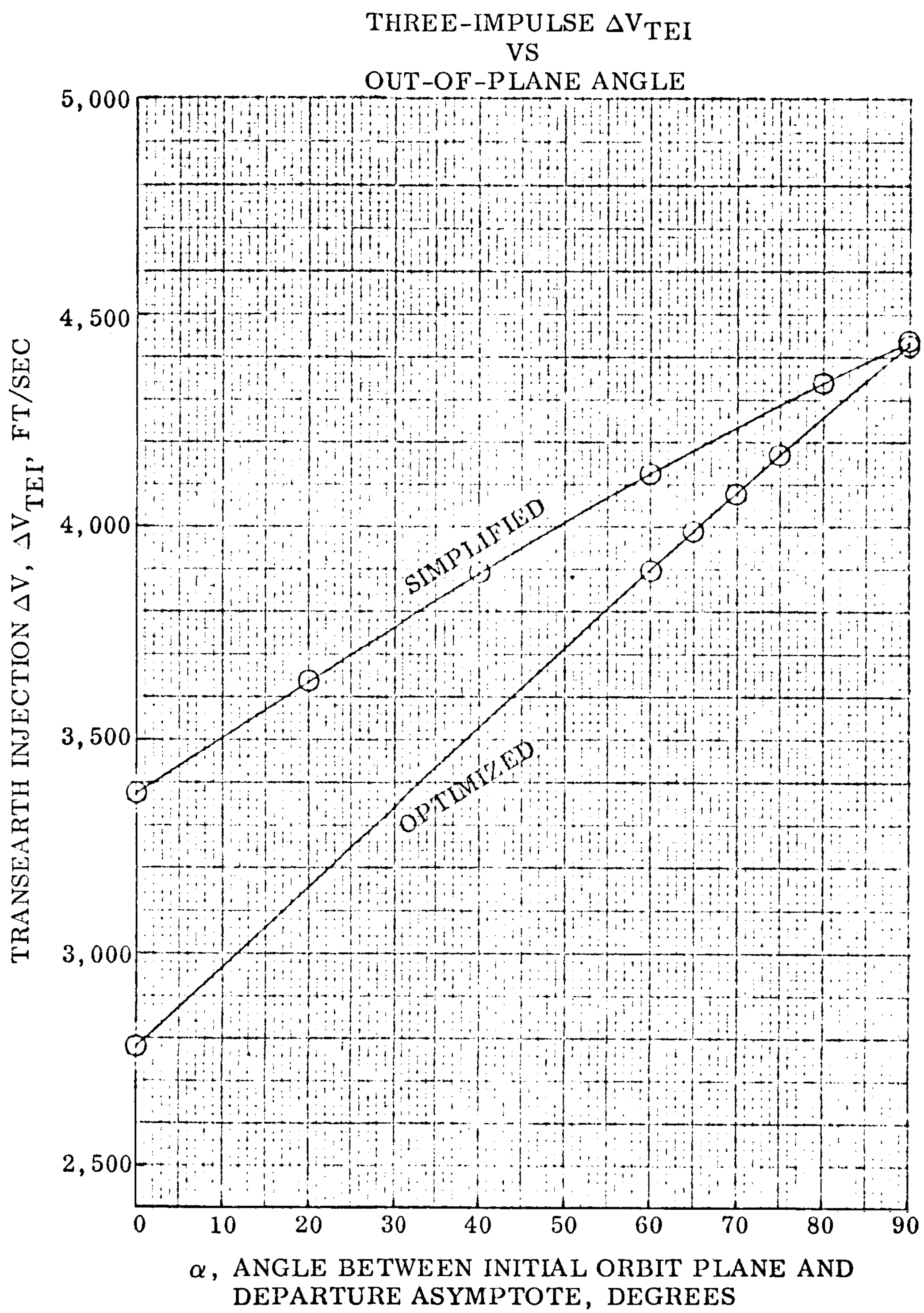

THREE-IMPULSE ΔV_{TEI}
VS
OUT-OF-PLANE ANGLE
TRANSEARTH INJECTION ΔV, ΔV_{TEI}, FT/SEC
5,000
4,500
4,000
3,500
3,000
2,500
SIMPLIFIED
OPTIMIZED
0
10
20
30
40
50
60
70
80
90
α, ANGLE BETWEEN INITIAL ORBIT PLANE AND
DEPARTURE ASYMPTOTE, DEGREES

