



LIFETIME AND TRANSITION PROBABILITIES OF $np^4 - (n + 1) p$ STATES OF Ne II, A II AND Kr II

S. H. Koozekanani
and
G. L. Trusty

The Ohio State University

ElectroScience Laboratory

(formerly Antenna Laboratory)
Department of Electrical Engineering
Columbus, Ohio 43212

REPORT 1093-42
17 September 1968

Grant Number NsG-74-60

FACILITY FORM 602	N 68-36653	
	(ACCESSION NUMBER)	(THRU)
	14	1
	(PAGES)	(CODE)
CR 97198	04	
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)	

National Aeronautics and Space Administration
Office of Grants and Research Contracts
Washington, D. C. 20546

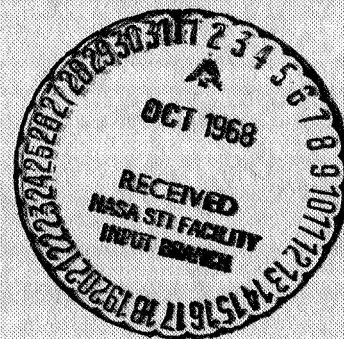
GPO PRICE \$ _____

CSFTI PRICE(S) \$ _____

Hard copy (HC) _____

Microfiche (MF) _____

ff 653 July 65



NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

The Government has the right to reproduce, use, and distribute this report for governmental purposes in accordance with the contract under which the report was produced. To protect the proprietary interests of the contractor and to avoid jeopardy of its obligations to the Government, the report may not be released for non-governmental use such as might constitute general publication without the express prior consent of The Ohio State University Research Foundation.

REPORT
by
The Ohio State University ElectroScience Laboratory
(Formerly Antenna Laboratory)
Columbus, Ohio 43212

Sponsor National Aeronautics and Space Administration
 Office of Grants and Research Contracts
 Washington, D. C. 20546

Grant Number NsG-74-60

Investigation of Receiver Techniques and Detectors for
 Use at Millimeter and Submillimeter
 Wave Lengths

Subject of Report Lifetime and Transition Probabilities of
 $np^4 - (n + 1) p$ States of Ne II, A II and Kr II

Submitted by S. H. Koozekanani
 G. L. Trusty
 ElectroScience Laboratory
 Department of Electrical Engineering

Date 17 September 1968

ABSTRACT

Life time as well as transition probabilities of the first p-excited states of neon⁺, argon⁺, and krypton⁺ have been calculated.

LIFETIME AND TRANSITION PROBABILITIES OF
 $np^4 - (n + 1)p$ STATES OF NeII, ArII AND KrII

Lifetime and transition probabilities of some of the first excited "p" states of neon II, argon II and krypton II^{1, 2, 3} for the configuration of $np^4 - (n + 1)p$, have been calculated. This information is needed to determine the possible efficiencies of current and proposed laser systems and to aid in the identification of the energy levels involved.

The exact mixed wave functions $\Psi_i(E_i)$ corresponding to the experimentally found energy levels E_i can be expressed in terms of purely LS coupled wave functions ϕ_i , with ϕ_i corresponding to the LS coupled states of the form $|p^4 \overline{LS}; l_{ex} s; L_j S_j J M_i\rangle$, i. e.,

$$(1) \quad \Psi_i(E_i) = \sum_j a_{ij} \phi_j.$$

where \overline{LS} are the orbital and spin angular momentum of the p^4 core electrons while $L_j S_j$ are the total orbital and spin angular moments. $\underline{J} = \underline{L} + \underline{S}$ and M is the projection of J on the z -axis. l_{ex} is the excited p electron with its spin s . The functions $\Psi_i(E_i)$ were found by treating the radial integrals as adjustable parameters so as to obtain the best fit with the experimentally found energy levels. This was done by choosing arbitrarily close test values for the parameters $[^1D]$, $[^3P]$ core energy states, F_2 , G_0 , G_2 direct and exchange integrals and ζ_{np} and $\zeta_{l_{ex}}$ corresponding to the spin orbit interaction terms of the np^4 core and $(n + 1)p$ excited electron.

