International Atomic Energy Agency
SYMPOSIUM ON THE PRODUCTION OF ELECTRICAL ENERGY BY MEANS OF MHD-GENERATORS
Warsaw, 24-30 July 1968

N69-13150 SM-107/137

END EFFECT IN MHD DUCTS WITH NON-CONDUCTING PARTITIONS *

by

I. V. Lavrentyev
D.V. Efremov Research Institute for Electrophysical Equipment, Leningrad, USSR

As has been shown previously [ref. #1, 2, 3], the introduction into the end zones of an MHD duct of non-conducting partitions whose plane is parallel to the induction vectors of the magnetic field and the velocity of the fluid improves its electrical characteristics (increasing the efficiency and the output voltage and raising the power to be extracted).

For the purpose of qualitative evaluation of the effect of introducing the partition on the integral characteristics of a duct, two problems were solved whose results are also presented in the report.

1. Let us examine a flat duct of constant cross section with insulating walls and two electrodes *MN* and *KL* (figure 1.a).

A $(n - 1)$ and a $(m - 1)$ partition is introduced respectively into the duct on the left and on the right of the electrodes. It is assumed that the partitions are non-conducting, infinitely thin, and one end approaches infinity (points A_k and C_i in figure 1.a). We shall assume that the electrical conductivity of the medium σ is constant. The distribution of velocity is given in the form $\vec{V} = (V, 0, 0)$ $(V = \text{const})$, and the magnetic field is expressed by the relationship

$$\vec{B} = (0,\ 0,\ -B(x)) \qquad (1.1)$$

$$B(x) = B_0\bar{B}(x),\ \bar{B}(x) \to 0 \qquad \text{when } x \to \infty$$

Then at small magnetic Reynolds numbers for determining the electrical characteristics of the duct in question, it is necessary to solve the following system of equations and boundary conditions [ref. #4]:

$$j_x = -\sigma\frac{\partial\varphi}{\partial x},\quad j_y = -\sigma\frac{\partial\varphi}{\partial y} + VB(x),\quad \nabla^2\varphi = 0 \qquad (1.2)$$

$\varphi = \varphi_1$ at electrode *MN*

$\varphi = \varphi_2$ at electrode *KL*

$\frac{\partial\varphi}{\partial} = VB(x)$ at the insulating walls and partitions.

Here $\vec{j}$ is the density of the current, φ is the electrical potential, φ_1 and φ_2 are potential values at the electrodes, We shall introduce, into the band with slots along the partition corresponding to the exterior

Cat. 03 Pages - 10 Code - 1

TRANSLATED FOR JET PROPULSION LAB, NAS 7-100, 1968 No. CR-97886 of the duct, an analytic function of complex variable $z = x + iy$

$$F = \varphi + i\psi \tag{1.3}$$

where ψ is a function of the current that satisfies the ratios:

$$\frac{\partial\psi}{\partial x} = -\frac{\partial\varphi}{\partial y}, \quad \frac{\partial\psi}{\partial y} = \frac{\partial\varphi}{\partial x} \tag{1.4}$$

Let us further represent the indicated band of plane z as the upper half-plane of plane $t = \tau + i\nu$ with the aid of the conform conversion [ref.#5]:

$$z = f(t) = \sum_{k=1}^{n} \frac{h_k}{\pi} \ln(t - a_k) - \sum_{i=1}^{m} \frac{\ell_i}{\pi} \ln (t - c_i) + C \tag{1.5}$$

where the designations h_k and ℓ_i are clear from figure 1.a, and a_k and c_i are point coordinates on the τ axis, corresponding to points A_k and C_i of plane z. The points on the τ axis with the coordinates b_j and d_s which are found from equation

$$\sum_{k=1}^{n} \frac{h_k}{t-a_k} - \sum_{i=1}^{m} \frac{\ell_i}{t-c_i} = 0$$

correspond to the ends of the partitions (points B_j and D_s of plane z). The parameters a_k, b_j, c_i, d_s, and C are defined from the values of Re $f(d_j)$ and Re $f(d_s)$ known from the assignment of the duct geometry and its position relative to the selected system of coordinates. Three parameters (a_k, b_j, c_i, d_s) are arbitrarily selected. If one of the parameters a_k or c_i is equal to ∞, the formulas remain in force if the corresponding components are selected If one of the parameters b_j or d_s is equal to ∞, then the formulas do not change. Without disrupting the generalities, it is possible to consider that the points (-1, $-k$, k, 1) (figure 1.b)($k < 1$) will correspond to the points of the electrodes on the τ axis.

