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Final Report

SUBOPTIMAL FILTERING

Part 2:

COMPENSATION FOR MODELING ERRORS IN
ORBIT DETERMINATION PROBLEMS

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FOREWORD

This final report is in four parts:

Part 1: ADAPTIVE FILTERING

Part 2: COMPENSATION FOR MODELING ERRORS IN ORBIT DETERMINATION PROBLEMS

Part 3: LIMITED MEMORY OPTIMAL FILTERING

Part 4: TEST-BED COMPUTER PROGRAM

The first three parts describe several suboptimal filter concepts developed under this Contract. A number of these filters have been simulated in the rectilinear orbit problem. These simulations are described therein. In order to provide a more realistic environment for testing these suboptimal filters, a more general test-bed computer program is under development. This program enables the simulation of real observation schedules and combined effects of dynamical model errors in three-dimensional satellite motion. This program is briefly described in Part 4.

Interim Report

COMPENSATION FOR MODELING ERRORS IN ORBIT DETERMINATION PROBLEMS

Stanley F. Schmidt

COMPENSATION FOR MODELING ERRORS
IN ORBIT DETERMINATION PROBLEMS

By

Stanley F. Schmidt[*]

INTRODUCTION

Orbit determination is considered herein as the problem of processing tracking data
for purposes of determining the position and velocity (orbit) of a spacecraft. The
problem considered is one in which only small time lags between the data acquisition
and the processed data (orbit) are permitted. Such orbit determination problems, which
are sometimes called estimation of state or filtering, occur in any real time system used
for command and control of spacecraft.

The orbit determination problem requires a dynamical model for use in calculating
the position and velocity time history from given initial conditions. A measurement
model is also required for use in calculating the estimated measurement. The orbit
is determined by calculating that set of initial conditions which cause the residuals
(differences between actual and computed measurements) to be small in some defined
sense.

Factors such as computation speed requirements (which may force simplifications in the
model), computational accuracies, and lack of knowledge, cause errors to exist in the
dynamic and measurement models. The dynamic model error generally has the
characteristic of causing a divergence from error analysis predictions of orbit
determination accuracy. This divergence occurs when the fundamental error source
or sources cause a secular growth in the deviation between the real and computed
position and velocity vectors for identical initial conditions. The divergence is

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displayed by the inability to compute initial conditions which provide small residuals.

Measurement model errors are generally not as serious as dynamic model errors. This results from the fact that the measurement error is bounded. As a matter of fact, measurement model errors tend to have less effect on orbit determination accuracy as more and more measurements are processed.

The deleterious effects of modeling errors in Kalman filters when used for state estimation or orbit determination problems have been reported by numerous authors. (See, for example, References 1-5.) Methods for controlling the divergence are also found in these references. These methods vary in complexity from a somewhat artificial increase in the a priori covariance matrix diagonal terms [8] to the more sophisticated modeling of additional error sources which are included but not solved for in the orbit determination process. This latter technique is described in [6] and used in [1], [7], [9].

A somewhat different method of compensation which does not directly augment the a priori covariance matrix is described in [2] and also considered here. This method involves a deliberate design of a non-optimal filter which attaches more weight to the most recent measurements. The method used in [5] is similar to [2] but is a more sophisticated and complicated approach for forcing current residuals to remain small.

As a result of the equivalences between the Kalman filter, the maximum likelihood (ML) and weighted least squares filters (WLS) [11] one would expect that dynamic model errors would cause significant degradation in orbit determination accuracy for the ML and WLS filters also.

The existence of these degradations can be readily proven for the M L or WLS filters. For reasons unknown to the writer they have never been widely publicized in the engineering literature, even though ML and WLS filters have been used for many more years than the Kalman filters. The ML and WLS filters, which are usually equivalent in implementation, use algorithms which process batches of data (large numbers of points) simultaneously. One may, by throwing out batches in the past, force residuals to remain reasonable for the orbit used for command and control purposes. This readily implemented and obvious manual procedure which attains reasonable results may be the reason why little has been publicized on accuracy degradation in the use of ML and WLS filters.

Reference [4] describes an implementation which is equally applicable to Kalman, WLS and ML filters and which is, basically, the discarding of past data. This moving window, or limited memory, filter is also referred to in Reference [11], which gives Reference [12] as the originating source. The algorithm for deciding when to discard past data is not covered, and presumably is a manually imple- mented decision in Reference [4]. The present paper discusses some alternatives which may be automated, based on the ideas of Reference [2].

A factor which should be recognized is that a strong difference exists between the real time orbit determination problem and the post-flight analysis problem. In the latter case we are interested in finding the dynamic model which predicts the orbit and fits the measurements over the entire flight. It is the very existence of unexplained residual behavior which provides the source of new information and discovery in the post-flight analysis of a spacecraft mission.

The post-flight problem is basically a problem of model identification and the dynamic model compensation described herein is not applicable, since it can destroy the basic information one is trying to obtain. See Reference [10] as an example of post-flight analysis procedures for model identification.

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The intent of this paper is first, to present the fundamental characteristics of dynamic and measurement model errors in real time orbit determination systems. Second, two fundamentally different techniques (which can give equivalent results) for dynamic model compensation will be described. Third, a review of the model compensation algorithms for recursive (Kalman) type filters will be given. Fourth, a discussion of the available implementation of algorithms for model compensation in batch data (ML or WLS) filters will be described. Finally, an approach which should lead to an automatic means of weighting out past information in batched data filters will be discussed.

PROBLEM DEFINITION

The problem of orbit determination is defined here as:

1. A true set of dynamical equations describing the motion of the spacecraft is believed to be representable by a set of vector differential equations.

$$\dot{X} = F(X, C, U, t) \tag{1}$$

We have an assumed set of equations describing the motion given by

$$\dot{\hat{X}} = \hat{F}(\hat{X}, \hat{C}, \hat{U}, t) \tag{2}$$

where

$\hat{X}$ = A 6-vector describing the assumed position and velocity

$\hat{C}$ = A vector describing assumed constants in the equations of motion, such as planetary masses

$\hat{U}$ = A vector of forcing functions whose effect in terms of specific force is theoretically measurable by onboard accelerometers. Forces created by drag, venting, thrust and gas leaks fall into this category.

t^* = The independent variable

2.   A vector, Y , of tracking data measurements is available which we believe is relatable to the spacecraft position and velocity by the form

$$Y = G(X, V, t) + q(t) \tag{3}$$

We have an assumed model of these measurements given by

$$\hat{Y} = \hat{G}(\hat{X}, \hat{V}, t) + \hat{q}(t) \tag{4}$$

where

$\hat{V}$ = A vector describing assumed constants and/or time-varying states in the measurement model.

$\hat{q}$ = The assumed random error in the measurement.

Before proceeding further in the definitions some discussion of the notation and meaning of the phrases "believed to be" and "assumed to be" is in order.

"Believed to be" is defined as meaning: in conformance with such scientific knowledge as we possess at the present time.

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*   Ephemeris time is considered herein as the independent variable. A note of importance is associated with the interpretation of this quantity when we are dealing with measurements in which a different time base and propagation delay is involved. See Reference [10] for more details on this problem.

"Assumed to be" is used in the context of known approximations, such as truncations of infinite series representations, and so on.

The $\wedge$ symbol on the functions $\hat{F}(\ )$ and $\hat{G}(\ )$ is intended to imply the approximation involved because of our limited ability in describing reality in the form of mathematical models.

The $\wedge$ symbol on the vectors $\hat{X}$, $\hat{C}$, $\hat{U}$, and $\hat{V}$ is implied to mean our best estimate of the quantities.

3. We may have an initial estimate of the position-velocity vector at a time t_o which will be defined as

$$\hat{X}(t_o) = \hat{X}_o \tag{5}$$

4. Other initial conditions and time-varying forcing functions which are assumed are

$$\hat{C}(t_o) = \hat{C}_o \tag{6}$$

$$\hat{V}(t_o) = \hat{V}_o \tag{7}$$

$$\hat{U}(t_o) = \hat{U}_o^*(t) \tag{8}$$

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* As a result of our limited knowledge and known approximation in computations, we may desire to consider a portion of $U(t)$ to be a random forcing function. This will be discussed later in this paper as a possible means of dynamic model compensation.

