DEPARTMENT OF ELECTRICAL ENGINEERING

CONTRACT NAS8-20399

TECHNICAL REPORT

AN INVESTIGATION OF THE DECOUPLING EFFECTS IN A MAGNETIC FORMING BERYLLIUM COIL ASSEMBLY

ENGINEERING AND INDUSTRIAL RESEARCH STATION

MISSISSIPPI STATE UNIVERSITY

STATE COLLEGE, MISSISSIPPI

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and M. E. Davis

June, 1968

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ABSTRACT

A beryllium coil assembly was developed by the National Aeronautics and Space Administration, George C. Marshall Space Flight Center in Huntsville, Alabama. The primary purpose of the hammer coil is to form metallic materials and reduce fatigue and work hardening.

This study was made to determine the force exerted by the hammer coil on a plate as it becomes decoupled from the coil.

The mutual coupling between the hammer coil and plate as a function of distance of separation was determined. Differential equations were derived which described the currents in a two ring hammer coil and plate circuit. The solution of these equations was determined by the analog computer. With the currents determined, the force of repulsion between the hammer coil and plate was

determined. The induced current reached a maximum value of $(7)10^3$ amperes in the smallest ring, while a value of $(5.66)10^3$ amperes was reached in the second ring. This yielded a force of repulsion of approximately 27 10 newtons/meter on the smallest ring.

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CHAPTER I

INTRODUCTION

The beryllium coil assembly or the hammer coil is a device used at the George C. Marshall Space Flight Center in Huntsville, Alabama, for shaping and smoothing metal structures. The coil possesses the ability to minimize work hardening and structural fatigue by the use of intense magnetic fields produced by the discharge from a capacitor bank. The coil utilizes the principle of a large current changing rapidly with respect to time to create a time-varying magnetic field. Thus, a current is induced into a metallic plate placed in the field of the coil. This current causes an opposing magnetic field which results in a repulsive force between the hammer coil and the sheet of metal called the plate.

In previous studies, the hammer coil's forces and currents have been studied from the standpoint of a stationary plate against the hammer coil. However, in reality the plate is moving away from the coil as a function of time. Hence, the currents induced into the metal plate and the repulsive force between the coil and plate will be a function of time.

In this study, the hammer coil and plate current will be determined as the plate becomes decoupled from the coil for a two ring coil. Again, concentric rings will be used to approximate the spiral coil in the hammer coil. Concentric rings with the same radii as the rings in the hammer coil will be assumed in the plate. The capacitance will be considered negligible in this study due to the results of previous studies.[1] Thus, a circuit of resistance and inductance will represent the hammer coil and plate. From this circuit, a reaction between the hammer coil and plate will be explained by equations utilizing mutual coupling as a function of distance. Thus, a voltage is induced in the plate due to the space rate of change of mutual coupling. These equations will have current as the dependent variable and time as the independent variable. Next, the solution of these equations will be obtained from the analog computer. With the time varying currents determined, the equation[2]

$$F = \frac{u_0 i i_r l_r}{2\pi R} \tag{1}$$

can be used to determine the force between the hammer coil and the two rings in the plate as a function of time, where u_0 is the permeability of air, i is the current in the hammer coil, i_r is the current in ring r of the plate, l_r is the circumference of ring r in the plate, and R is the distance of separation between the hammer coil and plate.

CHAPTER II

PROCEDURE

A. DESCRIPTION OF HAMMER COIL AND PLATE

The hammer coil, which consists of a spiral shaped coil, is assumed to be composed of sixteen concentric rings. Likewise, the plate consists of sixteen concentric rings with identical radii of the rings in the hammer coil. This number was chosen to represent the sixteen revolutions of the coil. Figure 1 illustrates the circuit of the coil and plate for a two ring beryllium coil assembly. This diagram could be extended to the case of sixteen concentric rings, but this study is concerned only with the two smallest rings of the hammer coil. The circuit is made up of a primary and secondary. The primary consists of a voltage source, resistor, and inductors. The voltage source represents the capacitor bank; the resistor represents the resistance of the two coils; and the inductors represent the inductance of each coil turn. The secondary is composed of the same elements. This is analogous to a simple transformer, except the secondary is allowed to move as a function of time. Thus, a time varying mutual coupling between the primary and secondary exists.

B. MUTUAL INDUCTANCE OR COUPLING OF HAMMER COIL AND PLATE

Figure 2 shows two concentric rings of radius "a" and radius "b" separated by a distance "d". The equation expressing the

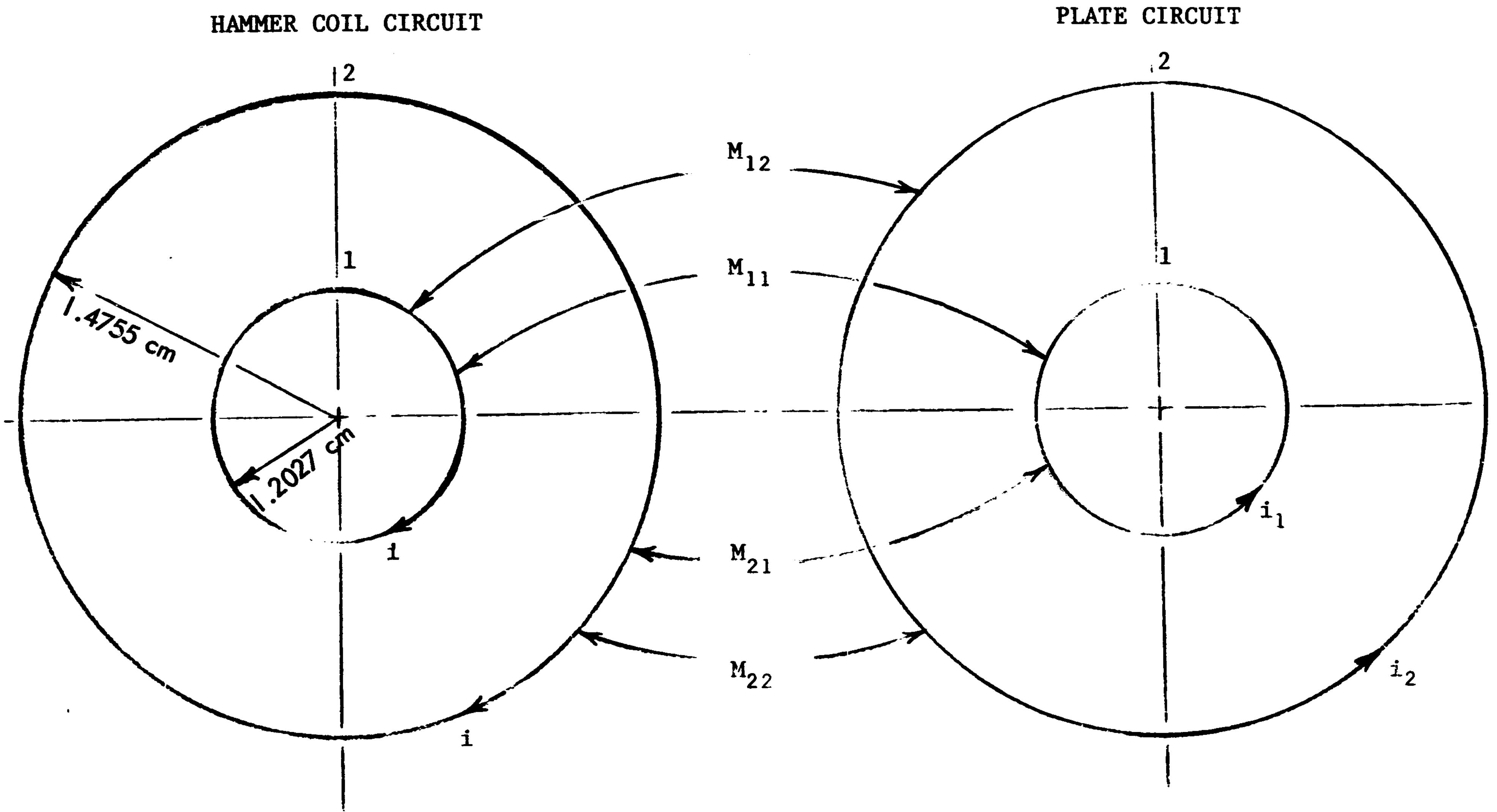


Figure 1. The Representation of the Mutual Inductance and Currents Associated with the Two Smallest Rings of the Hammer Coil and the Plate.

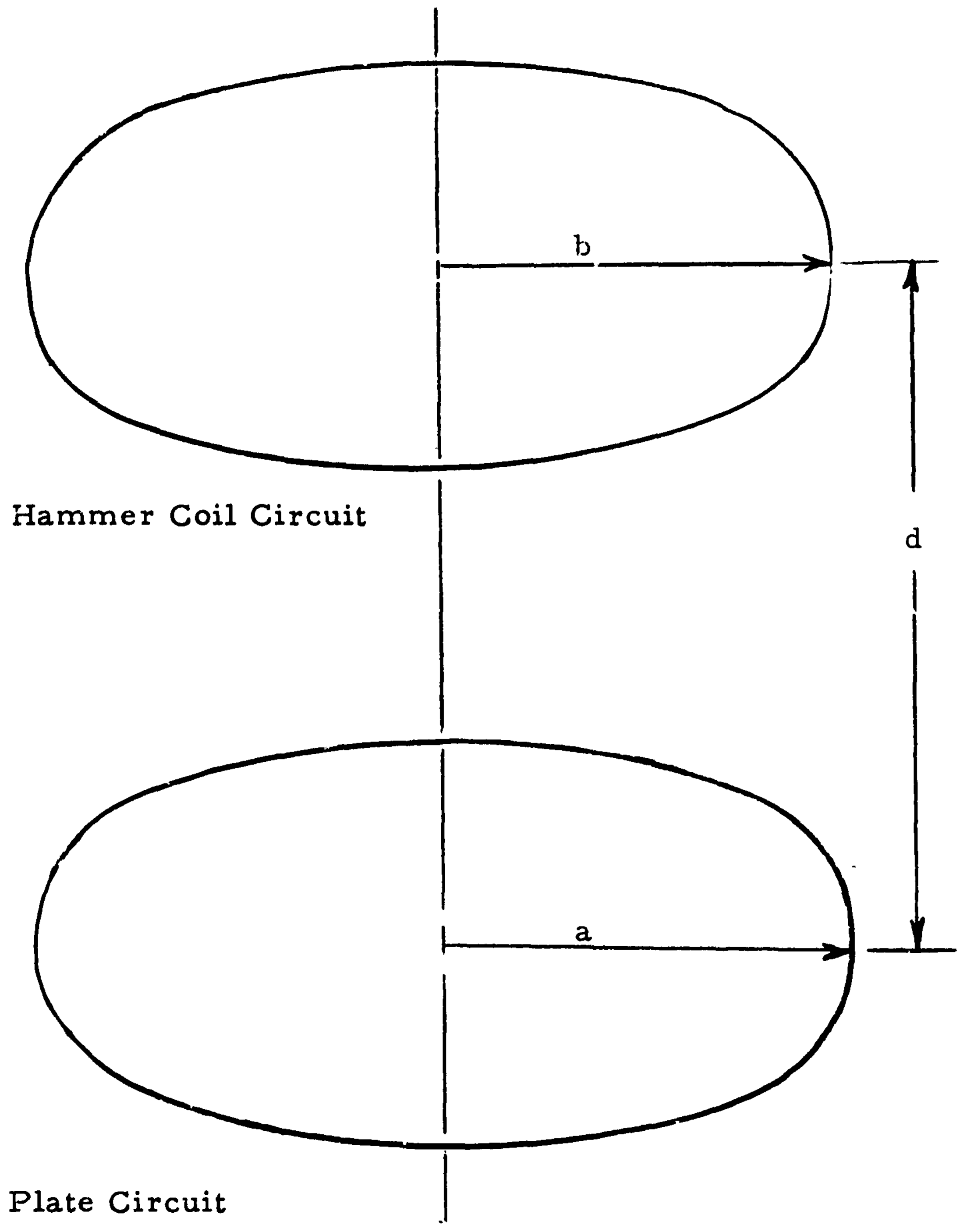


Figure 2. Diagram of Circuit Used for Calculation of Mutual Coupling Between Hammer Coil and Plate.

mutual inductance between the two rings is known as the Neumann form[3] and is given by

$$M = u\sqrt{d^2 + (a+b)^2}\left[(1 - \frac{k^2}{2})K(k) - E(k)\right], \quad (2)$$

where

$$k = 2\sqrt{\frac{ab}{d^2+(a+b)^2}}, \quad (3)$$

and

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\phi}\, d\phi , \quad (4)$$

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}. \quad (5)$$

Equations (4) and (5) are known as the complete elliptic integrals of the first and second kinds, respectively. For a specific value of k calculated from Equation (3), the value of the elliptic integrals are found in mathematical tables.[4] Thus, direct substitution into Equation (2) gives any combination of the mutual coupling between ring "b" in the hammer coil and ring "a" in the plate as a function of distance of separation "d".

