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A STUDY OF DIGITAL TECHNIQUES FOR SIGNAL PROCESSING

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Summary

Work during the initial half of the grant period centered on three areas in the general field of digital processing signals. These included

1. Recursive Techniques for Digital Signal Processing
2. Data smoothing and compression
3. Computer simulation of low error rate communication systems.

In addition, an intensive effort was begun in the general area of digital processing for adaptation of communication systems.

The work on recursive techniques has concentrated on extending in two directions recent prior work on the detection of binary signals in additive noise[1]:

a) the inclusion of colored noise with numerator dynamics into the previous digital formulation,

b) the effect of approximating necessary differential operations by sample differences. We report here in detail only on topic a). The work on topic b) will be reported on in detail in the next report.

In the recursive technique the test statistic on which signal decisions are based is generated recursively by means of a first-order difference equation. This results in minimizing the memory requirements and programming effort. Furthermore, the highest matrix inversion is dependent only on the statistics of the noise and not on the number of samples taken. This simplification is made possible by assuming that the additive noise is generated as the solution of a linear differential equation driven by white noise. In the prior work referenced, denominator dynamics only are assumed for the differential equation. The numerator dynamics mentioned above refer to this linear differential

equation associated with the noise.

The work on data smoothing and compression has also been concentrated in two areas. The first involves an often-neglected, although highly significant, problem, that of the appropriate design of a buffer at the transmitter to handle the adaptive nature of the smoothed data flow. In most data compression schemes successive groups of incoming data samples are used to predict the following samples to come. If the error between predicted and actual sample values is below a specified threshold, no data is sent. Hopefully this results in significant compression of the data rate. Various compressive schemes used in practice include zero-order predictors, interpolators, etc. The problem under study here is that of controlling the error threshold adaptively to satisfy the often conflicting requirements of maximum compression, tolerable buffer storage size, and minimal mean-squared error between the discrete input data and the reconstructed compressed discrete output data at the receiver. An iterative dynamic programming algorithm[2] has been adopted here for obtaining the optimum controller.

As the second area of work on data compression we have attempted to initiate a comparative study of the various types of compressive schemes used in practice. The interpolators and zero-order predictors noted above are usually quite difficult to analyze theoretically, so that they are usually designed on an intuitive and cut-and-try basis. Some limited amount of computer simulation in an attempt to evaluate the performance of various compressors has been reported on in the literature, but this doesn't provide much insight into compressor operation. The work described below in the body of the report represents a first attempt to gain some insight into the performance of a few data compression schemes.

The work on computer simulation of low error rate digital communication systems, begun under a recently-concluded NSF grant, has successfully demonstrated that computer running time in the simulation process may be decreased at least an order of magnitude by using the methods of Extremal Statistics[3] to estimate low probabilities of error. This work was reported on at the recent 1968 Spring Joint Computer Conference[4].

The work begun just recently on digital processing in adaptive communications has focussed on adaptive equalizers for time-varying random channels. Most work done in this area in the past few years has concentrated on the use of tapped delay line structures for equalization of telephone channels. In our projected activity we are interested in much more general applications - to time-varying channels, as noted; to antenna array processors; to digital processing jointly at transmitter and receiver, etc. We are also stressing the digital processing aspect, using algorithmic formulations of the problem, and employing more rapidly converging search procedures.

Details of this past activity appear in the sections following.

1. <u>Recursive Techniques for Digital Signal Processing</u>

In a recent paper[1] Pickholtz and Boorstyn described a recursive approach to signal detection. The scheme was based on the following. The received signal was converted into a vector Markov process which was then sampled. The recursive structure of the digital processor followed readily. Of concern here are two aspects of this problem. First, in order to form a vector Markov process derivatives of the incoming signal are usually required. Investigations have been conducted into replacing these differentiation operations with approximating digital operations, such as differences. These studies, including simulation results, indicate that it is possible to replace derivatives with differences without adversely affecting performance. Details will appear in the next report.

Secondly, the previous paper considered a special type of noise - that generated by a linear differential equation driven by white noise. A more general noise description would include numerator dynamics. Work has been initiated extending the recursive approach in this direction.