5. The errors in the various assumed variables are considered as describable by

$$E(X - \hat{X}_o) = E(\tilde{x}_o) = 0 \tag{9}$$

$$E(\tilde{x}_o, \tilde{x}_o^T) = \text{Covariance Matrix} = P_o \tag{10}$$

$$E[\hat{q}(t)] = 0 \tag{11}$$

$$E(\hat{q}\, \hat{q}^T) = Q(t) \tag{12}$$

$$E(C - \hat{C}_o) = E(\tilde{c}_o) = 0 \tag{13}$$

$$E(\tilde{c}_o\, \tilde{c}_o^T) = C_o \tag{14}$$

$$E[U(t) - \hat{U}_o(t)] = E[\tilde{u}(t)] = 0 \tag{15}$$

$$E(\tilde{u}\, \tilde{u})^T = U_o(t) \tag{16}$$

$$E(V - \hat{V}_o) = E(\tilde{v}) = 0 \tag{17}$$

$$E(\tilde{v}\, \tilde{v}) = V_o \tag{18}$$

The use of the expected value operator E in equations 9-18 requires some discussion. Nominal usage of the operator implies an ensemble average. In equation (9) if errors in $\hat{X}$ are considered as injection errors, we may consider the ensemble average to represent an infinite number of launch vehicles of the same type launched from the same location.

If the basic causes of injection errors are random in this ensemble, then statement (9) may be justified.

It is far more difficult to justify equation (13) for we may need to consider an ensemble as meaning the universe. It is perhaps better to consider the initial values as estimates and the covariance matrix as confidence levels in these estimates, avoiding the problems of defining the ensemble. This, in fact, will be how we will use these quantities.

The objective of the orbit determination problem considered herein is to find:

An algorithm for use in a given computer which processes the required tracking data and provides an estimate of the position and/or velocity which is sufficiently accurate for real time command and control purposes.

Factors which must be considered in this problem include

(a) Computer speed

(b) Computer memory availability

(c) Computer errors

(d) Tracking data availability

(e) Schedule of when the operating solution is required

as well as many other practical considerations.

Remarks

The factors given above are an attempt to state some of the practical considerations which arise in any orbit determination problem. Theory is used to provide approaches for finding a practical solution. By no means will this solution be optimal in the

theoretical sense. This is because exact mathematical models of all quantities, as well as practical performance indices which consider all the factors, are not available.

THEORETICAL CONSIDERATIONS

To obtain a practical solution of the orbit determination problem in any specific example, existing theory and practical experience is used to find algorithms for the filtering. Known implementations will be briefly reviewed in order to show why some form of dynamic model compensation is inevitably required. The algorithms for estimation will be stated without proof since the theory on which they are based has had wide publication. Before reviewing these algorithms, a few more fundamental definitions are required.

If the gradient of equation (1) is taken with respect to X, C, and U we may obtain a set of time-varying linear differential equations

$$\dot{x} = F(t)\overset{*}{x}(t) + B(t) c + D(t) u(t) \qquad (19)$$

where

$$F(t) = \nabla_X F(X, C, U, t) \Big|_{X(t) = \hat{X}(t)}$$

$$B(t) = \nabla_C F(X, C, U, t) \Big|_{C(t) = \hat{C}(t)}$$

$$D(t) = \nabla_U F(X, C, U, t) \Big|_{U(t) = \hat{U}(t)}$$

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* Lower case letters are used as meaning small deviations from the corresponding upper case value. That is, a vector Z is considered as $Z = \hat{Z} + z$.

Equation (19) is known as the variational equation and, since it is linear, the general solution may be written in the form

$$x(t) = \Phi(t;t_o) \, x(t_o) + \Phi_c(t;t_o) \, c + \int_{t_o}^{t} \Phi(t;\tau) \, D(\tau) \, u(\tau) \, d\tau \qquad (20)$$

The transition matrix $\Phi(t;t_o)$ and the sensitivity to constant forces or control can be found by solving

$$\dot{\Phi} = F(t)\Phi \qquad\qquad \Phi(t_o;\, t_o) = I \qquad\qquad (21)$$

$$\dot{\Phi}_c = F(t)\Phi_c + B(t)(I) \qquad \Phi_c(t_o;\, t_o) = 0 \qquad\qquad (22)$$

The integral term of equation (20) may be solved by considering $u(t)$ as constant over small time increments and using equation (22) in the form

$$\dot{\Phi}_u = F(t) \, \Phi_u + D(t) \, I \qquad \Phi_u(t_o + n\Delta;\, t_o + n\Delta) = 0 \qquad (23)$$

Equation (23) will only be used for the "random" part of any unknowns in the forces since constant forces may be represented by solutions of (22).

It is usually convenient to consider an expanded state vector of form

$$\hat{Z} = \begin{pmatrix} \hat{X} \\ \hat{C} \\ \hat{U}_C \end{pmatrix} \qquad \begin{matrix} - \text{ position and velocity} \\ - \text{ constant terms in equations of motion} \\ - \text{ constant forces} \end{matrix}$$

We may then write

$$z(t) = \Phi_z(t;\, t_o) \, z(t_o) + \int_{t_o}^{t} \Phi_z(t;\tau) \, D(\tau) \, u_r(\tau) \, d\tau \qquad (24)$$

where u_r = random forces.

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The gradient of the measurement equation, **(3)**, is taken with respect to X and V to find linear equations for the i-th measurement

$$y_i = h_i x + g_i v + q_i^* \qquad (25)$$

where

$$h_i = \nabla_X G(X, V, t_i) \Big|_{X(t) = \hat{X}(t)} \qquad (26)$$

$$g_i = \nabla_X G(X, V, t_i) \Big|_{V = \hat{V}} \qquad (27)$$

We may desire to relate the vector of measurements to the vector, z_o, at a fixed epoch. This may be done by using (24)** and (25) which we will define in the form

$$y = H z_o + G v + q \qquad (28)$$

In (28) the vector of y may consist of measurements at different time points. The vector z_o can be at a fixed epoch or not, depending on how we formulate the filter equations. This fact will be used in the algorithms described later.

As can be seen by referring to equation (28), it may also be desirable to consider a further expansion of the state to include the measurement parameters, V. Then

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* The random error in measurement is assumed to be small and therefore considered to give a small random deviation from a noiseless measurement.

** The random variation of u(t) will be omitted here. If it exists, one may modify q to include the effect.

(28) would be written in the equivalent form

$$y = (H \ G) \begin{pmatrix} z_o \\ v \end{pmatrix} + q \triangleq \underline{H} \, \underline{z}_o + q \tag{29}$$

In the case where it is meaningful or necessary, we may write the equation for time updating the covariance matrix $P_{\underline{z}}(t)$ in the form

$$P_{\underline{z}}(t) = \Phi_{\underline{z}}(t; t_o) \, P_{\underline{z}}(t_o) \, \Phi_{\underline{z}}^T(t; t_o) + U_r \tag{30}$$

The matrix U_r in (30) gives the added uncertainty caused by random forcing functions in the time interval $t - t_o$.

The WLS Filter

One of the most widely used filters for orbit determination is the weighted least squares (WLS) filter. In implementation, this filter is the same as the maximum likelihood filter (ML) and no distinction will be considered herein.

The algorithm, as generally used, is written as an iteration equation in the form

$$\hat{Z}_{n+1} = \hat{Z}_n + \left[P_{z_o}^{-1} + \underline{H}_n^T \underline{Q}^{-1} \underline{H}_n \right]^{-1} \left[\underline{H}_n^T \underline{Q}^{-1}(Y - \hat{Y}_n) + P_{z_o}^{-1}(Z_o - \hat{Z}_n) \right] \tag{31}$$

$$= \hat{Z}_n + \Delta \hat{Z}_n$$

The iteration may be terminated by various means of which the most obvious is when $\Delta \hat{Z}_n < \epsilon$. The vector ϵ is set at a pre-determined tolerance level.

