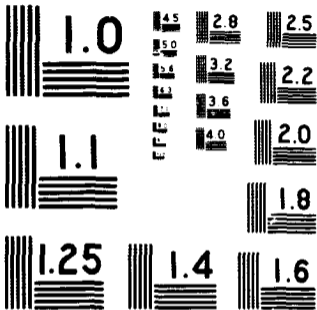


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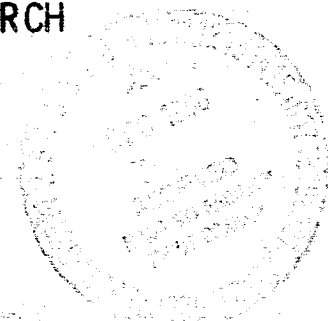
DIFFUSION APPROXIMATIONS IN COLLECTIVE RISK THEORY

BY  
DONALD L. IGLEHART

TECHNICAL REPORT NO. 111  
July 8, 1968

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DEPARTMENT OF OPERATIONS RESEARCH  
AND  
DEPARTMENT OF STATISTICS  
STANFORD UNIVERSITY  
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DEPARTMENT OF OPERATIONS RESEARCH

and

DEPARTMENT OF STATISTICS

STANFORD UNIVERSITY

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## NON-TECHNICAL SUMMARY

Collective risk theory is concerned with the random fluctuations of the total assets of an insurance company. The company has an initial capital  $u$  and policyholders pay a gross risk premium of  $a$  per unit time. At random instants of time, claims are made against the company for random amounts. The principal objectives of the theory are to obtain the distributions of the total assets of the company at time  $t$  and the time to ruin. Calculations of these distributions are usually very involved and use difficult analytical methods. In this paper explicit approximations are obtained for these distributions.

Approximations of the type obtained here can be developed for other applied models which involve the sum of a random number of random variables. In particular, a compound Poisson process is a special case of the processes considered in this paper.

# DIFFUSION APPROXIMATIONS IN COLLECTIVE RISK THEORY<sup>1/</sup>

by

Donald L. Iglehart  
Stanford University

## 1. Introduction

Collective risk theory is concerned with the random fluctuations of the total assets, the risk reserve, of an insurance company. Consider a company which only writes ordinary insurance policies such as accident, disability, fire, health, and whole life. The policyholders pay premiums regularly and at certain random times make claims to the company. A policyholder's premium, the gross risk premium, is a positive amount composed of two components. The net risk premium is the component calculated to cover the payments of claims on the average, while the security risk premium, or safety loading, is the component which protects the company from large deviations of claims from the average and also allows an accumulation of capital. When a claim occurs the company pays the policyholder a positive amount called the positive risk sum.

On the other hand, an insurance company which only writes life annuity policies represents the mirror image of the situation described above. In this case the company pays out an annuity regularly to the policyholder while the death of a policyholder places the corresponding reserve at the company's disposal. For this case we speak of negative risk premiums and negative risk sums.

For many insurance companies, of course, one would find both

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positive and negative premiums and risk sums present. As a mathematical model for this situation we shall assume that claims occur at the jumps of a renewal process,  $\{N(t) : t \geq 0\}$ , and that the successive risk sums form a sequence of independent, identically distributed random variables,  $\{X_i : i = 1, 2, \dots\}$ , with  $E\{X_i\} = \mu > 0$ . Furthermore, we shall assume that the initial risk reserve of the company is  $u > 0$  and that the policyholders pay a gross risk premium of  $\underline{a} > 0$  per unit time. If we let the risk reserve at time  $t$  be  $X(t)$  and let  $S_0 = 0$ ,  $S_i = X_1 + \dots + X_i$ , then we have

$$(1) \quad X(t) = u + at - S_{N(t)}, \quad t > 0 .$$

Also we define  $T$ , the first time the company has a non-positive risk reserve, by

$$T = \inf\{t > 0 : X(t) \leq 0\} .$$

The principal problems of collective risk theory have been to calculate the distributions of  $X(t)$  and  $T$ , usually for the case where  $\{N(t) : t \geq 0\}$  is a Poisson process. Many of the results for these distributions are complicated expressions which have been obtained by involved analytical methods. For a comprehensive treatment of the theory up to 1955 the reader should consult Cramér (1955). A more recent account of portions of the theory is available in Takács (1967), Chapter 7.

