General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Data Analyses in Connection with the
NATIONAL GEODETIC SATELLITE PROGRAM
Contract No. NSR 09-015-018

Quarterly Progress Report No. 12
for the period January 1, 1968, through March 31, 1968

Principal Investigator: Dr. C. A. Lundquist
Project Administrator: Mr. R. W. Martin

May 1967

Prepared for
National Aeronautics and Space Administration
Washington, D. C. 20546

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

Data Analyses in Connection with the
NATIONAL GEODETIC SATELLITE PROGRAM
Contract No. NSR 09-015-018

Quarterly Progress Report No. 12
for the period January 1, 1968, through March 31, 1968

Principal Investigator: Dr. C. A. Lundquist
Project Administrator: Mr. R. W. Martin

May 1967

Prepared for
National Aeronautics and Space Administration
Washington, D. C. 20546

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

805-7

NATIONAL GEODETIC SATELLITE PROGRAM

Quarterly Progress Report No. 12
Contract No. NSR 09-015-018
for the period January 1, 1968, through March 31, 1968

Data Analysis

E. M. Gaposchkin devoted much of his time during this quarter toward improving the Geos 1 data results that were presented in his paper "Numerical Results from Geos 1." The paper derives station coordinates in the SAO system for several stations in the SECOR, GRARR, and TRANET doppler networks and for miscellaneous optical stations. Retitled "Dynamical Determination of Station Locations Using Geos 1 Data," the paper is included as an appendix to this report.

Also, a good portion of the computation effort this quarter was directed toward an analysis of data to redetermine the zonal harmonics from observation files spanning very long periods of time. This redetermination will refine existing values, paying particular attention to reference systems and homogeneity of data. The project should be completed in late May.

Data Reduction

At a meeting in March, Gaposchkin, Dr. C. A. Lundquist, and Dr. K. Lambeck decided that it would be analytically advantageous for the dynamical geodesy program to fill in gaps in satellite orbital inclinations where possible. For this reason, the low-inclination (28°) Satellite 1960 13A was scheduled for reduction analysis, to be followed immediately by a high-inclination (144°) file of observations of Satellite 1965 78A. The end-of-the-quarter status of normal and select observation files used for geodetic studies is given in Tables 1 and 2. We decided to postpone reduction of the backup 1965 78A file (Normal File No. 30) and the 1965 63A file (Select File No. 30) in favor of a number of simultaneous observations. The simultaneous observations chosen will extend the SAO geometric network to the new SAO stations in Argentina, Brazil, Ethiopia, and Greece (see Table 3). Since a geometric station-position solution is, of course, an important adjunct to any standard-earth determination, we will devote some 3 to 4 months of photoreduction effort to this solution.

International Participation

In February, SAO scientists met with scientists from the French National Space Agency (CNES) and the Paris Observatory, during which time we informally agreed upon a program of cooperative observing of the laser reflecting satellites (including Geos 2) similar to last year's successful activity. There was also an exchange of observational data resulting from the previous activity. SAO now has on file more than 78,000 French laser range observations, as shown in Table 4. The French are continuing to process their optical and doppler data.

The French analysis emphasizes the regional geometric aspect of satellite geodesy, while SAO's program stresses the dynamic technique. Hence, it was agreed that the French would coordinate simultaneous laser-optical observations in the European area, while SAO would coordinate intensive global observing periods for dynamic geodesy. Visibility patterns for the six satellites were studied to determine an optimum observing schedule for this year (see Table 5).

Observatories in Helsinki, Finland; Riga, Latvia; and Uzhgorod, USSR, have reported successful Geos 2 flash observations.

Table 1. Normal files.

No.	Satellite	Period	Total frames	Status of file
1	1965-63A	November 1965	120	in analysis
2	1965-63A	December 1965	119	in analysis
3	1965-63A	March 1966	145	in analysis
4	1965-32A	September 1965	116	in analysis
5	1965-32A	October 1965	111	in analysis
6	1965-32A	November 1965	112	in analysis
7	1965-32A	December 1965	88	in analysis
8	1965-32A	January 1966	153	in analysis
9	1965-81A	October 1965	23	in analysis
10	1965-81A	November 1965	93	in analysis
11	1965-81A	December 1965	82	in analysis
12	1965-81A	January 1966	118	in analysis
13	1965-81A	February 1966	41	in analysis
14	1965-81A	March 1966	90	in analysis
15	1965-81A	April 1966	7	in analysis
16	1965-81A	May 1966	140	in analysis
17	1965-81A	June 1966	152	in analysis
			*	
18	1965-89A	November 1965	110	in analysis
19	1965-89A	December 1965	191	in analysis
20	1965-89A	January 1966	87	in analysis
21	1965-89A	February 1966		in analysis
22	1965-89A	March 1966	187	in analysis
23	1965-89A	April 1966	231	in analysis
24	1965-89A	May 1966	70	in analysis
25	1965-89A	June 1966	29	in analysis
26	1965-89A	July 1966	125	in analysis
27	1965-89A	August 1966	301	in analysis
28	1965-89A	September 1-24, 1966	74	in analysis
29	1965-78A	October 28-November 25, 1966	487	in analysis
30	1965-78A	December 31-January 20, 1967	80	scheduled for measuring

*The following Geos A observations are only passive.