In equation (31) the partials $\underline{H}_n$ are usually re-computed on each successive trial so that the linearization involved is about the trajectory (orbit) found by the preceding trial.

Equation (31) is given in the form where initial estimates of all the states, $\hat{Z}_o$, and the covariance matrix of errors in these estimates, P_{z_O}, are assumed available. If $P_{z_O}^{-1}$ is set zero and $\underline{H}^T\underline{Q}^{-1}\underline{H}$ is invertible, then if (31) converges, an estimate of the state, $\hat{Z}$, is obtained which is dependent only on the measurement vector, Y. Also involved for either case, however, are the assumed equations for the models.

In practice the WLS filter is implemented either at a fixed epoch (fixed anchor point) for the estimate, $\hat{Z}$, or at an anchor point which shifts as later batches of data are included. If a batch is defined as all the data from one pass of a given ground tracking station, for example, then the moving anchor point might be taken as near the time of acquisition of the spacecraft from the station. The anchor point could be made to shift for each new batch and equation (30) used for updating the covariance matrix.

The moving anchor point WLS filter has obvious computational speed advantages over the fixed anchor point system. Its disadvantages lie in the necessity of requiring P_{z_o} to be a meaningful quantity. This statement is made for a system designed such that, once a batch is processed, the raw data in this batch is never reprocessed.

The Kalman Filter

The Kalman Filter is usually mechanized in a form which processes the measurement at a given time point sequentially. The covariance matrix and the state are then updated to the time of the next set of measurements.

The algorithm is conveniently stated in two parts:

1. Between measurements

$$\hat{X}(t) = \hat{X}(t_o) + \int_{t_o}^{t} \hat{\dot{X}}(\tau) \, d\tau \qquad (32)$$

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$$P_z(t) = \Phi_z P_z(t_o) \Phi_z^T + U_r \tag{33}$$

2. At measurements

$$\hat{X}_a = \hat{X}_b + K(Y - \hat{Y}) \tag{34}$$

$$P_a = P_b - K H P_b \tag{35}$$

$$K = P_b H^T (H P_b H^T + Q)^{-1} \tag{36}$$

In (34)-(36) the subscripts a and b mean after or before the measurement, respectively.

Equation (35) may also be written in the form

$$P_a = \left[I - K H \right] \left[P_b \right] \left[1 - H^T K^T \right] + K H Q H^T K^T \tag{37}$$

The number of numerical operations in (37) is large by comparison to (35); however, in (37) numerical errors should not cause the covariance matrix to become non-positive definite [11].

Alternate forms can be derived for (33) and (35) when the U_r of (33) is non-existent. For example, if one lets

$$W W^T = P \tag{38}$$

then

$$W(t) = \Phi W(t_o) \tag{39}$$

and

$$W_a(t) = W_b\left[I - \frac{W_b^T H^T H W_b}{d}\right]$$ (40)

$$d = (HPH^T + Q)\left(1 + \sqrt{\frac{Q}{HPH^T + Q}}\right)$$

Equations (39)-(40) are only valid for processing measurements one at a time. This formulation (38) to (40) is generally referred to as the square root matrix. It has definite numerical accuracy advantages over the other methods [13].

Many other variations are possible such as integrating a differential equation for updating the P or W between measurements, etc.

As indicated in the discussion of the Kalman filter, no iteration is implied. There are many different ways of implementing an iteration to aid in assuring that the equations are used in a linear region about the desired solution.

The Kalman filter is normally used for onboard computations where computer memory and speeds do not necessarily allow storage of a lot of past measurements. Iteration is not possible in these applications and potential problems with the linearized equations exist.

Batched Data Implementation

The Kalman filter may also be implemented in a form similar to equation (31). If we let

$\bar{z}_n$ = cumulative deviation from $Z_0(t)$ for iteration No. n

$\bar{z}_0 = 0$

$\hat{z}$ = cumulative deviations as caused by the measurements

Then we write the equations in two parts

1. Observation Processing

$$\hat{z}_{m+1} = \hat{z}_m + P_m H_n^T (H_n P_m H_n^T + Q)^{-1} [Y_m - \hat{Y}_n - H_n (\hat{z}_m - \bar{z}_n)] \qquad (41)$$

$$P_{m+1} = P_m - P_m H_n^T (H_n P_m H_n^T + Q)^{-1} H_n P_m \qquad (42)$$

m = measurement number, $m = 1 \cdots k$

n = iteration number. Subscript n means evaluation for the trajectory determined on the n-th trial.

2. Start of an iteration, m = 1

$$\hat{Z}_{n+1} = \hat{Z}_n + \hat{z}_k - \bar{z}_n \qquad (43)$$

$$\bar{z}_{n+1} = \hat{z}_k \qquad (44)$$

$$P_m = P_{z_o} \qquad (45)$$

$$\hat{z}_m = 0 \qquad (46)$$

The logic in equations (41) to (46) is for incremental processing and accumulation of each measurement contribution to the next nominal trajectory. In the WLS filter all measurements are processed simultaneously.

The Kalman formulation permits one to readily implement a linearity test as described below.

Let w = the three position elements of $\Phi_m(\hat{z}_m - \bar{z}_n)$. The magnitude of w, $|w|$, then is the position deviation from the nominal trajectory at the m-th measurement.

By testing for and keeping

$$|w| < a \ R_c \qquad (47)$$

where

$\quad R_c$ = distance from central body

$\quad a$ = a small tolerance number

one has a means of insuring the accuracy of the Φ matrix.

By testing for and keeping

$$|w| < b \ R_s \qquad (48)$$

where

$\quad R_s$ = slant range vector to a tracking station

$\quad b$ = a tolerance number

one has a means of insuring the accuracy of the H matrix.

In theory, the Kalman formulation for batch data processing, outlined above, should give equivalent results to the WLS implementation for the case where P_{z_0} is

defined. A possible advantage of the Kalman formulation is the ability to test for possibilities of nonlinear effects early in the batch of measurements. Iteration over these measurements to satisfy linearity tolerance is then performed before proceeding to additional measurements. Matrix inversion is also avoided in this mechanization. Studies are under way to determine whether or not some inherent advantages of speed and convergence are possible with this approach.

DISCUSSION OF PROBLEMS IN
REAL TIME ORBIT DETERMINATION

The previous section outlined some of the algorithms we have at our disposal from theoretical considerations. In the real-time problem we will inevitably desire to minimize the number of unknowns (state variables) in the formulation. This is necessary from computational speed and memory considerations. Also, too many state variables can cause potential difficulties, and time to review the implications of each new solution is usually only available in post-flight analysis. For the balance of this paper it will be assumed that these considerations lead to solving for position and velocity vectors only. If such a constraint is imposed on the number of state variables, then it is easy to show that model errors generally make it impossible to find a solution for the position and velocity vectors which fits the actual data. This can be seen by considering the WLS solution for initial position and velocity.

We have then assumed models

$$\dot{\hat{X}} = F(X, t) \tag{49}$$

$$\hat{Y} = \hat{G}(\hat{X}, t) \tag{50}$$

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Equation (49) imposes a constraint on the time history of the solution for $\hat{X}$ if $\hat{X}_0$ is given.

Equation (50) gives the functional relationship for computing the observation from the estimate of $\hat{X}$. The real measurements obey reality rather than (49) and (50). Hence, if one uses equations (49) and (50), a value of $\hat{X}_0$ for which the residuals $Y - \hat{Y}$ are small and random about zero mean, over all time, implies there are no errors in the model equation.

