NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1230

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Contents

Figures

Abstract

The design of low-voltage capacitor-discharge circuits for firing hot bridgewire (HBW) squibs requires that squib *thermal* and circuit *electrical* time-constants be considered as interrelated. These functional relationships are analyzed by use of an electro-thermal analogy with one composite time-constant. The effects of secondary characteristics not considered in the simple analogy are also discussed.

Firing Squibs by Low-Voltage Capacitor Discharge for Spacecraft Application

I. Introduction

The use of a capacitor-discharge firing circuit to initiate hot-bridgewire-type (HBW) squibs reliably requires, among other things, that the electrical time-constant of the firing circuit be considered in relationship to the thermal time-constant of the squib. This report presents a review of the basic theory, defines the primary electro-thermal parameters, and presents a practical example of the design of a capacitor-discharge firing circuit. No attempt is made to compare the efficacy of using a capacitor-discharge-type firing circuit vs either a constant-voltage or constant-current power supply. It is presumed that a capacitor-discharge-type subsystem has been chosen to initiate HBW squibs. This choice could be required by low power availability, as from spacecraft solar cells, or other system restraints, such as direct-current isolation from the main battery.

A review is presented of the readily available, commonly used *constant-current* characteristics of HBW squibs, including Bruceton[1] derived *all-fire* and *no-fire* sensitivity data. The empirical relationship between the constant current into the bridgewire and the time to fire is examined, and an analytical solution is defined by means of an electro-thermal analogy. The interrelationship of current, time, energy, and power involved in the electrical sensitivity of the squib is examined in the light of the resulting equations.

Test results for a modern 1-A/1-W squib at typical conditions of applied power and reliability (R)–confidence (C) are compared with the analytical solution. A capacitor-discharge energy supply is then substituted for the constant-current power supply in the basic analogy and the subsequent equations examined. An analysis of some of the more important relationships in the capacitor-discharge configuration follows, and the results are applied in the design of a practical subsystem.

II. Typical Characteristics

The majority of present HBW squib-firing circuits use either constant-current or constant-voltage power supplies. It is, therefore, natural that most published data on squib-firing sensitivity is related to constant power.

[1]Unless otherwise specified, the data reported here were derived from Bruceton tests.

The squib engineer, when using a constant-current circuit, is concerned with two basic parameters: (1) the level of either the constant current or voltage that must be applied and (2) the time at which the squib will subsequently fire.

A common method employed to determine this reliable initiation level is the all-fire test, in which the results of some 50 or more test-firing attempts are statistically analyzed — a mean firing level is calculated and extrapolated to a specified reliability–confidence level. For example, with a typical 1-A/1-W squib, a current of 3.40 A might result as the 0.999 R/0.90 C all-fire level for all other squibs from the same lot as the test units. In the all-fire test, a current pulse (e.g., 10 to 50 ms) is generally used. After completion of the tests, the investigator can state that he is (for example) 90% confident that 999 out of 1000 of the squibs from this lot will fire within 10 ms when supplied with 3.40-A constant current. The engineer may then place an additional margin on both the current *level* and the *time* the current is applied. In our example, he may then require that the firing circuit deliver a minimum constant current of 5.0 A for a minimum of 50 ms. Thus, an adequate margin for system reliability is ensured.

It is interesting to note that the nominal firing energy (I^2Rt) delivered to a squib with a 1-Ω bridgewire is typically from 35 to 80 mJ and that the power (I^2R) delivered is of the order of 25 W. However, other tests may be run for 5-min periods and designated as no-fire tests. The resulting upper-current limit at the specified 0.999 R/0.90 C level (which will be called the sure-fire level) is generally 25% less than the 10-ms pulse all-fire level. However, in the no-fire test, the 0.999 R/0.90 C *no-fire* level is of primary concern. For a 1-A/1-W squib, this is typically in the range of 1.2 to 1.8 A.

If time-to-fire vs current is monitored during both types of tests and the results plotted on logarithmic paper, the relationship appears as in Fig. 1. The firing time appears to decrease as current increases, and there is a constant-power level below which firing never occurs. These tests provide an estimate of the constant-current all-fire and no-fire levels.

The test data provoke further investigation into the time to fire vs current relationship. Figure 2 illustrates the results of 17 firings conducted with a group of 1-A/1-W squibs between 2.0 and 10.0 A (in 0.5-A increments). If a best fit is drawn through the firing points, an empirical relationship between current and time-to-fire is graphically established. As noted, the time-to-fire decreases as the firing current increases, and the firing time approaches infinity as the current approaches some real level. Furthermore, the apparent energy required to fire the squib decreases with increasing current and appears to approach a constant value. In this figure, the equivalent current value where the energy approaches a constant value is approximately 7.0 A. This indicates a