With the initially chosen test parameters $[^1D]_0, [^3P]_0 \dots$ etc. and the Hamiltonian

$$(2) \quad H = -\sum \frac{\hbar^2}{2m} \nabla^2 - \sum_i \frac{e^2}{r_i} + \sum_{i>j} \frac{e^2}{r_{ij}} + \sum_i \xi(r) l_i \cdot S_i,$$

the matrix $[H_0]$ was constructed having elements $[H_0]_{ij}$ where

$$(3) \quad [H_0]_{ij} = \langle \phi_i | H | \phi_j \rangle_0$$

In Eq. (3), the subscript naught designated that in the calculation of the elements $[H_0]_{ij}$ the first set of test parameters are used.

A matrix $[M_0]$ is found such that,

$$(4) \quad [M_0]^{-1} [H_0] [M_0] = [D_0]$$

with $[D_0]$ being a diagonal matrix. If the choice of the parameters were appropriate $[D_0]$ will differ little from the experimentally found energy levels. If not, a new matrix equation is constructed having the form,

$$(5) \quad [M_0^{-1}] [H] [M_0] = [E]$$

where $[E]$ is again a diagonal matrix having for its diagonal elements the experimental energy levels E_i , $[M_0]$ is the matrix which was used to diagonalize $[H_0]$, and $[H]$ is the Hamiltonian matrix with its elements expressed in terms of the radial parameters⁴, i.e.,

$$(6) \quad [H]_{ij} = \langle \phi_i | H | \phi_j \rangle = a_{ij} [{}^1D] + b_{ij} [{}^3P] + C_{ij} F_2 + d_{ij} G_0 + \dots$$

Multiplying out $[M_0^{-1}] [H] [M_0]$ and equating it to $[E]$ one obtains, taking the diagonal terms only, a linear set of nineteen simultaneous equations in terms of the seven parameters $[{}^1D]$, $[{}^3P]$, F_2 , G_0 , G_2 , ζ_{np} and $\zeta_{l ex}$. Solving the equations and least-square-fitting the results, we obtain a new set of parameters $[{}^1D]_1$, $[{}^3P]_1$, ... etc. These parameters are now used to obtain the matrix $[H_1]$, with matrix elements $[H_1]_{ij}$ in which the set of the newly found values of the radial parameters have been substituted, i. e.,

$$[H_1]_{ij} = a_{ij} [{}^1D]_1 + b_{ij} [{}^3P]_1 + C_{ij} (F_2)_1 + d_{ij} (G_0)_1 + \dots$$

Once again we find a matrix $[M_1]$ which diagonalizes $[H_1]$ giving the diagonal matrix $[D_1]$ i. e.,

$$(7) \quad [M_1]^{-1} [H_1] [M_1] = [D_1]$$

If again the elements $[D_1]$ would differ much from the experimentally found energy levels E_i the process is repeated by solving the simultaneous equation obtained from the matrix equation,

$$(8) \quad [M_1]^{-1} [H] [M_1] = [E]$$

where again the elements of the matrix H are given by Eq. 6. A new set

of parameters $[^1D]_2$, $[^3P]_2$. . . etc., are found. The process is repeated until the parameters $[^1D]_n$, $[^3P]_n$. . . etc. converge and the matrix equation $[M_n]^{-1} [H_n] [M_n] = [D_n] \approx [E]$.

If the initial set of test parameters are close to the actual values, the process would terminate after a few cycles, and the parameters would converge to fixed values. If not, the convergence is not easily attained. In the above calculations the states arising from the 1S core were not taken into consideration. However, their contribution to the states $^2P_{\frac{1}{2}}$, $^2P_{\frac{3}{2}}$, $^2P_{\frac{1}{2}}$ and $^2P_{\frac{3}{2}}$ can be calculated by perturbation theory. The

shift in energy would be $\Delta E = - \frac{H^2_{ij}}{E_i - E_j}$. Table one gives the values of the parameters 1D , 3P , F_2 , G_0 , . . . etc. using Hartree Fock⁶ wavefunctions and also the values which were obtained by the above fitting techniques. The two sets of values are more or less very close. Table II gives the theoretical and experimentally found energy levels and the mixing coefficients a_{ij} . Table III gives the transition probabilities for the states arising from the $np^4 - (n+1)p$ configuration of neon, argon, and krypton to the lower states of $np^4 - (n+1)s$ configuration. Finally Table IV gives the lifetimes of some of the higher states of interest.

