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THE ATMOSPHERIC PRESSURE AT THE SURFACE OF MARS

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Summary

Photometric observations of eclipses of Phobos by Mars were carried out by a new photoelectric scanning technique for the purpose of estimating Martian atmospheric pressure on the surface of the planet. An upper limit of 30 mb for the pressure at a mean temperature of 200° K was obtained. The observations show strong evidence of particle absorption at high levels in the atmosphere. The data suggest that the absorption level which causes the so-called "blue haze" is not higher than 10 km above the surface of the planet. The question as to what value should be adopted for the surface pressure on Mars can be solved by first answering the question as to what distribution of solid particles in the atmosphere is the most probable.

Introduction

Determination of the Martian surface pressure has been attempted by numerous investigators. The earlier measurements are reviewed by de Vaucouleurs (1). Recently Dollfus (2, 3) has published his measurements of polarization across the disk at different phase angles and different colors. The principal difficulty affecting all of these photometric and polarimetric investigations is that of distinguishing between the amount of light scattered by the Martian surface, by the solid particles in the atmosphere and by the atmosphere itself.

The surface pressure can be computed also from the pressure broadening of the lines in the Martian band spectrum of CO_2. The first results of the spectroscopic method (4) disagree by an order of magnitude from previously accepted values, which were based on photometry and polarimetry. Other spectroscopic measurements made by numerous investigators in 1964 and 1965 have confirmed the disagreements mentioned above.

This difference of pressure represents a very substantial difference in the cost of landing vehicles on Mars; at a very low pressure, rockets must be used; while at higher pressure, one could use parachutes.

Another way of determining the density of the Martian atmosphere is from photoelectric measurements of the eclipses of the Martian satellite Phobos. Such measurements are extremely difficult. Phobos is fainter than Mars by a factor of 10^5 and at the time of egress or ingress not less than 4 to 5 seconds of arc distant from the Martian limb. Theoretically, an occultation of Phobos or of a star could also be used for such measurements. Unfortunately, the phase of Mars does not exceed one second of arc and an occultation of a star brighter than tenth magnitude seldom occurs.

Since the author believed that the chance of successfully making eclipse measurements would be small and to avoid any loss of light, he decided against the use of color filters. The forthcoming Martian oppositions should provide much better opportunities to make these measurements, because of the shorter distance between the earth and Mars. Such photometric data should give more information about the nature of the solid particles in the Martian atmosphere.

Equipment and Observations

Precise photoelectric measurements of Phobos near Mars cannot be made by using a circular diaphragm in conventional photoelectric equipment. The scattered light of Mars in the earth's atmosphere and in the optical system of a telescope rules out the possibility of making successful measurements of this kind.

New equipment, described as a photoelectric area scanner, was therefore developed (5). A scanning speed for the moving slit of ten scans per second was used throughout the program. This number of scans was found to permit good resolution because the moments of best seeing can be captured in this way by single scans. On the other

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hand, if the number of scans is too great, the total number of photoelectrons for a single scan will prove inadequate.

The slit width was adjusted to 50 microns and the length of the slit to 500 microns. In scanning, the slit was moved from a position nine seconds of arc away from the limb of Mars to within an angular distance of about one second. When the equipment is adjusted in this way and used with the new 61-inch astrometric reflector of the U. S. Naval Observatory in Flagstaff, one gets, according to conditions of the sky, between 30 and 50 photoelectrons for a single scan across the image of Phobos. This fact indicates the importance of using a telescope with large aperture, good images and a minimum of scattered light. Of the telescopes available in Flagstaff, the new 61-inch astrometric reflector of the U. S. Naval Observatory best satisfied these requirements. The author wishes to express his appreciation to Kaj Aa. Strand and the staff of the Naval Observatory Flagstaff Station for the privilege of using this fine telescope and for their most effective assistance during the course of the observations. He is equally grateful to Arthur A. Hoag and to the astronomers of Kitt Peak National Observatory for the privilege of using the 84-inch telescope on two nights during the second run of observations in June 1965.

The experience obtained with these two telescopes during two different parts of the year and in two different locations—Flagstaff and Kitt Peak—shows that the highest percentage of errors will be introduced by the scattered light of Mars in the neighborhood of the image of Phobos. Neither the small number of photoelectrons in a single scan and the resultant statistical scattering, nor the effects of seeing are of such paramount importance. The bad seeing affects the accuracy of the observations much more indirectly by increasing the scattered Martian light in the earth's atmosphere than directly by the broadening of the image of Phobos.

The amount of scattered light changes, within a good approximation, exponentially with the distance from the light source in the sky. The distance from Phobos to the limb of Mars is close to one-half of the Martian diameter. This means that the ratio of scattered light to the brightness of Phobos changes appreciably with the distance of Mars from earth. There are many other factors, too, which should be taken into consideration, such as the darkness of the night sky, the content of dust in the air and not least, configuration in the sky described by the orbital elements of earth, Mars and Phobos. The author wishes to express his appreciation for the very valuable assistance given him by Raynor L. Duncombe of the U. S. Naval Observatory for providing the ephemeris of Phobos.

As mentioned in an earlier paper (5), each scan is displayed on the screen of an oscilloscope and photographed by a continually moving film camera. To spare film material and to make the reductions more convenient, approximately four consecutive scans were contained within every single sweep of the oscilloscope. Figure 1 shows an enlargement of the film with a set of four scans.

The scattered light causes the background noise to increase as the slit approaches the planet. This is clearly seen on each scan in the figure. One can also recognize the response due to Phobos near the middle of the scan. When not in eclipse a signal from Phobos was detectable in about two out of three scans. Also, at ten minute intervals, a time mark was displayed on the oscilloscope.

Reduction of the Observations

As described in the previous section, the brightness of Phobos when measured from a single scan has a very low accuracy. The author tried to use three different methods of reduction for this very extensive observational material.

