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Technical Report 32-1318

Transport Equations for Gases and Plasmas Obtained by the 13-Moment Method: a Summary

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Abstract

A summary of results obtained from 13-moment solutions to the Boltzmann equations for a gas mixture is presented. The simplifications that can be achieved when one of the components has a small mass are discussed. A calculation of electron–atom collision parameters for noble gas atoms with Ramsauer cross sections (A, Kr, Xe) is also given.

Transport Equations for Gases and Plasmas Obtained by the 13-Moment Method: a Summary

I. 13-Moment Solutions

The equations of transport derived by the 13-moment procedure for a tenuous monatomic gas mixture are given in this section. The notation is mostly consistent with that of Chapman and Cowling (Ref. 1); a list of symbols is given in the Nomenclature. The basic method and details of evaluation of the integral term in the Boltzmann equation are given by Grad (Ref. 2). English subscripts are used to denote specie; Greek subscripts are used as tensor indices.

The Boltzmann equation for specie i is

$$\frac{\partial f_i}{\partial t} + \mathbf{c}_i \cdot \frac{\partial f_i}{\partial \mathbf{r}} + \mathbf{F}_i \cdot \frac{\partial f_i}{\partial \mathbf{c}_i} = \sum_j \left(\frac{\partial_e}{\partial t}\right)_j f_i$$

Let ϕ be any function of the molecular velocities and physical coordinates with an average given by

$$n_i \bar{\phi}_i \equiv \int \phi f_i \, d\mathbf{c}_i$$

The rate of change of this average due to encounters with specie j is (prime denotes values after encounter)[1]

$$\begin{aligned} n_i \Delta_j \bar{\phi}_i &\equiv \int \phi \left(\frac{\partial_e}{\partial t}\right)_j f_i \, d\mathbf{c}_i \\ &= \int \phi \, (f'_i f'_j - f_i f_j) \, g_{ij} \, b \, db \, d\epsilon \, d\mathbf{c}_i \, d\mathbf{c}_j \\ &= \int (\phi' - \phi) \, f_i f_j \, g_{ij} \, b \, db \, d\epsilon \, d\mathbf{c}_i \, d\mathbf{c}_j \end{aligned}$$

The mass velocity $\mathbf{u}$ is given by

$$\rho \mathbf{u} = \sum_i \rho_i \bar{\mathbf{c}}_i$$

Let $\mathbf{C}_i \equiv \mathbf{c}_i - \mathbf{u}$; the drift velocity $\mathbf{w}_i \equiv \bar{\mathbf{C}}_i$. The body force will be taken as

$$\mathbf{F}_i = \frac{e_i}{m_i} (\mathbf{E} + \mathbf{c}_i \times \mathbf{B}) + \mathbf{g}$$

Let

$$\mathbf{F}'_i = \frac{e_i}{m_i} (\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathbf{g}$$

$$\boldsymbol{\omega}_i \equiv \frac{e_i}{m_i} \mathbf{B}$$

$$\gamma_i \equiv \frac{k T_i}{m_i}$$

[1]See Ref. 1.

The electric current density $\mathbf{j} \equiv \sum_i n_i e_i \mathbf{w}_i$. The net charge density $q_c \equiv \sum_i n_i e_i$. The stress tensor and heat flux vector are

$$\overline{\overline{P}}_{i_{\alpha\beta}} \equiv \rho_i \overline{C_{i_\alpha} C_{i_\beta}} = p_i \delta_{\alpha\beta} + \overline{\overline{\tau}}_{i_{\alpha\beta}}$$

where

$$p_i = \frac{1}{3} \rho_i \overline{C_i^2} \equiv \rho_i \gamma_i$$

$$\mathbf{q}_i \equiv \frac{1}{2} \rho_i \overline{C_i^2 \mathbf{C}_i} = \mathbf{h}_i + \frac{5}{2} p_i \mathbf{w}_i$$

The equation of transport of ϕ is obtained by multiplying each term of the Boltzmann equation by ϕ and integrating:

$$\frac{D}{Dt}(n_i \bar{\phi}_i) + n_i \bar{\phi}_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \frac{\partial}{\partial \mathbf{r}} \cdot (n_i \overline{\phi_i \mathbf{C}_i})$$
$$- n_i \left[\overline{\frac{D\phi_i}{Dt}} + \overline{\mathbf{C}_i \cdot \frac{\partial}{\partial \mathbf{r}} \phi_i} + \left(\mathbf{F}'_i - \frac{D\mathbf{u}}{Dt} \right) \cdot \overline{\frac{\partial \phi_i}{\partial \mathbf{C}_i}} \right.$$
$$\left. + \boldsymbol{\omega}_i \cdot \overline{\frac{\partial \phi_i}{\partial \mathbf{C}_i} \times \mathbf{C}_i} - \overline{\frac{\partial \phi_i}{\partial \mathbf{C}_i} \mathbf{C}_i} : \frac{\partial}{\partial \mathbf{r}} \mathbf{u} \right] = n_i \sum_j \Delta_j \bar{\phi}_i$$

In the 13-moment approximation it is assumed

$$f_i = f_i^{(0)} \left[1 + \frac{1}{\gamma_i} \mathbf{w}_i \cdot \mathbf{C}_i + \frac{1}{2\gamma_i p_i} \overline{\overline{\tau}}_i : \mathbf{C}_i \mathbf{C}_i \right.$$
$$\left. + \frac{1}{5\gamma_i p_i} \left(\frac{1}{\gamma_i} C_i^2 - 5 \right) \mathbf{h}_i \cdot \mathbf{C}_i \right]$$

where $f_i^{(0)}$ is the local Maxwellian distribution:

$$f_i^{(0)} = n_i (2\pi \gamma_i)^{-3/2} \exp\left(- \frac{1}{2\gamma_i} C_i^2 \right)$$

Cases to be considered are $\phi = m_i, m_i \mathbf{C}_i, m_i \mathbf{C}_i \mathbf{C}_i$, and ½ $m_i C_i^2 \mathbf{C}_i$. The resulting transport equations are:

$$\frac{D\rho_i}{Dt} + \rho_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_i \mathbf{w}_i) = 0 \qquad (1)$$

$$\mathbf{w}_i + \boldsymbol{\omega}_i \tau_{i(1)} \times \mathbf{w}_i + \mathcal{C}_i^{(1)} \frac{\mathbf{h}_i}{p_i} + \tau_{i(1)} \boldsymbol{\Gamma}_i =$$
$$\mathbf{R}_{i(1)} + \sum_{j \neq i} \left(\mathcal{A}_{ij}^{(1)} \mathbf{w}_j + \mathcal{B}_{ij}^{(1)} \frac{\mathbf{h}_j}{p_j} \right) \qquad (2)$$

$$\frac{3}{2} \frac{Dp_i}{Dt} + \frac{5}{2} p_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{q}_i - \rho_i \mathbf{w}_i \cdot \left(\mathbf{F}'_i - \frac{D\mathbf{u}}{Dt} \right)$$
$$+ \overline{\overline{\tau}}_i : \frac{\partial}{\partial \mathbf{r}} \mathbf{u} = - \rho_i \sum_{j \neq i} a_{ij}^{(1)} \frac{3k(T_i - T_j)}{(m_i + m_j)} \qquad (3)$$

$$\overline{\overline{\tau}}_i + 2 \{ \boldsymbol{\omega}_i \tau_{i(2)} \times \overline{\overline{\tau}}_i \} + \eta_i \overline{\overline{E}}_i = \overline{\overline{R}}_{i(2)} + \sum_{j \neq i} \frac{\rho_i}{\rho_j} \mathcal{A}_{ij}^{(2)} \overline{\overline{\tau}}_j \qquad (4)$$

$$\mathbf{h}_i + \boldsymbol{\omega}_i \tau_{i(3)} \times \mathbf{h}_i + \frac{5}{2} p_i \mathcal{C}_i^{(3)} \mathbf{w}_i + \lambda_i \mathbf{D}_i =$$
$$\mathbf{R}_{i(3)} + \sum_{j \neq i} \left(\frac{\rho_i}{\rho_j} \mathcal{A}_{ij}^{(3)} \mathbf{h}_j + \frac{5}{2} p_i \mathcal{B}_{ij}^{(3)} \mathbf{w}_j \right) \qquad (5)$$