The essential part of the recursive receiver is to convert the incoming signal plus noise $[r(t) = s(t) + y(t)]$ into a vector Markov process in such a manner that information is not destroyed. If this is done by a linear processor then the output of this device is $\underline{\rho}(t) = \underline{\sigma}(t) + \underline{\eta}(t)$ where the noise component $\underline{\eta}(t)$ is to be Markov. Furthermore we insist that r(t) be recoverable from $\underline{\rho}(t)$. Because of the linearity we need only consider the noise term. In the original work $\underline{\eta}(t)$ consisted of the derivatives of y(t) as well as y(t) itself and satisfied both of the above requirements.

We now consider y(t) to be generated by the following differential equation

$$\frac{dy^n}{dt^n} + \sum_{k=0}^{n-1} \alpha_K(t) \frac{dy^k}{dt^k} = \sum_{\ell=0}^{n-1} \beta_K(t) \frac{dw^\ell}{dt^\ell}$$

where w(t) is white Gaussian noise. It is possible to find a state vector $\underline{x}(t)$ for this system such that the first component $x_1(t) = y(t)$. This vector is the solution of

$$\dot{\underline{x}}(t) = A(t)\, \underline{x}(t) + \underline{b}(t)\, w(t)$$

$$y(t) = \underline{c}^T\, \underline{x}(t)$$

where $\underline{c} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

Although $\underline{x}(t)$ is Markov it cannot be obtained from $y(t)$ alone - either $w(t)$ or $\underline{x}(t_0)$ is needed in addition (neither are available). We consider next the best mean-square estimate of $\underline{x}(t)$ given the input $y(s)$, $s \leq t$. Thus

$$\hat{\underline{x}}(t) = E\left[\underline{x}(t) \mid y(s),\ s \leq t\right].$$

Since $\hat{x}_1(t) = y(t)$, $y(t)$ can be recovered from $\hat{\underline{x}}(t)$ - it is reversible. Furthermore because of the Gaussian assumption $\hat{\underline{x}}(t)$ is obtained by a linear operation on $y(t)$. Finally we shall show that $\hat{\underline{x}}(t)$ is Markov and thus letting $\underline{\eta}(t) = \hat{\underline{x}}(t)$ satisfies our requirements.

Proof: For some fixed T let $\underline{z}(t) = E\left[\underline{x}(T) / y(s),\ s \leq t\right]$. $\underline{z}(t)$ is a Martingale. $\underline{x}(t)$ can be written in terms of the state transition matrix $\Phi(t,u)$.

$$\underline{x}(T) = \Phi(T,t)\ \underline{x}(t) + \int_t^T \Phi(T,u)\ \underline{b}(u)\ w(u)\ du.$$

Then $\underline{z}(t) = \Phi(T,t)\ \hat{\underline{x}}(t) + \int_t^T \Phi(T,u)\ \underline{b}(u)\ E\left[w(u) \mid y(s),\ s \leq t\right] du.$

But the last term is zero since $w(t)$ is white and $u > t$. Thus $\underline{z}(t) = \Phi(T,t)\ \hat{\underline{x}}(t)$ or $\hat{\underline{x}}(t) = \Phi^{-1}(T,t)\ \underline{z}(t)$. To show Markovity consider

$$E\left[\hat{\underline{x}}(t) \mid \underline{x}(r),\ r \leq s\right] = E\left[\hat{\underline{x}}(t) \mid \underline{z}(r),\ r \leq s\right]$$

$$= \Phi^{-1}(T,t)\ E\left[\underline{z}(t) \mid \underline{z}(r),\ r \leq s\right] = \Phi^{-1}(T,t)\ \underline{z}(s)$$

because $\underline{z}(t)$ is a Martingale.

Next $E\left[\hat{\underline{x}}(t) \mid \underline{x}(s)\right] = E\left[\hat{\underline{x}}(t) \mid \underline{z}(s)\right] = \Phi^{-1}(T,t)\ E\left[\underline{z}(t) \mid \underline{z}(s)\right] = \Phi^{-1}(T,t)\ \underline{z}(s).$

Thus $E\left[\hat{\underline{x}}(t) / \underline{x}(r),\ r \leq s\right] = E\left[\hat{\underline{x}}(t) / \underline{z}(s)\right]$, and $\hat{\underline{x}}(t)$ since Gaussian is Markov.