Since Equation (2) has no stipulations on the distance of separation, the constant mutual coupling between rings can be calculated by setting $d = 0$. This will put ring "a" inside ring "b" in Figure 2. The constant mutual coupling between rings is denoted as M'_{mn}, where m is the ring in the coil, and n is the ring in the plate.

C. DIFFERENTIAL EQUATIONS THAT DESCRIBE THE CURRENTS IN THE HAMMER COIL AND PLATE

By application of Kirchhoff's laws to the loops in Figure 3, the differential equations relating the currents in the hammer coil and plate can be determined. Thus, three equations with three unknown

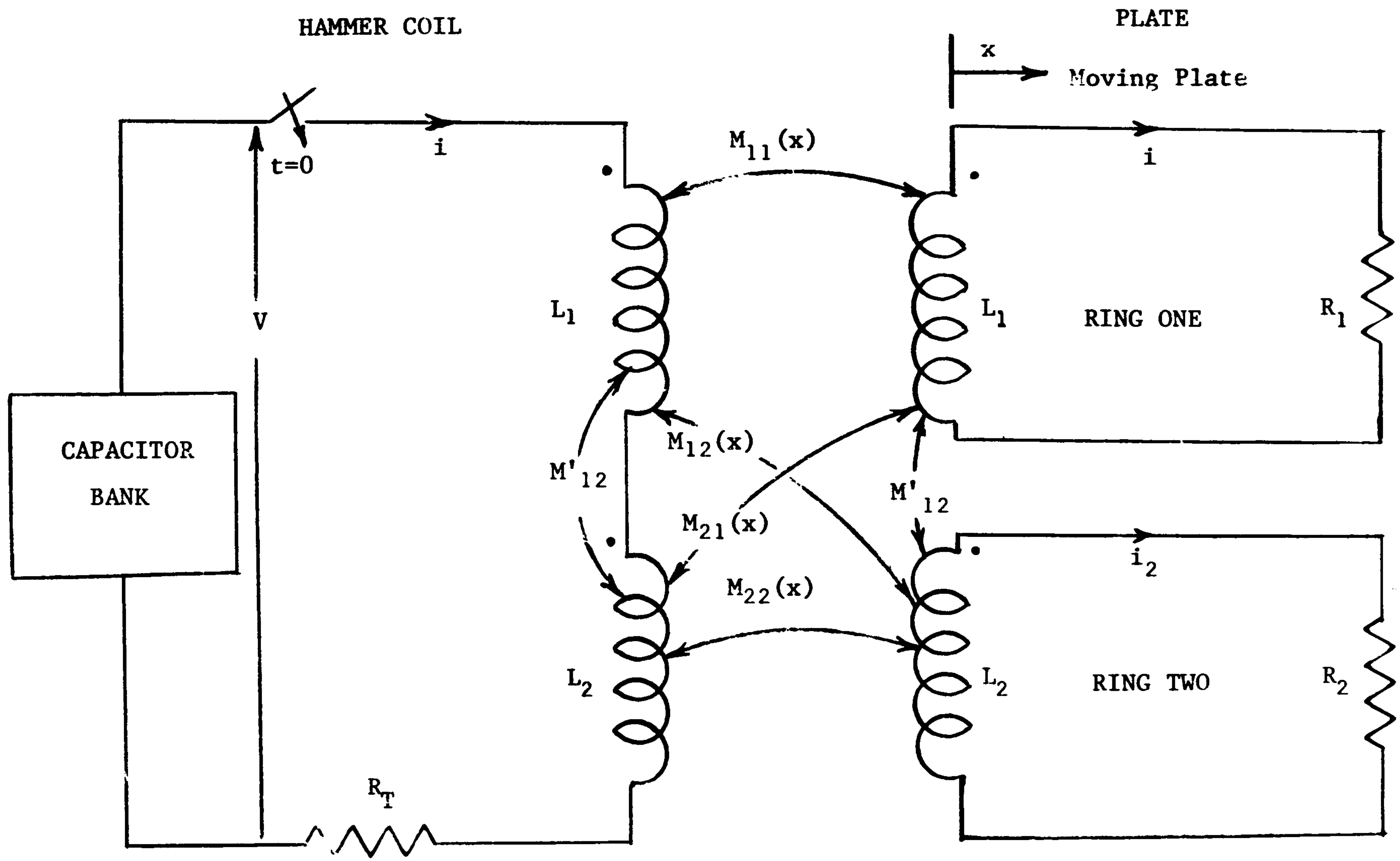


Figure 3. Circuit used to Write Differential Equations that Describe the Hammer Coil and Movable Plate Assembly.

currents will result. As will be seen later in the text, the differential equations will have time-varying coefficients due to the movement of the secondary or plate.

D. SOLUTION OF DIFFERENTIAL EQUATIONS THAT DESCRIBE HAMMER COIL

The analog computer which is located in Patterson Engineering Laboratories at Mississippi State University is used in the solution of the previously mentioned differential equations. Time and magnitude scaling will be applied to the three equations, and the time varying coefficients in the equations will be generated by the analog computer. Electronic multipliers will be used to multiply the generated function by the dependent variables.

E. FORCE OF REPULSION BETWEEN HAMMER COIL AND PLATE

The force between two conductors is repulsive if the currents are in opposite directions. The equation is given by

$$F = \frac{u_0 i i_r l_r}{2\pi R}, \tag{6}$$

where F is the force in newtons on length l_r of the conductor. In this equation, R, i, and i_r are functions of time. Thus, the force F is a function of time.

CHAPTER 111

APPLICATION OF PROCEDURE

A. DETERMINATION OF CIRCUIT PARAMETERS

From Equations (2) through (5), the mutual coupling between any ring in the hammer coil and any ring in the plate can be calculated. As an example, the mutual coupling $M_{12}(x)$ between ring one in the hammer coil and ring two in the plate will be calculated for a distance of seperation of 0.1 centimeters. From Equation (2),

$$k = 2\sqrt{\frac{ab}{d^2+(a+b)^2}} \tag{7}$$

where k is the constant used to calculate the elliptic integrals of Equations (4) and (5). a= radius of ring 1 in plate = 1. 2027 centimeters, and b = radius of ring 2 in the hammer coil = 1. 4755 centimeters, and d = distance of seperation = o. 1 centimeters. Thus,

$$k = 2\sqrt{\frac{(1.2027)(1.4755)}{(.1)^2 + (1.2027 + 1.4755)^2}}$$

$$k = 0.9941.$$

Appendix A gives the computer program used in calculating the value of k for any combination of a, b, and d.

For this value of k, the elliptic integrals in Equations (4) and (5) can be calculated from a handbook of tabulated values.[5] Hence, E(k) = E(.9941) = 1.016 and K(.9941) = 3.62. From Equation (2), the mutual coupling $M_{12}(x)$ for d = 0.1 is calculated as follows:

$$M_{mn}(x) = u \sqrt{d^2 + (a+b)^2} \left[(1 - \frac{k^2}{2})K(k) - E(k) \right] \qquad (8)$$

Since both rings are assumed to be in air, $u = u_o =$ permeability of air $= 4\pi(10^{-9})$ henrys/centimeter.

$$M_{12}(.1) = 4\pi(10^{-9}) \sqrt{(.1)^2 + (1.2027 + 1.4755)^2} \left[(1 - \frac{.9941}{2})3.62 - 1.016 \right]$$

$$M_{12}(.1) = (2.7459)10^{-8} \text{ henrys}$$

It should be noted that $M_{12}(x) = M_{21}(x)$, or in general $M_{mn}(x) = M_{nm}(x)$. Table 1 on page 24 is a tabulation of values of mutual coupling between ring one in the coil and all sixteen rings in the plate for a distance of 1, 2, 3, and 4 centimeters. Table 2 on page 25 gives the values of mutual coupling between the coil and plate for all possible combinations with distances of separation of .1, .5, 1.0, 2.0, 3.0, and 4.0 centimeters. The computer program used in calculating the mutual coupling is shown in Appendix B. Plots of mutual coupling versus the distance of separation for the three smallest rings in the hammer coil and plate are shown in Figures C-1 through C-6 of Appendix C.

Next, the constant mutual coupling M'_{mn} between concentric rings within the coil will be considered. Equations (2) through (5) can again be used with the exception that d = 0. Table 3 gives a tabulation of the computed values of the constant mutual coupling between the three smallest concentric rings in the hammer coil.

The self inductance in each ring of the hammer coil has been determined in earlier studies.[6] A list of these values are shown in Table 4.

Finally, the values of resistance are considered. Since the remainder of this study is concerned only with the two smallest rings of the hammer coil and plate, the values of resistance will not be tabulated. From previous studies[7] the resistance of ring one in the hammer coil is approximately $(2)10^{-4}$ ohms and $(3)10^{-4}$ ohms for ring two. Since the resistance of the plate may vary from one material to another, a conservative value of resistance of ring one and two in the plate is approximately $(3)10^{-4}$ ohms and $(9)10^{-4}$ ohms, respectively.

B. DETERMINATION OF DIFFERENTIAL EQUATIONS

The hammer coil and plate circuit is represented by the circuit in Figure 3. From this circuit there are four space rates of change of mutual inductance, M_{11}, M_{12}, M_{21}, and M_{22}, where $M_{21} = M_{12}$. The constant mutual coupling between rings is denoted as M'_{12} for both the hammer coil and plate.

The voltage drop due to mutual coupling is given by $\pm \frac{d}{dt}(Mi)$.[8] In the case of a stationary secondary or plate, the mutual inductance is constant, and the drop is represented as $\pm M\frac{di}{dt}$. However, in this study, the secondary or plate is moving with a velocity. Therefore,

$$\frac{d}{dt}(Mi) = i(t)\frac{dM(t)}{dt} + M(t)\frac{di(t)}{dt}. \qquad (9)$$

The first term on the right of Equation (9) may be written as

$$i(t)\frac{dM(t)}{dt} = i(t)\frac{dM(x)}{dx}\frac{dx}{dt}. \qquad (10)$$

Equation (9) now becomes:

$$\frac{d}{dt}(Mi) = i(t)\frac{dM(x)}{dx}\frac{dx}{dt} + M(t)\frac{di(t)}{dt}. \quad (11)$$

The expression $\frac{dx}{dt}$ which appears in Equation (11), is the velocity with which the plate moves away from the hammer coil due to the repulsive force between them. From earlier studies, a peak velocity of 2000 meters per second occurs. Therefore, this study will assume a linear displacement as a function of time or $x = (2)10^5 t$ centimeters, (12)

and $\frac{dx}{dt} = (2)10^5$ centimeters/second. (13)

From Figure 3, the voltage equation around the hammer coil can be written as

$$V = R_T i + (L_1 + L_2)\frac{di}{dt} + 2M'_{12}\frac{di}{dt} - M_{11}(t)\frac{di_1}{dt} + i_1\frac{dM_{11}}{dx}\frac{dx}{dt} - M_{12}(t)\frac{di_2}{dt} + i_2\frac{dM_{12}}{dx}\frac{dx}{dt} - M_{21}(t)\frac{di_1}{dt} + i_1\frac{dM_{21}}{dx}\frac{dx}{dt} - M_{22}(t)\frac{di_2}{dt} + i_2\frac{dM_{22}}{dx}\frac{dx}{dt}. \quad (14)$$

The equation for ring one in the plate is written as follows:

$$0 = R_1 i_1 + L_1\frac{di_1}{dt} + M'_{12}\frac{di_2}{dt} - M_{11}(t)\frac{di}{dt} + i\frac{dM_{11}}{dx}\frac{dx}{dt} - M_{21}(t)\frac{di}{dt} + i\frac{dM_{21}}{dx}\frac{dx}{dt}. \quad (15)$$

Next, the equation for ring two in the plate is

$$0 = R_2 i_2 + L_2\frac{di_2}{dt} + M'_{12}\frac{di_1}{dt} - M_{12}(t)\frac{di}{dt} + i\frac{dM_{12}}{dx}\frac{dx}{dt} - M_{22}(t)\frac{di}{dt} + i\frac{dM_{22}}{dx}\frac{dx}{dt}. \quad (16)$$

The terms of the form $M(t)\frac{di}{dt}$ in Equations (14) through (16) are all negative. Since a repulsive force exists between the hammer coil and plate, the currents in the hammer coil must be in opposition to those in the plate. Thus, an opposing flux field is present which makes the mutual terms negative. Since the slope $\frac{dM}{dx}$ is negative, these terms have a plus sign in order to represent the circuit with opposing flux linkage.

