TG–986
APRIL 1968

Technical Memorandum

ROLL RESONANCE FOR A GRAVITY-GRADIENT SATELLITE

by J. M. WHISNANT and D. K. ANAND

THE JOHNS HOPKINS UNIVERSITY ▪ APPLIED PHYSICS LABORATORY
8621 Georgia Avenue, Silver Spring, Maryland 20910

Operating under Contract NOw 62-0604-c with the Department of the Navy

ABSTRACT

Previous studies of gravity-gradient satellite attitude stabilization showed that large roll angles could occur for a partially shadowed orbit. The cause was determined to be a resonance effect due to solar radiation pressure. Here an expression for the solar torque for roll on a dumbbell-shaped satellite is presented. The amplitude of the torque is shown to be a function of the angle between the satellite-sun line and the normal to the orbit plane. For circular orbits, an expression is derived to determine for what position of the sun relative to the orbit plane the resonance effect is a maximum. For orbits of modest eccentricity, the amount of orbit shadowed as a function of sun-orbit orientation is determined. The persistence of the resonance effect for retrograde orbits is discussed.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
SILVER SPRING, MARYLAND

ABSTRACT

Previous studies of gravity-gradient satellite attitude stabilization showed that large roll angles could occur for a partially shadowed orbit. The cause was determined to be a resonance effect due to solar radiation pressure. Here an expression for the solar torque for roll on a dumbbell-shaped satellite is presented. The amplitude of the torque is shown to be a function of the angle between the satellite-sun line and the normal to the orbit plane. For circular orbits, an expression is derived to determine for what position of the sun relative to the orbit plane the resonance effect is a maximum. For orbits of modest eccentricity, the amount of orbit shadowed as a function of sun-orbit orientation is determined. The persistence of the resonance effect for retrograde orbits is discussed.

Introduction

Pre- and post-launch studies of the geodetic satellite (GEOS-A), launched in November 1966, indicated that large roll angles could occur when the orbit was partially shadowed [1]. An examination of the solar radiation pressure forcing function for roll showed that it had a component with twice orbital frequency. Since the natural frequency of roll for a gravity-gradient satellite is also twice orbital, the occurrence of resonance is clear. Furthermore, the amplitude of the roll librations was shown to be a function of Ω, the angle between the projection of the earth-sun line onto the equatorial plane and the line of nodes. The purpose of this note is to derive an expression for the value of Ω which maximizes the twice orbital component of the roll forcing function.

Analysis

The solar forcing function φ in roll for a satellite whose geometry is similar to that of a dumbbell is given by [1] as

$$\varphi = \varphi_o \sin i \sin \Omega \qquad (1)$$

where i is the orbital inclination and the satellite is in the sunlit part of the orbit with the earth-sun line lying in the equatorial plane. φ_o is a function of the solar flux and physical properties of the satellite as well as a weak function of the satellite's attitude. For small librational angles, it may be considered constant. If the sun is allowed to have a declination, δ_s, then the forcing function is

$$\varphi = \varphi_o \cos \eta \tag{2}$$

where η is interpreted to be the angle between the earth-sun line and the normal to the orbit plane. Using an equatorial coordinate system and zero right ascension of the sun, its direction cosines are $(\cos \delta_s, 0, \sin \delta_s)$. The direction cosines of the normal to the orbit plane are then given by $(\sin i \sin \Omega, -\sin i \cos \Omega, \cos i)$ so that

$$\cos \eta = [\sin i \sin \Omega \cos \delta_s + \cos i \sin \delta_s]. \tag{3}$$

If the orbit is partially shadowed then

$$\varphi = \begin{cases} \varphi_o \cos \eta & \text{satellite in sunlight} \\ 0 & \text{satellite in shadow} \end{cases} \tag{4}$$

as shown in Figure 1.

The amplitude of the second harmonic when equation (4) is expanded by a Fourier series is

$$\varphi(2\theta) = \frac{\varphi_o}{\pi} [\cos \eta \sin \beta(\eta)] \tag{5}$$

where β is the amount of orbit shadowed in radians and θ is the argument of satellite latitude.