Our objective in this paper is to obtain approximations for the

distributions of  $X(t)$ ,  $T$ , and other functionals of  $X(t)$  by applying the theory of weak convergence of probability measures on function spaces. We shall define a sequence of risk reserve process  $\{X_n(t) : t \geq 0\}$ ,  $n = 1, 2, \dots$ , and show that these processes converge weakly to a Brownian motion process with a drift. For this limit process we shall be able to explicitly calculate the density of the first passage time corresponding to the random variable  $T$ . The distributions of the random variables derived from the limit process are then proposed as approximations for the process  $X_n(t)$  for large  $n$ .

This paper is organized as follows. In Section 2 a number of concepts and results required from weak convergence theory are introduced. Section 3 contains a proof of the convergence of a sequence of risk reserve processes. Finally in Section 4 the distributions of quantities derived from the limiting Brownian motion process are discussed.

## 2. Preliminaries on Weak Convergence

In this section we shall assemble those concepts and results from weak convergence theory which we shall need for our application to collective risk theory. For a comprehensive development of weak convergence of probability measures we recommend the excellent book of Billingsley (1968).

Let  $S$  be a metric space and  $\mathcal{S}$ , the class of Borel sets, be the  $\sigma$ -field generated by the open sets of  $S$ . If  $P_n$  and  $P$  are probability measures on  $\mathcal{S}$  which satisfy

$$\lim_{n \rightarrow \infty} \int_S f dP_n = \int_S f dP$$



for every bounded, continuous, real-valued function  $f$  on  $S$ , we shall say that  $P_n$  converges weakly to  $P$  as  $n \rightarrow \infty$  and write  $P_n \Rightarrow P$ . In the case where  $S = R^k$ ,  $k$ -dimensional Euclidean space, weak convergence is equivalent to ordinary weak convergence of the distribution functions associated with  $P_n$  to that associated with  $P$ . However, for the function spaces we shall consider weak convergence is a deeper concept.

For the applications to be discussed in this paper it is convenient to introduce the following terminology used in [1]. Let  $X$  be a measurable mapping from the probability space  $(\Omega, \mathcal{B}, \mathcal{P})$  into a metric space  $S$ ; measurability of  $X$  means that  $X^{-1}\mathcal{S} \subset \mathcal{B}$ . We shall call  $X$  a random element of  $S$ . In particular, if  $S = R^1$ , we call  $X$  a random variable; if  $S = R^k$ , we call  $X$  a random vector; and if  $S$  is a function space, we call  $X$  a random function. The distribution of  $X$  is the probability measure  $P = \mathcal{P}X^{-1}$  on  $(S, \mathcal{S})$ . We shall say a sequence  $\{X_n\}$  of random elements of  $S$  converges in distribution to the random element  $X$ , and write

$$X_n \Rightarrow X ,$$

if the distribution  $P_n$  of  $X_n$  converges weakly to the distribution  $P$  of  $X$ :  $P_n \Rightarrow P$ . While this definition requires that the range  $S$  and topology be the same for the random elements  $X, X_1, X_2, \dots$ , the domains  $(\Omega, \mathcal{B}, \mathcal{P})$  may be different.

One of the most useful results in weak convergence theory for applications is the continuous mapping theorem which is an analog of the

Mann-Wald theorem for the Euclidean case; cf. [1], Section 5. Let  $h$  be a measurable mapping of  $S$  into another metric space  $S'$  with  $\sigma$ -field  $\mathcal{G}'$  of Borel sets. Each probability measure  $P$  on  $(S, \mathcal{G})$  induces on  $(S', \mathcal{G}')$  a unique probability measure  $Ph^{-1}(A) = P(h^{-1}A)$  for  $A \in \mathcal{G}'$ . Let  $D_h$  be the set of discontinuities of  $h$ . Then we have

Theorem 1. If  $P_n \Rightarrow P$  and  $P(D_h) = 0$ , then  $P_n h^{-1} \Rightarrow Ph^{-1}$ .

In applications often we take  $S' = R^1$  and thus  $h$  is a functional on the random elements of  $S$ .

Now let  $S$  be a separable metric space with metric  $m$  and  $X_n$  and  $Y_n$  be random elements of  $S$  with the same domain  $(\Omega, \mathcal{B}, P)$ . Then, since  $S$  is separable,  $m(X_n, Y_n)$  is a random variable; cf. [1], Appendix II. We say that  $m(X_n, Y_n)$  converges in probability to 0 and write  $m(X_n, Y_n) \xrightarrow{P} 0$  if

$$P\{m(X_n, Y_n) \geq \epsilon\} \rightarrow 0$$

for each positive  $\epsilon$ . Then we have the useful result

Theorem 2. If  $X_n \Rightarrow X$  and  $m(X_n, Y_n) \xrightarrow{P} 0$ , then  $Y_n \Rightarrow X$ .

