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Stability of a Pure Electron Plasma
in Cylindrical Geometry*

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ABSTRACT

The stability of a cylindrical cold plasma consisting of electrons only is investigated analytically. The geometry is similar to that of the proposed heavy ion plasma accelerator. The dispersion equation is derived and discussed for the case that the plasma rotates like a rigid body. The resulting instabilities are found to be much more violent than those studied by Buneman et al. for a straight geometry. The underlying physics is discussed.

I. INTRODUCTION

The properties of grossly non-neutral plasmas have received considerable attention recently because of several possible applications.[1,2,3] One of these,[1] the heavy ion plasma accelerator (HIPAC), makes use of an electron cloud which is suspended by a strong magnetic field in a metal cylinder of toroidal geometry. The ultimate failure or success of such a device will critically depend on a favorable stability behavior of the electron plasma in such a geometry.

The macroscopic stability for a cold electron plasma in a carthesian slab geometry has been investigated by Buneman, Levy, and Linson.[4] Their results were very encouraging: A long wave length diocotron instability can be avoided by a suitable geometry which does not allow such waves and the cyclotron instability becomes unimportant because the growth rate is proportional to $\exp(-2/q)$ which becomes insignificant if $q = O(10^{-2})$. q is defined as $q = \omega_p^2/\omega_c^2$ ($\omega_p^2 = 4\pi e^2 n/m$; $\omega_c = eB/mc$).

Although these results apply strictly only to a carthesian plasma slab (Fig. 1(a)), it has been assumed to hold--at least order of magnitude-wise--also for the HIPAC geometry (Fig. 1(b)). There are, however, important differences:

The slab geometry has two plasma boundaries, the zylindrical geometry only one. The second boundary condition is that for $r = 0$ the disturbed potential has to be finite. As a consequence, the diocotron instability disappears for a purely azimuthal disturbance if inertia effects are neglected. This has been studied by Timofeev[5,6] and Levy.[7]

Another important difference is that in Fig. 1(a) there will always be a "slipping" in the beam, i.e., the $\underline{E} \times \underline{B}$ drift will always depend on x. In Fig. 1(b), however, the plasma may rotate like a solid body with no internal "slipping". These differences may have profound consequences on the oscillation modes and in particular on the growth rates of the instabilities. It is for this reason that this investigation has been undertaken.

The remainder of this paper will proceed as follows: In Section II we state the basic equations and the equilibrium solutions. Section III deals with the derivation of the dispersion equation. It turns out that it can be written as an algebraic equation involving only Bessel functions. We then discuss the dispersion equation and the resulting instabilities for $q \ll 1$. Finally, the differences of the results in the two geometries are discussed.

II. EQUILIBRIUM

We approximate the toroidal geometry of HIPAC by a straight cylinder and periodic boundary conditions at $z = o$ and $z = z_o$. Typical toroidal effects due to curved field lines and the inhomogeneity of the magnetic field are thus excluded. However, it is generally believed that this is a reasonably good approximation to zeroth order (compare the literature on stellerator geometry, e.g., Ref.8).

Consider in this geometry a cold nonrelativistic electron plasma, which is described by the continuity equation, the equation of motion, and Poisson's equation:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\underline{v}) = 0; \tag{2.1}$$

$$m\frac{d\underline{v}}{dt} = -e\left(-\nabla\Phi + \frac{1}{c}\,\underline{v}\times\underline{B}\right); \tag{2.2}$$

$$\nabla^2\Phi = 4\pi e n. \tag{2.3}$$

The equilibrium solution is given by

$$n^\circ = n^\circ(r)$$

$$\underline{v} = V(r)\underline{e}_\varphi, \text{ where } V = -c\,\frac{E_r^\circ}{B};\quad E_r^\circ = -\frac{\partial\Phi^\circ}{\partial r};$$

and $$E_r^\circ(r) = -\frac{4\pi e}{r}\int_0^r n^\circ(r)\, r dr.$$

If we assume a constant density for $r \le a$, we find

$$V = \tfrac{1}{2}q\ \omega_c\ r \equiv \sigma r,\ \sigma = \tfrac{1}{2}q\ \omega_c;\ \text{where } q = \omega_p^{\ 2}/\omega_c^{\ 2}.$$

It will turn out that this assumption will make the dispersion equation particularly simple.