Figure 8

THREE-IMPULSE ΔV_{TEI} VS. OUT OF PLANE

ANGLE AND TE FLIGHT TIME

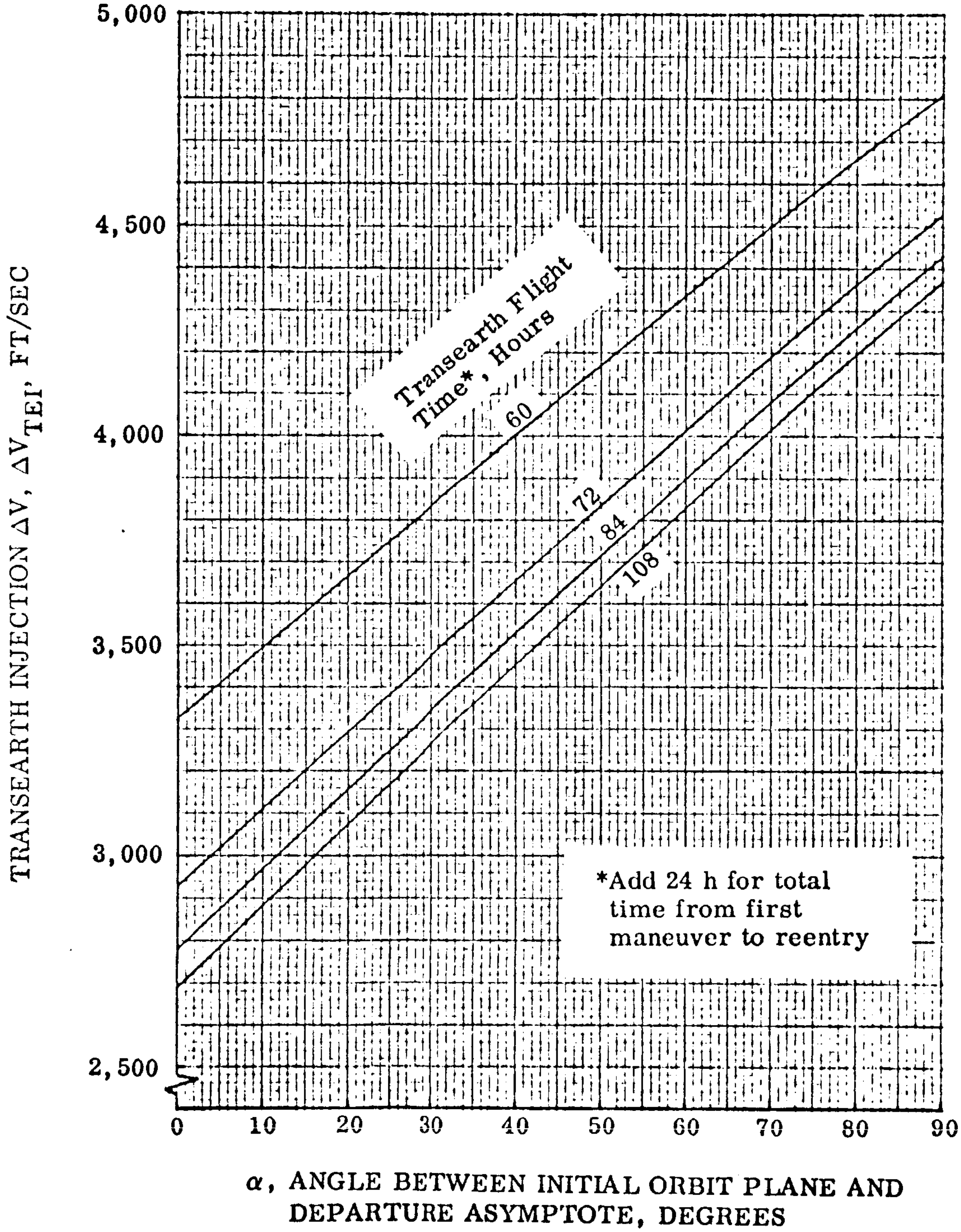


Figure 9

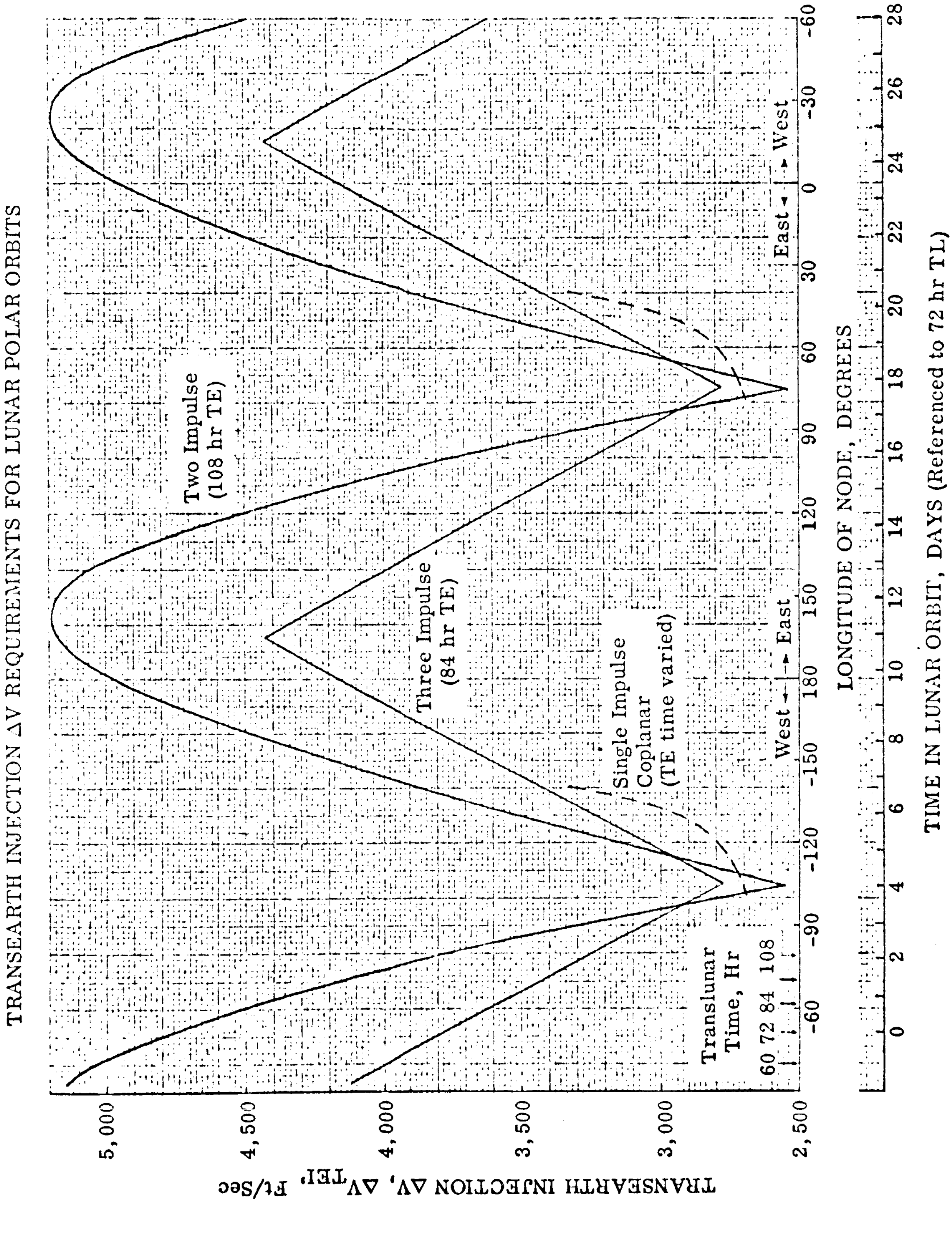

TRANSEARTH INJECTION ΔV REQUIREMENTS FOR LUNAR POLAR ORBITS
Two Impulse
(108 hr TE)
Three Impulse
(84 hr TE)
Single Impulse
Coplanar
(TE time varied)
Translunar
Time, Hr
60 72 84 108
West ← | → East
East ← | → West
-60 -90 -120 -150 180 150 120 90 60 30 0 -30 -60
LONGITUDE OF NODE, DEGREES
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
TIME IN LUNAR ORBIT, DAYS (Referenced to 72 hr TL)
5,000
4,500
4,000
3,500
3,000
2,500
TRANSEARTH INJECTION ΔV, ΔV_{TEI}, Ft/Sec