TABLE I

	Ne		Ar		Kr	
	Calculated Cm ⁻¹	Fitting Cm ⁻¹	Calculate Cm ⁻¹	Fitting Cm ⁻¹	Calculate Cm ⁻¹	Fitting Cm ⁻¹
F ₂ (P _{core} - P _{ex})	422.	433	424	365	415	397
G ₀ (P _{core} - P _{ex})	4166.	2452	2614	1752	2176	1802
G ₂ (P _{core} - P _{ex})	160.	167	122.5	86	111	171
ζ _{P_{core}}	1104	790	1198	1075	2593	2929
ζ _{p-ex}	63	-33	107	187	251	570
[³ P]	—	252853	—	159918	—	140669
[¹ D]	—	278205	—	173491	—	152937
F ₂ (P _{core} - P _{core})	4757	4225	2827	2262	2488	2044

TABLE II-a

Experimental and theoretical energy levels of Ne II taken from Moore⁷, tables of atomic energy levels and this calculation. The mixing coefficients a_{ij} are those given in Eq. (1).

Ne II States	E _{exp} Cm ⁻¹	E _{theor} Cm ⁻¹	⁴ D	⁴ P	⁴ S	² D	² P	² S	² P ₁	² D ₁	² F ₁
⁴ D _{1/2}	249110.8	248652.5	0.99976								0.21956
² F _{1/2}	274411.3	274228.2	0.21956							0.01720	-0.99976
⁴ P _{1/2}	246194.8	246314.4	0.10198	0.99415		0.03104					0.00042
⁴ D _{3/2}	249448.0	249217.8	0.98594	-0.09696		-0.13543				0.00271	0.01307
² D _{3/2}	251013.3	251158.4	-0.13166	0.04497		-0.98950				-0.03385	0.01966
² F _{3/2}	274366.9	274229.0	0.01038	0.00010		-0.02106				-0.00511	-0.99971
² D _{5/2}	277346.1	277407.1	-0.00883	-0.01525		-0.03379				0.99926	-0.00449
⁴ P _{3/2}	246417.4	246617.2	0.09174	0.99253	0.06423	0.00625	-0.04273		0.02033	0.00773	
⁴ D _{1/2}	249697.7	249553.3	-0.99010	0.09102	0.01201	0.09658	0.04061		-0.00908	-0.00448	
² D _{1/2}	251524.7	251713.3	0.10309	-0.00654	0.01349	0.97118	0.20859		-0.04265	0.02494	
² P _{1/2}	254167.0	253922.7	0.02353	0.05218	-0.11802	-0.21197	0.94040		-0.23011	-0.02373	
⁴ S _{1/2}	252956.0	253072.9	-0.00743	0.05933	-0.99058	0.04020	-0.11599		0.00933	0.00375	
² P _{1/2}	276278.6	276396.3	-0.00025	-0.00729	-0.01897	-0.00409	0.23087		0.96654	-0.10978	
² D _{3/2}	277327.6	277411.8	0.00721	0.00697	0.00196	0.02967	-0.04370		-0.10217	-0.99331	
⁴ P _{1/2}	246599.9	246861.7	0.05282	0.99729			-0.02210	-0.04454	-0.01249		
⁴ D _{3/2}	249841.8	249749.7	0.99720	-0.05422			-0.04947	-0.00301	0.01401		
² F _{1/2}	254294.0	254197.1	0.04669	0.00333			0.89586	-0.37847	-0.22804		
² S _{1/2}	252800.8	252885.9	0.02493	0.04932			0.36197	0.92444	-0.10641		
² P _{1/2}	276514.1	276598.9	0.00137	0.00587			-0.25193	-0.01308	-0.96764		

TABLE II-b

Experimental and theoretical energy levels of Ar II taken from Minnhagen⁸ and from this calculation. The mixing coefficients of a_{ij} are those given in Eq. (1).