Expressions in the above equations are given as follows:

$$\boldsymbol{\Gamma}_i \equiv \frac{1}{\rho_i} \left(\frac{\partial p_i}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{r}} \cdot \overline{\overline{\tau}}_i \right) + \frac{D\mathbf{u}}{Dt} - \mathbf{F}'_i$$

$$\overline{\overline{E}}_i = 2\overline{\overline{\epsilon}} + \frac{4}{5} \frac{1}{p_i} \left\{ \frac{\partial \mathbf{q}_i}{\partial \mathbf{r}} \right\} - \frac{2}{p_i} \left\{ \left(\mathbf{F}'_i - \frac{D\mathbf{u}}{Dt} \right) \rho_i \mathbf{w}_i \right\}$$

where

$$\overline{\overline{\epsilon}} \equiv \left\{ \frac{\partial \mathbf{u}}{\partial \mathbf{r}} \right\}$$

$$\mathbf{D}_i \equiv \frac{\partial T_i}{\partial \mathbf{r}} - \frac{2m_i}{5kp_i} \left(\mathbf{F}'_i - \frac{D\mathbf{u}}{Dt} \right) \cdot \overline{\overline{\tau}}_i + \frac{2}{5} \frac{T_i}{p_i} \frac{\partial}{\partial \mathbf{r}} \cdot \overline{\overline{\tau}}_i$$
$$+ \frac{7}{5} \frac{1}{p_i} \overline{\overline{\tau}}_i \cdot \frac{\partial T_i}{\partial \mathbf{r}}$$

$$\mathbf{R}_{i(1)} \equiv - \tau_{i(1)} \left[\frac{1}{\rho_i} \frac{D}{Dt} (\rho_i \mathbf{w}_i) + \mathbf{w}_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \mathbf{w}_i \cdot \frac{\partial}{\partial \mathbf{r}} \mathbf{u} \right]$$

$$\overline{\overline{R}}_{i(2)} \equiv - \tau_{i(2)} \left[\frac{D}{Dt} \overline{\overline{\tau}}_i + \overline{\overline{\tau}}_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + 2 \left\{ \overline{\overline{\tau}}_i \cdot \frac{\partial}{\partial \mathbf{r}} \mathbf{u} \right\} \right]$$

$$\mathbf{R}_{i(3)} \equiv - \tau_{i(3)} \left[\frac{D}{Dt} \mathbf{q}_i + \mathbf{q}_i \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \mathbf{q}_i \cdot \frac{\partial}{\partial \mathbf{r}} \mathbf{u} \right.$$
$$\left. + \mathbf{q}_i \cdot \left(\frac{4}{5} \overline{\overline{\epsilon}} + \frac{2}{3} \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} \overline{\overline{U}} \right) \right] - \frac{5}{2} p_i \left(\frac{\tau_{i(3)}}{\tau_{i(1)}} \right) \mathbf{R}_{i(1)}$$

where $\overline{\overline{U}}$ is the unit tensor.

For any two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, the brace expression is defined as

$$\{ \mathbf{V}_1 \mathbf{V}_2 \}_{\alpha\beta} \equiv \frac{1}{2} (V_{1_\alpha} V_{2_\beta} + V_{1_\beta} V_{2_\alpha}) - \frac{1}{3} \delta_{\alpha\beta} V_{1_\sigma} V_{2_\sigma}$$

Also $(\mathbf{V} \times \overline{\overline{\tau}}_i)_{\alpha\beta} = \epsilon_{\alpha\sigma\rho} V_\sigma \overline{\overline{\tau}}_{i\rho\beta}$ where $\epsilon_{\alpha\sigma\rho}$ is the permutation tensor. Other parameters are:

$$\mathcal{A}_{ij}^{(n)} \equiv a_{ij}^{(n)} \tau_{i(n)}$$
$$\mathcal{B}_{ij}^{(n)} \equiv b_{ij}^{(n)} \tau_{i(n)}$$
$$C_i^{(n)} \equiv \nu_{i(n)} \tau_{i(n)}$$

(The a's, b's, and ν's are characteristic collision frequencies; τ's are collision times.)

The viscosity coefficient, $\eta_i \equiv p_i \tau_{i(2)}$. The heat conduction coefficient is

$$\lambda_i \equiv \frac{5k}{2m_i} p_i \tau_{i(3)}$$

$$\frac{1}{\tau_{i(1)}} \equiv \frac{1}{m_i} \sum_{j \neq i} \frac{\mu_{ij}}{\tau_{ij}}$$

$$a_{ij}^{(1)} \equiv \frac{\mu_{ij}}{m_i \tau_{ij}}$$

$$b_{ij}^{(1)} \equiv \frac{\mu_{ij} \gamma_j \zeta_{ij}}{\gamma_{ij} m_i \tau_{ij}}$$

$$\nu_{i(1)} \equiv \frac{\gamma_i}{m_i} \sum_{j \neq i} \frac{\mu_{ij} \zeta_{ij}}{\gamma_{ij} \tau_{ij}}$$

$$\frac{1}{\tau_{i(2)}} \equiv \frac{3}{5} \frac{A_{ii}^*}{\tau_{ii}} + \frac{2}{m_i} \sum_{j \neq i} \left[1 - \frac{\mu_{ij}}{m_i}\left(1 - \frac{3}{5} A_{ij}^*\right) + \zeta_{ij}\xi_{ij} \right] \frac{\mu_{ij}}{\tau_{ij}}$$

$$a_{ij}^{(2)} \equiv \frac{2}{m_i} \left[\frac{\mu_{ij}}{m_i}\left(1 - \frac{3}{5} A_{ij}^*\right) - \zeta_{ij}\xi_{ij} \right] \frac{\mu_{ij}}{\tau_{ij}}$$

$$\frac{1}{\tau_{i(3)}} \equiv \frac{2}{5} \frac{A_{ii}^*}{\tau_{ii}} + \frac{1}{m_i} \sum_{j \neq i} \left[3\left(\frac{\mu_{ij}}{m_j}\right)^2 + \frac{8}{5} \frac{\mu_{ij}}{m_i} \frac{\mu_{ij}}{m_j} A_{ij}^* + \frac{5}{2}\left(\frac{\mu_{ij}}{m_i}\right)^2 \beta_{ij} + \left(\frac{4}{5} \frac{\mu_{ij}}{m_i} (8E_{ij}^* - 7A_{ij}^*) + \left(11 - 16\frac{\mu_{ij}}{m_i}\right)\zeta_{ij}\right)\xi_{ij} + 3\left(\frac{5}{2}\beta_{ij} - 1 - 2\zeta_{ij}\right)\xi_{ij}^2 \right] \frac{\mu_{ij}}{\tau_{ij}}$$