The recursive receiver will now consist of a device that estimates $\underline{x}(t)$ given y(t) [this will actually operate on r(t) to yield $\underline{\rho}(t)$] - some form of a Kalman filter. The remainder of the receiver will parallel the original work. This study is continuing.

2. <u>Data Smoothing and Compression</u>

a. Optimal Adaptive Control for Data Compression Systems.

The object here is to determine an optimal controller to minimize the mean squared error between discrete input data, x_n, and reconstructed compressed discrete output data, y_n. We plan to evaluate the optimum controller solution and minimum normalized rms error for several data models and compressor algorithms. The present plan is to use a uniformly distributed independent data model and the uniformly distributed first order Markov data model. The compressor algorithms under consideration are both the zero and first order predictors and interpolators. Tables or curves will be obtained from computer runs and will list the compressor aperture, K, vs. buffer fill or state, S, and the minimum normalized rms error for several values of the following parameters: b, the number of input amplitude bits, L, the buffer length, and C, the transmission ratio. With these data, the designer will be able to select suitable parameters to satisfy his data compression and rms error requirements.

As a check on the optimal controller solution, the data models and data compression system using the corresponding optimal controller solution will be simulated on a computer to measure the actual normalized rms error. As a test of the sensitivity of the optimal controller solution to the input data statistics, the first order Markov data will be fed into systems optimized for independent data, and independent data will be fed into systems optimized for first order Markov data. Finally, real telemetry data will be obtained, if possible, and fed into these optimized systems to determine the practical use of the chosen data models.

Progress thus far consists of the following: The controller-buffer system has been modeled as a discrete Markov process and a method of solution adopted using an iterative dynamic programming algorithm based on the work of Howard[2]. Once the statistics of the input process x_n, n=0,1,... and the compressor algorithm have been specified, an optimum controller can be determined using this technique.

For the case of uniformly distributed independent input data and the zero order prediction compressor algorithm, the problem of determining the optimum controller has been solved. The solution is in an iterative form and is best computed on a general purpose digital computer for all possible values of interest of the various parameters noted above. The iterative solution has been programmed in FORTRAN on a digital computer and is presently being debugged. The status of the programming is: writing in FORTRAN completed; program cards punched, verified and listed; diagnostic errors are being eliminated from initial compilations. In order to check the program results, one test case for the specific values of b=1, L=4, and C=2 was computed by hand yielding a mean squared error, $\overline{e^2}$=1/26, and the following controller rules:

S(Buffer State)	K(Aperture)	
0	0	b=1
1	0	L=4
2	0	C=2
3	1	$\overline{e^2}$=1/26

For the case of uniformly distributed first order Markov input data and the zero order prediction compression algorithm, the solution of the problem of determining the optimum controller is presently being attempted. The complexity of the mathematics in the Markov case is many times greater than that of the independent case.

b. Comparison of data compression schemes.

It is quite apparent that if signals to be transmitted were stationary and perfectly bandlimited, there would be no data compression problem. One would sample at the Nyquist rate and transmit these samples, or coded versions of them. No further data compression would be possible. Real signals do not behave this way, however. They are generally not bandlimited and their statistics are either not known or varying in a nonstationary manner. How does one then perform data compression? One good engineering technique is to first oversample to some extent and then use various types of predictive techniques to reduce the redundancy. Such methods are now well known and have been summarized in the literature.[5]

But these various cut-and-try procedures offer no real insight into the problem of data compression. In an attempt to analyze these various schemes and others that may be developed we have chosen to pick some signal models that deviate from the perfectly bandlimited one, providing some complexity of structure to make them meaningful yet sufficiently simple to enable analysis to be carried out. The two simplest models reported on here are a stationary gaussian random process, an a discrete Nth order Markov process. For both classes of models and attempt is made to relate the mean-squared reconstruction error to sampling interval, for various compressor schemes.