Figure 10

LUNAR POLAR ORBIT ΔV_{TEI} AS A FUNCTION OF TRANSEARTH FLIGHT TIME AND LONGITUDE OF NODE

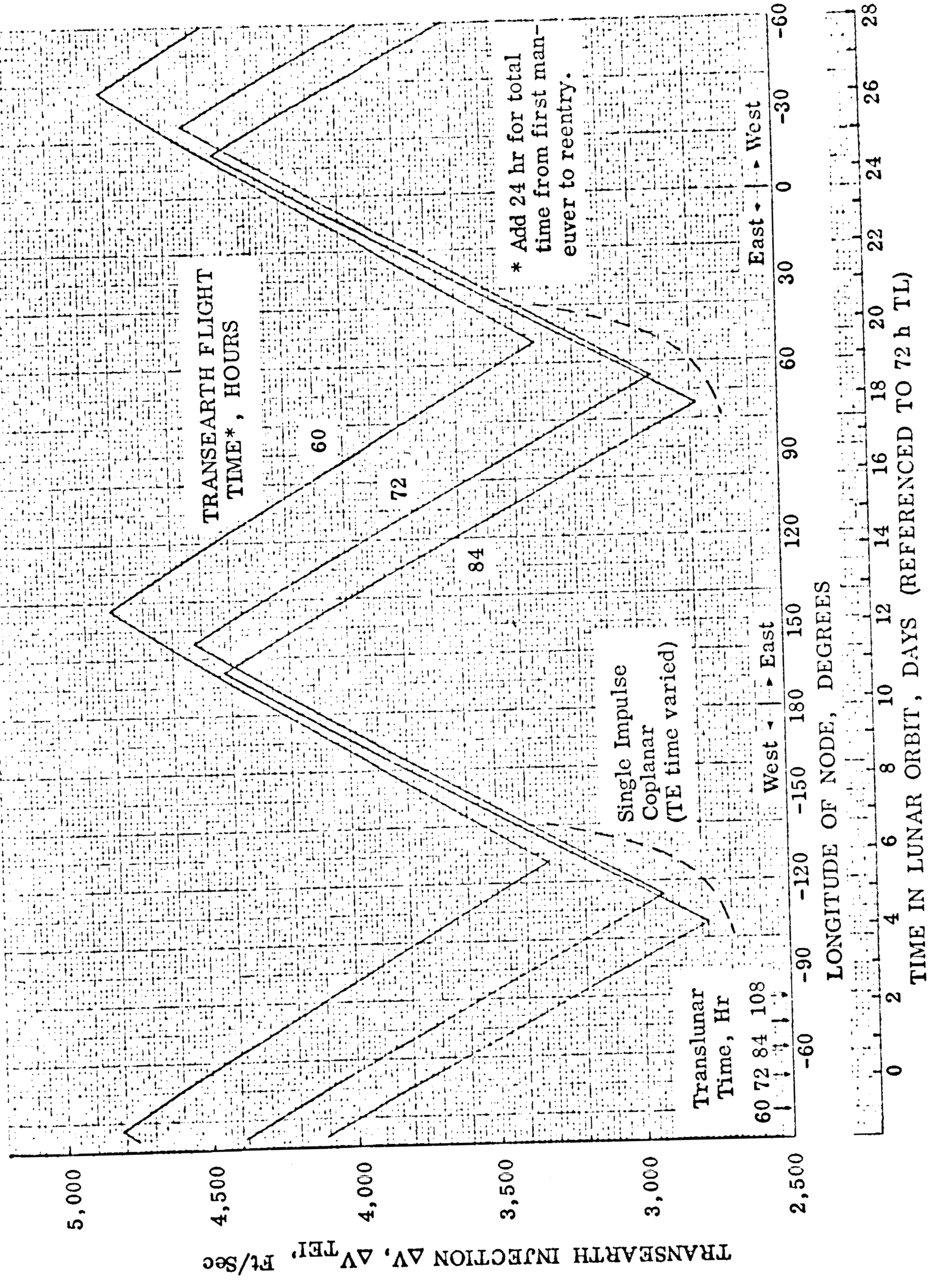


Figure 11

LOSM PAYLOAD CAPABILITY VS SM VELOCITY REQUIREMENTS AFTER PAYLOAD SEPARATION