$$a_{ij}^{(3)} \equiv \frac{1}{m_i} \left[\left(\frac{\mu_{ij}}{m_i}\right)^2 \left(3 - \frac{8}{5} A_{ij}^* + \frac{5}{2}\beta_{ij}\right) + \left(\frac{4}{5}\frac{\mu_{ij}}{m_i}(8E_{ij}^* - 7A_{ij}^*) - 16\frac{\mu_{ij}}{m_i}\zeta_{ij}\right)\xi_{ij} + 3\left(\frac{5}{2}\beta_{ij} - 1 - 2\zeta_{ij}\right)\xi_{ij}^2 \right] \frac{\mu_{ij}}{\tau_{ij}}$$

$$b_{ij}^{(3)} \equiv \frac{2}{m_i} \left[\frac{1}{2}\left(\frac{\mu_{ij}}{m_i}\right)^2 \zeta_{ij} - 2\frac{\mu_{ij}}{m_i}\left(1 - \frac{2}{5}A_{ij}^*\right)\xi_{ij} + \frac{3}{2}\zeta_{ij}\xi_{ij}^2 \right] \frac{\gamma_{ij}\mu_{ij}}{\gamma_i \tau_{ij}}$$

$$\nu_{i(3)} \equiv \frac{2}{m_i} \sum_{j \neq i} \left[\frac{1}{2}\left(\frac{\mu_{ij}}{m_i}\right)^2 \zeta_{ij} + \left(1 - 2\frac{\mu_{ij}}{m_i}\left(1 - \frac{2}{5}A_{ij}^*\right)\right)\xi_{ij} + \frac{3}{2}\zeta_{ij}\xi_{ij}^2 \right] \frac{\gamma_{ij}\mu_{ij}}{\gamma_i \tau_{ij}}$$

where

$$\gamma_{ij} \equiv \gamma_i + \gamma_j$$

$$\xi_{ij} \equiv \frac{\gamma_i}{\gamma_{ij}} - \frac{\mu_{ij}}{m_i} = \frac{k(T_i - T_j)}{\gamma_{ij}(m_i + m_j)}$$

$$\zeta_{ij} \equiv \frac{6}{5} C_{ij}^* - 1$$

$$\beta_{ij} \equiv 1 - \frac{12}{25} B_{ij}^*$$

and μ_{ij} is the reduced mass.

The time between collisions of a particle of specie i with those of specie j, τ_{ij}, is given by:

$$\frac{1}{\tau_{ij}} \equiv \frac{16}{3}\left(\frac{\gamma_{ij}}{2\pi}\right)^{1/2} n_j q_{ij}$$

The diffusion cross section q_{ij} and cross section ratios are defined using the $\Omega_{ij}^{(l)}(r)$ integrals of Chapman and Cowling (Refs. 1 and 3).

$$q_{ij} \equiv \left(\frac{2\pi}{\gamma_{ij}}\right)^{1/2} \Omega_{ij}^{(1)}(1)$$

$$A_{ij}^* \equiv \frac{\Omega_{ij}^{(2)}(2)}{2\Omega_{ij}^{(1)}(1)}$$

$$B^*_{ij} \equiv \frac{5\Omega^{(1)}_{ij}(2) - \Omega^{(1)}_{ij}(3)}{3\Omega^{(1)}_{ij}(1)}$$

$$C^*_{ij} \equiv \frac{\Omega^{(1)}_{ij}(2)}{3\Omega^{(1)}_{ij}(1)}$$

$$E^*_{ij} \equiv \frac{\Omega^{(2)}_{ij}(3)}{8\Omega^{(1)}_{ij}(1)}$$

$$\Omega^{(l)}_{ij}(r) \equiv \left(\frac{\gamma_{ij}}{2\pi}\right)^{1/2} \int_0^\infty Q^{(l)}_{ij} \mathcal{g}^{2r+3} \exp(-\mathcal{g}^2)\, d\mathcal{g}$$

where

$$\mathcal{g} = \frac{g_{ij}}{(2\gamma_{ij})^{1/2}}$$

(g_{ij} is the relative velocity before encounter)

$$Q^{(l)}_{ij} = \int (1 - \cos^l \chi)\, b\, db\, d\epsilon$$

$$= 2\pi \int_0^\pi I_{ij}(\chi)(1 - \cos^l \chi) \sin\chi\, d\chi$$

where I_{ij} is the differential scattering cross section ($= a_{ij}/g_{ij}$, Refs. 1 and 3).

The definitions of A^*_{ij}, B^*_{ij}, and C^*_{ij} are congruent to those of Ref. 3.

Summing Eqs. (1), $\rho_i/\tau_{i(1)}$ times (2), and (3) over all species gives the overall conservation equations (note that $\sum_i \rho_i \mathbf{w}_i = 0$):

$$\frac{D\rho}{Dt} + \rho \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} = 0 \qquad (1')$$

$$\rho \frac{D\mathbf{u}}{Dt} + \frac{\partial p}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{r}} \cdot \overline{\overline{\tau}} = \rho \mathbf{g} + q_c(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathbf{j} \times \mathbf{B} \qquad (2')$$

$$\frac{3}{2}\frac{Dp}{Dt} + \frac{5}{2} p \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u} + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{q} + \overline{\overline{\tau}} : \frac{\partial}{\partial \mathbf{r}} \mathbf{u} = \mathbf{j} \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \qquad (3')$$

where

$$\rho \equiv \sum_i \rho_i$$

$$p \equiv \sum_i p_i$$

$$\overline{\overline{\tau}} \equiv \sum_i \overline{\overline{\tau}}_i$$

$$\mathbf{q} \equiv \sum_i \mathbf{q}_i$$

The continuum limit is obtained by neglecting $\mathbf{R}_{i(1)}$, $\overline{\overline{R}}_{i(2)}$, and $\mathbf{R}_{i(3)}$. Since $\tau_{i(n)} \lesssim \tau_{i(1)}$, this means $\tau_{i(1)} \frac{D}{Dt}$ must be small.

Extension of the transport theory to polyatomic gases with internal degrees of freedom is discussed in Ref. 3. A formal discussion of the theory for polyatomic gases with solution by the Chapman–Enskog expansion procedure has been given in a recent paper (Ref. 4). A quantum treatment of transport theory has also been discussed recently (Ref. 5).

II. Small Mass Approximation

A component of small particle mass, specifically an electron, is now considered. In this section the subscript e will denote electrons, i ions only, n neutral particles only, and j, k any particle except an electron. The equations of transport will be taken in the continuum limit.

First, Eq. (4) for the electron viscous stress tensor is examined. The relative order of magnitude of terms is estimated. The parameter $a^{(2)}_{ek} \approx (\tau_{ek})^{-1}$.

$$|\overline{\overline{\tau}}_e| \gg \frac{\rho_e}{\rho_k} \mathcal{A}^{(2)}_{ek} |\overline{\overline{\tau}}_k| \approx \frac{\rho_e}{\rho_k} \frac{\tau_{e(2)}}{\tau_{ek}} |\overline{\overline{\tau}}_k|$$

if

$$|\overline{\overline{E}}_e| \gg \left(\frac{m_e}{m_k}\right)^{1/2} |\overline{\overline{E}}_k|$$

This requires

$$1 \gg \left(\frac{m_e}{m_k}\right)^{1/2}$$

and

$$|n_e e_e \mathbf{w}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})| \gg \left(\frac{m_e}{m_i}\right)^{1/2} |n_i e_i \mathbf{w}_i \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})|$$

Thus within an error of $O\ (\sqrt{m_e/m_k})$

$$\overline{\overline{\tau}}_e + 2\{\mathbf{H}_{e(2)} \times \overline{\overline{\tau}}_e\} = -\eta_e \overline{\overline{E}}_e$$

where $\mathbf{H}_{r(s)} \equiv \boldsymbol{\omega}_r \tau_{r(s)}$. Note that $\mathbf{H}_{e(s)}$ is antiparallel to $\mathbf{B}$ and that ξ_{ke} and ξ_{ek} are of order m_e/m_k.

