USER MANUAL

FOR

GERT SIMULATION PROGRAM

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A. Alan B. Pritsker

ABSTRACT

This report describes the procedures for using a digital computer program for simulating GERT networks. The simulation program can accommodate GERT networks which have EXCLUSIVE-OR, INCLUSIVE-OR and AND logical operations associated with the input side of a node and deterministic or probabilistic operations associated with the output side of a node. The branches of the GERT network are described in terms of a probability that the branch is realized and a time to perform the activity represented by the branch. (Time is used throughout this report to represent an additive variable.) The time associated with a branch can be a random variable.

The results obtained for the GERT simulation program are:

(1) The probability that a node is realized;

(2) The average time to realize a node;

(3) An estimate of the standard deviation of the time to realize a node;

(4) The minimum time observed to realize a node;

(5) The maximum time observed to realize a node; and

(6) A histogram of the times to realize a node.

Normally the above information is obtained for each sink node of the network. The program is written to permit the above information to be obtained for any node specified by the input data.

The GERT simulation program is written in GASP IIA, a FORTRAN - based simulation language. The program has been exercised on the IBM 1130

which has a FORTRAN IV operation set more restrictive than the general FORTRAN IV operation set. Therefore, the GERT simulation program can be used on most computers which have FORTRAN IV compilers. On the IBM 1130 with 8192 words of core storage, GERT networks with up to 30 nodes and 40 branches can be simulated.

The program described in this report has been submitted to COSMIC.

USER MANUAL FOR

GERT SIMULATION PROGRAM

The digital computer program for simulating GERT networks involves the use of the GASP IIA simulation language. Knowledge of simulation techniques and GASP IIA is not necessary in order to use the GERT simulation program. This manual describes the input formats for describing a GERT network for insertion into the computer. The definitions and procedures employed in the GERT simulation program are contained in a companion report. (19) The GERT simulation program after receiving the network to be simulated from the input data and the number of simulations of the network to be performed, automatically performs a simulation by selecting branches to be traversed based on random numbers and the time it takes to perform the activities represented by the branches based on samples from the distributions inserted as input information. Statistics are automatically collected on nodes as specified by the user of the program. When all simulations requested by the user have been completed, a summary report which describes the network simulated and the final results is printed. The final results include the following statistics:

(1) The probability that a specified node is realized;

(2) The average time to realize the specified node;

(3) An estimate of standard deviation of the time to realize the specified node;

(4) The minimum time observed to realize the specified node;

(5) The maximum time observed to realize the specified node; and

(6) A histogram of the times to realize the specified node.

The user of GERT simulation program need only be familiar with the input specifications in order to simulate a GERT network. The next section of this report details the input variables and the formats associated with them. Examples to illustrate the input data and the output reports from the program are presented at the end of this report.

Program Operating Procedure

The control cards for the program are the standard FORTRAN control cards for the computer being used. The input data cards will be described below in the order in which they must be read into the computer. Each data card will be described separately in terms of the fields required. The variable in each field will be described and the format and number of variables which must be inserted will be given. The first six data cards are special to the GERT simulation program. Data cards 7 through 19 are standard GASP IIA data cards. The use of GASP IIA necessitates some duplication of input data.

Data Card 1

Field 1. The largest node number of the network (I2, 1). The smallest node number permitted is 2. The largest node number permitted on the IBM 1130 version is 31.

Field 2. Number of nodes on which statistics are to be collected (I2, 1). The maximum value for the IBM 1130 version is 5.

Field 3. Number of source nodes (I2, 1). A maximum of 5 is permitted in the IBM 1130 version.

Field 4. Number of sink nodes that must be realized before the network is realized (I2, 1). A maximum of 5 is permitted in the IBM 1130 version.

Data Card 2

Field 1. The node numbers for all source nodes (40 I2, the number of source nodes). For example, if there is a 2 in Field 3 of Data Card 1 and if the source node numbers are 2 and 6 then 0206 would appear in columns 1 through 4 of data card 2.

Field 2. The node numbers for which statistics are to be collected (40 I2, the number of nodes on which statistics are to be collected).

Data Card 3

Field 1. The number of branches required to release each node. (40 I2, the largest node number assigned minus 1).

A value must be given for each number starting with node 2 and going to the largest node number assigned. Even if there is no node number 3, a value must be given to the number of releases associated with node number 3. Node numbers must be greater than or equal to 2 and the first value on card number 3 is the number of releases associated with node number 2.

If the number of releases assigned to a node is greater than 1, the AND logical operation is associated with the node. If the number of releases associated with a node is 1, the input side of the node is described by an OR operation. For the GERT simulation program, no distinction is made between INCLUSIVE-OR nodes or EXCLUSIVE-OR nodes. The simulation automatically accounts for the difference in these node types.

Data Card 4

Field 1. The node type or output characteristics of each node. As in data card 3, all numbers from 2 to the largest node number assigned must be given node type designators (40I2, the largest node number assigned minus 1).

Four codes are used to describe the output characteristics of a node. These codes are:

(1) The node has a deterministic output and each branch emanating from the node will be realized when the node is realized. However, once the node is realized it cannot be realized again and, therefore, the activities emanating from it cannot be caused to occur more than once.

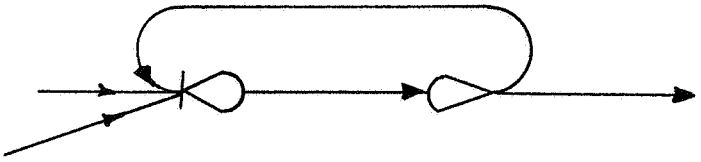
(2) The output side of the node is probabilistic and only one of the branches emanating from the node is caused to occur when the node is realized. Again, the node can only be realized once.

(3) The node has a deterministic output and can be realized many times. Once the node is realized, it only takes one release

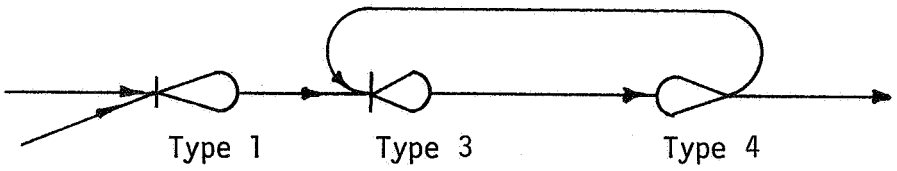
for it to be realized again.

(4) The node has a probabilistic output side and can be realized many times.

This characterization of nodes can cause an additional node to be required to describe a network. For example, if three branches are incident on a node, one of which is a feedback branch, it is necessary to specify the operations involved by two nodes. The first node would be a node type 1 or 2 and the second would be a node type 3 or 4. The condition described above is shown below:



must be drawn as



The above discussion also illustrates an anomoly with regard to the EXCLUSIVE-OR operation associated with an input side of a node. If a branch is fedback to an EXCLUSIVE-OR node, it activates the node even though the EXCLUSIVE-OR node has been realized previously. The EXCLUSIVE-OR operation only applies to branches which are incident to the node and can occur in any order. Since the feedback branch can only occur after a forward branch, the feedback branch also causes the node

to be realized.

Data Card 5

Field 1. The lower limit for the histogram to be obtained for each node on which statistics are being gathered. The lower limits are put on the card in the same order as the nodes are listed in Field 2 of Data Card 2. The value assigned will be the lower limit of the second cell of the histogram. The first cell of the histogram will contain the number of times the node was realized in a time lower than the lower limit given. (8F10.2, the number of nodes on which statistics are to be collected.)

Data Card 6

Field 1. The width of each cell of the histogram. The widths are put on the card in the same order as the nodes are listed in Field 2 of Data Card 2. (8F10.2, the number of nodes on which statistics are to be collected.)