CASE I-A & B (2300 LBS IN SECTOR 1) BLOCK II

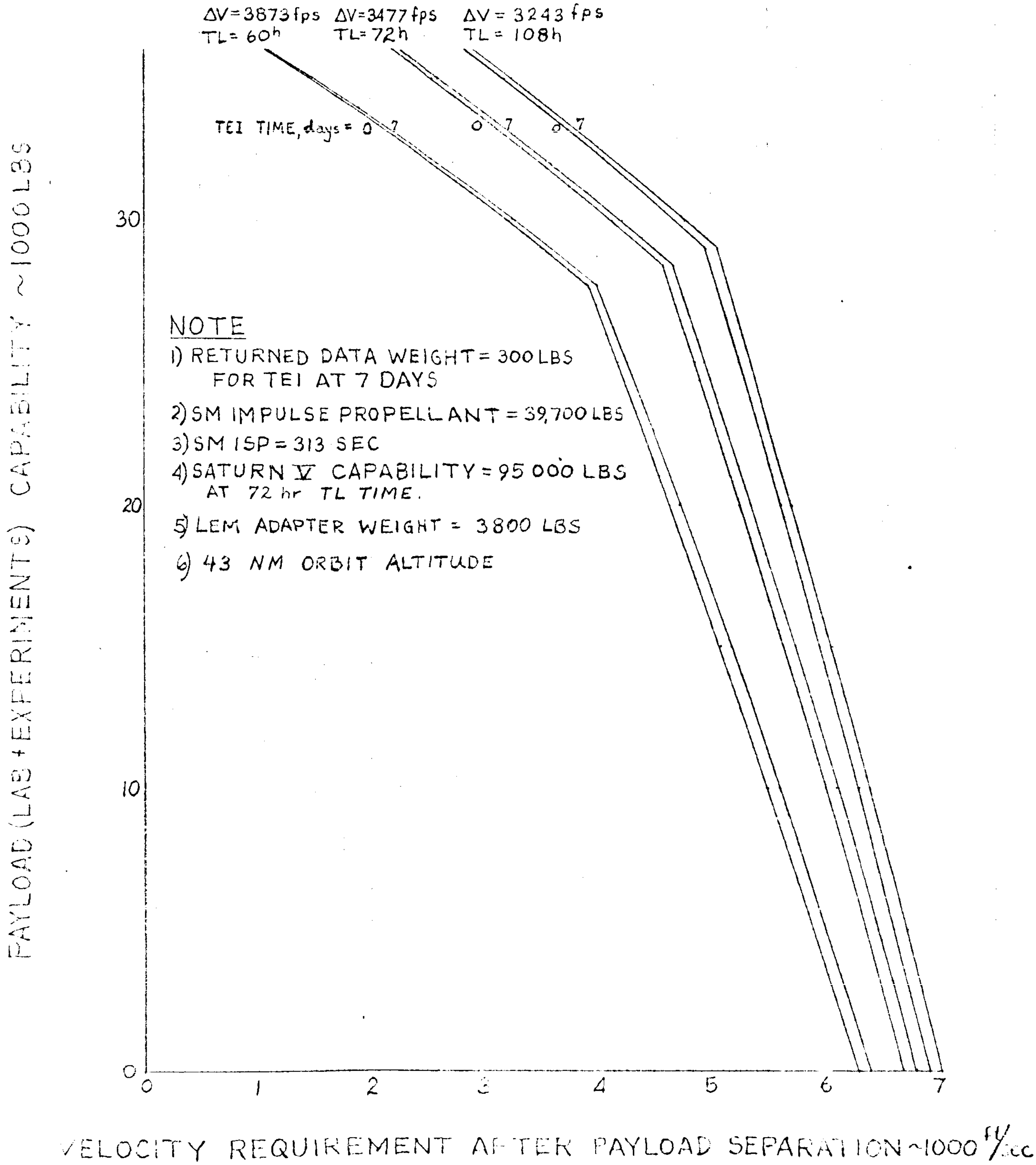


Figure 12

LOSM PAYLOAD CAPABILITY VS SM VELOCITY REQUIREMENTS AFTER PAYLOAD SEPARATION
CASE II-A&B (1500 LBS IN SECTOR I), BLOCK II