Dropping subscripts, the solution to this equation is

$$-\frac{1+4H^2}{\eta}\overline{\overline{\tau}}_{\alpha\beta} = (1+2H^2)\overline{\overline{E}}_{\alpha\beta} - (\epsilon_{\alpha\gamma\sigma}H_\gamma \overline{\overline{E}}_{\sigma\beta} + \epsilon_{\beta\gamma\sigma} H_\gamma \overline{\overline{E}}_{\sigma\alpha}) + \frac{1-2H^2}{1+H^2}(H_\alpha H_\gamma \overline{\overline{E}}_{\gamma\beta} + H_\beta H_\gamma \overline{\overline{E}}_{\gamma\alpha}) + 2\epsilon_{\alpha\gamma\sigma}\epsilon_{\beta\rho\nu}H_\gamma H_\rho \overline{\overline{E}}_{\sigma\nu} - \frac{3}{1+H^2} \cdot (\epsilon_{\alpha\gamma\sigma}H_\gamma H_\rho H_\beta \overline{\overline{E}}_{\rho\sigma} + \epsilon_{\beta\gamma\sigma}H_\gamma H_\rho H_\alpha \overline{\overline{E}}_{\rho\sigma}) + \frac{6}{1+H^2}(H_\alpha H_\beta H_\gamma H_\sigma \overline{\overline{E}}_{\gamma\sigma})$$

where

$$H^2 \equiv H_\sigma H_\sigma$$

Next, Eq. (5) is considered. The parameters $a_{ek}^{(3)} \approx (\tau_{ek})^{-1}$,

$$\mathcal{B}_{ke}^{(3)} \approx (m_e/m_k)^{3/2},\ \mathcal{B}_{kj}^{(3)} \lesssim (|\zeta_{kj}| + |\xi_{kj}|).$$

Therefore,

$$|\mathbf{h}_e| \gg \frac{\rho_e}{\rho_k} \mathcal{A}_{ek}^{(3)} |\mathbf{h}_k| \approx \frac{\rho_e}{\rho_k}\frac{\tau_{e(3)}}{\tau_{ek}} |\mathbf{h}_k|$$

if

$$|\mathbf{D}_e| \gg \left(\frac{m_e}{m_k}\right)^{3/2} |\mathbf{D}_k|$$

$$|\mathcal{C}_e^{(3)}| \gg \left(\frac{m_e}{m_k}\right)^{5/2}$$

or

$$k\,|\mathbf{D}_e| \gg \left(\frac{m_e}{m_k}\right)^{5/2} \frac{m_e |\mathbf{w}_e|}{\tau_{ek}}$$

and

$$|\mathcal{C}_e^{(3)}\ \mathbf{w}_e| \gg \frac{m_e}{m_k}\sum_j (|\zeta_{kj}| + |\xi_{kj}|)\,|\mathbf{w}_j|$$

or

$$k\,|\mathbf{D}_e| \gg \left(\frac{m_e}{m_k}\right)^2 \frac{m_k \sum_j (|\zeta_{kj}| + |\xi_{kj}|)\,|\mathbf{w}_j|}{\tau_{ek}}$$

It is conceivable under certain plasma conditions that both $|\mathcal{C}_e^{(3)}|$ and $|\mathbf{D}_e|$ may be zero, but this is an extremely unlikely situation.

Equation (5) for electrons can be closely approximated as

$$\mathbf{h}_e + \mathbf{H}_{e(3)} \times \mathbf{h}_e = -\lambda_e \mathbf{D}_e - \frac{5}{2}p_e\left[\mathcal{C}_e^{(3)}\ \mathbf{w}_e - \sum_k \mathcal{B}_{ek}^{(3)}\ \mathbf{w}_k\right]$$

The equation $\mathbf{V} + \mathbf{H}_{r(s)} \times \mathbf{V} = \mathbf{W}$ inverts into

$$\mathbf{V} = \frac{1}{1+(H_{r(s)})^2}[\mathbf{W} + \mathbf{H}_{r(s)}(\mathbf{H}_{r(s)} \cdot \mathbf{W}) - \mathbf{H}_{r(s)} \times \mathbf{W}] \equiv \overline{\overline{A}}_{r(s)} \cdot \mathbf{W}$$

In many plasmas $|\mathbf{w}_e| \gg |\mathbf{w}_k|$ because of the high electron mobility, and the $\mathbf{w}_k$ terms can be dropped. This requires

$$|\mathcal{C}_e^{(3)}| \gg \left(\frac{m_e}{m_k}\right)^{1/2} |H_{e(s)}\ \mathcal{B}_{ek}^{(3)}|$$

Since $\mathcal{C}_e^{(1)} \approx \mathcal{C}_e^{(3)}$, if $|\mathcal{C}_e^{(3)}| \gtrsim |\zeta_{ek}|$, then

$$\left|\mathcal{C}_e^{(1)}\frac{\mathbf{h}_e}{p_e}\right| \gg \left|\mathcal{B}_{ek}^{(1)}\frac{\mathbf{h}_k}{p_k}\right|$$

to the same approximations as before. In any event, the condition

$$|\mathbf{w}_e| \text{ or } |\tau_{e(1)}\boldsymbol{\Gamma}_e| \gg \left|\mathcal{B}_{ek}^{(1)}\frac{\mathbf{h}_k}{p_k}\right|$$

is easily met if, say,

$$|\mathbf{w}_e| \gg \frac{m_e}{m_k}|\zeta_{ek}|\frac{\lambda_k}{p_k}|\mathbf{D}_k|$$

or

$$|\nabla T_e| \gg \frac{5}{2}|\zeta_{ek}|\left(\frac{m_e}{m_k}\right)^2\frac{\tau_{k(3)}}{\tau_{ek}}|\nabla T_k|$$

where

$$\frac{\tau_{k(3)}}{\tau_{ek}} \lesssim \left(\frac{m_e}{m_k}\right)^{-1/2}$$

or

$$|\mathbf{E}+\mathbf{u}\times\mathbf{B}| \gg \frac{5}{2}|\zeta_{ek}|\left(\frac{m_e}{m_k}\right)^2\frac{\tau_{k(3)}}{\tau_{ek}}\frac{k}{|e_e|}|\nabla T_k|$$

In Eq. (2) for electrons, the $\mathbf{h}_k$ terms are small. If $1 \gg (m_e/m_k)^{1/2} H_{e(s)}$, the $\mathbf{w}_k$ terms can also be dropped since $\mathcal{A}^{(1)}_{ek} \lesssim 1$.

If the Hall parameter $H_{e(s)}$ is not too large, the electron transport equations uncouple from the equations for the rest of the species. If the Hall parameter is large, the $\mathbf{w}_i$ terms must be included. The equations of the other species are now examined to see to what extent they are independent of the electron fluxes.