In the first, he selected the best scans and estimated the area on the film given by photoelectrons from Phobos. This did not produce good

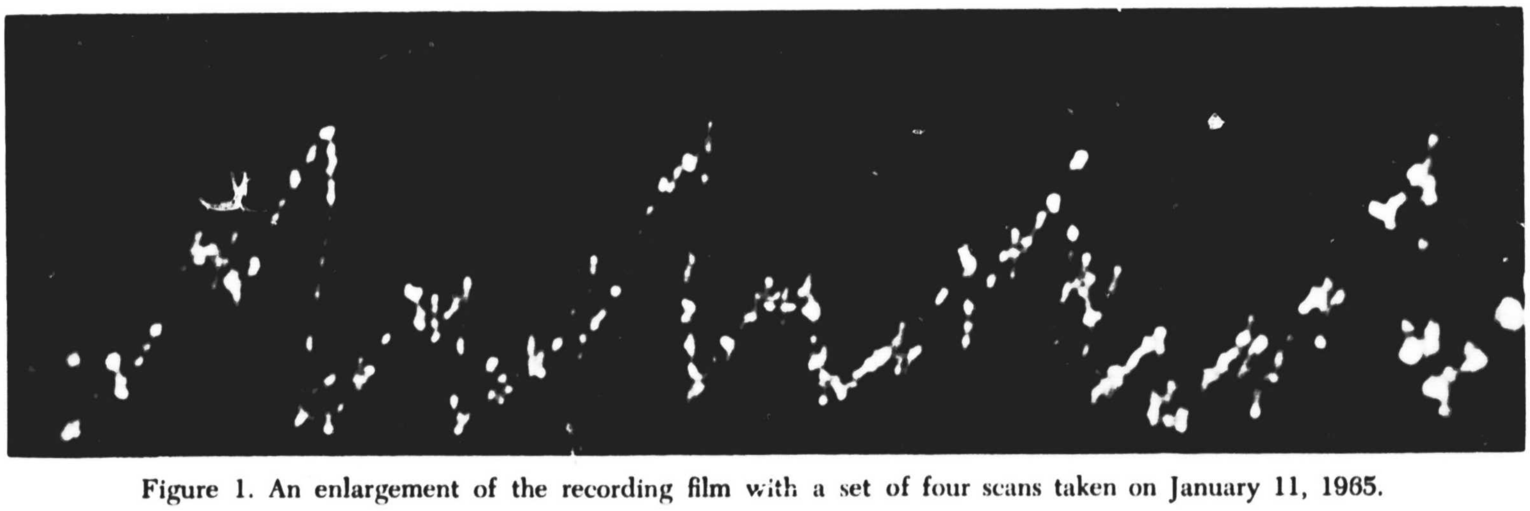

Figure 1. An enlargement of the recording film with a set of four scans taken on January 11, 1965.

results, since it was difficult to decide which scans were the best ones.

In the second method, he tried to average out the statistical fluctuations by making composite prints of 25 consecutive scans and to get a mean from these. The results were fair.

The third method, which proved to be the fastest one and also gave the best means, consisted of visually estimating each mean from a corresponding set of about 100 consecutive scans. First it was decided which scans were useful or useless. Useless scans were those which showed too much scattered light because of bad seeing. On an average less than 25 percent of all scans were rejected. The proportion of useful scans provided the weight for each mean. In each useful scan, the area given by photoelectrons from Phobos was estimated. The signal intensity was estimated as 0, 1, 2, or 3. To prevent systematic errors and the accumulation of personal errors on any one part of the film, each set of 100 scans was estimated many times regardless of its position on the film. Also, the estimations were made without any knowledge on the part of the observer regarding the phase of the eclipse.

The mean value between five to eight of such sets, according to their weight, was formed in order to get all observational points of equal mean error. This resulted in about one observational point for each minute of time. This kind of reduction of the material obtained in January, 1965, gives useful accuracy compatible with an acceptable time constant which, in turn, is of course closely related to the eclipse duration. The eclipses observed in January are very favorable from this point of view. They are all grazing eclipses in contrast to the eclipses observed in June. It is assumed for purposes of calibration that the transparency of the earth's atmosphere and the sensitivity of the equipment were constant within a range of ±[illegible] percent

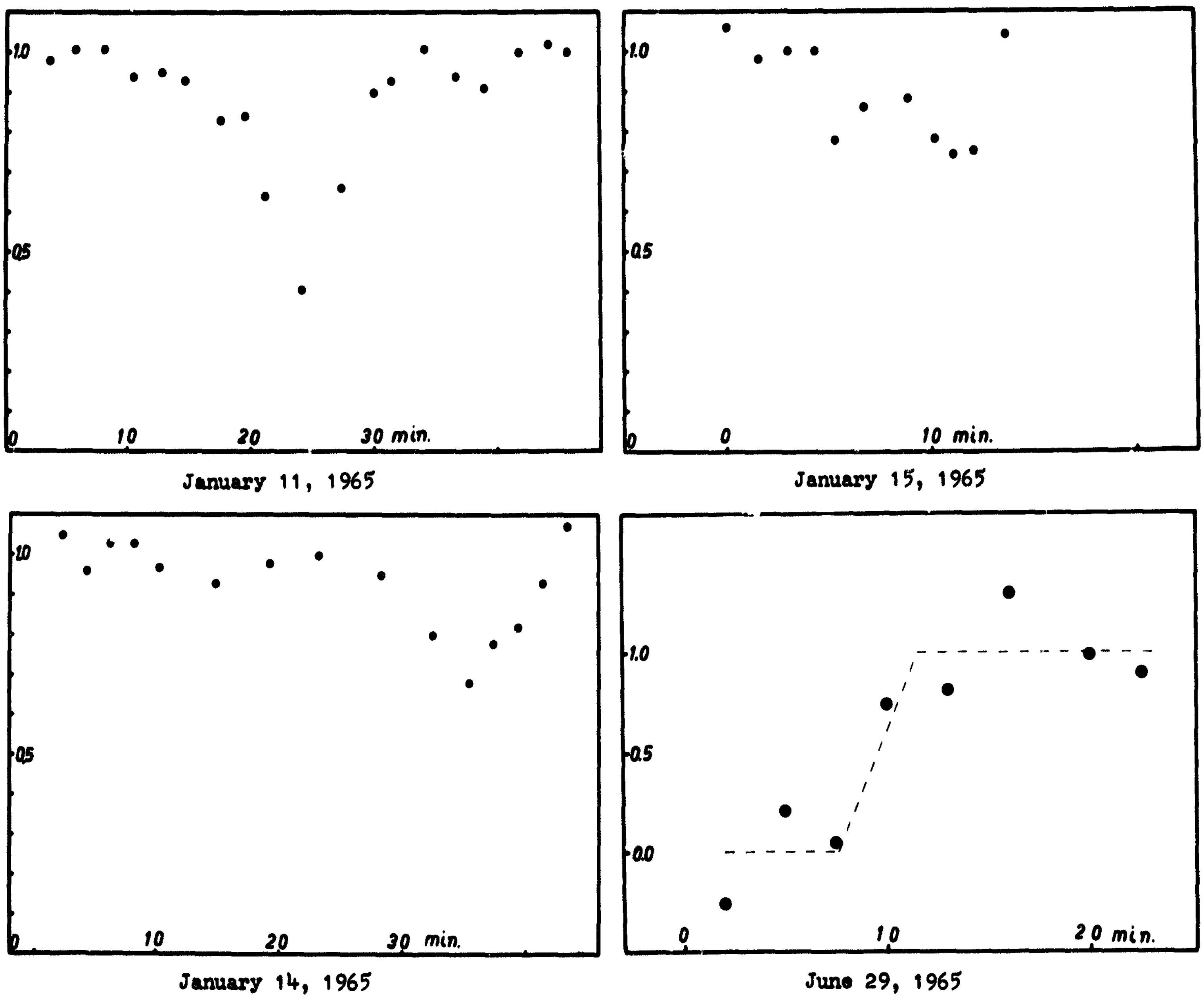

Figure 2. Eclipses of Phobos observed in January and June 1965.