Data Card 7

Field 1. The analysts name (6A2,1)

Field 2. The project number (I4,1)

Field 3. The month number (I2,1)

Field 4. The day number (I2,1)

Field 5. The year (I4, 1)

Field 6. The number of times the network is to be simulated (I4,1).

Data Card 8

Field 1. The number of activities with different time characteristics (I5,1)

Field 2. The number of nodes on which statistics are to be collected (I5,1)

Field 3. The number of nodes on which statistics are to be collected. Field 3 is the same as Field 2. It is required because standard GASP IIA input formats are being used. (I5,1)

Field 4. Blank (I5,1)

Field 5. The number of branches in the network plus the number of activities which can occur simultaneously. The maximum number that can be accommodated on the IBM 1130 is 40 (I5,1).

Field 6. A three (3) (I5,1).

Field 7. The largest node number assigned in the network (I5,1)

Field 8. A twenty-two (22) (I5,1)

Field 9. A one (1) (I5,1)

Data Card 9

Field 1. The number of cells to be included in each histogram minus 2. The number of cells does not include 1) cell 1

which counts the number of times the node is realized in a time less than the lower limit, nor 2) the last cell when the upper limit (computed from the lower limit plus the number of cells times the width of each cell) is exceeded. (I5, the number of nodes on which statistics are collected).

Data Card 10

Field 1. A set of ones. (I5, the largest node number assigned with 10 values on a card). For example, if the largest node number is 24, then 3 cards are required with four ones on the third card.

Data Card 11

Field 1. A set of ones. (I5, the largest node number assigned with 10 values on a card.) These cards are identical to the cards required for Data Card 10.

Data Card 12

The parameters associated with the distribution of the time to perform each activity. One card is required for each activity with a different time characterization. The number of cards is specified by Data Card 8, Field 1. The cards must be arranged by ascending activity number and the activities must be numbered consecutively or blank cards appropriately placed. Four distribution types are available in the IBM 1130 version of the GERT simulation program. These types are:

(1) Constant

(2) Normal

(3) Uniform

(4) Erlang

The fields required are dependent on the distribution type of the activity.

For distribution type 1:

Field 1. The constant time (F10.4,1)

For distribution type 2:

Field 1. The mean value (F10.4,1)

Field 2. The minimum value (F10.4,1)

Field 3. The maximum value (F10.4,1)

Field 4. The standard deviation (F10.4,1)

For distribution type 3:

Field 1. Not Used (F10.4,1)

Field 2. The minimum value (F10.4,1)

Field 3. The maximum value (F10.4,1)

Field 4. Not Used (F10.4,1)

For distribution type 4:

Field 1. The mean divided by the value given to Field 4 (F10.4,1)

Field 2. The minimum value (F10.4,1)

Field 3. The maximum value (F10.4,1)

Field 4. The number of exponential deviates to be included in the sample obtained from the Erlang distribution (F10.4,1)

If Field 4 is set equal to 1, an exponential deviate will be obtained from distribution type 4.

Samples are obtained from the distributions such that if a sample is less than the minimum value, the sample value is given the minimum value. Similarly, if the sample is greater than the maximum value, the sample value is assigned the maximum value. This is not sampling from a truncated distribution but sampling from a distribution with a given probability of obtaining the minimum and maximum values. The GERT simulation program has been written to facilitate the adding of other distribution types. FORTRAN subroutines for obtaining deviates from other distribution types are available (14,23).

Data Card 13

Field 1. A zero (I5,1)

Field 2. A one (I5,1)

Field 3. A zero (I5,1)

Field 4. A one (I5,1)

Field 5. A 0.0 (F10.3,1)

Field 6. Blank (F10.3,1)

Field 7. The random number seed (I4,1)

Data Card 14

Field 1. A "-" in column 9 and a "1" in column 10.

Data Card 15

Field 1. A one in column 10 (I10,1)

Field 2. A zero in column 20 (I10,1)

Field 3. A zero in column 30 (I10,1)

Field 4. A zero in column 40 (I10,1)

Data Card 16

Field 1. A 0.0 (F10.4,1)

Data Cards 17 and 18

For each activity associated with the network, two data cards are required. One in the format for Data Card 17 and one in the format for Data Card 18. For Data Card 17 the fields are:

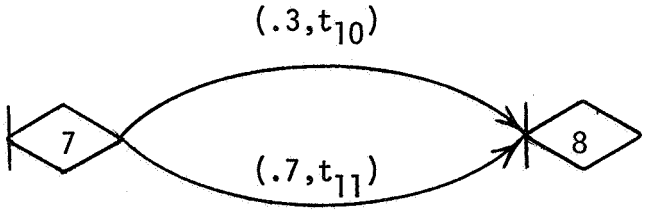
Field 1. Start node (I10,1)

Field 2. End node (I10,1)

Field 3. The activity number (I10,1)

Field 4. The distribution type (I10,1)

For each activity the Data Card type 18 contains only one field. This field contains the cumulative probability that the activity emanating from the start node as described in the Data Card type 17 will be taken. For a deterministic node,(node types 1 and 3), this probability will always be 1.0. For probabilistic nodes (node types 2 and 4), a cumulative probability is inserted in Field 1 (F10.4,1). An illustration of Data Cards 17 and 18 for two branches from a probabilistic node is given below.



Data Card Number	Field 1	Field 2	Field 3	Field 4
17	7	8	10	2
18	.0.3			
17	7	8	11	4
18	1.0			

For the example, the distribution type and activity numbers were selected arbitrarily and t_{10} is normally distributed and t_{11} is Erlang distributed. Note in data card 18, a 1.0 is used, not 0.7, since the cumulative probability is required.

Data Card 19

Field 1. A zero (I10,1)

A Data Card 19 is required after all Data Cards 17 and 18 are inserted. It specifies the end of the data cards.

Multiple networks can be analyzed by stacking the data cards as described above, one after another. No blank cards should separate the data cards for each network. A blank card is required to indicate the end of all networks to be simulated.

Limitations on the Variables for the IBM 1130 GERT Simulation Program

The largest node number must be less than or equal to 31. The number of nodes on which statistics are collected must be less than or equal to 5.

The number of activities (branches) which can be included is dependent on core storage availability. By using the LOCAL storage concept of the IBM 1130, the number of activities can be increased at the expense of computer execution time. The changes required to increase the number of activities are:

1. The DIMENSION card of the main program so that the dimension of NSET is 5*ID and for QSET is ID where ID is the number of activities plus the maximum number of simultaneous end of activities events; and
2. Data Card 8, Field 5 where ID is initialized.

The maximum number of sets of parameters for the distributions associated with an activity is 40. However different branches can have the same activity number. Subprograms which can be put in LOCAL are: DATAN, SUMRY, OTPUT, MONTR, HISTO, COLCT, RNORM, ERLNG, and RMOVE.

The GERT simulation program has been written in general terms and if a larger core memory is available, only the COMMON and DIMENSION statements associated with the program need be changed in order to remove the limitations given above.

Examples

Four examples will be presented. The purpose of the examples is to illustrate the input required to utilize the GERT simulation program and the output obtained from the GERT simulation program. For each example the network will be illustrated, the input required for the GERT simulation program presented and the output obtained will be displayed. Details of the method for performing the simulation is contained in a companion report (19).

Example 1

The first example involves the analysis of a simple network with one self loop as shown in Fig. 1. The start node is node 2 since all node numbers must have a value of 2 or greater. The probabilities associated with each branch of the network are shown as the first entry of the 2-tuple given above each branch. The second entry is the time variable where the subscript identifies the parameter set associated with the time variable. This convention will be followed throughout all the examples. For Example 1, all times will be considered as constants and $t_1 = 1$, $t_2 = 2$, and $t_3 = 3$. The characteristics of the network will be described in terms of the data cards required as input to the GERT simulation program.