Analysis of equation (5) with reference to equation (3) indicates that

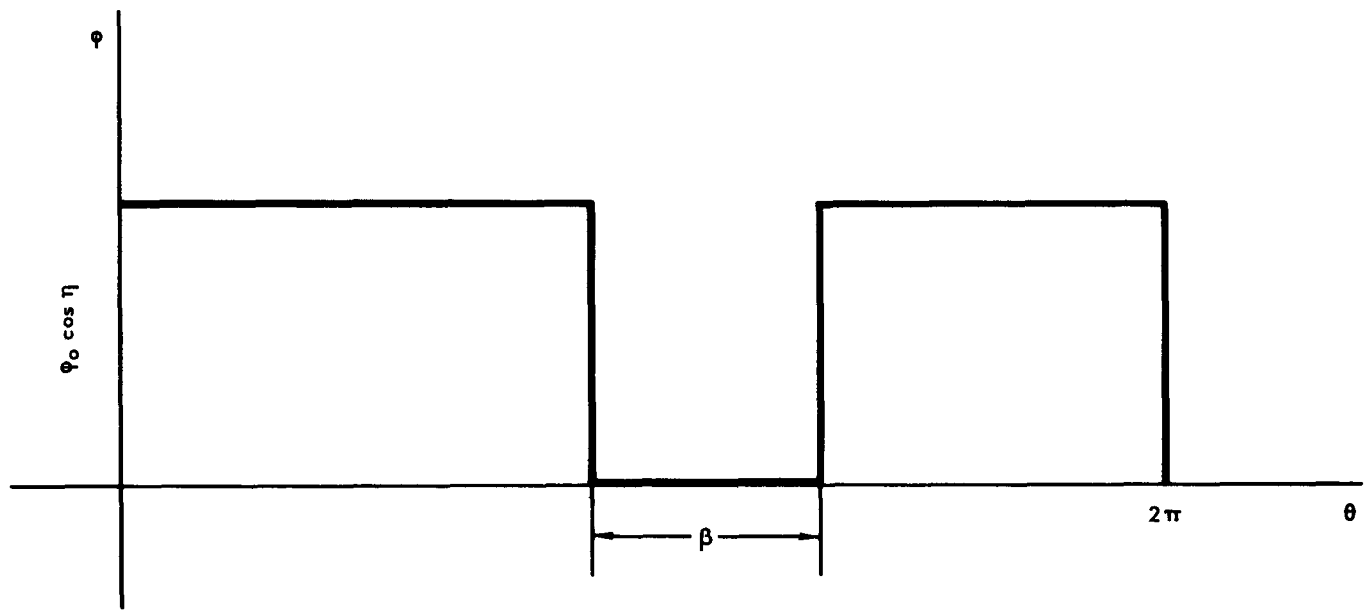

Fig. 1 SOLAR TORQUE FOR ROLL DURING ONE NODAL PERIOD

the variable in (5) is Ω for any given satellite, orbit, and epoch. It is to be noted that since β is a function of η, it also is a function of Ω, as will be shown.

For circular orbits Patterson [2] gives a formula for the time that a satellite spends in the sun. This expression, which neglects penumbra effects, is used to obtain

$$\beta = \pi - 2 \sin^{-1} \left| \frac{F}{\sin \eta} \right| \tag{6}$$

where

$$F = (1 - 1/a^2)^{1/2}$$

and a is the semi-major axis in units of earth radii.

Sustituting the above into (5) yields

$$\varphi(2\theta) = \frac{\varphi_o}{\pi} \left[\cos \eta \frac{2F}{\sin^2 \eta} (\sin^2 \eta - F^2)^{1/2} \right] \tag{7}$$

Since we are interested in maximizing (7), we set $d\varphi/d\Omega = 0$ and using equation (3) obtain, after some algebra,

$$\Omega \Big|_{\substack{Max \\ Roll}} = \sin^{-1} \left\{ \frac{1}{\sin i \cos \delta_s} \left[(2a^2 - 1)^{-1/2} - \cos i \sin \delta_s \right] \right\} \tag{8}$$

The above equation provides the nodal angle that will give the largest roll libration with twice orbital frequency. As an example, consider a satellite which has a = 1.2, i = 74° and $\delta_s = 0$. The variation of the "twice orbital roll" amplitude for a partially shadowed orbit is shown in Figure 2.