III. DISPERSION EQUATION

If we disturb the equilibrium, we may write the perturbed quantities as:

$$\begin{Bmatrix} \Phi(r) \\ n(r) \\ \underline{u}(r) \end{Bmatrix} e^{i\omega t + i\nu\varphi + ikz}$$

Linearizing in the usual way, the equation of motion can now be written as:

$$i(\omega + \nu\, V/r)u_r + (\omega_c - V/r)u_\varphi = \frac{e}{m}\,\frac{\partial\Phi}{\partial r} \quad ;$$

$$(\omega_c - \frac{\partial V}{\partial r})u_r - i(\omega + \nu\, V/r)u_\varphi = -\frac{e}{m}\,\frac{i\nu}{r}\,\Phi \quad ;$$

$$i(\omega + \nu\, V/r)u_z = \frac{e}{m}\, ik\Phi \quad .$$

The determinant D of this system is given by:

$$D = (\omega_c - \frac{V}{r})(\omega_c - \frac{\partial V}{\partial r}) - (\omega + \nu\, V/r)^2.$$

We can easily solve for the velocities u_r and u_φ:

$$D\,u_r = \frac{e}{m} i\, [(\omega + \nu \frac{V}{r})\Phi' - (\omega_c - \frac{V}{r})\frac{\nu}{r}\Phi],$$

$$D\,u_\varphi = \frac{e}{m} i\, [(\omega_c - \frac{\partial V}{\partial r})\Phi' - (\omega + \nu \frac{V}{r})\frac{\nu}{r}\Phi], \qquad (3.1)$$

$$u_z = \frac{e}{m} \frac{k}{\omega + \nu \frac{V}{r}} \Phi .$$

If $n^\circ(r)$ is a constant, D and all coefficients in Eq. (3.1) become constants and we obtain for the divergence of $\underline{u}$:

$$\nabla \cdot \underline{u} = \frac{e}{m} \frac{i(\omega + \nu\sigma)}{D} \left\{ \frac{1}{r}(r\Phi')' - \frac{\nu^2}{r^2}\Phi + \frac{k^2 D}{(\omega + \nu\sigma)^2}\Phi \right\} \qquad (3.2)$$

where D is now given by:

$$D = (\omega_c - \sigma)^2 - (\omega + \nu\sigma)^2 .$$

From the continuity equation, we derive the perturbed density:

$$n = -\frac{1}{i(\omega + \frac{\nu}{r} V)} \{\frac{\partial n^\circ}{\partial r} u_r + n^\circ \nabla \cdot \underline{u}\} \tag{3.3}$$

Poisson's equation can now be integrated for the case of constant density along r from $r = a - \epsilon$ to $a + \epsilon$, $\epsilon \ll 1$, and we obtain at once the boundary condition:

$$\frac{\partial \Phi_{out}}{\partial r} - \frac{\partial \Phi_{in}}{\partial r} = \frac{4\pi e n^\circ}{i(\omega + \nu\sigma)} u_r(r = a). \tag{3.4}$$

Φ_{out} and Φ_{in} refer to the potential outside or inside of the boundary. The only term which survives the limiting process $\epsilon \to 0$ is the term with n°, which degenerates into an δ-function. All other terms remain finite.