Since we are performing a weighted least squares fit in this example, one can hypothesize the typical residual behavior of the solution for errors in (49) and (50). A pictorial representation is shown in Figure 1.
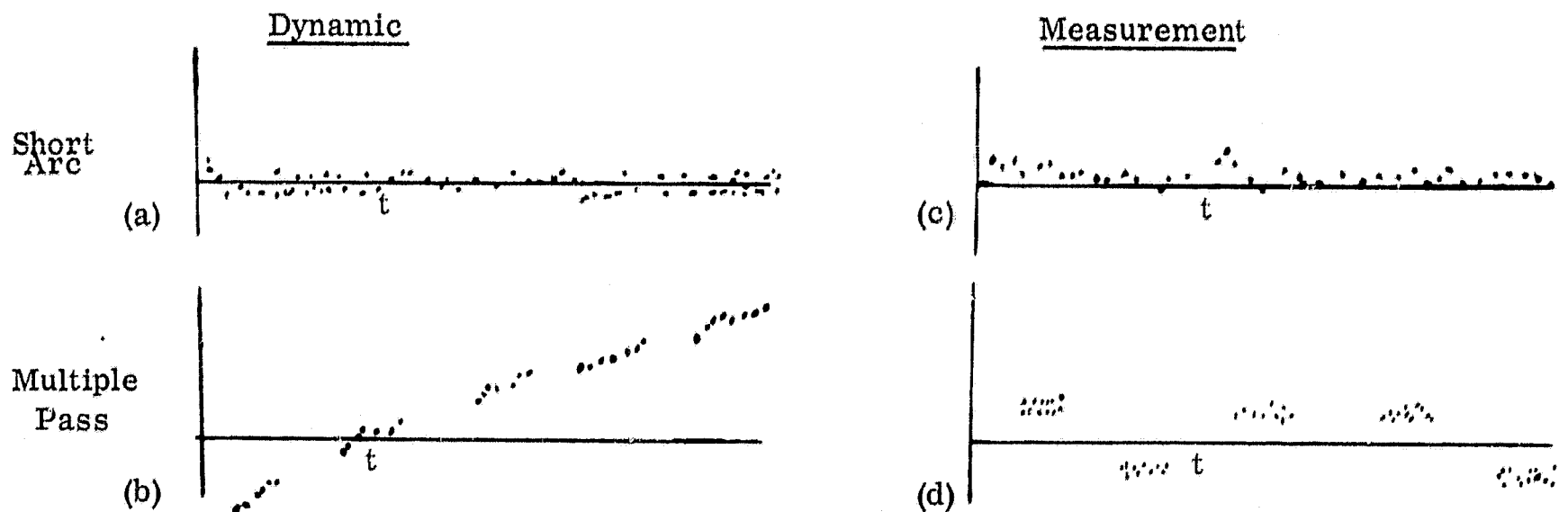

Figure 1. Characteristics of Residuals Resulting from Model Errors

As shown in Figure 1, for a short time arc, dynamic model errors [errors in equation (49)] will not prevent a solution for $\hat{X}_o$ where residuals are essentially random with zero mean. This can be seen by considering a Taylor series solution of (49). Let $\hat{\underline{X}}_o$ = 3 position components at the beginning of a data arc. Then

$$\hat{\underline{X}}(t - t_o) = \hat{\underline{X}}_o + \hat{\dot{\underline{X}}}_o (t - t_o) + \hat{\ddot{\underline{X}}}_o \left(\frac{t - t_o}{2!}\right)^2 + \dots \dots \tag{51}$$

In equation (51) $\hat{\underline{X}}_o$ and $\hat{\dot{\underline{X}}}_o$ are free. Hence the constraining effect of the dynamic model is not significant when $(t - t_o)$ is small.

The underlined terms of the infinite series (equation 51) are dependent on the dynamic model; hence, errors in the dynamic model will become effective over long time arcs. We therefore will not be able to find a value of $\hat{\underline{X}}_o$ and $\hat{\dot{\underline{X}}}_o$ which fits , for example, over a multiple pass. Dynamic model errors for data over long time arcs should cause the effects shown in Figure 1b.

Measurement errors such as biases will also have only a small effect, for short time arcs, as shown in Figure 1c. Only when we deal with measurements of many different types, over a short arc, should we expect to have biased offset residuals when measurement model errors exist in processing. This latter results from the fact that no single values of $\hat{\underline{X}}_o$ and $\hat{\dot{\underline{X}}}_o$ can be selected which fit multiple measurements (more than 6) in the presence of biases or other measurement model errors. This type of error, however, is, in general, bounded. Hence, if we take a long data arc over many stations, we can expect these errors to cause residuals as shown in Figure 1d. Since the solution is constrained to obey the dynamic model, measurement model errors will have less and less effect on orbit determination accuracy as more data is processed.

The real problem, of course, is comprised of all these effects. Dynamic errors may have both periodic and secular effects on the residuals. Hence, Figure 1 can only indicate trends to be expected. Also, in the real situation what is meant by short arc and multiple pass is not readily defined.

Figure 2 depicts general trends of the effects of measurement and dynamic model errors on orbit determination accuracy.
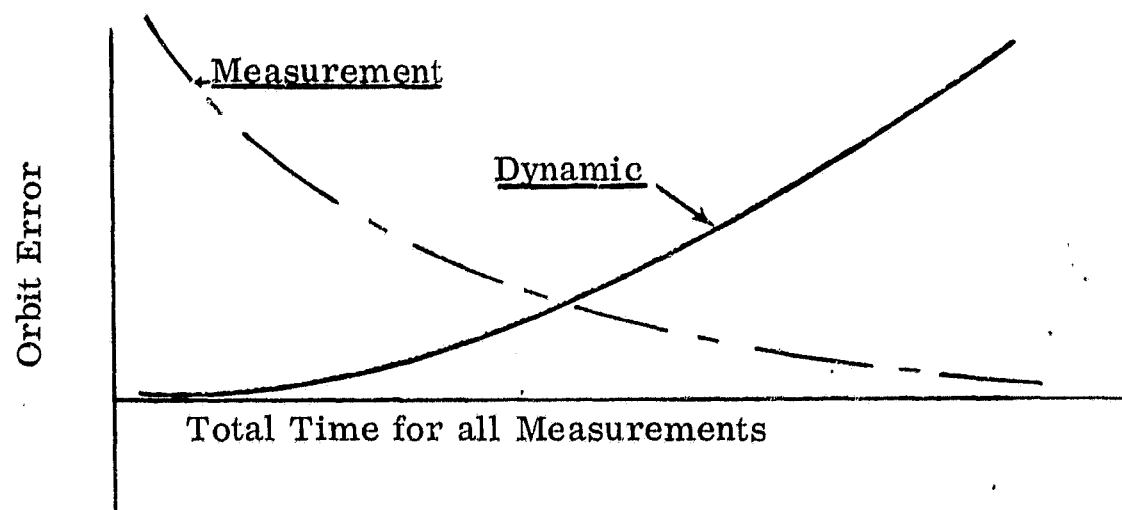

Figure 2   Orbit Error Trends Resulting from Model Errors

The figure is intended to indicate that dynamic errors cause an ever-increasing orbit determination error as more data points, over a large time arc, are included. Measurement errors have the opposite effect.

One would desire to have an algorithm which somehow gives the best balance between these two effects. This will be called dynamic model compensation and will be discussed in the next section. We should recognize, however, that both measurement and dynamic model errors do exist and the resulting effect of the dynamic compensation can cause the measurement errors to be dominant as far as their contribution to the orbit determination accuracy.

DYNAMIC MODEL COMPENSATION

As was indicated in the previous section, some means of preventing the orbit determination errors from growing indefinitely is necessary. It should be reasonably obvious that one way of preventing this growth is to totally remove or gradually reduce the influence of past measurements in determining the orbit for current usage. Let us assume that totally false measurements (outliers) caused by malfunctions can be removed. The remaining measurements can then be trusted to give the theoretical value plus an error term. It is reasonably likely that upper bounds may be placed on the error. That is, the error which is caused by calibration, atmospheric effects, receiver noise, and so forth, cannot be larger than some specified number. If this is the case, then the error in the computed measurement should be no worse than this upper bound.