By combining terms and letting $L_m = L_1 + L_2 + 2M'_{12}$, Equations (14) through (16) become:

$$V = R_T i + L_m \frac{di}{dt} + \frac{d}{dx}(M_{11} + M_{21}) \frac{dx}{dt} i_1 - (M_{11} + M_{21}) \frac{di_1}{dt}$$
$$+ \frac{d}{dx}(M_{12} + M_{22}) \frac{dx}{dt} i_2 - (M_{12} + M_{22}) \frac{di_2}{dt}. \qquad (17)$$

Ring 1 in plate: $0 = R_1 i_1 + L_1 \frac{di_1}{dt} + M'_{12} \frac{di_2}{dt} + \frac{d}{dx}(M_{11} + M_{21}) \frac{dx}{dt} i$

$$- (M_{11} + M_{21}) \frac{di}{dt}. \qquad (18)$$

Ring 2 in plate: $0 = R_2 i_2 + L_2 \frac{di_2}{dt} + M'_{12} \frac{di_1}{dt} + \frac{d}{dx}(M_{12} + M_{22}) \frac{dx}{dt} i$

$$- (M_{12} + M_{22}) \frac{di}{dt}. \qquad (19)$$

In Equations (17) through (19), the primed mutual terms are the only mutuals that are constant.

The terms $(M_{11} + M_{12})$ and $(M_{12} + M_{22})$ that appear in these equations are approximated by their um at each distance of separation. Thus, by use of Table 2, these terms are calculated and tabulated in Table 5. A plot of this combined mutual coupling versus distance of separation is shown in Figures C-7 and C-8 of Appendix C, along with a decaying exponential curve which approximates the actual curve.

Next, from the tables and plots previously mentioned the actual values for the terms in Equations (17), (18), and (19) are substituted. Since the voltage in the hammer coil is assumed constant during the time the plate is under the influence of the force, V = 2000 volts will be used in this study.

Hammer coil: $$2000 = (5)10^{-4}i + (.5)10^{-6}\frac{di}{dt} + \frac{d}{dx}(6.6(10^{-8})e^{-1.5x})2\cdot10^{5}i_1 - (6.6(10^{-8})e^{-1.5x})\frac{di_1}{dt} + \frac{d}{dx}(8(10^{-8})e^{-1.6x})2(10^{5})i_2 - (8(10^{-8})e^{-1.6x})\frac{di_2}{dt}. \quad (20)$$

Ring 1 in plate: $$0 = (3)10^{-4}i_1 + (.2)10^{-6}\frac{di_1}{dt} + (2.8)10^{-8}\frac{di_2}{dt} + \frac{d}{dx}(6.6(10^{-8})e^{-1.5x})2(10^{5})i - (6.6(10^{-8})e^{-1.5x})\frac{di}{dt}. \quad (21)$$

Ring 2 in plate: $$0 = (9)10^{-4}i_2 + (.3)10^{-6}\frac{di_2}{dt} + (2.8)10^{-8}\frac{di_1}{dt} + \frac{d}{dx}(8(10^{-8})e^{-1.6x})2(10^{5})i - (8(10^{-8})e^{-1.6x})\frac{di}{dt}. \quad (22)$$

Finally, substituting Equation (12) into Equations (20), (21), and (22) and taking the derivative yields:

Hammer coil: $$2000 = (5)10^{-4}i + (.5)10^{-6}\frac{di}{dt} - (19.8)10^{-3}e^{-3(10^{5})t}i_1 - (6.6)10^{-8}e^{-3(10^{5})t}\frac{di_1}{dt} - (25.6)10^{-3}e^{-3.2(10^{5})t}i_2 - (8)10^{-8}e^{-3.2(10^{5})t}\frac{di_2}{dt}. \quad (23)$$

Ring 1 in plate: $$0 = (3)10^{-4}i_1 + (.2)10^{-6}\frac{di_1}{dt} + (2.8)10^{-8}\frac{di_2}{dt} - (19.8)10^{-3}e^{-3(10^{5})t}i - (6.6)10^{-8}e^{-3(10^{5})t}\frac{di}{dt}. \quad (24)$$

Ring 2 in plate: $$0 = (9)10^{-4}i_2 + (.3)10^{-6}\frac{di_2}{dt} + (2.8)10^{-8}\frac{di_1}{dt} - (25.6)10^{-3}e^{-3.2(10^{5})t}i - (8)10^{-8}e^{-3.2(10^{5})t}\frac{di}{dt}. \quad (25)$$

C. DETERMINATION OF CURRENTS

The method used for solving for the three currents in Equations

(23), (24), and (25) for this study is the analog computer. These three equations must first be solved in terms of the highest derivative. Since the self inductive terms are constant and contain the highest order derivative, the equations are set as follows:

Hammer coil: $(.5)10^{-6}\frac{di}{dt} = 2000 - (5)10^{-4}i + (19.8)10^{-3}e^{-3(10^5)t}\, i_1$

$+ (6.6)10^{-8}e^{-3(10^5)t}\frac{di_1}{dt} + (25.6)10^{-3}e^{-3.2(10^5)t}\, i_2$

$+ (8)10^{-8}e^{-3.2(10^5)t}\frac{di_2}{dt}.$ (26)

Ring 1 in plate: $(.2)10^{-6}\frac{di_1}{dt} = -(3)10^{-4}\, i_1 - (2.8)10^{-8}\frac{di_2}{dt}$

$+ (19.8)10^{-3}e^{-3(10^5)t}\, i + (6.6)10^{-8}e^{-3(10^5)t}\frac{di}{dt}.$ (27)

Ring 2 in plate: $(.3)10^{-6}\frac{di_2}{dt} = -(9)10^{-4}\, i_2 - (2.8)10^{-8}\frac{di_1}{dt}$

$+ (25.6)10^{-3}e^{-3.2(10^5)t}\, i + (8)10^{-8}e^{-3.2(10^5)t}\frac{di}{dt}.$ (28)

Now, the equations are considered for time scaling since time scaling should preceed magnitude scaling.[9] To accomplish a time scale change it is only necessary to make the following substitution for the independent variable:

$$t = \frac{T}{K} \quad (29)$$

If K is greater than unity, the solution is slowed by a factor of K. If K is less than unity, the problem is speeded up by a factor of $\frac{1}{K}$. Therefore, the original equations can be time scaled by the following substitutions:

$$dt = \frac{dT}{K}, \quad (30)$$

$$\text{and} \quad \frac{d}{dt} = \frac{d}{dT/K} = K\frac{d}{dT}, \tag{31}$$

$$\text{or in general} \quad \frac{d^n}{dt^n} = K^n \frac{d^n}{dT^n}, \tag{32}$$

where n is the order of the derivative.[10]

From Equations (27) and (28) it is obvious that the plate circuit is coupled to the hammer coil circuit only by the terms containing the decaying exponentials. In order to bring this rapid response within the bandwidth of the computer, a time scale of $K = 10^5$ will be used to slow the problem down. Thus, the following substitutions will be made in Equations (26), (27), and (28).

$$t = 10^{-5}T \tag{33}$$

$$\text{and} \quad \frac{d}{dt} = 10^5 \frac{d}{dT}. \tag{34}$$

Substituting Equations (33) and (34) into Equations (26), (27), and (28) gives:

Hammer coil: $(5)10^{-2}\frac{di}{dT} = -(5)10^{-4}\, i + (19.8)10^{-3}e^{-3T}\, i_1$

$$+(6.6)10^{-3}e^{-3T}\frac{di_1}{dT} + (25.6)10^{-3.2T}\, i_2 + (8)10^{-3}e^{-3.2T}\frac{di_2}{dT}. \tag{35}$$

Ring 1 in plate: $(2)10^{-2}\frac{di_1}{dT} = -(3)10^{-4}\, i_1 -(2.8)10^{-3}\frac{di_2}{dT}$

$$+(19.8)10^{-3}e^{-3T}\, i + (6.6)10^{-3}e^{-3T}\frac{di}{dT}. \tag{36}$$

Ring 2 in plate: $(3)10^{-2}\frac{di_2}{dT} = -(9)10^{-4}\, i_2 - (2.8)10^{-3}\frac{di_2}{dT}$

$$+(25.6)10^{-3}e^{-3.2T}\, i + (8)10^{-3}e^{-3.2T}\frac{di}{dT}. \tag{37}$$

Next, the previous equations are considered for magnitude scaling. Magnitude scaling is a choice between the problem variables and the computer variables. In order to assure that the voltages appearing at the outputs of the amplifiers will fall within the desired range, the scale factors are chosen as follows:

1 volt = 1 ampere for i, i_1, and i_2.

1 volt = 1 ampere/second for $\frac{di}{dT}$, $\frac{di_1}{dT}$, and $\frac{di_2}{dT}$.

To assure that the voltages at the outputs of the amplifiers are above the noise level and below the saturation level of the amplifiers, Equations (35), (36), and (37) are multiplied by 10^{-2}. Also, in order to preserve amplifiers, Equations (36) and (37) are multiplied by -1. For simplicity the following substitutions will be made:

$$X = 10^{-4}\, i$$
$$X_1 = 10^{-4}\, i_1 \qquad (38)$$
$$X_2 = 10^{-4}\, i_2.$$

With previously discussed changes, Equations (35), (36), and (37) become:

Hammer coil: $5\dot{X} = 20 - .05X + 1.98e^{-3T}\, \dot{X}_1 + .66e^{-3T}\, X_1 + 2.56e^{-3.2T}\, \dot{X}_2 + .8e^{-3.2T}\, X_2.$ (39)

Ring 1 in plate: $-2\,\dot{X}_1 = .03\, X_1 + .28\, X_2 - 1.98e^{-3T}\, \dot{X} - .66e^{-3T}\, X.$ (40)

Ring 2 in plate: $-3\,\dot{X}_2 = .09\, X_2 + .28\, X_1 - 2.56e^{-3.2T}\, \dot{X} - .8e^{-3.2T}\, X.$ (41)

Equations (39), (40), and (41) are set up on the analog computer by the diagram shown in Figure 4. The decaying exponentials that represent the time-varying mutual coupling are generated as shown in Figure 5.[11] "A" is set equal to 100 volts in Figure 5 in order to give a maximum of 100 volts to the input to the electronic multipliers. It should be remembered that the multipliers divide the product of the

Note: $+100e_1 = 100e^{-3t}$

and $+100e_2 = 100e^{-3.2t}$

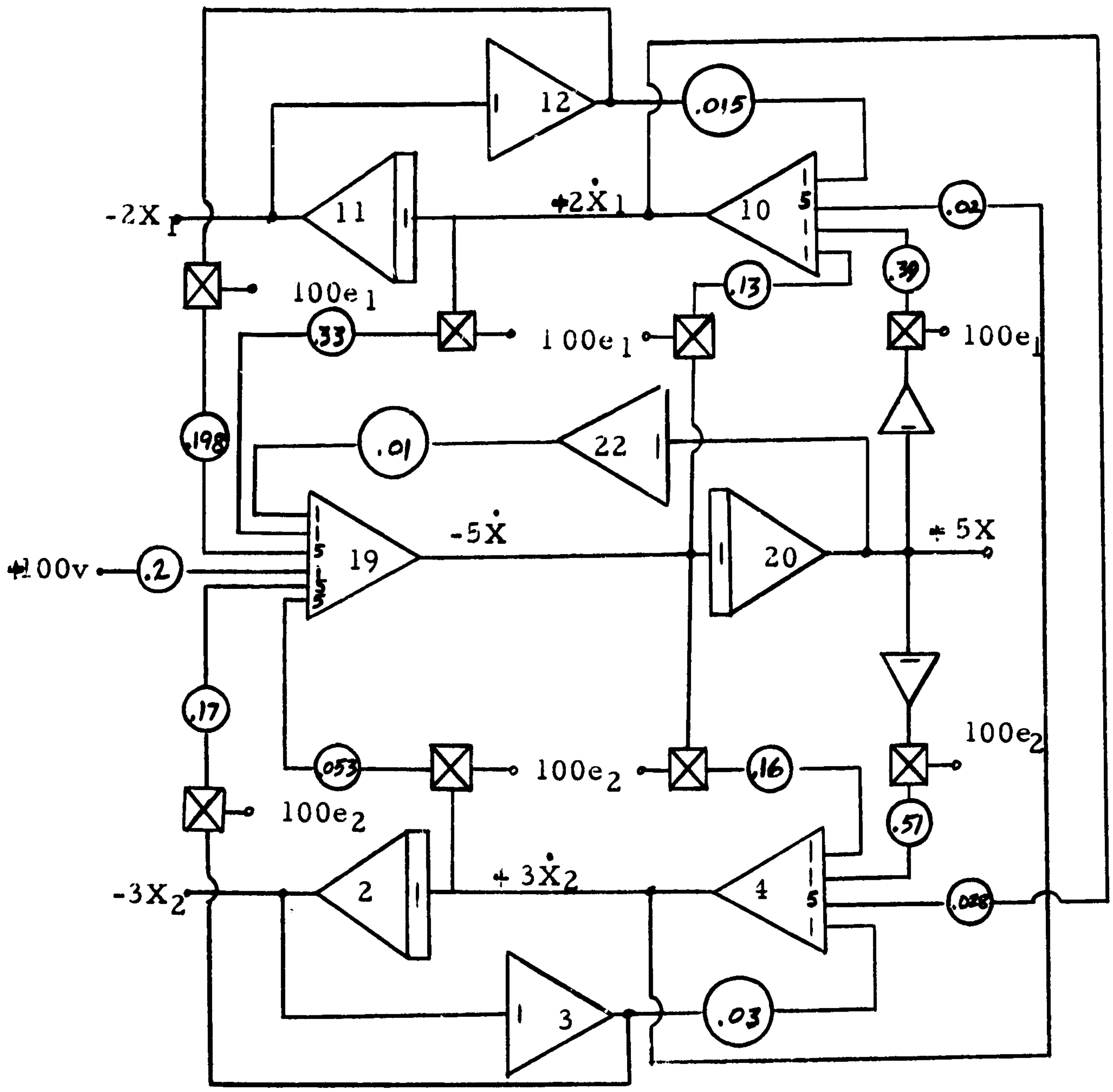


Figure 4. Computer Diagram Used to Solve the Differential Equations that Describe the Hammer Coil and Plate Assembly.