For an eccentric orbit, Escobal [3] gives the shadow equation as a quartic in the true anomaly which cannot in general be solved analytically. Here expressions are derived to reflect the first-order effects of eccentricity, ϵ. For the case $\delta_s = 0$ (this simplifies the algebra without imposing constraints on the applicability), the arguments of latitude, θ_I and θ_0, at which the satellite enters and exits from the earth's shadow satisfy

$$\cos\theta\cos\Omega - \cos i\sin\theta\sin\Omega = -(1 - 1/r^2)^{1/2}, \quad \text{where} \qquad (9)$$

r is the magnitude of the radius vector to the satellite, and for perigee at $\theta_p = \pi$ is given by

$$r = a(1 - \epsilon^2) / (1 - \epsilon\cos\theta). \qquad (10)$$

Expanding the right-hand side of (9) and retaining only terms of first order in eccentricity yield

$$(1 - 1/r^2)^{1/2} = F + \frac{\epsilon}{a^2}\frac{1}{F}\cos\theta \qquad (11)$$

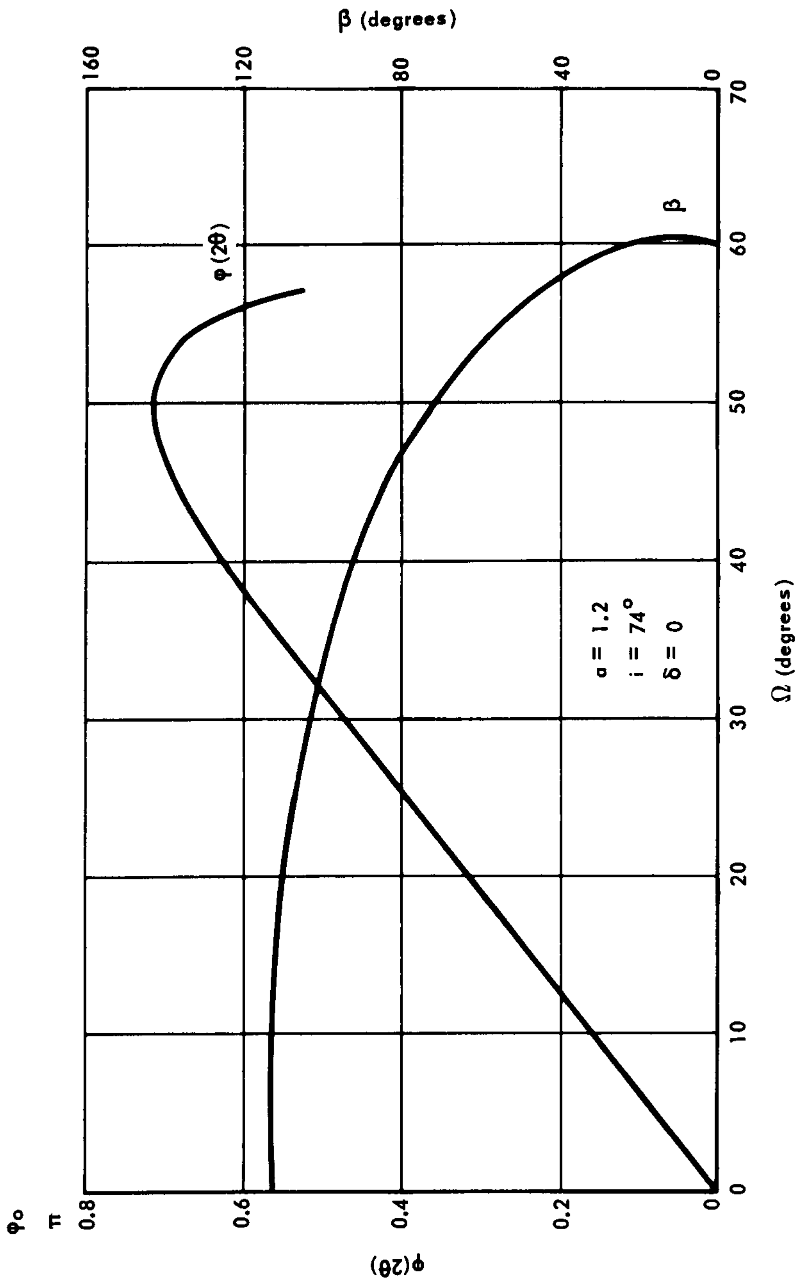

Fig. 2 TWICE ORBITAL ROLL FORCING FUNCTION FOR A PARTIALLY SHADOWED ORBIT

where F has been defined previously. Substituting (11) into (9) and solving for β $(= \theta_0 - \theta_I)$, we obtain