The input data to analyze the network shown in Fig. 1 is presented in Fig. 2. Each line in Fig. 2 represents one data card. The data card number is shown on the left of the figure and is not part of the input data. The first data card specifies that the largest node

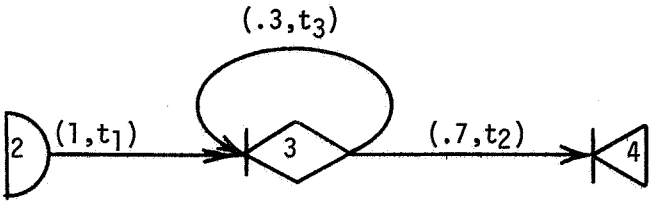


Fig. 1 The Network for Example 1

Data Card Type — COLUMNS

	2	10	20	30	40	50	80
1	04010101						DAT1 10
2	0204						DAT1 20
3	000101						DAT1 30
4	010401						DAT1 40
5		3.0					DAT1 50
6		3.0					DAT1 60
7		PRITSKER	2	730 19681000			DAT1 70
8		3 1	1 0	30 3	4 22	1	DAT1 80
9		20					DAT1 90
10		1 1	1 1				DAT1 100
11		1 1	1 1				DAT1 110
12		1.0					DAT1 120
12		2.0					DAT1 130
12		3.0					DAT1 140
13		0 1	0 1	0.0	400.	0567	DAT1 150
14		-1					DAT1 160
15		1	0	0	0		DAT1 170
16		0.0					DAT1 180
17		2	3	1	1		DAT1 190
18		1.0					DAT1 200
17		3	4	2	1		DAT1 210
18		0.7					DAT1 220
17		3	3	3	1		DAT1 230
18		1.0					DAT1 240
19		0					DAT1 250

Fig. 2 The Input Data for Example 1

number is 4, there is one node on which statistics are to be collected, there is one source node, and it is only required to realized one sink node in order to realize the network. On data card type 2, the source node is specified as node 2 and the sink node as node 4. In this case, statistics will be collected on the time to realize node 4 and the network will be realized when node 4 is realized. On data card type 3 the number of releases associated with nodes 2, 3 and 4 are given as 0, 1 and 1. On data card type 4, node 2 is defined to have a deterministic output size (node type 1). Node 3 has a probabilistic output side and can be realized more than once (node type 4), and node 4 is shown with a deterministic output side (for sink nodes the output description of the node is immaterial). On data card types 5 and 6, the lower limit and width of the histogram are given as 3.0 and 3.0. The above completes the specialized data cards required for the GERT simulation program. The data required for GASP IIA is contained in data card types 7 through 19.

Data card type 7 presents identification information and the number of simulations of the network to be performed. This is given in the last data field and is 1000. On data card type 8, the first field specifies the number of cards of type 12 to be read. For Example 1, there are three parameter sets to be read. In Field 5 of data card type 8, a value of 30 is used as the maximum number of branches in the network plus the number of simultaneous activities which can occur. By using 30 instead of 40, all subroutines mentioned in the preceeding section need not be inserted in LOCAL. This causes a faster execution

time for the simulation program. Data card types 9 through 11 follow the input specification presented previously. The three cards of type 12 provide the data for the constant time associated with the three activities of the network. These must be inserted in the order in which the activities are defined. Thus activity 1 has a constant time of 1 time unit, activity 2 has a constant time of 2 time units, and activity 3 has a constant time of 3 units. The information that these times are constant is contained in the activity descriptions which are provided by data card type 17.

Data cards 13 through 16 are in the standard form presented in the previous section. The initial random number seed is 0567 as shown in data card type 13. On data card types 17 and 18, the information associated with each branch of the network is provided. On the cards sequenced DAT1 190 and DAT1 200, a branch from node 2 to node 3 is described. This activity has parameters described by parameter set 1 and has a distribution of type 1. The probability of taking the branch from node 2 to node 3 is one as seen from the type 18 data card shown in card number DAT1 200. On the card sequenced as DAT1 220, the probability of going from node 3 to 4 is given as 0.7. On the card sequenced as DAT1 240 a value of 1.0 is shown. To obtain the probability of going from node 3 to node 3, 0.7 must be subtracted from the 1.0 since the cumulative probability must be inserted in data card type 18. A zero is inserted in column 10 of data card 19 to indicate the end of the data.

The first output obtained for the GERT simulation program is an echo check of the parameter set and the GASP storage area (Fig. 3) which contains

SIMULATION PROJECT NO. 1 BY PRITSKER

DATE 7/ 30/ 1968 RUN NUMBER 1

PARAMETER NO. 1	1.0000	0.0000	0.0000	0.0000
PARAMETER NO. 2	2.0000	0.0000	0.0000	0.0000
PARAMETER NO. 3	3.0000	0.0000	0.0000	0.0000

GASP JOB STORAGE AREA DUMP AT 0.0000 TIME UNITS

NSET

1	0	0	0	7777	9999
2	3	1	1	7777	9999
3	4	2	1	4	9999
4	3	3	1	7777	3
5	0	0	0	6	9999
6	0	0	0	7	5
7	0	0	0	8	6
8	0	0	0	9	7
9	0	0	0	10	8
10	0	0	0	11	9
11	0	0	0	12	10
12	0	0	0	13	11
13	0	0	0	14	12
14	0	0	0	15	13
15	0	0	0	16	14
16	0	0	0	17	15
17	0	0	0	18	16
18	0	0	0	19	17
19	0	0	0	20	18
20	0	0	0	21	19
21	0	0	0	22	20
22	0	0	0	23	21
23	0	0	0	24	22
24	0	0	0	25	23
25	0	0	0	26	24
26	0	0	0	27	25
27	0	0	0	28	26
28	0	0	0	29	27
29	0	0	0	30	28
30	0	0	0	8888	29

QSET

1	0.000000E 00
2	0.100000E 01
3	0.700000E 00
4	0.100000E 01
5	0.000000E 00
6	0.000000E 00
7	0.000000E 00
8	0.000000E 00
9	0.000000E 00
10	0.000000E 00
11	0.000000E 00
12	0.000000E 00
13	0.000000E 00
14	0.000000E 00
15	0.000000E 00
16	0.000000E 00
17	0.000000E 00
18	0.000000E 00
19	0.000000E 00
20	0.000000E 00
21	0.000000E 00
22	0.000000E 00
23	0.000000E 00
24	0.000000E 00
25	0.000000E 00
26	0.000000E 00
27	0.000000E 00
28	0.000000E 00
29	0.000000E 00
30	0.000000E 00

Fig. 3 Echo Check Before Simulation Starts

the network description in coded form. When this echo check is printed, the input data has been completely read by the computer. The GASP manual (22) provides instructions for reading the echo check and gives the defintions of the GASP error codes. When the simulation is completed a report is printed which gives the network description and the activity parameters for the network. The report obtained from the simulation for Example 1 is presented in Fig. 4. The final results are printed on a separate page as shown in Fig. 5. The results are for 1000 simulations of the network presented in Fig. 1 and show that the probability of reaching node 4 is 1.0 and that the mean time to realize node 4 was 4.299 with a standard deviation of 2.3356. The GERT EXCLUSIVE-OR program (11) was used to analyze the network of Fig. 1 and the results obtained were a mean value of 4.288 and a standard deviation of 2.360. A comparison of the histogram shown in Fig. 5 with the theoretical probabilities is shown below:

Time Node 4 is Realized	Observed Number	Theoretical Number
3	695	700.00
6	212	210.00
9	69	63.00
12	18	18.90
15	2	5.67
18	3	1.70
21	1	0.51