$$e_o = Ae^{-10kt}$$

$$\frac{de_o}{dt} = -10kAe^{-10kt} = -10ke_o$$

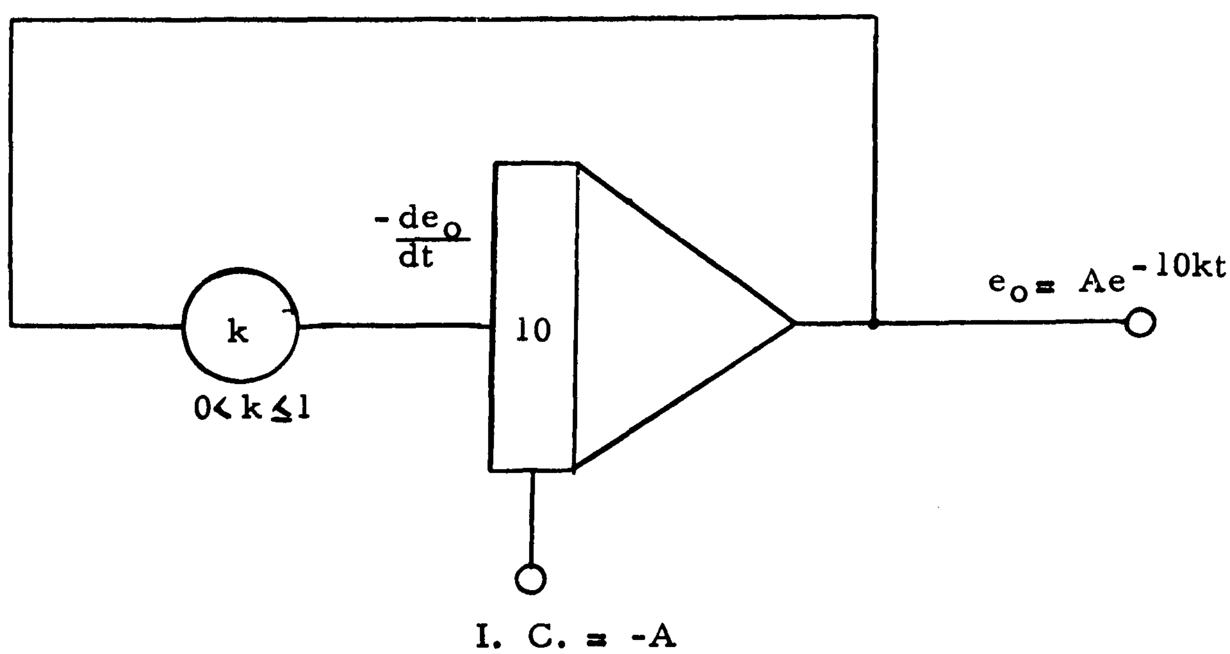


Figure 5. Computer Diagram Illustrating the Generation of a Decaying Exponential, Ae^{-10kt}.

inputs by 100. The results found from the analog computer are shown in Figure 7 in the Analysis of Results chapter.

D. DETERMINATION OF FORCE

Currents flowing in opposite directions in two conductors have a force of repulsion between them given by the relationship,[12]

$$F_R = \frac{u_o i \, i_r l_r}{2rR}, \tag{42}$$

where $u_o = 4\pi(10^{-7})$ henrys/meter

$i =$ current in hammer coil in amperes

$i_r =$ current in ring r of plate in amperes

$R =$ distance of separation between average radius in hammer coil and rings in plate in meters

$l_r =$ circumference of ring r in plate in amperes

$F_R =$ force of repulsion in R direction in newtons

From Table 4, the radius of ring 1 and ring 2 is given by

$$r_1 = .012027 \text{ meters}$$
$$r_2 = .014755 \text{ meters} \tag{43}$$

Now, the circumference l_r in Equation (42) is

$$l_1 = 2\pi r_1 = 6.28(.012) = .0754 \text{ meters,}$$
$$\text{and } l_2 = 2\pi r_2 = 6.28(.0148) = .0930 \text{ meters.} \tag{44}$$

The same current flows through the two rings of the hammer coil. Therefore, an average radius midway between ring 1 and ring 2 is assumed to carry the current as shown in Figure 6. "a" is midway between radius 1 and radius 2.

$$a = \frac{r_2 - r_1}{2}$$

$$a = \frac{.0148 - .0120}{2} = .0014 \text{ meters} \tag{45}$$

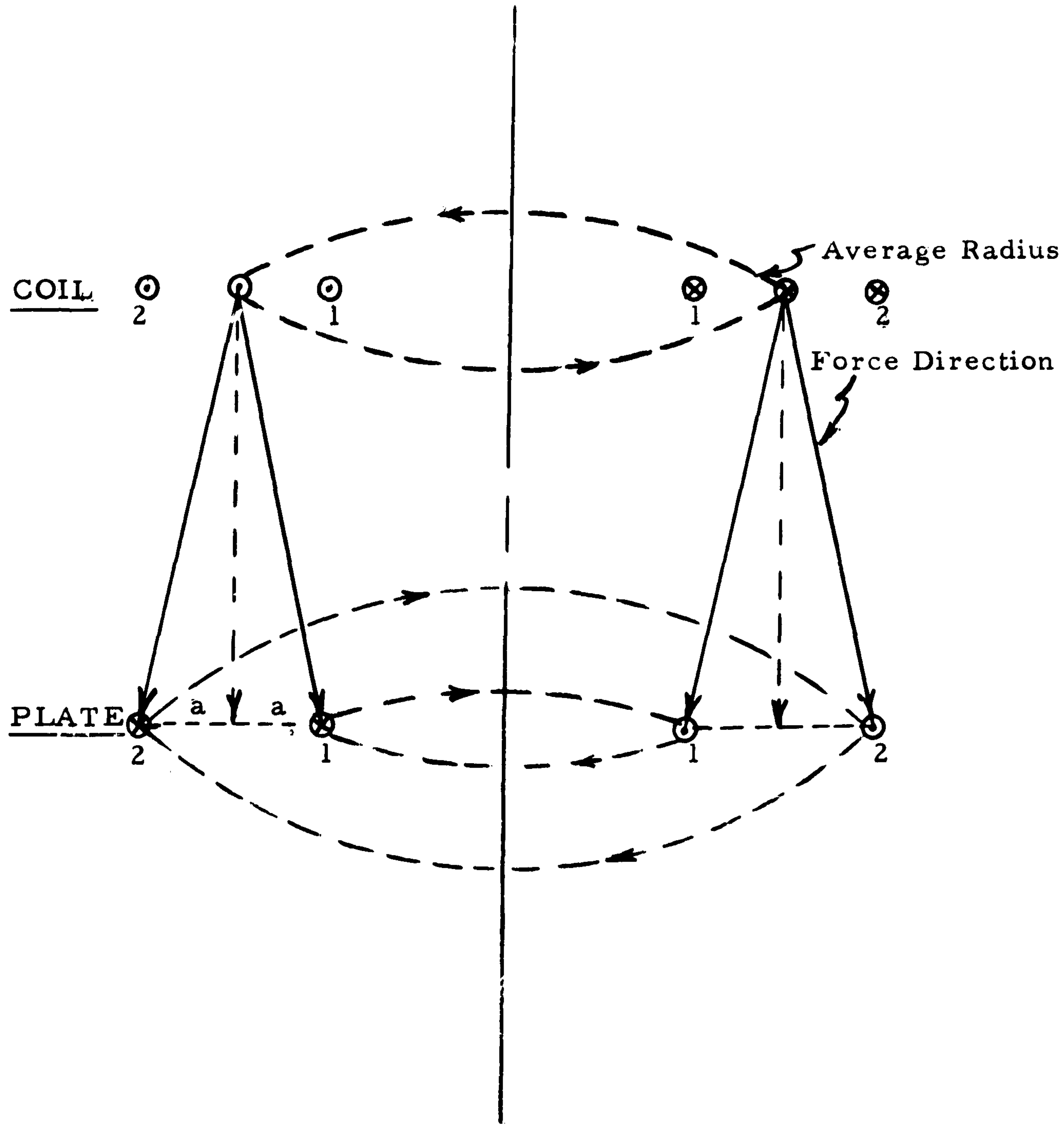


Figure 6. Diagram Illustrating the Forces of Repulsion Between the Hammer Coil and Plate.

The distance R in Figure 6 is calculated by

$$R = \sqrt{x^2 + a^2}, \tag{46}$$

where $x = 2000t + .001$. The term .001 accounts for the initial separation between the hammer coil and plate due to the potting of the coil in the hammer. Now, Equation (46) becomes:

$$R = \sqrt{\left[2(10^3)t + .001\right]^2 + (.0014)^2}$$

$$R = \sqrt{4(10^6)t^2 + 4t + 3.0(10^{-6})}$$

Since the conductors are concentric rings, the forces in the "a" direction in Figure 6 will cancel. Thus, the plate moves only in the x direction. This force is given by

$$F_{x_r} = \frac{x}{R} F_R \tag{48}$$

In order to determine the force per unit length of conductor, Equation (48) is divided by l_r.

$$\frac{F_{x_r}}{l_r} = \frac{u_o\, i\, i_r\, x}{2\pi R^2}. \tag{49}$$

The force exerted on ring 1 in the plate is determined as follows:

$$\frac{F_{x1}}{l_1} = \frac{4\pi(10^{-7})\, i\, i_1 \left[2(10^3)t + .001\right]}{2(3.14)\left[4(10^6)t^2 + 4t + 3.0(10^{-6})\right]}$$

$$\frac{F_{x1}}{l_1} = \frac{2.0(10^{-7})\left[2(10^3)t + .001\right] i\, i_1}{4(10^6)t^2 + 4t + 3.0(10^{-6})} \tag{50}$$

Force on ring 2: $$\frac{F_{x2}}{l_2} = \frac{2.0(10^{-7})\left[2(10^3)t + .001\right] i\, i_2}{4(10^6)t^2 + 4t + 3.0(10^{-6})} \tag{51}$$

Table 6 shows the values of the force per unit length of conductor as time increases. Figure 7 shows a plot of force per unit length of conductor versus time for ring 1 and ring 2 in the plate.

CHAPTER IV

ANALYSIS OF RESULTS

In Chapter III the mutual coupling between two concentric rings with various radii was calculated for a specific set of radii and distance of separation. Because of these tedious calculations, the IBM 360 computer was employed. The programs used are shown in Appendix A and B. The information needed for this program is the radii of the two concentric rings and the distance of separation between them. Because of the numerous possible combinations of mutual coupling, the combinations were limited to those shown in Tables 1 through 3.

From Table 2 plots of the mutual coupling versus distance of separation were made for all combinations of mutual coupling in a three ring hammer coil. These plots are shown in Figures C-1 through C-6. Notice that the smallest distance between the concentric rings is 0.1 centimeters. This is due to the recession or potting of the coil in the head of the hammer. As might be expected the mutual coupling follows the curve of a decaying exponential out to a distance of 3 centimeters. After a distance of separation of 3 centimeters the mutual coupling is very small as compared with its initial value of approximately $3(10^{-8})$ to $7(10^{-8})$ henrys.

