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INTERPRETATION OF GEOMAGNETIC Pc3,4 PULSATIONS

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SUMMARY

Discussed in this paper are the possible Alfvên resonances in sections of tubes of force bounded on one side by the weak field region in the neighborhood of neutral points at magnetosphere boundary and on the other side - by the Earth's surface. It is shown that if resonance excitation on the fundamental harmonic is more probable in the tubes of force closest to neutral points, resonances on the second harmonic may result more effecive with the transition to deeper layers of the magnetosphere. The resonance on the fundamental harmonic is identified with the Pc4 pulsations, while the one on the second harmonic is identified with Pc3.

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The possibility of formation of a standing Alfvên wave in section of magnetic tubes of force bounded on one side by the weak field region(i.e. "thickening" on the tube of force) near neutral points lying on magnetosphere boundary and on the other by the Earth's surface, was discussed in [1]. The resonance oscillations thus occurring may correspond to type-Pc4 geomagnetic pulsations. In tubes of force passing at a certain distance from neutral points connected with the latter "thickenings" smooth out, the reflection factor of their resonant wave decreases; for a certain critical thickening, when the wavelength is more effective than the length of the surrounding sector, the resonance at these wavelengths will generally be impossible, although shorter wave oscillations may still effectively reflect and resonate. This effect may result in that standing Alfvên waves with fundamental period, linked by us with Pc4-pulsations in the 45 - 150 sec period range in tubes of force more remote from neutral points, resonances will be possible only on higher harmonics; it will be natural to attempt to link the latter with the Pc3-pulsations (10-45 sec period range).

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* INTERPRETATSIYA GEOMAGNITNYKH PUL'SATSIY Pc3,4.

In order to estimate quantitatively the effectiveness of the proposed mechanism we must know the course of the index of refraction along the line of force. In the Alfvén region the square of the index of refraction has the form

$$n^2 = 4\pi c^2 \frac{\rho}{H^2}, \tag{1}$$

where c is the speed of light, ρ is the density of the magnetospheris plasma, H is the intensity of the external magnetic field. The variation of the quantity $1/H^2$ along the line of force may be computed by using the expression for the magnetic field in the form of the sum of the dipole part of the Earth's magnetic field and of the first 12 terms of expansion in spherical functions of the field from currents flowing along the magnetosphere boundary [2]. Limiting ourselves to the consideration of the field only in the solar meridian plane and choosing the Descartes system of coordinates with axis x directed along the Earth's magnetic moment toward the North pole, and axis y, directed at the Sun, we shall obtain

$$\begin{aligned}
H_x = {} & 0{,}315\,\frac{-2x^2 + y^2}{(x^2 + y^2)^{5/2}} - g_1^0 + \sqrt{3}\,g_2^1 y - 3g_3^0 x^2 + \\
& + \left(\frac{3}{2} g_3^0 - \frac{\sqrt{15}}{2} g_3^2\right) y^2 + 3\sqrt{10}\,g_4^1 x^2 y - \left(3\sqrt{\frac{5}{8}}\,g_4^1 - \sqrt{\frac{35}{8}}\,g_4^3\right) y^3 - 5g_5^0 x^4 + \\
& + \left(15g_5^0 - \frac{3}{2}\sqrt{105}\,g_5^2\right) x^2 y^2 - \left(\frac{15}{8} g_5^0 - \frac{\sqrt{105}}{4} g_5^2 + \frac{\sqrt{315}}{8} g_5^4\right) y^4 + \\
& + 5\sqrt{21}\,g_6^1 x^4 y + \left(\frac{15}{2}\sqrt{21}\,g_6^1 - \frac{3}{2}\sqrt{210}\,g_6^3\right) x^2 y^3 + \\
& + \left(\frac{5}{8}\sqrt{21}\,g_6^1 - \frac{3}{16}\sqrt{210}\,g_6^3 - \frac{\sqrt{1386}}{16} g_6^5\right) y^5, \\
H_y = {} & -0{,}315\,\frac{3xy}{(x^2 + y^2)^{5/2}} + \sqrt{3}\,g_2^1 x + (3g_3^0 - \sqrt{15}\,g_3^2)\,xy + \\
& + \sqrt{10}\,g_4^1 x^3 - \left(9\sqrt{\frac{5}{8}}\,g_4^1 - 3\sqrt{\frac{35}{8}}\,g_4^3\right) xy^2 + (10g_5^0 - \sqrt{105}\,g_5^2)\,x^3 y - \\
& - \left(\frac{15}{2} g_5^0 - \sqrt{105}\,g_5^2 + \frac{\sqrt{315}}{2} g_5^4\right) xy^3 + \sqrt{21}\,g_6^1 x^5 - \\
& - \left(\frac{15}{2}\sqrt{21}\,g_6^1 - \frac{3}{2}\sqrt{210}\,g_6^3\right) x^3 y^2 + \\
& + \left(\frac{25}{8}\sqrt{21}\,g_6^1 - \frac{15}{16}\sqrt{210}\,g_6^3 + \frac{5}{16}\sqrt{1386}\,g_6^5\right) x y^4,
\end{aligned} \tag{2}$$