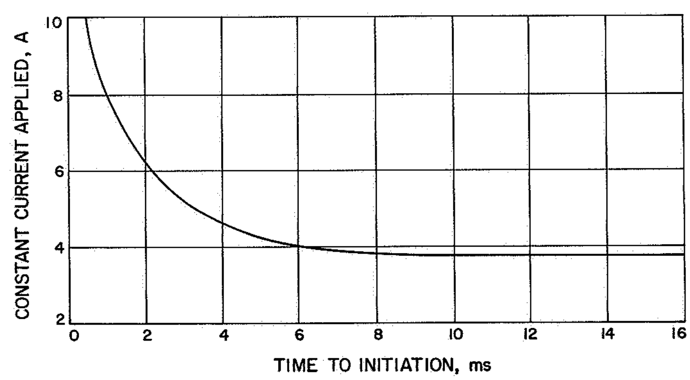

Fig. 2. Time to initiation vs applied constant current

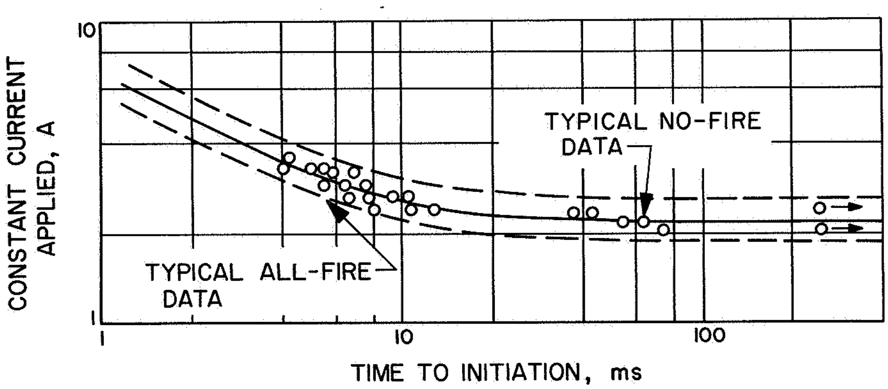

Fig. 1. Typical Bruceton results

practical constant energy-to-fire region from 8 A up to the highest test firing current of 10.0 A. The energy here is a calculated 75.0 mJ. This relationship is more apparent when plotted as energy-to-fire vs current (Fig. 3).

Without an analytical solution to this empirical relationship, practical use of the graphic method can be made. From one lot of squibs intended for flight use, a simple series of test firings in the all-fire and no-fire regions can provide good relative test data. For example, above the expected all-fire level, six or so firings at each

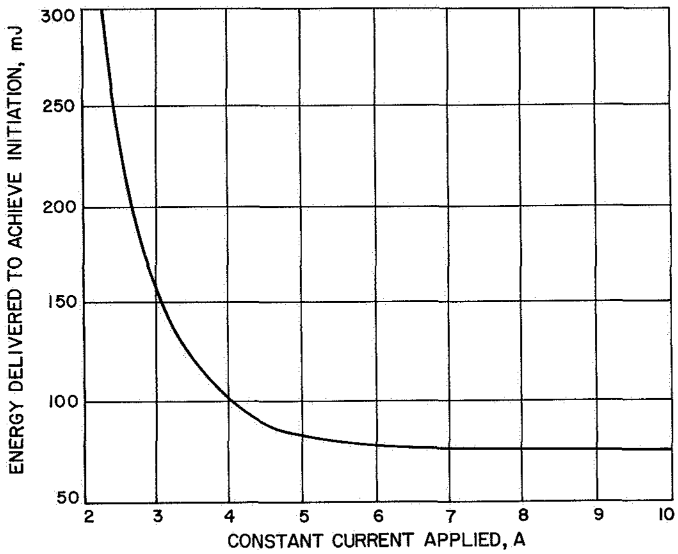

Fig. 3. Initiation energy vs applied constant current

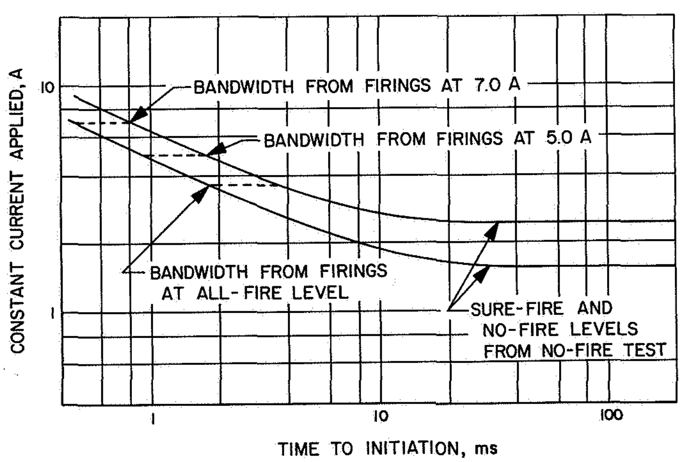

Fig. 4. Empirical sensitivity characteristics

of three or four current levels can provide a statistical mean and appropriate reliability–confidence limits of the energy to fire and yield a good representation of either the constant energy or transition region or both. A modified no-fire test can be conducted in the no-fire region and a 50% firing line, and appropriate reliability levels can be calculated.

Figure 4 shows the results of this type of testing. By fitting a curve between these two regions, an empirical function with a bandwidth can be established. This relationship can be used for monitoring lot-to-lot uniformity and sensitivity, to measure relative sensitivity degradation of the squibs after abuse, and to provide the ordnance user with a more thorough knowledge of how a particular squib should function.