GERT SIMULATION PROJECT 1 BY PRITSKER
DATE 7/ 30/ 1968

NETWORK DESCRIPTION

START NODE	END NODE	ACTIVITY NUMBER	DISTRIBUTION TYPE	PROBABILITY
2	3	1	1	1.0000
3	4	2	1	0.7000
3	3	3	1	0.3000

ACTIVITY PARAMETERS

ACTIVITY NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0000	0.0000	0.0000
2	2.0000	0.0000	0.0000	0.0000
3	3.0000	0.0000	0.0000	0.0000

Fig. 4. The Network Description and Activity Parameters for Example 1

GERT SIMULATION PROJECT 1 BY PRITSKER
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FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
4	1.0000	4.2990	2.3356	3.0000	21.0000

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
4	3.	3.0	0	695	212	69	18	2	3	1	0	0	0
			0	0	0	0	0	0	0	0	0	0	0

Fig. 5. Final Results For 1,000 Simulations of the Network Given in Fig. 1

Example 2

The network for Example 2 is shown in Fig. 6, and is taken from reference (11) with slight modifications. The network has 2 source nodes and 2 sink nodes, and will be used to describe the input data for networks with multiple source nodes.

First the network will be analyzed with one source node, node 9 and two sink nodes only one of which must be realized in order to realize the network. The input data for this network is shown in Fig. 7. The fact that only one source node is being used is specified by Field 3 of data card type 1. The source node is defined as node 9 in data card type 2, Field 1. The information that only one sink node must be realized to realize the network is given by Field 4 of data card type 1. Statistics are to be collected on two nodes as defined in Field 2 of data card type 1 and Field 2 of data card type 2 defines nodes 7 and 8 as the nodes on which statistics are to be collected. Even though the branch from node 2 to node 4 is not being used in this simulation since node 2 was not specified as a source node it can be included as part of the input information as shown by the cards sequenced as DAT2 270 and 280. The program automatically starts the simulation from source node 9 and does not schedule activities from node 2.

Figure 8 presents the network description and the activity parameters which are printed out by the GASP simulation program. Figure 9 presents the final report obtained from the GASP simulation program for 400 simulations of the network described in Fig. 6 when the source node is node 9. Note that the final report does not specify the source node

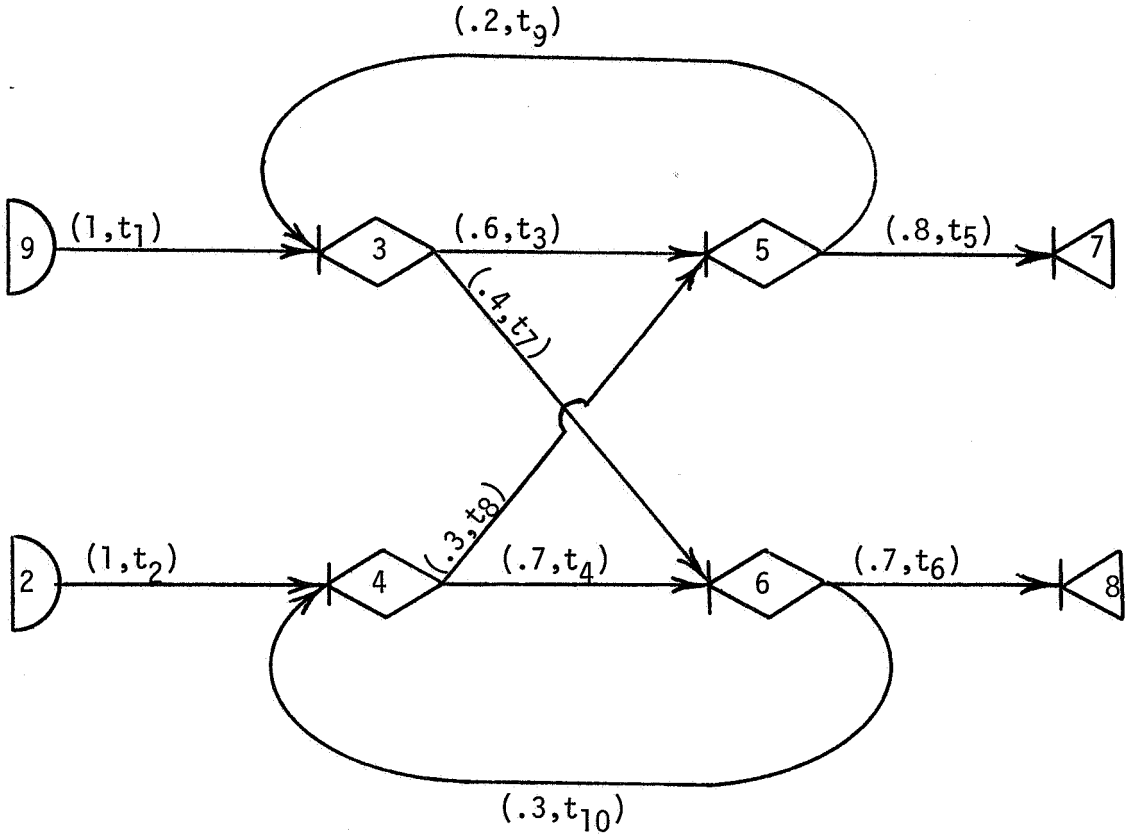


Fig. 6 The network for example 2

Data Card Type	COLUMNS 2 10	1 0	2 0	3 0	4 0	5 0	8 0
1	09020101						DAT2 05
2	090708						DAT2 10
3	0001010101010100						DAT2 20
4	0100404040410101						DAT2 30
5		2.0	2.0				DAT2 40
6		2.0	2.0				DAT2 50
7	PRITSKER	2	7301968040	0			DAT2 60
8	10	2	2 0	30	3 9	22 1	DAT2 70
9	20	20					DAT2 80
10	1	1	1 1	1	1	1 1 1 1	DAT2 90
11	1	1	1 1	1	1	1 1 1 1	DAT2 100
12	1.0						DAT2 110
12	1.0						DAT2 120
12	2.0						DAT2 130
12	2.0						DAT2 140
12	0.0						DAT2 150
12	0.0						DAT2 160
12	4.0						DAT2 170
12	5.0						DAT2 180
12	7.0						DAT2 190
12	6.0						DAT2 200
13	0	1	0 1	0.0	400.	0567	DAT2 210
14		-1					DAT2 220
15		1	0	0	0		DAT2 230
16		0.0					DAT2 240
17		9	3	1	1		DAT2 250
18		1.0					DAT2 260
17		2	4	2	1		DAT2 270
18		1.0					DAT2 280
17		3	5	3	1		DAT2 290
18		0.6					DAT2 300
17		3	6	7	1		DAT2 310
18		1.0					DAT2 320
17		4	5	8	1		DAT2 330
18		0.3					DAT2 340
17		4	6	4	1		DAT2 350
18		1.0					DAT2 360
17		5	7	5	1		DAT2 370
18		0.8					DAT2 380
17		5	3	9	1		DAT2 390
18		1.0					DAT2 400
17		6	8	6	1		DAT2 410
18		0.7					DAT2 420
17		6	4	10	1		DAT2 430
18		1.0					DAT2 440
19		0					DAT2 450

Fig. 7 Input data for Example 2 With Node 9 as the Source Node

GERT SIMULATION PROJECT 2 BY PRITSKER
DATE 7/ 30/ 1968

NETWORK DESCRIPTION

START NODE	END NODE	ACTIVITY NUMBER	DISTRIBUTION TYPE	PROBABILITY
2	4	2	1	1.0000
3	5	3	1	0.6000
3	6	7	1	0.4000
4	5	8	1	0.3000
4	6	4	1	0.7000
5	7	5	1	0.8000
5	3	9	1	0.2000
6	8	6	1	0.7000
6	4	10	1	0.3000
9	3	1	1	1.0000