As an example, suppose we consider the measurement of the propagation delay of an rf signal between transmittal and reception from a spacecraft. It is reasonable to assume that we can compute the two-way range from this measurement and know the upper bound on the error in this quantity. If the residual is greater than this error, one must conclude that too much emphasis has been placed on past data in establishing the computed orbit. The writer does not mean to imply that such "worst case" considerations are to be used. These remarks are only made to give some reasons as to why we must impose a reasonable degree of trust in the basic measurements or we will be unable to devise any means of compensation.

Two different approaches are possible in obtaining a means of weighting out the influence of past data in the determination of the current orbit. These both involve formulation of the problem in a form where past data is included by using an a priori covariance matrix which is associated with the a priori orbit. The two approaches are as follows:

1. Increase the a priori covariance matrix. Arguments for using this approach are as follows:

Equations of motion which are invalid have been used to update the estimate of state. One should therefore increase the a priori covariance matrix in accordance with the errors involved in the time-updating of the estimate. The difficulties in using this approach lie in defining the real error sources. Their formulation can also become extremely complex. Hence, for practical usages, it is perhaps better to say that pseudo-errors are introduced to cause an increase in the a priori covariance matrix. These pseudo-errors can be of two types:

    a. random forcing functions

    b. errors attributed to inaccuracies of constants in the equations of motion.

2. Overweight the most recent data. In this approach it is also recognized that the a priori covariance matrix may be overly optimistic. This matrix, however, is not modified on the basis of adding the effects produced by pseudo-errors in the dynamic model. Instead, a non-optimal filter algorithm is adopted which attaches a greater significance to the recent observations than the optimal filter does. The a posteriori covariance matrix is modified to conform with the non-optimal algorithm.

Either of these basic approaches has many variations to suit any specific problem. Although the philosophy is different for the two, they both should have the desired effect of reducing the error in the current estimate of the orbit.

Figure 3 shows the expected results with and without using compensation. Also shown is the optimum solution which theoretically is attainable if modeling and computational errors were nonexistent.
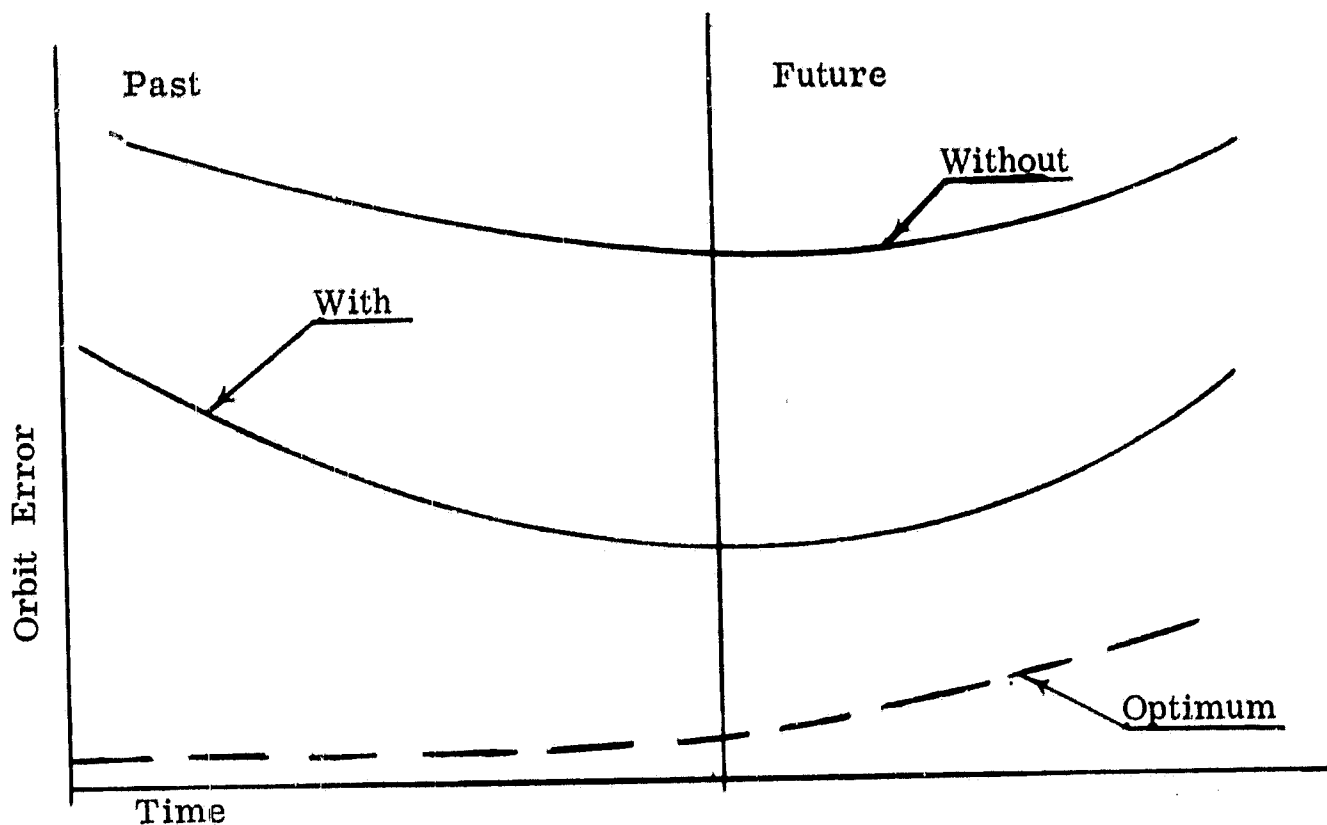
Figure 3   Expected Trends in Orbit Error

Figure 3 is intended to imply that compensation, if properly applied, can give smaller orbit errors than a mechanization which assumes that the dynamical model is perfect. It also should be recognized that improper application of compensation can give poorer results.

Dynamic Model Compensation for Sequential (Kalman) Filters

The examples which will be given here for model compensation are for the approach which overweights the most recent data.

If we consider the sequential filter given by equation (32)-(36), then to implement this approach we consider modifications of the equations for treating measurements (34)-(36).

One such modification is shown below. This modification is only valid for processing observations one at a time.

$$\hat{X}_a = \hat{X}_b + (PH^T + \underline{H^T QK/HH^T}) (Y - \hat{Y}_b)/(HPH^T + Q) \tag{52}$$

K in equation (52) is a control gain.

Now assume linearity prevails and therefore

$$H\hat{X}_a = \hat{Y}_a = \text{the computed measurement after the observation}$$

Multiplying (52) by H gives:

$$H\hat{X}_a = H\hat{X}_b + \left(\frac{HPH^T + QK}{HPH^T + Q} \right) \left(Y - \hat{Y}_b \right) \tag{53}$$

Note in (53) that for $K = 1$ the estimate of the observation is equal to the observation. Hence, the error in this component of the estimate of state is no worse than the error in the measurement. For $K = 0$ we have an optimal filter.

When processing mixed type measurements, for example range, azimuth, and elevation, the control gain, K, should be different for the various observations. One reasonable choice for this example is

$$K = K_r \text{ for range}$$

$$K = K_{az} \text{ or } K_{el} /(\text{range from station})^2 \text{ for azimuth and elevation measurements}$$

K_r, K_{az}, and K_{el} can all be different. Such a choice reduces the influence of angular measurements in the non-optimal part of the filter as distances from the tracking station increase.

The covariance matrix of errors in $\hat{X}_a$ is found by taking

$$E(X - \hat{X}_a)(X - \hat{X}_a)^T = P_a$$

This operation applied to equation (52) gives

$$P_a = P_b - P_b H^T H P_b \Big/ (H P_b H^T + Q) + K^2 Q^2 H^T H \Big/ \left[(H P_b H^T + Q)(H H^T)^2\right]$$
(54)

Comparison of equation (54) with equations (35) and (36) shows the degradation in accuracy from optimal filtering caused by the control gain, K.