Problem (1.2) can be solved by various methods. The first method includes the fact that we initially represent the upper half-plane by a rectangle, and then we solve the Laplace equation for the potential under the condition that the value of the potential is given on two sides of the rectangle, and the value of its normal derivative is on the order of two. The solution of the problem is obtained then in the form of a trigonometric series. Thus, in the paper [ref. #6], the problem was solved for a duct without partitions. It should be noted that this method of solution is suitable only in the case when there is a total of 2 electrodes. The second method is included in the use of the Keldysh-Sedov formula for this problem. Actually, problem (1.2) is reduced to finding the following analytic function in the upper half-plane

$$F_1(t) \equiv F[z(t)] = \varphi_1 + i\psi_1 \tag{1.7}$$

whose real part is assigned in the sectors $|\tau| \in (k,1)$ and whose imaginary part in the remaining section of the τ axis. Since $F_1(t)$ is limited on the τ axis and it is always possible to define it so that it disappears at infinity ($t = \infty$), then the solution is written in the following manner [ref. #7]:

$$F_1(t) = \frac{R(t)}{i\pi}\left\{\varphi_1 \int_{-1}^{-k} \frac{d\tau}{R(\tau)(\tau-t)} + \varphi_2 \int_{k}^{1} \frac{d\tau}{R(\tau)(\tau-t)}\right\} + \tag{1.8}$$

$$\frac{R(t)}{\pi}\left\{\int_{-k}^{k} \frac{\psi(\tau)\,d\tau}{R(\tau)(\tau-t)} + \int_{-\infty}^{-1} \frac{\psi(\tau)\,d\tau}{R(\tau)(\tau-t)} + \int_{1}^{\infty} \frac{\psi(\tau)\,d\tau}{R(\tau)(\tau-t)}\right\}$$

In fulfilling the conditions of solubility:

$$\varphi_1 \int_{-1}^{-k} \frac{\tau^{j-1}d\tau}{R(\tau)} + \varphi_2 \int_{k}^{1} \frac{\tau^{j-1}d\tau}{R(\tau)} + i\left\{\int_{-k}^{k} \frac{\psi(\tau)\tau^{j-1}d\tau}{R(\tau)} + \int_{-\infty}^{-1} \frac{\psi(\tau)\tau^{j-1}d\tau}{R(\tau)} + \int_{1}^{\infty} \frac{\psi(\tau)\tau^{j-1}d\tau}{R(\tau)}\right\} = 0 \tag{1.9}$$

where $j = 1, 2$; $R(t) = \sqrt{(t+1)(t+k)(t-k)(t-1)}$, and the values of the roots are considered positive when $\tau > 1$. Condition (1.9), when $j = 2$, defines φ_1 and φ_2 at a given difference of potentials at the electrodes $\varphi_1 = \varphi_2 = U$ and when $j = 1$ yields the expression for the total current at the electrodes. The last condition ($j = 1$) makes it possible to easily find one of the basic values that determines the electrical characteristics of the duct, i.e., the dimensionless integral resistance of the duct ϕ^{-1}. Actually, assuming $\psi = 0$ in the sectors $|\tau| \varepsilon (1, \infty)$ and $\psi = \text{const}$ in the sector $\tau \varepsilon (-k, k)$ from (1.9) when $j = 1$, we find

$$\phi = K(k')/2K(k) \tag{1.10}$$

where $K(k)$ is the total elliptic integral of the first kind, $k' = \sqrt{1-k^2}$. The parameter k (see figure 1.b) is defined by the formula of the conform conversion of the internal duct to the upper half-plane $\nu > 0$ and is only a function of the duct geometry. Let us note that formula (1.10) is justified for a duct of arbitrary form having 2 electrodes, it is necessary only that its exterior be represented as an upper half-plane $\nu > 0$; then the points corresponding to the ends of the electrodes will go over to points τ_1, τ_2, τ_3, and τ_4 of the τ axis, and then by means of an additional linear conversion of them, it is always possible to transfer them to points $(-1, -k, k, 1)$.