where the coefficients g_m^n are brought out in [2], and x, y are expressed in Earth's radii. Assigning ourselves a point on the Earth's surface with coordinates (x_0, y_0), and utilizing the evident relations

$$x_1 = x_0 - \Delta l \cdot \frac{H_x^0}{\sqrt{(H_x^0)^2 + (H_y^0)^2}} \qquad y_1 = y_0 - \Delta l \cdot \frac{H_y^0}{\sqrt{(H_x^0)^2 + (H_y^0)^2}} \qquad (3)$$

$H_x^0 = H_x(x_0, y_0)$, $H_y^0 = H_y(x_0, y_0)$; Δl being the small distance between the points (x_0, y_0) and (x_1, y_1) along the line of force, we may successively construct the entire line of force, computing simultaneously the magnetic field along it according to (2). Defining $\Delta l = 0{,}01\ R_E$ (R_E being the radius of the Earth) and postulating the distance to magnetosphere boundary from the solar side to be r_b = 10 R_E (this quantity determines g_m^n [2]), we have computed the variation of the magnetic field along a few lines of force in the vicinity of the neutral point (at r_b = 10 R_E the latter is linked with the line of force crossing the Earth's surface at the geomagnetic latitude $\Phi \approx 83°$). The variation of $1/H^2$ along four lines of force (corresponding to Φ = 82, 81, 80 and 79°) of interest to us is shown in Fig.1, where the distance is plotted in the abscissa along the line of force and in Earth's radii, counted from the region of minimum value of H.

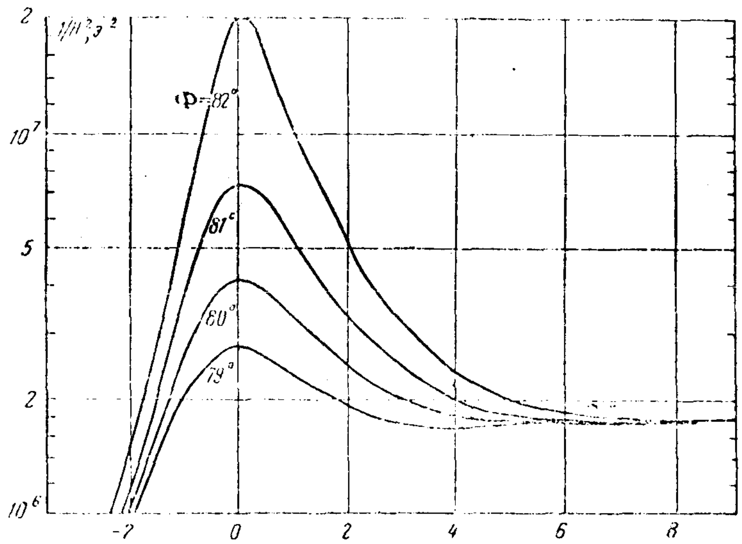

Fig.1

For the determination of the course of refractive index along the line of force, it remains to define the plasma density distribution in the magnetosphere. When interpreting whistlers, the density in the magnetosphere is usually assumed to be proportional to $r^{-3} - r^{-4}$ (r being the geocentric distance). However, our calculations are substantially simplified only in case of plasma distribution along the line of force determined by the curves of Fig.2 (N being the concentration of charged particles and z the distance along the line of force in R_E, counted from the region of field intensity minimum); the plasma density determined by these curves decreases proportionally to $r^{-5} - r^{-6}$ over the greater part of line of force trajectory. Such a distribution presupposes overrated values of charged particle concentrations at small distances from Earth and underrated at large distances by comparison with the probable values [3]. This is well seen in Fig.2, where the upper curves correspond to geocentric distance $r = 2R_E$, the overrated values taking into account the sharp plasma density drop beyond the limits of the "knee". Taking advantage of (1), it is now possible to plot the dependence of the magnitude of refractive index's square along the line of force.