Before proceeding further it is relevant to consider a simple example using this non-optimal filter. This example is given in reference [2]. Let the actual and assumed models be given by:

Actual Model	Assumed Model
$\dot{x} = .1 = $ scalar	$\dot{\hat{x}} = 0$
$y = x$	$\hat{y} = \hat{x}$

Let $Q = 1$.

$P(0+) = Q$ (after first measurement)

$x(0+) = 10$ (after first measurement)

$\hat{x}(0+) = x(0+)$
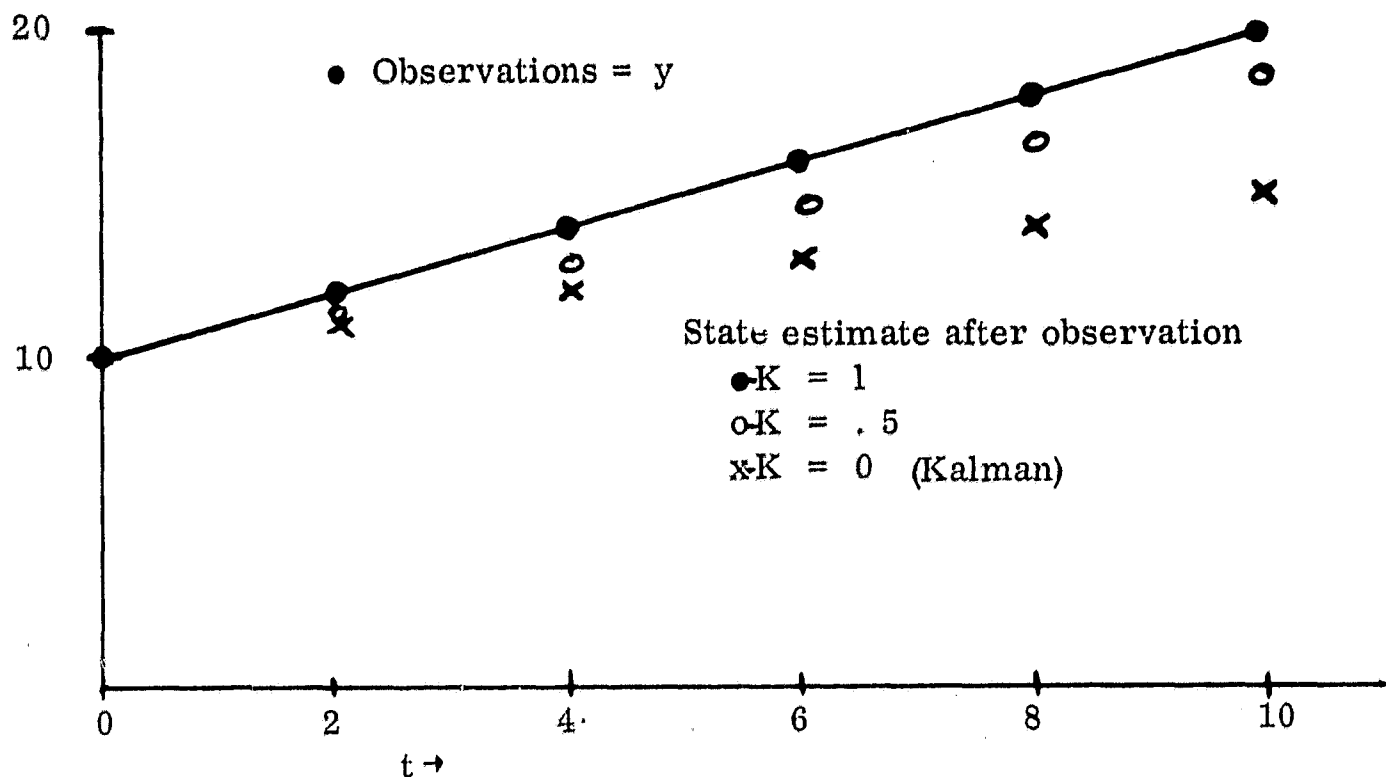
Figure 4 shows the results for 3 values of K.

Figure 4. Example Results Using Model Compensation in a Kalman Filter

The error in estimate of x for the Kalman filter grows indefinitely with time. The growth arises because we have assumed $x(t)$ = a constant, while in reality $x(t)$ = a constant + a time-dependent term. The modified filter (K $\neq$ 0) has an error growth between measurements. However, if K = 1 the error in $\hat{x}$ after the measurement is included is no greater than the measurement error. For K < 1, the error in estimate, after each measurement is processed, reaches a constant offset from the measured value. Hence, use of this non-optimal filter tends to lock the estimate of state to the recent observations.

Another advantage of this modification is that errors in defining $P_0(t_0)$ and Q are not as significant in causing errors in the estimate as they are in the Kalman filter.

Equivalent results to those given in Figure 4 could also have been obtained by introducing a pseudo-random forcing function. This can readily be seen since random forcing functions cause an added growth to P between measurements. Prior to including the n-th measurement

$$P_n(-) = P_{n-1} + U_r$$

If U_r were made extremely large compared to Q then the results corresponding to K = 1 of Figure 4 would be obtained.

Another modification (see reference [2]) which might be used is to let

$$\hat{X}_a = \hat{X}_b + s\,(P_b H^T)(Y - \hat{Y})/(H P_b H^T + Q) \tag{55}$$

$$P_a = P_b - (2s - s^2)P_b H^T H\, P_b/(H P_b H^T + Q) \tag{56}$$

The scalar s of equations (55) and (56) is made a function of available quantities such that it is, in general, larger than unity. Hence the more recent measurements are weighted more heavily than in the optimal filter (s = 1).

If, for example, s were selected as

$$s = 1 + KQ/H P_b H^T \tag{57}$$

Then for K = 1, the computed value of the measurement, $H\hat{X}_a$, after the measurement is included, is

$$H \hat{X}_a = H \hat{X}_b + Y - \hat{Y} = Y \qquad (58)$$

This result is identical to that given in equation (53) for K = 1.

The writer is not aware of any studies of this modification of Kalman filtering.

The algorithm (56) may be factored to obtain a modification of the square root matrix implementation [equations (38)-(40)]. Hence, this type of model compensation in square root mechanizations is permissible. The pseudo-random forcing function method or equation (54) appear to require too many added calculations to make them useful for the square root implementation.

Before leaving the subject of model compensation for sequential Kalman filters, it is desirable to consider the general characteristics shown in Figure 5.
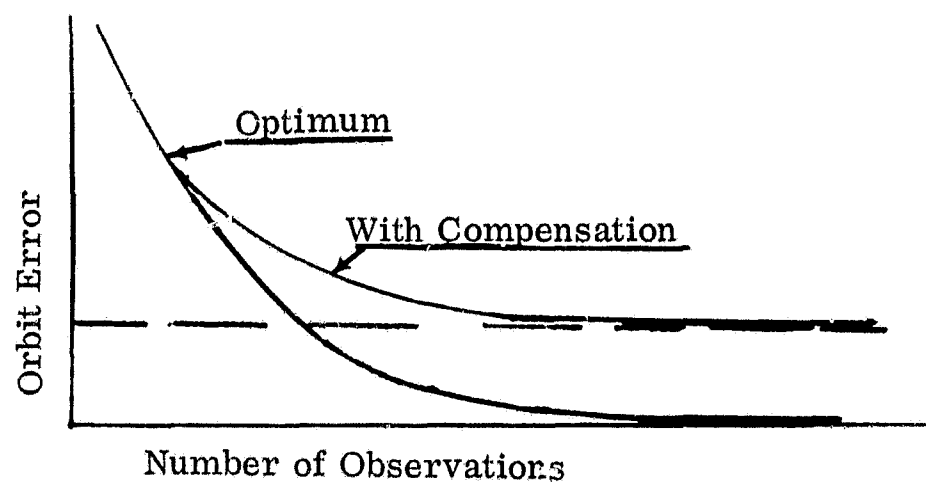

Figure 5. General Characteristics of Model Compensation

If model errors do not exist and random forcing functions are negligible, the error in the optimum solution asymptotically approaches zero as the number of data points goes to infinity. Any of the compensation schemes discussed thus far for the same

idealized situation gives the characteristic behavior shown in Figure 5. That is, the orbit error asymptotically approaches some finite value which is dependent on the control gains used in giving a higher weight to recent measurements.