ACTIVITY PARAMETERS

ACTIVITY NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0000	0.0000	0.0000
2	1.0000	0.0000	0.0000	0.0000
3	2.0000	0.0000	0.0000	0.0000
4	2.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000
7	4.0000	0.0000	0.0000	0.0000
8	5.0000	0.0000	0.0000	0.0000
9	7.0000	0.0000	0.0000	0.0000
10	6.0000	0.0000	0.0000	0.0000

Fig. 8 Network Description and Activity Parameters for the Input Data of Fig. 7

GERT SIMULATION PROJECT 2 BY PRITSKER
DATE 7/ 30/ 1968

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
7	0.6025	5.9419	6.4708	3.0000	42.0000
8	0.3975	8.9182	7.1794	5.0000	51.0000

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
7	2.	2.0	0	190	0	0	0	0	26	0	9	0	6
			0	7	0	0	0	2	0	0	0	0	1
8	2.	2.0	0	0	107	0	0	0	26	13	0	0	4
			3	0	1	0	2	1	0	0	0	1	1

Fig. 9 Final Results from GERT Simulation Program Using Input Data of Fig. 7

Field 1 of Data Card 2 involves both nodes 9 and 2. Data Card 2 would be as shown below:

09020708

The project number as given in Field 2 of Data Card 7 is now changed to 22. The final result for 400 simulations of the network with 2 source nodes is shown in Fig. 11.

A word of caution should be given here about attempting to simulate the network of Fig. 6 with 2 source nodes and 2 sink nodes and requiring that both sink nodes be realized for the network to be realized. Since both source nodes can lead to a single sink node, it is possible that the 2 sink nodes will never be realized during a simulation of the network. When this occurs, a GASP error type 87 will be reported. It is the analyst's responsibility to prepare networks for which all sink nodes which must be realized can be realized. That is, if it is required to realize three sink nodes before realizing the network, then all three sink nodes must be realized for every simulation of the network.

Example 3

The network selected for Example 3 is given in Fig. 12. It is given from the GASP manual (22) and is a standard PERT-type network. The input data for the network shown in Fig. 12 is given in Fig. 13. The time variables of the branches are assumed to be normally distributed. The parameters for the normal distribution are given on the data cards

GERT SIMULATION PROJECT 21 BY PRITSKER
DATE 7/ 30/ 1968

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
7	0.3400	10.2794	6.6885	6.0000	40.0000
8	0.6600	6.0757	6.6665	3.0000	49.0000

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
7	2.	2.0	0	0	0	87	0	0	0	33	0	0	0
			10	2	0	1	2	0	0	0	0	1	0
8	2.	2.0	0	200	0	0	0	38	1	0	9	9	0
			0	3	1	0	0	1	1	0	0	0	2

Fig. 10 Final Results of GERT Simulation Program When Node 2 is the Source Node for the Network Given in Fig. 6

GERT SIMULATION PROJECT 22 BY PRITSKER
DATE 7/ 30/ 1968

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
7	0.5425	3.8156	2.9475	3.0000	30.0000
8	0.4575	4.2131	2.0091	3.0000	13.0000

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
7	2.	2.0	0	187	0	22	0	0	2	3	1	0	0
			1	0	0	0	1	0	0	0	0	0	0
8	2.	2.0	0	106	66	0	0	10	1	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0

Fig. 11 Final Results When Nodes 2 and 9 of Fig. 6 are Source Nodes

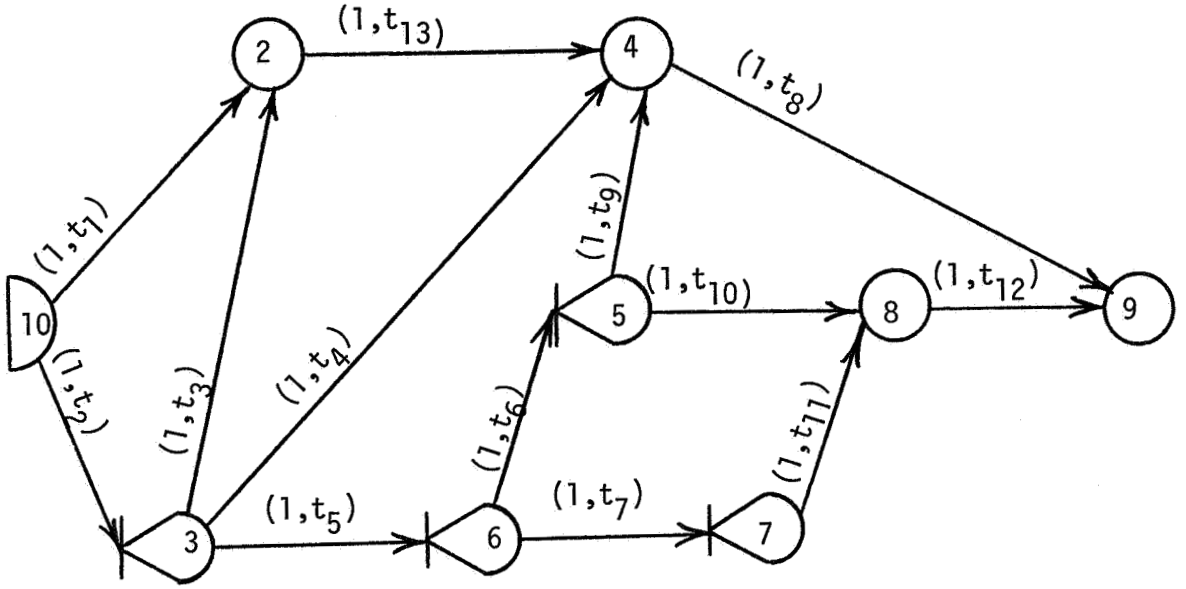
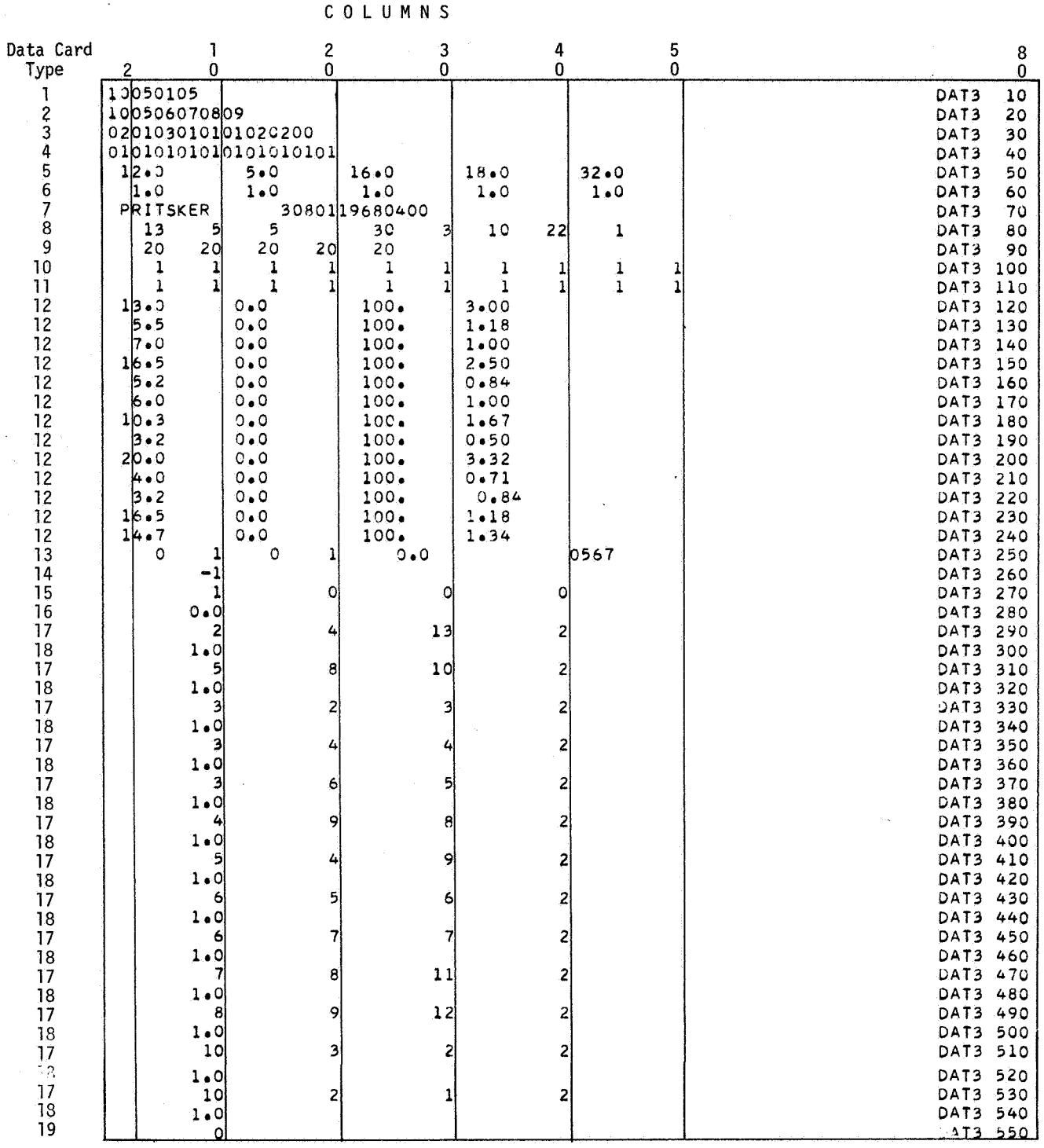