over a period from 10 minutes before the start of an eclipse until 10 minutes after its end. Also, the sky brightness was subtracted from the readings for Phobos. Because sometimes the statistical distribution of photoelectrons from the sky had formed peaks, similar to peaks made by Phobos, it was necessary to get the readings for the sky during the time when Phobos was not detectable in the sky background. This means that Phobos was then closer than one second of arc to the limb of Mars. The ratio between readings for Phobos and for the sky gives a simple way of judging the accuracy of observations on different nights. A comparison of observations made on different nights in January showed no significant difference from one night to another. The ratio was of the order of 0.9. In June it was generally smaller, namely of the order of 0.25. This large difference can be explained primarily by the different content of solid particles in the earth's atmosphere. In springtime, there are seasonal storms in the whole Arizona desert. A second reason for it lies in the difference in the brightness of Phobos. In June, Phobos was 0.7 magnitude fainter than in January. Unfortunately, the progress of the eclipses in June was appreciably faster than for the grazing eclipses observed in January. In order to have the same resolution of the light curve, the time constant for each mean observational point in June should have been one-third of that used in January. Therefore, the June observations are not very suitable for studying the loss of light in the Martian atmosphere.

Eclipses observed in January and one of the best observations obtained in June are shown in Figure 2.

The eclipses in June are very convenient for checking the predicted time of immersion in and emersion of Phobos from the Martian shadow. The observations in January show effective disagreement between the calculated position of Phobos in its orbit as given by the orbital elements of Struve (6) and the observed time of eclipses. The observations in June agree very well with the results from January. According to these observations, Phobos has advanced in its orbit nearly 40 degrees beyond its ephemeris position. This, of course, strongly indicates that the orbital elements of Phobos n.ed to be revised.

Fundamental Relationships

The loss of light measured in magnitudes at an elevation z above the Martian surface at the sunlight passes through the Martian atmosphere is given by the geometrical phase of the eclipse, the molecular scattering of light in the atmosphere, the refraction of light and the possible absorption by solid particles.

In general, all computations were made on the assumption that the diameter of Phobos is negligible. Since the diameter of Phobos is generally quoted as about 15 km, and since the height of the Martian atmosphere involved in this experiment can be ten times as great, the diameter of Phobos can, for our purposes, be considered negligibly small. Also the surface brightness of Phobos can be considered uniform and, since the observations usually encompassed only 15 minutes of time, one can safely assume that the rotation of Phobos has not influenced the results. However, it cannot be assumed that the diameter of the sun is also negligible.

Geometrical Considerations. The sun was divided into 20 equal segments parallel to the Martian horizon, and the brightness of each of these segments was expressed in units of the total solar brightness, taking into account the effects of limb darkening. The loss of light for all segments together was used as a total loss at an elevation z of the center of the sun above the Martian surface.

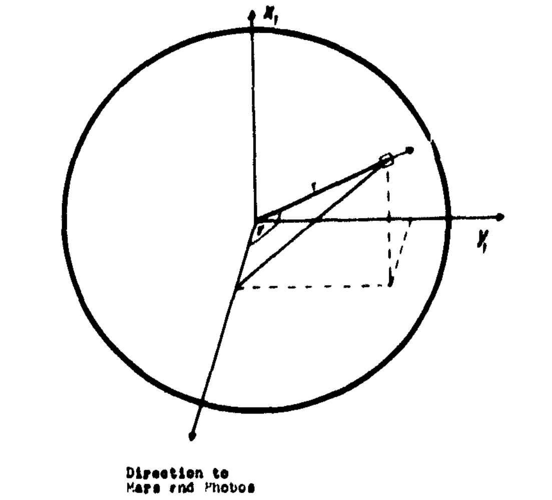

Figure 3. Geometrical relations pertaining to the brightness of solar segments.

The calculation of the segments mentioned above was carried out in the following way:

The brightness of a surface element on the sun is given by a linear limb darkening law,

$$I = \text{const}\ (1 + \beta' \cos \gamma) \qquad (1)$$

where β' = Limb darkening coefficient
γ = Angle between the direction to Mars and Phobos and the perpendicular to the sun's surface
r = Radius of the sun

and from Figure 3

$$\sin \gamma = \frac{\sqrt{x_1^2 + y_1^2}}{r}$$

following

$$\cos\gamma = \sqrt{1 - \frac{x_1^2 + y_1^2}{r^2}}$$

by transformation

$$\frac{x_1}{r} = x\,, \qquad \frac{y_1}{r} = y$$

the formula (1) will be

$$I = \text{const}\left(1 + \beta'\sqrt{1 - (x^2 + y^2)}\right)$$

By assuming that the brightness of a segment is given by the product of a function A(x) and a width of the segment dx, the total light intensity of the sun will be

$$\int_{-1}^{+1} A(x)\, dx = 1$$

The function A(x) dx expressed as a fraction of the total light intensity of the sun is given by

$$A(x) = \frac{\displaystyle\int_{y=0}^{\sqrt{1-x^2}} \left(1 + \beta'\sqrt{1-(x^2+y^2)}\right) dy}{\displaystyle\int_{x=-1}^{+1}\int_{y=0}^{\sqrt{1-x^2}} \left(1 + \beta'\sqrt{1-(x^2+y^2)}\right) dy\, dx}$$

These are simple integrals and their solution is

$$A(x) = \frac{\frac{2}{\pi}\sqrt{1-x^2} + \frac{\beta'}{2}(1-x^2)}{1 + \frac{2}{3}\beta'}$$

The brightness of the first 10 solar segments, I_s, is given in Table I. These values are symmetrical with respect to the other ten segments.