The metric spaces  $S$  of interest in this paper are  $C[0,1]$  and  $D[0,1]$ . The space  $C[0,1]$  (which we shall abbreviate as  $C$ ) is the space of all real-valued continuous functions on  $[0,1]$  with the uniform metric,  $\rho(x,y) = \max_{0 \leq t \leq 1} |x(t) - y(t)|$ . Let  $\mathcal{C}$  denote the class of Borel sets of  $C$ . With this metric  $C$  is a complete, separable metric space. The space  $D[0,1]$ , abbreviated  $D$ , is the space of all

real-valued functions  $x(t)$  on  $[0,1]$  that are right-continuous and have left limits:

- (i) for  $0 < t < 1$ ,  $x(t+) = \lim_{s \rightarrow t} x(s)$  exists and  $x(t) = x(t+)$ ;  
(ii) for  $0 < t \leq 1$ ,  $x(t-) = \lim_{s \rightarrow t} x(s)$  exists.

Skorohod (1956) has introduced the following topology on  $D$ . Let  $\Lambda$  denote the class of strictly increasing, continuous mappings of  $[0,1]$  onto itself. For  $\lambda \in \Lambda$ ,  $\lambda(0) = 0$  and  $\lambda(1) = 1$ . For  $\lambda \in \Lambda$  let<sup>2/</sup>  
 $\|\lambda\| = \sup_{s \neq t} \left| \log \frac{\lambda t - \lambda s}{t - s} \right|$ . The metric  $d(x,y)$ , for  $x$  and  $y$  in  $D$ , is defined to be the infimum of those positive  $\epsilon$  for which there exists a  $\lambda \in \Lambda$  such that

$$\|\lambda\| \leq \epsilon$$

and

$$\sup_t |x(t) - y(\lambda t)| \leq \epsilon.$$

With this metric  $D$  is a complete separable metric space.

A useful result connecting weak convergence in  $C$  and  $D$  was recently obtained by Liggett and Rosén (1968).

Theorem 3. (Liggett and Rosén). Let  $\{X_n\}$  be a sequence of random functions in  $(D,d)$ ,  $\{Y_n\}$  a sequence of random functions in  $(C,\rho)$ , and  $X$  a random function in  $(C,\rho)$ . If  $d(X_n, Y_n) \xrightarrow{P} 0$ , then

$$X_n \Rightarrow X \text{ in } (D,d) \text{ if and only if}$$

$$Y_n \Rightarrow X \text{ in } (C,\rho).$$

<sup>2/</sup> For functions  $\lambda \in \Lambda$  we shall write  $\lambda t$  for  $\lambda(t)$ .

To prepare for our application we state a special case of a functional central limit theorem of Prohorov (1956), Theorem 3.1. Let  $X_1^{(n)}, \dots, X_n^{(n)}$  be a triangular array of random variables defined on some probability space  $(\Omega, \mathcal{B}, \mathcal{P})$ , which are independent and identically distributed for each value of  $n = 1, 2, \dots$ . Furthermore, assume that  $E[X_1^{(n)}] = 0$ ,  $\sigma^2[X_1^{(n)}] = \sigma_n^2 > 0$ ,  $\sigma_n^2 \rightarrow \sigma^2 > 0$ , and that  $E[(X_1^{(n)})^{2+\epsilon}]$  is bounded in  $n$  for some  $\epsilon > 0$ . Now construct the random functions  $Y_n$  in  $C$  by setting

$$Y_n(t, \omega) = \frac{S_{[nt]}^{(n)}(\omega)}{\sqrt{n}\sigma} + (nt - [nt]) \frac{X_{[nt]+1}^{(n)}}{\sqrt{n}\sigma}$$

for  $[nt]n^{-1} \leq t < ([nt]+1)n^{-1}$ . Then we have

Theorem 4. (Prohorov).  $Y_n \Rightarrow W$ , where the distribution of  $W$  is Wiener measure and  $W(0) = 0$ .

Now define  $X_n(t, \omega) = \frac{S_{[nt]}^{(n)}(\omega)}{\sqrt{n}\sigma}$  for  $0 \leq t \leq 1$ . Since  $d(X_n, Y_n) \xrightarrow{P} 0$

and  $W$  is in  $(C, \rho)$ , Theorem 3 implies that  $X_n \Rightarrow W$ .