For the domain inside the boundary, we combine Eqs. (2.3), (3.3), and (3.2) to obtain:

$$\frac{1}{r} (r\Phi')' - \frac{\nu^2}{r^2} \Phi - \varkappa^2 \Phi = 0, \tag{3.5}$$

with $$\varkappa^2 = k^2 D\left(1 - \frac{\omega_p^{\,2}}{(\omega + \nu\sigma)^2}\right) (D + \omega_p^{\,2})^{-1} . \tag{3.6}$$

The only admissable solution is:

$$\Phi_{in} = cI_{\nu}\,(\kappa r). \tag{3.7}$$

$I_{\nu}(z)$ is the modified Bessel function of the first kind. For the vacuum region $a \le r \le R$, we assume for simplicity that R is infinite. Then the solution of $\nabla^2 \Phi_{out} = 0$ is given by:

$$\Phi_{out} = hK_{\nu}\,(kr),$$

where $K_{\nu}(z)$ is the modified Bessel function of second kind. The continuity of the potential requires:

$$cI_{\nu}\,(\kappa a) = hK_{\nu}\,(ka). \tag{3.8}$$

If we now divide the boundary condition (3.4) by Eq. (3.8), we obtain the desired dispersion relation:

$$ka\,\frac{d\,\ell n K_{\nu}(ka)}{d(ka)} = \kappa a\,\frac{d\,\ell n I_{\nu}(\kappa a)}{d(\kappa a)}\left(1 + \frac{\omega_p^2}{D}\right) - \frac{\omega_p^2}{D}\,\frac{\omega_c - \sigma}{\omega + \nu\sigma}\,\nu \quad ; \tag{3.9}$$

The simplicity of Eq. (3.9) contrasts the complexity of the dispersion equation found by Buneman et al.[4] This is due to the absence of slipping, in our case.

We try now to get some insight into the possible wave modes and instabilities of the dispersion equation just derived.

IV. INSTABILITIES

We shall make use of the asymptotic expressions for the logarithmic derivatives of the Bessel functions.

For $z \to 0$, we have:

$$\frac{d}{dz} \ln I_\nu(z) = \frac{\nu}{z} + \frac{1}{2(\nu + 1)} +,..; \quad A_\nu \equiv -\frac{d}{dz} \ln K_\nu(z) = +\frac{\nu}{z} +..; \qquad (4.1)$$

For $z \to \infty$,

$$\frac{d}{dt} \ln I_\nu(z) = 1 - \frac{1}{2z} +..; \quad A_\nu \equiv -\frac{d}{dz} \ln K_\nu(z) = +1 + \frac{1}{2z} +...; \qquad (4.2)$$

It is useful to introduce the following quantities:

$$y = \frac{\omega + \nu\sigma}{\omega_c - \sigma} \quad \text{and} \quad \hat{q} = \frac{\omega_p^2}{(\omega_c - \sigma)^2} = \frac{q}{(1 - \frac{1}{2} q)^2} . \qquad (4.3)$$

If ω has an imaginary part, so has y.

For $q \ll 1$, the cyclotron frequency is characterized by $y \approx 1$, the diocotron oscillation is mostly found at $y = O(\hat{q})$. Note that $q \ll 1$ implies $\hat{q} \approx q$.

With Eq. (4.3), we can write the dispersion Eq. (3.9) as:

$$-y(1 - y^2)A_\nu + \eta = \frac{\varkappa a}{ka} \frac{d\,\ell n I_\nu(\varkappa a)}{d(\varkappa a)} (1 + \hat{q} - y^2); \qquad ((4.4)$$

with $\varkappa^2 a^2 = k^2 a^2 \dfrac{(1 - y^2)(y^2 - \hat{q})}{(1 + \hat{q} - y^2)\, y^2}$, $A_\nu = - \dfrac{d\,\ell n K_\nu(ku)}{d(ka)} > 0$,

and $\eta = \hat{q}\ \nu/ak.$

Let's first consider the case, $k = 0$. Eq. (4.4) yields with Eq. (4.1) the solutions:

$$y_1 = 1; \quad y_{2/3} = -1/2 \pm 1/2\,(1 + 2\hat{q})^{1/2}. \qquad (4.5)$$

Alternatively, the 3 solutions can be written as:

$$\omega_1 / \omega_c = 1 - \frac{1}{2}(\nu + 1)q; \quad \omega_2 / \omega_c = -1 - \frac{1}{2}\,\nu q, \qquad (4.6)$$

and $\omega_1/\omega_c = \frac{q}{2}(1 - \nu), \quad (k = 0,\ \nu \pm 0).$

q is completely arbitrary. Clearly, the first two solutions are cyclotron oscillations, whereas the third is the diocotron branch. The diocotron mode has been considered for a wider class of density distributions by Timofeev[6] who showed that for monotonically decreasing distributions, this branch is always stable.

We specialize now for small $\hat{q}$, and allow (ak) to be different from 0, yet so small that

$$1 \gg \hat{q} \gg (ak)^2.$$

The result is that the solutions remain still stable. However, if (ak) grows further, so that we have

$$1 \gg (ak)^2 \gg \hat{q} \,, \tag{4.7}$$

a more interesting behavior develops.

If we insert two of the roots (4.5), namely,

$$y_2 = -1 - \frac{1}{2}\hat{q} \quad \text{and} \quad y_3 = \frac{1}{2}\hat{q} \quad \text{into} \quad \varkappa^2 a^2 \,,$$

we find that

$$|\varkappa^2 a^2| \approx \frac{k^2 a^2}{\hat{q}} \gg 1. \tag{4.8}$$

Thus, we have to use now the asymptotic form (4.2) for $\frac{d}{d(\varkappa a)} \ell n I_\nu(\varkappa a)$.

With the assumption (4.8), our dispersion assumes the form:

$$+y(y^2 - 1)A_\nu + \eta = \left[(y^2 - 1)(y^2 - \hat{q})(y^2 - 1 - \hat{q})\right]^{1/2} \qquad (4.9)$$

We consider now the diocotron branch and look for a solution $|y| \ll 1$. Eq. (4.9) can be written as:

$$y = \eta/A_\nu + y^3 \pm \frac{1}{A_\nu} \left[-(1 - y^2)(\hat{q} - y^2)(1 - y^2 + \hat{q})\right]^{1/2} . \qquad (4.10)$$

For $ka \ll 1$, we have $A_\nu = \nu/ka \gg 1$ and $\eta/A = \hat{q}$.

Eq. (4.10) is of the form $y = f(y)$, and we may find a solution by iteration, if $|f'(y)| < 1$ in the considered domain of y. For example, we have $|f'(\hat{q})| = O(q^{1/2}/A_\nu) < 1$.

In this way, we find:

$$y = \hat{q} \pm i \frac{ka}{\nu} \hat{q}^{1/2} \tag{4.11}$$

which is clearly unstable.

If we now check our assumption $|\varkappa^2 a^2| \gg 1$, it turns out that it is rigorously satisfied only if $(ka)^2/\nu^2 \ll \hat{q}$. Simultaneously, Eq. (4.7) must hold and, therefore, the relation $1 \ll (ka)^2/q \ll \nu^2$. This excludes $\nu = 1$ and 2 but we may expect that they, too, are unstable.

The opposite case is $ka \gg 1$, $A_\nu = 1$. We find from Eq. (4.10):

$$\eta^2/\hat{q} = (1 - y^2)\ (2y^2 + 2\frac{\eta}{q}\ y - 1 - \hat{q}).$$

The solutions $y = -\frac{1}{2}\ \frac{\nu}{ka}\ \pm \left(1/2 + 1/4\ (\frac{\nu}{ka})^2\right)^{1/2}$,

violate our assumption (4.8) as $(\varkappa a)^2$ becomes of order one and are erroneous. This means that we cannot explore this part of the diocotron mode with our simple approach, but have to apply more sophisticated means.