$$\beta = \sin^{-1} [\frac{2F}{\Delta^2} (\Delta^2 - F^2)^{1/2}] \tag{12}$$

where

$$\Delta^2 = (\cos \Omega + \frac{\epsilon}{F a^2})^2 + (\cos i \sin \Omega)^2 \quad .$$

Using this expression in (5), the amplitude of the second harmonic becomes

$$\varphi(2\theta) = \frac{\varphi_0}{\pi} [\cos \eta \frac{2F}{\Delta^2} (\Delta^2 - F^2)^{1/2}] . \tag{13}$$

Although (13) is similar to (7), setting $d\varphi/d\Omega = 0$ no longer yields a closed-form solution. However, any of several numerical techniques are applicable. Figure 3 shows $\beta = \beta(\Omega)$ using both (6) and (12) and compares them with β obtained by numerically solving Escobal's equation.

The selection of the correct orbital parameters becomes particularly significant when we consider a retrograde orbit. In such an orbit the sense of nodal precession P_Ω and the precession of the sun (which is about 0.985 degrees/day) is the same. Furthermore, $P_\Omega = f(i,a,\epsilon)$ [4], and i,a,ϵ could be so selected that the magnitude of nodal precession becomes equal to that of the sun. If indeed this happens and the orbit is partially shadowed the condition of roll resonance can persist near its maximum value for long periods (Figure 4). We note, however, that for prograde orbits the precessional sense of Ω is opposite to that of the the sun, and roll resonance will not persist for long periods since the fraction of shadowed orbit is itself going to vary.

