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GENERAL RELATIVITY IN SATELLITE ORBITS

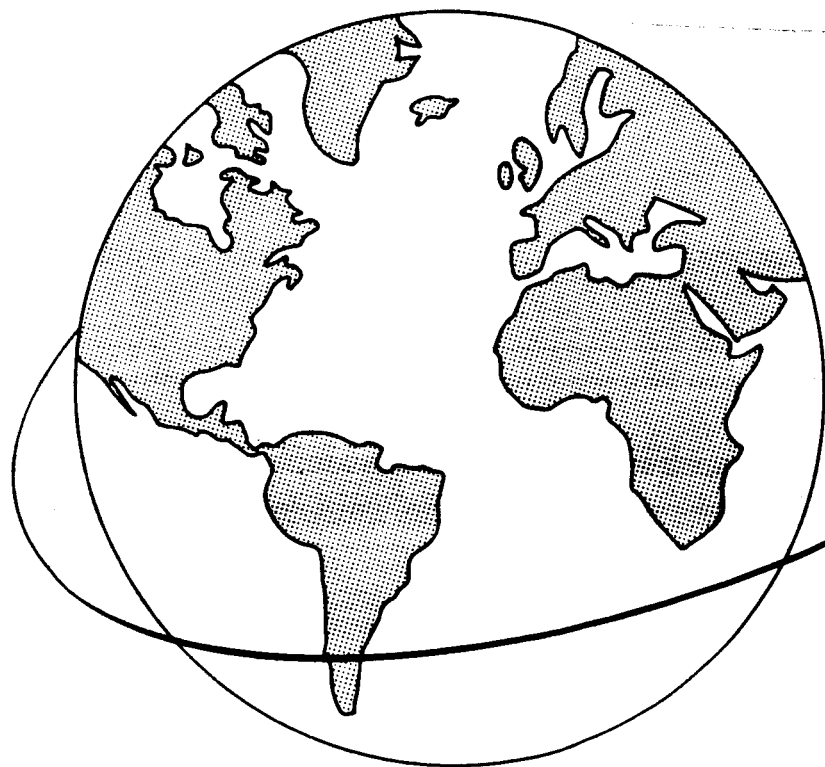
E. M. GAPOSCHKIN AND J. P. WRIGHT

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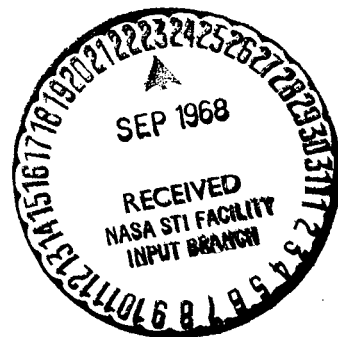
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A MEASURABLE EFFECT OF GENERAL RELATIVITY IN
SATELLITE ORBITS

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ABSTRACT

The reduction of data for satellites having a one-way Doppler system should include the general relativistic terms. There is a general relativistic correction term, which should be used in obtaining the velocity of a satellite from the observed frequency, that is periodic and proportional to the eccentricity. The effect should be observable for satellites with an eccentricity greater than 0.05. We should also be able to measure the effect directly by using single and double Doppler systems simultaneously from the same earth location.

RÉSUMÉ

L'analyse des données dans le cas de satellites ayant un système Doppler à sens unique devrait comprendre les termes généraux de relativité. Il y a un terme général de correction, terme relativiste, qui devrait être employé pour obtenir la vitesse d'un satellite à partir de la fréquence observée, c'est à dire un terme périodique et proportionnel à l'excentricité. L'effet devrait être observable pour des satellites à excentricité plus grande que 0,05. Nous devrions aussi pouvoir mesurer l'effet directement en employant en même temps des systèmes Doppler simples et doubles à un même endroit sur la terre.

КОНСПЕКТ

Редукция данных для спутников имеющих одностороннее доплеровское устройство должна включать общие релятивистические члены. Имеется общий релятивистический корректирующий член который должен быть употребляем для получения скорости спутника на основании наблюдаемой частоты являющейся периодической и пропорциональной эксцентриситету. Явление должно быть наблюдаемым для спутников с эксцентриситетом превышающим 0,05. Мы также должны иметь возможность непосредственно измерять это явление одновременно из одной и той же земной точки с помощью одиночных и двойных доплеровских систем.

A MEASURABLE EFFECT OF GENERAL RELATIVITY IN SATELLITE ORBITS

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There are some close earth satellites that carry RF transmitter instrumentation. By use of the Doppler effect, the radiation from this transmitter, which is received at ground stations, is used to infer the line-of-sight velocity component. The satellite navigation system known as TRANET and the series of TRANET satellites use the Doppler system. These Doppler data have been successfully used for almost a decade in orbit analysis, geodetic research, and navigation.

The transmitting oscillators in the satellites seem to have a stability of better than 1 part in 10^{10} . This has been established by laboratory experiments (Sykes, Smith, and Spencer, 1962) and by position analyses by Newton (1966) and E. M. Gaposchkin (unpublished).

Since the oscillators (which are clocks) will run at different rates (Landau and Lifshitz, 1962; Clemence and Szebehely, 1967) in different potential fields, the calculated Doppler velocity attributed to the satellite will be different depending upon whether one uses Newtonian or relativistic (Einsteinian) theory to reduce the data of the one-way Doppler satellites, such as TRANET observations. But this effect will not be present for two-way Doppler satellites such as the Goddard Range and Range Rate Doppler satellites (Kruger, 1965). In fact, simultaneous one-way and two-way Doppler data should differ by the relativistic effect.

