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# GEOS 1 OBSERVATIONS AT MALVERN, ENGLAND

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## ABSTRACT

The Smithsonian Astrophysical Observatory is collaborating with other agencies active in optical satellite tracking. Results from Geos 1 flash observations obtained by these agencies have been implemented in current SAO research programs on satellite geodesy. Proper implementation of these observations requires a detailed description of observation techniques and data processing applied by these agencies.

This report deals with the methods used by the Royal Radar Establishment, Great Malvern, England. Accuracy estimates and some geodetic results are also presented.

## RÉSUMÉ

L'Observatoire Astrophysique du Smithsonian (SAO) travaille en collaboration avec d'autres organismes qui s'occupent du repérage des satellites par moyens optiques. Les résultats des observations des éclairs lumineux de Geos 1, obtenus par ces organismes, ont été appliqués aux programmes de recherche que le SAO poursuit dans le domaine de la géodésie par satellite. La mise en application appropriée de ces résultats nécessite une description détaillée des techniques d'observation et des procédés d'analyse de données, employés par ces organismes.

Le rapport ci-dessous traite des méthodes utilisées par le Royal Radar Establishment, Great Malvern, Grande-Bretagne. Des estimations concernant la précision, ainsi que certains résultats géodésiques, sont inclus.

# КОНСПЕКТ

Смитсониан Астрофизическая Обсерватория сотрудничает с другими учреждениями активными в оптическом слежении за спутниками. Результаты наблюдений вспышек Геоса 1 полученные этими учреждениями были применены к текущим программам исследований в области спутниковой геодезии в САО. Надлежащее применение этих наблюдений требует детального описания методов наблюдений и обработки данных употребляемых этими учреждениями.

В этом докладе рассматриваются методы употребляемые Королевским Радиолокационным Учреждением в Грэйт Малверн, Англии. Также приведены оценки точности и некоторые геодезические результаты.

# GEOS 1 OBSERVATIONS AT MALVERN, ENGLAND

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## 1. INTRODUCTION

Since the launching of the first artificial earth satellite in 1957, the Smithsonian Astrophysical Observatory (SAO) has published many reports on orbital theory, satellite geodesy, and related subjects. A milestone was reached with the publication of Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth (Lundquist and Veis, 1966), which gives a detailed account of all the methods applied and results found by 1966 in SAO's Satellite Geodesy Program. The observational data from which these geodetic parameters have been determined came solely from Baker-Nunn cameras.

The forthcoming improvement of the 1966 Standard Earth will be based not only upon new observational data from the Baker-Nunn cameras, but also upon observations by other tracking systems, such as Doppler and laser. In addition, observations from a limited number of optical tracking stations not strictly belonging to the SAO network will be incorporated into the new Standard Earth determination.

One of these optical stations is located at the Royal Radar Establishment (RRE) in Great Malvern, England. The Malvern station has already been closely cooperating with SAO for several years by making simultaneous observations with the Baker-Nunn cameras in Spain, Norway, and Iran.

This work was supported in part by Contract NSR 09-015-018 and Grant NsG 87-60 from the National Aeronautics and Space Administration.

Malvern is also one of the international participants in the U. S. National Geodetic Satellite Program (NGSP), which is operated under the auspices of NASA.

The Geos 1 flash observations resulting from this participation are of importance to SAO for two reasons: they will serve very well for the new determination of the Standard Earth parameters, and will connect the European Datum to the World Net. The latter determination is one of the scientific programs conducted at SAO.

The implementation of these observations in the SAO research programs requires a full description of the observing techniques and data-processing methods at Malvern, which are presented in this report. Joseph Hewitt has been so kind as to write the sections pertaining to his camera and his procedures in data processing.

It has been extremely useful to obtain this extensive information first hand, instead of having an interpretation by our own staff members. At the same time, it has opened the way for issuing publications jointly with scientists who are closely collaborating but not directly associated with SAO. This is the first example of such a jointly prepared report. Several others, also dealing with procedures at foreign optical satellite-tracking stations, are in preparation.

# 2. CAMERA AND TIMING EQUIPMENT

## 2.1 Camera

Satellite observations at Malvern, England, are made with a field-flattened f1 Schmidt camera of 610-mm aperture (Hewitt, 1965, p. 10). Since the camera has no facilities for tracking the satellite, it is used in the fixed mode only. The optical system of 610-mm focal length consists of an aspheric plate and a spherical mirror, as in the normal Schmidt system, with the addition of a three-element field-flattening lens mounted in front of the photographic plate. The field is of 10° diameter, but the measurements of the images are restricted to a field of 8° diameter. The photographic emulsion is coated on high-quality plate glass, 6 mm thick; the glass plates are approximately 200 × 150 mm to allow for the recording of fiducial marks.

The camera has two shutters. A large 5-bladed iris capping shutter, mounted immediately in front of the aspheric plate, is carried on a separate turntable concentric with that carrying the optical system but isolated from it. This shutter is used to make exposures of the star trails and to code the breaks in the satellite track when the camera is recording passive satellites. The shutter opens (or closes) in approximately 100 msec. The time at which a star exposure is made is obtained by recording both the time when the shutter is near the middle of the opening cycle and when it is in the corresponding position in the closing cycle; the mean of these gives the time of the star exposure. The duration of the exposure can be set to 0.3, 0.6, or 1.2 sec; the choice depends upon the speed of the photographic emulsion being used. The shutter is programmed to produce one or two star images before the satellite pass, and one, two, or three images after the pass. The duration of the star exposure is the same for all the star images on one plate.

The second shutter is a rotating sector mounted so that the blade of the shutter passes through the gap between the field-flattening lens and the photographic plate. This shutter produces the breaks in the recorded track of a passive satellite. The speed of the shutter can be varied in discrete steps and, depending upon the velocity of the satellite, is set to produce breaks in the track approximately 100 $\mu$ long. The time is recorded every revolution of the shutter at the point where the blade is about to enter the field of the camera. This point is defined by a small phototransistor unit with a very narrow slit. Four fiducial marks are recorded on the photographic plate; two of these define the position of the leading edge of the sector shutter when the time is recorded. From the measurements of these two fiducial marks and those of a satellite break, an increment of time is computed and added to the corresponding recorded time. The calculation of the time increment assumes that the sector shutter rotates at a uniform velocity during any one rotation. Tests show that the rotation of the shutter has an r. m. s. variation of $\pm 0.05\%$ of the period; for example, at 1 rev/sec the variation is $\pm 0.5$ msec. As the shutter completely sweeps across the field of the camera in one-fourth a revolution, the actual error in the computed time interval due to the variation in the shutter speed is less than this.

The sector shutter is not used and does not rotate when observations of active satellites are being made.

## 2.2 Timing Equipment

The timing equipment is a quartz clock driven by a 1-MHz oscillator accurate to 2 parts in $10^9$. The time is displayed to 0.0001 sec in digital form by means of digitrons (Nixie tubes). The decimal seconds change only when a timing pulse is received; this pulse transfers the count to storage tubes and hence to the digitrons. The display is recorded photographically every time a pulse is received from the camera. To correlate the photographically recorded times with the breaks in the satellite track, the breaks are coded by means of the capping shutter and the times are recorded with an identical code.

The timing pulses are derived directly from the blades of the shutters, and errors in the recorded times are due to delays in the pulses between the camera and the timing equipment. These delays are approximately 25 μsec; no correction for them is applied. In the case of passive satellites, there is the additional error due to the variations in the speed of the sector shutter, as described above.

The clock is checked against the MSF 60-kHz time transmissions. Since this transmitter is within 80 miles of the Malvern camera site, the travel time of the signals can be calculated quite accurately. There is, however, an error in the setting of the signal when the time check is being carried out; this error is in the order of 0.1 to 0.2 msec. Taking all the errors into account, it is estimated that the times are determined to ±0.5 msec.

The internal accuracy of the camera system derived from the measurements of star images and breaks in the satellite track is 1 arcsec.

# 3. PLATE-MEASUREMENT AND REDUCTION PROCEDURES AT MALVERN

## 3.1 Plate-Measurement Procedure

The plates are measured at Malvern on a Zeiss 30 × 30 coordinate measuring machine. They are measured independently by two operators. Each operator makes two measurements of the star images, one direct and one reversed through 180° (reversal is by means of a prism on the measuring machine). The difference between the means of the two readings of each operator is used as an assessment of the quality of the star image. The observational equations in the calibration routine are weighted by use of a value for the weight derived from this difference. The measured star position is the average of all four readings.

The satellite images or breaks are measured in a similar way, but each operator now measures an image twice in the direct position and twice in the reverse position. The difference between the means of the four readings of each operator is again used to derive a weighting factor, but this is not used in the computation of the final directions from a single camera orientation. The measurement of the satellite image is the mean of all eight readings.

The means of the measurements of the star and satellite images are corrected for

1. The difference between the magnitudes of the x and y scales.
2. The nonrectangularity of the axes according to the expressions

$$x' = ax \quad ,$$

$$y' = y - bx \quad ,$$

where x, y are the actual measurements, $x'$, $y'$ are the true measurements, and a, b are constants predetermined from test measurements on the Zeiss comparator. Other comparator errors such as periodic errors in the micrometers and nonstraightness of the guides have been found to be insignificant.

### 3.2 Timing Corrections

The clock at the observing site is monitored against the MSF 60-kHz radio time signals; at least one time check is made each day. The modulated time signal (UTC) is transmitted only between 14.30 and 15.30 UT. Since May 1966 an interrupted carrier time signal has been transmitted during the remaining 23 hours. These interrupted carrier signals are now used in monitoring the clock, and checks are made during the period of a night's observation.

The times recorded from the camera-shutter pulses are corrected for the clock error. This error is determined by interpolation of the errors from the time checks made 1 or 2 days on either side of the observation. Any step change in the UTC signal is noted, and the necessary adjustment is made. The recorded time is also corrected for propagation and receiver delays.

The star times are further converted to UT1 by use of the provisional values for this correction issued by the Royal Greenwich Observatory, England.

In the absence of information, no corrections are made to the times of the flashes from an active satellite, Geos 1. With passive satellites, after the recorded time has been corrected for the clock error and propagation delays, the shutter-sweep correction described earlier is applied. The final corrected satellite time is in UTC. The correction for light time is discussed later.

### 3.3 Reduction of Star Positions

The Smithsonian Astrophysical Observatory Star Catalog is used for the star positions. Before March 1966, when the SAO catalog was received, three or four Geos 1 plates were reduced with star positions from the AGK 2 Catalog.

The catalog positions are reduced to the apparent position of date. Precession from the equinox of the catalog (1950.0) to the beginning of the nearest Besselian year is first applied by means of the rigorous trigonometric formulas and by use of the values of $\zeta_0$, z, and $\theta$ derived from the expressions, as given by Newcomb (Wilkins, 1961, p. 30):

$$\zeta_0 = (2304''.250 + 1''.396\ T_0)\ T + 0''.302\ T^2 + 0''.018\ T^3 ,$$

$$z = \zeta_0 + 0''.791\ T^2 ,$$

$$\theta = (2004''.682 - 0''.853\ T_0)\ T - 0''.426\ T^2 - 0''.042\ T^3 ,$$

where T and $T_0$ are measured in tropical centuries, the initial epoch is $1900.0 + T_0$, and the final epoch is $1900.0 + T_0 + T$. The apparent place at the the epoch of observation is then computed by use of the Independent Day Numbers taken from the Astronomical Ephemeris. Correction for proper motion is also made at this stage. Finally, the star positions are corrected for diurnal aberration given by

$$\alpha' = \alpha + 0''.32 \cos\phi \cos h \sec\delta \quad ,$$

$$\delta' = \delta + 0''.32 \cos\phi \sin h \sin\delta \quad ,$$

where $\alpha'$, $\delta'$ are the corrected and $\alpha$, $\delta$ are the uncorrected values of right ascension and declination, h is the hour angle, and $\phi$ is the latitude of the station.

## 3.4 Plate-Reduction Procedure

The plate is reduced using the projective relationships

$$\begin{bmatrix} x - x_p \\ y - y_p \\ c \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \end{bmatrix} ,$$

where x, y are the measured coordinates of the photographic image, $x_p$, $y_p$, c are the coordinates of the principal point of the camera, u, v, w are the coordinates of the object point corresponding to the measured image, and A is a $3 \times 3$ matrix, the elements of which are functions of the orientation of the camera (Schmid, 1953; Brown, 1957; and Currie, 1965). The effects of optical distortion are removed from the measured plate coordinates by introducing expressions for the radial distortion and for the off-center corrections (also called decentering distortion or tangential distortion) (Brown, 1964).