We turn our attention now to instabilities in the neighborhood of the cyclotron frequency. It is convenient to introduce:

$$y^2 = 1 + \epsilon, \ \epsilon \ll 1; \qquad y = \delta(1 + \tfrac{1}{2}\epsilon), \ \delta = \pm 1;$$

and to write Eq. (4.9) as an equation for ϵ, which we assume to be small. After squaring both sides, we obtain:

$$(A_\nu{}^2 - 1)\epsilon^2 + 2(\delta A_\nu \eta + \tfrac{1}{2}q)\epsilon + \eta^2 = 0. \qquad (4.12)$$

In deriving Eq. (4.12), we neglected terms of order ϵ and q compared to 1. From the solution

$$\epsilon = \frac{\delta A_\nu \eta + \frac{1}{2}q}{A_\nu{}^2 - 1} \pm \frac{1}{A_\nu{}^2 - 1}\left[y^2 + \frac{q^2}{4} + \delta A \eta q\right]^{1/2}, \qquad (4.13)$$

we see that one solution with $\delta = +1$ is always stable. The other is stable if:

$$A_\nu < \frac{\nu}{ka} + \frac{1}{4}\,\frac{ka}{\nu}\,. \tag{4.14}$$

Let's consider the case $ka \ll 1$. Eq. (4.1) for A_ν is now no longer sufficient to determine stability because the leading terms just cancel. We have to take into account more terms in the expension of A_ν for small argument (compare Ref. 9). For $\nu = 1$, we have:

$$A_1 = \frac{1}{ka}\left\{1 - (ka)^2 \ln\frac{ka}{2} - (ka)^2(\gamma + \tfrac{1}{2})\right\}, \tag{4.15}$$

where $\gamma = 0.577$ is Euler's constant. This yields:

$$ka < 0.532, \; (\nu = 1) \tag{4.16}$$

as condition for instability. Eq. (4.13) becomes:

$$\epsilon = q\left\{1 \pm i(ka)^2\left[-\ln\frac{ka}{2} - (\gamma + \tfrac{3}{4})\right]^{1/2}\right\}\,. \tag{4.17}$$

The upper bound for (ka) in (4.16) is already beyond the limit of validity of our expansion for A_1, because the second and third term in the bracket of (4.15) should be small compared to one.

For $\nu = 2$, we find:

$$A_2 = \frac{2}{ak}\left\{1+ \frac{1}{4}\,(ak)^2 + \frac{1}{8}\,(ak)^4 \;\ell n\,\frac{ak}{2} + \frac{\gamma}{8}\,(ak)^4.\right\}$$

The condition for instability becomes now:

$$(ka)^2 \,(\ell n\,\frac{ka}{2} \; + \gamma) > -\,\frac{3}{2}\,. \tag{4.18}$$

This inequality is satisfied for any ka. Thus, the $\nu = 2$ mode is also unstable for all small ak. Eq. (4.13) becomes for this case:

$$\epsilon = q\frac{ka}{2}\left\{1 \pm i\frac{\sqrt{3}}{2}\;\;\frac{ka}{2}\left[1 + \frac{3}{2}\,(ka)^2(\ell n\,\frac{ka}{2} + \gamma)\right]^{1/2}\right\}\,. \tag{4.19}$$

The opposite case, ka >> 1 can be treated for all modes simultaneously. Using (4.2), we obtain from (4.14):

$$2\nu - \sqrt{2\nu} < ka < 2\nu + \sqrt{2\nu}. \tag{4.20}$$

For ϵ, we obtain:

$$\epsilon = -\frac{1}{2}q\,\left(ka \pm i\sqrt{-(ka\;-\;2\nu)^2 + 2\nu}\right). \tag{4.21}$$

If $kaq \ll 1$, but $ka \gg 1$, all our assumptions are satisfied, including (4.8). This instability seems to be particularly dangerous because, being a short wave length instability, it cannot be stabilized by choosing a particular geometry. Because of the short wave length, finite Larmor radius effects may become important, but it cannot be said, at the present stage of the investigation, if these effects have a stabilizing or de-stabilizing tendency.