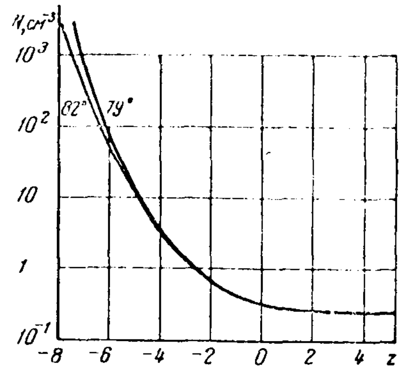

Fig.2

As in Figures 1 and 2, the distance along the line of force is plotted in Fig.3 along the abscissa axis; this distance is counted in Earth's radii starting from the region of field intensity minimum. The curves $n^2(z)$, shown in Fig.3 may be fairly well approximated by symmetrical curves of the form

$$n^2(z) = n_0^2\left(1 + 4M\frac{e^{mz}}{(1+e^{mz})^2}\right), \tag{4}$$

where $M = (n_{max}^2/n_0^2 - 1)$, n_{max} and n_0 are respectively the maximum and minimum (at origin and infinity) values of the index of refraction; m is a coefficient characterizing the rate of refractive index's variation. The numerical values of **parameters M and m, obtained** with the approximation are compiled in Table 1.

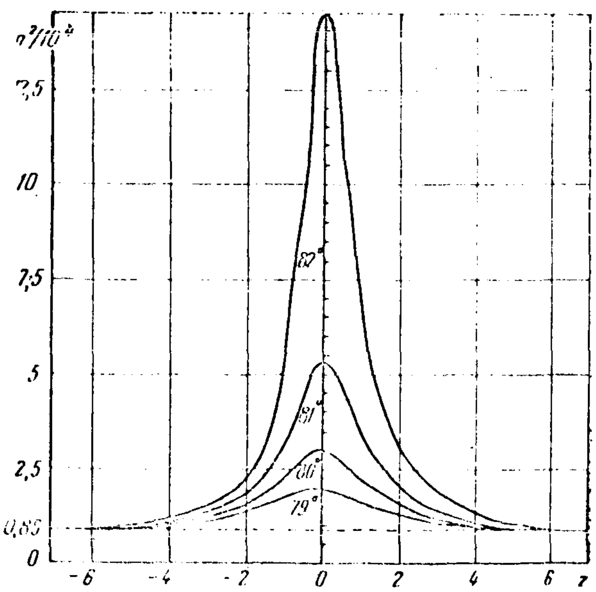

Fig.3

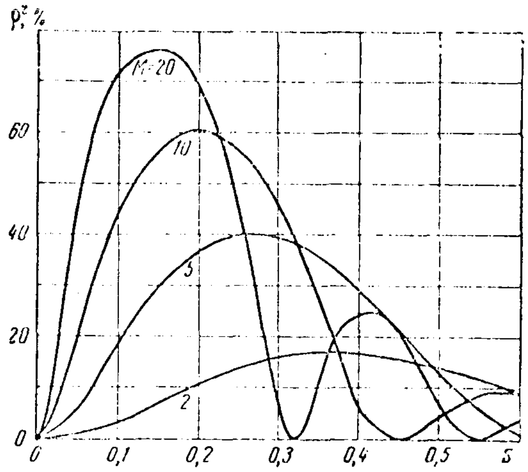

Fig.4

The index of refraction by energy from the region with index of refraction of the form (4) is given in [4]; in the case taking place here of normal wave incidence, it may be written in the form

$$\rho^2 = \frac{1}{1 + \frac{\mathrm{sh}^2 \pi s}{\cos \pi d}}, \tag{5}$$

where

$$s = \frac{4\pi n_0}{mcT}, \qquad d = \frac{1}{2}\sqrt{1 + 4s^2 M},$$

T is the period of the considered harmonic oscillations. Taking into account that $n_0^2 \approx 0.86 \cdot 10^4$ (see Fig.4), we obtain $s \approx 3{,}87 \cdot 10^{-8} / mT$. Substituting here the obtained values of m and assuming the period of the fundamental harmonic identified by us with Pc4 at $T_1 = 90$ sec. and the period of the second harmonic at $T_2 = 45$ sec., it is possible to obtain the value of parameter s in all cases of interest to us (see Table 1).