TABLE I
Brightness of Solar Segments

No.	I_s	No.	I_s
1	0.011	6	0.059
2	.023	7	.065
3	.034	8	.068
4	.045	9	.070
5	0.052	10	0.073

The progress of the geometrical eclipse (neglecting the effects of the Martian atmosphere) can be determined as a function of z and i (see Figure 4) by simple geometrical relations involving the

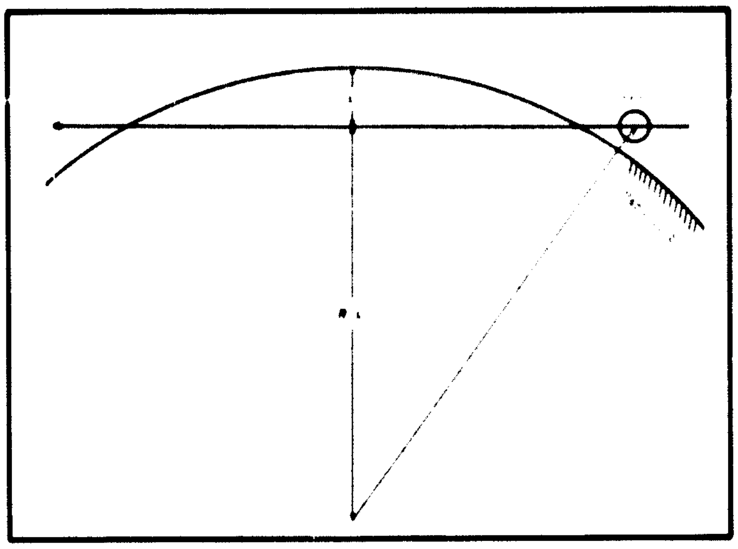

Figure 4. The solar eclipse by Mars as seen from Phobos.

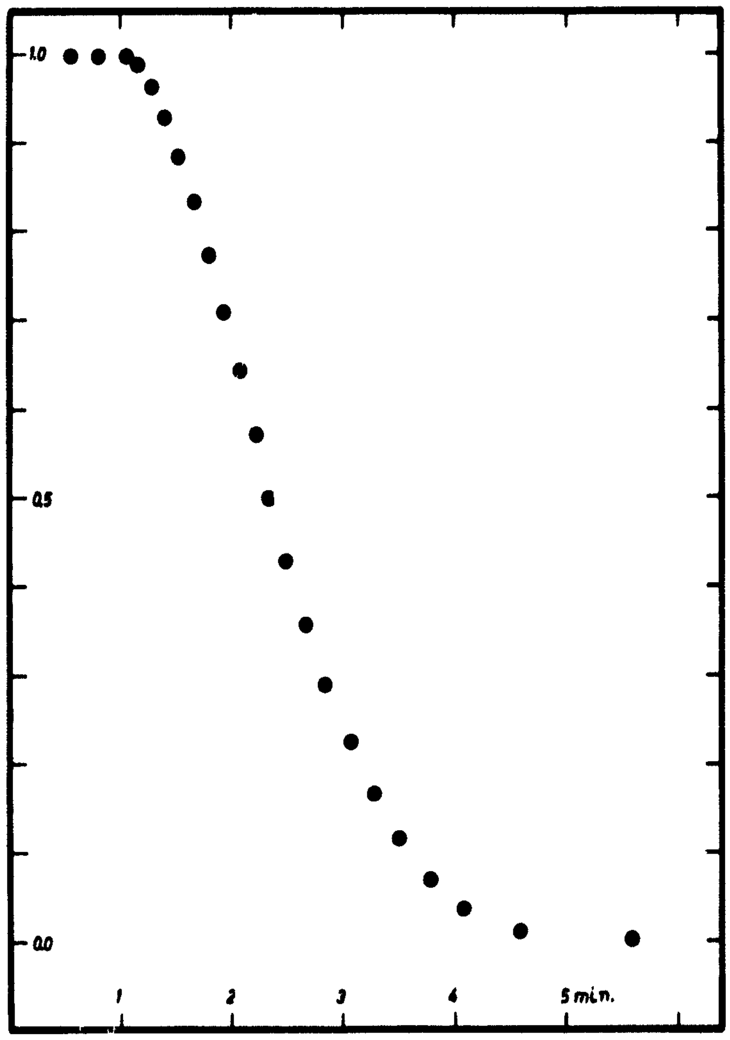

Figure 5. Computed light curve of Phobos for $i = r$ when effects of Martian atmosphere are neglected.

radius of Mars, the orbital elements of Phobos, and the sun's distance.

Figure 5 shows the computed light curve of Phobos (neglecting the effects of Martian atmosphere) in a light intensity scale as a function of time for

$$i = r$$

The exact value of t is unknown. The error in t is directly related to the error in estimating z in Figure 9. This parameter may be kept variable by moving the observational data in a horizontal direction in Figure 9 to produce the best fit for different atmospheric models for Mars. Of course, the error in t will be most serious only for very low values of z.

Molecular Scattering. The second term to be considered in the determination of the loss of light in the Martian atmosphere is the molecular scattering. Let $k\delta$ be the absorption coefficient for one Martian air mass; this term would correspond to astronomical extinction. Consequently, the absorption will be a product of $k\delta$ and the total mass of Martian atmosphere at a certain level above the surface of the planet and parallel to it. The total absorption would be

$$k\delta M,$$

where M is the total mass of gases measured in units of one Martian air mass and can be computed in the following way:

Let R = Radius of Mars in kilometers
z = Altitude above surface of Mars in kilometers
e = Base of natural logarithm.

Let us also assume that the change in the density of the atmosphere with height follows an exponential law.

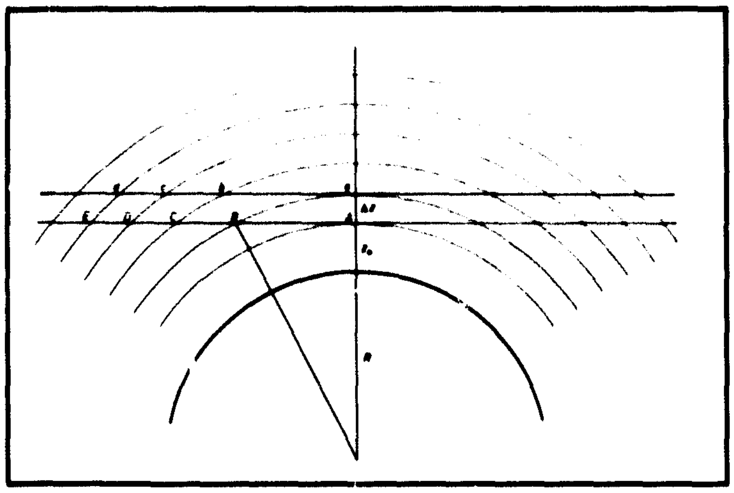

Figure 6. Geometrical relations for computation of total air mass perpendicular to the radius of the planet.

Represented in Figure 6 is a light beam with a vertical cross section Δz as it traverses the Martian atmosphere. It is divided into n regions according to their density. The volume of each region should be multiplied by its density function. The sum of all n products divided by the similar sum made for a light beam perpendicular to the surface of the planet gives the total mass of gases at the level z measured in units of one Martian air mass.