III. Analytical Considerations

Analytical solutions to the mechanics of supplying electrical energy to the bridgewire, the heating of the bridgewire, and the consequent explosive initiation have been examined previously in published literature.[2] However, a brief review will facilitate the interpretation of the capacitor-discharge-energy delivery mode related to the basic equations.

Electrical energy is supplied to the bridgewire. This energy may be delivered at a constant rate (i.e., constant power) or delivered as a function of time (i.e., power as a function of time). The electrical energy for constant current is simply I^2Rt, in joules. This is the amount of energy delivered to the squib in time t. This energy is received by the bridgewire, the temperature of which is increased in proportion to the amount received. This bridgewire has a specific heat capacity – i.e., a specific number of joules of energy are required to raise its temperature by 1°C. The dimensions of the specific heat capacity used here are J/°C. The bridgewire also has a specific thermal resistance, which unfortunately is not infinitely large. This factor results in heat loss away from the bridgewire, through the ceramic, pins, match head, etc. As the bridgewire temperature increases, a proportionally higher number of joules per second, or watts, is lost by heat transfer away from the bridgewire.

Figure 5a illustrates the typical HBW squib; and Fig. 5b indicates a thermal representation of this squib. The bridgewire has an effective thermal capacitance and

[2]See Bibliography.

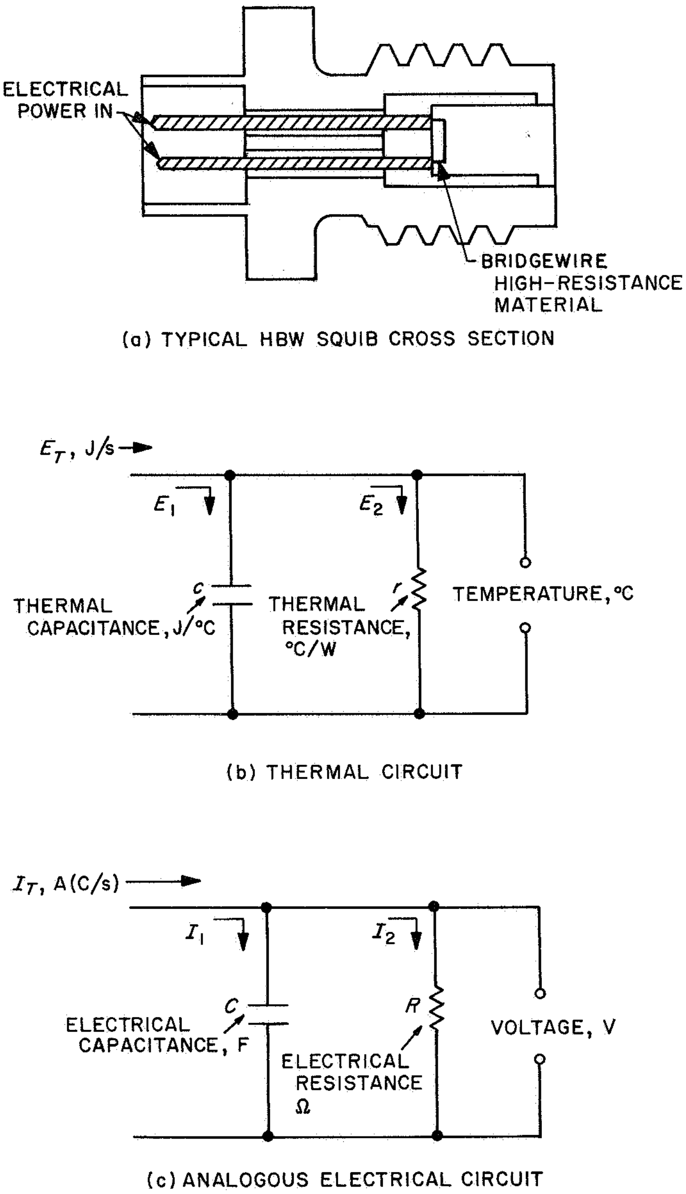

Fig. 5. HBW squib: thermal and analogous electrical circuits

a thermal resistance. Power into the bridgewire can be considered in joules per second and the thermal potential in degrees Celsius. This thermal circuit is directly analogous to the capacitor–resistor electrical circuit shown in Fig. 5c and is the basic electro-thermal analogy.

This electrical analog of the thermal characteristics of a squib indicates power input (J/s) as current (A, or C/s). This energy raises the bridgewire temperature (i.e., charges the capacitor) while, also, supplying energy to the heat-loss paths (i.e., the electrical resistor) in direct proportion to the temperature. The heat loss in W/°C is represented as $1/R$. Equivalent electrical and thermal units are tabulated below.

Electrical unit	Thermal equivalent
Ampere A (coulomb/ second C/s)	Watt W (joule/second J/s)
Volt V	Degree temperature Celsius (°C)
Ohm Ω (volt/coulomb/ second V/C/s)	Degree Celsius/watt °C/W (°C/J/s)
Mho	Watt/degree Celsius W/°C

The equations associated with Fig. 5c can be handled as follows:

For total current,

$$I_T = I_1 + I_2$$

For current to the capacitor,

$$I_1 = \frac{cd\theta}{dt} = c\dot{\theta}$$

For current to the resistor,

$$I_2 = \frac{\theta}{r}$$

$$I_T = c\dot{\theta} + \frac{\theta}{r}$$

or

$$\dot{\theta} + \frac{\theta}{rc} = \frac{I_T}{c}$$

Where $I_T \cong$ power in,

$$\dot{\theta} + \frac{\theta}{rc} = \frac{P(t)}{c} \tag{1}$$

is the basic equation.