The radial distortion is expressed as

$$\delta_x = (x - x_p) \sum_{i=1}^{n} h_i \, r^{2i} ,$$

$$\delta_y = (y - y_p) \sum_{i=1}^{n} h_i \, r^{2i} ,$$

where $r^2 = (x - x_p)^2 + (y - y_p)^2$ and the $h_i$ are the distortion coefficients. The off-center corrections are

$$\Delta x = (-\sin \chi) \sum_{i=1}^{n} j_i \, r^{2i} ,$$

$$\Delta_y = (\cos\chi)\sum_{i=1}^{n} j_i\, r^{2i} \quad .$$

The projective relationship now becomes

$$\begin{bmatrix} x + \delta_x + \Delta_x - x_p \\ y + \delta_y + \Delta_y - y_p \\ c \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad .$$

It has been found that only two terms in h and two in j need be retained in the computation.

The coordinates of the stars used in the calculations are the standard coordinates $\xi$ and $\eta$ in a plane perpendicular to the optical axis of the camera and at unit distance from the projection center. The coordinates u, v, w in the projective equation are replaced, therefore, by $\xi$, $\eta$, and 1.

The computing program contains 11 parameters: the three angles of the camera orientation, the three coordinates of the camera principal point, two distortion coefficients, and three parameters of the off-center corrections. At present, all the plates reduced at Malvern treat the 11 parameters as unknowns. Consequently, approximately 120 star images, distributed fairly uniformly over the plate, are measured. About 60 of these are taken from precalibration exposures and the remainder from postcalibration exposures. Images of stars with magnitudes between 6 and 8.5 are selected for measurement, the stars being identified by reference to star atlases and the SAO star catalog.

The possibility of using predetermined values of the distortion parameters is being considered. If this can be done without a significant decrease in accuracy, only 40 or 50 star images divided between the precalibration and postcalibration exposures will be measured in future plate reductions.

The star positions taken from the catalog are reduced to the apparent positions at the epoch of the observation, as already described. A correction for atmospheric refraction is then applied to each star position. The astronomical refraction is computed by the method of Garfinkel (1944), expressed as

$$R = K_0 \sum_{i=0}^{4} C_i \, t^{2i+1} \quad ,$$

where R is the astronomical refraction, and

$$C = \sin z\{[\cos^2 z + \gamma_0{}^2(\mu_0{}^2 - \sin^2 z)]^{1/2} + \gamma_0(\mu_0{}^2 - \sin^2 z)^{1/2}\}^{-1} \quad ,$$

z being the true zenith distance of the star. If we take

$$P = (\text{atmospheric pressure in mm})/760 \quad ,$$

and

$$T = (\text{temperature in } ^\circ\text{A})/273,$$

then

$$\mu_0 = 1 + \frac{0.0002924\, P}{T}$$

$$\gamma_0 = 8.2223\, T^{-1/2} \quad ,$$

$$K_0 = 4952''\, PT^{-3/2} \quad ,$$

$$B_0 = 0.03916\, PT^{-1/2} \quad ,$$

$$C_0 = 0.2 \quad ,$$

$$C_1 = \frac{2}{15} + \frac{1}{5} B_0 \quad ,$$

$$C_2 = \frac{2}{35} + \frac{17}{35} B_0 + \frac{2}{5} B_0{}^2 \quad ,$$

$$C_3 = \frac{1}{70} + \frac{87}{385} B_0 + 0.88091\, B_0{}^2 \quad ,$$

$$C_4 = \frac{1}{630} + \frac{188}{1430} B_0 + 0.975\, B_0{}^2 \quad .$$

The standard coordinates of the stars are now calculated with the optical axis of the camera used as the origin. The approximate direction of this axis in right ascension and declination is obtained from the angles of azimuth and elevation to which the camera was set during the observation (Currie, 1965).

The projective equations are solved by the method described in detail by Schmid (1953) and others. The equations are converted into linear functions of x and y by means of a Taylor expansion about approximate values of the parameters. The normal equations are formed from the linear observation equations, each observation equation being weighted according to the value of the weight derived from its plate measurements. The final values of the parameters are obtained from the normal equations, by use of an iterative technique.

In the method used at Malvern, some of the expressions for the partial differentials in the Taylor expansion differ from those normally given in the descriptions of the method.

1. The differentials with respect to $x_p$ and $y_p$ usually neglect the terms containing the distortion coefficients. If the first distortion coefficient $h_1$, or the principal distance c, is large, it is possible that the iterative process will not converge. In fact, the process will converge only if $|2h_1\ c^2| < 1$. In the Malvern Schmidt cameras, $c \simeq 0.6$ m and $h_1 \simeq 1.8\ m^{-2}$. Hence, $|2h_1\ c^2|$ is approximately equal to 1.3, and therefore terms in $h_1$ have been included in the expansions of the partial differential.

2. The normal equations are ill conditioned because there is correlation between the plate center $x_p$, $y_p$ and the two camera orientation angles $\theta$ and $\phi$: A slight shift of the plate center $x_p$, $y_p$ can be almost exactly compensated by a small shift in the angles $\theta$ and $\phi$. This ill-conditioning has been minimized in the computer programs used at Malvern by modification of the partial differentials with respect to $\theta$ and $\phi$.

These modifications are discussed in detail by Currie (1965).

The normal equations are solved in two stages. In the first stage, only the three camera-orientation parameters are treated as unknowns. This enables any gross errors in an individual star position and its associated plate measurements to be quickly detected and rejected. After each iteration any star image is rejected if the residual of the observation equation is greater than three times the r.m.s. error. This stage in the solution of the normal equations is completed when the weighted sum of the squares of the residuals in an iteration is reduced by less than 0.5 of the weighted sum of the squares of the residuals in the previous iteration.

The second stage takes account of all the unknown parameters. In the plates that have so far been reduced at Malvern, the 11 parameters specified earlier have been treated as unknowns. But any parameter can be taken as being predetermined. Star images with large residuals are rejected on the same criterion as before. The second stage of the iterative process is complete when the weighted sum of the squares of the residuals in the last iteration is reduced by less than 0.01 of the weighted sum of those in the previous iteration.

### 3.5 Reduction of the Satellite Measurements

Before the directions to the satellite are calculated, the measured plate coordinates of the satellite images are corrected for the differences in magnitude of the comparator scales, nonrectangularity of the comparator axis, radial distortion, and off-center errors. These corrected measurements are then fitted to polynomials as a check for errors in the measurements and in the recorded time for each image. The coordinate axes of the plate measurements are parallel to the edges of the plate, and these axes are rotated to bring the x axis parallel to the direction of the satellite track before fitting the polynomials.

The plate measurements are first fitted to a second-order polynomial in x with respect to y. This checks for large errors in the plate readings, and a satellite point is rejected if its residual is greater than three times the r.m.s. error. The measurements are then fitted to a third-order polynomial in time with respect to x, by means of which an error in the x coordinate or in the time of a satellite position can be detected. Erroneous satellite positions are rejected on the same criterion as before. In the case of passive satellites, corrections for the sweep of the sector shutter will have been applied to the recorded satellite times.

For a passive satellite, the time of the observation at the camera is corrected for satellite aberration if the slant range is known. The correction takes the form

$$t = t_s - \frac{r}{c} \quad ,$$

where t is the corrected satellite time, $t_s$ is the satellite time at the camera, r is the range of the satellite in meters, and c is the velocity of light ($2.997 \times 10^8$ m/sec).

For the Geos 1 flashing satellite, no correction is made to the quoted times of the flashes; therefore, the time given is the nominal time of the beginning of the flash.

The direction of each satellite image is now computed. The refracted direction in the alt-azimuth system is first determined by use of the projective relationship in the form

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = B \begin{bmatrix} x - x_p \\ y - y_p \\ c \end{bmatrix} \quad .$$

The matrix B is the transpose of the direction cosine matrix with respect to east, north, and zenith, obtained from the plate calibration. A correction for refraction is then applied to the plate measurements by means of the expressions

$$x' = x - \left( B_{31} - \frac{xw}{u^2 + v^2 + w^2} \right) mr \quad ,$$

$$y' = y - \left( B_{32} - \frac{yw}{u^2 + v^2 + w^2} \right) mr \quad ,$$

where $B_{31}$, $B_{32}$ are elements of the B matrix and

$$m = (u^2 + v^2 + w^2)(u^2 + v^2)^{-1/2} \quad .$$

If the height of the satellite is unknown, then r is the astronomical refraction calculated in the same manner as for the star positions, by means of the Garfinkel formulas. However, if the height of the satellite is known and it is desired to correct for the parallactic refraction, the expression for r becomes

$$\left[\frac{(\mu - 1)\sin z}{1 + \frac{h}{a}} - R \cos \frac{\mu \sin z}{1 + \frac{h}{a}}\right] \left[\frac{\cos z}{1 + \frac{h}{a}} - \cos \frac{\mu \sin z}{1 + \frac{h}{a}}\right]^{-1} \quad ,$$

where

$$\mu = 1 + \frac{0.0002924\ P}{T} \quad ,$$

P = (atmospheric pressure in mm)/760 ,

T = (temperature in ° A)/273 ,

h = satellite height in km ,

a = radius of the earth in km ,

z = zenith distance ,

R = astronomical refraction .

The unrefracted direction ratio u, v, w is converted to a direction ratio $u'$, $v'$, $w'$ relative to a set of axes having the X axis parallel to the local meridian, the Z axis parallel to the celestial pole, and the Y axis in the direction of negative hour angle. The local meridian is defined by the Local Apparent Sidereal Time (LAST) $\theta$, calculated from the formula

$$\theta = \theta_0 + \Delta\theta + 1.0027379093\ t - \lambda \quad ,$$

where $\theta_0$ is the Greenwich Apparent Sidereal Time (GAST) at 0 hr, taken from the Astronomical Ephemeris, $\Delta\theta$ is the interpolated difference between the equation of the equinoxes for GAST at 0 hr UT before and after the epoch of the observation, t is the time of the observation corrected to UT1 using the provisional correction given by the Royal Greenwich Observatory, and $\lambda$ is the longitude of the observing station (positive west).

For a large, passive spherical satellite that has a perfectly reflecting surface, a correction for the phase of the satellite can be applied. This correction is calculated as follows:

Let L, M, N be the sun's direction cosines derived from the expression

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ,$$

where X, Y, Z are the rectangular coordinates of the sun, given in the Astronomical Ephemeris, and a is the LAST at the epoch of the observation. Also, let $\ell$, m, n be the direction cosines to the satellite image. Then the direction cosines of the bisector $\lambda$, $\mu$, $\nu$ are

$$\frac{\ell - L}{S} , \quad \frac{m - M}{S} , \quad \frac{n - N}{S} ,$$

where $S = [2 - 2(\ell L + mM + nN)]^{1/2}$. The new coordinates of the satellite are now given by

$$x = R\ell + r\lambda ,$$

$$y = Rm + r\mu ,$$

$$z = Rn + r\nu ,$$

where R is the slant range of the satellite, and r is the radius of the satellite. The coordinates are then converted to direction cosines.

The direction cosines are now converted to Greenwich hour angle and declination through the use of the observer's longitude, the Greenwich meridian being defined in a similar way to the local meridian.

Because the precise time of a flash from the Geos 1 satellite is unknown and any correction applied to the nominal time of the flash would also have to be applied to the right ascension, the directions to the flashes have been given in Greenwich hour angle and declination, from which the right ascension is readily calculated once the correct time of the flash is available.

In the case of passive satellites, the direction to the satellite is given in right ascension and declination by means of the expression

$$RA = GAST - h \quad ,$$

where GAST is given by

$$\theta = \theta_0 + \Delta\theta + 1.0027379093\, t \quad ,$$

and $\theta_0$, $\Delta\theta$, and t have the same definitions as in the formula for LAST.

## 3.6 Output from the Computer Programs

At present, the camera calibration and the reduction of the satellite measurements are carried out in two separate computer programs.

### 3.6.1 Camera-calibration program (see Appendix A)

The camera-calibration program outputs the following data:

1. The code number of the plate.

2. The star number and the x and y residuals (in microns) of rejected stars.

3. The coordinates of the principal point $x_p$, $y_p$, c.

4. The fiducial coefficients by means of which the principal point $x_p$, $y_p$ can be calculated from the measurements of the fiducial marks. These coefficients have been derived from the computed $x_p$, $y_p$ and the fiducial measurements.

5. The radial distortion coefficients $h_1$, $h_2$.

6. A direction cosine matrix with respect to the local east, north, and zenith. The zenith is defined by the LAST, derived from UT1 and the observer's latitude. This matrix is the B matrix used to compute the satellite directions from the plate measurements.

7. The off-center corrections, the angle $\chi$ (in degrees), and the coefficients $j_1$ and $j_2$.

8. The weighted r.m.s. residual of the x measurements of the star images, of the y measurements, and of the sum of the x and y measurements.