The equations used in obtaining this total mass of gas at a level z are as follows. The sum of all products of the volume of each region multiplied by its density function is:

$$\left[(\overline{AB})\frac{\Delta z}{2}\cdot e^{-\beta(z+\frac{\Delta z}{2})}\right]+\left[(\overline{BC}+\overline{ab})\frac{\Delta z}{2}\cdot e^{-\beta(z+\frac{\Delta z}{2}+\Delta z)}\right]+\left[(\overline{CD}+\overline{bc})\frac{\Delta z}{2}\cdot e^{-\beta(z+\frac{\Delta z}{2}+2\Delta z)}\right]+\ldots\ldots$$

$$(\overline{AB}) = \sqrt{(R+z+\Delta z)^2-(R+z)^2} = \sqrt{2(R+z)\Delta z}$$

$$(\overline{BC}+\overline{ab}) = \sqrt{2}\sqrt{2(R+z)\Delta z}$$

$$(\overline{CD}+\overline{bc}) = (\sqrt{3}-1)\sqrt{2(R+z)\Delta z}$$

$$\left(\frac{\Delta z}{2}\sqrt{2(R+z)\Delta z}\, e^{-\beta(z+\frac{\Delta z}{2})}\right)\left[1+\sqrt{2}\,e^{-\beta\Delta z}+\sqrt{3}\,e^{-2\beta\Delta z}-e^{-2\beta\Delta z}+\sqrt{4}\,e^{-3\beta\Delta z}-\sqrt{2}\,e^{-3\beta\Delta z}+\sqrt{5}\,e^{-4\beta\Delta z}-\sqrt{3}\,e^{-4\beta\Delta z}+\ldots\right]$$

$$M = \frac{2\frac{\Delta z}{2}\sqrt{2(R+z)\Delta z}\, e^{-\beta(z+\frac{\Delta z}{2})}\left(e^{\beta\Delta z}-e^{-\beta\Delta z}\right)\sum_{1}^{n}\sqrt{n}\, e^{-n\beta\Delta z}}{\Delta z\cdot\Delta z\sum_{1}^{n} e^{-\beta\left[\frac{2(n-1)+1}{2}\Delta z\right]}}$$

$$M = \frac{\sqrt{2(R+z)\Delta z}}{\Delta z}\cdot e^{-\beta z}\,\frac{e^{\beta\Delta z}-e^{-\beta\Delta z}}{\frac{1}{1-e^{-\beta\Delta z}}}\cdot\sum_{1}^{n}\sqrt{n}\, e^{-\beta n\Delta z}$$

and for $\Delta z = 1$ kilometer

$$M = \sqrt{2(R+z)}\,e^{-\beta z}(e^{\beta}-e^{-\beta})\,(1-e^{-\beta})\sum_{1}^{n}\sqrt{n}\, e^{-\beta n} \qquad (2)$$

The function $f(\beta) = \sum_{1}^{n}\sqrt{n}\, e^{-\beta n}$

was computed on an IBM 1620 computer. The running number n was so chosen that

$$\sum_{1}^{n+1}\sqrt{n}\, e^{-\beta n}-\sum_{1}^{n}\sqrt{n}\, e^{-\beta n} < 0.0001$$

β	$f(\beta)$
0.050	79.02
.052	74.49
.054	70.38
.056	66.63
.058	63.21
.060	60.06
.080	38.94
0.100	27.81

and β is a known relation

$$\beta = \frac{m\, g\, m_H}{j\, T}$$

m = Mean molecular weight
m_H = Mass of the hydrogen atom
j = Boltzmann's constant
g = Gravitatonal acceleration on the surface of Mars
T = Absolute temperature

Assuming that the Martian atmosphere has the same molecular weight and the same dielectrical constant as the earth's atmosphere it can be written

$$\frac{k'_{♂}}{k_{♁}} = \frac{M_{♂}}{M_{♁}} \qquad (3)$$

$M_{♂}$ means one Martian air mass, similarly $M_{♁}$ = one earth air mass. The formula (3) can be transformed for the surface pressure by

$$P_{♁} \approx M_{♁}\, g_{♁} \quad \text{and} \quad P_{♂} \approx M_{♂}\, g_{♂}$$

to

$$\frac{k'_{♂}}{k_{♁}} = \frac{P_{♂}\, g_{♁}}{P_{♁}\, g_{♂}} \qquad (4)$$

and $k'_{♂}$ should be corrected for different molecular weights and different dielectrical constants of gases on Mars by using Rayleigh's formula of scattering:

$$k = \frac{8\pi^3(\varepsilon-1)^2}{3N\lambda^4} = \frac{8\pi^3 m(\varepsilon-1)^2}{3\rho\lambda^4}$$

ε = Dielectric constant of gas
N = Number of molecules
m = Mean molecular weight
ρ = Density of gas

$k_{♂}$ should also be multiplied with the correction factor

$$\frac{m_{♁}\,(\varepsilon_{♁}-1)^2}{m_{♂}\,(\varepsilon_{♂}-1)^2}.$$

The formula (4) can now be expressed as

$$P_{♂} = 1.5\times10^3\, \frac{k_{♂}\, m_{♁}\,(\varepsilon_{♁}-1)^2}{m_{♂}\,(\varepsilon_{♂}-1)^2} \qquad (5)$$

We assume that $k_{♁} = 0^m254$ for the spectral range given by the spectral response of solar radiation, the absorption in the earth's atmosphere and the spectral response of the photometer. Generally, the wavelength dependence of absorption by a gas or by solid particles can be very different. From this point of view, the effective wavelength for gas absorpton will be different from that for absorption by solid particles for the same broad spectral range. For this reason, it is better to present the total spectral range of the optical system by a diagram, Figure 7, instead of using the term effective wavelength.

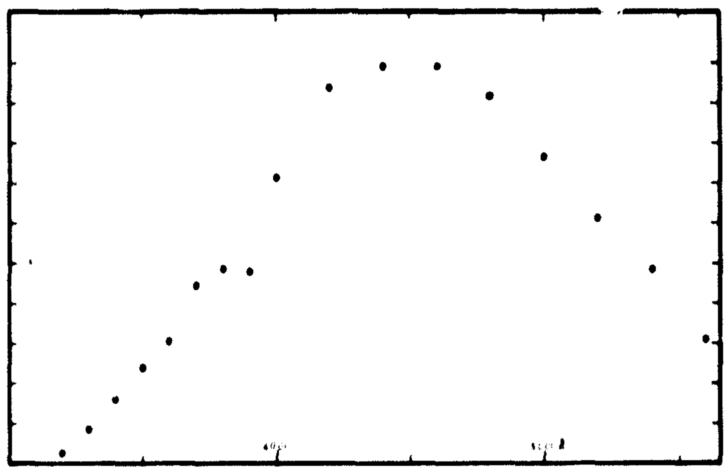

Figure 7. The relative spectral response of the photometric system used in this experiment.