(1) Gaussian process x(t). Here we assume a power spectral density

$$S_x(\omega) = \frac{2}{1+\omega^2}$$

and autocorrelation function

$$R_x(t) = e^{-|t|}$$

Assume samples are taken uniformly every T sec. Several methods of reconstructing the original signal have been compared. These include various sampling techniques as well as predictive techniques.

(a) sin x/x reconstruction. Here the transmitted samples are passed through a lowpass filter with bandwidth B = 1/2T. It is then found that the mean-squared reconstruction error is given by

$$\overline{e^2} = 2\left(1 - \frac{2}{\pi} \tan^{-1} \frac{\pi}{T}\right)$$

(b) pre-filtering to $B = \frac{1}{2T}$ prior to sampling, then the same reconstruction method as above. The mean-squared error is then found to be one-half that above:

$$\overline{e^2} = \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\pi}{T}\right), \text{ with pre-filtering.}$$

(c) An alternate reconstruction technique, due to Tufts[6], that minimizes $\overline{e^2}$ where pre-filtering is not used. For this case

$$\overline{e^2} = \coth T - \frac{1}{T}$$

(d) A finite Karhunen-Loève representation of the random process x(t). Here the representation x(t) is given by

$$\hat{x}(t) = \sum_{n=1}^{N} a_n \varphi_n(t) \qquad 0 \le t \le \mathcal{T}$$

where a $\mathcal{T}$- second piece of x(t) is taken and the $\varphi_n(t)$'s are fixed orthonormal functions. The N discrete numbers a_n are the numbers to be transmitted and are random variables. The orthogonal functions $\varphi_n(t)$ are chosen to minimize the mean-squared error power

$$\overline{e^2} = \frac{1}{\tau} E\left\{\int_0^{\tau} [\hat{x}(t) - x(t)]^2 dt\right\}$$

The resultant expression for $\overline{e^2}$ vs. τ/N (time between samples for this technique) is obtained as the solution of transcendental equations, and is shown plotted in Fig. 1 for $\tau = 4$ and 10 sec. Also plotted for comparison are the sampling techniques mentioned above.

Note that for this particular power spectrum the pre-filtered, sin x/x reconstruction technique is the best of the four shown, but is very closely followed by the Karhunen-Loève technique.

How do these techniques compare with the ad hoc compressive schemes that are used in practice? As noted earlier these are in general hard to analyze, but Ehrman[7] has obtained the approximate theoretical mean time between transmitted samples for three common techniques[5] - the floating aperture zero-order predictor, the zero-order interpolator, and the fan (first order) interpolator, assuming a gaussian Markov signal source as done here. His results, valid only for small sampling intervals on the average, and adapted to the example taken here, are

$$\overline{e^2} = 0.67\ E(T) \text{ floating aperture predictor}$$

$$\overline{e^2} = 0.33\ E(T), \text{ zero-order predictor,}$$

$$\overline{e^2} = 0.28\ E(T), \text{ fan interpolator.}$$

In all three cases, $E(T) \ll 1$, the half aperture widths Δx are assumed small compared to the standard deviation of the signal process (which is unity here), and the reconstruction error is assumed uniformly distributed between $\pm \Delta x$.

As a comparison with the sampling and Karhunen-Loeve techniques described above, we may assume T and $\tau/N \ll 1$ in the equations given earlier. It is then found that

$\overline{e^2} = \frac{2}{\pi^2} T = 0.203\, T$, sampling with pre-filtering and $\frac{\sin x}{x}$ reconstruction.

$\overline{e^2} = 0.405\, T$, sampling without pre-filtering and with $\frac{\sin x}{x}$ reconstruction,

$\overline{e^2} = 0.333\, T$, sampling without pre-filtering and using Tuft's optimum reconstruction,

$\overline{e^2} \doteq \frac{2}{\pi^2} \frac{\tau}{N} = 0.203 \frac{\tau}{N}$, Karhunen-Loève expansion.