If the power input is considered constant, the equation becomes:

$$\dot{\theta} + \frac{\theta}{rc} = \frac{I_K^2 R}{c} \tag{2}$$

The term I_K represents the constant current (in amperes) delivered to the bridgewire (of resistance R). On integration, Eq. (2) becomes

$$\theta = I_K^2 Rr\,(1 - e^{-t/rc}) \tag{3}$$

Equation (3) represents the normal curve utilized for squibs on application of constant current (the effect of the bridgewire temperature coefficient of resistivity is considered later). Examination of this equation as time approaches zero and as time approaches infinity gives Eqs. (4) and (5).

As $t \to 0$:

$$\theta = I_K^2 Rt/c, \quad \theta c = I_K^2 Rt \tag{4}$$

which is the energy to fire as $t \to 0$.

As $t \to \infty$:

$$\theta = I_\infty^2 Rr$$

or

$$\theta c = I_\infty^2 R\, rc = E_t \to 0 \tag{5}$$

Equations (3), (4), and (5) represent a simplified solution to the relationship of time-to-fire vs current.

If θ_f (firing temperature) is assumed to be a relative constant for one lot of squibs and t is a function of I_K, the equation identifying this relationship becomes:

$$I_K = \left[\frac{\theta_f}{Rr\,(1 - e^{-t/rc})}\right]^{1/2} \tag{6}$$

This equation, when graphically plotted as time-to-fire vs current, matches the empirically derived function shown in Fig. 4 and is the simplified analytical solution. A graphical representation of Eq. (3), displaying bridgewire temperature as a function of time at several different values of current input, is shown in Fig. 6. Note that, as time approaches zero, the temperature becomes a simple linear function of time ($I_K^2 Rt/c$) and that, as time approaches infinity, the temperature approaches some upper limit ($I_\infty^2 Rr$).

There are several test techniques that can be applied to make practical use of this analysis of a squib's electrical initiation characteristics. But primarily, this is a tool by which the ordnance user can, with a relatively small quantity of test squibs, accomplish three tasks: He can (1) identify the relative sensitivity characteristics of a particular lot of squibs; (2) determine relative firing reliabilities and confidence levels at different constant current inputs; and (3) measure the relative increase or decrease in sensitivity of squibs after subjection to any of several environmental abuse tests. For example, the *sure-fire* level and the energy to fire as $t \to \infty$ may be found to increase or decrease after electrostatic discharge or after several short pulses of all-fire current.

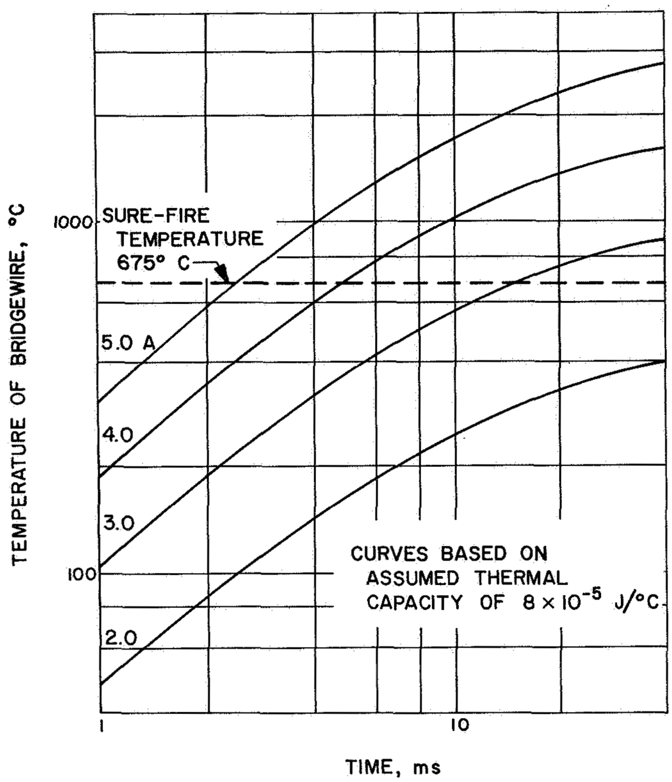

Fig. 6. Bridgewire temperature vs time

One interesting approach consists of calculating the electro-thermal efficiencies of a particular lot of squibs. Figure 7 shows such a plot of the ratio of time-to-fire to the thermal time-constant of a squib vs the percentage of delivered energy that is heat loss. As an example, it is easily determined that if our typical squib has a thermal time-constant of 8.0 ms and a sufficiently high level of constant current is used in firing to achieve a time-to-fire of 2.0 ms, the heat loss away from the bridgewire is approximately 10%. This means that if the current were 5.0 A the energy delivered would be 50 mJ ($I_K^2 Rt$). Thus, 45 mJ would have been expended in increasing the temperature of the bridgewire and 5 mJ expended in heat losses. Note that, as the firing time approaches the