Ar II States	Exp E_{n1} Cm	Etheor ^r Cm	⁴ D	⁴ P	⁴ S	² D	² P	² S	² P _{1/2}	² D'	² F'
⁴ D _{7/2}	157233.93	157176.6	0.99839								0.05680
² F' _{7/2}	170530.31	170582.7	0.05680								-0.99839
⁴ P _{5/2}	155043.07	155128.2	0.18291	0.98216		0.02312				0.03949	0.00299
⁴ D _{5/2}	157673.32	157556.9	0.86726	-0.15036		-0.47202				-0.00525	0.04919
² D _{3/2}	158730.21	158710.3	-0.46181	0.10820		-0.87867				-0.04014	0.03689
² F' _{5/2}	170400.94	170364.6	-0.02553	0.00160		0.05688				-0.03043	0.99759
² D' _{5/2}	173393.38	173432.7	0.02203	0.03525		0.03701				-0.99794	-0.03204
⁴ P _{3/2}	155351.03	155383.5	0.16724	0.96996	0.11854	-0.1561	-0.11366		0.05959	0.02102	
⁴ D _{3/2}	158167.71	158007.6	0.91426	-0.18070	-0.04193	-0.31306	-0.17205		0.04422	0.01261	
² D _{1/2}	159393.31	159244.3	0.36791	0.00919	0.00421	0.80472	0.44766		-0.12847	0.00763	
² P _{3/2}	160239.35	160354.7	0.01598	0.09361	0.06872	-0.50253	0.81016		-0.27314	-0.05244	
⁴ S _{3/2}	161048.64	161012.7	0.01607	-0.13135	0.98834	0.01899	-0.03572		0.06355	0.00052	
² P _{1/2}	172213.80	172213.8	-0.00095	0.01493	0.05133	0.0218	-0.30918		-0.94747	0.05873	
² D' _{1/2}	173347.83	173349.9	0.01702	0.01413	0.00192	0.02951	-0.06202		-0.04059	-0.99657	
⁴ P _{1/2}	155708.02	155781.9	0.09789	0.99105			-0.06195	-0.05442	0.03795		
⁴ D _{1/2}	158428.03	158331.0	-0.98840	0.10630			0.09918	0.01678	-0.04050		
² P _{1/2}	159706.46	159869.7	-0.11005	-0.07824			-0.80018	-0.49034	0.31788		
² S _{1/2}	161089.31	161306.5	-0.03676	0.01627			-0.46980	0.86912	0.14933		
² P' _{1/2}	172816.21	172726.4	-0.00350	-0.01162			0.35401	0.03085	0.93465		

TABLE II-C

Experimental and theoretical energy levels of Kr II taken from Moore⁷, tables of atomic energy levels and this calculation. The mixing coefficients a_{ij} are those given in Eq. (1).

	Exp Cm ⁻¹	E _{theor} Cm ⁻¹	⁴ D	⁴ P	⁴ S	² D	² P	² S	² P ^o	² D ^o	² F ^o
⁴ D _{7/2}	135783.18	136046.4	0.98818								0.15329
² F _{1/2}	149704.55	149720.5	0.15329								-0.98818
⁴ P _{3/2}	133925.65	133855.3	0.35157	0.92956		-0.00191			0.10898		0.02069
⁴ D _{5/2}	136071.00	136913.7	0.69899	-0.26376		-0.64380			-0.04663		0.15873
² D _{3/2}	140118.99	139794.1	-0.61630	0.24015		-0.73946			-0.08975		0.08744
² F _{3/2}	149173.42	149096.5	-0.06087	0.00875		0.17647			-0.05814		0.98066
² D _{1/2}	152316.26	152377.9	0.06542	0.09275		0.08704			-0.98717		-0.07092
⁴ P _{1/2}	134288.44	134032.7	0.29227	0.86841	0.24557	-0.06918	-0.25507		0.15819	0.07257	
⁴ D _{3/2}	138381.35	138171.7	0.70387	-0.33387	-0.23513	-0.42128	-0.39352		0.05687	-0.04741	
² D _{5/2}	141995.68	142424.3	0.15404	-0.21173	0.13765	0.80116	-0.43082		0.25429	0.14271	
² P _{1/2}	140137.15	140117.4	0.62449	0.04557	0.03686	0.32338	0.67428		-0.21368	-0.04111	
⁴ S _{1/2}	141722.72	141508.3	0.05458	-0.29324	0.91550	-0.23728	0.06388		0.10487	-0.03901	
² P _{3/2}	150203.48	150519.5	-0.00713	0.02327	0.16055	0.07215	-0.32003		-0.91809	0.15208	
² D _{1/2}	152191.86	152172.6	0.04903	0.03093	0.01389	0.09900	-0.18255		-0.08685	-0.97251	
⁴ P _{1/2}	135783.03	135638.4	0.20533	0.92476			-0.20833	-0.19012	0.15203		
⁴ D _{1/2}	140163.25	139549.3	-0.85103	0.01265			-0.28601	-0.40360	0.17577		
² F _{1/2}	139103.36	139042.2	-0.47019	0.37937			0.56136	0.46584	-0.32072		
⁴ S _{1/2}	142363.55	142592.5	-0.11170	0.00795			-0.59032	0.76087	0.24507		
² P _{1/2}	152240.97	152094.4	-0.00571	-0.02610			0.45955	0.07095	0.88491		

TABLE III

Transition probabilities between 3p-3s levels of Ne II, 4p-4s levels of Ar II and 5p-5s levels of Kr II, each with a $p^4 [^3P]$ core. A in sec^{-1} .