Note that sampling with pre-filtering and the Karhunen-Loève expansion give almost identical results, and that they are better than the predictor and interpolators. It must also be pointed out the the latter techniques require the transmission of timing information, so that the advantage of sampling is even greater than indicated above. We do not mean to imply, however, that the aperture techniques are inferior; there may be other signals on which they perform better than sampling. The adaptivity and relative simplicity of the aperture techniques are factors in their favor.

(2) <u>Discrete state Markov source.</u>

An N-state Markov chain has been assumed, and both the zero-order predictor and the zero-order interpolator have been considered. Two cases have been analyzed: first, with the aperture width less than the spacing between adjacent source levels, so that there is no reconstruction error; second, with the aperture greater than the source level spacing, with ± 1 level reconstruction error allowed.

The compressibility of the source symbols, as measured by the entropy in bits/source symbols, is then found for the zero-order predictor to be given by

$$H_{zop} = \frac{H_x + H_R}{1+E(n)} \quad \text{bits/source}$$

where

H_x is the information content of the transmitted sample values,

H_R is the information content of the run length between transmitted samples,

and $E(n)$ is the average run length.

Average run lengths for a zero-order predictor with no allowable error operating on a discrete state Markov source have previously been computed by Stanley and Liu[8]. Using their results, and assuming as an example, a particular 15 state Markov chain possessing a uniform stationary distribution, and characterized by a relatively large probability of remaining in the same state or going to an adjacent state, the source entropy is calculated to be

2.73 bits/symbol.

For a zero-order predictor, with no reconstruction error, we also find

$$E(n) = 1.05 \text{ symbols}$$

$$H_x = \log_2 15 = 3.9/\text{bits}$$

$$H_R = 2.04 \text{ bits}$$

$$H_{zop} = 2.91 \text{ bits/source symbol.}$$

Thus, a zero-order predictor used with efficient coding can achieve a transmission rate fairly close to the entropy of this particular source.

This method has been extended to the case where the aperture is greater than the source level spacing. For example, suppose ± 1 level reconstruction error is allowed. The probability of a run of length n within the aperture about a level is easily calculated. Once the run length proabilities have been found the entropy may be computed as above. It has also been shown that the run length probabilities for a zero-order interpolator may be written in terms of powers of sub matrices of the Markov transition matrix. For the example considered above, the expected run lengths following one of the "interval" states of the source for the zero-order predictor and interpolator with ± 1 level error are

$$E(n)_{zop} = 2.10 \text{ symbols},$$

$$E(n)_{zoi} = 2.62 \text{ symbols}.$$

While the work described above has barely scratched the surface of the problem, it at least provides a little insight into the performance of the simple, data compression techniques.

3. Computer simulation of low error rate digital communication systems

The average error rate serves as a very common measure of performance for digital communication systems, with a probability of error of less than 10^{-5} a desirable goal in most system design. With such low error rates, however, the usual Monte Carlo simulation techniques require very large numbers of simulation runs, a costly procedure. Using the methods of extremal statistics[3] to estimate low probabilities of error we have been able to reduce the usual number of simulation runs by at least an order of magnitude, a highly encouraging result.

The field of extremal statistics is concerned with the occurrence of rare events, exactly the problem encountered in simulating low error rate communication systems. Previous applications to communications have emphasized the analysis of data obtained from existing systems[9,10]. Thus, use has been made, in analyzing these data, of special plotting paper developed by Gumbel[3]. Our approach has differed in assuming from the beginning that all calculations were to be made by a high speed computer, that time was of the essence, and that we were interested in applying the theory to the simulation of broad classes of systems.

Extremal statistics relies on the fact that many common probability distribution functions are asymptotically exponential: the probability of exceeding a specified value out on the tail of the distribution is of the form $P(x>x_o) = \frac{1}{n} e^{-\alpha_n(x_o-u_n)}$. The two parameters α_n and u_n depend on the underlying distribution; n represents the number of samples available of the random variable under study, and is assumed very large. The two parameters may also be shown to be closely related to the asymptotic (large numbers of samples) maxima of the random variable under study.