Table 2. Select files.

No.	Satellite	Period	Total frames to be reduced		Status
1	1959-01A	November 1965	304		in analysis
2	1959-01A	December 1965	274		in analysis
3	1959-01A	January 1966	372		in analysis
4	1959-01A	February 1966	195		in analysis
5	1959-01A	March 1966	400		in analysis
6	1959-01A	May 1966	487		in analysis
7	1959-01A	June 1966	667		in analysis
8	1962-60A	December 1965	526		in analysis
9	1962-60A	January 1966	406		in analysis
10	1962-60A	February 1966	375		in analysis
11	1962-60A	March 1966	450		in analysis
12	1962-60A	April 1966	442		in analysis
13	1965-32A	March 1966	718		in analysis
14	1965-32A	April 1966	625		in analysis
15	1965-32A	May 1966	533		in analysis
16	1966-05A	April 1966	932		in analysis
			(1)	(2)*	
17	1965-89A	November 1965	757	402	in analysis
18	1965-89A	July 10-August 6, 1966	3851		in analysis
19	1965-89A	Sept. 25-October 23, 1966	3719	220	in analysis
20	1967-14A	April 30-June 3, 1967	2659		in analysis
21	1967-11A	April 16-May 20, 1967	886		in analysis
22	1965-89A	February 26-March 25, 1967	728		in analysis
23	1967-11A	February 19-March 25, 1967	476		in analysis
24	1967-14A	February 19-March 25, 1967	651		in analysis
25	1964-64A	February 26-March 25, 1967	318		in analysis
26	1965-32A	March 12-April 29, 1967	945		in analysis
27	1964-64A	May 7-June 3, 1967	293		in analysis
28	1960-13A	December 24-February 2, 1967	1011		in analysis
29	1960-13A	June 2-July 28, 1967	767		in analysis
30	1965-63A	September 1-October 13, 1967	329**		scheduled

*Geos observations have been divided into (1) flash and (2) passive categories.

**Field statistics: Number of measurable images will be about three times larger.

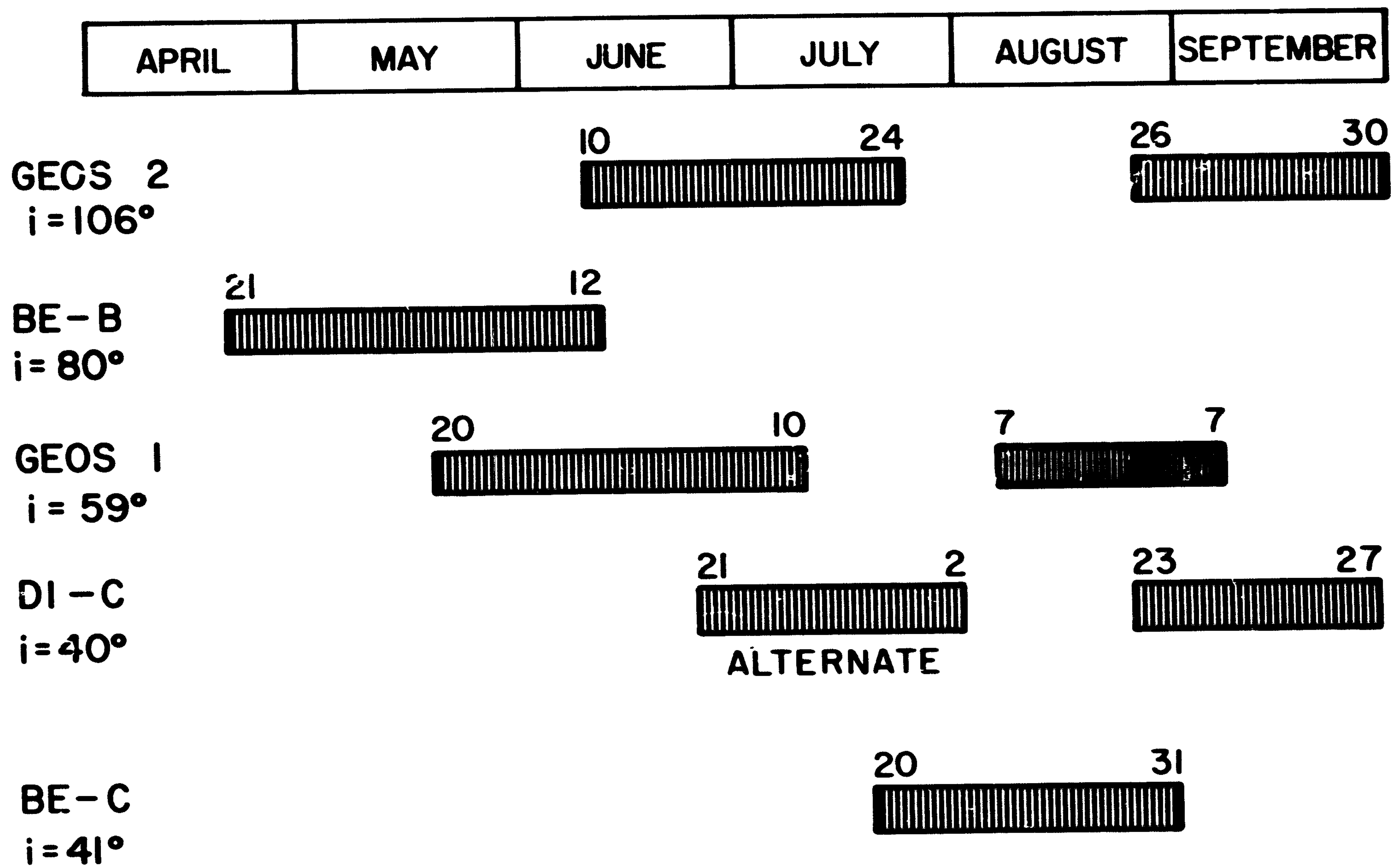
Table 3. Simultaneous observations to be reduced for geometric analysis.