2. Let us now examine the case of the distribution of partitions of practical interest. The geometry of the duct in question is clear in figure 2.a. In order to simplify the solution, we shall consider that the electrodes are semi-infinite, i.e., we shall examine the left half of the real duct as having sufficiently long symmetrically distributed electrodes. In practice, the formulas obtained below can be used when $c > 1$, where $c = 2\lambda/H$, 2λ is the length of the electrodes, H is the width of the duct. Actually, as was shown [ref. #6] for a duct without partitions, formulas that take into consideration the influence of the end effects on the electrical characteristics of the duct when $c > 1$ in practice do not differ from the corresponding formulas obtained in the assumption that the electrodes are infinitely long. In our case here, the introduction of partitions into the duct can only reduce the

scale of the zone of inhomogeneity of the electrical field at the input or output of the duct, therefore, we shall consider that all formulas that will be obtained below are of course justified when $c > 1$. Thus, the problem is reduced to the solution of the system of equations of (1.2) with the following boundary conditions:

$$\frac{\partial \varphi}{\partial x} = 0 \quad \text{when} \quad |x| \to \infty$$

$$\frac{\partial \varphi}{\partial y} = 0 \text{ when } x = -\infty, \quad \varphi = U/2 \quad \text{when} \quad y = H, \ \mu < x < \infty$$

$$\frac{\partial \varphi}{\partial y} = \frac{U}{H} \text{ when } x = \infty, \quad \varphi = -U/2 \quad \text{when} \quad y = 0, \ \mu < x < \infty \qquad (2.1)$$

$$\frac{\partial \varphi}{\partial y} = VB(x) \text{ on the insulating walls and partitions.}$$

$$B(x) \to 0 \text{ when } x \to -\infty$$

An example of this problem can be conveniently demonstrated by yet another method of solution.

Let us introduce the analytic function

$$f(z) = p + iq = \frac{\partial \varphi}{\partial y} + i\frac{\partial \varphi}{\partial x} \qquad (2.2)$$

Then problem (2.1) is reduced to finding this function that satisfies the boundary conditions: $q = 0$ on the electrodes and $p = VB(x)$ on the insulating walls and partitions. In order to solve the formulated problem, we shall represent conformally the band $|x| < \infty$, $0 < y < H$ with slots along the partitions as the upper half-plane of plane $t = \tau + i\nu$ (figure 2.b) by means of the following formula [ref. #5]:

$$z = \frac{H}{k\pi} \ln Tn(t) \qquad (2.3)$$

where

$$Tn(t) = \frac{1}{2}[(t + \sqrt{t^2-1})^n + (t - \sqrt{t^2-1})^n]$$

is Chebyshev's polynomial.

Here points $(\tau_k, 0)$ and $(\tau'_m, 0)$ of plane t will correspond to points D_k and B_m of plane z, where

$$\tau_k = \cos\theta_k, \ \theta_k = \frac{\pi}{2n} + \frac{k\pi}{n}; \ Tn(\tau_k) = 0, \ k = 1, \ldots, n$$

$$\tau'_m = \cos\theta'_m, \ \theta'_m = \frac{m\pi}{n}; \ \frac{dTn(\tau'_m)}{d\tau} = 0, \ m = 1, \ldots, n-1 \qquad (2.4)$$

The correspondence of the remaining points is clear from figure 2.a,b, and the value a is found from the equation:

$$Tn(a) = \exp\left(\frac{\mu n\pi}{H}\right) \qquad (2.5)$$

The analytic function $f(z)$ from (2.2) in the region of the complex variable t will change into function

$$f_1(t) = p_1 + iq \tag{2.6}$$

satisfying the boundary conditions:

$$\begin{gathered} p = VB(\tau) \quad \text{at} \quad A_-A_+ \quad \tau \varepsilon (-a, a) \\ q = 0 \quad \text{at } CA_- \text{ and } A_+C \;, \quad a < |\tau| < \infty \\ f_1(\tau_k) = 0 \quad \text{and} \quad f_1(\infty) = U/H \end{gathered} \tag{2.7}$$