If one is now interested in estimating small probabilities of error, say of the order of 10^{-3} or 10^{-4}, it may be possible instead to first estimate much higher probabilities, say 10^{-2}. If the exponential approximation is valid one should then be able to extrapolate down to the desired probability. Instead of the usual number of samples required to estimate a small probability of error, one can work with a much smaller number. One major difficulty, however, is that with the underlying probability density function unknown (as it generally is in complex systems to be simulated), α_n and u_n are unknown and must be estimated.

In the computer simulations carried out the results were nonetheless quite encouraging: including computer time necessary to estimate the parameters an order of magnitude in computer time was saved over the normal estimation procedures. In these computer experiments known statistics were first generated to determine the applicability of the extremal approach, including estimation of the parameter. These statistics included the gaussian (normal), Rayleigh, and exponential density functions. Various methods of estimating α_n and u_n, including maximum likelihood iteration, were compared. Twenty samples were found to be sufficient to estimate α_n and u_n: the estimates of α came within 10% of the known values, while those of u were within 3%, well within the confidence limits calculated theoretically. Using n=500 samples, the total number of samples required was $500 \times 20 = 10^4$. With this number we were able to successfully extrapolate down to probabilities of error of 10^{-4} and even lower (with rapidly decreasing accuracy of course). This compares with previous estimation of probabilities of 10^{-3} using 10^4 samples.

The methods studied were then applied to the simulation of two digital feedback communication systems of considerable current interest. One is a binary signalling system, using sequential decision feedback. The other is an M-ary PAM system, with information feedback. Both systems may have potential usefulness in space-ground communications. Details of the feedback schemes are included in the work reported on at the 1968 Spring Joint Computer Conference[4]. In both cases simulation results agreed well with calculated approximate system performances.

Further study beyond the work reported on has focussed on the tradeoff possible between n, the number of samples used, and N, the number needed to estimate α_n and u_n. A minimal total number of samples, nN, may readily be seen to exist for given underlying probability distribution, confidence limits, and the range of probability of error to be estimated. For decreasing n allows better estimation of α_n and u_n as N increases, but the range of extrapolation over which one would expect the asymptotic exponential approximation to hold decreases. Similarly, as n increases the range of the desired probability of error is approached more closely, but α_n and u_n are less accurately estimated.

REFERENCES

1. R. Pickholtz & R.R. Boorstyn, A Recursive Approach to Signal Detection, IEEE Trans. on Info. Theory, Vol. IT-14, no. 3, May, 1968, pp. 445-450.
2. R.A. Howard, "Dynamic Programming & Markov Processes", MIT Press, Cambridge, Mass., 1960.
3. E.J. Gumbel, "Statistics of Extremes", Columbia University Press, New York, 1958.
4. M. Schwartz & S.H. Richman, Extremal Statistics in Computer Simulation of Digital Communication Systems, Proceedings, 1968 Spring Joint Computer Conference, Atlantic City, N.J., May, 1968, pp. 483-489.
5. Special Issue on Redundancy Reduction, Proc. IEEE, Vol. 55, no. 3, March, 1967.
6. D.W. Tufts, "A Sampling Theorem for Stationary Random Processes", MIT Research Lab. of Electronics, Cambridge, Mass., Quarterly Progress Report. 53, pp. 87-93, April 15, 1959.
7. L. Ehrman, "Analysis of Some Redundancy Removal Bandwidth Compression Techniques", Proc. IEEE, Vol. 55, no. 3, pp. 278-287, March, 1967.
8. T.P. Stanley & B. Liu, "Data Compression for Discrete Markov Signals", Proc. Nat'l Electronics Conference, Vol. XXIII, pp. 534-539, 1967.
9. E.C. Posner, The Application of Extreme-Value Theory to Error-free Communications, Technometrics, Vol. 7, no. 4, November, 1965, pp. 517-529.
10. J.C. Ashlock & E.C. Posner, The Application of the Statistical Theory of Extreme Values to Space craft Receivers, National Telemetering Conference, May, 1966.