Line	Number of observations
New Mexico — Peru	4
South Africa — Iran	14
South Africa — Ethiopia	30
Spain — Curaçao	7
Spain — Florida	13
Spain — Ethiopia	30
Spain — Brazil	20
Spain — Malvern, United Kingdom	24
Japan — Hawaii	9
Japan — Johnston Island	8
India — Ethiopia	30
Peru — Brazil	30
Peru — Comodoro Rivadavia, Argentina	30
Iran — Ethiopia	30
Iran — Greece	30
Florida — Cold Lake, Alberta	12
Villa Dolores — Comodoro Rivadavia, Argentina	30
Hawaii — Cold Lake, Alberta	10
Ethiopia — Greece	30
Brazil — Comodoro Rivadavia, Argentina	30
Brazil — Greece	10

Table 4. 1967 cooperative French laser observations.

Satellite		Haute Provence France	Stephanion Greece	Coulomb-Bechar Algeria
DI-D	Ranges	5025	10933	7785
	Passes	107	147	61
DI-C	Ranges	6871	11453	4141
	Passes	113	138	45
BE-B	Ranges	2305	2336	892
	Passes	54	53	13
BE-C	Ranges	3975	5685	689
	Passes	50	67	12
Geos 1	Ranges	6207	7407	3092
	Passes	118	90	19

Table 5. 1968 laser satellite intensive observing schedule.



APPENDIX

DYNAMICAL DETERMINATION OF STATION LOCATIONS USING GEOS 1 DATA

E. M. Gaposchkin

April 1968

DYNAMICAL DETERMINATION OF STATION LOCATIONS USING GEOS 1 DATA

E. M. Gaposchkin

1. INTRODUCTION

The Smithsonian Astrophysical Observatory (SAO) has for some years been actively engaged in a geodesy program. This field of study is in the process of expansion through the acquisition of new sources of data, new methods of data analysis, and the combination of satellite geodesy with classical techniques. The general background, basic concepts, and broad context of this program will be discussed in separate papers by Lundquist and Rolff.

Our most recent significant achievement was the publication in 1966 of the Smithsonian Institution Standard Earth (Lundquist and Veis, 1966), which was based entirely on Baker-Nunn camera observations. Before this work was accomplished, we had already recognized that a wider distribution of data and a greater variety of data types would improve geodetic results.

We are encouraged in the expansion of our program by the investigations currently pursued by the Applied Physics Laboratory of Johns Hopkins University (APL) and by the Naval Weapons Laboratory (NWL) with the use of electronic TRANET doppler data. It is reassuring that the geodetic results obtained by SAO, APL, and NWL are in reasonably good agreement. The combination of the data from these sources is a logical advance in geodetic studies. Moreover, additional sources of data are becoming available. Smaller observatories and geodetic institutes can participate in global geodetic investigations with an illuminated satellite. Newer electronic systems such as the Goddard Range and Range Rate (GRARR) and the SECOR systems are beginning to acquire data in fairly large amounts, and the development of laser tracking provides greater accuracy.

This work was supported in part by contract NSR 09-015-018 from the National Aeronautics and Space Administration.

The first step in combining different kinds of data to solve the geodetic problem involves selection of additional data types, establishment of suitable variances, reduction to a uniform time and spatial reference system, and, as part of the last item, adoption of an initial set of station locations. A geodetic satellite such as Geos 1 is ideal for these tasks. Each of the cooperating agencies has acquired data from this satellite, and its orbital characteristics are such that we can be reasonably confident in using a dynamical theory.

2. SCOPE AND GOALS OF THIS STUDY

First, and most important, we have used data from a variety of tracking systems (optical directions, range, and range rate) to determine station locations in a geocentric Cartesian coordinate system. Second, having performed this adjustment, we have obtained some measure of the validity, accuracy, and potential usefulness of these data-acquisition systems for future work.

Our approach to the problem is the so-called dynamical method, which is discussed in detail in the Smithsonian Institution Standard Earth. Some use will be made of simultaneous optical observations to ascertain the validity of the positions of the optical stations determined in our reference system.

I assert that the results given here are only an indication of the accuracy of the system, and in the final analysis, a dynamical theory cannot be used to calibrate an observing system with an accuracy greater than 100 m. Such a calibration can be performed only by intercomparison.