In collective risk theory, however, we are interested in sums of a random number of random variables. Let  $\{Y_i\}$  be a sequence of independent, identically distributed positive random variables with  $E(Y_1) = \lambda^{-1} > 0$ . The random variable  $Y_1$  will represent the time between the  $(i-1)^{st}$  claim and the  $i^{th}$  claim. Let  $N(t)$ , the number of claims (renewals) in time  $t$ , be defined by

$$N(t) = \max\{k : \sum_{i=1}^k Y_i \leq t\} \quad t \geq 0,$$

with  $N(t) = 0$  if  $Y_1 > t$ . The random function  $N(nt)$ ,  $0 \leq t \leq 1$ , is therefore an element of  $D$  for  $n = 1, 2, \dots$ . It follows immediately from the functional central limit theorem for  $N(n \cdot)$ , [1, Theorem 18.3], that  $N(n \cdot)/n \Rightarrow \Lambda$ , where  $\Lambda$  is the constant-valued random function whose value at  $t$  is  $\lambda t$ . Now let  $Z_n(t, \omega) = S_{N(nt)}^{(n)} / \sqrt{n} \sigma$ . An immediate application of the method used in [1, Theorem 18.1] yields,

Theorem 5.  $Z_n \Rightarrow W \circ \Lambda$ , where  $W \circ \Lambda(t) = W(\lambda t)$ .

It should be noted that for this theorem the random elements  $X_n$  and  $N(n \cdot)/n$  need not be independent.

### 3. Weak Convergence of Risk Reserve Processes

In this section we shall construct a sequence of risk reserve processes  $\{X_n(t)\}$  and then use the results of Section 2 to prove that they converge weakly. For the  $n^{\text{th}}$  process let  $u_n > 0$  be the initial risk reserve,  $a_n > 0$  be the gross risk premium per unit time, and  $\{X_i^{(n)}\}$  be the sequence of independent identically distributed risk sums defined on  $(\Omega, \mathcal{B}, \mathcal{P})$  with  $E[X_i^{(n)}] = \mu_n > 0$  and  $\sigma^2[X_i^{(n)}] = \sigma_n^2 > 0$ . Assume that the claims occur at the jumps of the renewal process  $\{Y_i\}$  described in Section 2 also defined on  $(\Omega, \mathcal{B}, \mathcal{P})$ . Finally, the process  $X_n(t)$  is obtained from (1) by compressing the original time scale by a factor of  $n^{-1}$ . Thus we have

$$X_n(t) = u_n + a_n \cdot nt = S_{N(nt)}^{(n)} \quad 0 \leq t \leq 1,$$

where  $S_0^{(n)} = 0$  and  $S_i^{(n)} = X_1^{(n)} + \dots + X_i^{(n)}$ . Clearly,  $X_n$  is an

element of  $D[0,1]$ . With this set-up we can proceed to the proof of our main result.

Theorem 6. If  $u_n = un^{-1/2} + o(n^{-1/2})$ ,  $a_n = an^{-1/2} + o(n^{-1/2})$ ,  $\mu_n = \mu n^{-1/2} + o(n^{-1/2})$ ,  $\sigma_n^2 \rightarrow \sigma^2 > 0$ , and  $E[(X_1^{(n)})^{2+\epsilon}]$  is bound in  $n$  for some  $\epsilon > 0$ , then

$$n^{-1/2} X_n \Rightarrow u + \Gamma + \sigma \lambda^{1/2} W,$$

where  $\Gamma$  is the constant-valued random function whose value at  $t$  is  $(a - \mu\lambda)t$ .

Proof: Applying Theorem 5, we have that

$$n^{-1/2} [S_{N(n\cdot)}^{(n)} - \mu_n N(n\cdot)] \Rightarrow \sigma W \circ \Lambda$$

Since  $n^{-1/2} \mu_n N(n\cdot) \Rightarrow M$ , where  $M(t) = \lambda \mu t$  with probability 1, an application of Theorems 1 and 2 implies that

$$-n^{-1/2} S_{N(n\cdot)}^{(n)} + M \Rightarrow -\sigma W \circ \Lambda = \sigma W \circ \Lambda.$$

Consequently, we have immediately the fact that

$$n^{-1/2} X_n \Rightarrow u + \Gamma + \sigma W \circ \Lambda, \text{ but}$$

since  $\lambda^{1/2} W$  has the same distribution as  $W \circ \Lambda$  the proof of the theorem is complete.

