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The Effect of Bulk Motion on
the Synchrotron Radiation Rate

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Recent criticisms [Epstein and Feldman, 1967; Takakura and Uchida, 1968] of the synchrotron emission power formula $P = 2e^4 H_1^2 \gamma^2 / 3m_0^2 c^3$ [Westfold, 1959] have largely been countered in the elegant analysis by Scheuer [1968]; however, Scheuer's analysis and his conclusion that the original result stands are not rigorously true when bulk motion occurs. The departures from the classical result to be discussed here will be significant only for nonstationary sources (and especially for resolved nonstationary sources). The results have actually been in the literature for some time [Robertson, 1938; Rees, 1966] and apply no matter what the emission mechanism; it is precisely because Scheuer's conclusion seems to disagree with analyses such as those of Robertson and Rees that comment is called for.

Briefly, I shall argue that the surest method for obtaining the proper radiation rate for an ensemble of relativistic particles in bulk (mean) motion, is to calculate the power emitted in the center of momentum system, and then to transform to the laboratory system. The case of emission that is isotropic in the rest frame was treated by Robertson [1938] and clearly leads to a brightening (increased power) when the object approaches the observer. Epstein and Feldman [1967] treated the case of a certain

anisotropic emitter (one electron) and found a brightening by a factor $(\sin \alpha)^{-2}$, where α is the angle between the guiding center velocity (taken to be approaching) and the line of sight. Now Scheuer asserts that the apparent luminosity of any synchrotron-radiating source may be found by multiplying (power per electron, according to the conventional formula) by (mean number of electrons in the source with appropriate pitch angle to radiate toward the observer). This rule is valid for steady sources that do not move, but I shall adduce two examples for which it fails. The first example is that of an isotropic distribution of electrons in a tangled magnetic field, the whole in motion. This object emits isotropically in its rest frame, so Robertson's [1938] law of luminosities may be applied. According to this law, one finds the red-shift z of the object by the law [Jackson, 1962]

$$1 + z = (1 - \beta \cos \theta) / (1 - \beta^2)^{\frac{1}{2}} = \lambda' / \lambda_0 \quad (1)$$

where $\vec{c\beta}$ is the center-of-mass velocity of the object, directed at the angle θ to the line of sight, $\theta = 0$ corresponding to approach. Then if L_0 is the luminosity in the rest frame of the object, the observed luminosity is

$$L' = L_0 / (1 + z)^4 \quad (2)$$

This formula may be interpreted as follows: two powers of $(1 + z)$ are due to the relativistic transformation [Landau and Lifshitz, 1962] of plane-wave Fourier components emitted by the object (Hubble's [1936] "energy effect" and "number effect.") The remaining two powers of $(1 + z)$ are due to the aberration of the emitted waves, which become peaked in the forward direction. Since relativistic electrons emit in the forward direction, proper attention to the transformation of the electron distribution to the frame of the observer will disclose a change by the factor $(1 + z)^{-2}$ in the number with appropriate pitch angle to radiate toward the observer, as compared with the number that would be so oriented if the source had no mean motion. Substituting in Scheuer's rule (above), one would find a luminosity change $L' = L_0 / (1 + z)^2$, which is incorrect. (In all this discussion, z may be negative, leading to brightening.) Hubble's "energy effect" and "number effect" are omitted.

The second example is Woltjer's [1966] model for a quasi-stellar source, also cited by Scheuer. If the object is in steady state, Scheuer's statement is valid, that the classical radiation law may be applied in the manner he suggests. Consider, however, if (as must happen in a quasi-stellar source) there is sudden injection of an additional supply of electrons. In Woltjer's model, the electrons all stream out from the center nearly radially, along the magnetic lines of force. I assume, without loss of generality,

that the sudden injection occurs near the center, and that the injected electrons stream outward relativistically. We see only those very close to a line of sight passing through the center. Let us consider that when they reach a distance R from the center, they have lost most of their energy; R is then more or less the radius of the QSO (at least the continuum-emitting part of it.) The total energy radiated may be found by multiplying the classical, invariant radiation rate by the actual transit time $R/(c \cos \bar{\alpha})$, where $\bar{\alpha}$ is the mean pitch angle (≈ 0) of the electrons. But this is seen in the foreshortened time [Epstein and Feldman, 1967] $R \sin^2 \bar{\alpha} / (c \cos \bar{\alpha})$. Thus, the observed luminosity is increased by the factor $(\sin \bar{\alpha})^{-2}$ and the time during which the increase is seen is correspondingly reduced. In steady state, the reduction of observed electron lifetime exactly compensates the increase in luminosity, validating Scheuer's rule. But in non-steady objects, there can be observable effects. Similarly, in an object that can be resolved, such as the M87 jet, relativistic motions could produce not only brightening and dimming, but apparent motions faster than the speed of light, in the manner described by Rees [1966].

In passing, I should like to point out a feature of Woltjer's model that he seems to have passed over (although he did discuss the time-foreshortening mentioned here.) Since we see only those electrons near a line of sight through the center of the object, its

apparent size is reduced markedly. For example, a typical angle of 15° or 20° between the magnetic field and the emitted photons, as suggested by Woltjer, leads to an observed diameter less than the physical diameter by a factor 3 or 4. Since Woltjer suggests that the time-foreshortening allows linear sizes larger by an order of magnitude or two than the ones usually required by fluctuation observations, this factor of 3 or 4 may be relevant in avoiding conflict with angular sizes set by interferometer or scintillation measurements.

When the termination of emission by an electron is due to passage out of the region of strong magnetic field, the results are the same; Scheuer's rule may be applied to steady sources that are not moving, but careful attention must be paid to the transformation of angles and time scales in non-steady or moving sources. Here, moving sources are not taken to include those which have only the Hubble recession. This recession has a quite different effect on luminosity [Robertson, 1938]. In conclusion, for non-steady or moving sources there is no simple rule. Epstein and Feldman's argument is useful where there is smooth streaming of a non-steady sort, but does not seem useful for the case of electrons in a disordered, moving magnetic field. The latter case can be treated by Robertson's method if it radiated isotropically, but if the disorder is insufficient to allow that assumption, the best method

would probably be to analyze the system in its rest frame and then transform to the laboratory frame.

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REFERENCES

- Epstein, R. I., and P. A. Feldman, *Ap. J. (Letters)*, 150, L109 (1967).
- Hubble, E. P., *The Realm of the Nebulae*, Yale University Press, New Haven, 1936, pp. 182-202.
- Jackson, J. D., *Classical Electrodynamics* (New York: John Wiley and Sons), 1962, p. 364.
- Landau, L. D., and E. M. Lifshitz, *The Classical Theory of Fields* (Reading, Mass., The Addison-Wesley Publishing Co.), 1962, p. 127.
- Rees, M., *Nature*, 211, 468 (1966).
- Robertson, H. P., *Z. Astrophys.*, 15, 69 (1938).
- Scheuer, P. A. G., *Ap. J. (Letters)*, 151, L139 (1968).
- Takakura, T., and Y. Uchida, *Astrophys. Letters* 1, 147 (1968).
- Westfold, K. C., *Ap. J.*, 130, 241 (1959).
- Woltjer, L., *Ap. J.*, 146, 597 (1966).