3. REFERENCE SYSTEMS AND ORBITAL ACCURACIES

The 1966 Smithsonian Institution Standard Earth forms the basis for this analysis. The coordinate systems are briefly as follows. The inertial reference frame is referred to the equinox of 1950.0 and the equator of date.

The terrestrial reference frame is referred to the mean pole of 1900. 0 to 1905. 0 and the longitude of the mean observatory at Greenwich. The coordinates of the SAO Baker-Nunn cameras expressed in this terrestrial system are the C6 coordinates of the Standard Earth. The relation between these two frames of reference is given by the measured values of the time UT1 and the position of the pole.

The usefulness of the dynamical method hinges exclusively upon the accuracy of the orbital ephemeris. This, in turn, depends on the accuracy of the orbit theory itself, which includes uncertainties in the earth gravity-field model adopted, and on the accuracy with which the orbital elements of the satellite can be determined. Unfortunately, Geos 1 was in an orbit that is resonant with some of the 12th-order tesseral harmonics. Therefore, before any attempt can be made to use the dynamical method, these harmonics must be determined quite accurately.

The important harmonics with which Geos 1 is resonant are ℓ, m = 12, 12; 13, 12; 14, 12; and 15, 12. One satellite is not adequate for the determination of the eight numerical parameters. Fortunately, we have observations of another satellite, 1960 ι2, resonant with the same harmonics and of essentially different orbital characteristics. The required harmonics can be determined by the combined use of these two satellites.

Table 1 gives the orbital characteristics of Geos 1 and 1960 ι2, with other relevant information. The first step, then, is to determine the resonant gravity-field harmonics from optical observations of these two satellites. The harmonic coefficients determined in this way are shown, with additional geodetic information, in Table 2.

The question of the accuracy of the reference orbits can be answered, in part, by the range observations acquired by the SAO laser tracking system collocated with the SAO Baker-Nunn camera at Organ Pass, New Mexico (station 9001). The collocation eliminates any problem of possible timing-system differences or errors in the station coordinates. If we use the reference orbits computed without the laser observations and compare the computed ranges with the laser observations, we get a measure of the accuracy in an absolute sense of the orbit theory. The mean value of 20 m agrees quite well with previous estimates of the orbital accuracy and must be taken to be the accuracy we can expect.

Table 1. Characteristics of Geos 1 and 1960 ι2

		Geos 1	1960 ι2
a		8.073861 Mm	7.971380 Mm
e		0.070941	0.0114367
I		59°.38020	47°.231275
n		11.967616 rev day^{-1}	12.197092 rev day^{-1}
$\sqrt{\bar{C}_{\ell,m}^{2} + \bar{S}_{\ell,m}^{2}}$ (maximum amplitude)			
ℓ	m		
12	12	60 meters	7 meters
13	12	490 meters	360 meters
14	12	90 meters	26 meters
15	12	310 meters	630 meters
Period of perturbation:		7.1 days	14.5 days

Table 2. Geodetic constants

Velocity of light	c	2.997925×10^{10} cm sec^{-1}
Gravitational constant times earth mass	GM	3.986013×10^{20} cm^3 sec^{-2}
Semimajor axis of the earth	a_e	6.378155×10^6 m
Zonal harmonics	J_n	Kozai solution to J_{14}*
Tesseral harmonics	$\overline{C}_{\ell,m}\overline{S}_{\ell,m}$	M1 solution* with the following changes: $\overline{C}_{13,12} = -6.848 \times 10^{-8}$ $\overline{S}_{13,12} = 6.57 \times 10^{-8}$ $\overline{C}_{14,12} = 0.261 \times 10^{-8}$ $\overline{S}_{14,12} = -2.457 \times 10^{-8}$ $\overline{C}_{15,12} = -7.473 \times 10^{-8}$ $\overline{S}_{15,12} = -1.026 \times 10^{-8}$

*From Lundquist and Veis (1966).

The mean elements of these reference orbits are plotted in Figure 1. We note that the semimajor axis has a consistent variation of not more than 10 m. The eccentricity and inclination show the long-period effect of the earth's oblateness; this effect has a period of 550 days. Including the laser observations in the orbit determination does not change the values of the elements to any significant extent, and the mean value of the range residuals computed with respect to these orbits is 10 m.

As stated in Lundquist and Veis (1966), the internal consistency of the fundamental Baker-Nunn coordinates is 15 m. The orbital ephemeris is computed for 1-month arcs and has an accuracy of 20 m. Therefore, we cannot hope to determine the station positions to an accuracy better than 15 to 20 m.