Fig. 12 The network for example 3



COLUMNS

Data Card Type	1 2 0	2 0	3 0	4 0	5 0	8 0
1	10050105					DAT3 10
2	10050607080 9					DAT3 20
3	02010301010102C200					DAT3 30
4	01010101010101010101					DAT3 40
5	12.0	5.0	16.0	18.0	32.0	DAT3 50
6	1.0	1.0	1.0	1.0	1.0	DAT3 60
7	PRITSKER	30801	19680400			DAT3 70
8	13 5	5	30 3	10 22	1	DAT3 80
9	20 20	20 20	20			DAT3 90
10	1 1	1 1	1 1	1 1	1 1	DAT3 100
11	1 1	1 1	1 1	1 1	1 1	DAT3 110
12	13.0	0.0	100.	3.00		DAT3 120
12	5.5	0.0	100.	1.18		DAT3 130
12	7.0	0.0	100.	1.00		DAT3 140
12	16.5	0.0	100.	2.50		DAT3 150
12	5.2	0.0	100.	0.84		DAT3 160
12	6.0	0.0	100.	1.00		DAT3 170
12	10.3	0.0	100.	1.67		DAT3 180
12	3.2	0.0	100.	0.50		DAT3 190
12	20.0	0.0	100.	3.32		DAT3 200
12	4.0	0.0	100.	0.71		DAT3 210
12	3.2	0.0	100.	0.84		DAT3 220
12	16.5	0.0	100.	1.18		DAT3 230
12	14.7	0.0	100.	1.34		DAT3 240
13	0 1	0 1	0.0		0567	DAT3 250
14	-1					DAT3 260
15	1	0	0	0		DAT3 270
16	0.0					DAT3 280
17	2	4	13	2		DAT3 290
18	1.0					DAT3 300
17	5	8	10	2		DAT3 310
18	1.0					DAT3 320
17	3	2	3	2		DAT3 330
18	1.0					DAT3 340
17	3	4	4	2		DAT3 350
18	1.0					DAT3 360
17	3	6	5	2		DAT3 370
18	1.0					DAT3 380
17	4	9	8	2		DAT3 390
18	1.0					DAT3 400
17	5	4	9	2		DAT3 410
18	1.0					DAT3 420
17	6	5	6	2		DAT3 430
18	1.0					DAT3 440
17	6	7	7	2		DAT3 450
18	1.0					DAT3 460
17	7	8	11	2		DAT3 470
18	1.0					DAT3 480
17	8	9	12	2		DAT3 490
18	1.0					DAT3 500
17	10	3	2	2		DAT3 510
18	1.0					DAT3 520
17	10	2	1	2		DAT3 530
18	1.0					DAT3 540
19	0					DAT3 550

Fig. 13 The Input Data for Example 3

sequenced from DAT3 120 to DAT3 240. In Field 4 of data card type 17 a 2 indicates that the times are normally distributed. The network description and the activity parameters obtained from the GERT simulation program are shown in Fig. 14. The final results for 400 simulations of the network are shown in Fig. 15.

In this example statistics were collected on nodes 5, 6, 7, 8 and 9. This is accomplished by specifying that there are five nodes on which statistics are to be collected and that it is required to realize all five nodes before the network is realized. This is specified in data card type 1.

Example 4

The network of Example 4 is taken from reference (7) and is shown in Fig. 16. This network has 19 nodes numbered from node 2 to node 20 and 32 branches. The number of different parameters associated with the times for the branches is only 4. Therefore, only 4 parameter sets need be used with this network. The example illustrates the input when branches have common parameter sets. This is seen in data card type 17, Field 3 where the activity number is the same for many of the branches. This network also illustrates a simulation of a network involving the different logical operations associated with GERT networks. The input data for this network is shown in Fig. 17. The network description and activity parameters is given in Fig. 18. The final results for 400 simulations of the network is presented in Fig. 19.

GERT SIMULATION PROJECT 3 BY PRITSKER
DATE 8/ 1/ 1968

NETWORK DESCRIPTION

START NODE	END NODE	ACTIVITY NUMBER	DISTRIBUTION TYPE	PROBABILITY
2	4	13	2	1.0000
3	2	3	2	1.0000
3	4	4	2	1.0000
3	6	5	2	1.0000
4	9	8	2	1.0000
5	8	10	2	1.0000
5	4	9	2	1.0000
6	5	6	2	1.0000
6	7	7	2	1.0000
7	8	11	2	1.0000
8	9	12	2	1.0000
10	3	2	2	1.0000
10	2	1	2	1.0000

ACTIVITY PARAMETERS

ACTIVITY NUMBER	PARAMETERS 1	2	3	4
1	13.0000	0.0000	100.0000	3.0000
2	5.5000	0.0000	100.0000	1.1800
3	7.0000	0.0000	100.0000	1.0000
4	16.5000	0.0000	100.0000	2.5000
5	5.2000	0.0000	100.0000	0.8400
6	6.0000	0.0000	100.0000	1.0000
7	10.3000	0.0000	100.0000	1.6700
8	3.2000	0.0000	100.0000	0.5000
9	20.0000	0.0000	100.0000	3.3200
10	4.0000	0.0000	100.0000	0.7100
11	3.2000	0.0000	100.0000	0.8400
12	16.5000	0.0000	100.0000	1.1800
13	14.7000	0.0000	100.0000	1.3400

Fig. 14 The Network Description and Activity Parameters for Example 3

GERT SIMULATION PROJECT 3 BY PRITSKER
DATE 8/ 1/ 1968

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
5	1.0000	16.8135	1.6645	11.3974	22.2840
6	1.0000	10.7760	1.3891	6.4889	14.9853
7	1.0000	21.0646	2.2413	14.8873	27.6319
8	1.0000	24.2792	2.3347	17.8410	32.0453
9	1.0000	42.1309	2.7951	34.3539	51.4105