Refraction. The third term in the determination of the loss of light is caused by the refraction of the sunlight as it passes through the Martian atmosphere.

In Figure 8 there are:
ω = Angle of refraction
$\overline{QH}$ = L = Distance, sun–Mars
$\overline{HZ_0}$ = R = Radius of Mars
$\overline{Z_0Z}$ = z = Height above the Martian surface
$\overline{UY}$ = dS = Ring surface produced by refracted light beam in the distance $\overline{HW}$ from Mars
$\overline{U'Y'}$ = dS' = Ring surface produced by unrefracted light beam (by absence of atmosphere)
$\overline{HW}$ = l
$\overline{HQ'}$ = L'

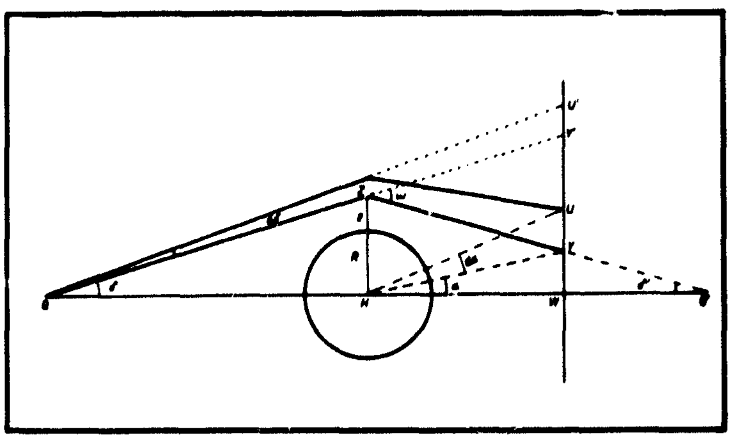

Figure 8. Geometrical relations for the calculation of the loss of light caused by the refraction of the sunlight as it passes through the Martian atmosphere.

The loss of light is given by the ratio of the two ring surfaces dS and dS'

$$\frac{dS'}{dS} = \frac{2\pi(L+l)^2\delta d\delta}{2\pi l^2 \alpha d\alpha} \qquad (6)$$

The angles δ and δ' are very small and from the relations

$$\delta = \frac{R+z}{L};\ \delta' = \frac{R+z}{L'} = \omega - \delta$$

it can be written

$$\alpha = \frac{YW}{l} = \frac{L'+l}{l}\delta' = \frac{L'\delta'}{l} - \delta' = \frac{R+z}{l} - \omega + \delta,$$

$$\alpha = \frac{R+z}{l} + \frac{R+z}{L} - \omega$$

and by substitution

$$l+L = \frac{lL(\pi_a+\pi_0)}{R} \quad \text{because of } \pi_a = \frac{R}{L};\ \pi_0 = \frac{R}{l},$$

$$\alpha = (\pi_a+\pi_0)\left(1+\frac{z}{R}\right) - \omega$$

$$\frac{d\alpha}{dz} = \left(\frac{L+l}{L\cdot l} - \frac{d\omega}{dz}\right);\ \frac{d\delta}{dz} = \frac{1}{L}.$$

Further by substitution of the formula (6) it will be

$$\frac{dS'}{dS} = \frac{(L+l)^2\frac{R+z}{L^2}}{l^2\left[(\pi_a+\pi_0)\left(1+\frac{z}{R}\right) - \omega\right]\left[\frac{L+l}{L\cdot l} - \frac{d\omega}{dz}\right]},$$

and further

$$\frac{dS'}{dS} = \frac{\frac{R+z}{L^2}\left(\frac{lL(\pi_a+\pi_0)}{R}\right)^2}{l^2(\pi_a+\pi_0)^2\left[1+\frac{z}{R}-\frac{\omega}{\pi_a+\pi_0}\right]\left[\frac{1}{R}-\frac{d\omega}{dz}\frac{1}{\pi_a+\pi_0}\right]},$$

$$\frac{dS'}{dS} = \frac{1}{\frac{R}{R+z}\left[1+\frac{z}{R}-\frac{\omega}{\pi_a+\pi_0}\right]\left[1-\frac{d\omega}{dz}\frac{R}{\pi_a+\pi_0}\right]}$$

With regard to the small value of

$$\frac{z}{R},\ \frac{1}{1+\frac{z}{R}} = 1-\frac{z}{R},$$

$$\frac{dS'}{dS} = \frac{1}{\left[1-\frac{\omega}{\pi_a+\pi_0}\left(1-\frac{z}{R}\right)\right]\left[1-\frac{d\omega}{dz}\frac{R}{\pi_a+\pi_0}\right]};$$

in consideration of the small value of ω and π_0

it can be written $$\frac{dS'}{dS} = \frac{1}{1-\frac{d\omega}{dz}\frac{R}{\pi_0}}$$

and finally

$$\frac{dS'}{dS} = 1 + \frac{d\omega}{dz}\frac{R}{\pi_0}. \qquad (7)$$

ω is calculated by the following procedure using the formula of Laplace

$$M = \frac{\mu(R+z)}{\mu_0(R+z_0)}\frac{\omega}{c\beta}$$

and, for the desired accuracy,

$$\frac{\mu(R+z)}{\mu_0(R+z_0)} \approx 1,$$

where:

μ Index of refraction at level $z_0+\Delta z = z$

μ_0 Index of refraction at level z_0

$$c = \frac{\mu-1}{\rho};\ c_♂ = c_\oplus$$

should be adopted.

The ratio of ω for earth and Mars will be

$$\frac{\omega_♂}{\omega_\oplus} = \frac{\beta_♂ M_♂}{\beta_\oplus M_\oplus}$$ and, using $mNg \approx P$,

$$\frac{\omega_♂}{\omega_\oplus} = \frac{\beta_♂ P_♂ g_\oplus}{\beta_\oplus P_\oplus g_♂}, \qquad (8)$$

where $\omega_\oplus$ = 56 minutes of arc on the earth's surface, from

$$\frac{d\omega}{dz} = \frac{d\omega}{dP}\frac{dP}{dz},\ P = P_0e^{-\beta z},\ \frac{dP}{dz} = -P_0\beta e^{-\beta z},$$

we obtain $\frac{d\omega}{dz} = -\omega\beta e^{-\beta z}$;

finally, the formula (7) will be

$$\frac{dS'}{dS} = 1 - \frac{R}{\pi_0}\omega_♂\beta e^{-\beta z}.$$

A second influence of refraction is to increase the light flux because the value l (see Figure 4) will apparently decrease with the amount of refraction ω. The value ω is very small and correction can be made using formula 8.

Solid Particles. The last term in the determination of loss of light in the Martian atmosphere represents additional absorption caused by solid particles.