Table 3 gives the constant mutual coupling between adjacent rings in the hammer coil and between adjacent rings in the plate for a three ring configuration. Again, the programs in Appendices

$M_{mn}(x)$	Mutual Coupling Between Hammer Coil and Plate $X10^{-8}$ henrys			
	Separation of 1 cm.	Separation of 2 cm.	Separation of 3 cm.	Separation of 4 cm.
M_{11}	.7565	.2221	.1145	.0328
M_{12}	.9199	.3270	.1263	.1002
M_{13}	.9909	.4102	.1873	.0865
M_{14}	.9763	.4812	.1589	.1665
M_{15}	.9769	.4874	.2542	.1603
M_{16}	.9038	.5271	.3020	.1810
M_{17}	.8456	.5005	.3198	.1569
M_{18}	.8247	.5465	.3632	.2274
M_{19}	.7278	.5445	.3984	.2463
$M_{1\,10}$	.7063	.6144	.3610	.2222
$M_{1\,11}$	.6434	.5284	.3526	.2120
$M_{1\,12}$	.6284	.4825	.3320	.2142
$M_{1\,13}$	.5968	.4825	.3978	.2278
$M_{1\,14}$	.4714	.4234	.3493	.2516
$M_{1\,15}$	.4542	.4127	.3422	.2122
$M_{1\,16}$	.5639	.4818	.3593	.2513

Table 1. Computed Values of the Mutual Coupling Between the Hammer Coil and Plate for a Separation of One, Two, Three, and Four Centimeters in All Sixteen Rings.

Distance of Separation in Centimeters	M_{mn}	Mutual Coupling Between Hammer Coil and Plate $X10^{-8}$ henrys	M_{mn}	Mutual Coupling Between Hammer Coil and Plate $X10^{-8}$ henrys
0.1	M_{11}	3.8891	M_{13}	2.0384
0.5	M_{11}	1.5973	M_{13}	1.5681
1.0	M_{11}	0.7565	M_{13}	0.9909
2.0	M_{11}	0.2221	M_{13}	0.4102
3.0	M_{11}	0.1145	M_{13}	0.1873
4.0	M_{11}	0.0382	M_{13}	0.0865
0.1	M_{12}	2.7459	M_{22}	5.2148
0.5	M_{12}	1.6979	M_{22}	2.2663
1.0	M_{12}	0.9199	M_{22}	1.1819
2.0	M_{12}	0.3270	M_{22}	0.4373
3.0	M_{12}	0.1263	M_{22}	0.1458
4.0	M_{12}	0.1002	M_{22}	0.1350
0.1	M_{23}	3.6373	M_{33}	6.6390
0.5	M_{23}	2.2663	M_{33}	3.0436
1.0	M_{23}	1.3916	M_{33}	1.7251
2.0	M_{23}	0.5590	M_{33}	0.6897
3.0	M_{23}	0.2759	M_{33}	0.3715
4.0	M_{23}	0.1550	M_{33}	0.2014

Table 2. Computed Values of the Mutual Coupling Between the Hammer Coil and Plate for the Three Smallest Concentric Rings.

M'_{mn}	Values of Elliptic Integral of First Kind, K(k)	Values of Elliptic Integral of Second Kind, E(k)	Constant Mutual Coupling Between Concentric Rings X 10^{-8} henrys
$M'_{12} = M'_{21}$	1.017	3.68	2.8342
$M'_{13} = M'_{31}$	1.044	3.09	2.0538
$M'_{23} = M'_{32}$	1.012	3.86	3.7751

Table 3. Computed Values of the Elliptic Integrals and the Constant Mutual Coupling Between the Three Smallest Concentric Rings in the Hammer Coil.

Ring Segment	Radius in Centimeters	Inductance $\times 10^{-6}$ henrys
L_1	1.2027	.2070
L_2	1.4755	.2949
L_3	1.7483	.3866
L_4	2.0213	.4803
L_5	2.2936	.5745
L_6	2.5667	.6673
L_7	2.8395	.7572
L_8	3.1123	.8420
L_9	3.3848	.9199
L_{10}	3.6581	.9866
L_{11}	3.9306	1.0456
L_{12}	4.2034	1.0879
L_{13}	4.4762	1.1113
L_{14}	4.7490	1.1081
L_{15}	5.0216	1.0578
L_{16}	5.2944	——

Table 4. Tabulated Values of the Inductances and Radii of the Sixteen Concentric Rings Which Approximate the Hammer Coil.

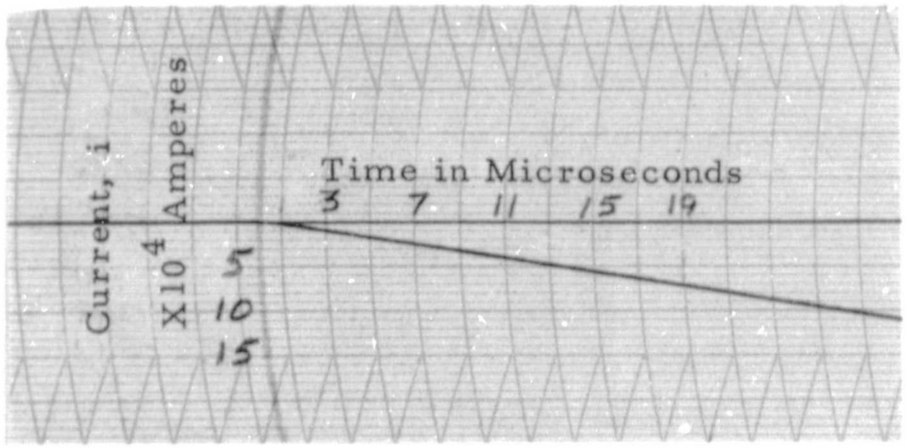
Distance of Separation in Centimeters	$M_{11} + M_{12}$ X10^{-8}henrys	$M_{21} + M_{22}$ X10^{-8}henrys
0.1	6.6350	7.9607
0.5	3.2952	3.9642
1.0	1.6764	2.1018
2.0	0.5491	0.7643
3.0	0.2408	0.2721
4.0	0.1330	0.2352

Table 5. Calculated Values of the Combined Mutual Coupling Between the Hammer Coil and Plate.

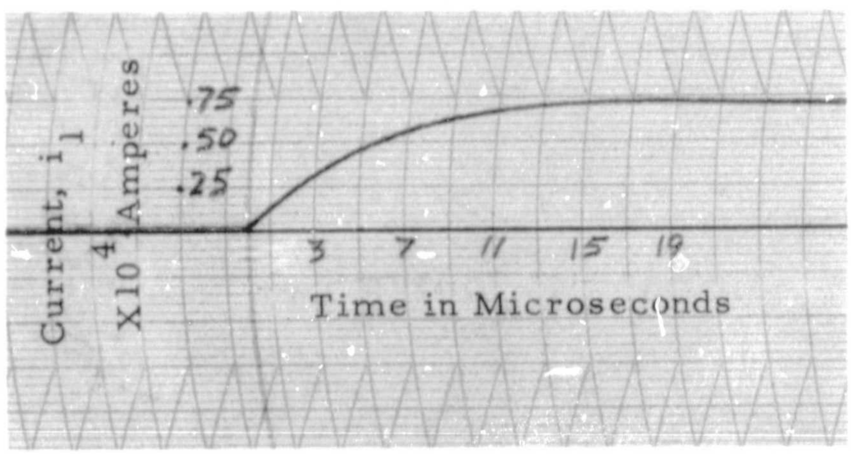
A and B were used by using a distance of separation equal to zero. This causes the rings to be in the same plane with each other. Due to the complexity of the problem, a two ring hammer coil and plate was analyzed.

Table 5 gives the combined values of the mutual coupling which resulted in Equations (17), (18), and (19). Figures C-7 and C-8 give plots of the combined mutual coupling as a function of distance of separation for the two ring hammer coil used in this study. Decaying exponentials of $6.6(10^{-8})e^{-1.5x}$ and $(8)10^{-8}e^{-1.6x}$ henrys were used to approximate the actual curves.

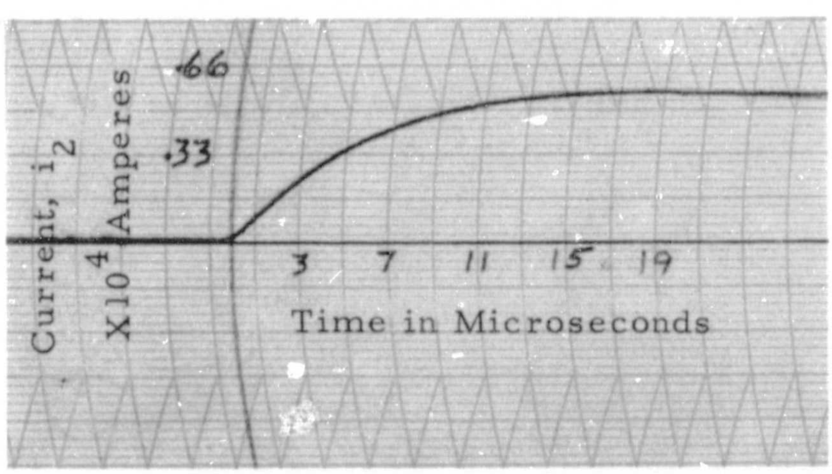
The solutions of Equations (23), (24), and (25), which were implemented by the analog computer diagram in Figure 4, are shown in Figure 7. It was observed that the constant mutual coupling terms could be neglected with an error resulting of approximately 8 percent of the maximum response. Also, the mutual terms $-(19.8)10^{-3}e^{-3(10^5)t}\, i_1$ and $-(25.6)10^{-3}e^{-3.2(10^5)t}\, i_2$ can be neglected entirely without any observable error. In the center and bottom recordings of Figure 7, the currents in the plate are found to peak at 19 microseconds. The slow fall time is due to the very small resistance of the plate. In the top recording, the current in the hammer coil i is linear during the period of 19 microseconds. This current would reach a steady state value if the analog computer were allowed to run, and if it would not overload. But after 19 microseconds, the plate has moved 3.8 centimeters by Equation (12), and the currents in the plate have reached a maximum value. Thus, the equations are only needed during the time when the maximum



Computer Recording of Current i in Hammer Coil.
Scale: 50 volts/centimeter - 2.5 centimeters/second
Output: 5X



Computer Recording of Current i_1 in Plate.
Scale: 1 volt/centimeter - 2.5 centimeters/second
Output: $2X_1$



Computer Recording of Current i_2 in Plate.
Scale: 1 volt/centimeter - 2.5 centimeters/second
Output: $3X_2$

Figure 7. Computer Recordings of Currents Versus Time in Hammer Coil and Plate.

current is induced into the plate circuit, which is during the period of 19 microseconds as seen from Figure 7.

From Equations (50) and (51) the force per unit length is calculated for several values of time and tabulated in Table 6. From this table plots of the force exerted on ring 1 and ring 2 in the plate are made as shown in Figure 8. It is seen from these plots that the maximum force of 2710 newtons/meter on ring 1 and 2080 newtons/meter on ring 2 is developed at 13 microseconds. After this time the force decreases. Thus, the force peaks 6 microseconds before the currents in the plate peak at values of $0.7(10^4)$ amperes in ring 1 and $0.566(10^4)$ amperes in ring 2. Also, it is seen that the force on ring one in the plate is larger than the force exerted on ring two. This is expected from observation of the shape of the plate after impact from the hammer coil. After impact the center of the plate is the farthest point from the initial position. Hence, Figure 8 gives the force exerted on ring 1 and ring 2 of the plate as a function of time for a two ring hammer coil.

Time in Microseconds	Current in Hammer Coil i (10^4 amperes)	Current in Ring 1 of Plate, i_1 (10^4 amperes)	Current in Ring 2 of Plate, i_2 (10^4 amperes)	Force on Ring 1 in Plate, F_{x_1} newtons/meter	Force on Ring 2 in Plate, F_{x_2} newtons/meter
1	0.5	.10	.083	273	227
3	1.0	.29	.22	797	604
5	2.0	.44	.33	1570	1180
9	3.6	.60	.50	2260	1885
13	5.0	.67	.53	2490	1950
15	6.0	.69	.54	2710	2080
17	6.5	.69	.56	2550	2070
19	7.0	.70	.57	2530	2050

Table 6. Experimental Values of Currents and Forces in the Two Ring Hammer Coil and Plate at Several Increments of Time.