9. The r.m.s. directional error, which is the estimated error in the computed direction of the camera axis, defined as

$$\sigma_{par} = \sigma_{x+y} \left(\frac{P}{2n}\right)^{1/2} ,$$

where $\sigma_{x+y}$ is the weighted r.m.s. residual of the sum of the x and y measurements, par is the number of parameters in the solution, and n is the number of star images.

10. A covariance matrix, the inverse of the matrix of the normal equations with the elements multiplied by $\sigma_{x+y}$.

11. A table, which gives for each star the star number, the residuals in x and y (in microns), the weight, the average of the x measurements and of the y measurements (in meters), and the distance (in meters) of the star image from the computed principal point $x_p$, $y_p$.

3.6.2 Computation of the satellite directions (see Appendix B)

The output of the satellite direction program is in two parts. The first part gives:

1. The code number of the plate.

2. The average of the x measurements of a satellite image (in meters), the weight of the x measurement, and the average of the y measurement (in meters) and its weight. These data are given in tabular form with one row for each satellite image.

3. The number and the y residual (in microns) of any satellite image rejected from the polynomial in x with respect to y.

4. The coefficients (in ascending powers of x) of the polynomial in x.

5. The number and the x residual (in microns) of any satellite image rejected from the polynomial in t (the time of the satellite image) with respect to x.

6. The coefficients (in ascending powers of z) of the polynomial in t.

7. The weighted r.m.s. residuals from the x, t polynomial and from the y, x polynomial.

8. A table giving, for each satellite image, the number, the residual from the x, t polynomial, and the residual from the y, x polynomial.

The second part gives all the data on the satellite directions:

9. The date of the observation.

10. The latitude and longitude of the observing station.

11. A summary of the corrections that have been applied.

12. A list giving, for each satellite image, the time of the observation, the azimuth and elevation with reference to the local set of axes, east, north, and zenith, and, for the flashes of Geos 1, the Greenwich hour angle and declination. For each satellite direction, the refraction correction that has been applied is also given.

# 4. PLATE-MEASUREMENT AND REDUCTION PROCEDURES OF MALVERN PLATES AT SAO

The procedures used in the measurement and reduction of K-50 and Malvern plates at SAO are, as much as possible, the same as the techniques used for processing Baker-Nunn films. Therefore, most of the publications concerning this subject will apply with no change. There are three principal ways in which plate reductions differ from Baker-Nunn reductions. First, since the projection is not azimuthly equidistant, it is necessary to know the right ascension and declination of the plate center. Second, a correction must be made to the reduced satellite position because the satellite and star images are not taken simultaneously. Lastly, the method of chopping the satellite trail to obtain measurable breaks varies from camera to camera. A modified version of the Baker-Nunn reduction program has been written to handle the differences mentioned above. The final reduced positions are in exactly the same format as the Baker-Nunn observations.

## 4.1 Comparator Measurements

The plates are measured on a Mann comparator, model 422D. Three pointings are made on each star image and six on each satellite image. Fiducial marks used in the calculation of the interior orientation of the camera and for the computation of chopping-shutter time corrections are also measured. The stage is then rotated 180°, and the star and satellite measurements repeated in the same way. No corrections are applied to the comparator settings. It is assumed that over the area normally measured the errors are reasonably linear and will be accounted for in the final Turner's reduction method. The number of stars measured varies from approximately 15 to 25, depending on the length of the series of satellite images to be measured and on the availability of stars. The stars, therefore, are located in a narrow band across the plate. If both precalibration and postcalibration star images are measured, two separate computer runs must be made with the present reduction program.

## 4.2 Star Catalog

The SAO star catalog tapes are used. The tape is updated each year for proper motion, and the tape for the year in which the observation was made is used. The epoch of the stars is always 1950.0 and the final reduced positions are in the 1950.0 system.

## 4.3 Plate-Reduction Procedure

The individual settings on an image are averaged, and the scatter of the measurements computed. If the scatter is too high, the measurements are rejected. Once the measurements are accepted, no attempt is made to give them individual weights in the solution. Catalog positions of the stars are treated as being exact, and the program assumes that all the errors are in the measurements. The scatter of the satellite measurements is used in the calculation of an error figure for the final satellite position.

The chopping-shutter time corrections for passive satellites photographed by the Malvern camera are computed by use of parameters supplied by Mr. Hewitt.

The direct star and satellite measurements are normalized by the equations

$$X'_D = X_D - X_{SD} \quad ,$$

$$Y'_D = Y_D - Y_{SD} \quad ,$$

where $X_D$, $Y_D$ are the measured coordinates of the image, $X'_D$, $Y'_D$ the normalized coordinates, and $X_{SD}$, $Y_{SD}$ the measured coordinates of one of the satellite images that has been chosen to be the new origin of coordinates. The reverse measurements are normalized by the equations

$$X'_R = X_{SR} - X_R \quad ,$$
$$Y'_R = Y_{SR} - Y_R \quad ,$$

where $X_{SR}$ and $Y_{SR}$ are the coordinates of the satellite image on reverse that corresponds to the satellite used as the origin of the direct measurements. The procedure reverses the sign of the reverse measurements in preparation for matching. The direct and reverse measurements are then matched by a method that essentially picks the reverse image closest to each of the direct images. The match is then subjected to a least-squares adjustment using the equations

$$X'_D = a_0 + a_1 X'_R + a_2 Y'_R \quad ,$$
$$Y'_D = b_0 + b_1 X'_R + b_2 Y'_R \quad ,$$

where the a's and the b's are parameters to be adjusted. Any measurements with too high residuals are rejected. The direct and reverse measurements are then combined by the formulas

$$X_c = \frac{X'_D + X'_R}{2} \quad ,$$

$$Y_c = \frac{Y'_D + Y'_R}{2} \quad ,$$

where $X_c$ and $Y_c$ are the combined coordinates.

If we use field orbits for the satellite being reduced, an approximate position and position angle for the satellite can be calculated for the time of observation. The program looks up all the stars within about 3° of the predicted position from the SAO star tape and computes standard coordinates to the scale of the camera by use of a gnomonic projection with the satellite as plate center. The measured star positions are matched with the catalog positions by the same procedure whereby the direct and reverse measurements were matched. A least-squares adjustment is made by means of the formulas

$$X_c = a_0 + a_1\xi + a_2\eta \quad ,$$
$$Y_c = b_0 + b_1\xi + b_2\eta \quad ,$$

where $\xi$ and $\eta$ are the standard coordinates of the catalog positions. Any measurements with too high residuals are rejected.

This last least-squares adjustment usually gives a poor fit of the measured coordinates to the standard coordinates because the optical axis of the camera is not being used as the plate center. In the case of the Malvern camera, there is also distortion present due to the field-flattening lens.

The standard coordinates $\xi$, $\eta$ at unit focal length can be related to the measured coordinates $X_c$, $Y_c$ by the parameters

$$\theta,\ \phi,\ \psi,\ x_p,\ y_p,\ c \quad ,$$

where $\theta$, $\phi$, $\psi$ are the rotation angles relating the axes of the measured and standard coordinates, $x_p$, $y_p$ are the coordinates of the principal point of the camera, and c is the focal length. If distortion is present, two radial distortion coefficients $h_1$, $h_2$ may also be added. The measured coordinates can be calculated from the standard coordinates as follows:

$$\begin{bmatrix} m \\ n \\ q \end{bmatrix} = A \begin{bmatrix} \xi \\ \eta \\ 1 \end{bmatrix} \quad ,$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ,$$

and $\begin{bmatrix} m \\ n \\ q \end{bmatrix}$ is the standard coordinate vector, expressed relative to the optical axis of the camera. The multiplication of $\begin{bmatrix} m \\ n \\ q \end{bmatrix}$ by $c/q$, where c is the focal length of the camera, gives the intersection of the vector $\begin{bmatrix} m \\ n \\ q \end{bmatrix}$ with the measured plate. The application of the distortion and addition of the coordinates of the principal point give the computed value for the measured coordinates:

$$X = \frac{mc}{q}(1 + h_1 R^2 + h_2 R^4) + x_p \quad ,$$

$$Y = \frac{nc}{q}(1 + h_1 R^2 + h_2 R^4) + y_p \quad ,$$

where

$$R^2 = \left(\frac{mc}{q}\right)^2 + \left(\frac{nc}{q}\right)^2 \quad .$$

If we start with approximate values for the parameters, we can solve for corrections to the parameters using the equations

$$\begin{bmatrix} \frac{\partial X_1}{\partial \theta} & \frac{\partial X_1}{\partial \phi} & \frac{\partial X_1}{\partial \psi} & \frac{\partial X_1}{\partial x_p} & \frac{\partial X_1}{\partial y_p} & \frac{\partial X_1}{\partial c} & \frac{\partial X_1}{\partial h_1} & \frac{\partial X_1}{\partial h_2} \\ \frac{\partial Y_1}{\partial \theta} & \cdots & & & & & & \\ \frac{\partial X_2}{\partial \theta} & & & & & & & \\ \frac{\partial Y_2}{\partial \theta} & & & & & & & \\ \vdots & & & & & & & \end{bmatrix} \begin{bmatrix} d\theta \\ d\phi \\ d\psi \\ dx_p \\ dy_p \\ dc \\ dh_1 \\ dh_2 \end{bmatrix} = \begin{bmatrix} X_{c_1} - X_1 \\ Y_{c_1} - Y_1 \\ X_{c_2} - X_2 \\ Y_{c_2} - Y_2 \\ \vdots \end{bmatrix}$$

where $X_1$, $Y_1$, $X_2$, $Y_2$, ... are the computed values of the measured coordinates; $d\theta$, $d\phi$, $d\psi$, $dx_p$, $dy_p$, $dc$, $dh_1$, $dh_2$ are the corrections to be added to the parameters; and $X_{c_1}$, $Y_{c_1}$, $X_{c_2}$, $Y_{c_2}$, ... are the measured combined coordinates for each image. The computer program calculates the partial derivatives numerically, using the expression

$$\frac{\partial X_1}{\partial \theta} = \frac{X_1(\theta + \Delta\theta) - X_1(\theta - \Delta\theta)}{2\Delta\theta} \quad ,$$

which gives a better approximation to the derivative at the point $\theta$ than does the expression

$$\frac{\partial X_1}{\partial \theta_1} = \frac{X_1(\theta + \Delta\theta) - X_1(\theta)}{\Delta\theta} \quad .$$

When the partial derivatives are taken, each parameter is incremented by an amount that is several orders of magnitude smaller than the value of the parameter, but not smaller than the accuracy to which the parameter must be determined. Usually, nine-tenths of the correction is added to the parameter on each iteration to help keep the solution from oscillating.

When the parameters have been determined, the right ascension and declination of the plate center are computed as follows: The vector giving the coordinates of the plate center relative to the standard coordinate axes is

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = A^T \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \quad ,$$

where $A^T$ is the transpose of A. The standard coordinates of the plate center are then $\xi = u/w$ and $\eta = v/w$. The right ascension and declination are computed from $\xi$, $\eta$ and used as the plate center to compute new standard coordinates for the stars. The coordinates of the plate center $x_p$, $y_p$ are subtracted from all measured points and the distortion is removed:

$$X'_c = \frac{X_c - x_p}{1 + h_1 R^2 + h_2 R^4} \quad ,$$

$$Y'_c = \frac{Y_c - y_p}{1 + h_1 R^2 + h_2 R^4} \quad ,$$

where $X'_c$, $Y'_c$ are the undistorted combined coordinates, and $R^2 = (X_c - x_p)^2 + (Y_c - y_p)^2$.

Since this $R^2$ is not the same $R^2$ used to derive $h_1$ and $h_2$, there is a slight error that could be removed by iteration of the above equation.

In the case of the Malvern camera, the values $h_1 = -2.0795$ and $h_2 = -132.0938$ (unit of length of plate measurements in meters) are supplied to SAO along with the relationship of the plate center to the fiducial marks. The focal length is also known accurately, so that the values of $x_p$, $y_p$, c, $h_1$, $h_2$ can be enforced by the omission of the corresponding columns in the matrix of partial derivatives when the corrections to the parameters are being solved for. The Malvern Geos plates recently measured by SAO were reduced by solving only for the exterior elements of orientation $\theta$, $\phi$, $\psi$ and by enforcing the rest of the parameters.

We could have solved for the satellite positions using the eight parameters that have been derived. However, since no corrections have been applied to the comparator readings, there is the possibility of scale changes or nonorthogonality of the axes of the measuring machine, which have not been taken into account up to this point. With standard coordinates expanded about the optical axis of the camera and with the distortion removed from the plate readings, we can now do a simple Turner's method of reduction of the form

$$x = A + B\xi + C\eta \quad ,$$

$$y = D + E\xi + D\eta \quad ,$$

where x, y are the measured coordinates, $\xi$, $\eta$ the corresponding standard coordinates, and A, B, C, D, E, F the parameters to be adjusted. The six parameters will absorb any linear errors that may exist in the measurements. This is the form of reduction used on Baker-Nunn films, and it is convenient to have the final reduction of plates in the same form.