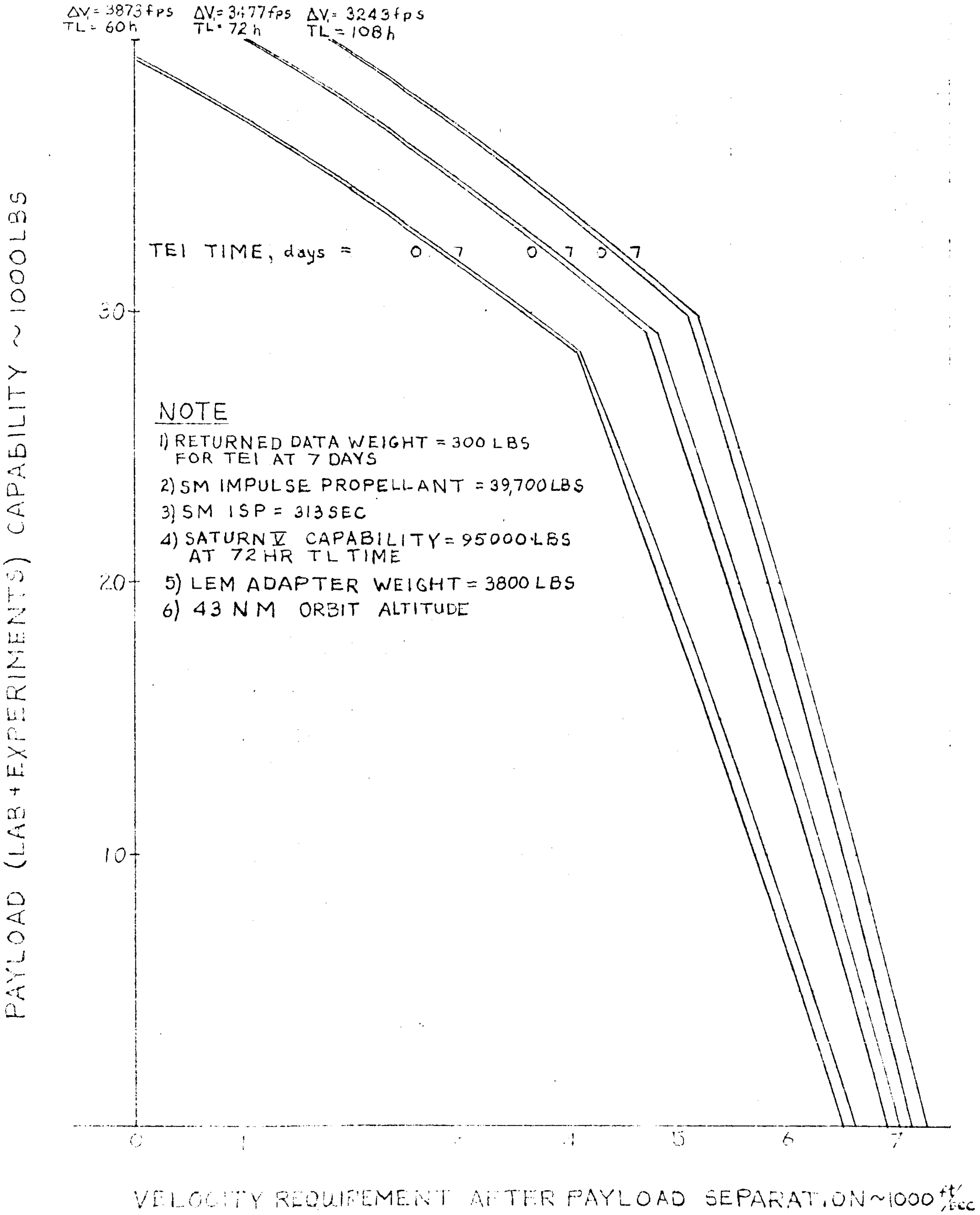


Figure 13

LOSM PAYLOAD CAPABILITY VS SM VELOCITY REQUIREMENTS AFTER PAYLOAD SEPARATION