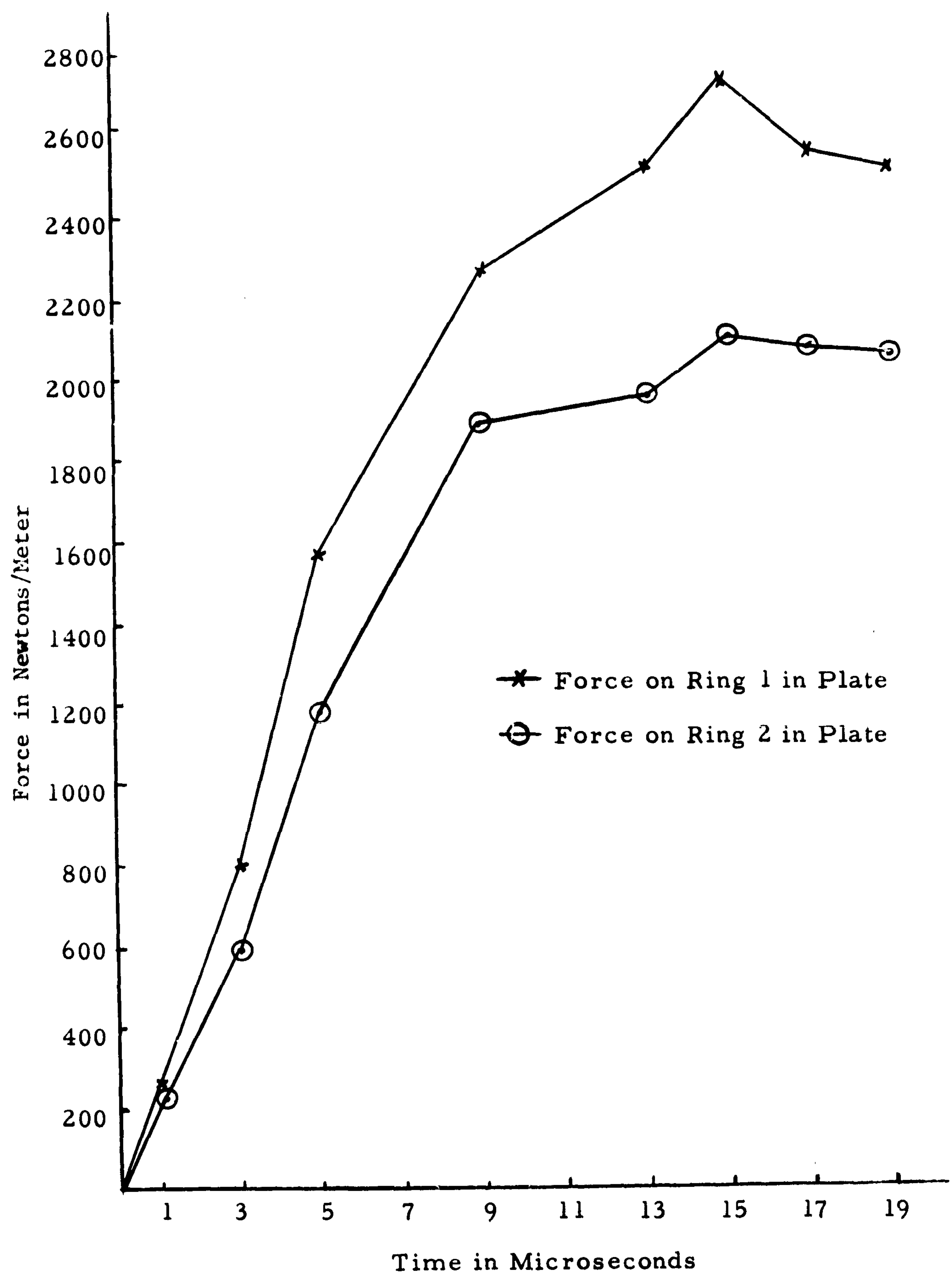


Figure. 8. Plot of Force Exerted on Ring One and Two in the Plate Versus Time.

CHAPTER V

CONCLUSION

The purpose of this study was to determine the force exerted by a two ring hammer coil on a metallic plate as the plate was allowed to move. This process has been previously called magnetomotive forming. The parameters chosen were typical of present day hammer coils.

These forces were determined by assuming that the hammer coil and plate circuit consisted of concentric ring segments, each with the same center line. The currents induced in the plate were explained by the presence of mutual coupling between two concentric rings. These mutual couplings were determined by an equation called the Neumann form.

The maximum force exerted on ring 1 in the plate was found to be 2710 newtons/meter, while a maximum force of 2080 newtons/meter was found in ring 2 of the plate. These maximum forces occurred at 13 microseconds, as compared with a maximum induced current occurring at 19 microseconds.

The maximum current in ring 1 of the plate was $0.7(10^4)$ amperes as compared with $0.566(10^4)$ amperes in ring 2 of the plate. At 19 microseconds the current in the hammer coil was $7.0(10^4)$ amperes. For a 16 ring hammer coil, this current would be smaller due to the added resistance of the other rings.

The next step in the study of the hammer coil shoud be to determine how the force varies with the elasticity of the metallic plate. This would require a knowledge of the strength of materials such as yield and break points of a metallic material.

When one considers the equations which describe the beryllium coil assembly and the formed material for both the two and three ring approximations, certain characteristics should be noticed.

In the case of the two rings, three equations result which have seventeen terms after simplification. There are eight time-varying coefficients present in the seventeen terms; therefore, eight analog multipliers are required for solution. For the complete two ring problem, eleven amplifiers, eight multipliers and fifteen potentiometers are required.

When the three ring case is considered, four equations which have fifty-one terms result. Twenty-five of these terms have time-varying coefficients which in turn require twenty-five multipliers; thus, one can see that the number of multipliers increases quite rapidly with increasing number of rings. Since the analog computer at Mississippi State has only one-hundred multipliers, the solution to the full sixteen rings in the hammer coil cannot be instrumented.

In view of the limitation illustrated above, it is suggested that the alternate approach, that is the digital computer program, be pursued to a more suitable number of rings.

APPENDICES

APPENDIX A

```
C  C  CALCULATION OF E FOR MUTUAL
      DIMENSION R(16), D(100)
      READ (1, 10) R
6     FORMAT (14X1HA19X1HB19X1HD19X1HE19X1HF)
10    FORMAT (F10.0)
      READ (1, 2) I
2     FORMAT (I3)
      DO 3 N  1,I
3     READ (1, 10) D(N)
      DO 4 L  1,I
      DO 4 N  1, 16
      K= N
      DO 4 M  K, 16
      E= 2.*SQRT(R(N)*R(M)/(D(L)**2+(R(N)+R(M))**2))
      F= 57.2958*ATAN(E/(SQRT(1.0-E**2)))
4     WRITE (3, 93) R(N), R(M), D(L), E, F
93    FORMAT (1X5E20.8)
      STOP
      END
```

APPENDIX B

```
C C  CALCULATION OF MUTUAL TERMS
9    READ (1, 10) A, B, D, E, F
10   FORMAT (5F10.0)
     WRITE (3, 6)
6    FORMAT (13X1HA16X1HB16X1HD16X1HE16X1HF16X1HM)
     C=2.0*SQRT (A*B/(D*D+(A+B)**2))
     HAMER=12.567E-9*SQRT(D*D+(A+B)**2)*((1.-C*C/2.)*F-E)
     WRITE (3, 4) A, B, D, E, F, HAMER
4    FORMAT (1X6E17.8)
     GO TO 9
     STOP
     END
```

APPENDIX C

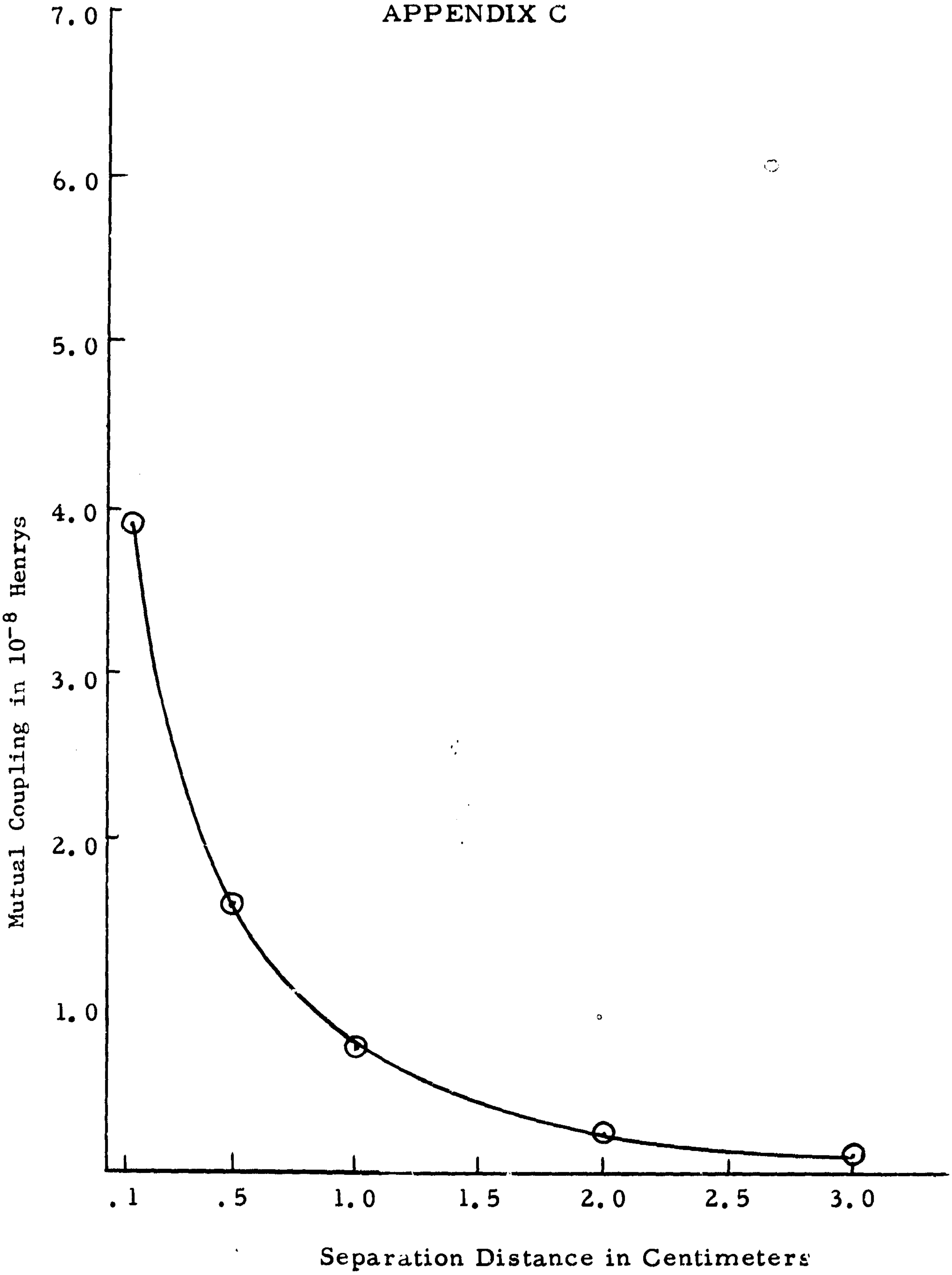


Figure C-1. A Plot of the Mutual Coupling Between Ring One of the Hammer Coil and Ring One in the Plate as a Function of Distance of Separation.

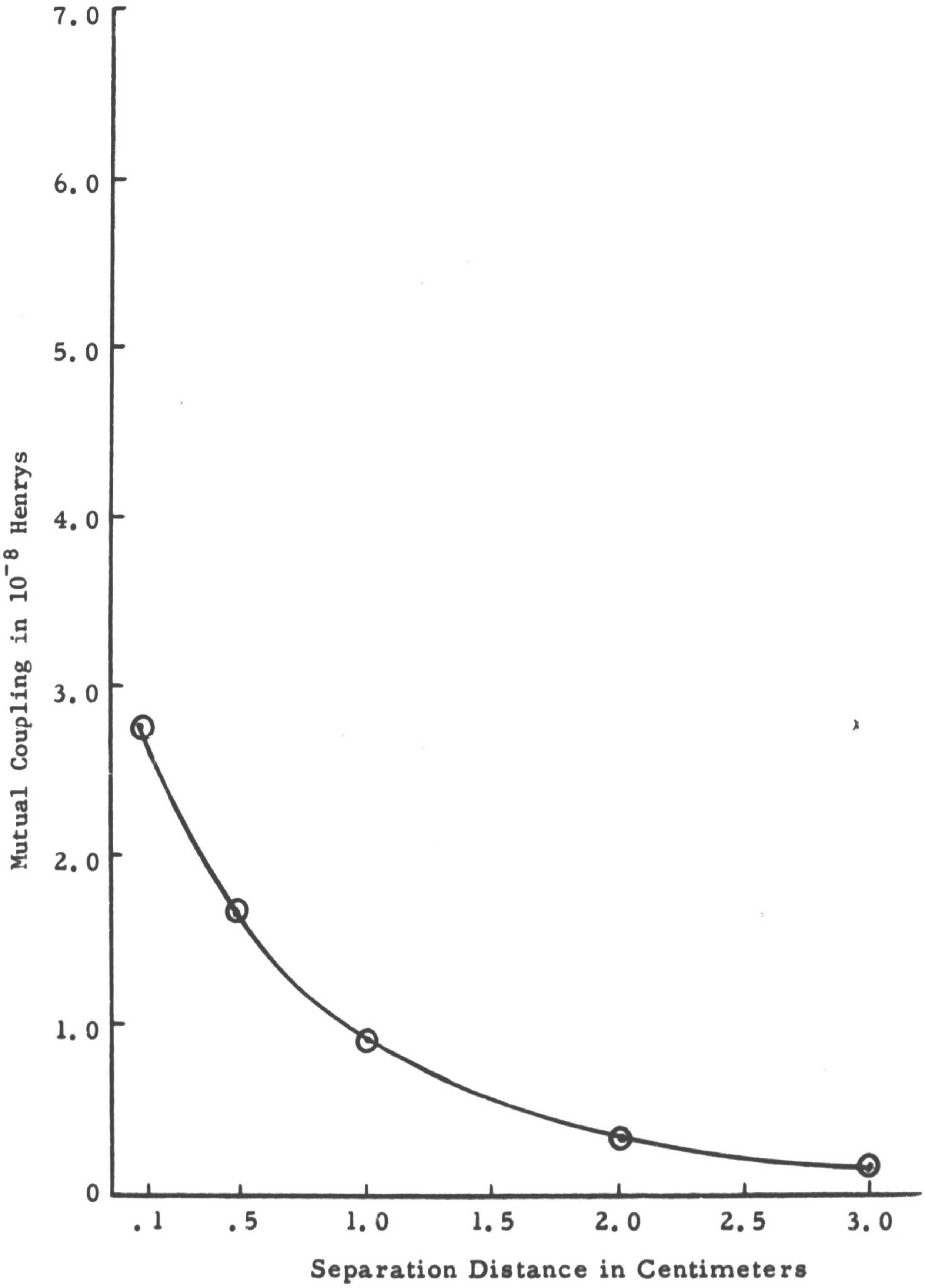


Figure C-2. A Plot of the Mutual Coupling Between Ring One of the Hammer Coil and Ring Two in the Plate as a Function of Distance of Separation.