Generally, the value of such possible absorption as a function of z can be computed as a difference between the observed and calculated absorption for a given pressure on the Martian surface. From the amount of the absorption, G, by solid particles a relative step-by-step distribution across the atmosphere can be obtained by a simple computation of the path length of the light beam for each atmospheric region with its uniform concentration of the particles. The results can be compared after making different assumptions regarding atmosphere, temperature and chemical composition.

Discussion

The data in Figure 2, except for the eclipses observed in June, was condensed into a single diagram, Figure 9. The total absorption in magnitudes is plotted against the height above the Martian surface for the light beam passing between the sun's center and Phobos.

If there are no solid particles in the atmosphere of Mars, then it would be possible from the absorption at very high altitudes, 60 km or more, to have a very sensitive criterion for the mean molecular weight of gases at an assumed temperature in the upper atmosphere, because the absorption curve for high altitudes depends strongly on β.

Figure 9 shows immediately that some additional absorption by solid particles must be present, at least at high altitudes. The absorption measured at 100 km, for example, could not be produced at all by an extensive atmosphere with more than 100 mb pressure on the surface. Even if the atmosphere extends beyond a height of 150 km, the molecular absorption at a height of 100 km could still not be measured.

Therefore, the absorption at 90 km can be neglected (for gas only), with the assumption of a reasonable upper limit and a surface pressure of 80 mb. If the total absorption at this level is due to solid particles, an assumption necessary to explain this absorption, the assumed pressure of 80 mb will be too high to explain the absorption curve at the low level (below 60 km). By successive approximation, changing the pressure (to lower and lower values) and by changing the distribution of the solid particles in the atmosphere, the observed absorption curve was brought into very good agreement with the computed absorption, see Figure 9, for the following properties of the atmosphere:

Surface pressure ($65\%\ N_2 + 30\%\ CO_2 + 5\%\ A$)	$P = 30$ mb
Mean molecular weight	$m = 33$
Refraction angle on the surface	$\omega = 2\rlap{.}'6$
Temperature, mean value	$T = 200°$ K
Rayleigh extinction for one Martian air mass	$k\delta = 0\rlap{.}^m030$
Extinction caused by solid particles from the elevation of 10 km to the top of the atmosphere perpendicular to Martian surface	$G_0 = 0\rlap{.}^m038$

For the present reductions, however, a most probable value for the mean molecular weight, according to spectroscopic measurements and the very low probability of the presence of significant amounts of the light gases H_2 and He can be adopted. The amount of argon is supposed to have been generated by a radiogenic process during the last 5 x 10^9 years. A change in the assumed mean temperature of $\pm 20°$ K in this model corresponds to a change in the pressure of ∓ 5 mb at the surface.

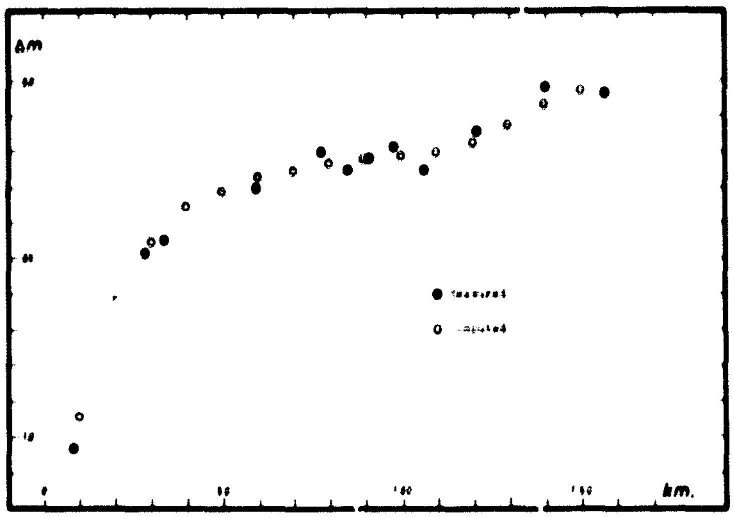

Figure 9. Measured and computed total loss of light in magnitudes as a function of the height above the Martian surface.

The distribution of solid particles which corresponds to this atmospheric model is presented in Figure 10. Also, shown in this same figure is the temperature change with height published by J. Chamberlain (7). It shows the same trend as the assumed relative concentration of solid particles. Of course, it is also assumed that these have the same optical properties everywhere in the Martian atmosphere. Significant is the highest density of particles at an elevation of 120 to 130 km. This is the elevation of the mesopause, the coolest region in the Martian atmosphere. This suggests that some sort of condensing process is taking place at these high levels.

Unfortunately, the estimation of the surface pressure in this way is not definitive for pressures lower than 30 mb. This means that it is possible to find another special distribution of solid particles which would produce good agreement between the observed and calculated values of absorption for all altitudes in the atmosphere for any given pressure below 30 mb. Of course these difficulties are not theoretical in nature. They are caused only by the fact that the computed light variations according to height for different pressures below 30 mb with their corresponding distribution of solid particles, differ so little from one another that the attainable accuracy of observation cannot distinguish between them.

Figure 11 shows a very different distribution of solid particles than that shown in Figure 10. For

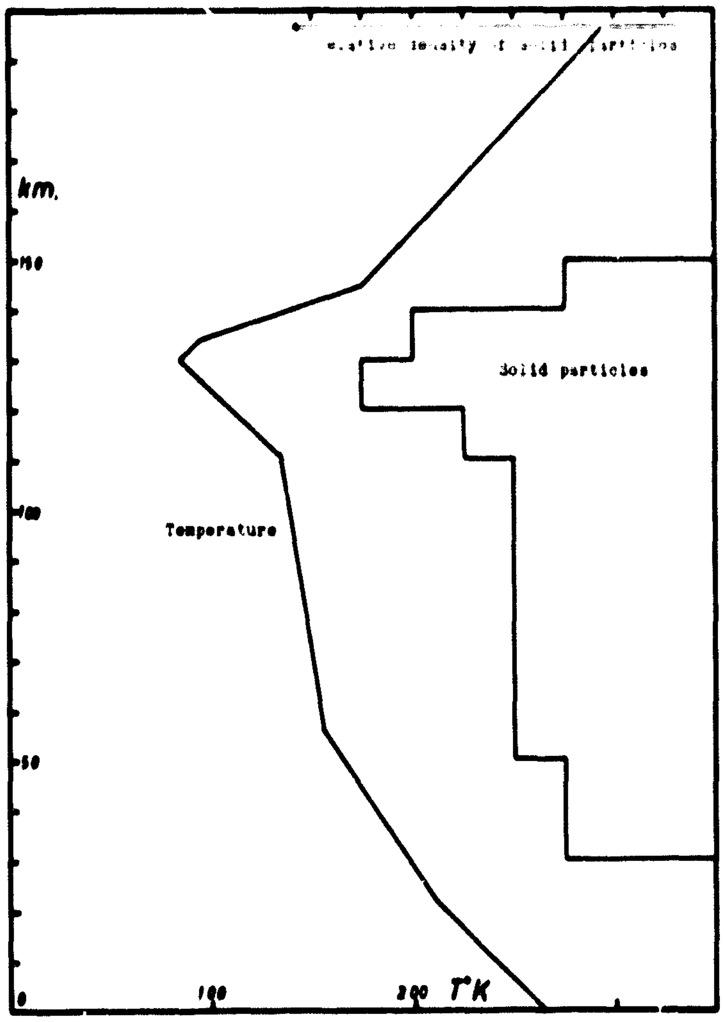

Figure 10. The distribution of solid particles as a function of the height in the Martian atmosphere corresponding to a 30 mb atmospheric model.