The right ascension and declination of the satellite are computed from the plate constants, derived from Turner's method. A correction must now be made for the fact that the satellite image was not taken at the same time as the star images. For this to be done, the reduced satellite position is updated from 1950.0 to the date of the observation. If the time between the satellite and star images is short, as it is with the K-50 camera and the Malvern camera, the position is updated for precession only.

With the satellite position expressed in the epoch of the observation, the right ascension of the satellite position is corrected by the formula

$$\alpha' = \alpha + (T_{sat} - T_{star}) \times 1.002737909 \quad ,$$

where $\alpha'$ is the corrected right ascension, $\alpha$ is the old right ascension, $(T_{sat} - T_{star})$ is the time difference between the satellite and star times, and 1.002737909 is the sidereal time constant. The satellite position is then converted back to the epoch 1950.0, and the correction for annual aberration is applied. The final position is therefore 1950.0 with annual aberration applied as for Baker-Nunn observations. A variance-covariance matrix and the semimajor axis of the error ellipse are computed for each satellite position.

During the reduction process, no corrections are applied for diurnal aberration, atmospheric refraction, parallactic refraction, or phase. It is assumed that the configuration used is small enough so that the effects of differential refraction will be largely absorbed in the reduction by Turner's method.

Multiple observations are always tested on a separate program after the reduction; a polynomial fit is made of the positions versus time in order to detect any gross errors such as a speck of dust or a star measured instead of the satellite.

## 4.4 Geos Time Corrections

Several time corrections are applied to the Geos flash time. From graphs supplied by the Applied Physics Laboratory of Johns Hopkins University, a correction is applied for the error in the running of the on-board clock. Also, 0.75 msec is added to the flash time to correct for the delay in the flash after the pulse is triggered. For the satellite time to be the time of arrival of the signal at the station, the travel time of light is computed by dividing the range in megameters by the velocity of light (approximately 300 Mm/sec) and adding it to the satellite time. All these corrections affect the satellite position slightly, since the satellite position must be corrected for the time difference between the satellite and star exposures. At present, these corrections are added after the reduction in a separate program to produce the final observation. The satellite position is corrected by the same technique described in the discussion of the reduction program.

## 5. COMPARISON OF PROCEDURES

Two plates, numbers 96a2 and 105a5, have been measured at SAO and at RRE. Results from these measurements, acquired by both the SAO and the RRE methods of reduction, are listed in Table 1. The investigation has been extended to compare the results from these two plates when computed with various changes in the parameters. The seven results in each column refer to the seven Geos 1 flash observations, in one sequence.

Table 1 lists the complete results, tabulating only the second of time in right ascension and the seconds of arc in declination. The first three columns list the reduced results based on the RRE plate measurements. The results in the remaining columns are based upon the measurements made by SAO.

Column a gives the results when all the parameters, including the distortion, are allowed to vary. This is a method that has been used in reducing the 20 Geos 1 plates at RRE. In column b, the distortion is held fixed at the values that are being used by SAO, namely, $h_1 = -2.0795$, $h_2 = -132.0938$; in column c, the distortion has been held fixed at the new values derived from the last 24 plates reduced at Malvern, namely, $h_1 = -1.78222$, $h_2 = -263.73$. These new distortion values are at first sight alarming, but within the group of 24 plates there have been values for $h_1$ and $h_2$ varying from -1.20 and -692.6 to -2.20 and -48.8, respectively. With a field of narrow angle, the distortion curve is not well defined, and the balance between the $h_1$ and $h_2$ terms depends very much on the errors in plate measurements.

Column d gives the results obtained by the SAO reduction method. Columns e and f give the results from the SAO measurements reduced at RRE, the distortion being fixed at the values in column e and at the new values in column f. Finally, the SAO measurements were divided into three groups and reduced with distortion terms fixed at the new values. These results are given in columns g1, g2, and g3.

Table 1. Results of plate reductions

| RRE Measurements | | | SAO Measurements | | | | | |
|---|---|---|---|---|---|---|---|---|
| a | b | c | d | e | f | gl | g2 | g3 |
| | | | | Plate 96a2 | | | | |
| | | | | Right Ascension (sec) | | | | |
| 42.257 | 42.342 | 42.315 | 42.176 | 42.175 | 42.164 | 42.163 | | |
| 23.901 | 23.970 | 23.971 | 23.848 | 23.877 | 22.887 | 23.980 | | |
| 12.229 | 12.263 | 12.286 | 12.213 | 12.262 | 12.289 | 12.245 | 12.153 | |
| 07.257 | 07.268 | 07.286 | 07.098 | 07.147 | 07.166 | | 07.192 | |
| 09.287 | 09.285 | 09.278 | 09.072 | 09.118 | 09.113 | | 09.129 | 09.115 |
| 18.715 | 18.692 | 18.668 | 18.315 | 18.320 | 18.301 | | | 18.333 |
| 35.138 | 35.088 | 35.074 | 34.969 | 34.929 | 34.925 | | | 34.944 |
| | | | | Declination (arcsec) | | | | |
| 05.47 | 06.24 | 06.07 | 06.11 | 05.87 | 05.72 | 05.30 | | |
| 55.20 | 55.93 | 55.98 | 56.68 | 56.33 | 56.40 | 56.26 | | |
| 30.61 | 30.77 | 31.01 | 33.08 | 32.68 | 32.89 | 33.64 | 33.17 | |
| 47.22 | 46.98 | 47.13 | 47.89 | 47.48 | 47.60 | | 47.64 | |
| 42.85 | 42.50 | 42.42 | 43.75 | 43.44 | 43.35 | | 43.40 | 43.58 |
| 15.44 | 15.13 | 14.93 | 15.22 | 15.09 | 14.90 | | | 15.05 |
| 23.44 | 23.19 | 23.11 | 25.08 | 24.95 | 24.92 | | | 24.68 |
| | | | | Plate 105a5 | | | | |
| | | | | Right Ascension (sec) | | | | |
| 13.497 | 13.535 | 13.513 | 13.719 | 13.735 | 13.720 | 13.744 | | |
| 01.091 | 01.093 | 01.097 | 01.134 | 01.166 | 01.173 | 01.180 | | |
| 46.019 | 45.994 | 46.013 | 46.017 | 46.052 | 46.075 | 46.090 | 46.078 | |
| 28.250 | 28.224 | 28.238 | 28.220 | 28.262 | 28.277 | | 28.274 | |
| 07.802 | 07.794 | 07.791 | 07.817 | 07.864 | 07.862 | | 07.862 | 07.826 |
| 44.800 | 44.805 | 44.796 | 44.797 | 44.843 | 44.833 | | | 44.826 |
| 19.284 | 19.285 | 19.286 | 19.233 | 19.277 | 19.277 | | | 19.270 |
| | | | | Declination (arcsec) | | | | |
| 35.47 | 35.34 | 35.62 | 36.51 | 36.57 | 36.72 | 36.46 | | |
| 43.56 | 43.61 | 43.60 | 44.53 | 44.75 | 44.71 | 44.55 | | |
| 57.76 | 57.74 | 57.59 | 57.74 | 57.95 | 57.80 | 57.48 | 58.07 | |
| 14.42 | 14.10 | 14.08 | 14.43 | 14.66 | 14.66 | | 15.09 | |
| 40.11 | 39.57 | 39.76 | 39.30 | 39.37 | 39.59 | | 39.98 | 39.13 |
| 08.89 | 08.49 | 08.73 | 08.33 | 08.38 | 08.64 | | | 08.19 |
| 46.95 | 47.03 | 47.06 | 46.75 | 46.61 | 46.60 | | | 46.68 |

In the absence of knowledge of the correct position of the satellite, the reductions using the RRE measurements and those using the SAO measurements have been separately compared. Table 2 gives the mean of columns a, b, and c in Table 1 and the differences between the calculated results and the mean for each condition used in the reduction. Likewise, the mean of columns d, e, and f in Table 1 and the differences are given. Columns g1, g2, and g3 give the differences between the results from grouping and the SAO mean.

There is nothing to be gained from a discussion of the results in detail. At the bottom of each column, given by a sequence of seven flashes, the mean difference has been recorded without regard to sign, and from these differences certain conclusions can be drawn. First, in the case of the RRE measurements, fixing the value of distortion produces a difference in the position of the satellite of 0.020 to 0.040 sec of time in right ascension and 0.2 to 0.4 arcsec in declination. Second, with fixed distortion, the change in value of the distortion parameters has produced only a very small difference in the position of the satellite. This conclusion is also shown by columns e and f with the SAO measurements.

Computing by the SAO and the RRE methods (see columns d and e in Table 1) produces a difference in the satellite position of 0.030 to 0.040 sec of time in right ascension and 0.2 to 0.3 arcsec in declination. The results in columns g1 to g3 indicate that there is no advantage in dividing the track into small areas. In fact, there appears to be a decrease in accuracy. This is probably due to the small number of stars used in each group (approximately 10) and the corresponding increased dependence on the accuracy of each individual star measurement.

Finally, in Table 3 the differences between the means from the SAO measurements and those from the RRE measurements are given. The mean differences are approximately three times greater than those obtained from the various changes in the distortion parameters. It would therefore appear that larger differences are obtained from one plate measurer to another,

## Table 2. Differences in plate reductions (calculated minus mean)

| RRE Measurements | | | | SAO Measurements | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean | a | b | c | Mean | d | e | f | g1 | g2 | g3 |
| | | | | **Plate 96a2** | | | | | | |
| | | | | **Right Ascension (sec)** | | | | | | |
| 42.305 | -0.048 | +0.037 | +0.010 | 42.172 | +0.004 | +0.003 | -0.008 | -0.009 | | |
| 23.947 | -0.046 | +0.023 | +0.024 | 23.871 | -0.023 | +0.006 | +0.016 | +0.109 | | |
| 12.259 | -0.030 | +0.004 | +0.027 | 12.255 | -0.042 | +0.007 | +0.034 | -0.010 | -0.102 | |
| 07.270 | -0.013 | -0.002 | +0.016 | 07.137 | -0.039 | +0.010 | +0.029 | | +0.055 | |
| 09.283 | +0.004 | +0.002 | -0.005 | 09.101 | -0.029 | +0.017 | +0.012 | | +0.028 | +0.014 |
| 18.692 | +0.023 | 0.000 | -0.024 | 18.312 | +0.003 | +0.008 | -0.011 | | | +0.021 |
| 35.100 | +0.038 | -0.012 | -0.026 | 34.941 | +0.028 | -0.012 | -0.016 | | | +0.003 |
| Mean diff. | 0.029 | 0.011 | 0.019 | | 0.024 | 0.009 | 0.018 | | 0.039 | |
| | | | | **Declination (arcsec)** | | | | | | |
| 05.94 | -0.47 | +0.35 | +0.13 | 05.90 | +0.21 | -0.03 | -0.18 | -0.60 | | |
| 55.70 | -0.50 | +0.23 | +0.28 | 56.47 | +0.21 | -0.14 | -0.07 | -0.21 | | |
| 30.80 | -0.19 | -0.03 | +0.21 | 32.88 | +0.20 | -0.20 | +0.01 | +0.76 | +0.29 | |
| 47.11 | +0.11 | -0.13 | +0.02 | 47.66 | +0.23 | -0.18 | -0.06 | | -0.02 | |
| 42.59 | +0.26 | -0.09 | -0.17 | 43.51 | +0.24 | -0.07 | -0.16 | | -0.11 | +0.06 |
| 15.17 | +0.27 | -0.04 | -0.24 | 15.07 | +0.15 | +0.02 | -0.17 | | | -0.02 |
| 23.25 | +0.19 | -0.06 | -0.14 | 24.98 | +0.10 | -0.03 | -0.06 | | | -0.30 |
| Mean diff. | 0.29 | 0.13 | 0.17 | | 0.19 | 0.10 | 0.10 | | 0.26 | |
| | | | | **Plate 105a5** | | | | | | |
| | | | | **Right Ascension (sec)** | | | | | | |
| 13.515 | -0.018 | +0.020 | -0.002 | 13.725 | -0.006 | +0.010 | -0.005 | +0.019 | | |
| 01.094 | -0.003 | -0.001 | +0.003 | 01.158 | -0.024 | +0.008 | +0.015 | +0.022 | | |
| 46.009 | +0.010 | -0.015 | +0.004 | 46.048 | -0.031 | +0.005 | +0.027 | +0.042 | +0.030 | |
| 28.237 | +0.013 | -0.013 | +0.001 | 28.253 | -0.033 | +0.009 | +0.024 | | +0.021 | |
| 07.796 | +0.006 | -0.002 | -0.005 | 07.848 | -0.031 | +0.016 | +0.014 | | +0.014 | -0.022 |
| 44.800 | 0.000 | +0.005 | -0.004 | 44.824 | -0.027 | +0.019 | +0.009 | | | +0.002 |
| 19.285 | -0.001 | 0.000 | +0.001 | 19.262 | -0.029 | +0.015 | +0.015 | | | +0.008 |
| Mean diff. | 0.007 | 0.008 | 0.003 | | 0.026 | 0.012 | 0.016 | | 0.020 | |
| | | | | **Declination (arcsec)** | | | | | | |
| 35.48 | -0.01 | -0.14 | +0.14 | 36.60 | -0.09 | -0.03 | +0.12 | -0.14 | | |
| 43.59 | -0.03 | +0.02 | +0.01 | 44.66 | -0.13 | +0.09 | +0.05 | -0.11 | | |
| 57.70 | +0.06 | +0.04 | -0.11 | 57.83 | -0.09 | +0.12 | -0.03 | -0.35 | +0.24 | |
| 14.30 | +0.22 | -0.10 | -0.12 | 14.58 | -0.15 | +0.08 | +0.08 | | +0.51 | |
| 39.81 | +0.30 | -0.24 | -0.05 | 39.42 | -0.12 | -0.05 | +0.17 | | +0.56 | -0.29 |
| 08.70 | +0.19 | -0.21 | +0.03 | 08.45 | -0.12 | -0.07 | +0.19 | | | -0.26 |
| 47.01 | -0.06 | +0.02 | +0.05 | 46.65 | +0.10 | -0.04 | -0.05 | | | +0.03 |
| Mean diff. | 0.12 | 0.11 | 0.07 | | 0.11 | 0.07 | 0.10 | | 0.36 | |