Theorems 5, and 6 have been stated and proved for random elements of  $D[0,1]$ . A similar theory could be developed for random elements

of  $D[0, N]$  using the metric  $d_N$  which is defined like  $d$  but over the interval  $[0, N]$ . A topology has been developed by Stone (1963) for the space  $D[0, \infty)$  which essentially requires convergence for each metric  $d_N$ ,  $N > 0$ . Using the results of [8] together with the necessary and sufficient condition for weak convergence given by Skorohod (1956) we have versions of Theorems 5 and 6 which are valid for  $D[0, \infty)$ , that is the stochastic processes are defined for all values of  $t \geq 0$ . From here on we shall use this version of Theorem 6.

#### 4. The Distribution of Functionals of the Limit Process

The principal contribution of functional central limit theorems, such as Theorem 6, is that they enable one to obtain limit theorems for a large class of functionals of the process. Collective risk theory has been mainly concerned with the functionals which represent the total assets of the insurance company at time  $t$ , namely  $X_n(t)$ , and of the time to ruin,  $T_n$ , which is defined as

$$T_n = \inf\{t > 0 : X_n(t) \leq 0\}$$

if the set  $\{t \geq 0 : X_n(t) \leq 0\}$  is not empty and  $+\infty$  otherwise.

Since the projection  $\pi_t : D[0, \infty) \rightarrow \mathbb{R}$  which is defined by  $\pi_t(x) = x(t)$  is measurable and continuous almost everywhere with respect to the measure on  $D[0, \infty)$  corresponding to  $u + \Gamma + \sigma\lambda^{\frac{1}{2}}W$ , an application of Theorem 1 yields

Theorem 7. Under the hypotheses of Theorem 6, the

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathcal{P}\{n^{-\frac{1}{2}}X_n(t) \leq x\} &= \Pr\{u + (a-\mu\lambda)t + \sigma\lambda^{\frac{1}{2}}W(t) \leq x\} \\ &= \frac{1}{\sqrt{2\pi\sigma^2\lambda t}} \int_{-\infty}^x \exp\left\{-\frac{(y-[u+(a-\lambda\mu)t])^2}{2\sigma^2\lambda t}\right\} dy . \end{aligned}$$

Similar results could be obtained, of course, for the finite-dimensional distributions of  $n^{-\frac{1}{2}}X_n$ .

Turning now to the time to ruin problem, we define the random variable

$$T = \inf\{t > 0 : u + (a-\lambda\mu)t + \sigma\lambda^{\frac{1}{2}}W(t) \leq 0\}$$

if the set is non-empty and  $+\infty$  otherwise. Again the mapping  $\tau : D[0, \infty) \rightarrow R^1 \cup \{+\infty\}$  defined by  $\tau(x) = \inf\{t > 0 : x(t) \leq 0\}$  if the set is non-empty and  $+\infty$  otherwise is measurable and almost everywhere continuous with respect to the measure on  $D[0, \infty)$  corresponding to  $u + \Gamma + \sigma\lambda^{\frac{1}{2}}W$ . Thus another application of Theorem 1 yields

Theorem 8. Under the conditions of Theorem 6, the

$$\lim_{n \rightarrow \infty} \mathcal{P}\{T_n \leq t\} = \Pr\{T \leq t\} ,$$

where the density  $f_T$  of  $T$  is given by

$$f_T(t) = \frac{c^{-1}e^{-bc}}{\sqrt{2\pi}} t^{-\frac{3}{2}} \exp\left\{-\frac{1}{2}[c^2t^{-1} + (bc)^2t]\right\} , \quad t > 0 ,$$

with  $b = (a-\lambda\mu)/\sigma\lambda^{\frac{1}{2}}$  and  $c = u/\sigma\lambda^{\frac{1}{2}}$ . Furthermore, the probability of



ultimate ruin for the limit process is given by

$$\Pr\{T < \infty\} = \exp\{-2 bc\} .$$

The density of  $T$  and the probability of ultimate ruin are easily obtained from the Laplace transform of the distribution of  $T$  given by Darling and Siegert (1953).

In a similar manner one could develop a limit theorem for the distribution of the first time  $X_n(t)$  reached a specified level, conditional on ruin not having occurred. The Laplace transform of the limiting distribution is given in [3].

Theorems 7 and 8 give explicit expressions for the limiting behavior of the distributions of greatest concern for collective risk theory. They immediately suggest the use of the limit distribution as an approximation for the distributions associated with  $X_n(\cdot)$  for  $n$  large. It would be interesting to compare these approximations with those given, for example, in [2]. While there is considerable similarity in the functional forms of the two sets of approximations, those given in this paper seem to be more natural and are valid for a larger class of underlying distributions.

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