The parameters $a^{(2)}_{ke} \approx (m_e/m_k)^2\,(1/\tau_{ke})$. Thus

$$|\overline{\overline{\tau}}_k| \gg \frac{\rho_k}{\rho_e}\mathcal{A}^{(2)}_{ke}|\overline{\overline{\tau}}_e| \approx \left(\frac{m_e}{m_k}\right)^2\frac{\rho_k}{\rho_e}\frac{\tau_{k(2)}}{\tau_{ke}}|\overline{\overline{\tau}}_e|$$

if $|\overline{\overline{E}}_k| \gg \dfrac{\rho_e}{\rho_k}|\overline{\overline{E}}_e|$. This requires $1 \gg \rho_e/\rho_k$ and

$$kT_e|\overline{\overline{\epsilon}}| \gg \frac{\rho_e}{\rho_k}|e_e\mathbf{w}_e(\mathbf{E}+\mathbf{u}\times\boldsymbol{B})|$$

Similarly,

$$a^{(3)}_{ke} \approx \left(\frac{m_e}{m_k}\right)^3\frac{1}{\tau_{ke}}$$

and

$$|\mathbf{h}_k| \gg \frac{\rho_k}{\rho_e}\mathcal{A}^{(3)}_{ke}|\mathbf{h}_e| \approx \left(\frac{m_e}{m_k}\right)^3\frac{\rho_k}{\rho_e}\frac{\tau_{k(3)}}{\tau_{ke}}|\mathbf{h}_e|$$

if

$$|\mathbf{D}_k| \gg \frac{\rho_e}{\rho_k}|\mathbf{D}_e|$$

and

$$1 \gg \left(\frac{m_e}{m_k}\right)^{1/2}\mathcal{C}^{(3)}_e$$

The first requires

$$|\nabla T_k| \gg \frac{\rho_e}{\rho_k}|\nabla T_e|$$

and

$$k|\nabla T_k| \gg \frac{\rho_e}{\rho_k}\left|e_e\frac{\overline{\overline{\tau}}_e}{p_e}\cdot(\mathbf{E}+\mathbf{u}\times\mathbf{B})\right|$$

Note that

$$\mathcal{B}^{(3)}_{ke} \approx \left(\frac{m_e}{m_k}\right)^2$$

It is seen that the equations for $\overline{\overline{\tau}}_k$ and $\mathbf{h}_k$ for heavy species of non-negligible concentration, $1 \gg \rho_e/\rho_k$, may be independent of electron fluxes $\mathbf{w}_e$, $\overline{\overline{\tau}}_e$, and $\mathbf{h}_e$. If the gradients of the mass velocity and temperatures T_k are small, separation does not occur, but then the fluxes $\overline{\overline{\tau}}_k$ and $\mathbf{h}_k$ are correspondingly small. Coupling of drift velocities remains.

As an example, calculation of the drift velocities and heat flux vectors for a three-species plasma (e, i, n) will be given. Let $e \equiv |e_e|$. The usual quasineutrality approximation is made: $en_e = e_i n_i$. The "ionization ratio" is defined as $\alpha \equiv (\rho_e + \rho_i)/\rho$ and the electrical conductivity as $\sigma \equiv (e^2 n_e \tau_{e(1)})/m_e$. Let $\mathbf{V}_e \equiv (\mathbf{w}_e - \mathbf{w}_i)$ and $\mathbf{V}_i \equiv (\mathbf{w}_i - \mathbf{w}_n)$. Then

$$\mathbf{j} = -en_e\mathbf{V}_e$$

$$\mathbf{w}_e = \left(1-\frac{\rho_e}{\rho}\right)\mathbf{V}_e + (1-\alpha)\mathbf{V}_i \doteq \mathbf{V}_e + (1-\alpha)\mathbf{V}_i$$

$$\mathbf{w}_i = (1-\alpha)\mathbf{V}_i - \frac{\rho_e}{\rho}\mathbf{V}_e \doteq (1-\alpha)\mathbf{V}_i$$

$$\mathbf{w}_n = -\alpha\mathbf{V}_i - \frac{\rho_e}{\rho}\mathbf{V}_e \doteq -\alpha\mathbf{V}_i$$

Momentum equations (2) and (2′), using the small mass approximation, give

$$\boldsymbol{\Gamma}_e + \boldsymbol{\omega}_e \times \mathbf{w}_e + \frac{\mathbf{V}_e}{\tau_{ei}} + \frac{\mathbf{V}_e + \mathbf{V}_i}{\tau_{en}} + \nu_{e(1)} \frac{\mathbf{h}_e}{p_e} = 0$$

and

$$\mathbf{V}_i = \frac{\zeta_{in}}{\gamma_{in}} \left(\gamma_n \frac{\mathbf{h}_n}{p_n} - \gamma_i \frac{\mathbf{h}_i}{p_i} \right)$$
$$- \epsilon \left[\mathbf{V}_e + \zeta_{en} \frac{\mathbf{h}_e}{p_e} + (1 - \alpha)\boldsymbol{\omega}_e \tau_{en} \times \mathbf{V}_e \right]$$
$$+ \epsilon \frac{\tau_{en}}{\rho_e} [\alpha \nabla p - \nabla(p_e + p_i) + \alpha \nabla \cdot \overline{\overline{\tau}} - \nabla \cdot (\overline{\overline{\tau}}_e + \overline{\overline{\tau}}_i)]$$

where

$$\epsilon \equiv \frac{n_e}{n_i} \frac{m_e}{\mu_{in}} \frac{\tau_{in}}{\tau_{en}} \leq O\left(\sqrt{\frac{m_e}{\mu_{in}}}\right)$$

Terms of $O(\epsilon)$ are neglected; terms of $O(\epsilon\, H_{e(s)})$ are retained.

Heat fluxes are given by the following equations:

$$\mathbf{h}_e + \mathbf{H}_{e(3)} \times \mathbf{h}_e = -\lambda_e \mathbf{D}_e + \alpha_T \frac{kT_e}{e} \mathbf{j}$$
$$- \frac{5}{2} \frac{kT_e}{e} \epsilon\, \zeta_{en} (1 - \alpha) \mathbf{H}_{e(3)} \times \mathbf{j}$$

where

$$\alpha_T \equiv \frac{5}{2} \mathcal{C}_e^{(3)}$$

is the electron thermal diffusion ratio.

$$\mathbf{h}_i (1 - \mathcal{A}_{in}^{(3)} \mathcal{A}_{ni}^{(3)}) + \mathbf{H}_{i(3)} \times \mathbf{h}_i \simeq -\left(\lambda_i \mathbf{D}_i + \frac{\rho_i}{\rho_n} \mathcal{A}_{in}^{(3)} \lambda_n \mathbf{D}_n \right)$$
$$+ \frac{5}{2} p_i \left\{ \left(\mathcal{B}_{ie}^{(3)} + \frac{\gamma_n}{\gamma_i} \mathcal{A}_{in}^{(3)} \mathcal{B}_{ne}^{(3)} \right) \mathbf{V}_e \right.$$
$$\left. + \left[\frac{\gamma_n}{\gamma_i} \mathcal{A}_{in}^{(3)} (\mathcal{B}_{ne}^{(3)} + \mathcal{B}_{ni}^{(3)}) - \mathcal{B}_{in}^{(3)} \right] \mathbf{V}_i \right\}$$

$$\mathbf{h}_n \simeq -\lambda_n \mathbf{D}_n + \frac{\rho_n}{\rho_i} \mathcal{A}_{ni}^{(3)} \mathbf{h}_i$$
$$+ \frac{5}{2} p_n \left[\mathcal{B}_{ne}^{(3)} \mathbf{V}_e + (\mathcal{B}_{ne}^{(3)} + \mathcal{B}_{ni}^{(3)}) \mathbf{V}_i \right]$$

(In the preceding two expressions, $\nu_{k(3)}$ was replaced by $\sum_{r \neq k} b_{kr}^{(3)}$.) It is easily seen that

$$|\mathbf{h}_k| \leq |\lambda_k \mathbf{D}_k| + p_k \left[\left(\frac{m_e}{m_k} \right)^{3/2} |\mathbf{V}_e| + (|\zeta_{in}| + |\xi_{in}|) |\mathbf{V}_i| \right]$$