rather than from the changes in the parameters that have been investigated. Certainly, the differences between the SAO method of reduction and that of RRE are less than the differences resulting from remeasurement of the plates by other people.

Table 3. Differences of means

| SAO and RRE measurements | | |
|---|---|---|
| | Right ascension | Declination |
| | Plate 96a2 | |
| | -0.133 | -0.04 |
| | -0.076 | +0.77 |
| | -0.004 | +2.08 |
| | -0.133 | +0.55 |
| | -0.182 | +0.92 |
| | -0.380 | -0.10 |
| | -0.159 | +1.73 |
| Mean diff. | 0.152 | 0.88 |
| | Plate 105a5 | |
| | +0.210 | +0.12 |
| | +0.064 | +1.07 |
| | +0.039 | +0.13 |
| | +0.016 | +0.38 |
| | +0.052 | -0.39 |
| | +0.024 | -0.25 |
| | -0.023 | -0.36 |
| Mean diff. | 0.061 | 0.39 |

There are other factors involved in the difference between the SAO and RRE reductions, i.e., columns a, b, c, and d, e, f. Different stars were measured, and approximately five times as many stars were measured by RRE. These factors may contribute to the difference between the two reductions.

A new method is in use at Malvern for the derivation of the plate center from the fiducial marks. This method is based on the principle of finding the length of the perpendicular from a point upon a straight line.

If $x_c$, $y_c$ are the coordinates of the center and $x_1$, $y_1$ and $x_2$, $y_2$ are the coordinates of the two fiducial marks, then the x coefficient is

$$x_{c0} = \frac{(x_c - x_1)(x_2 - x_1) + (y_c - y_1)(y_2 - y_1)}{[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}} \quad ,$$

and the y coefficient is

$$y_{c0} = \frac{(y_c - y_1)(x_2 - x_1) - (x_c - x_1)(y_2 - y_1)}{[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}} \quad .$$

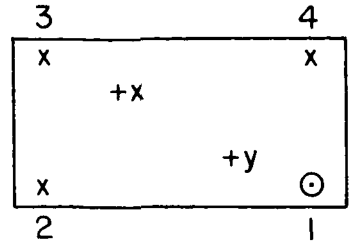
Each pair of fiducial marks gives a value for the center, and the mean value can be taken as the position of the plate center ($x_c$, $y_c$), where, for fiducial marks 1 and 2 (1, 2),

$$x_c = \frac{X_{c0_{12}}(x_2 - x_1) - Y_{c0_{12}}(y_2 - y_1)}{[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}} + x_1 \quad ,$$

and

$$y_c = \frac{X_{c0_{12}}(y_2 - y_1) + Y_{c0_{12}}(x_2 - x_1)}{[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}} + y_1 \quad .$$

As the measurer looks at the glass side of the plate, the fiducial marks are measured in a clockwise direction (see sketch), and the values of the coefficients averaged over 24 plates, starting with the rotating fiducial mark, are:




| | $x_{c0}$ | $y_{c0}$ |
|---|---|---|
| 1, 2 | $9.7193483 \times 10^{-2}$ | $-5.6592840 \times 10^{-2}$ |
| 2, 3 | $5.2395102 \times 10^{-2}$ | $-7.8257511 \times 10^{-2}$ |
| 3, 4 | $7.8054681 \times 10^{-2}$ | $-5.2053614 \times 10^{-2}$ |
| 4, 1 | $4.0510558 \times 10^{-2}$ | $-8.3864855 \times 10^{-2}$ . |

All measurements are in meters.

The foregoing study on the comparison between the RRE and the SAO results has been presented by Mr. Hewitt. The differences found between the reduction methods of the two agencies are small. They may have a systematic character in some instances, however. Some examples are:

1. RRE applies a correction to the plate measurements due to different scales in the x and y directions. A deviation from orthogonality is also accounted for. SAO does not apply these corrections directly to the plate measurements. Instead, they are expected to be accounted for by applying Turner's method of plate reduction.

However, if a plate is measured at SAO and reduced at Malvern (columns e and f in Tables 1 and 2), the sources of error just mentioned are not accounted for at all. Hence, columns e and f are liable to a systematic error.

2. A correction for diurnal aberration is applied at RRE, but not at SAO.

3. RRE determines azimuth and elevation of the satellite image with respect to a local terrestrial reference system first. Right ascension and declination are then found by transformation of the local system. This transformation may cause systematic errors. SAO does not make use of this transformation, but rather determines right ascension and declination in a direct manner.

In addition to the comparison tests between plates 96a2 and 105a5, RRE compared results obtained from a third plate.

The third plate was measured and reduced at Malvern; 120 reference stars were used and all 11 parameters were treated as unknowns in the reduction program. The resulting flash coordinates are on the reference lines in Figure 1.

Then, the third plate was completely reread by an independent pair of measurers and reduced in the same way as above. The differences found in the flash coordinates resulting from this remeasurement are indicated by curves 1 in Figure 1.

Moreover, some additional tests with plate 105a5 were performed at RRE.

On the reference line are the flash coordinates, obtained from plate 105a5, measured at Malvern and reduced with all 11 parameters unconstrained.

Curves 2 give the differences in the results in right ascension and declination from the same plate measured and reduced at SAO.

Curves 3 give the differences when the plate, measured at Malvern, is reduced using 120 stars and predetermined distortion coefficients.

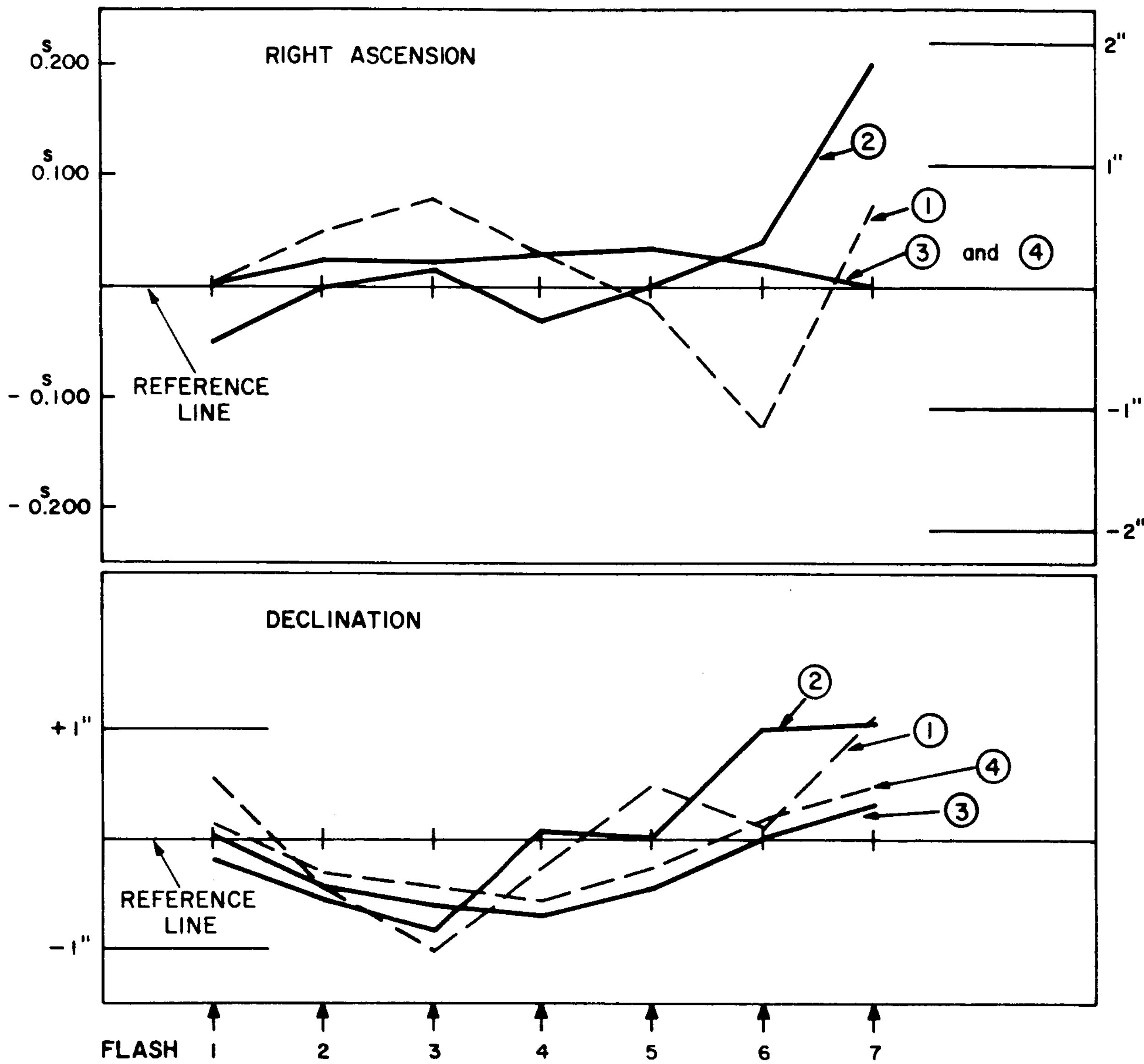



Figure 1. Comparison of SAO and RRE results from two additional plates.

Curves 4 give the differences when the reduction is made using 50 stars and with predetermined distortion coefficients.

Figure 1 shows again that differences in results using independent measurers appear to be more important than the difference obtained by minor changes in the reduction method.

Table 4 summarizes the corrections applied to the RRE plates of Geos 1 observations.

Table 4. Geos 1 observations: Summary of corrections applied to RRE plates during measurement and reduction

| Item | Applied at RRE | Applied at SAO |
|---|---|---|
| Linear difference in x and y scales of measuring machine | Directly to measured coordinates | Indirectly, by using Turner's method |
| Orthogonality of x and y directions of measuring machine | Directly to measured coordinates | Indirectly, by using Turner's method |
| Planetary aberration | No | No |
| Parallactic refraction | No | No |
| Differential refraction | No (refraction is applied to all stars in full) | Indirectly, by using Turner's method |
| Annual aberration | Yes, to stars only | Yes, to satellite only |
| Diurnal aberration | Yes, to stars | No |
| Flashing time | Prediction time is not altered in reduction program | Prediction time is not altered in reduction program |

## 6. GEODETIC RESULTS

After the precise reduction, the observations were used to correct the preliminary station coordinates of Malvern ($x = +3.920160$, $y = -0.134757$, $z = +5.012706$). This station-coordinate improvement was carried out at SAO. Both the dynamical and the geometrical methods, as described by Aardoom, Girnius, and Veis (1966) and Izsak and Gaposchkin (1966), were used.