	Neon II		Argon II		Krypton II	
	Pure L's	Mixed	Pure L's	Mixed	Pure L's	Mixed
$^4D_{7/2} - ^4P_{5/2}$	197.81×10^6	197.71×10^6	146.53×10^6	146.06×10^6	168.15×10^6	164×10^6
$^2D_{7/2} - ^4P_{5/2}$	0	0.28	0	4.67	0	5.28
$^2D_{7/2} - ^4P_{3/2}$	0	4.12	0	31.65	0	91.27
$^2D_{7/2} - ^2P_{3/2}$	143.30	140.31	103.65	80.03	140.97	77.08
$^4D_{5/2} - ^4P_{5/2}$	61.37	43.08	46.53	18.91	52.37	4.59
$^4D_{5/2} - ^4P_{3/2}$	135.98	149.75	97.25	90.69	89.85	68.27
$^4D_{5/2} - ^2P_{3/2}$	0	2.20	0	19.70	0	31.39
$^4P_{5/2} - ^4P_{5/2}$	101.86	114.65	79.28	96.24	91.38	122.90
$^4P_{5/2} - ^4P_{3/2}$	41.20	28.96	30.06	14.84	27.86	4.29
$^4S_{5/2} - ^4P_{5/2}$	142.05	124.62	116.11	87.65	167.69	71.85
$^4S_{5/2} - ^4P_{3/2}$	90.42	93.83	70.32	77.64	87.52	93.20
$^4S_{5/2} - ^4P_{1/2}$	44.00	52.87	33.11	49.39	32.51	69.82
$^4S_{5/2} - ^2P_{3/2}$	0	1.41	0	0.088	0	0.26
$^4S_{5/2} - ^2P_{1/2}$	0	1.17	0	0.13	0	7.28
$^2P_{3/2} - ^4P_{5/2}$	0	0.59	0	2.80	0	10.91
$^2P_{3/2} - ^4P_{3/2}$	0	3.29	0	0.0073	0	44.51
$^2P_{3/2} - ^4P_{1/2}$	0	1.31	0	0.088	0	24.04
$^2P_{3/2} - ^2P_{3/2}$	166.49	119.05	106.91	36.64	117.77	78.98
$^2P_{3/2} - ^2P_{1/2}$	31.30	62.62	18.56	69.40	16.23	0.039
$^2D_{7/2} - ^4P_{5/2}$	0	0.20	0	1.60	0	0.036
$^2D_{7/2} - ^4P_{3/2}$	0	1.00	0	12.59	0	0.53
$^2D_{7/2} - ^4P_{1/2}$	0	1.35	0	8.83	0	17.24
$^2D_{7/2} - ^2P_{3/2}$	25.27	52.23	19.01	61.98	30.15	0.74
$^2D_{7/2} - ^2P_{1/2}$	118.08	91.00	82.01	29.97	107.19	105.87