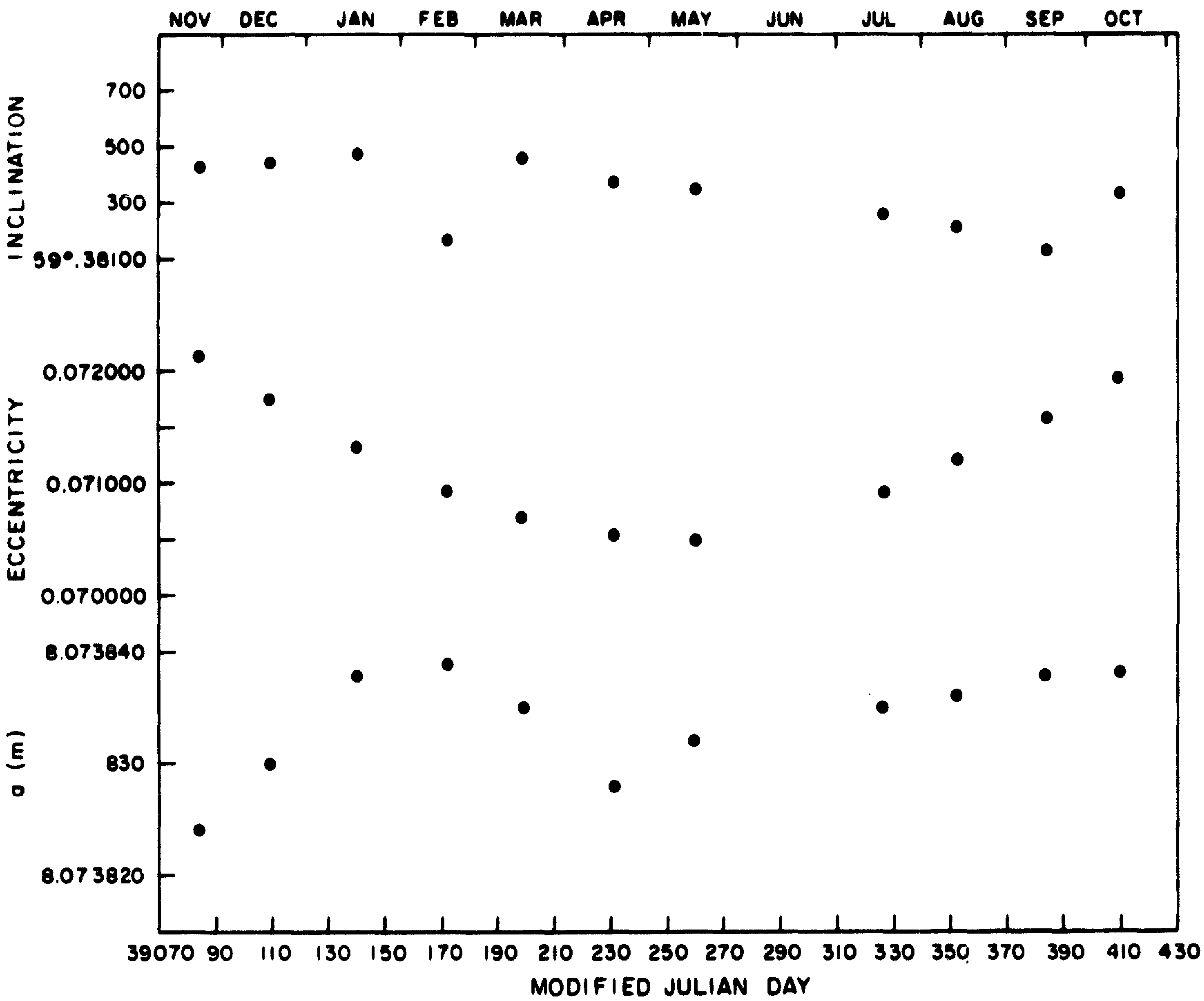


Figure 1. Mean inclination, eccentricity, and semimajor axis of Geos 1 for November 1965 to October 1966.

4. DETERMINATION OF THE LOCATIONS OF NINE MISCELLANEOUS OPTICAL SITES

The flashing light of Geos 1 was observed by several observatories. The coordination of the flashes removed the problems of mixed time systems. The observation of a flash sequence provided a set of points. We reduced each flash sequence to a synthetic observation for use in the dynamical determination of the station coordinates. These observations seemed to have an accuracy of nearly 1 arcsec.

The essential results are given in Table 3. The initial coordinates, the corrections, and the resulting coordinates are shown in the columns labeled X, Y, and Z. The numbers of synthetic observations are also given. For all stations except 9113, a direction to this previously unknown station from an SAO station had been determined by the use of simultaneous observations. In general, these directions are determined from other satellites as well. If the distance between the stations is known, this direction would suffice to determine the station coordinates uniquely. If we adopt the dynamically determined position to compute the distance, we can compute the location. The equivalent corrections from this method are also given in Table 3. This calculation is merely a consistency check.

In general, the agreement is good when there are sufficient observations. The coordinates for Rosamund seem well determined. The three stations at Cold Lake, Harvestua, and Johnston Island were determined in the Standard Earth. In each case the data were few and were acquired from geodetically less useful satellites. Since Geos 1 is essentially a better satellite, more orbital arcs are used here, and the agreement with the directions is good, the coordinates determined from Geos 1 for Cold Lake and Harvestua are preferable to the earlier results. Nevertheless, the number of observations is marginal, and these coordinates can be considered only provisional. Since

Table 3. Dynamical determination of station coordinates (optical observations of Geos 1)

