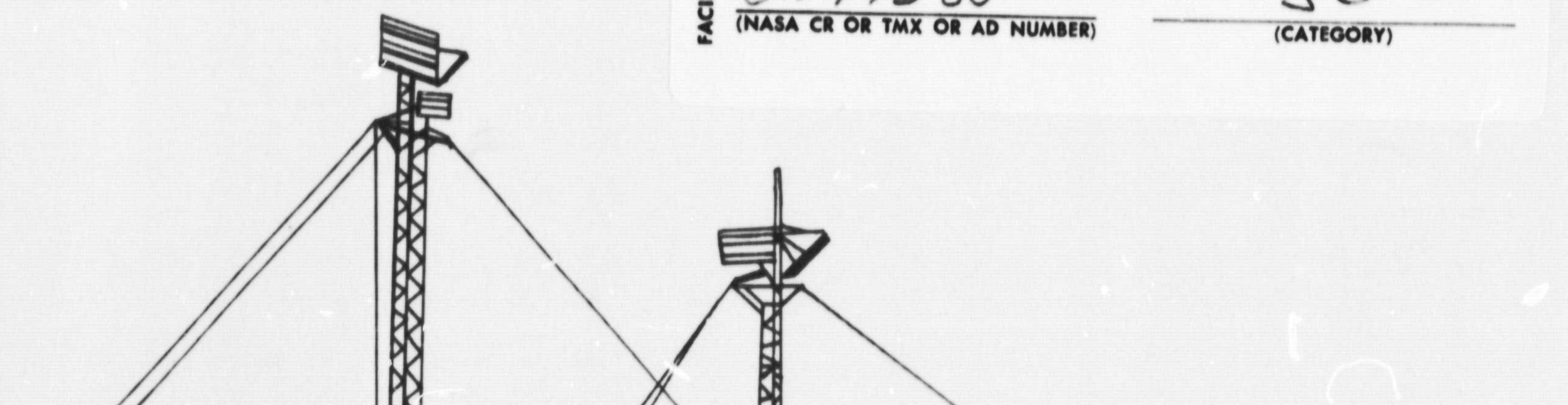
THE PREABLATION HEATING OF METEOROIDS

Z. CEPLECHA and A. POSEN



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CONSIDERATIONS OF CONDUCTION AND RADIATION ON THE PREABLATION HEATING OF METEOROIDS

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FIGURE CAPTION

ABSTRACT

The equation of heat conduction is solved for the early part of the meteor trajectory of a homogeneous spherical blackbody meteoroid; both radiation and heat conductivity are considered. It is possible to compute the air density at which a given surface temperature of the meteoroid is reached.

RÉSUMÉ

L'équation de la conduction de la chaleur est résolue pour la première phase de la trajectoire météorique d'un météore qui serait un corps noir sphérique et homogène; nous considérons à la fois la radiation et la conductibilité thermique. Il est possible de calculer la densité de l'air à laquelle est atteinte une température de surface donnée du météore.

КОНСПЕКТ

Решено уравнение теплопроводности для начальной части метеорной траектории однородного сферического метеорного черного тела; рассматриваются обои излучение и теплопроводность. Возможен расчет плотности воздуха при которой достигается данная температура метеорного тела.

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1. INTRODUCTION

The rate of heating of a meteor body during the early stage of its penetration into the atmosphere will determine the height at which ablation becomes significant. The complete solution of the heating problem is related to the observed beginning heights of meteors. This problem was solved by Levin (1961) and by Ceplecha and Padevět (1961). In the latter paper, the two cases of heat conductivity and radiation cooling of the surface were computed separately. The present solution considers the two processes together for spherical meteoroids.

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2. FORMULATION OF THE PROBLEM

The surface temperature at which ablation begins should fall somewhere between the melting and the boiling points for the body. At what height, then, is this temperature reached for a homogeneous sphere with radius r_0, density δ, specific heat c, and heat conductivity λ, radiating as a blackbody? We shall assume a randomly oriented, rapidly rotating meteoroid whose kinetic energy is converted to heat uniformly distributed on its surface. Then the following differential equation of heat conduction applies:*

$$\frac{\partial \tau}{\partial t} - \frac{2\beta^2}{r} \frac{\partial \tau}{\partial r} - \beta^2 \frac{\partial^2 \tau}{\partial r^2} = 0 \quad , \tag{1}$$

where

$$\beta^2 = \frac{\lambda}{\delta c} \tag{2}$$

and

$$\tau = \tau(r, t) = T - T_0 \quad . \tag{3}$$

The initial condition can be written

$$\tau(r, -\infty) = 0 \quad . \tag{4}$$

The boundary conditions are

$$\tau(0, t) = \text{finite value} \tag{5}$$

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*The list of mathematical symbols used in this paper is presented in the Appendix.