Since $\zeta_{in}^2 \ll 1$ and $|\zeta_{in} \xi_{in}| \ll 1$, the dependence of $\mathbf{h}_k$ and $\mathbf{h}_e$ on the drift velocities can be ignored in estimating $\mathbf{V}_i$ for Ohm's law. Let $\Delta_0 = 5/2\, \mathcal{C}_e^{(1)} \mathcal{C}_e^{(3)}$ and $\delta_0 = (1 - \alpha)^2 \epsilon\, \tau_{en}/\tau_{e(1)}$. Ohm's law is

$$\mathbf{j} - \mathbf{j} \times \mathbf{H}_{e(1)} - \delta_0 (\mathbf{j} \times \mathbf{H}_{e(1)}) \times \mathbf{H}_{e(1)} - \Delta_0 \overline{\overline{A}}_{e(3)} \cdot \mathbf{j} =$$
$$-en_e (1 - \alpha) \left[\frac{\zeta_{in}}{\gamma_{in}} \left(\gamma_n \frac{\mathbf{h}_n}{p_n} - \gamma_i \frac{\mathbf{h}_i}{p_i} \right) \right.$$
$$\left. + \epsilon\, \zeta_{en} \frac{\lambda_e}{p_e} \overline{\overline{A}}_{e(3)} \cdot \mathbf{D}_e \right] \times \mathbf{H}_{e(1)}$$
$$+ \sigma \left\{ \mathbf{E} + \mathbf{u} \times \mathbf{B} - \alpha_T \frac{k}{e} \overline{\overline{A}}_{e(3)} \cdot \mathbf{D}_e \right.$$
$$+ \frac{1}{en_e} (\nabla p_e + \nabla \cdot \overline{\overline{\tau}}_e) - \frac{\delta_0}{(1 - \alpha) en_e}$$
$$\cdot [\alpha \nabla p - \nabla (p_e + p_i) + \alpha \nabla \cdot \overline{\overline{\tau}} - \nabla$$
$$\left. \cdot (\overline{\overline{\tau}}_e + \overline{\overline{\tau}}_i)] \times \mathbf{H}_{e(1)} \right\}$$

The terms involving δ_0, ϵ, and ζ_{in} are small if $1 \gg (m_e/m_k)^{1/2} H_{e(s)}$.

The results given above agree with those of the equal temperature calculation of Zhdanov (Ref. 6) and the calculation for $\alpha = 1$ of Herdan and Liley (Ref. 7) if these authors' assumptions are considered. Ohm's law for the simplest plasma is quite complex; for a many-component plasma it would be more so. It is doubtful that an explicit expression for the electric current density can be found for many plasmas. Note that in the limit of zero Hall parameter, the effective conductivity is $\sigma/(1 - \Delta_0)$. The factor $(1 - \Delta_0)$ corresponds to the correction due to the second approximation for the diffusion coefficient in Chapman and Cowling (Ref. 1).

III. Collision Parameters

Calculation of the collision parameters dependent on the $\Omega_{ij}^{(l)}(r)$ integrals for center-of-force and Lennard–Jones potentials are discussed in Refs. 1 and 3. A few comments are now given on charged particle and charge exchange interactions. Finally, some calculations of parameters are given for noble gas atoms exhibiting the Ramsauer effect.

The effects of distant (grazing) simultaneous encounters are more important than close binary encounters when charged particles interact with the inverse square potential. However, it is well known that the solution of the Fokker–Planck equation describing such grazing encounters is the same as that given by the binary collision theory of Chapman and Cowling, provided the same cutoff distance for integration over the impact parameter, the Debye length, is used for both (Ref. 8). Thus the binary collision theory is used to calculate collision parameters for particles interacting under the shielded Debye potential. The Debye length λ_D is given by (cgs units):

$$\lambda_D^{-2} = 4\pi \sum_i \frac{n_i e_i^2}{kT_i}$$

Let

$$\Lambda_{ij} \equiv 3\lambda_D \frac{\mu_{ij}\gamma_{ij}}{|e_i e_j|}$$

The collision parameters for charged particles are approximately (within an error of a few percent)

$$q_{ij} = \frac{\pi}{2}\left(\frac{e_i e_j}{\gamma_{ij}\mu_{ij}}\right)^2 \ln \Lambda_{ij}$$

$$A^*_{ij} = 1 - \frac{1}{2 \ln \Lambda_{ij}} = 2E^*_{ij}$$

$$B^*_{ij} = 1$$

$$C^*_{ij} = \frac{1}{3}$$

When an ion collides with an atom of its parent specie with sufficiently high relative velocity, charge exchange can occur. The cross section $Q^{(1)}_{ij}$ for this process may be much larger than the polarization cross section. A discussion of this effect and also the effect of multiple interaction potentials is given by Mason et al (Ref. 9). Cross sections and/or mobilities are given in Refs. 10 and 11.

In considering collisions between electrons and the heavier noble gas atoms, the cross sections $Q^{(1)}_{ea}$ and $Q^{(2)}_{ea}$ must be obtained from experimental data. Let κ denote the wave number of the electron ($\kappa = 2\pi\, m_e g/h$). Curves of $Q^{(1)}_{ea}(\kappa)$ are given by Frost and Phelps (Ref. 12).

The ratio $R(\kappa)$ is defined as follows: $Q^{(2)}_{ea}(\kappa) = R(\kappa) \cdot Q^{(1)}_{ea}(\kappa)$. For the energy range from about 1 to 10 eV[2] this ratio was determined by numerical integration of the differential cross-section data given by Ramsauer and Kollath (Ref. 13). For low energies ($E \leq 10^{-2}$ eV) the effective-range partial-wave phase shifts given by O'Malley (Ref. 14) were used to calculate $R(\kappa)$. If $\delta_L(\kappa)$ is the phase shift of the partial wave of angular momentum quantum number L, then near the Ramsauer cross section minimum ($\sin \delta_0 = 0$):

$$R(\kappa) = \frac{(6/5) + (50/21)\xi^2}{3 + 4\xi\,|\eta| + 5\xi^2}$$

where

$$\xi \equiv \sin \delta_2 / \sin \delta_1 \leq O(1)$$

$$\eta \equiv \cos(\delta_2 - \delta_1)$$

A plot of this function is given in Fig. 1.

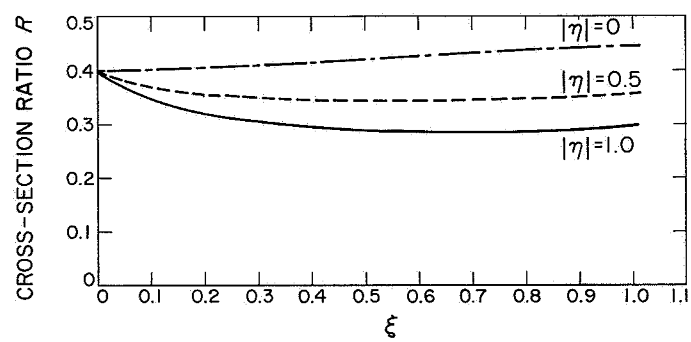

Fig. 1. Ratio of monoenergetic cross sections near Ramsauer minimum

At the minimum, it is expected that $|\eta| \simeq 1$ and ξ is small. Thus R is nearly equal to 1/3. With this value, a smooth curve was fitted to cover the energy range between the O'Malley theory and the Ramsauer–Kollath data. For energies above 10 eV, the $R(\kappa)$ curve was extended by a straight line in a manner based on values calculated using the phase shifts given by Holtsmark (Refs. 15 and 16) for argon and krypton. The functions $R(\kappa)$ used in this paper are given in Fig. 2.