The following corrections (in meters) were found:

| | Method | Satellites | Observations | $\Delta x$ | $\Delta y$ | $\Delta z$ |
|---|---|---|---|---|---|---|
| A | Dynamical | Geos 1 | 20 sequences | +34 | - 2 | +42 |
| B | Geometrical | Geos 1 | 9 sequences | -10 | +10 | +15 |
| C | Geometrical | Various passive | 16 | +39 | +16 | +41 |
| D | Geometrical | B + C | $2\sigma$ cutoff | +16 | + 9 | +21 |
| E | Geometrical | B + C | $3\sigma$ cutoff | +25 | +14 | +30 |

The letters B, C, D, and E refer to simultaneous observations with the Baker-Nunn camera in San Fernando, Spain. The chord distance between these two stations (Malvern and San Fernando) is derived from the station coordinates as determined by the dynamical method under A, and has been kept fixed in B through E.

The 20 plates in A were measured and reduced at RRE, Malvern.

Nine of these plates, with a total of 56 flashes, were simultaneous with San Fernando. They are used for solution B.

In solution C, the plates involved were measured and reduced at SAO. All single observations having a larger residual than $2\sigma$ were not used for the solution of $\Delta x$, $\Delta y$, and $\Delta z$.

In solution D, all single observations having a residual larger than $2\sigma$ were rejected for the correction determination; 73 single observations were used here.

In solution E, all single observations, 79 in total, were used. The normal $3\sigma$ cutoff was applied.

Preliminary results show an internal accuracy of 1 arcsec and better in the direction between San Fernando and Malvern, in each of the solutions B, C, D, and E.

The present number of observations, however, is too small for any definitive statements on the Malvern coordinates and their accuracy to be made.

## 7. CONCLUSIONS

Malvern plates have been measured and reduced at RRE and at SAO. The reduction methods applied at both agencies are slightly different, but it has been demonstrated that the ultimate observational results agree within 0.2 or 0.3 arcsec, provided they are based upon the same measurement of the plates. Remeasurement of the plates results in occasional differences in the flash coordinates of the order of 1 arcsec.

The geodetic results obtained so far give another indication of the great accuracy of the Malvern observations, whether they are measured and reduced at RRE or at SAO. Furthermore, it has been demonstrated that application of fixed, instead of varying, distortion coefficients does not introduce any significant change in the results.

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# APPENDIX A

## CAMERA CALIBRATION PROGRAM OUTPUT DATA

| | | | | |
|---|---|---|---|---|
| 1. | 114 a6 | Star No. | x res. | y res. |
| 2. | reject | 2 | -0 | 16 |
| | reject | 65 | 13 | -1 |
| | reject | 67 | 3 | 12 |

3. x = -4·9912 549$_{10}$-1
   y = 2·0018 764$_{10}$-1
   c = 6·1090 961$_{10}$-1

4. fiducial coefficients

| | | |
|---|---|---|
| 4·0378 14$_{10}$-2 | -8·3991 759$_{10}$-2 | constants for this plate |
| 9·7335 320$_{10}$-2 | -5·0560 945$_{10}$-2 | computed from measure- |
| 5·2408 397$_{10}$-2 | -7·8170 536$_{10}$-2 | ments of fiducial marks |
| 7·7959 886$_{10}$-2 | -5·2015 648$_{10}$-2 | and plate constants. |

5. distortion terms
   -2·1066 804$_{10}$+0 $h_1$
   -1·1078 385$_{10}$+2 $h_2$

6. direction cosines

| | | |
|---|---|---|
| -6·1692 508$_{10}$-1 | -5·8605 845$_{10}$-1 | 5·2529 839$_{10}$-1 |
| 6·3745 765$_{10}$-1 | -7·6354 135$_{10}$-1 | -1·0321 030$_{10}$-1 |
| 4·6157 469$_{10}$-1 | 2·7118 278$_{10}$-1 | 8·4463 525$_{10}$-1 |

7. off centre corrections
   phi 9·7397 470$_{10}$+1
   coefficients
   8·3648 158$_{10}$-3
   -3·7516 668$_{10}$+0

8. weighted r·m·s· residual = 3·672$_{10}$+0 3·740$_{10}$+0 3·706$_{10}$+0 microns
9. r·m·s· directional error = 7·744$_{10}$-1 microns

10. covariance matrix

| | | | | | |
|---|---|---|---|---|---|
| 6·8$_{10}$-13 | 3·1$_{10}$-14 | 2·1$_{10}$-12 | 3·0$_{10}$-13 | 5·0$_{10}$-12 | 3·9$_{10}$-13 |
| 3·1$_{10}$-14 | 5·4$_{10}$-13 | -1·6$_{10}$-12 | 3·6$_{10}$-13 | -1·1$_{10}$-12 | 1·5$_{10}$-11 |
| 2·1$_{10}$-12 | -1·6$_{10}$-12 | 1·1$_{10}$-9 | -3·4$_{10}$-12 | 2·9$_{10}$-10 | -1·8$_{10}$-10 |
| 3·0$_{10}$-13 | 3·6$_{10}$-13 | -3·4$_{10}$-12 | 4·9$_{10}$-11 | 6·8$_{10}$-11 | 4·0$_{10}$-11 |
| 5·0$_{10}$-12 | -1·1$_{10}$-12 | 2·9$_{10}$-10 | 6·8$_{10}$-11 | 9·3$_{10}$-9 | 2·6$_{10}$-11 |
| 3·9$_{10}$-13 | 1·5$_{10}$-11 | -1·8$_{10}$-10 | 4·0$_{10}$-11 | 2·6$_{10}$-11 | 5·2$_{10}$-9 |
| 8·2$_{10}$-9 | -1·8$_{10}$-9 | 3·2$_{10}$-6 | -1·1$_{10}$-8 | 7·5$_{10}$-7 | -4·2$_{10}$-7 |
| -3·8$_{10}$-6 | 5·2$_{10}$-8 | -1·2$_{10}$-3 | 4·2$_{10}$-6 | -3·3$_{10}$-4 | 1·9$_{10}$-4 |
| 1·6$_{10}$-8 | -1·2$_{10}$-7 | 1·4$_{10}$-7 | 8·2$_{10}$-8 | 3·1$_{10}$-6 | 2·9$_{10}$-6 |
| -1·5$_{10}$-9 | -2·4$_{10}$-10 | -6·6$_{10}$-9 | -1·4$_{10}$-9 | -6·5$_{10}$-8 | -1·2$_{10}$-8 |
| 6·0$_{10}$-7 | 1·2$_{10}$-7 | 3·0$_{10}$-6 | 3·6$_{10}$-7 | 6·8$_{10}$-6 | 6·5$_{10}$-6 |

| | | | | |
|---|---|---|---|---|
| 8·2$_{10}$-9 | -3·8$_{10}$-6 | 1·6$_{10}$-8 | -1·5$_{10}$-9 | 6·0$_{10}$-7 |
| -1·8$_{10}$-9 | 5·2$_{10}$-8 | -1·2$_{10}$-7 | -2·4$_{10}$-10 | 1·2$_{10}$-7 |
| 3·2$_{10}$-6 | -1·2$_{10}$-3 | 1·4$_{10}$-7 | -6·6$_{10}$-9 | 3·0$_{10}$-6 |
| -1·1$_{10}$-8 | 4·2$_{10}$-6 | 8·2$_{10}$-8 | -1·4$_{10}$-9 | 3·6$_{10}$-7 |
| 7·5$_{10}$-7 | -3·3$_{10}$-4 | 3·1$_{10}$-6 | -6·5$_{10}$-8 | 6·8$_{10}$-6 |
| -4·2$_{10}$-7 | 1·9$_{10}$-4 | 2·9$_{10}$-6 | -1·2$_{10}$-8 | 8·5$_{10}$-6 |
| 1·0$_{10}$-2 | -4·2$_{10}$+0 | -4·7$_{10}$-4 | -3·4$_{10}$-5 | 1·8$_{10}$-2 |
| -4·2$_{10}$+0 | 1·7$_{10}$+3 | 3·9$_{10}$-1 | 1·7$_{10}$-2 | -9·2$_{10}$+0 |
| -4·7$_{10}$-4 | 3·9$_{10}$-1 | 4·2$_{10}$-2 | -1·7$_{10}$-5 | 1·9$_{10}$-4 |
| -3·4$_{10}$-5 | 1·7$_{10}$-2 | -1·7$_{10}$-5 | 4·6$_{10}$-6 | -2·0$_{10}$-3 |
| 1·8$_{10}$-2 | -9·2$_{10}$+0 | 1·9$_{10}$-4 | -2·0$_{10}$-3 | 1·0$_{10}$+0 |

11.

| true residuals | | | weight | average readings | | radius |
|---|---|---|---|---|---|---|
| 1 | -3 | 1 | $9.95_{10}-1$ | $-5.4379\ 02_{10}-1$ | $2.0614\ 37_{10}-1$ | $4.51_{10}-2$ |
| 2 | 0 | 0 | $0.0_{10}-37$ | $-5.3945\ 87_{10}-1$ | $1.8825\ 22_{10}-1$ | $4.21_{10}-2$ |
| 3 | 7 | -0 | $1.17_{10}+0$ | $-5.3208\ 38_{10}-1$ | $2.0671\ 53_{10}-1$ | $3.36_{10}-2$ |
| 4 | -7 | 5 | $9.86_{10}-1$ | $-5.2363\ 80_{10}-1$ | $2.3496\ 21_{10}-1$ | $4.25_{10}-2$ |
| 5 | 1 | 2 | $1.18_{10}+0$ | $-5.3611\ 36_{10}-1$ | $1.9478\ 00_{10}-1$ | $3.74_{10}-2$ |
| 6 | 1 | -1 | $1.24_{10}+0$ | $-5.2690\ 37_{10}-1$ | $2.0724\ 76_{10}-1$ | $2.87_{10}-2$ |
| 7 | 7 | 3 | $1.13_{10}+0$ | $-5.3279\ 78_{10}-1$ | $1.9008\ 05_{10}-1$ | $3.51_{10}-2$ |
| 8 | -0 | 0 | $8.45_{10}-1$ | $-5.2412\ 83_{10}-1$ | $2.1513\ 69_{10}-1$ | $2.91_{10}-2$ |
| 9 | 4 | -2 | $8.95_{10}-1$ | $-5.1966\ 76_{10}-1$ | $2.2741\ 62_{10}-1$ | $3.41_{10}-2$ |
| 10 | 0 | 2 | $1.45_{10}+0$ | $-5.1636\ 93_{10}-1$ | $2.3478\ 05_{10}-1$ | $3.87_{10}-2$ |
| 11 | 7 | 2 | $1.25_{10}+0$ | $-5.3320\ 71_{10}-1$ | $1.7291\ 75_{10}-1$ | $4.36_{10}-2$ |
| 12 | -1 | 2 | $1.00_{10}+0$ | $-5.2116\ 61_{10}-1$ | $2.0157\ 62_{10}-1$ | $2.21_{10}-2$ |
| 13 | -1 | 2 | $1.06_{10}+0$ | $-5.1725\ 36_{10}-1$ | $2.1305\ 98_{10}-1$ | $2.22_{10}-2$ |
| 14 | 2 | 0 | $1.05_{10}+0$ | $-5.1605\ 18_{10}-1$ | $2.1655\ 40_{10}-1$ | $2.35_{10}-2$ |
| 15 | 1 | -0 | $1.01_{10}+0$ | $-5.1876\ 29_{10}-1$ | $2.0813\ 62_{10}-1$ | $2.12_{10}-2$ |
| 16 | -0 | 1 | $1.06_{10}+0$ | $-5.2369\ 21_{10}-1$ | $1.9253\ 33_{10}-1$ | $2.57_{10}-2$ |
| 17 | 1 | 2 | $7.75_{10}-1$ | $-5.2407\ 23_{10}-1$ | $1.8883\ 26_{10}-1$ | $2.74_{10}-2$ |
| 18 | -2 | 4 | $8.53_{10}-1$ | $-5.1264\ 30_{10}-1$ | $2.2046\ 66_{10}-1$ | $2.44_{10}-2$ |
| 19 | 1 | 6 | $9.33_{10}-1$ | $-5.2553\ 23_{10}-1$ | $1.8050\ 17_{10}-1$ | $3.29_{10}-2$ |
| 20 | 3 | 7 | $9.78_{10}-1$ | $-5.1342\ 29_{10}-1$ | $2.1367\ 20_{10}-1$ | $1.97_{10}-2$ |
| 21 | -4 | -2 | $1.17_{10}+0$ | $-5.1854\ 27_{10}-1$ | $1.9666\ 04_{10}-1$ | $1.97_{10}-2$ |
| 22 | 0 | 2 | $1.11_{10}+0$ | $-5.2201\ 41_{10}-1$ | $1.8446\ 14_{10}-1$ | $2.78_{10}-2$ |
| 23 | -3 | -4 | $1.42_{10}+0$ | $-5.0707\ 37_{10}-1$ | $2.2471\ 29_{10}-1$ | $2.58_{10}-2$ |
| 24 | 2 | -6 | $9.99_{10}-1$ | $-5.0861\ 91_{10}-1$ | $2.1583\ 50_{10}-1$ | $1.83_{10}-2$ |
| 25 | 2 | 1 | $1.19_{10}+0$ | $-5.0913\ 20_{10}-1$ | $2.1302\ 96_{10}-1$ | $1.63_{10}-2$ |
| 26 | -4 | -7 | $1.03_{10}+0$ | $-5.0167\ 94_{10}-1$ | $2.3432\ 33_{10}-1$ | $3.42_{10}-2$ |
| 27 | 3 | 7 | $7.93_{10}-1$ | $-5.1874\ 10_{10}-1$ | $1.7663\ 61_{10}-1$ | $3.06_{10}-2$ |
| 28 | -6 | -4 | $1.29_{10}+0$ | $-5.0060\ 42_{10}-1$ | $2.2294\ 37_{10}-1$ | $2.28_{10}-2$ |
| 29 | -2 | -3 | $9.61_{10}-1$ | $-5.2171\ 28_{10}-1$ | $1.6023\ 40_{10}-1$ | $4.59_{10}-2$ |
| 30 | 0 | -1 | $1.31_{10}+0$ | $-5.0562\ 03_{10}-1$ | $2.0417\ 01_{10}-1$ | $7.62_{10}-3$ |
| 31 | -0 | -9 | $8.88_{10}-1$ | $-4.9765\ 01_{10}-1$ | $2.2173\ 39_{10}-1$ | $2.16_{10}-2$ |
| 32 | 5 | 0 | $1.01_{10}+0$ | $-5.0262\ 66_{10}-1$ | $2.0059\ 54_{10}-1$ | $3.52_{10}-3$ |
| 33 | -4 | -1 | $8.83_{10}-1$ | $-5.0902\ 58_{10}-1$ | $1.7997\ 86_{10}-1$ | $2.25_{10}-2$ |
| 34 | 0 | 0 | $0.0_{10}-37$ | $-4.9099\ 50_{10}-1$ | $2.3674\ 78_{10}-1$ | $3.75_{10}-2$ |
| 35 | 3 | 4 | $9.48_{10}-1$ | $-5.0120\ 59_{10}-1$ | $1.9673\ 09_{10}-1$ | $4.03_{10}-3$ |
| 36 | -3 | -1 | $9.55_{10}-1$ | $-4.9519\ 54_{10}-1$ | $2.1467\ 34_{10}-1$ | $1.50_{10}-2$ |
| 37 | 2 | -4 | $9.72_{10}-1$ | $-5.0551\ 04_{10}-1$ | $1.7943\ 82_{10}-1$ | $2.17_{10}-2$ |
| 38 | -2 | 2 | $9.74_{10}-1$ | $-4.9362\ 60_{10}-1$ | $2.1122\ 43_{10}-1$ | $1.23_{10}-2$ |
| 39 | 1 | -1 | $1.03_{10}+0$ | $-4.8377\ 74_{10}-1$ | $2.4290\ 13_{10}-1$ | $4.54_{10}-2$ |
| 40 | 0 | 0 | $1.06_{10}+0$ | $-4.9491\ 06_{10}-1$ | $2.0471\ 72_{10}-1$ | $6.20_{10}-3$ |
| 41 | -6 | 3 | $1.12_{10}+0$ | $-5.0480\ 17_{10}-1$ | $1.7445\ 52_{10}-1$ | $2.63_{10}-2$ |
| 42 | 2 | -1 | $1.07_{10}+0$ | $-4.9956\ 45_{10}-1$ | $1.8881\ 98_{10}-1$ | $1.14_{10}-2$ |
| 43 | 2 | 9 | $9.12_{10}-1$ | $-5.0956\ 21_{10}-1$ | $1.5949\ 99_{10}-1$ | $4.20_{10}-2$ |
| 44 | -6 | 5 | $9.62_{10}-1$ | $-4.9560\ 56_{10}-1$ | $1.9279\ 34_{10}-1$ | $8.18_{10}-3$ |
| 45 | -2 | -4 | $8.00_{10}-1$ | $-5.0319\ 43_{10}-1$ | $1.7108\ 32_{10}-1$ | $2.94_{10}-2$ |