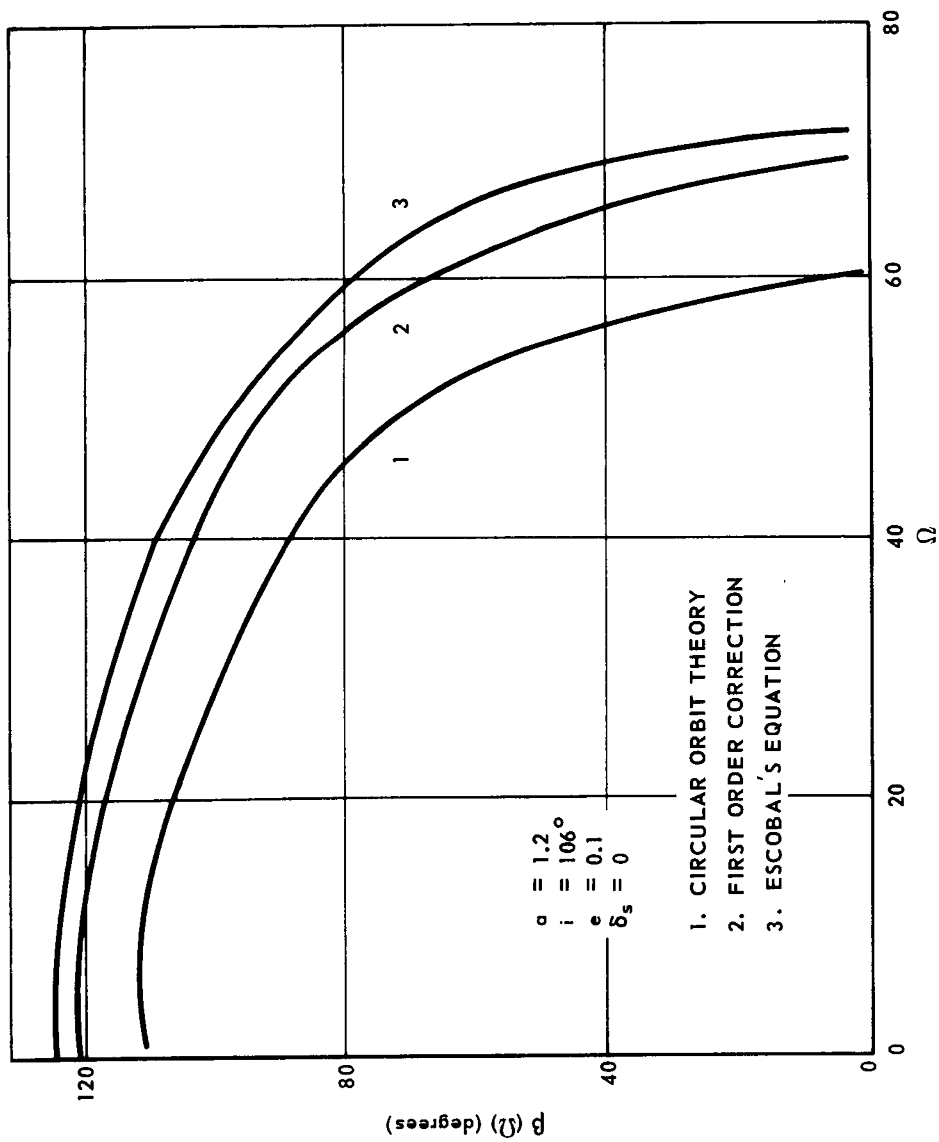

Fig. 3 SHADOWING FOR AN ECCENTRIC ORBIT

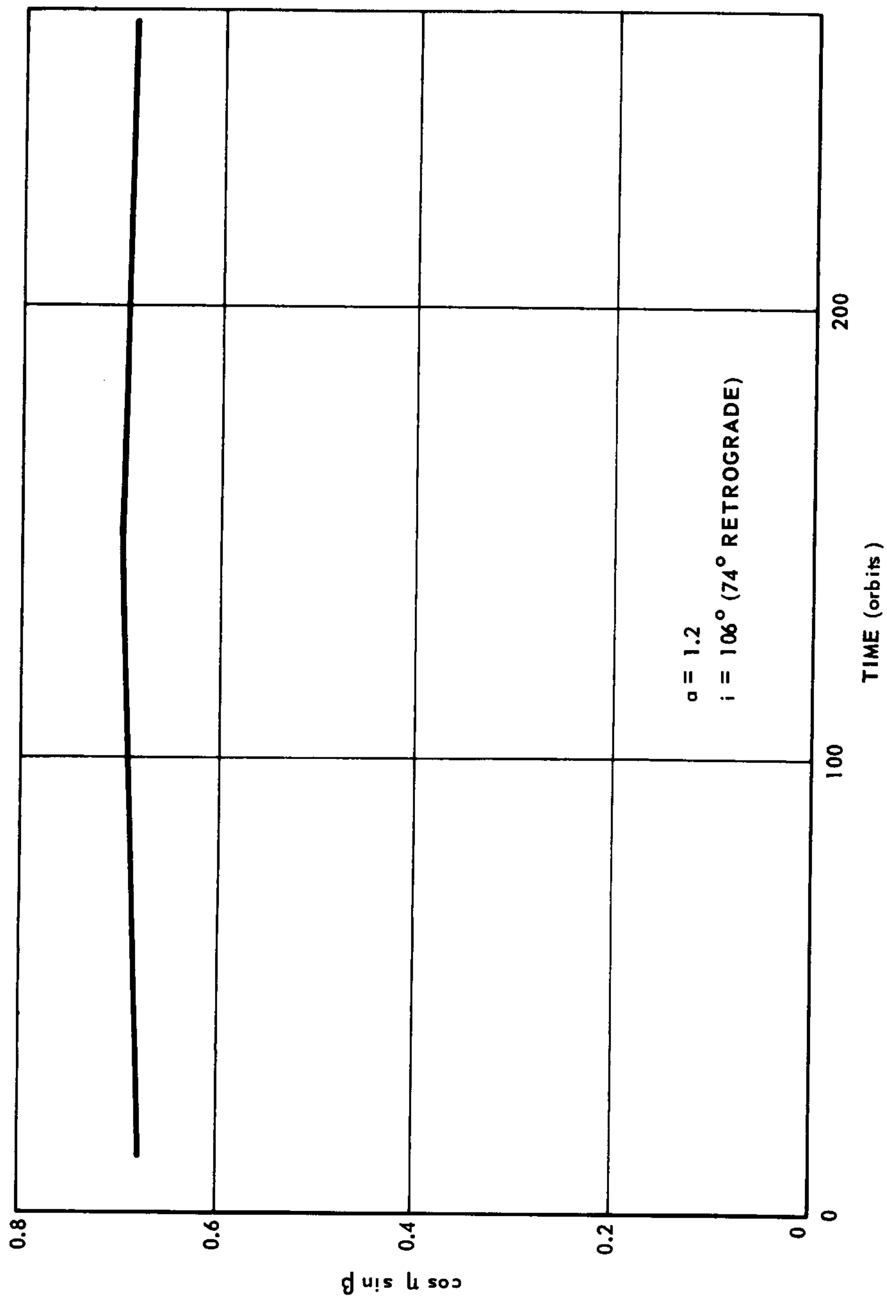

Fig. 4 TIME VARIATION OF THE RESONANCE EFFECT

REFERENCES

1. Pisacane, V. L., Pardoe, P. P., and Hook, B. J., "Stabilization System Analysis and Performance of the GEOS-A Gravity-Gradient Satellite," Journal of Spacecraft and Rockets, Vol. 4, No. 12, Dec. 1967, p. 1626.

2. Patterson, G. B., "Graphical Method for Prediction of Time in Sunlight for a Circular Orbit," ARS Journal, Vol. 31, No. 3, March 1961, p. 441.

3. Escobal, P. R., "Orbital Entrance and Exit From the Shadow of the Earth," ARS Journal, Vol. 32, No. 12, Dec. 1962, p. 1940.

4. King-Hele, D., Theory of Satellite Orbits in an Atmosphere, Butterworths, London, 1964.

ACKNOWLEDGMENT

The authors would like to acknowledge a helpful discussion with Robert E. Jenkins of the Applied Physics Laboratory.

INITIAL DISTRIBUTION EXTERNAL TO THE APPLIED PHYSICS LABORATORY*

The work reported in TG-986 was done under Navy Contract NOw 62-0604-c. This work is related to Task I41, which is supported by NASA.

ORGANIZATION	LOCATION	ATTENTION	No. of Copies
DEPARTMENT OF DEFENSE			
DDC	Alexandria, Va.		20
Department of the Navy			
NAVPLANTREPO	Silver Spring, Md.		1
NAVORDSYSCOM	Washington, D. C.	ORD-9132	2
U.S. GOVERNMENT AGENCIES			
NASA	Washington, D. C.	J. Rosenberg, Code SAG	1
UNIVERSITIES			
Canaan College	Canaan, New Hampshire	P. Pardoe	1

Requests for copies of this report from DoD activities and contractors should be directed to DDC, Cameron Station, Alexandria, Virginia 22314 using DDC Form 1 and, if necessary, DDC Form 55.

*Initial distribution of this document within the Applied Physics Laboratory has been made in accordance with a list on file in the APL Technical Reports Group.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) The Johns Hopkins Univ., Applied Physics Lab. 8621 Georgia Avenue Silver Spring, Maryland	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP n. a.

3. REPORT TITLE

Roll Resonance for a Gravity-Gradient Satellite

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

J. M. Whisnant and D. K. Anand

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1968	11	4

8a. CONTRACT OR GRANT NO. NOw 62-0604-c b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) TG-986 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	NASA

13. ABSTRACT

Previous studies of gravity-gradient satellite attitude stabilization showed that large roll angles could occur for a partially shadowed orbit. The cause was determined to be a resonance effect due to solar radiation pressure. An expression for the solar torque for roll on a dumbbell-shaped satellite is presented herein. The amplitude of the torque is shown to be a function of the angle between the satellite-sun line and the normal to the orbit plane. For circular orbits, an expression is derived to determine for what position of the sun relative to the orbit plane the resonance effect is a maximum. For orbits of modest eccentricity, the amount of orbit shadowed as a function of sun-orbit orientation is determined. The persistence of the resonance effect for retrograde orbits is discussed.

14. KEY WORDS

Gravity-gradient stabilization

Resonance

Orbit shadowing