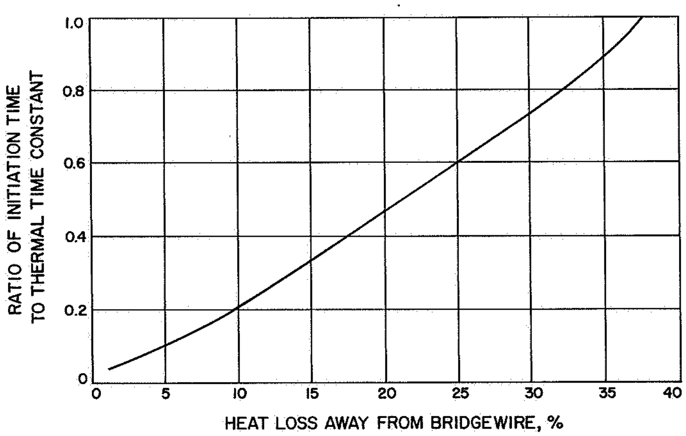

Fig. 7. Heat loss vs ratio of initiation time to thermal time-constant

thermal time-constant, the heat loss approaches 1/2.718, or 37%.

Also, the all-fire and no-fire current bands may be plotted as energy retained within the bridgewire vs time (Fig. 8). A typical 1-A/1-W squib is used for this example. The band limits of 1.4-A no-fire and 2.6-A

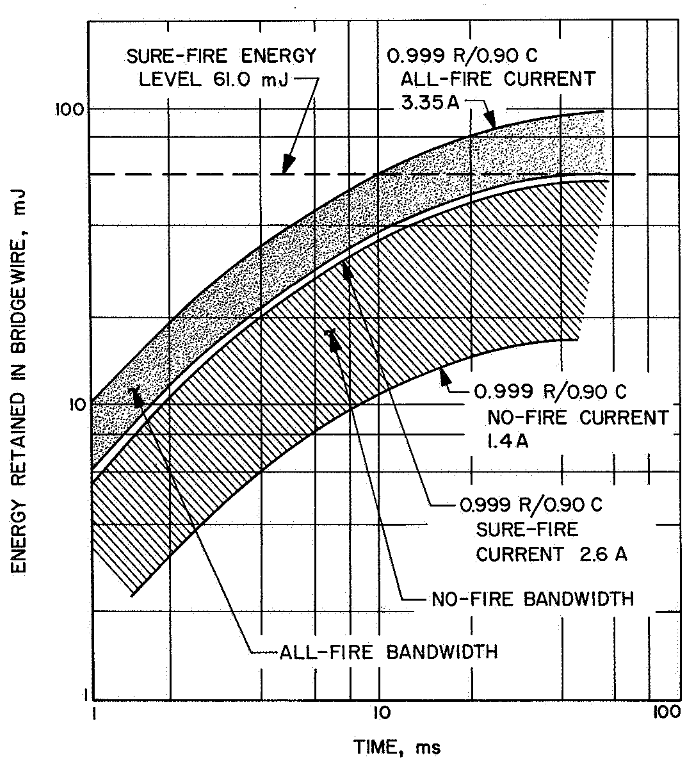

Fig. 8. Bruceton results plotted as bridgewire energy vs time

sure-fire are shown. The 2.6-A constant-current input approaches a limit of 61.0 mJ. This can be interpreted as the sure-fire energy (retained in the bridgewire) to achieve reliable initiation. The thermal time-constant of this squib is $61.0/2.6^2$, or 9.0 ms.

It should be apparent that there is a correlation between energies derived by the analytical solution and the typical test results (at the 0.999 R/0.90 C all-fire limits). No correlation is apparent at the 50% levels or the no-fire levels. The no-fire test data indicate that the sure-fire energy within the bridgewire is 61.0 mJ (2.6 A). The 10.0-ms all-fire results also indicate an all-fire energy within the bridgewire of approximately 61.0 mJ. An analysis of the energy margin, if the recommended firing current is 5.0 A, indicates a margin of (150 − 61.0)/61.0, or 146% (within 10.0 ms).

These sensitivity characteristics of a typical 1-A/1-W squib, the nominal current levels at which they are fired and the relative reliability margins will be used in the practical design of a capacitor-discharge firing circuit.

IV. Capacitor Discharge Analysis

The electro-thermal analogy permitted a definition of the temperature of the bridgewire as a function of the power in. The previous derivations were for constant current into the bridgewire. This equation, however, should be valid for any power input as a function of time.

The instantaneous current from a simple capacitor discharge circuit is:

$$i = I_0 \, e^{-t/RC} \tag{7}$$

The instantaneous power, therefore, is:

$$P(t) = I_0^2 R_s \, e^{-2t/RC} \tag{8}$$

By substitution of this value in Eq. (1),

$$\dot{\theta} + \frac{\theta}{rc} = \frac{I_0^2 R_s \, e^{-2t/RC}}{c} \tag{9}$$

which, upon integration, becomes

$$\theta = I_0^2 R_s \frac{RC\,r}{2rc - RC} \left(e^{-t/rc} - e^{-2t/RC}\right) \tag{10}$$

In this equation the quantity $e^{-2t/RC}$ approximates the fractional energy left after firing (as a charge on the capacitor) and the quantity $(1 - e^{-t/rc})$ approximates the fraction of energy dissipated as heat losses (away from the bridgewire). Thus, if at some time t, the quantity $(e^{-t/rc} - e^{-2t/RC})$ equals, say, 0.9 − 0.3, then 30% of the energy originally stored on the capacitor remained undischarged; 10% of the energy delivered was dissipated as heat losses; and 0.9 to 0.3, or 60%, of the stored energy was utilized in heating the bridgewire.