and

$$\lambda \left(\frac{\partial \tau}{\partial r}\right)_{r = r_0} + \sigma (\tau + T_0)^4 = \frac{\Lambda \rho v^3}{8} \quad . \tag{6}$$

Equations (1) and (4) assume a more suitable form under the transformation

$$u = \tau r \quad . \tag{7}$$

Then

$$\frac{\partial u}{\partial t} - \beta^2 \frac{\partial^2 u}{\partial r^2} = 0 \tag{8}$$

and

$$u(r, -\infty) = 0 \quad , \tag{9}$$

with the relative temperature of the meteoroid surface τ_B at the time $t = 0$. Then

$$u(r_0, 0) = r_0 \tau_B \quad ; \tag{10}$$

thus,

$$u = \exp (wr + \beta^2 w^2 t) \tag{11}$$

is a solution of equation (8) but is not consistent with equation (9). This suggests a general solution of the form

$$u = k_1 \exp(w_1 r + \beta^2 w_1^2 t) + k_2 \exp(w_2 r + \beta^2 w_2^2 t) \quad , \tag{12}$$

where

$$w_1 \neq w_2 \quad . \tag{13}$$

Equation (5) can be satisfied by

$$w_1^2 = w_2^2 \quad ; \tag{14}$$

this implies

$$w_1 = -w_2 \ (= w) \tag{15}$$

and

$$k_1 = -k_2 \ (= k) \quad . \tag{16}$$

Then we can write

$$u = k \left[\exp (wr) - \exp (-wr) \right] \exp (\beta^2 w^2 t) \tag{17}$$

and, from equation (10),

$$k = \frac{r_0 \tau B}{\exp (wr_0) - \exp (-wr_0)} \quad . \tag{18}$$

Consequently, the general solution of equation (1) has the form

$$\tau = \frac{r_0 \tau B \left[\exp (wr) - \exp (-wr) \right]}{r \left[\exp (wr_0) - \exp (-wr_0) \right]} \exp (\beta^2 w^2 t) \tag{19}$$

and is to be combined with equation (6), in which ρ and v are functions of time only. The drag equation

$$\frac{dv}{dt} = - \frac{3\Gamma}{4r_0\delta} \rho v^2 \tag{20}$$

yields the expression for velocity

$$v = v_\infty \exp\left(- \frac{3\Gamma\rho}{4r_0\delta b \cos Z_R}\right) \tag{21}$$

if we use

$$\frac{d\rho}{dh} = - b\rho \tag{22}$$

and

$$\frac{dh}{dt} = - v \cos Z_R \quad . \tag{23}$$

Then, substituting equations (19) and (21) into equation (6), we have

$$\lambda\tau_B w \coth (wr_0) - \frac{\lambda\tau_B}{r_0} + \sigma(\tau_B + T_0)^4 = \frac{\Lambda\rho_B v_\infty^3}{8} \exp\left(- \frac{9\Gamma\rho_B}{4r_0\delta b \cos Z_R}\right) . \tag{24}$$

The solution of equation (24) for ω is ρ_β at time t = 0. The solution appropriate to this problem should be valid from t = $-\infty$ to t just beyond t = 0. To approximate this condition, we solve equation (24) simultaneously with the first time derivative of equation (6), substituting equations (19) and (21), evaluated at t = 0:

$$\lambda\tau_B\beta^2 w^3 \coth(wr_0) - \frac{\lambda\tau_B\beta^2 w^2}{r_0} + 4\sigma\tau_B\beta^2 w^2(\tau_B + T_0)^3$$

$$= \frac{\Lambda\,\rho_B v_\infty^4}{8}\left(b\cos Z_R - \frac{9\Gamma\rho_B}{4\,r_0\delta}\right)\exp\left(-\frac{9\Gamma\rho_B}{2\,r_0\delta\, b\cos Z_R}\right) \quad . \qquad (25)$$

Equations (24) and (25) express the solution of equation (1) with initial and boundary conditions from equations (4), (5), and (6) at time $t = 0$ when $\tau = \tau_B$ and $\rho = \rho_B$. They can be solved numerically for the two unknowns ρ_B and w. That is, for a given meteoroid ($\lambda, \delta, c, r_0, v_\infty, \cos Z_R$), we can compute for any surface temperature τ_B the corresponding air density ρ_B at which this temperature is reached. The whole set of solutions for one meteoroid will give us the temperature as a function of time at meteoroid penetration into the atmosphere.