Thus, we have come to a mixed boundary value problem for the harmonic function f_1, while f_1 has the properties of a polar type at points corresponding to the ends of the partitions. We shall first plot the function f_{10} corresponding to theuniform problem ($p = 0$ at A_-A_+). It is easy to see that

$$f_{10} = \frac{U}{Hg(t)} + \frac{1}{\Pi(t)}\left[\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m}{t-\tau_m}\right] \tag{2.8}$$

where

$$g(t) = \sqrt{\frac{t-a}{t+a}}, \quad \Pi(t) = \sqrt{(t-a)(t+a)}$$

In order to obtain a solution of the initial problem, it is sufficient to add some special solution of the non-uniform problem (2.7) to f_{10}. As this special solution one can take the function [ref. #7]:

$$f_{11} = \frac{VB_0}{i\pi g(t)} \int_{-a}^{a} \frac{g(\tau)\bar{B}(\tau)\,d\tau}{\tau - t} \tag{2.9}$$

then, finally, we shall obtain the following expression for f_1 solving the initial problem:

$$f_1(t) = \frac{VB_0}{i\pi g(t)} \int_{-a}^{a} \frac{g(\tau)\bar{B}(\tau)\,d\tau}{\tau - t} + \frac{U}{Hg(t)} + \frac{1}{\Pi(t)}\left[\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma'_m}{t - \tau_m}\right] \tag{2.10}$$

The constants γ'_0 and γ'_m are found from the condition $f_1(\tau_k) = 0$, i.e., they satisfy the system of equations:

$$\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m}{\tau_n - \tau_m} = (J - k)(a + \tau_k) + F_k \tag{2.11}$$

where $\gamma_0 = \gamma'_0/B_0V$, $\gamma_m = \gamma'_m/B_0V$ $k = U/HB_0V$ is the coefficient of load

$$J = \frac{}{\pi}\int_0^{\pi} \bar{B}(\vartheta)\,d\vartheta, \quad F_k = \frac{a\cos\vartheta_k}{\pi}\int_0^{\pi} \bar{B}(\vartheta)\,\text{ctg}\,(\vartheta-\vartheta_k)\,d\vartheta \tag{2.12}$$

$$\cos\vartheta = \tau/a, \quad \cos\vartheta_k = \tau_k/a, \quad \bar{B}(\vartheta) \equiv \bar{B}[x(\vartheta)]$$

From (2.10) with the use of (2.11) and (2.12) we obtain the following expression for the current extracted from the electrodes:

$$I = \sigma\mathcal{E}\left(cG_1 - k\phi + \beta_1\right) \tag{2.13}$$

where $\mathcal{E} = B_0VH$ is the Faraday e.m.f.,

$$G_1 = \lambda^{-1}\int_{\mu}^{\lambda+\mu}\bar{B}(x)\,dx \tag{2.14}$$

$$\phi = c + \frac{2\ln 2}{\pi} + \frac{2\mu}{H} - \frac{2}{\pi}\ln a$$

$$\beta_1 = \frac{4}{n\pi^2}\int_0^a \arcsin\left(\frac{\tau}{a}\right)\bar{B}(\tau)\,d\ln T_n(\tau) \tag{2.15}$$

Then the electric power taken from the electrodes will be equal to:

$$P_1 = k\mathcal{E}I \tag{2.16}$$

The electromagnetic power developed in the duct is defined in the following manner:

$$P_2 = 2\int_{-\infty}^{\mu+\lambda}dx\int_0^H dy\,\left(\vec{J}\times\vec{B}\right)\vec{V} = \frac{2\sigma\mathcal{E}^2}{H}\int_{-\infty}^{\lambda}\bar{B}(x)\,dx - \sigma\mathcal{E}^2ckG_1 + \int_s VB(x)\varphi(x)\,dx, \tag{2.17}$$

where $\int_s$ designates the integration along the insulating walls and both sides of the partitions. Omitting the intermediate calculations, we shall present the final expression for the power developed

$$P_2 = \sigma\mathcal{E}^2[c(G_2 - kG_1) - k\beta_1 + \beta_2], \tag{2.18}$$

where

$$G_2 = \lambda^{-1}\int_{\mu}^{\lambda\,\mu}\bar{B}^2(x)\,dx$$

$$\beta_2 = \frac{2}{H}\int_{-\infty}^{\mu}\bar{B}^2(x)\,dx - \frac{2}{n^2\pi^3}\int_0^{\pi}d\vartheta\left(\int_{\vartheta}^{\pi}\bar{B}\left(\frac{\vartheta'}{2}\right)d\ln T_n\left(\frac{\vartheta'}{2}\right)\right)\int_0\bar{B}\left(\frac{\vartheta_1}{2}\right)\operatorname{ctg}\frac{\vartheta_1-\vartheta}{2}\,d\ln T_n\left(\frac{\vartheta_1}{2}\right) \tag{2.19}$$