The Ramsauer–Kollath data for Kr indicates the possibility of a rapid oscillation of R with energy. As pointed out in Ref. 12, however, transport coefficients are insensitive to such variations. For purposes of comparisons,

[2] $E = 13.6\,(\kappa a_0)^2$ eV where a_0 is the Bohr radius.

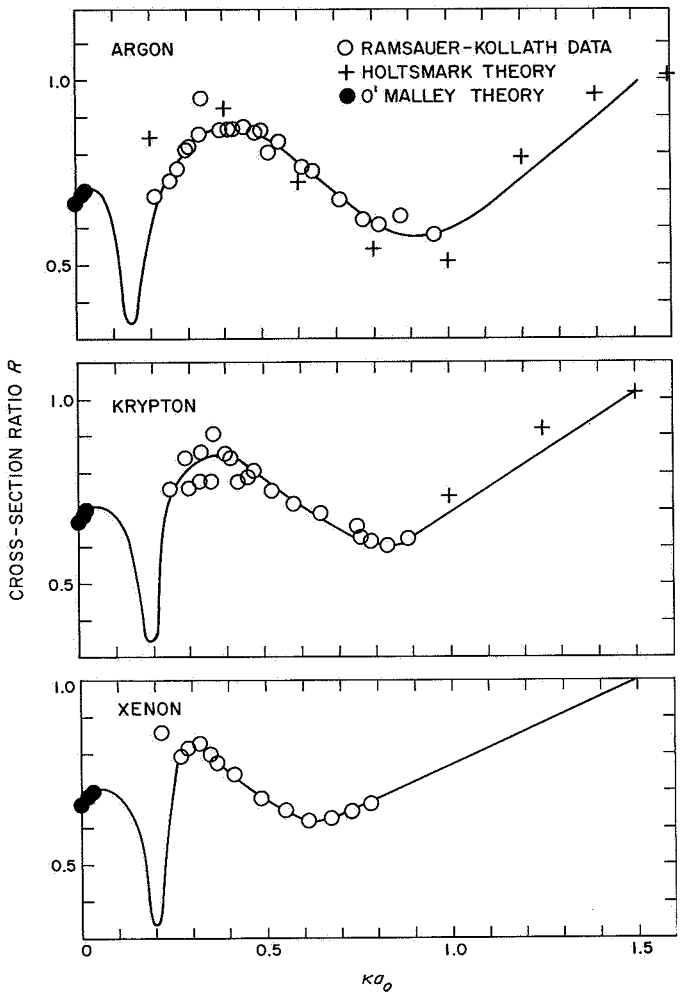

Fig. 2. Ratio of monoenergetic cross sections $R(\kappa)$

another parameter is defined:

$$D^*_{ea} \equiv \frac{3\Omega^{(2)}_{ea}(1)}{2\Omega^{(1)}_{ea}(1)}$$

This ratio and the other ratios are defined so that they approach unity as the (electron) temperature approaches zero. Note that the weighting function $\mathscr{g}^{2r+3} \exp(-\mathscr{g}^2)$ has a maximum at $\kappa a_0 \simeq 1/2(T_e/10^4\ °K)^{1/2}$ and becomes small quite rapidly above this value. Values of $Q^l(\kappa)$ for very large and very small κ are relatively unimportant. Ratios in the temperature range $5000°K \leq T_e \leq 25{,}000°K$ are principally determined from the Ramsauer–Kollath data and Frost–Phelps cross sections. The straight line approximation to $R(\kappa)$ for the higher energies should be adequate for these calculations. Values of certain ratios (A^*_{ea}, D^*_{ea}, E^*_{ea}) at lower temperatures are expected to be somewhat in error because $R(\kappa)$ is imprecisely known near the Ramsauer cross section minimum.

For center-of-force interactions,

$$D^*_{ij} = A^*_{ij}/C^*_{ij}$$

and

$$E^*_{ij} = \frac{(5C^*_{ij} - B^*_{ij})A^*_{ij}}{4C^*_{ij}}$$

The integrals were performed numerically on an IBM 7094 computer. Tables 1, 2, and 3 list the computed values of the ratios and q_{ea}. The average momentum transfer cross sections are also presented in Fig. 3. The ratios for A are plotted in Fig. 4. For purposes of comparison only, ratios based on the Holtsmark (Ref. 15) phase functions $\delta_L(L = 0, 1, 2)$ are given in Fig. 4. They are not expected to be as accurate as those of Table 1. Fifth- and sixth-order polynomials were fitted to the listed values of the phase angles.

Since the electron temperature appears in the exponent in the Maxwellian weighting function and since the monoenergetic cross sections vary rapidly with energy because of the Ramsauer effect, the ratio variations scale with the logarithm of T_e and the functions vary rapidly

Table 1. Averaged momentum transfer cross section and cross-section ratios for argon

$T_e \times 10^{-4}$, °K	$q_{ea}/4\pi a_0^2$	A^*_{ea}	B^*_{ea}	C^*_{ea}	D^*_{ea}	E^*_{ea}
0.02	0.623	0.704	1.52	0.686	1.04	0.483
0.04	0.283	0.545	1.42	0.562	1.01	0.315
0.06	0.164	0.487	1.18	0.555	0.967	0.313
0.08	0.115	0.519	0.844	0.637	0.916	0.460
0.10	0.094	0.647	0.430	0.788	0.877	0.756
0.12	0.088	0.841	0.026	0.965	0.866	1.13
0.14	0.090	1.05	−0.290	1.12	0.880	1.49
0.16	0.097	1.23	−0.498	1.25	0.911	1.79
0.18	0.107	1.38	−0.615	1.33	0.949	2.01
0.20	0.120	1.49	−0.672	1.39	0.987	2.17
0.3	0.206	1.76	−0.616	1.47	1.13	2.47
0.4	0.307	1.82	−0.479	1.46	1.20	2.46
0.5	0.415	1.82	−0.376	1.44	1.24	2.40
0.6	0.526	1.80	−0.304	1.42	1.25	2.32
0.7	0.638	1.76	−0.252	1.41	1.25	2.24
0.8	0.751	1.72	−0.212	1.40	1.25	2.17
1.0	0.978	1.64	−0.140	1.39	1.22	2.02
1.2	1.20	1.56	−0.058	1.37	1.19	1.88
1.4	1.42	1.49	0.040	1.35	1.16	1.76
1.6	1.63	1.42	0.153	1.33	1.13	1.65
1.8	1.83	1.36	0.272	1.31	1.10	1.56
2.0	2.01	1.31	0.392	1.29	1.07	1.48
2.4	2.33	1.23	0.617	1.24	1.04	1.35
2.8	2.58	1.17	0.811	1.20	1.01	1.25

at the lower temperatures. This means the collisional frequencies corresponding to the various electron–atom transport processes will also vary more rapidly at the lower temperatures than at the higher temperatures. There is no essential distinction in the behavior of the functions for the three gases.