TABLE 1

Φ	M	m, $cm^{-1} \cdot 10^{-8}$	T = 90 sec		T = 45 sec	
			s	ρ^2, %	s	ρ^2, %
82°	15	3,4	0,13	69	0,25	55
81	1,9	2,5	0,17	33	0,34	37
80	2,4	2,1	0,20	15	0,11	20
79	1,2	1,8	0,21	5	0,48	9

As may be seen from (5), the reflection factor ρ^2 oscillates with the variation of s and will approach zero as $s \to 0$, as well as $s \to \infty$. The oscillation is induced by the fact that the waves, reflected from the region of growth and drop of $n^2(z)$, as s varies (for example on account of wave frequency variation) will be at times in phase and in other in opposite phase. The decrease of ρ^2 for great and very small s is linked in the first case with the decrease of the gradient dn/dz at a distance of the order of the wavelength, and in the second case with wave seepage through the region of fast n(z) variation in wavelengths great by comaprison with the effective extension of this region. The curves $\rho^2(s)$ are plotted in Fig.4 for 4 cases of M. Using these curves one may estimate the wave's reflection factor for various laws of $n^2(z)$ variations of the form (4). It may be seen from Fig.4 too that for curves $n^2(z)$ characterized by approximately identical parameters m, the reflection factor maximum at decrease $M = (n_{max}^2/n_0^2 - 1)$ (which is equivalent to "thickening" decrease on the line of force) is shifted toward the side of greater s (i.e. smaller oscillation periods). The exact numerical values of ρ^2, computed in cases of interest to us according to (5), are compiled in Table 1, from which it may be seen that in the cases considered by us, the excitation of resonance oscillations on the second harmonic, rather than on the first, becomes indeed more probable as parameter M decreases.

If resonance oscillations with fundamental period are reliably linked with Pc4 oscillations, the resonance oscillations in higher harmonics will be naturally linked with Pc3 oscillations. Inasmuch as the assumed excitation mechanisms of both types of oscillations are entirely identical, the similarity between the latter is also naturally explained [5, 6]: both Pc3 and Pc4 represent basically a summer daytime phenomenon, their oscillations increasing toward the morning-evening side of the Earth in magnetoquiet time and decreasing with the rise of magnetic disturbance with amplitudes of both types of oscillations rising toward high latitudes. From the standpoint of the developed model the explanations of these regularities in the case of Pc4 is given in [1]; similar regularities of Pc3 are explained in an entirely similar manner.

types of scillations rising toward high latitudes. From the standpoint of the developed model the explanation of these regularities in the case of Pc4 is given in [3]; similar regularities of Pc3 are explained in an entirely similar manner.

Let us now pause at some differences between Pc3 and Pc4.

1) If Pc3 oscillations are usually observed in magnetodisturbed days. Pc4 occur mainly in magnetoquiet days [5]. Within the framework of our own model this may be explained by the fact that in magnetoquiet time intervals the disturbances are found to be localized near the neutral points, inducing there Pc4, without reaching the "concealed" tubes of force somewhat deeper in the magnetosphere, which are responsible for Pc3. As the magnetic activity increases, shocks along the magnetosphere become stronger and more frequent; because of continuous irregular shift of neutral points, resonance in their immediate vicinity may result to be ineffective (Pc4 cannot be excited); to the contrary, the irregular disturbances spreading deep into the magnetosphere may excite the tubes of force responsible for Pc3. Simultaneous excitation of Pc3 and Pc4 may naturally be expected, which is observed in reality quite often indeed [5].

2) The average period of Pc4 constitutes 80 - 100 sec, while that of Pc3 is of 20 to 30 sec [5, 6], i.e., the periods differ by more than a factor of two. This difference is explained in particular by the fact that, as was noted earlier, Pc3 are excited mainly in perturbed times, when the magnetosphere is contracted, and the resonating segments of tubes of force are shortened by comparison with the magnetoquiet period for which excitation of Pc4 is characteristic.

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