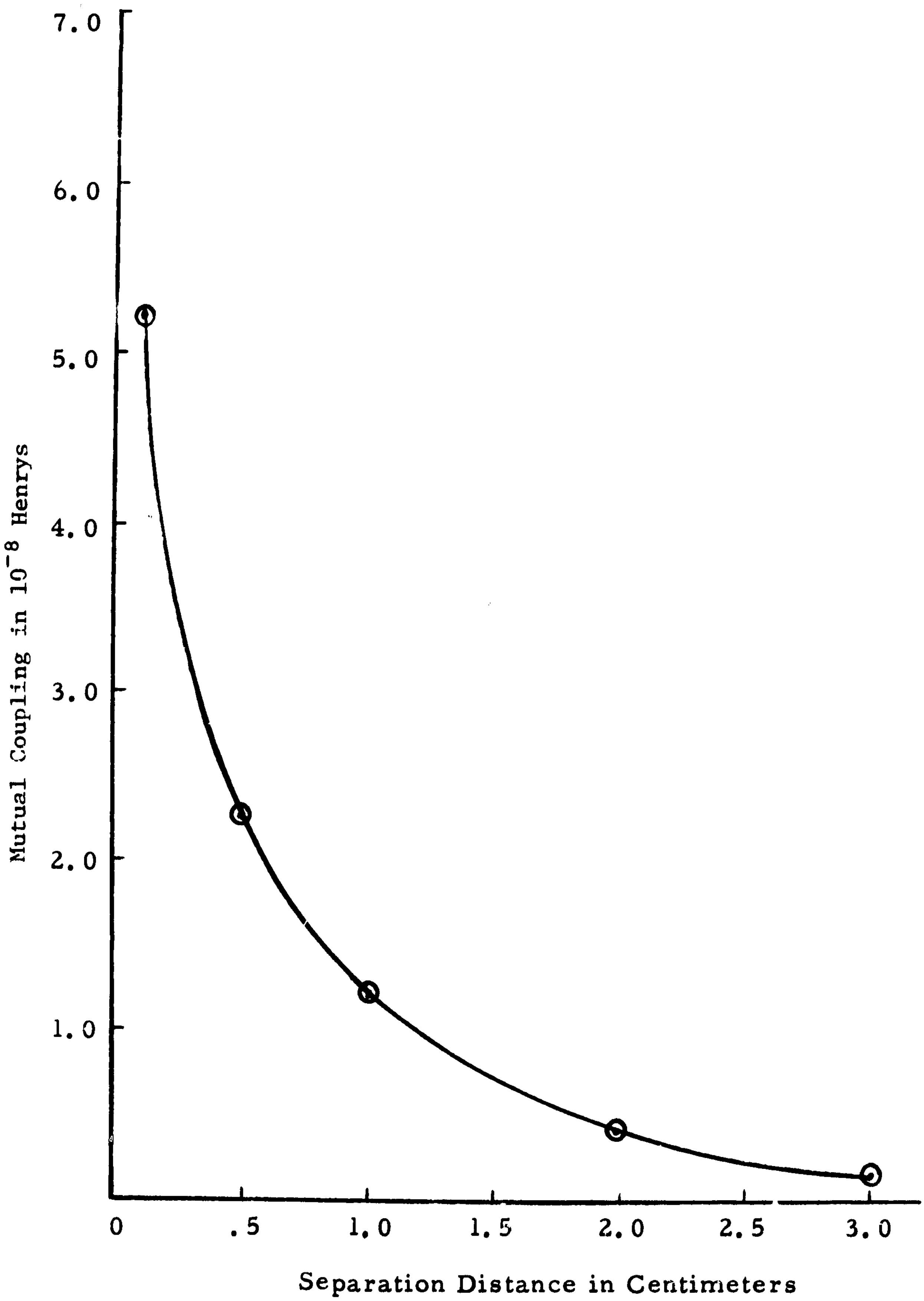


Figure C-3. A Plot of the Mutual Coupling Between Ring Two of the Hammer Coil and Ring Two in the Plate as a Function of Distance of Separation.

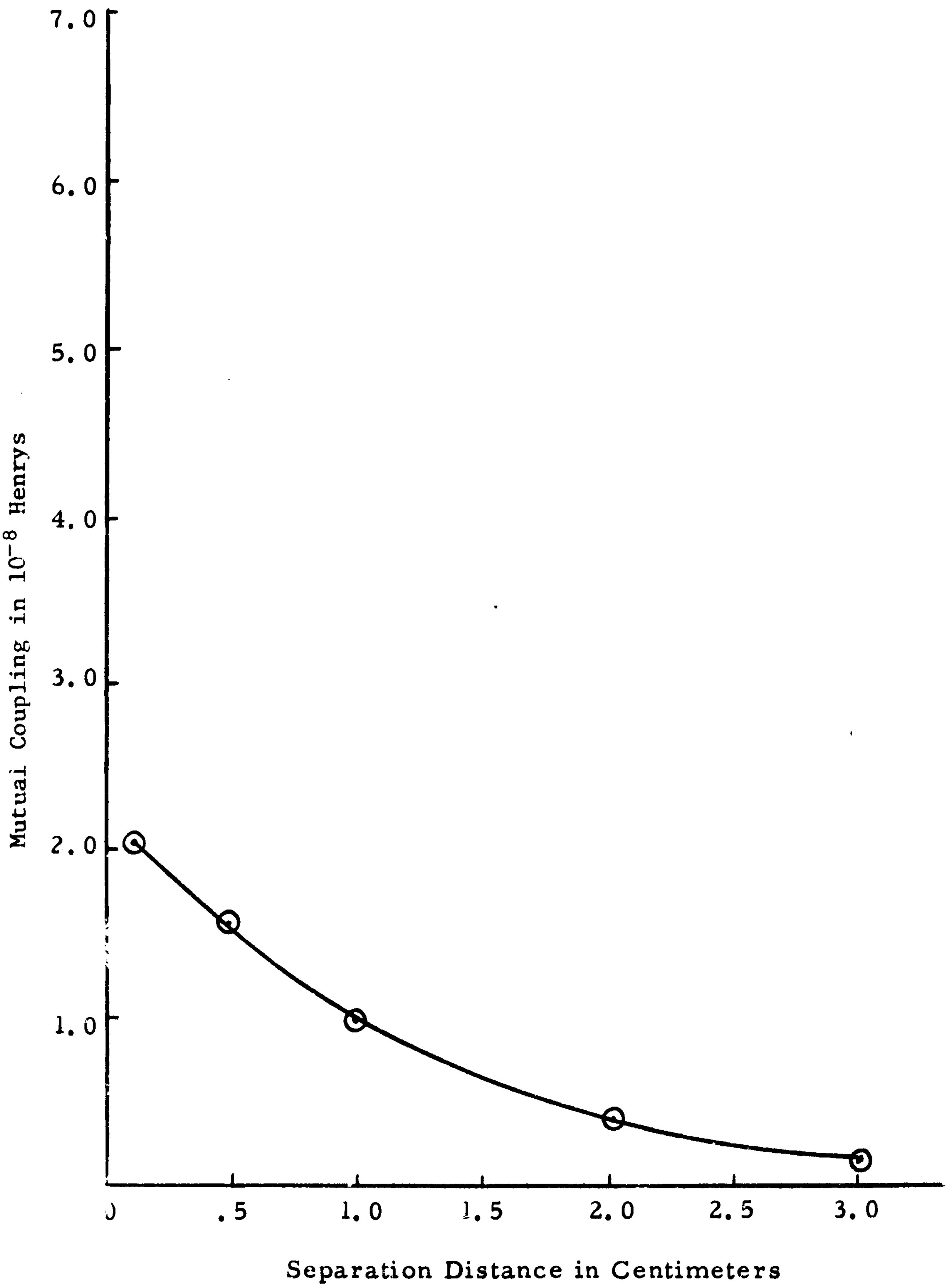


Figure C-4. A Plot of the Mutual Coupling Between Ring One of the Hammer Coil and Ring Three in the Plate as a Function of Distance of Separation.

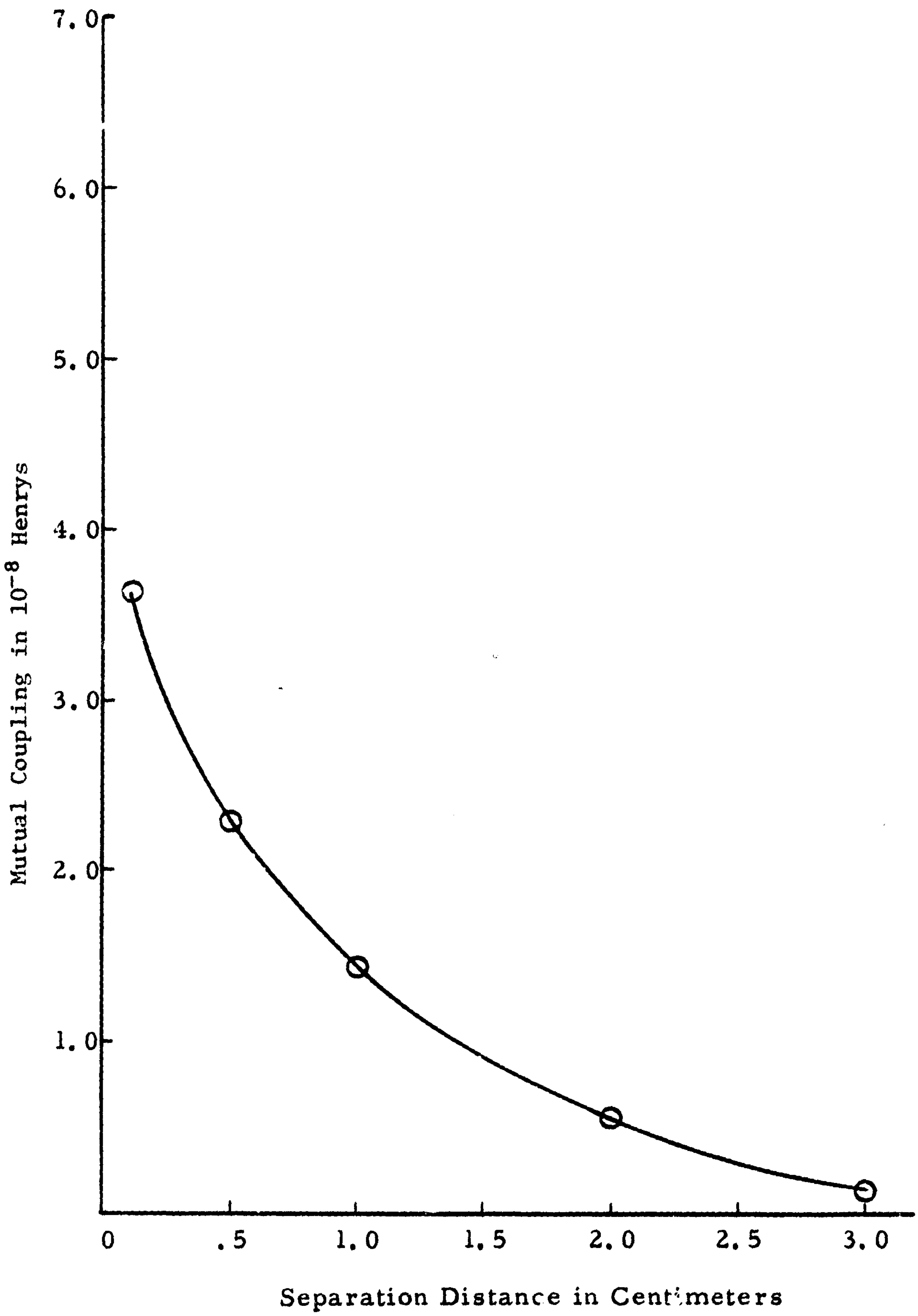


Figure C-5. A Plot of the Mutual Coupling Between Ring Two of the Hammer Coil and Ring Three in the Plate as a Function of Distance of Separation.