Station	X (Mm)	Y (Mm)	Z (Mm)	Correction from the direction between stations dx (m)	dy (m)	dz (m)	Stations	Number of synthetic observations
9113 Rosamund California	-2.450064 +28 -2.450036	-4.624412 -24 -4.624388	3.635023 11 3.635034					243
9114 Cold Lake Canada	-1.264846 -13 -1.264859	-3.466880 27 -3.466853	5.185464 -8 5.185456	-5	46	-3	9001, 9009, 9010, 9012	28
9115 Harvestua Norway	3.121265 1 3.121266	0.592600 20 0.592620	5.512684 18 5.512702	7	17	13	9004	34
9117 Johnston Island Pacific	-6.007395 72 -6.0073[illegible]	-1.111893 80 -1.111813	1.825725 -10 1.825715	-22	39	-14	9012	17
9050 Agassiz Massachusetts	1.489724 23 1.489747	-4.467505 13 -4.467492	4.287291 -21 4.287270	12	12	-8	9001, 9009, 9010	41
9066 Zimmerwald Switzerland	4.331312 10 4.331322	0.567475 15 0.567490	4.633124 15 4.633139	11	25	-2	9004	109
9074 Riga Latvia	3.183913 -161 3.183752	1.421510 -51 1.421459	5.322773 -13 5.322760	-53	3	53	9004	15
9080 Malvern England	3.920160 34 3.920194	-0.134757 -2 -0.134759	5.021706 42 5.012748	25	14	30	9004	20
8015 Haute Provence France	4.578321 4 4.578325	0.457957 2 0.457959	4.403167 63 4.403230	13	13	52	9004	77

for Johnston Island the agreement between the direction and the dynamical determination is poor, and since there are so few observations, this determination must be considered unreliable. The remaining stations are all new. Haute Provence and Zimmerwald are clearly well determined. Because of the small number of observations at Agassiz and Malvern, the determination would have to be provisional, but the good agreement between the direction and the dynamical determination is very encouraging. Riga is a first attempt.

5. DETERMINATION OF THE SECOR RANGE STATIONS

The Goddard Data Bank provided us with more than 20,000 observations from four stations. These data were obtained from as many as 17 passes from each station. In our opinion, much of this large volume of data was redundant. We therefore removed 9 of every 10 observed points, rather than fitting polynomials to the 10 points to compute a synthetic observation, because the data from each pass were extremely coherent. The noise level from the mean was 5 m or less. Nothing would have been gained by the use of synthetic points.

SECOR data have a range ambiguity of 256 m. This is because the equipment is so constructed that the range is determined from the properties of an electromagnetic wave with a 256-m wavelength. The analysis must provide the range to within that accuracy. Therefore, in the determination of station locations, we used the residuals modulo 256 m. Hence, we never computed a residual greater than 128 m. We rejected residuals greater than 100 m.

Table 4 summarizes the data available. It details the standard errors (σ) and the corrections to the station locations computed. Because of the small number of passes available and the standard error relative to the rejection criterion, we consider this determination unacceptable. In Table 4 the corrections are resolved into the height component because this sometimes provides an insight into possible problems with an ionospheric or elevation correction. This is not the case here. Table 5 gives the initial coordinates used for these stations.

Table 4. Dynamical determination of station coordinates (SECOR)

	Station			
	5001 Herndon Virginia	5333 Greenville Mississippi	5648 Ft. Stewart Georgia	5861 Homestead Florida
January '66 passes	4	5	5	4
February '66 passes	1	1	1	1
March '66 passes	6	5	3	5
April '66 passes	6	6	-	5
Total passes	17	16	9	15
Number of observations	742	641	219	550
σ (m)	48	53	54	54
Corrections (m)				
dx	10	-9	8	-10
dy	5	4	-31	20
dz	29	12	-48	27
dh	18	3	11	-7

Range ambiguity of 256 m removed

Maximum residual accepted = ±100 m

The small number of passes would not allow a very good determination of the station locations. However, the size of the standard error comes from the data set itself. Either difficulties in converting the time systems or systematic errors in the data seem the most likely reasons for the large standard error.

Table 5. Initial coordinates for SECOR stations (Mm)

Station	X	Y	Z
5001 Herndon, Virginia	1.088856	-4.842927	3.991836
5333 Greenville, Mississippi	-0.085002	-5.327944	3.493472
5648 Ft. Stewart, Georgia	0.794688	-5.360041	3.353082
5861 Homestead, Florida	0.963463	-5.679723	2.728118

6. DETERMINATION OF THE GRARR STATIONS

The GRARR system provides both distance and velocity measurements. Geos 1 was observed from three stations during the interval of our precise orbits. By far the largest amount of data came from the station at Rosman, North Carolina.

It was found that careful data selection was necessary. We obtained the raw data directly from Goddard and developed our own reduction methods and rejection criteria. During that phase of the analysis we were in close contact with the Goddard Intercomparison Effort, and we were fortunate to be able to incorporate their findings into our analysis. We found polynomial fitting to short intervals (e.g., 20 sec) valuable for two reasons: First, the smoothed or synthetic points provided significantly better results for station-coordinate determination than did the raw data points used "en masse." Second, the standard error of the curve fit proved to be an excellent rejection criterion. For the range rate data we used virtually all the data available. For the range data, a rejection criterion of 8 m in the curve fit satisfactorily discriminated good from bad passes. The 8-m criterion should not be interpreted as the accuracy of the data; it is only a measure of the internal consistency of the data for a short interval.