Equation (10) can be differentiated and, by setting $d\theta/dt = 0$, the time at maximum possible bridgewire temperature (i.e., the time beyond which the squib will never fire, because it is then cooling off) can be obtained:

$$t_{max} = \frac{\ln (2rc/RC)}{(2/RC) - (1/rc)} \qquad (11)$$

This t_{max} can be used when examining the optimum values for a capacitor-discharge circuit.

By use of a typical 1-A/1-W squib as an example, it is interesting to examine and compare the theoretical bridgewire temperature (or bridgewire energy) as a function of time when either constant-current power is applied or when capacitor-discharge power is applied.

Our typical squib exhibits the following characteristics from the 5-min no-fire tests: no-fire level of 0.999 R/0.90 C, 1.40 A; mean no-fire level, 2.00 A; and sure-fire level of 0.999 R/0.90 C, 2.60 A. From a practical standpoint, this means that all of these squibs will fire when 2.60-A constant current is applied for 5 min. Thus, the sure-fire power level is 6.76 W, and all of the squibs in this lot will have increasing bridgewire temperatures until initiation (when 2.6 A minimum, constant current is applied).

When tested with a constant-current input higher than the all-fire level (which might be 3.4 A), the 0.999 R/0.90 C energy-to-fire level is 75.0 mJ at 7.0 A. The theoretical thermal time-constant of the squib is the energy-to-fire as $t \to 0$ divided by the power below which the squib never fires as $t \to \infty$. In this case, this is simply 75.0/6.76, or 11.1 ms. The time at which the squib fires with 7.0 A is $75.0/7^2$, or 1.53 ms. By use of Fig. 8 to calculate efficiency, the ratio $t/rc = 1.53/11.1 = 0.138$, and the heat loss is 6.4%. Therefore, the approximate constant energy as $t \to 0$ is 0.936×75 or 70.0 mJ. A more exact thermal time-constant can now be calculated as $70.0/6.76 = 10.34$ ms. For calculation purposes, rc will be taken as 0.010 s, the constant-power level as 6.76 W, and the constant-energy level as 67.6 mJ.

This same squib when energized from a capacitor discharge power supply can be examined by utilizing Eq. (9) and the energy in the bridgewire plotted vs time (Fig. 9). The assumptions are made that the minimum initial current (I_0) is 15 A, the bridgewire resistance R_s is 1.0 Ω, the total circuit resistance R is 2.5 Ω, and the thermal time-constant rc is 10 ms. Four different values of capacitances from 200 to 600 μF are plotted.

With capacitor energy applied, the bridgewire temperature initially increases at a faster rate than with the application of constant power. However, unlike the constant-power input, the bridgewire temperature reaches a maximum value and then decreases. It should be noted that, as the electrical time-constant (i.e., the capacitance) is increased, the energy becomes flatter and approaches the pattern of the constant-current curves in Fig. 8. As a first approximation of the minimum capacitance value used in this example, the basic equation can be set equal to 67.7 mJ and solved for the capacitance, resulting in approximately 285 μF. Initiation would occur at 1.25 ms. However, the bridgewire would be at this energy level instantaneously, only — then would begin to cool off. It can be seen that the 600-μF capacitor would provide an energy margin of (136 − 68)/68, or 100%.

It is hoped that the foregoing discussion will be sufficient to demonstrate how constant-current squib parameters might be used for the sizing of capacitor-discharge circuits. One typical design approach will be described and the effects of secondary characteristics not considered in the simple analogy briefly discussed.

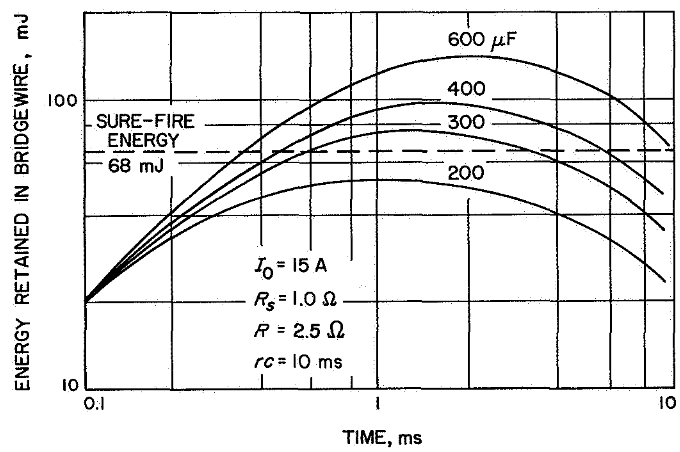

Fig. 9. Bridgewire energy vs time

The practical design of a capacitor-discharge firing circuit entails a number of parameters. However, the following exercise indicates one approach wherein the relationship between the electrical and thermal time-constants becomes quite apparent. Equations (7) and (11) can be combined to yield:

$$\left(\frac{i_f}{I_0}\right)^2 = \psi^{1/(1-\psi)} \qquad (12)$$

where $\psi = RC/2rc$.