Note that the special integral with Hilbert's nucleus enters into the expression for β_2. It is possible to calculate this integral for the arbitrary function $\bar{B}(x)$ by expanding the function

$$f(\vartheta) = \bar{B}\left(\frac{\vartheta}{2}\right)\frac{d\ln Tn(\vartheta/2)}{d\vartheta}$$

into the Fourier series

$$f(\vartheta) = \sum_{m=1}^{\infty} b_m \sin m\vartheta \tag{2.20}$$

where

$$b_m = \frac{2}{\pi}\int_0^{\pi} \bar{B}(\vartheta/2)\ \sin m\vartheta\ d\ln Tn(\vartheta/2) \tag{2.21}$$

Substituting (2.20) into (2.19) and integrating, having used the relationship [ref. #5]:

$$\int_0^{2\pi} \sin m\vartheta_1\ \mathrm{ctg}\,\frac{\vartheta_1-\vartheta}{2}\,d\vartheta_1 = 2\pi\cos m\vartheta,$$

we shall obtain

$$\beta_2 = \frac{2}{H}\int_{-\infty}^{\mu} \bar{B}^2(x)\,dx - \frac{8}{n^2\pi^3}\sum_{m=1}^{\infty}\frac{A_m^2}{m}, \tag{2.22}$$

where

$$A_m = \int_0^{\pi/2} \bar{B}(\vartheta)\ \sin 2m\vartheta\ d\ln Tn(\vartheta)$$

When $n = 1$ (duct without partitions) expressions (2.15) and (2.22) for β_1 and β_2 coincide with the corresponding expressions obtained in a previous paper [ref. #6], and when $n = 2$, those obtained in [ref. #8] If the partitions approach right up to the electrode zone ($\mu = 0$), then

$$\beta_1 = \beta_{10}/n \quad \text{and} \quad \beta_2 = \beta_{20}/n \tag{2.23}$$

where β_{10} and β_{20} are the corresponding [appropriate] coefficients for the subduct (width H/n).

Let us examine the practically important case fo the distribution of the magnetic field

$$\bar{B}(x) = \begin{cases} 1 & \text{when } 0 < x < \mu \\ 0 & \text{when } x < 0 \end{cases} \tag{2.24}$$

then (2.15) and (2.19) will assume the form:

$$\beta_1 = \frac{2\mu}{H} + \frac{4}{n\pi^2}\int_{\arccos 1/a}^{0} \ln Tn(\vartheta)\,d\vartheta, \tag{2.15'}$$

$$\beta_2 = \frac{2\mu}{H}\cdot\frac{2}{\pi}\arcsin\frac{1}{a} + \left(1 - \frac{2}{\pi}\arcsin\frac{1}{a}\right)\beta_1 - \frac{4}{n^2\pi^3}\sqrt{a^2-1}\int_{\arccos 1/a}^{0}\frac{\ln Tn(\vartheta)\,d\vartheta}{a^2\cos^2\vartheta - 1} -$$

$$- \frac{4}{n^2\pi^3}\int_{\arccos 1/a} \ln Tn(\vartheta)\sum_{k=1}^{n}\frac{F_k}{a(\cos\vartheta - \cos\vartheta_k)}\,d\vartheta \tag{2.19'}$$

Figures 3 and 4 show the dependence of β_1 and β_2 on μ/H at various values of n calculated according to formulas (2.15') and (2.19'). From formulas

(2.15′) and (2.19′), it is possible to obtain an expression for the Joulean losses Q in the case where all walls of the duct are insulating, by going over to the limit when $\mu/H \to \infty$ ($a \to \infty$), here

$$Q = \sigma \mathcal{E}^2 \beta \qquad (2.25)$$

where

$$\beta = \lim_{\mu/H \to \infty} (\beta_2 - \beta_1) = \frac{4}{n^2\pi^3} \int_0^1 \frac{\ln^2 Tn(1/x)\,dx}{1 - x^2} + \frac{4}{n^2\pi^3} \sum_{m=1}^{n-1} \gamma_{m1} \int_0^1 \frac{\ln^2 Tn(1/x)\,dx}{(1-\tau'_m x)^2} \qquad (2.26)$$

$$\sum_{m=1}^{n-1} \gamma_{m1}/\tau_k - \tau'_m = -\sin \tau_k \operatorname{arccth}|\tau_k|^{-1}$$

As can be easily confirmed, when n = 1 and n = 2 (2.26) coincides with the corresponding expressions obtained in the papers [ref. #9, 10], the only difference being that the value β from (2.26) is twice as large as in [ref. #9, 10], because we take into account the total losses at the input and output of the magnetic field. Figure 5 shows the dependence of β on the number of partitions n calculated according to formula (2.26).