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
5	12.	1.0	1	4	10	40	65	105	83	55	23	10	3
			1	0	0	0	0	0	0	0	0	0	0
6	5.	1.0	0	0	1	8	23	84	107	100	55	16	6
			0	0	0	0	0	0	0	0	0	0	0
7	16.	1.0	5	9	17	41	54	72	74	58	30	22	10
			6	2	0	0	0	0	0	0	0	0	0
8	18.	1.0	1	2	9	13	45	51	59	75	59	39	19
			19	4	4	0	1	0	0	0	0	0	0
9	32.	1.0	0	0	0	1	3	5	16	25	35	56	60
			55	48	33	30	13	11	6	0	1	2	0

Fig. 15 Final Results for 400 Simulations of the PERT Type Network Given in Fig. 12

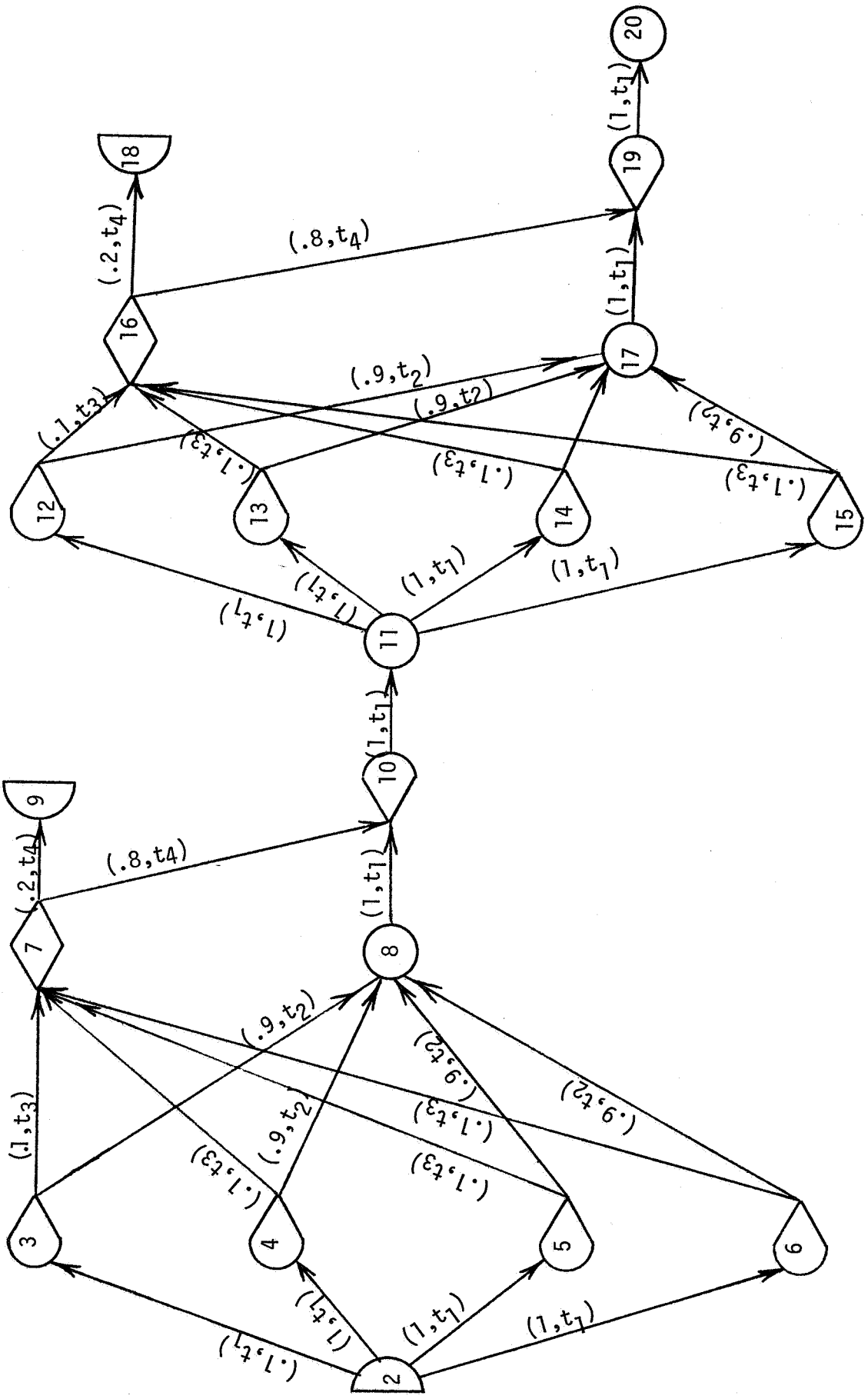


Fig. 16 The network for example 4

COLUMNS

Data Card Type	2	10	20	30	40	50	80
1	20030101						DAT4 10
2	02091820						DAT4 20
3	1 1 1 1 1	1 4 1 1 1	1 1 1 1 1	4 1 1 1			DAT4 30
4	1 2 2 2 2	2 1 1 1 1	2 2 2 2 2	1 1 1 1			DAT4 40
5		0.0	0.0	0.0			DAT4 50
6		5.0	5.0	5.0			DAT4 60
7	RONALD ENLOW	4 8	51968	400			DAT4 70
8	4 3	3 0	40 3	20	22	1	DAT4 80
9	20 20	20 0	0				DAT4 90
10	1 1	1 1	1 1	1	1	1 1	DAT4 100
10	1 1	1 1	1 1	1	1	1 1	DAT4 110
11	1 1	1 1	1 1	1	1	1 1	DAT4 120
11	1 1	1 1	1 1	1	1	1 1	DAT4 130
12	1.0						DAT4 140
12	3.0	0.0	6.0		1.0		DAT4 150
12	10.0	7.0	13.0		1.0		DAT4 160
12	12.0	6.0	18.0		2.0		DAT4 170
13	0 1	0 1	0.0		0.0	567	DAT4 180
14	-1						DAT4 190
15	1	0	0				DAT4 200
16	0.0						DAT4 210
17	2	3	1		1		DAT4 220
18	1.0						DAT4 230
17	2	4	1		1		DAT4 240
18	1.0						DAT4 250
17	2	5	1		1		DAT4 260
18	1.0						DAT4 270
17	2	6	1		1		DAT4 280
18	1.0						DAT4 290
17	3	7	3		2		DAT4 300
18	0.1						DAT4 310
17	3	8	2		2		DAT4 320
18	1.0						DAT4 330
17	4	7	3		2		DAT4 340
18	0.1						DAT4 350
17	4	8	2		2		DAT4 360
18	1.0						DAT4 370
17	5	7	3		2		DAT4 380
18	0.1						DAT4 390
17	5	8	2		2		DAT4 400
18	1.0						DAT4 410
17	6	7	3		2		DAT4 420
18	0.1						DAT4 430
17	6	8	2		2		DAT4 440
18	1.0						DAT4 450
17	7	9	4		2		DAT4 460
18	0.2						DAT4 470
17	7	10	4		2		DAT4 480
18	1.0						DAT4 490
17	8	10	1		1		DAT4 500
18	1.0						DAT4 510
17	10	11	1		1		DAT4 520
18	1.0						DAT4 530
17	11	12	1		1		DAT4 540
18	1.0						DAT4 550
17	11	13	1		1		DAT4 560
18	1.0						DAT4 570
17	11	14	1		1		DAT4 580
18	1.0						DAT4 590
17	11	15	1		1		DAT4 600
18	1.0						DAT4 610
17	12	16	3		2		DAT4 620
18	0.1						DAT4 630
17	12	17	2		2		DAT4 640
18	1.0						DAT4 650
17	13	16	3		2		DAT4 660
18	0.1						DAT4 670
17	13	17	2		2		DAT4 680
18	1.0						DAT4 690
17	14	16	3		2		DAT4 700
18	0.1						DAT4 710
17	14	17	2		2		DAT4 720
18	1.0						DAT4 730
17	15	16	3		2		DAT4 740
18	0.1						DAT4 750
17	15	17	2		2		DAT4 760
18	1.0						DAT4 770
17	16	18	4		2		DAT4 780
18	0.2						DAT4 790
17	16	19	4		2		DAT4 800
18	1.0						DAT4 810
17	17	19	1		1		DAT4 820
18	1.0						DAT4 830
17	19	20	1		1		DAT4 840
18	1.0						DAT4 850
19	0						DAT4 860