Equation (12) is plotted in Fig. 10 and can be used for design solutions.

First, a maximum allowable current to the squib is established; experience and available test techniques (in our example) suggest limiting this to approximately 22 A. For optimum design efficiency a maximum operating voltage consistent with this 22-A limit is desired. However, an upper limit of 150 V seems advisable to facilitate packaging and preclude high-voltage arcing. An examination of high-reliability industrial components with optimal size-to-weight ratios makes tantalum foil capacitors attractive, while 50-V maximum appears to be a practical upper limit for available hardware. Note that optimum design is predicated upon specific requirements and subsequent tradeoffs — i.e., size, weight, reliability, availability, cost, rated voltage, rated capacity, etc.

The 22-A I_0 maximum and 50-V V_0 maximum indicate a 2.3-Ω R minimum. Allowance of an anticipated ΔR of 0.7 Ω results in a maximum resistance of 3.0 Ω. This maximum resistance R will predicate our worst case. The basic squib characteristics of energy-to-fire as time approaches zero (E_0) and constant current required to fire as time approaches infinity (I_∞) are utilized to define the thermal time-constant rc. In our example, we find that a 0.999-reliability/90%-confidence examination of the squib indicates 0.75 J (E_0) and 2.8 A (I_∞). The rc product is E_0/I^2R_s, or 0.075/2.8^2, which is 9.6 ms for a 1-Ω squib. Note that the worst-case thermal time-constant is not the longest thermal time-constant possible, but it represents that squib requiring the highest energy E_0 and the highest firing current I_∞.

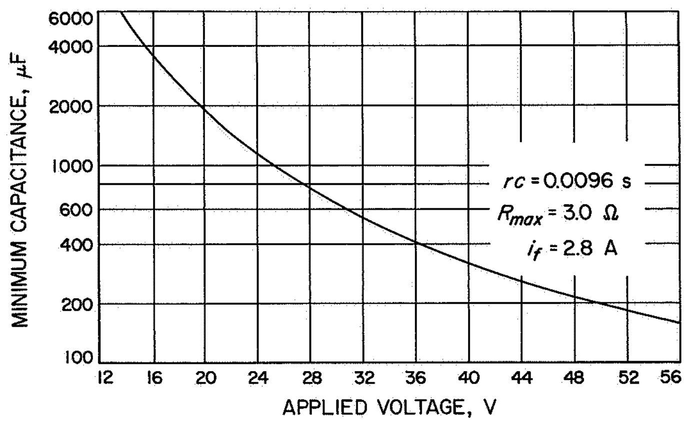

Fig. 11. Specific design parameters

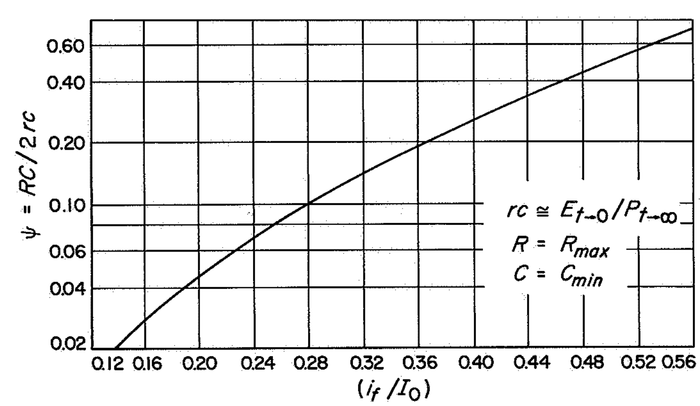

Fig. 10. General design parameters

Figure 10 can be used directly. However, it is convenient to replot, now that some parameters have been fixed. If

$$C = \frac{2rc}{R_{max}}\,\psi = \frac{2 \times 0.0096}{3}\,\psi = 0.00644\,\psi$$

and

$$V_0 = \frac{i}{i_f/I_0}\,R = \frac{2.8 \times 3}{i_f/I_0} = \frac{8.4}{i_f/I_0}$$

a replot of minimum capacitance vs minimum voltage can be established as in Fig. 11. At this time, the voltage regulation is examined and the upper voltage set as, say, 50 V. Thus, a ±10% regulated voltage would be 45.5 V, nominal or 41.0 V, minimum. From Fig. 11 it is determined that a capacitance of 310 μF is required. In this example, the capacitors available are reduced to 75% of nominal capacity at the lowest expected temperature; thus, 413 μF (nominal) is required.

The electronic design engineer must observe a number of precautions in the above procedure. Excessive voltage derating on polarized capacitors, for instance, may affect reliability adversely. A solid-state switch, such as a silicon-controlled rectifier, must be represented as a constant-voltage drop and a resistance. In this event the V_0 in our equation is reduced by this drop. The equivalent series resistance of the capacitors and other circuit resistance parameters must be known and controlled within limits. The capacitor reliability at peak discharge current must be examined.