To conclude this paragraph, we list the most important formulae derived, in terms of frequencies, together with their assumptions.

Diocotron Mode:

$$q \ll (ka)^2 \ll 1 : \omega/\omega_c = q(1 - \tfrac{\nu}{2}) \pm i \frac{ka}{\nu} q^{1/2}. \quad (\nu \gg 1) \qquad (4.11a)$$

$ka \gg 1$: Assumption (4.8) violated, no results.

Cyclotron Mode:

$q \ll (ka)^2 \ll 1$

$$\nu = 1: \ \omega/\omega_c = - [1 + \tfrac{1}{2}q \pm i \ (ka)^2 \sqrt{-\ln \frac{ka}{2} - (\gamma + \tfrac{3}{4})}] \qquad (4.17a)$$

Instability for $ka < 0.53$.

$$\nu = 2: \quad \omega/\omega_c = -[1 + \frac{q}{2} \pm iq \frac{\sqrt{3}}{16} (ka)^2 \sqrt{1 + \frac{3}{2} (ka)^2 (\ln \frac{ka}{2} + \gamma)}] \qquad (4.19a)$$

Instability for any small (ka).

$ka \gg 1$:

$$\omega/\omega_c = [1 + \frac{q}{\nu} (\nu - 1 - \frac{1}{4} ka) \pm i \frac{1}{4} q \sqrt{2\nu - (ka - 2\nu)^2}]; \qquad (4.21a)$$

Instability for $2\nu - \sqrt{2\nu} < ka < 2\nu + \sqrt{2\nu}$.

V. DISCUSSION

The results just quoted are quite discomforting in view of the proposed Heavy Ion Plasma Accelerator. In their basic paper, James, Levy, Bethe, and Feld quote the results of a corresponding calculation in straight geometry. One might conclude that similar results hold order of magnetude-wise also for the HIPAC geometry. The above calculation has shown that there is a substantial difference. This is readily seen if we compare the various growth rates.

For the diocotron instability, Buneman et al.[4] find in the long wavelength limit:

$$\mathrm{Im}\ \omega = ka \frac{\omega_p{}^2}{\omega_c} \quad ; \qquad ka \ll 1.$$

(Their equation 5.5)

From our Eq. (4.11a), we obtain:

$$\mathrm{Im}\ \omega = \frac{ka}{\nu}\, \omega_p$$

It has to be kept in mind that in these two formulae the same symbols mean different things. In the straight geometry, k is the wavelength along the y-axis; in cylindrical geometry, it is along the z-axis. a is the half-width of the plasma slab in straight geometry; here, it is the plasma radius. Actually, k in slab geometry resembles more our ν, but we have seen in our analysis that disturbances with $\nu = 0$ are all stable.

The cyclotron instability in slab geometry has a growth rate:

$$\mathrm{Im}\ \omega = \frac{\pi}{2e}\ \omega_c q e^{-\frac{2}{q}} \qquad k\, a \gg \frac{1}{2q}\ .$$

(Eq. (9.10) in Ref. 4)

The corresponding equation would be (4.21a) with

$$\mathrm{Im}\ \omega = \frac{1}{4}\ \omega_c q \sqrt{2\nu - (ka - 2\nu)^2}\ .$$

Whereas the first growth rate becomes insignificantly small, as $q \to 0$, the second remains appreciable. For example, for $n_e \sim 10^{10} cm^{-3}$, $q = 10^{-2}$, the e-folding times are of the order of microseconds or shorter.

What is the physical reason for this striking difference? The cyclotron instability is due to a resonance of the Doppler-shifted frequency of the wave with the electron gyrofrequency. In straight geometry, we have necessarily a "slip", i.e., the drift velocity of the electron beam varies from point to point. Therefore, a resonance may occur only in a very thin layer of the beam, all the rest is not quite in resonance. In our analysis, however, we considered a plasma which was rotating like a rigid column. If our wave of the form $\exp[i(\omega t + \nu\varphi + kz)]$ is in resonance at one layer, it is in resonance with the whole column. It is quite natural that we expect a stronger instability. It can be seen easily from (4.17a) and (4.19a) that the real frequency of the wave is just the gyration frequency of the electrons plus the rotation frequency of the plasma.