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In the framework of Einstein's theory, the ratio of the observed frequency (ν_{obs}) to the standard frequency on the earth (ν_{E}) would be (for a symmetrical earth)

$$\frac{\nu_{\text{obs}}}{\nu_{\text{E}}} \cong \left(1 - \frac{GM_{\text{E}}}{c^2 r_{\text{s}}} - \frac{1}{2} \frac{V_{\text{E}}^2}{c^2} + \frac{GM_{\text{E}}}{c^2 r_{\text{E}}} + \frac{1}{2} \frac{V_{\text{s}}^2}{c^2} \right) \Bigg/ \left(1 - \frac{1}{c} \frac{dr}{dt} \right) ,$$

where M_{E} is the mass of the earth, V_{E} the velocity of the earth's rotation at the observer, r_{s} and V_{s} are the geocentric distance and total velocity of the satellite, respectively, r is the distance from the observer to the satellite, G is the gravitational constant, and c is the speed of light. This expression includes the special relativistic transverse Doppler effect. In practice, the nonsphericity of the earth's potential is included in the reduction, and the rate of the earth clock will depend on the potential at the station.

We may rewrite the above expression in terms of the orbital parameters as

$$\frac{\nu_{\text{obs}} - \nu_{\text{E}}}{\nu_{\text{obs}}} + \frac{\nu_{\text{E}}}{\nu_{\text{obs}}} \left[\frac{2GM_{\text{E}} e}{a(1 - e^2) c^2} \cos f + \text{const terms} \right] = \frac{1}{c} \frac{dr}{dt} ,$$

where the quantities a , e , and f are the semimajor axis, eccentricity, and true anomaly of the orbit, respectively. In computing effects of this type, these parameters must be calculated in the framework of a relativistic theory or Newtonian theory although a , e , and f will be nearly the same numerically.

The resulting expression is separated into two parts. One is a constant term that is indistinguishable from a zero set error in the oscillator frequency. The second term is multiplied by the eccentricity and has a period of 1 revolution.

The amplitude of the periodic term for a typical geodetic satellite (Geos 1), with $a = 8.07 \times 10^6$ m and $e = 0.07$, is $\approx 2.5 \text{ cm sec}^{-1}$ in velocity or a total variation of 5 cm sec^{-1} for one orbit. There will be post-Newtonian corrections to the orbital parameters in the reduction of the orbit, but, since these are essentially constant terms, we can calculate the periodic effect using the Newtonian values of G , M_{E} , a , e , and c . It is much more difficult to detect a constant effect than a periodic effect.

A stability of 1 part in 10^{10} in the oscillator is equivalent to 3 cm sec^{-1} , and there is abundant evidence that the Doppler data can be acquired to an accuracy of 1 to 2 cm sec^{-1} with a two-frequency system, which removes the ionospheric effects (Willman and Tucker, 1968; Tucker and Fannin, 1968; Berbert, 1967; Applied Physics Laboratory, 1968).

It is quite clear, then, that this periodic term should be included in the reduction of the data.

With an improved capability in orbit calculation, it would be possible to use this effect to establish the validity of relativity, if that is required. To achieve this capability the gravity-field and atmospheric reductions and the ionospheric and tropospheric effects will have to be improved, as well as other effects that have the same spectral characteristics as the periodic term.

It seems, therefore, that we can make a positive statement on the validity of the terms in the formulation of Einstein's theory that predict the gravitational redshift. The calculations show that the residuals are reduced consistently if the general relativistic effect is included; and, unless there is an effect that gives errors in the Doppler velocity of the same magnitude and direction, the calculations show that the relativistic effect is real. At this stage we cannot choose between the various theories using the effect, but it may be possible to do this in the near future (Dicke, 1967).

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BIOGRAPHICAL NOTES

E. M. GAPOSCHKIN graduated in Electrical Engineering from Tufts University in 1957. He received a Degree of Numerical Analysis in 1959 from Cambridge University in England and is now working toward a doctorate in geophysics at Harvard University.

Since joining the staff at Smithsonian in 1959, Mr. Gaposchkin has held positions as programmer and Division Chief of the Computations Division, and is now a mathematician in the Research and Analysis Department. He has helped to develop the basic computer program used in all analyses of satellite motion.

His main interests include satellite geodesy and geophysics and applied mathematics.

JAMES P. WRIGHT received the B.S. degree from the University of Florida in 1956 and the Ph.D. degree in chemistry from the University of Chicago in 1961.

Dr. Wright joined the staff of the Smithsonian Astrophysical Observatory as an astrophysicist in 1964. From 1961 to 1963 he was a National Science Foundation - National Research Council Associate at the Institute for Space Studies, and in 1963 and 1964 he spent a year at the Mathematics Research Center, Madison, Wisconsin.

Dr. Wright's fields of investigation include general relativistic effects and their observation; high-energy astrophysics; and interpretation of X-ray and cosmic-ray background.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

The Reports are regularly distributed to all institutions participating in the U. S. space research program and to individual scientists who request them from the Publications Division, Distribution Section, Smithsonian Astrophysical Observatory, Cambridge, Massachusetts 02138.