		Neon II cont't.		Argon II cont't.		Krypton II cont't.	
		Pure L's	Mixed	Pure L's	Mixed	Pure L's	Mixed
$^4D_{\frac{1}{2}}$	$^4P_{\frac{5}{2}}$	10.48×10^6	4.97×10^6	8.26×10^6	0.47×10^6	11.60×10^6	12.58×10^6
	$^4P_{\frac{3}{2}}$	106.23	96.59	79.06	58.08	93.66	48.93
	$^4P_{\frac{1}{2}}$	80.54	93.00	57.72	65.89	51.96	41.17
	$^2P_{\frac{3}{2}}$	0	0.72	0	7.74	0	30.95
	$^2P_{\frac{1}{2}}$	0	0.58	0	3.79	0	3.66
$^4P_{\frac{3}{2}}$	$^4P_{\frac{3}{2}}$	67.11	79.86	51.03	67.56	61.83	92.73
	$^4P_{\frac{1}{2}}$	18.77	21.67	13.38	16.69	13.11	14.86
	$^4P_{\frac{1}{2}}$	56.72	41.97	38.69	20.49	26.90	4.76
	$^2P_{\frac{3}{2}}$	0	0.11	0	0.73	0	3.75
	$^2P_{\frac{1}{2}}$	0	0.040	0	0.052	0	0.055
$^2S_{\frac{1}{2}}$	$^4P_{\frac{3}{2}}$	0	0.82	0	0.0000053	0	0.41
	$^4P_{\frac{1}{2}}$	0	0.0018	0	0.32	0	2.34
	$^2P_{\frac{3}{2}}$	115.86	161.43	95.83	27.63	126.35	14.90
	$^2P_{\frac{1}{2}}$	54.30	9.24	41.81	98.32	45.17	115.01
$^4D_{\frac{1}{2}}$	$^4P_{\frac{3}{2}}$	33.68	25.84	25.55	14.40	36.51	24.72
	$^4P_{\frac{1}{2}}$	163.34	170.51	119.48	128.23	133.01	97.63
	$^2P_{\frac{3}{2}}$	0	0.12	0	0.50	0	34.71
	$^2P_{\frac{1}{2}}$	0	0.17	0	0.43	0	0.000026
$^4P_{\frac{1}{2}}$	$^4P_{\frac{3}{2}}$	119.75	124.82	88.10	94.35	102.62	106.05
	$^4P_{\frac{1}{2}}$	23.162	17.90	16.33	9.74	13.92	3.02
	$^2P_{\frac{3}{2}}$	0	0.20	0	0.41	0	5.47
	$^2P_{\frac{1}{2}}$	0	0.0045	0	0.020	0	0.16

TABLE IV

Lifetimes of some of the $3p$ of Ne II, $4p$ of A II and $5p$ of Kr II (in nanoseconds)

	Ne II	A II	Kr II
$^4D_{7/2}$	5.0579	6.8457	6.0841
$^4D_{5/2}$	5.1276	7.7325	9.5710
$^2D_{5/2}$	6.9104	8.5926	5.7454
$^4P_{5/2}$	6.9598	8.9995	7.8594
$^4D_{3/2}$	5.1102	7.3537	7.2848
$^2D_{3/2}$	6.8593	8.6801	7.8240
$^4P_{3/2}$	6.9615	9.4748	8.6004
$^2P_{3/2}$	5.3463	9.0649	6.2623
$^4S_{3/2}$	3.6508	4.6504	4.1172
$^4D_{1/2}$	5.0853	6.9650	6.3331
$^4P_{1/2}$	6.9967	9.5673	8.7047
$^2S_{1/2}$	5.8307	7.8869	7.3880
$^2P_{1/2}$	5.5149	9.2892	8.5162
$^2F'_{7/2}$	6.1745	8.0360	6.3348
$^2F'_{5/2}$	6.2084	7.9657	6.7005
$^2D'_{5/2}$	4.5647	5.5537	4.6621
$^2D'_{3/2}$	4.5505	5.4526	4.5695
$^2P'_{3/2}$	4.1451	5.3876	5.0771
$^2P'_{1/2}$	3.9419	4.7836	3.7494

REFERENCES

1. Garstang, R. H., Monthly Notices, Roy. Ast. Soc., 114, p. 118, 1954.
2. Koopman, D. W., "Line Strengths for Neutral and Singly Ionized Neon," J. of Op. Soc. of Am., Vol. 54, no. 11, November 1964.
3. Statz, H., F. A. Horrigan, S. H. Koozekanani, C. L. Tang and G. F. Koster, J. of Appl. Phys., Vol. 36, no. 7, p. 2278, July 1965.
4. See T. Yamanouchi and A. Amemiya, Proc. Phys. Soc. Japan, 1, p. 18, 1946, for the exchange and direct integral, and J. L. Tech, J. Res. Nat. Bur. Std., 67A, p. 555, 1963, for the spin orbit terms.
5. It can be shown that the equations arising from the off diagonal terms give no information.
6. Herman, F., and S. Steillman, "Atomic Structure Calculations," Prentice Hall Inc., Englewood Cliffs, N. J., 1963.
7. Moore, C. E., "Atomic Energy Levels," Circular of the Nat. Bur. of Std., U. S. Gov. Printing Office, Washington 25, D. C., 1949.
8. Minnhagen, L., "The Spectrum of Singly Ionized Argon, Ar II," Arkiv För Fysik, Band 25, nr. 19, Almquist and Wiksell, Stockholm, Sweden, 1963.