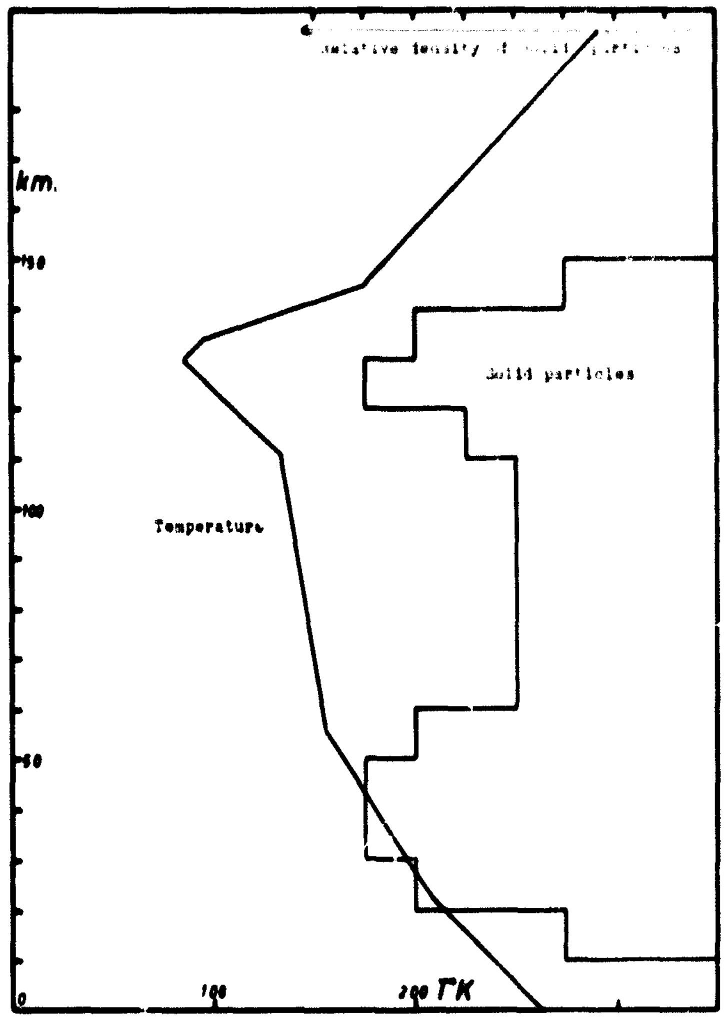

Figure 11. The distribution of solid particles as a function of the height in the Martian atmosphere corresponding to a 10 mb atmospheric model.

the distribution in Figure 11, a surface pressure of only 10 mb is assumed. The other properties of this atmospheric model are:

$m = 43(70\%\ CO_2 + 30\%\ A)$ $\quad k\delta = 0^m013$

$\omega = 1''1$ $\quad G_0 = 0^m052$

$T = 200°K$

This model should be near the lower limit of the atmospheric pressure. This distribution of solid particles shows a considerable difference from the temperature change with height. It has two maxima, the first, the larger one, at a height of 40 km, and the second at 120 to 130 km.

Table II shows for two different surface pressures the loss of light in magnitudes caused by a gaseous atmosphere, D, (including all effects discussed in this paper) and the amount of absorption by solid particles, G, as a function of height.

In conclusion, one might say that, with regard to the technique discussed in this paper, the value of the surface pressure on Mars can be determined only after the distribution of solid particles in the Martian atmosphere has been established.

Blue Haze. In this section, the problem of "blue haze" in the Martian atmosphere is discussed. The spectral range (see Figure 7) of the optical system was very broad. The total absorption caused by solid particles from an elevation of 10 km to the top of the atmosphere measured perpendicularly to

TABLE II

Absorption as a Function of Height for Two Assumed Surface Pressures.

z	10 mb		30 mb	
km	D	G	D	G
0	1.00	0.31	1.21	0.18
10	0.48	0.34	0.72	0.20
20	0.19	0.36	0.40	0.21
30	0.08	0.36	0.21	0.24
40	0.31	0.34	0.10	0.25
50	0.11	0.31	0.05	0.26
60	0.00	0.27	0.02	0.25
70	0.00	0.25	0.01	0.24
80		0.23		0.23
90		0.22		0.22
100		0.21		0.21
110		0.20		0.20
120		0.17		0.17
130		0.13		0.13
140		0.07		0.07
150		0.03		0.03

the surface is $0^m.038$ for the 30 mb model and $0^m.052$ for the 10 mb model. This absorption is comparable with the amount of zenith extinction by haze scattering found in the earth's atmosphere when measured at high altitude observatories.

λ(A)	Extinction
4000	$0^m.036$
4500	0.031
5000	0.027

The optical thickness of the total Martian atmosphere in blue light, according to the measurements made during the last two decades is approximately 0.1 to 0.2. In addition, scattering theory indicates that the optical thickness of the blue haze cannot be much greater than the same value, because otherwise the haze alone would reflect more blue light than the total light observed. Scattering by solid particles corresponding to extinction of only $0^m.038$ or $0^m.052$ would not produce such an appreciable effect. It seems that some additional extinction in the atmosphere below 10 km height caused by unknown blue haze material must be adopted. The observational method employed in this paper could not be used to distinguish between the thin absorption layer near the limb of the planet and the limb itself.

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REFERENCES

1. de Vaucouleurs, G., 1954, *Physics of the Planet Mars*, Faber and Faber, Ltd., London.
2. Dollfus, A., 1951, *Compt. Rend.*, **232**, 1066.
3. ———, 1957, *Suppl. Ann. Astrophys.*, **4**.
4. Kaplan, D., Münch, G., and Spinrad, H., 1964, *Ap.J.*, **139**, 1.
5. Rakos, K., 1965, *Applied Optics*, **4**, 1453.
6. Struve, H., 1911, *Sitzungsberichte* d. Akad. d. Wiss., Berlin, 1056.
7. Chamberlain, J., 1962, *Ap.J.*, **136**, 582.