CASE III-A (5000 LBS IN SECTOR 1) BLOCK II

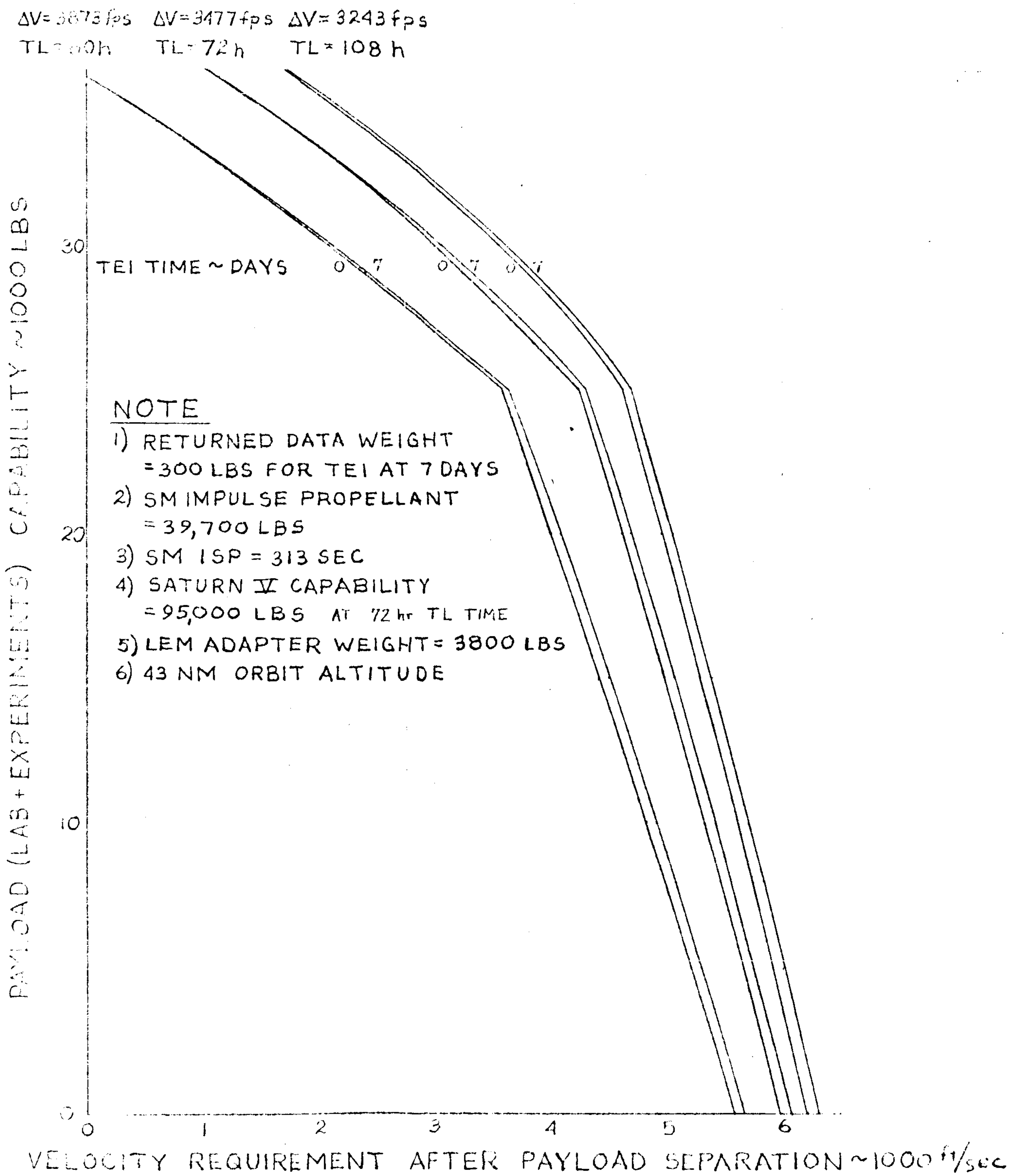


Figure 1[illegible]