This characteristic suggests the idea of using acceptable error tolerance levels as a means of defining a model compensation scheme for processing batched data. This idea will be investigated further in the next section.

Dynamic Model Compensation for Batched Data Mechanizations

As was previously mentioned, batched data processing is a method where, in general, a subset of the total number of measurements is simultaneously processed to determine the estimated orbit. The subset or batch will be assumed to be defined such that the beginning points of batches of data[*] occur sequentially in time. It will be assumed that the processing of batches will be carried out in a sequential manner. A further assumption is that once a batch is processed its influence on any subsequent batches will be contained in the estimate of state and the covariance matrix of errors of this estimate.

No generality of the subsequent material is lost if we also consider a moving anchor point implementation. That is, the estimate of state and the covariance matrix of errors in estimate of this state are always updated between batches. The updating is to some point close to a defined reference time of the new batch of data.

With the above definitions, the moving anchor point batched data filter is very similar in operation to the Kalman filter. First, a batch of measurements is processed where

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[*] The end point of the data within a batch may occur at a later time than the beginning point of the next batch. Factors such as the desire to include all measurements available may, however, define the batch as all data from all sources over a given time interval.

the estimate of state is referenced to a fixed time point. The covariance matrix of the errors in this estimate is also calculated. Second, the estimate of state and the covariance matrix of errors in this estimate are updated for processing the next batch of measurements.

Dynamic model compensation using pseudo-random forcing functions or pseudo-constant uncertainties in some equation of motion parameters is readily implemented. This implementation effects an added growth of the covariance matrix during the time propagation.

The data in the newest batch is thus given a higher weighting in calculating the new estimate of state than would exist if the pseudo-variables were omitted. This method is fundamentally sound if indeed one can define the correct growth of the covariance matrix. That is, the pseudo-error sources are real. This is one of the difficulties with use of this method. Control gains have to be selected which provide a realistic growth of the covariance matrix. Also, since the batch of data spans a given time interval, such an implementation is an approximation. If the pseudo-variables were real, then weighting of measurements within the batch must also be effected.

Let us consider the alternate method which overweights the data in the current batch. The exact method used will depend on the basic algorithm used in processing the data. If, for example, the Kalman formulation for batched data were used, then the modifications discussed in the previous section are applicable. An implementation of this type is under study; however, no results are available at this time.

It is the intent here to review the WLS mechanization equations and to show one way by which a reasonable modification can be derived.

The unmodified WLS algorithm for estimating position and velocity is

$$\hat{X}_{n+1} = \hat{X}_n + \left[P_{X_0}^{-1} + \underline{H}_n^T \underline{Q}^{-1} \underline{H}_n \right]^{-1} \left[\underline{H}_n \underline{Q}^{-1} (Y - Y_n) + P_0^{-1} (\hat{X}_0 - \hat{X}_n) \right]$$

(59)

In equation (59)

$P_{X_0}^{-1}$ describes the information about $\hat{X}_0$, the a priori estimate of state.

$\underline{H}_n^T \underline{Q}^{-1} \underline{H}_n$ describes the information contained in the measurement vector, Y.

If $P_{X_0}^{-1}$ is correct, no model errors exist, and the solution of (59) converges, then the covariance matrix of errors after processing the batch is

$$E^*(X - X_k)(X - X_k)^T = P_{X_k} = \left[P_{X_0}^{-1} + \underline{H}_k^T \underline{Q}^{-1} \underline{H}_k \right]$$

(60)

The quantity, k, in equation (60) means the k-th trial where convergence was attain .

The basic philosophy of trusting the data tells us that we should in some manner force Y to have a stronger influence on $\hat{X}_k$ than that given it in (59). Theoretically, this type of modification will also give a larger value of P_{X_k} than indicated by equation (60).

In using pseudo-variables we would compensate for both the above factors by making $P_{X_0}^{-1}$ smaller. Instead of using pseudo-variables to cause this reduction in $P_{X_0}^{-1}$, let us examine a procedure based on the idea of acceptable tolerance levels.

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* A linear system must be assumed for this derivation. Equation (60) is an approximation for the real situation.

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It appears reasonable from engineering considerations that one should be able to define a level of accuracy which is adequate for current command and control purposes. If the measurements in the current batch give a sufficiently accurate orbit, then there is no need to let $\hat{X}_o$ exert any influence on $\hat{X}_k$. If, however, the current batch only gives sufficient accuracy in certain components of $\hat{X}_k$, we must use some of the information contained in $\hat{X}_o$ to attain, if possible, the level of accuracy desired. These considerations lead to the following steps for a modification of equation (59).

1. Compute $\underline{H}^T \underline{Q}^{-1} \underline{H}$ and $\underline{H}^T \underline{Q}^{-1} (Y - \hat{Y})$

2. Modify $P_{x_o}^{-1} \rightarrow P_m^{-1}$ such that $(P_m^{-1} + \underline{H}^T \underline{Q}^{-1} \underline{H})^{-1}$ satisfies the tolerance levels.

3. Compute the solution for $\hat{X}$ using equation (59) with $P_{x_o}^{-1}$ replaced by P_m^{-1}.

4. Compute the solution for P_{x_k} using (60) with $P_{x_o}^{-1}$ replaced by P_m^{-1}.

5. Update to the next batch of data.

Step #2 in the above sequence presents a problem. The problem is in the definition of tolerance levels such that a simple algorithm can be used to carry out the calculations. This problem has not been solved.

The subsequent paragraphs will discuss some material which, it is believed, is relevant to finding a useful algorithm.

To gain some insight on the problem, consider how pseudo-variables effect a reduction in P_o^{-1} during a time update.

$$P_o(t) = \Phi P(t_o) \Phi^T + U_r \triangleq P_n(t) + U_r \tag{61}$$

If we write U_r in the form

$$U_r = \sum_{i=1}^{n} a_i^T b_i a_i \tag{62}$$

where

$$a_i = \text{a row vector}$$

$$b_i = \text{a scalar}$$

And if we compute

$$P_n^{-1}(t) = \Phi^{T^{-1}} P^{-1}(t_o) \Phi^{-1} \tag{63}$$

Then we may calculate

$$P_o^{-1}(t) = [P_n + U_r]^{-1} \tag{64}$$

by

$$P_{k-1}(t) = P_k^{-1} - P_k^{-1} a_k^T (a_k P_k^{-1} a_k^T + 1/b_k)^{-1} a_k P_k^{-1} \tag{65}$$

where

$$k = n, \ n-1, \ \cdots \cdots \ 1$$

and P_n^{-1} is given by (63)

The matrix identity

$$[A + v s v^T]^{-1} = A^{-1} - A^{-1}v(v^T A v + 1/s)^{-1}v^T A^{-1} \qquad (66)$$

where

$v =$ a column vector

$s =$ a scalar

is used in equation (65).

Equation (65) illustrates the manner in which pseudo-random forcing functions reduce the information matrix for the a priori estimate of state. In this instance, the vectors a_k and scalars b_k are defined by the manner in which the pseudo forcing functions enter the equations for a time update.