One typical spacecraft circuit utilized on the *Mariner* 1964 spacecraft involved a transformer-coupled ac input with rectification and limiting resistors to the capacitor charge bank. Silicon-controlled rectifiers were used for switching, and the limiting resistors were so sized that even a shorted squib would eventually result in less than the required holding current. Thus, no protection against squib shorts was necessary, and the resulting design incorporated a minimum of components. To provide isolation, and incidentally mechanize the measurement of current, 1-Ω load resistors were placed in series with each squib.

V. Secondary Characteristics

Of the parameters not previously considered, the most important appears to be the change of squib resistance with time. The use of one composite thermal time-constant for the squib instead of multiple time-constants appears to be of lesser importance; however, this factor must be remembered in any rigorous handling of the data as time approaches infinity. The thermal analog using multiple time-constants is not only cumbersome in its equations, but selection of the proper critical temperature–voltage element is somewhat open to question.

Consideration of the change of squib resistance (Δ) due to the temperature coefficient of resistivity (α) results in the following equations for constant-current firings.

$$c\dot{\theta} + \frac{\theta}{r} = I_K^2 R_s (1 + \alpha\theta) \tag{13}$$

$$\theta c = \frac{I_K^2 R_s \, rc}{1 - I_K^2 R_s \, \alpha \, r} \left\{ 1 - \exp\left[- (1 - I_K^2 R_s \, \alpha \, r) \, \frac{t}{rc} \right] \right\} \tag{14}$$

This equation can be examined at the firing temperature (θ_f), at which time $\alpha\theta = \Delta$. The quantity Δ is the fractional change of resistance from ambient to firing and can be obtained from test results. As time approaches infinity, Eq. (14) yields:

$$\theta_f c = I_\infty^2 R_s (1 + \Delta) \, rc \tag{15}$$

Combination of Eqs. (14) and (15) when $\alpha\theta_f = \Delta$ gives Eq. (16):

$$\dot{\theta}_f c = t \, \frac{R_s \left[\Delta (I_K^2 - I_\infty^2) - I_\infty^2 \right]}{\ln \{ (1 + \Delta) \left[(1 - I_\infty^2)/I_K^2 \right] \}} \tag{16}$$

Equation (16) can be relatively useful. Values for Δ, t, R_s, and I_K are measurable for a given squib. I_∞ can be obtained from test results for a particular squib lot. The corresponding capacitor discharge equation utilizing $\alpha\theta$ is complicated. The first approximation yields

$$c\dot{\theta} + \frac{\theta}{r} = I_0^2 R_s (1 + \alpha\theta) \, e^{-2t/RC} \tag{17}$$

$$\frac{\theta c}{I_0^2 R_s} = \frac{\exp \{ - t/rc - [(I_0^2 R_s \, \alpha r)/2] \, (1 - e^{-2t/RC}) \}}{2/RC - 1/rc + (I_0^2 R_s \, \alpha)/c} - \frac{e^{-2t/RC}}{2/RC - 1/rc + [(I_0^2 R_s \alpha \, e^{-2t/RC})/c]} \tag{18}$$

Equation (17) is not quite correct. It should be:

$$c\dot{\theta} + \frac{\theta}{r} = I_0^2 R_s (1 + \alpha\theta) \exp \left\{ \frac{-2t}{RC \, [1 + (\alpha\theta R_s)/R]} \right\} \tag{19}$$

As the parameters θ and t both appear in exponential form, no technique for integration occurs. This equation can be examined at the maximum temperature where $\dot{\theta} = 0$, $\theta \cong \theta_f$, and $\alpha\theta_f \cong \Delta$. However, the resulting equation

$$\left(\frac{I_f}{I_0} \right)^2 = \exp \left[\frac{-2t}{C \, (R + \Delta R_s)} \right] \tag{20}$$

is the basic capacitor-discharge equation and, having no boundaries for energy delivered, is not too meaningful.

The effect of α on our simplified analogy must be examined. In practice, the curve resulting from Eq. (6) is superimposed on the various constant-current firing data derived in testing. Any significant deviation of the data from this idealized curve would indicate second-order effects. Equations (16) and (18) could then be used for spot checks.

The simple analogy does appear to be adequate for sizing the typical 1-A/1-W squib utilized in this report. This squib does have a resistance change, but the equations supply counteracting factors. First, the derivation of E_0 from constant-current data neglected the effect of α and, thus, gave a value lower than actual. Second, the simplified analogy fails to account for the increasing squib resistance. Thus, although the calculated E_0 may be low, the squib receives more energy than calculated for a given current. This effect is enhanced since the increasing squib resistance not only increases the instantaneous power but, also, increases the portion of the available energy that the squib itself receives in the capacitor-discharge circuit. A final detrimental factor exists in that the increasing total loop resistance increases the electrical time-constant and produces a slower energy delivery, thereby increasing heat losses.

From the above, it can be seen that the simple analogy has its pitfalls and that test results are required on a given squib before final determination of the applicability of this approach can be made.

In summary, a short electrical time-constant for the circuit is desirable so that the energy being delivered at a fast rate can be used most effectively. A long thermal time-constant for the squib is desirable so that the initial electrical energy can be used primarily for heating the bridgewire and, thus, minimize the heat losses. The optimum design of a low-voltage capacitor-discharge firing circuit requires consideration of these time-constants in relation to each other, and considerable awareness of second-order effects, only a few of which are discussed here.

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