3. NUMERICAL SOLUTION OF EQUATIONS (24) AND (25)

Solving explicitly for w from equations (24) and (25), we get

$$w^2 = \frac{(v_\infty/\beta^2)\,[b \cos Z_R - (9\Gamma\rho_B/4r_0\delta)]}{\exp(9\Gamma\rho_B/4r_0\delta b \cos Z_R) + (8\sigma/\Lambda\rho_B v_\infty^3)\,(\tau_B + T_0)^3(3\tau_B - T_0)\exp(9\Gamma\rho_B/2r_0\delta b \cos Z_R)} \; ; \quad (26)$$

for ρ_B we get

$$\rho_B = \frac{b \cos Z_R}{F(w) + (9\Gamma/4r_0\delta)} \; , \quad (27)$$

where

$$F(w) = \frac{\Lambda\tau_B\beta^2 w^2 v_\infty^2 \left[\lambda w \coth(wr_0) - (\lambda/r_0) + 4\sigma(\tau_B + T_0)^3\right]}{8\left[\lambda\tau_B w \coth(wr_0) - \lambda\tau_B/r_0 + \sigma(\tau_B + T_0)^4\right]^2} \; . \quad (28)$$

Beginning with some chosen value of ρ_B, we get w from equation (26) and a new ρ_B from equation (27); then we proceed iteratively to a solution w, ρ_B. Starting values of ρ_B were chosen according to the predominant cooling effect: either by conductivity or by radiation. The meteoroid radius is a good criterion. Thus, we use for $\log r_0 \geq -1.5$ the starting value of

$$\rho_B = \frac{8\lambda\tau_B}{\Lambda v_\infty^3}\left\{\frac{(bv_\infty \cos Z_R)^{1/2}}{\beta} \coth\left[\frac{r_0}{\beta}(bv_\infty \cos Z_R)^{1/2}\right] - \frac{1}{r_0}\right\} \quad (29)$$

(Ceplecha and Padevět, 1961), and for $\log r_0 < -1.5$,

$$\sigma(\tau_B + T_0)^4 = \frac{\Lambda \rho_B v_\infty^3}{8} \exp\left(-\frac{9\Gamma\rho_B}{4r_0\delta b \cos Z_R}\right) \quad . \tag{30}$$

We solved equation (30) iteratively, beginning with

$$\rho_B = \frac{8\sigma\,(\tau_B + T_0)^4}{\Lambda v_\infty^3} \tag{31}$$

and computing

$$\rho'_B = \rho_B \exp\left(\frac{9\Gamma\rho_B}{4r_0\delta b \cos Z_R}\right) \quad . \tag{32}$$

Convergence is rapid in both stages of the computation.

4. NUMERICAL RESULTS

We chose meteoroids with three different hypothetical compositions (iron, stone, and porous meteoroids), three initial velocities (15, 30, and 60 km sec^{-1}), and two surface temperatures (τ_B = 1900° and 2300°; i.e., T_B = 2180°K and 2580°K). Some of the results from rather extensive tables are given in Figure 1. The logarithm of the air density (and height, taken from the U.S. Standard Atmosphere 1962 (1962)) is plotted against the logarithm of the radius of the meteoroid. The curves follow constant surface temperature.

We used the following constants throughout the computation:

$$\Lambda = 1,$$

$$\Gamma = 0.7,$$

$$\sigma = 5.67 \times 10^{-5},$$

$$T_0 = 280°K,$$

$$b = 1/6 \times 10^{-5},$$

and

$$\cos Z_R = 1 .$$

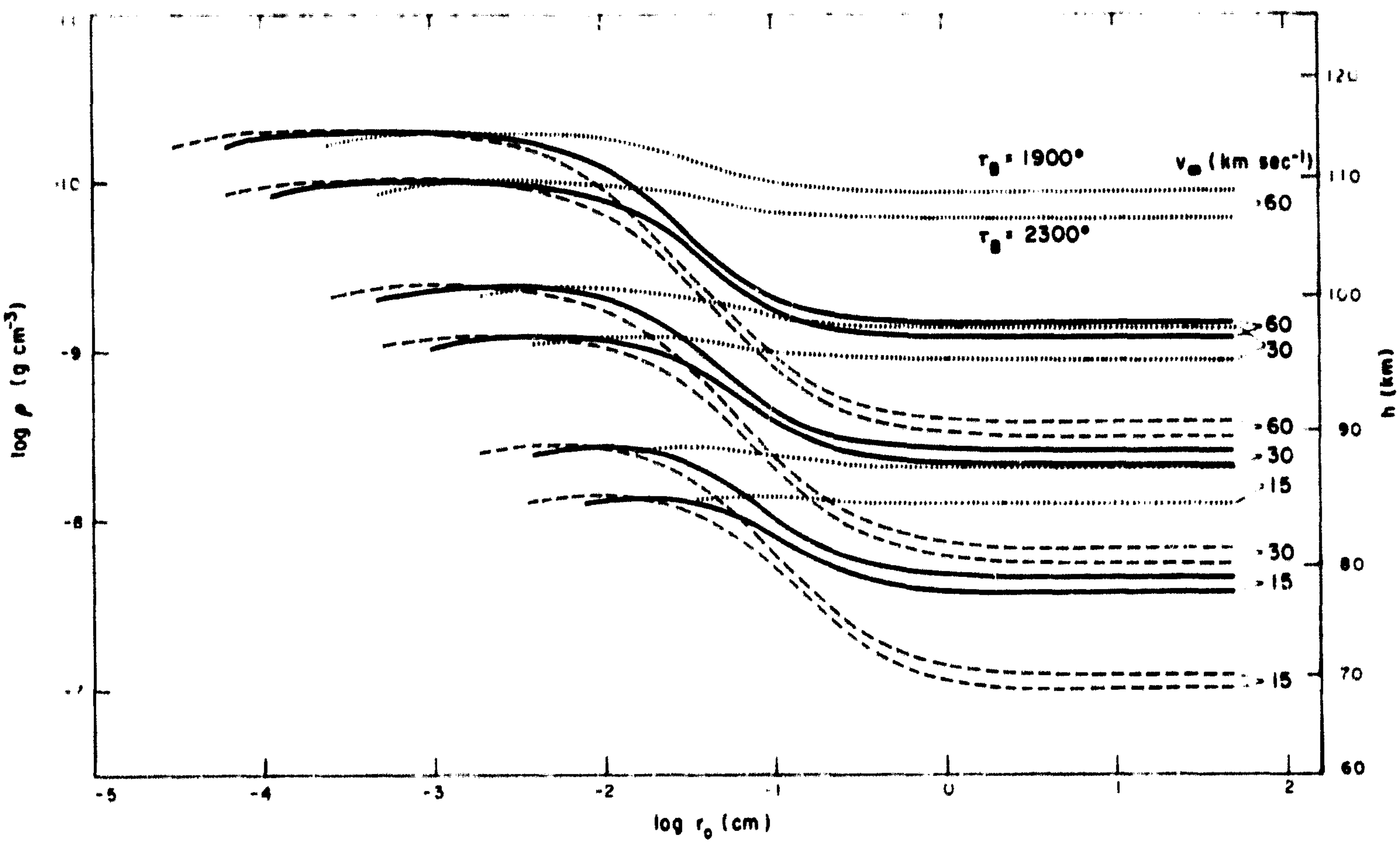

Figure 1. The air density and the height in the atmosphere, where a given surface temperature of a meteoroid is reached, are plotted against the radius of the meteoroid (in cgs units). Three velocities (15, 30, and 60 km sec^{-1}) are used. Two curves belong to each velocity and composition; the upper one is for temperature $\tau_B = 1900°$ ($T_B = 2180°K$), and the lower one is for $\tau_B = 2300°$ ($T_B = 2580°K$).

---------- iron composition ($\delta = 7.6$, $\lambda = 3 \times 10^6$, $c = 7 \times 10^6$)

———— stony composition ($\delta = 3.5$, $\lambda = 3.5 \times 10^5$, $c = 10^7$)

.... porous body ($\delta = 1$, $\lambda = 2 \times 10^4$, $c = 10^7$)

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APPENDIX

LIST OF MATHEMATICAL SYMBOLS USED

b	air-density gradient
c	specific heat of the meteoroid
F	function defined by equation (28)
h	height above sea level
k, k_1, k_2	integration constants
r	distance from the center of the meteoroid
r_0	radius of the meteoroid
t	time
T	absolute temperature
T_0	preatmospheric temperature of the body
u	function defined by equation (7)
v	velocity of the meteoroid
v_∞	initial velocity of the meteoroid
w, w_1, w_2	integration constants
Z_R	zenith distance of the radiant
β	defined by equation (2)
Γ	drag coefficient
δ	density of the meteoroid
λ	heat conductivity of the meteoroid
Λ	heat-transfer coefficient
σ	Stefan-Boltzmann constant

ρ	air density
ρ_B	air density at the height where the surface temperature τ_B is attained ($t = 0$)
τ	temperature relative to T_0, as defined by equation (3)
τ_B	surface temperature of the meteoroid, relative to T_0

BIOGRAPHICAL NOTES

ZDENĚK CEPLECHA received the RNDr. degree from Charles University, Czechoslovakia, in 1952 and the C.Sc. and D Sc. degrees from the Czechoslovak Academy of Sciences in 1955 and 1967, respectively.

Since 1951 Dr. Ceplecha has been an astrophysicist with the Astronomical Institute of the Czechoslovak Academy of Sciences in Ondřejov. In 1968 he held a National Research Council Postdoctoral Visiting Research Associateship at the Smithsonian Astrophysical Observatory.

Dr. Ceplecha's principal field of investigation is meteors and meteoroids.

ANNETTE G. POSEN received her B.A. in physics and mathematics from the University of Toronto in 1953. Since 1955 she has worked with Dr. Richard E. McCrosky, on the Harvard Meteor Project until 1964 and at SAO from 1964 to the present.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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