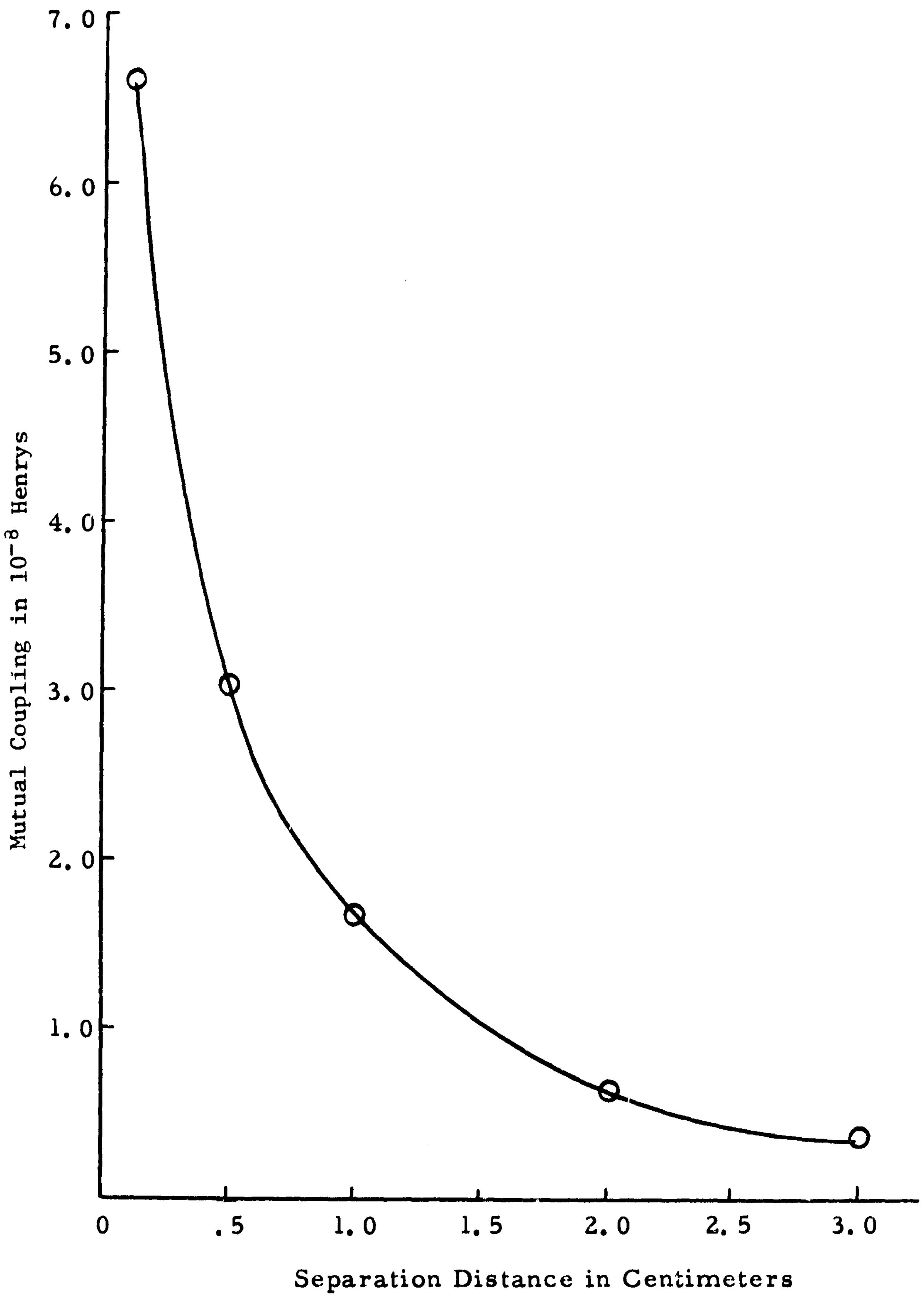


Figure C-6. A Plot of the Mutual Coupling Between Ring Three of the Hammer Coil and Ring Three in the Plate as a Function of Distance of Separation

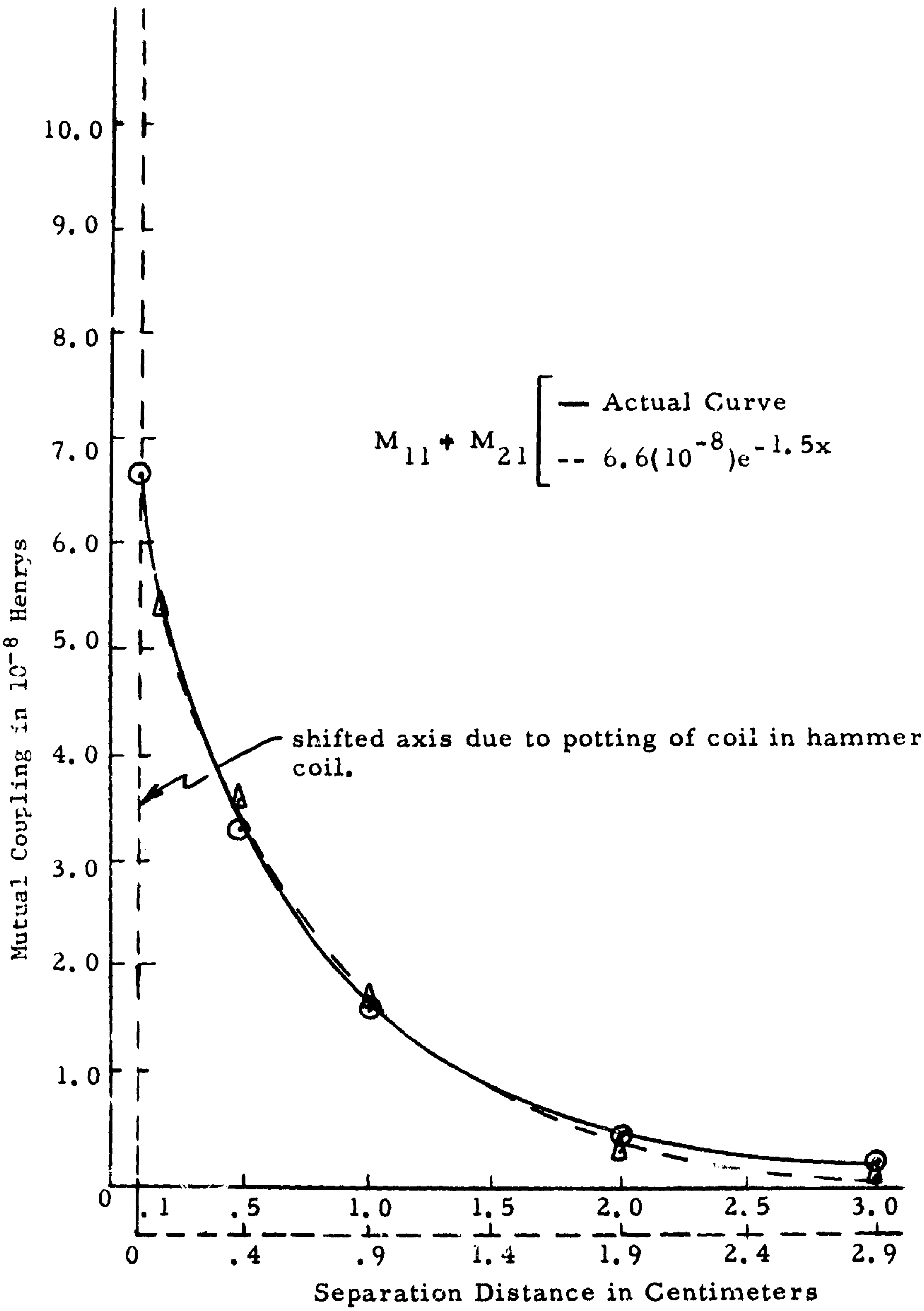


Figure C-7. A Plot of the Combined Mutual Coupling Between Ring One of the Hammer Coil and Rings One and Two of the Plate and Approximate Fitted Curve as a Function of Distance of Separation.

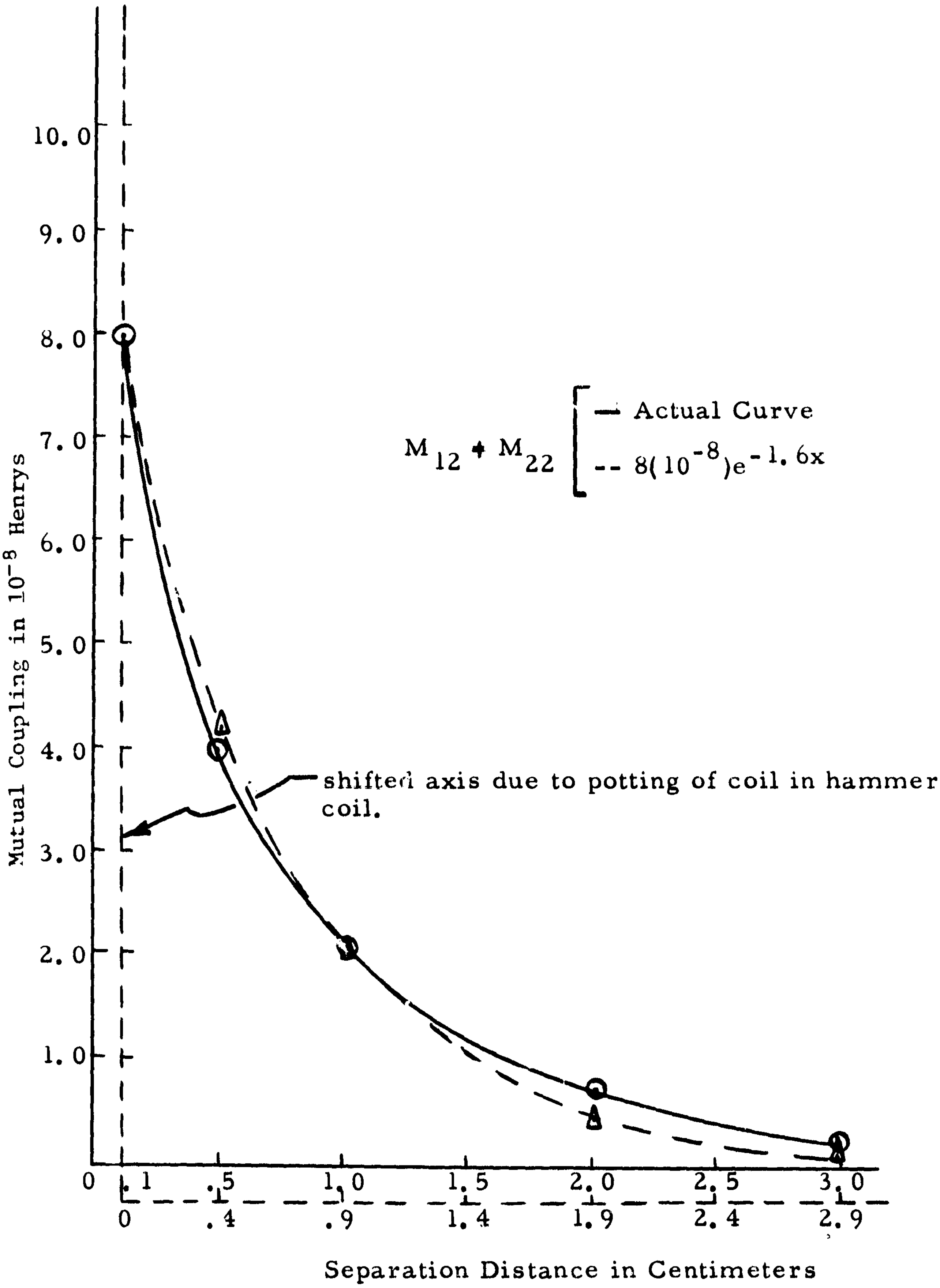


Figure C-8. A Plot of the Combined Mutual Coupling Between Ring Two of the Hammer Coil and Rings One and Two of the Plate and Approximate Fitted Curve as a Function of Distance of Separation.

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