| true residuals | | | weight | average readings | | radius |
|---|---|---|---|---|---|---|
| 46 | −4 | −0 | $1.58_{10}+0$ | $-4.86[illegible]7\ 59_{10}-1$ | $2.0332\ 32_{10}-1$ | $1.29_{10}-2$ |
| 47 | −4 | 8 | $1.14_{10}+0$ | $-4.9516\ 78_{10}-1$ | $1.7483\ 61_{10}-1$ | $2.57_{10}-2$ |
| 48 | −1 | −1 | $1.40_{10}+0$ | $-4.9129\ 14_{10}-1$ | $1.8019\ 66_{10}-1$ | $2.15_{10}-2$ |
| 49 | −0 | 5 | $8.23_{10}-1$ | $-4.9286\ 03_{10}-1$ | $1.6999\ 76_{10}-1$ | $3.08_{10}-2$ |
| 50 | −2 | −0 | $1.19_{10}+0$ | $-4.8549\ 55_{10}-1$ | $1.9005\ 63_{10}-1$ | $1.70_{10}-2$ |
| 51 | 4 | −2 | $1.02_{10}+0$ | $-4.8551\ 85_{10}-1$ | $1.8122\ 13_{10}-1$ | $2.33_{10}-2$ |
| 52 | 0 | 5 | $9.12_{10}-1$ | $-4.7492\ 79_{10}-1$ | $2.1109\ 13_{10}-1$ | $2.65_{10}-2$ |
| 53 | 4 | −3 | $1.32_{10}+0$ | $-4.8697\ 23_{10}-1$ | $1.7388\ 66_{10}-1$ | $2.90_{10}-2$ |
| 54 | 7 | 6 | $1.27_{10}+0$ | $-4.7461\ 44_{10}-1$ | $2.0337\ 14_{10}-1$ | $2.47_{10}-2$ |
| 55 | 2 | −0 | $1.13_{10}+0$ | $-4.7875\ 41_{10}-1$ | $1.8585\ 82_{10}-1$ | $2.49_{10}-2$ |
| 56 | −5 | −3 | $1.01_{10}+0$ | $-4.7591\ 39_{10}-1$ | $1.9416\ 82_{10}-1$ | $2.40_{10}-2$ |
| 57 | 6 | −2 | $1.08_{10}+0$ | $-4.7869\ 36_{10}-1$ | $1.7995\ 12_{10}-1$ | $2.88_{10}-2$ |
| 58 | 1 | −4 | $1.09_{10}+0$ | $-4.8162\ 39_{10}-1$ | $1.6981\ 25_{10}-1$ | $3.51_{10}-2$ |
| 59 | 1 | −1 | $1.13_{10}+0$ | $-4.8401\ 20_{10}-1$ | $1.6055\ 10_{10}-1$ | $4.24_{10}-2$ |
| 60 | 2 | 0 | $7.80_{10}-1$ | $-4.6211\ 45_{10}-1$ | $2.1852\ 23_{10}-1$ | $4.13_{10}-2$ |
| 61 | 4 | −3 | $1.16_{10}+0$ | $-4.7005\ 29_{10}-1$ | $1.8509\ 17_{10}-1$ | $3.28_{10}-2$ |
| 62 | 1 | −4 | $1.01_{10}+0$ | $-4.5868\ 43_{10}-1$ | $2.0660\ 78_{10}-1$ | $4.10_{10}-2$ |
| 63 | −3 | 2 | $1.07_{10}+0$ | $-4.6876\ 62_{10}-1$ | $1.7591\ 78_{10}-1$ | $3.89_{10}-2$ |
| 64 | −5 | −1 | $1.08_{10}+0$ | $-4.5981\ 27_{10}-1$ | $2.0027\ 09_{10}-1$ | $3.93_{10}-2$ |
| 65 | 0 | 0 | $0.0_{10}-37$ | $-4.6156\ 14_{10}-1$ | $1.8653\ 90_{10}-1$ | $4.00_{10}-2$ |
| 66 | −2 | 5 | $9.19_{10}-1$ | $-5.4437\ 54_{10}-1$ | $2.0449\ 45_{10}-1$ | $4.54_{10}-2$ |
| 67 | 0 | 0 | $0.0_{10}-37$ | $-5.4009\ 16_{10}-1$ | $1.8658\ 34_{10}-1$ | $4.32_{10}-2$ |
| 68 | −4 | −0 | $9.75_{10}-1$ | $-5.3267\ 41_{10}-1$ | $2.0503\ 85_{10}-1$ | $3.39_{10}-2$ |
| 69 | 2 | 7 | $9.35_{10}-1$ | $-5.2413\ 35_{10}-1$ | $2.3325\ 28_{10}-1$ | $4.15_{10}-2$ |
| 70 | −3 | −0 | $1.07_{10}+0$ | $-5.3673\ 20_{10}-1$ | $1.9310\ 69_{10}-1$ | $3.83_{10}-2$ |
| 71 | 2 | −4 | $1.03_{10}+0$ | $-5.2747\ 93_{10}-1$ | $2.0555\ 74_{10}-1$ | $2.89_{10}-2$ |
| 72 | 7 | 6 | $8.89_{10}-1$ | $-5.3342\ 30_{10}-1$ | $1.8840\ 46_{10}-1$ | $3.63_{10}-2$ |
| 73 | −4 | 4 | $1.17_{10}+0$ | $-5.2468\ 74_{10}-1$ | $2.1344\ 52_{10}-1$ | $2.88_{10}-2$ |
| 74 | −2 | 4 | $7.64_{10}-1$ | $-5.2019\ 77_{10}-1$ | $2.2570\ 95_{10}-1$ | $3.31_{10}-2$ |
| 75 | −6 | 9 | $9.35_{10}-1$ | $-5.1688\ 22_{10}-1$ | $2.3305\ 99_{10}-1$ | $3.74_{10}-2$ |
| 76 | 4 | −4 | $1.11_{10}+0$ | $-5.3388\ 55_{10}-1$ | $1.7121\ 77_{10}-1$ | $4.52_{10}-2$ |
| 77 | −3 | 8 | $1.15_{10}+0$ | $-5.2175\ 77_{10}-1$ | $1.9988\ 31_{10}-1$ | $2.26_{10}-2$ |
| 78 | 1 | −0 | $1.10_{10}+0$ | $-5.1781\ 20_{10}-1$ | $2.1134\ 60_{10}-1$ | $2.18_{10}-2$ |
| 79 | −6 | 3 | $1.25_{10}+0$ | $-5.1661\ 13_{10}-1$ | $2.1484\ 31_{10}-1$ | $2.28_{10}-2$ |
| 80 | 9 | −3 | $9.24_{10}-1$ | $-5.1932\ 78_{10}-1$ | $2.0642\ 86_{10}-1$ | $2.11_{10}-2$ |
| 81 | −2 | −4 | $1.27_{10}+0$ | $-5.2430\ 75_{10}-1$ | $1.9083\ 17_{10}-1$ | $2.69_{10}-2$ |
| 82 | −5 | −3 | $1.23_{10}+0$ | $-5.2470\ 20_{10}-1$ | $1.8713\ 09_{10}-1$ | $2.87_{10}-2$ |
| 83 | 3 | −1 | $8.63_{10}-1$ | $-5.1318\ 09_{10}-1$ | $2.1873\ 83_{10}-1$ | $2.33_{10}-2$ |
| 84 | −5 | −2 | $9.45_{10}-1$ | $-5.2618\ 59_{10}-1$ | $1.7879\ 34_{10}-1$ | $3.45_{10}-2$ |
| 85 | −1 | 4 | $1.24_{10}+0$ | $-5.1398\ 57_{10}-1$ | $2.1195\ 12_{10}-1$ | $1.90_{10}-2$ |
| 86 | −3 | −2 | $1.04_{10}+0$ | $-5.1914\ 26_{10}-1$ | $1.9495\ 47_{10}-1$ | $2.07_{10}-2$ |
| 87 | 2 | −1 | $1.50_{10}+0$ | $-5.2264\ 67_{10}-1$ | $1.8275\ 45_{10}-1$ | $2.93_{10}-2$ |
| 88 | −4 | −2 | $8.79_{10}-1$ | $-5.0760\ 73_{10}-1$ | $2.2297\ 68_{10}-1$ | $2.43_{10}-2$ |
| 89 | 4 | −4 | $9.20_{10}-1$ | $-5.0917\ 13_{10}-1$ | $2.1410\ 72_{10}-1$ | $1.72_{10}-2$ |
| 90 | 2 | 0 | $1.03_{10}+0$ | $-5.0969\ 24_{10}-1$ | $2.1130\ 20_{10}-1$ | $1.53_{10}-2$ |
| 91 | −1 | −8 | $1.00_{10}+0$ | $-5.0218\ 86_{10}-1$ | $2.3256\ 22_{10}-1$ | $3.25_{10}-2$ |
| 92 | 1 | 7 | $9.48_{10}-1$ | $-5.1939\ 61_{10}-1$ | $1.7491\ 94_{10}-1$ | $3.24_{10}-2$ |
| 93 | −3 | −3 | $1.06_{10}+0$ | $-5.0113\ 98_{10}-1$ | $2.2119\ 23_{10}-1$ | $2.11_{10}-2$ |
| 94 | −3 | −12 | $6.20_{10}-1$ | $-5.2241\ 62_{10}-1$ | $1.5849\ 17_{10}-1$ | $4.78_{10}-2$ |
| 95 | 3 | 2 | $1.12_{10}+0$ | $-5.0619\ 93_{10}-1$ | $2.0244\ 57_{10}-1$ | $7.42_{10}-3$ |