Table 6 details the results of the determination of three stations. Five points per pass were used. Clearly, the combination solution depends on the adopted uncertainties of the two kinds of data, and these were taken at 30 m for range and 15 cm sec^{-1} for range rate. In addition, the relative number of data points is important. We rejected residuals at 100 m for range and 45 cm sec^{-1} for range rate. Since the correction for station 4714 was larger than the rejection criterion, we performed a second iteration to verify convergence.

The results for station 4713 are quite reasonable; the data set was good. The initial coordinates were given in the North American datum. In addition, the effective correction of 16 m in height agrees with the determination by Brown (1967) from short-arc studies. The Madagascar and Australian coordinates must be considered preliminary at this stage. The small amount of data, the lack of a comparison, the high rejection rate of the data, and the lack of timing records all support this conclusion.

7. DETERMINATION OF THE TRANET DOPPLER STATIONS

The TRANET network provided data from 10 stations, generally 30 points per pass. The data were available through the entire period of precise orbits. The ionospheric correction had, of course, been removed. In addition, a preliminary frequency correction had been applied. These doppler data were treated in the same way as the Goddard range rate data.

Table 7 gives the corrections computed for the 10 sites. The initial coordinates were heterogeneous. As designated in Table 7, six stations were initially taken from an APL solution (H. Black, 1968, private communication); the remaining were taken from the Goddard directory. However, some comparisons are possible. Any solution for station coordinates computed solely with electronic data is indeterminate by one longitude. If the longitude of one station is fixed, a unique solution is possible. Therefore, for solutions to be compared, this rotation must be removed.

Table 6. Dynamical determination of station coordinates (GRARR)

Station	dX (Mm)	dY (Mm)	dZ (Mm)	dh (m)	Number of range observations	Number of range-rate observations	σ unit weight
4713 Rosman North Carolina	16	-3	19	15	4485	9764	0.90
Final coordinates	0.647192	-5.178336	3.365152				
4714 Madagascar	40	-95	-107	-6	124	1056	1.08
Final coordinates	4.091419	4.434122	-2.066035				
4715 Carnarvon Australia	-59	-34	-4	-5	225	1358	1.11
Final coordinates	-2.328264	5.299679	-2.669395				

	Range	Range Rate
Assumed accuracy	30 m	0.15 cm sec^{-1}
Rejection criterion	100 m	0.45 cm sec^{-1}

Table 7. Dynamical determination of station coordinates (TRANET)

Station	σ	Number of obs.	dX	dY	dZ
7014* Anchorage, Alaska	0.93	9014	-50	5	-17
7017 Tafuna, American Samoa	1.07	6108	-55	342	109
7019* McMurdo Scund, Antartica	0.96	3263	-42	66	-102
7100* South Point, Hawaii	0.97	18088	-25	15	54
7103* Las Cruces, New Mexico	0.91	19890	-25	16	74
7106* Lasham, England	0.99	23615	-54	-38	42
7111* Johns Hopkins University Baltimore, Maryland	0.85	24595	-34	-76	-21
7739 Shemya, Alaska	0.71	4986	31	226	7
7742 Beltsville, Maryland	0.85	3533	-41	-42	-23
7745 Stoneville, Mississippi	0.89	4279	-66	-35	-27

Assumed accuracy = 15 cm sec^{-1}; rejection criterion = 45 cm sec^{-1}.

*Initial coordinates APL 3.5 solution.

With several points given in two coordinate systems that differ by a rotation, this rotation can be determined. If we introduce the infinitesimal rotation (Goldstein, 1950, p. 124) $\mathcal{R}(d\Omega_1, d\Omega_2, d\Omega_3)$ such that

$$\mathcal{R}(d\Omega_1, d\Omega_2, d\Omega_3) = \begin{bmatrix} 1 & d\Omega_3 & -d\Omega_2 \\ -d\Omega_3 & 1 & d\Omega_1 \\ d\Omega_2 & -d\Omega_1 & 1 \end{bmatrix} ,$$

we want to find $d\Omega_i$ such that

$$\bar{X}_j^{SAO} = \mathcal{R}(d\Omega_i)\, \bar{X}_j^{APL} .$$

We have three candidates for such a computation and comparison: the APL 3.5 coordinates, a set of coordinates given by Anderle and Smith (1967), and a set attributed to Guier and Yionoulis (Anderle and Smith, 1967). These three sets of coordinates will be designated as X_j^{APL}, X_j^{A}, and X_j^{GY}, respectively. In each case, the subset of stations is different. During the comparison, the determinations of X_{7017}^{SAO} and X_{7019}^{SAO} showed large disagreement and were therefore not included in the determination of the relative positions of the reference systems. Table 8 gives the relative rotations of the reference systems in seconds of arc and the standard error of the determination in meters. The rotation in terms of meters at the surface of the earth is also included.