As one step in solving the problem given in Step #2, equation (66) provides a general means of removing information as is required in forming P_m^{-1} from $P_{x_o}^{-1}$. What we need to investigate is how to define a set of vectors, v_j, and scalars, s_j, such that the desired amount of information is extracted from $P_{x_o}^{-1}$. It appears reasonable to select the vectors v_j in accordance with

$$\left. \begin{array}{ll} v_j^T v_k = 0 & k \neq j \\[2em] v_j^T v_k = 1 & k = j \end{array} \right\} \qquad (67)$$

Since we are dealing with a 6-space (3 position components and 3 velocity components), 6 unit vectors, given by (67), will be selected.

The unit vectors can be arranged in a transformation matrix

$$6 \times 6 \atop T = [v_1 v_2 v_3 \cdots v_6] \tag{68}$$

If we perform the operation

$$P' = T[P_m^{-1} + \underline{H}^T \underline{Q}^{-1} \underline{H}]^{-1} T^T \tag{69}$$

then the diagonal elements of P' are variances of the quantities

$$\tilde{\omega} = \omega - \hat{\omega} \tag{70}$$

where

$$\tilde{\omega} = T \tilde{x} \tag{71}$$

We will consider the ω space as the coordinate frame in which we wish to express tolerance levels of orbit accuracy. This might, for example, be selected as a local tangent plane defined at the anchor point. T, of course, could be the identity matrix. In the latter case, we would select tolerance levels in the frame chosen for the covariance matrix.

The remaining factor is how to select the scalars s_i. One possibility is outlined below.

Let

$$B = \underline{H}^T \underline{Q}^{-1} \underline{H} \tag{72}$$

If we consider the operation

$$B_{k+1} = B_k - B_k v_k (v_k^T B_k v_k)^{-1} v_k^T B_k \qquad (73)$$

$$k = 1, 6$$

$$B_1 = B$$

then we note that the matrix B may be expressed as a sum of vector outer products.

If we let

$$b_k = B_k v_k \Big/ \sqrt{v_k^T B_k v_k} \qquad (74)$$

then

$$B = \sum_{k=1}^{6} b_k b_k^T \qquad (75)$$

One note in passing is that if $T = I$ in equation (68), then if the vectors b are ordered as shown in (76), a triangular*matrix is obtained

$$M = [b_1 \ b_2 \ b_3 \ \cdots \ b_6] \qquad (76)$$

In this instance

$$B = M M^T \qquad (77)$$

Hence, for this special case, the operation indicated by (73) and (74) provides a simple means of obtaining a square root matrix for an arbitrary positive definite symmetric matrix.

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* The elements of the matrix above the diagonal are identically zero.

A further note of importance is that $v_k^T B_k v_k$ must be a positive number. If zero (or some small negative values) are obtained, then the rank of the matrix B is less than 6. Since B represents the information matrix in the current batch of data, we have to add some information about the a priori estimate in this case in order to have a unique estimate of state. For this example, P_m^{-1} of step #2 cannot be zero.

These factors suggest the following algorithm for defining P_m^{-1}.

Let

ϵ_k = tolerance levels on the quantity , d_k, where

$$d_k = v_k^T B_k v_k + v_k^T A_k v_k \tag{78}$$

That is, it is desired that

$$d_k \geq \epsilon_k \tag{79}$$

We define the quantity A_k as

$$A_k = P_k^{-1} v_k (v_k^T P_k^{-1} v_k + \frac{1}{s_k})^{-1} v_k^T P_k^{-1} \tag{80}$$

and

$$P_{k+1}^{-1} = P_k^{-1} - A_k \tag{81}$$

Using (78)–(80) we determine the scalar, s_k, so that

$$(v_k^T P_k^{-1} v_k)^2 \Big/ (v_k^T P_k^{-1} v_k + 1/s_k) + v_k^T B_k v_k = \epsilon_k \tag{82}$$

If s_k from (82) is negative but

$$|1/s_k| < v_k^T P_k^{-1} v_k \tag{83}$$

we set $1/s_k = 0$ and define A_k and P_{k+1}^{-1} using equations (80) and (81). This corresponds to using all the information in the a priori estimate about the v_k vector direction.

If s_k from (82) is negative and

$$|1/s_k| > v_k^T P_k^{-1} v_k$$

we set

$$A_k = 0 \quad \text{and omit equation (81).}$$

This corresponds to sufficient information in the current batch for defining the v_k vector direction.

If s_k is positive, then we use A_k and P_k defined by (79) and (80) respectively. This corresponds to using some of the information about the v_k direction in the a priori estimate.

Following these steps we define

$$P_m^{-1} = \sum_{k=1}^{6} A_k \qquad (84)$$

The algorithm outlined above does not make a direct test on the covariance matrix of errors after the batch is included. Instead, tests are made about the information matrix. A similar effect to that given in equation (95) is obtained. The level of information reduction in the a priori matrix is, however, governed by considering the information in the batch of measurements.

The procedure outlined above for dynamic model compensation in WLS filters has not, to the writer's knowledge, been tested. Intuitively speaking, some advantages over the method which uses pseudo forcing functions are apparent. The primary advantage is that the amount of information extracted from the a priori matrix is governed by the information in the batch.

Certainly in any practical problem there will be difficulties associated with the assignment of the tolerance levels, ϵ_k. Also, this is only one of many algorithms which could be derived based on the same principles.

A more thorough investigation is obviously warranted before any definitive conclusions can be given.

CONCLUDING REMARKS

One of the fundamental problems of real time orbit determination, the retention of accuracy in the presence of mathematical model errors, has been reviewed. This review has shown the desirability of classifying the mathematical model errors in the following two types:

1. Dynamic (errors in the equations of motion)

2. Measurement (errors in the computation of the estimated measurement)

The dynamic model errors have been shown to cause a divergence from error analysis predictions in the errors in the estimated orbit. The divergence is displayed by the inability to find an orbit which fits the measurements over long time arcs.

The measurement model errors have the opposite effect on orbit determination accuracy. That is, increasing the length of the data arc and the number of measurements reduces the errors in the estimated orbit. As a result of these factors it has been shown possible to suppress the divergence caused by dynamic model errors by introducing dynamic model compensation. This compensation imposes a higher influence of the most recent measurements in the data reduction process.

Dynamic model compensation can be derived by either of the following methods:

1. Introduction of pseudo-variables which cause the covariance matrix of errors in estimate to grow with time.

2. Introduce a non-optimal filter which deliberately weights the most recent measurement heavier than that weighting obtained from optimal filter theory.

Algorithms for processing data by both Kalman and weighted least squares procedures have been outlined. The algorithms depend on both the type of filter (i.e., WLS or Kalman) and the method used in obtaining the dynamic model compensation.

Method (2) above appears to give algorithms which provide advantages over Method (1). The advantage lies in the fact that the information in the most recent data is used to determine how much past data is degraded. In the use of Method (1) this degradation is an a priori set value and hence can also be an additional model error. That is, the settings can be too large or too small unless they are made a function of the many variables of a tracking system.

Method (1) has the fundamental advantage of providing an optimal filter if the statistical modeling of the pseudo-variables are models of the actual error sources.

The use of acceptable tolerance levels for setting non-optimal filter control gains appears to have both theoretical and practical merits in state estimation problems. From practical considerations, there is always a finite level of accuracy which is adequate for the particular problem. From theoretical considerations, this modification in the performance index opens the field for a new approach in filter design. It appears reasonable to believe that filters designed with this new approach would be more appropriate for practical problems than those derived with current optimal filter theory.

The use of tolerance levels tends to remove some of the "black magic" in going from theory to practice. The tolerance levels must be set in any practical problem and the definition of appropriate values will depend on experimental results. In this sense, it is no better than any other compensation method. The significance lies in knowing what experiments need to be made. A methodical rather than pure cut and try approach for conducting the experiments therefore appears possible.

The author would like to emphasize again that the material in this paper is not applicable to post-flight analysis or more generally to the model identification problem. Improvements

in mathematical models and improvements in theory for solving such problems are definitely needed. The dynamic model compensation allows one to live with existing mathematical models if the attainable accuracy is sufficient. Advanced missions invariably demand increased accuracy, which justifies both improvements in instrumentation and improvements in data processing techniques.

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