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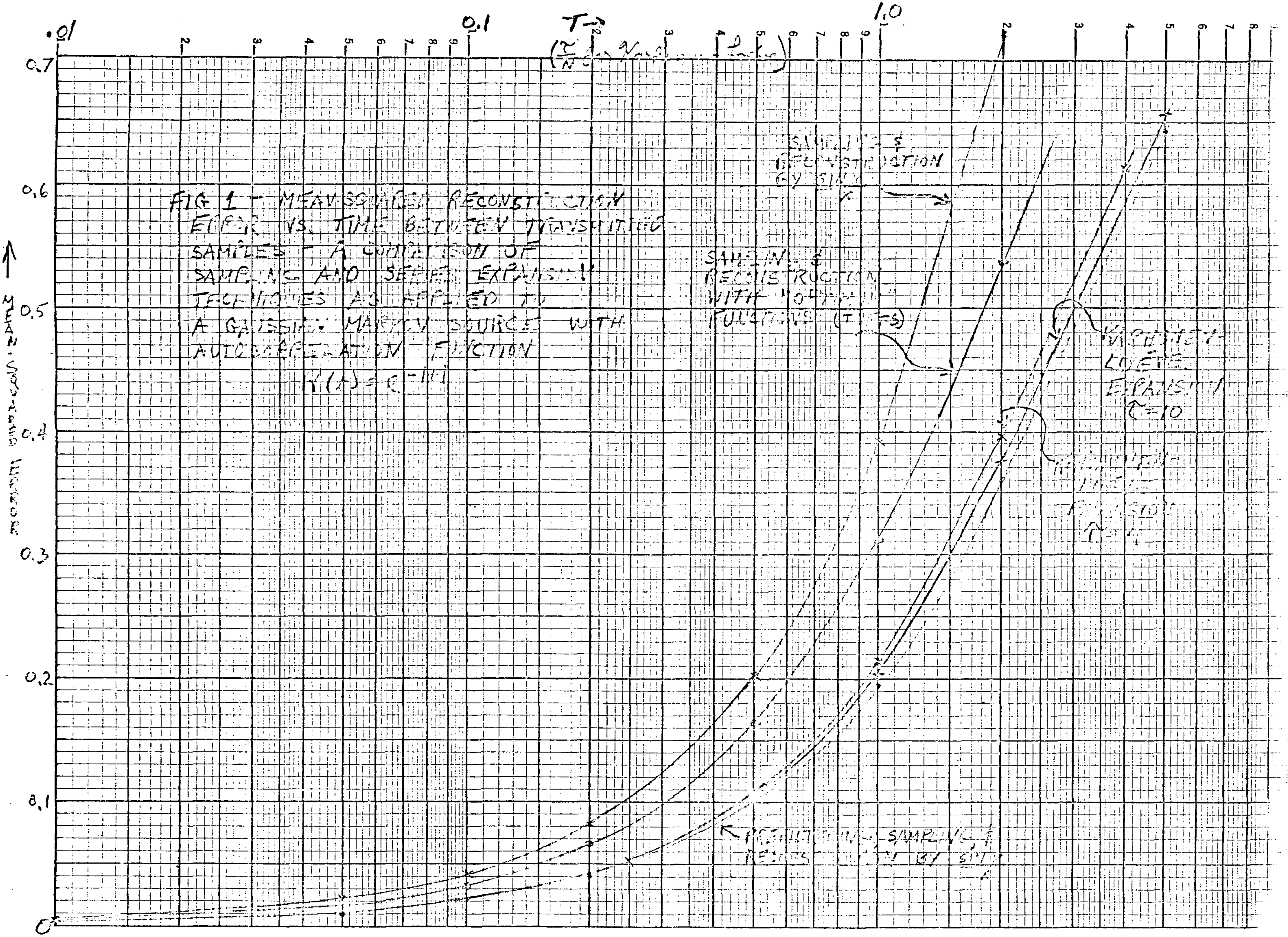

FIG 1 — MEAN SQUARED RECONSTRUCTION ERROR VS. TIME BETWEEN TRANSMITTED SAMPLES — A COMPARISON OF SAMPLING AND SERIES EXPANSION TECHNIQUES AS APPLIED TO A GAUSSIAN MARKOV SOURCE WITH AUTOCORRELATION FUNCTION $R(\tau) = e^{-|\tau|}$