# APPENDIX A (Cont.)

| true residuals | | | weight | average readings | | radius |
|---|---|---|---|---|---|---|
| 96 | 3 | -8 | 1.02$_{10}$+0 | -4.9818 94$_{10}$-1 | 2.1997 67$_{10}$-1 | 1.98$_{10}$-2 |
| 97 | 4 | -4 | 1.18$_{10}$+0 | -5.0321 71$_{10}$-1 | 1.9885 15$_{10}$-1 | 4.30$_{10}$-3 |
| 98 | -2 | -1 | 8.37$_{10}$-1 | -5.0966 43$_{10}$-1 | 1.7824 56$_{10}$-1 | 2.43$_{10}$-2 |
| 99 | 4 | 8 | 9.67$_{10}$-1 | -4.9150 12$_{10}$-1 | 2.3500 30$_{10}$-1 | 3.56$_{10}$-2 |
| 100 | 1 | 1 | 1.11$_{10}$+0 | -5.0180 67$_{10}$-1 | 1.9498 40$_{10}$-1 | 5.84$_{10}$-3 |
| 101 | -2 | 3 | 1.09$_{10}$+0 | -4.9575 33$_{10}$-1 | 2.1291 69$_{10}$-1 | 1.32$_{10}$-2 |
| 102 | 2 | -3 | 6.46$_{10}$-1 | -5.0615 13$_{10}$-1 | 1.7769 70$_{10}$-1 | 2.36$_{10}$-2 |
| 103 | -0 | 2 | 1.20$_{10}$+0 | -4.9419 04$_{10}$-1 | 2.0946 16$_{10}$-1 | 1.05$_{10}$-2 |
| 104 | -2 | -5 | 1.14$_{10}$+0 | -4.8428 48$_{10}$-1 | 2.4107 86$_{10}$-1 | 4.35$_{10}$-2 |
| 105 | 1 | -1 | 1.08$_{10}$+0 | -4.9549 02$_{10}$-1 | 2.0295 81$_{10}$-1 | 4.58$_{10}$-3 |
| 106 | -2 | -1 | 1.05$_{10}$+0 | -5.0545 04$_{10}$-1 | 1.7270 41$_{10}$-1 | 2.82$_{10}$-2 |
| 107 | 3 | -6 | 9.17$_{10}$-1 | -5.0017 93$_{10}$-1 | 1.8706 47$_{10}$-1 | 1.32$_{10}$-2 |
| 108 | 0 | -1 | 8.84$_{10}$-1 | -5.1025 81$_{10}$-1 | 1.5773 26$_{10}$-1 | 4.39$_{10}$-2 |
| 109 | 2 | -1 | 1.01$_{10}$+0 | -4.9620 53$_{10}$-1 | 1.9103 11$_{10}$-1 | 9.61$_{10}$-3 |
| 110 | 1 | -8 | 7.43$_{10}$-1 | -5.0385 10$_{10}$-1 | 1.6932 58$_{10}$-1 | 3.12$_{10}$-2 |
| 111 | -4 | 2 | 1.00$_{10}$+0 | -4.8716 10$_{10}$-1 | 2.0154 80$_{10}$-1 | 1.20$_{10}$-2 |
| 112 | -3 | 3 | 1.15$_{10}$+0 | -4.9581 57$_{10}$-1 | 1.7306 30$_{10}$-1 | 2.73$_{10}$-2 |
| 113 | 3 | 1 | 9.09$_{10}$-1 | -4.9192 26$_{10}$-1 | 1.7842 60$_{10}$-1 | 2.29$_{10}$-2 |
| 114 | -0 | 3 | 1.09$_{10}$+0 | -4.9351 89$_{10}$-1 | 1.6821 87$_{10}$-1 | 3.25$_{10}$-2 |
| 115 | 0 | 2 | 8.52$_{10}$-1 | -4.8610 63$_{10}$-1 | 1.8827 79$_{10}$-1 | 1.76$_{10}$-2 |
| 116 | 6 | -6 | 7.60$_{10}$-1 | -4.8614 85$_{10}$-1 | 1.7943 25$_{10}$-1 | 2.45$_{10}$-2 |
| 117 | 3 | -7 | 9.49$_{10}$-1 | -4.7549 71$_{10}$-1 | 2.0927 13$_{10}$-1 | 2.53$_{10}$-2 |
| 118 | 4 | -1 | 8.35$_{10}$-1 | -4.8762 03$_{10}$-1 | 1.7210 12$_{10}$-1 | 3.03$_{10}$-2 |
| 119 | -1 | 6 | 8.46$_{10}$-1 | -4.7520 96$_{10}$-1 | 2.0156 49$_{10}$-1 | 2.40$_{10}$-2 |
| 120 | 7 | -1 | 8.26$_{10}$-1 | -4.7937 09$_{10}$-1 | 1.8405 86$_{10}$-1 | 2.55$_{10}$-2 |
| 121 | 2 | -1 | 8.34$_{10}$-1 | -4.7651 15$_{10}$-1 | 1.9236 71$_{10}$-1 | 2.39$_{10}$-2 |
| 122 | 7 | -5 | 9.39$_{10}$-1 | -4.7932 70$_{10}$-1 | 1.7814 66$_{10}$-1 | 2.96$_{10}$-2 |
| 123 | -0 | -3 | 1.15$_{10}$+0 | -4.8228 14$_{10}$-1 | 1.6800 93$_{10}$-1 | 3.63$_{10}$-2 |
| 124 | -1 | -0 | 1.01$_{10}$+0 | -4.8469 20$_{10}$-1 | 1.5874 20$_{10}$-1 | 4.39$_{10}$-2 |
| 125 | 2 | 4 | 7.73$_{10}$-1 | -4.6267 98$_{10}$-1 | 2.1668 02$_{10}$-1 | 4.00$_{10}$-2 |
| 126 | 4 | 0 | 9.31$_{10}$-1 | -4.7067 73$_{10}$-1 | 1.8327 47$_{10}$-1 | 3.31$_{10}$-2 |
| 127 | -4 | -3 | 1.15$_{10}$+0 | -4.5927 72$_{10}$-1 | 2.0476 03$_{10}$-1 | 4.01$_{10}$-2 |
| 128 | -9 | 3 | 7.98$_{10}$-1 | -4.6941 48$_{10}$-1 | 1.7409 03$_{10}$-1 | 3.95$_{10}$-2 |
| 129 | -9 | 0 | 9.40$_{10}$-1 | -4.6041 50$_{10}$-1 | 1.9842 83$_{10}$-1 | 3.88$_{10}$-2 |
| 130 | 10 | -5 | 8.18$_{10}$-1 | -4.6218 82$_{10}$-1 | 1.8469 48$_{10}$-1 | 4.01$_{10}$-2 |

# APPENDIX B

## SATELLITE DIRECTION PROGRAM OUTPUT DATA

1. 114 a6

22 03 66

2.

| x | weight | y | weight |
|---|---|---|---|
| -5.0692 16$_{10}$-1 | 1.46$_{10}$+0 | 1.7252 81$_{10}$-1 | 3.65$_{10}$-1 |
| -5.0740 14$_{10}$-1 | 1.06$_{10}$+0 | 1.7848 77$_{10}$-1 | 9.86$_{10}$-1 |
| -5.0787 12$_{10}$-1 | 1.33$_{10}$+0 | 1.8440 93$_{10}$-1 | 1.28$_{10}$+0 |
| -5.0833 28$_{10}$-1 | 1.26$_{10}$+0 | 1.9030 56$_{10}$-1 | 1.32$_{10}$+0 |
| -5.0877 95$_{10}$-1 | 7.36$_{10}$-1 | 1.9618 09$_{10}$-1 | 1.03$_{10}$+0 |
| -5.0922 33$_{10}$-1 | 1.45$_{10}$+0 | 2.0203 66$_{10}$-1 | 7.52$_{10}$-1 |
| -5.0965 71$_{10}$-1 | 1.06$_{10}$+0 | 2.0787 73$_{10}$-1 | 5.62$_{10}$-1 |

4. y,x coefficients
-2.62167$_{10}$-7    1.42363$_{10}$-3    -7.70073$_{10}$-2

6. x,t coefficients
7.62939$_{10}$-6    2.42138$_{10}$-3    -1.31130$_{10}$-6    2.23517$_{10}$-8

7. weighted r.m.s.residuals,xt= 4.044$_{10}$+0    yx= 1.299$_{10}$+0    microns

8. true residuals

| | x,t | y,x |
|---|---|---|
| 1 | -6.9 | 0.5 |
| 2 | 0.3 | 0.3 |
| 3 | 4.7 | -0.2 |
| 4 | 2.3 | -1.7 |
| 5 | -2.6 | 2.4 |
| 6 | -4.3 | 0.1 |
| 7 | 1.0 | -1.0 |

1. 22 03 66

2. latitude    052 08 39.114
longitude(west)    001 58 00.508

reference stars reduced using ut1

3. corrections applied    astronomical refraction
time signal delays
not applied    parallactic refraction
light travel time(aberration)

4.

| nominal time | azimuth | elevation | gha | dec |
|---|---|---|---|---|
| 00 35 00.0000 | 104 47 09.80 | 59 30 41.67 | 21 36 25.585 | 36 56 10.29 |
| | | calc.ref(secs) | 35.129 | |
| 00 35 04.0000 | 103 46 40.64 | 59 17 35.19 | 21 33 53.724 | 37 10 22.36 |
| | | calc.ref(secs) | 35.436 | |
| 00 35 08.0000 | 102 47 15.44 | 59 04 01.71 | 21 31 21.436 | 37 24 16.17 |
| | | calc.ref(secs) | 35.748 | |
| 00 35 12.0000 | 101 48 49.23 | 58 50 01.37 | 21 28 48.538 | 37 37 53.10 |
| | | calc.ref(secs) | 36.080 | |
| 00 35 16.0000 | 100 51 24.20 | 58 35 33.05 | 21 26 14.969 | 37 51 10.98 |
| | | calc.ref(secs) | 36.428 | |
| 00 35 20.0000 | 099 54 57.86 | 58 20 41.54 | 21 23 40.922 | 38 04 13.28 |
| | | calc.ref(secs) | 36.779 | |
| 00 35 24.0000 | 098 59 32.05 | 58 05 25.32 | 21 21 06.315 | 38 16 57.58 |
| | | calc.ref(secs) | 37.146 | |

## BIOGRAPHICAL NOTES

JOSEPH HEWITT received his B.Sc. from the University of London and studied technical optics at Imperial College, London. He joined the Royal Radar Establishment, Ministry of Technology, in Malvern, Worcestershire, in 1942. He has worked in the field of colorimetry, visual optics, and optical instrumentation.

Mr. Hewitt is currently with a group working on the optical tracking of targets at the Royal Radar Establishment. He was made a Principal Scientific Officer in 1955. In 1956, his group entered the field of optical tracking of satellites and he was made responsible for the design and development of the Hewitt f1 Schmidt satellite camera.

JAN ROLFF received his degrees in geodesy from the Technological University, Delft, Netherlands, in 1946 and 1951.

He is now Executive Director of the Central Bureau for Satellite Geodesy. He has held this position since 1964, when the Bureau was established at the Smithsonian Astrophysical Observatory at the request of the International Union of Geodesy and Geophysics and the International Committee on Space Research.

He is also an active participant in the Observatory's own program of geodetic research.

DAVID A. ARNOLD received his A.B. in physics from Boston University in 1962. He worked as a physical science assistant at the U.S. Army Cold Regions Research and Engineering Laboratory in New Hampshire until 1964.

Mr. Arnold joined SAO in 1964 as an astrometric computer and was promoted to senior astrometric computer the following year. He is currently working as a project leader in the Photoreduction Division.

## NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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