Table 2. Averaged momentum transfer cross section and cross-section ratios for krypton

$T_e \times 10^{-4}$, °K	$q_{ea}/4\pi a_0^2$	A^*_{ea}	B^*_{ea}	C^*_{ea}	D^*_{ea}	E^*_{ea}
0.02	3.70	0.850	1.36	0.800	1.06	0.698
0.04	2.27	0.751	1.48	0.722	1.05	0.545
0.06	1.55	0.659	1.51	0.653	1.04	0.425
0.08	1.13	0.587	1.48	0.603	1.02	0.350
0.10	0.855	0.535	1.41	0.573	0.997	0.312
0.12	0.674	0.504	1.33	0.560	0.975	0.308
0.14	0.550	0.496	1.23	0.562	0.956	0.337
0.16	0.463	0.509	1.12	0.579	0.940	0.397
0.18	0.400	0.544	0.998	0.609	0.930	0.487
0.20	0.356	0.600	0.858	0.651	0.926	0.606
0.3	0.276	1.07	0.005	0.970	0.990	1.44
0.4	0.307	1.52	−0.713	1.27	1.10	2.16
0.5	0.392	1.74	−1.02	1.44	1.16	2.49
0.6	0.510	1.81	−1.04	1.51	1.19	2.54
0.7	0.649	1.81	−0.952	1.53	1.19	2.48
0.8	0.803	1.77	−0.830	1.53	1.18	2.38
1.0	1.14	1.67	−0.594	1.50	1.15	2.17
1.2	1.48	1.57	−0.393	1.47	1.11	1.99
1.4	1.83	1.49	−0.216	1.44	1.08	1.86
1.6	2.16	1.43	−0.054	1.40	1.06	1.76
1.8	2.48	1.38	0.096	1.37	1.04	1.68
2.0	2.78	1.35	0.236	1.35	1.03	1.61
2.4	3.31	1.29	0.485	1.29	1.01	1.51
2.8	3.74	1.25	0.699	1.24	1.01	1.42

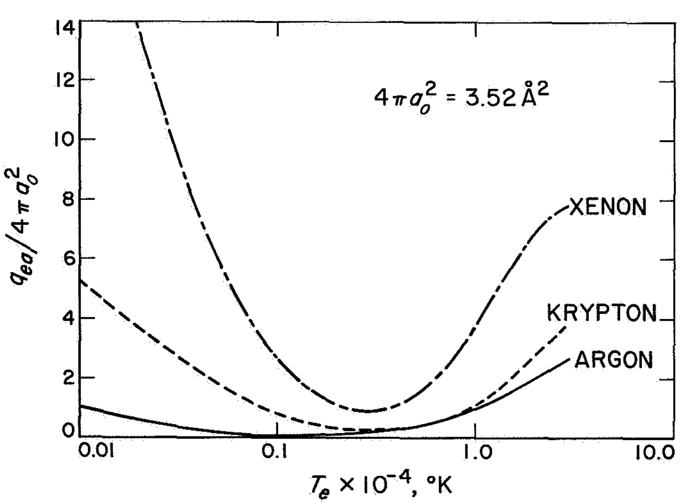

Fig. 3. Averaged momentum transfer cross sections

The relationship between the various ratios as given for center-of-force interactions is seen to hold very approximately.

Table 3. Averaged momentum transfer cross section and cross-section ratios for xenon

$T_e \times 10^{-4}$, °K	$q_{ea}/4\pi a_0^2$	A^*_{ea}	B^*_{ea}	C^*_{ea}	D^*_{ea}	E^*_{ea}
0.02	13.68	0.795	1.40	0.743	1.07	0.618
0.04	7.55	0.719	1.43	0.683	1.06	0.515
0.06	4.99	0.653	1.47	0.635	1.05	0.426
0.08	3.58	0.594	1.46	0.594	1.04	0.358
0.10	2.70	0.546	1.41	0.565	1.02	0.318
0.12	2.12	0.516	1.33	0.551	1.00	0.313
0.14	1.72	0.508	1.21	0.553	0.984	0.345
0.16	1.44	0.525	1.07	0.574	0.971	0.418
0.18	1.25	0.569	0.903	0.612	0.961	0.530
0.20	1.11	0.637	0.715	0.667	0.958	0.676
0.3	0.916	1.15	−0.244	1.05	1.01	1.56
0.4	1.09	1.49	−0.751	1.34	1.07	2.06
0.5	1.43	1.59	−0.842	1.45	1.09	2.18
0.6	1.85	1.59	−0.737	1.49	1.09	2.13
0.7	2.32	1.55	−0.560	1.48	1.07	2.04
0.8	2.80	1.51	−0.367	1.46	1.06	1.93
1.0	3.75	1.41	−0.010	1.41	1.03	1.74
1.2	4.61	1.34	0.276	1.35	1.01	1.60
1.4	5.35	1.29	0.496	1.30	1.00	1.49
1.6	5.97	1.24	0.664	1.25	1.00	1.40
1.8	6.47	1.21	0.794	1.21	1.00	1.34
2.0	6.87	1.19	0.898	1.17	1.00	1.29
2.4	7.43	1.15	1.05	1.11	1.01	1.21
2.8	7.74	1.13	1.16	1.07	1.03	1.14

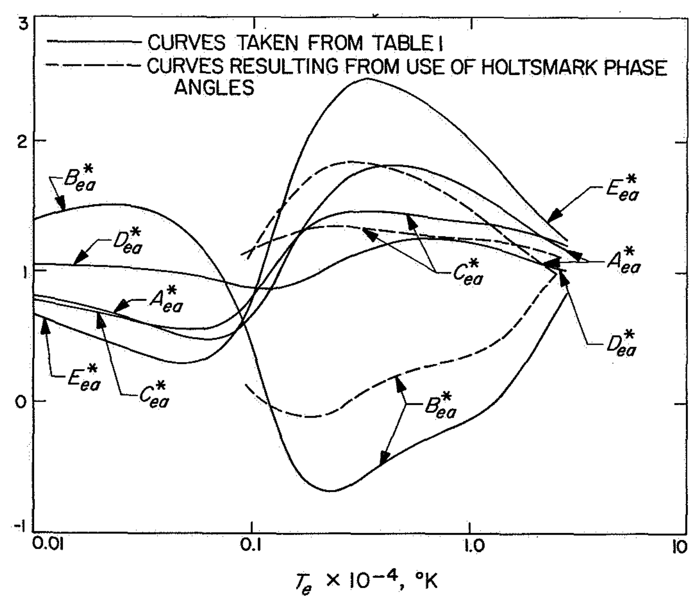

Fig. 4. Ratios of argon cross sections

Nomenclature

$\mathbf{B}$ magnetic field strength

b impact parameter

$\mathbf{C}$ peculiar molecular velocity

$\mathbf{c}$ molecular velocity

$\mathbf{E}$ electric field

e molecular charge

$\mathbf{F}$ molecular body force

f molecular distribution function

g relative impact velocity

$\mathbf{g}$ gravity acceleration vector

$\mathcal{G}$ relative impact velocity normalized by its mean

$\mathbf{H}$ Hall parameter

$\mathbf{h}$ heat flux vector relative to mean motion of a specie

$\mathbf{j}$ electric current density

k Boltzmann's constant

m molecular mass

n molecular number density

$\overline{\overline{P}}$ stress tensor

p pressure

Q monoenergetic cross section

q averaged momentum transfer cross section

$\mathbf{q}$ total heat flux vector

T temperature

$\mathbf{u}$ mass velocity

$\mathbf{w}$ drift velocity

α ionization ratio

α_T thermal diffusion ratio

$\overline{\overline{\epsilon}}$ rate of strain tensor

η viscosity coefficient

λ thermal conduction coefficient

λ_D Debye length

μ reduced mass

ξ temperature difference parameter

ρ mass density

σ electrical conductivity

$\overline{\overline{\tau}}$ viscous stress tensor

χ scattering angle

$\boldsymbol{\omega}$ cyclotron frequency

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