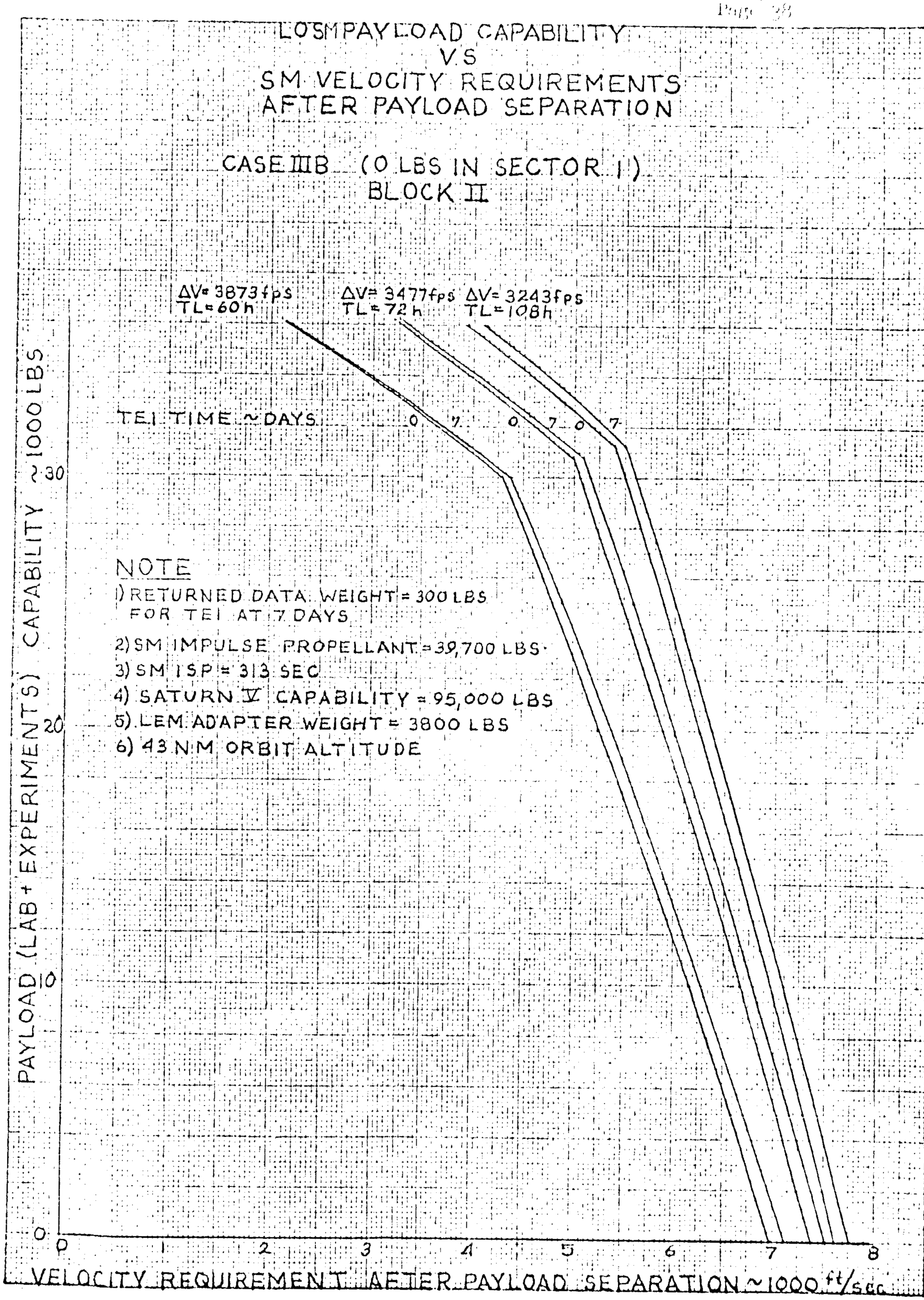

LOSM PAYLOAD CAPABILITY
VS
SM VELOCITY REQUIREMENTS
AFTER PAYLOAD SEPARATION
CASE IIIB (0 LBS IN SECTOR I)
BLOCK II
ΔV= 3873 fps
TL=60 h
ΔV= 3477 fps
TL=72 h
ΔV= 3243 fps
TL=108 h
TEI TIME ~ DAYS
0 7 0 7 0 7
NOTE
1) RETURNED DATA WEIGHT = 300 LBS FOR TEI AT 7 DAYS
2) SM IMPULSE PROPELLANT = 39,700 LBS
3) SM ISP = 313 SEC
4) SATURN V CAPABILITY = 95,000 LBS
5) LEM ADAPTER WEIGHT = 3800 LBS
6) 43 NM ORBIT ALTITUDE
PAYLOAD (LAB + EXPERIMENTS) CAPABILITY ~1000 LBS
0
10
20
30
0 1 2 3 4 5 6 7 8
VELOCITY REQUIREMENT AFTER PAYLOAD SEPARATION ~1000 ft/sec

Figure 15