Fig. 17 The Input Data for Simulating the Network Given in Fig. 16

GERT SIMULATION PROJECT 4 BY RONALD ENLOW
DATE 8/ 5/ 1968

NETWORK DESCRIPTION

START NODE	END NODE	ACTIVITY NUMBER	DISTRIBUTION TYPE	PROBABILITY
2	3	1	1	1.0000
2	4	1	1	1.0000
2	5	1	1	1.0000
2	6	1	1	1.0000
3	7	3	2	0.1000
3	8	2	2	0.9000
4	7	3	2	0.1000
4	8	2	2	0.9000
5	7	3	2	0.1000
5	8	2	2	0.9000
6	7	3	2	0.1000
6	8	2	2	0.9000
7	9	4	2	0.2000
7	10	4	2	0.8000
8	10	1	1	1.0000
10	11	1	1	1.0000
11	12	1	1	1.0000
11	13	1	1	1.0000
11	14	1	1	1.0000
11	15	1	1	1.0000
12	16	3	2	0.1000
12	17	2	2	0.9000
13	16	3	2	0.1000
13	17	2	2	0.9000
14	16	3	2	0.1000
14	17	2	2	0.9000
15	16	3	2	0.1000
15	17	2	2	0.9000
16	18	4	2	0.2000
16	19	4	2	0.8000
17	19	1	1	1.0000
19	20	1	1	1.0000

ACTIVITY PARAMETERS

ACTIVITY NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0000	0.0000	0.0000
2	3.0000	0.0000	6.0000	1.0000
3	10.0000	7.0000	13.0000	1.0000
4	12.0000	6.0000	18.0000	2.0000

Fig. 18 The Network Description and Activity Parameters for the Input Data Given in Fig. 17

GERT SIMULATION PROJECT 4 BY RONALD ENLOW
DATE 8/ 5/ 1968

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB.	MEAN	STD.DEV.	MIN.	MAX.
9	0.0825	23.7219	2.1037	18.9812	27.6837
18	0.0525	38.1142	9.0602	25.6621	50.3755
20	0.8650	24.1574	10.9763	11.7053	51.1187

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	0.	5.0	0	0	0	0	1	23	9	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
18	0.	5.0	0	0	0	0	0	0	5	6	0	4	5
			1	0	0	0	0	0	0	0	0	0	0
20	0.	5.0	0	0	0	137	34	0	41	97	7	8	16
			6	0	0	0	0	0	0	0	0	0	0

Fig. 19 Final Results for 400 Simulations of the Network Given in Fig. 16

BIBLIOGRAPHY ON GERT AND RELATED TOPICS

1. Brock, P. and S. M. Drezner, "A Problem Emanating from a Study of Feedback Loops in Stochastic Networks," The RAND Corporation, P 3276, February, 1966.

2. Deutsch, D., "The Theory of Blocking," Masters Report, Arizona State University, May, 1966.

3. Drezner, S. M. and A. A. B. Pritsker, Network Analysis of Countdown, The RAND Corporation, RM-4976-NASA, March, 1966.

4. Eisner, Howard, "A Generalized Network Approach to the Planning and Scheduling of a Research Program, " Journal of Operations Research Society, Vol. 10, No. 1, 1962.

5. Elmaghraby, Salah E., "An Algebra for the Analysis of Generalized Activity Networks," Management Science, Vol. 10, No. 3, April 1964, pp. 494-514.

6. _____. "On Generalized Activity Networks," Journal of Industrial Engineering, Vol. XVII, No. 11, November, 1966, pp. 621-631.

7. Enlow, R. A. and A. A. B. Pritsker, Planning R&D Projects Using GERT, NASA/ERC, NGR 03-001-034, Arizona State University, July 1968.

8. Graham, P., "Profit Probability Analysis of Research and Development Expenditures," The Journal of Industrial Engineering, Vol. XVI, No. 3, May-June, 1965, pp. 186-191.

9. Happ, W. W., "Flowgraph Techniques for Closed Systems," IEEE Transactions, AES, Vol. 2, May, 1966, pp. 252-264.

10. Hill, T. W., "System Optimization: A Sensitivity Approach Using GERT," unpublished Masters Research Report, Arizona State University, Tempe, Arizona, 1966 (Also in Bulletin #3, Industrial Engineering Research at Arizona State University, 1966).

11. Ishmael, P. C. and A. A. B. Pritsker, User Manual for GERT EXCLUSIVE-OR Program, NASA/ERC, NGR 03-011-034, Arizona State University, July 1968.

12. _____. Definitions and Procedures Employed in the GERT EXCLUSIVE-OR Program, NASA/ERC, NGR 03-001-034, Arizona State University, July, 1968.

13. Ledbetter, L. R., "A Study of Computational Aspects of Network Models for Planning and Control," Masters Thesis, Lehigh University, 1967.

14. Naylor, T. H.,et. al., Computer Simulation Techniques, New York: John Wiley & Sons, Inc., 1966.

15. Powell, G. E., "Development, Evaluation and Selection of a Dodge Continuous Sampling Plan When the Rectifying Operation is Not Perfect," Masters Thesis, Lehigh University, 1967.

16. McDonald, W. W., "Analysis of Stochastic Networks," in GRAPH THEORY, Significant Books in Operations Research, Management Information Services, Detroit, Michigan (Also Masters Thesis, Arizona State University, 1966).

17. Pritsker, A. A. B., GERT: Graphical Evaluation and Review Techniques, The RAND Corporation, RM-4973-NASA, April, 1966.

18. _____, User's Manual for GERT Simulation Program, NASA/ERC, NGR 03-001-034, Arizona State University, July 1968.

19. _____, Definitions and Procedures for GERT Simulation Program, NASA/ERC, NGR 03-001-034, Arizona State University, July 1968.

20. _____, and W. W. Happ, "GERT: Graphical Evaluation and Review Technique, Part I Fundamentals," The Journal of Industrial Engineering, Vol. XVII, No. 5, pp. 267, May, 1966.

21. _____, and G. E. Whitehouse, "GERT: Graphical Evaluation and Review Technique, Part II Probabilistic and Industrial Engineering Applications," Journal of Industrial Engineering, Vol. XVII, No. 6, June, 1966.

22. _____, and P. J. Kiviat, GASP II, A FORTRAN Based Simulation Language, Department of Industrial Engineering, Arizona State University, September 1967.

23. _____ _____, Simulation Using GASP II, to be published by Prentice-Hall, Inc., 1968.

24. Smith, R. L., "Stochastic Analysis of Personnel Movement in Formal Organizations," Ph.D. Dissertation, Arizona State University, August, 1967.

25. Thomas, J. H., "An Investigation into the Output Distribution of GERT Networks by the Use of Simulation", Masters Thesis, Lehigh University, 1967.

26. Werberger, G., "Optimality Considerations of GERT," Masters Thesis, Lehigh University, 1967.

27. Whitehouse, Gary E., "Extensions, New Developments, and Applications of GERT," Ph.D. Dissertation, Arizona State University, August, 1965.