The physical significance of the $d\Omega_3$ is a rotation in longitude and corresponds to the difference in the adopted longitude of the TRANET solutions and the longitude of the mean observatory; $d\Omega_1$ and $d\Omega_2$ would correspond to the differences in the adopted pole of the difference solutions. While SAO used observed values of the polar motion in its analysis, none of these data were used for the TRANET solutions (Black, 1968, private communication); hence, the resulting pole is defined by a mean of the data arcs used. The computed values are consistent in sign and magnitude with this interpretation.

The differences in the values of GM used in the solutions are small, as evidenced by the values adopted (Lundquist and Veis, 1966; Black, 1968, private communication):

$$\overline{X}_j^{A} \quad GM = 3.986010 \times 10^8 \ Mm^3 \ sec^{-2}$$

$$\overline{X}_j^{SAO} \quad GM = 3.986013 \times 10^8 \ Mm^3 \ sec^{-2}$$

$$\overline{X}_j^{GY} \quad GM = 3.986015 \times 10^8 \ Mm^3 \ sec^{-2} \quad .$$

Table 8. Relation between the various reference systems: SAO C6 coordinates (SAO); APL 3.5 coordinates (APL); Anderle coordinates (A); Guier and Yionoulis coordinates (GY).

	$d\Omega_1$ (arcsec)	$d\Omega_2$ (arcsec)	$d\Omega_3$ (arcsec)	$a_e d\Omega_1$ (m)	$a_e d\Omega_2$ (m)	$a_e d\Omega_3$ (m)	σ (m)
SAO-APL	-0.02	0.42	1.24	0	13	41	41
SAO-GY	-0.93	0.35	2.11	-28	10	65	35
SAO-A	-0.85	0.36	0.91	-26	11	28	18
A-GY	-0.08	-0.03	0.94	-2	-1	29	13

The standard errors of 18 m for the Anderle solution relative to the SAO solution and of 35 m for the Guier and Yionoulis solution relative to the SAO solution are quite satisfactory in view of the 13-m agreement between the A and the GY (Table 8). Considering that both the TRANET solutions also used other satellites and involved a further improvement of the frequency and tropospheric correction, their reliability is much enhanced. This is especially true for station 7019, which is at -77° latitude. Since Geos 1 is of 59° inclination, all the data used in our analysis were low passes to the north, which resulted in very poor geometry. The poor results from station 7017 cannot be attributed to its latitude. Table 9 provides the final coordinates determined from GEOS 1.

Table 9. Final coordinates of the TRANET stations

Station	X_j^{SAO}			$X_j^{SAO} - X_j^A$		
	X (Mm)	Y (Mm)	Z (Mm)	dx (m)	dy (m)	dz (m)
7014 Anchorage Alaska	-2.656183	-1.544326	5.570618	22	25	-23
7017 Tafuna American Samoa	-6.100005	-0.997366	-1.568560	19	-244	-73
7019 McMurdo Sound Antartica	-1.310712	0.310531	-6.213456	-11	25	-83
7100 South Point Hawaii	-5.504199	-2.224095	2.325278	-20	-3	-7
7103 Las Cruces New Mexico	-1.556251	-5.169461	3.387239	19	-23	10
7106 Lasham, England	4.005469	-0.071800	4.946720			
7111 Johns Hopkins University Baltimore, Maryland	1.122608	-4.823073	4.006486	11	-1	29
7739 Shemya, Alaska	-3.851550	0.397301	5.051523	-16	15	63
7742 Beltsville, Maryland	1.130731	-4.830861	3.994701	-4	-13	-8
7745 Stoneville, Mississippi	-0.085070	-5.327989	3.493425	-17	-14	-10

8. SUMMARY AND CONCLUSIONS

The geodetic satellite Geos 1 has been immensely successful in the determination of the locations of many new stations in the SAO C6 system. In some cases these coordinates are preliminary, in the sense that the determination is thought to be significantly worse than the 20-m accuracy that could be desired. Where stations had previously been determined by earlier and more comprehensive analysis, these results can be viewed as a confirmation of our technique, and an adjustment or average may provide somewhat more realistic results. The values determined for the relation between the SAO and the TRANET systems are considerably more reliable than any of the individual determinations. In any case, the station coordinates determined here are suitable for an initial set to be used in future large-scale solutions. It is quite clear that it is desirable and feasible to combine the SAO Baker-Nunn observations, other optical observations of good quality, and GRARR, TRANET, and laser observations in a comprehensive global solution for station coordinates and the gravity field with the use of a wide variety of satellites.

REFERENCES

ANDERLE, R. J. and SMITH, S. J.

1967. NWL-8 geodetic parameters based on doppler satellite observations. U. S. Naval Weapons Laboratory Tech. Rep. No. 2106, July, 56 pp. .

BROWN, D. C.

1968. GEOS A short arc optical survey of a sixteen station mid-North American net. Proceedings of the NASA GEOS Program Review Meeting 12-14 December 1967, ed. by Communications & Systems, Inc., vol. III, pp. 1-20.

GOLDSTEIN, H.

1950. Classical Mechanics. Addison-Wesley, Reading, Mass., 399 pp.

LUNDQUIST, C. A. and VEIS, G., eds.

1966. Geodetic parameters for a 1966 Smithsonian Institution Standard Earth. Smithsonian Astrophys. Obs. Spec. Rep. No. 200, 3 vols., 686 pp.