How well our assumption of a rigidly rotating plasma can be realized in an experiment remains to be seen. It may be that experimentally we always have a gently decreasing density distribution such that not the plasma as a whole, but only a thin layer, can resonate. Under these circumstances, we expect slow growth rates comparable to those found by Buneman.

This instability might flatten the density profile which then gives rise to a more violent instability in the flattened area. This in turn broadens the density plateau further. Thus, the effective decay time still might be quite short.

A stabilizing effect might be introduced by taking into account conducting walls. The radius R of Fig. (1b) would then be finite, whereas in our analysis we had assumed $R = \infty$.

Also, finite temperature should be taken into account by introducing a pressure term in Eq. (2.2) and adding an equation of state. This increases the degrees of freedom of our system, and the effect is expected to be de-stabilizing rather than stabilizing.

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FIGURE CAPTIONS

Figure 1a	Carthesian slab geometry as considered by Buneman, Levy, and Linson.
Figure 1b	Cylindrical geometry, which simulates the toroidal HIPAG - geometry. The cylindrical geometry has been considered in this work.

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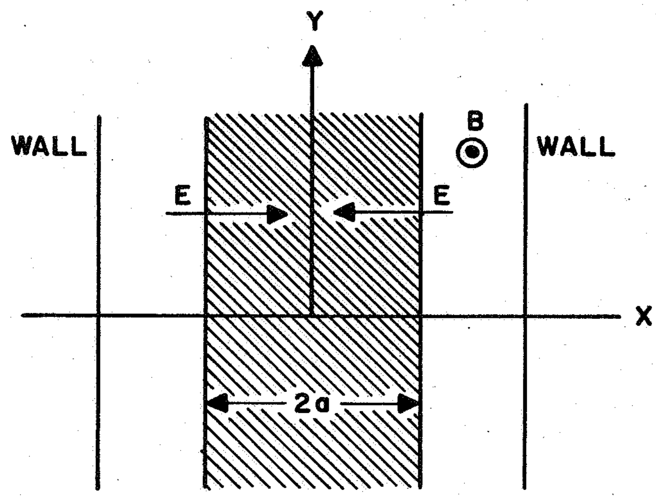

FIG. 1a

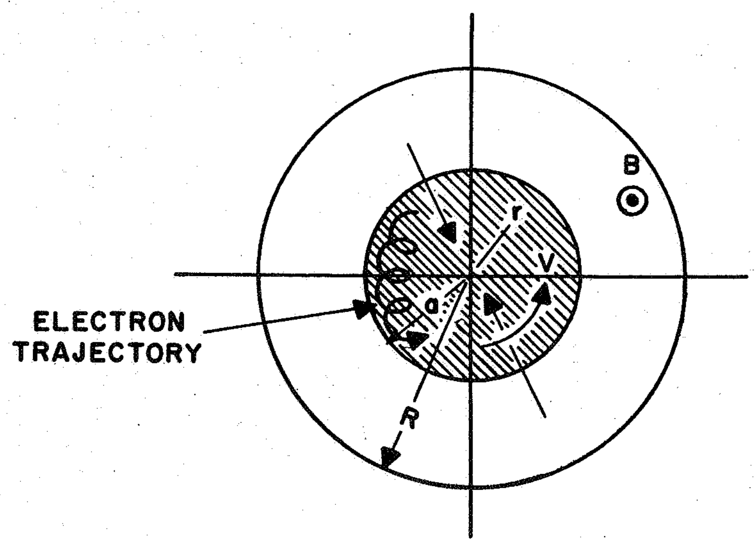

FIG. 1b

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