3. All the above examined problems pertain to the case when the partitions have infinite length. In actual constructions the partitions can not be infinitely long, moreover, in order to reduce the hydraulic resistance in the duct they are rationally made as short as possible. If several partitions of finite length are introduced into a duct, then it is not possible to obtain an analytic solution of the corresponding problems by virtue of the greater mathematical difficulties connected with the fact that the region corresponding to the interior of the duct becomes multiply connected. If then one pair of partitions of finite length is introduced into the duct symmetrically with respect to the electrodes, then using the conditions of symmetry, it is possible to solve the problem for a single connected region. We shall present some results of the solution of such a problem. The geometry of the duct is clear from figure 6.a, the electrodes are considered semi-infinite, the assumed tolerances are the same as at point 1.

Thus, it is necessary to solve system (1.2) with the following boundary conditions (figure 6.a):

$$\frac{\partial\varphi}{\partial x} = 0, \quad \frac{\partial\varphi}{\partial y} = 0 \quad \text{when} \quad x = -\infty$$

$$\frac{\partial\varphi}{\partial x} = 0, \quad \frac{\partial\varphi}{\partial y} = \frac{U}{2\delta} \quad \text{when} \quad x = \infty$$

$$\frac{\partial\varphi}{\partial y} = VB(x) \ \text{when} -\infty < x < 0, \quad y = 0$$

$$-\mu_2 < x < \mu_1 \quad y = \delta \qquad (3.1)$$

$$\varphi = 0 \ \text{when} \ y = \delta, \ -\infty < x < -\mu_2, \ -\mu_1 < x < \infty$$

$$\varphi = -U/2 \ \text{when} \ y = 0, \quad 0 < x < \infty.$$

Introducing the analytical function $f(z) = \frac{\partial\varphi}{\partial y} + i\frac{\partial\varphi}{\partial x} = p + iq$ and represent-the band $|x| < \infty$, $0 < y < \delta$ of plane z by means of formula $t = \exp(\pi z/\delta)$ on the upper half-plane of plane $t = \tau + i\nu$(figure 6.b) we come to the following problem: to find the function $f_1(t) \equiv f[z(t)] = p_1 + iq$ that satisfies the boundary conditions

$$p_1 = VB(\tau) \quad \text{at} \quad B_1A_1 \text{ and } O_1D_1$$

$$q_1 = 0 \quad \text{at} \quad C_1B_1,\ A_1O_1, \text{ and } D_1C_1.$$

Having applied the Keldysh-Sedov formula [ref. #7], we shall obtain the following expression for the function f_1:

$$f_1(t) = \frac{U}{2\delta g(t)} + h(t) + \frac{VB_0}{\pi i g(t)}\left\{\int_{-\ell}^{-m}\frac{\bar{B}(\tau)g(\tau)}{\tau - t}d\tau + \int_{0}^{1}\frac{\bar{B}(\tau)g(\tau)}{\tau - t}d\tau\right\} \qquad (3.2)$$

where

$$g(t) = \sqrt{\frac{(t+m)(t-1)}{t(t+\ell)}},\quad h(t) = \frac{\gamma_0 + \gamma_1 t}{\sqrt{(t+m)(t+\ell)t(t-1)}}$$

$$m = \exp(-\pi\mu_2/\delta), \quad \ell = \exp(-\pi\mu_1/\delta), \quad \nu = \mu_2 - \mu_1$$

The roots are considered positive when $\tau > 1$. From the boundary conditions of (3.1) it follows that $\gamma = 0$. The constant γ_1 is defined form the condition that the increase of the potential along the partition is equal to 0.

Formulas for β_1, β_2, and ϕ obtained from (3.2) at an arbitrary position of the partition are very cumbersome, therefore, they are not presented in the report. The basic conclusion which can be drawn from the analysis of the obtained solution includes the fact that the values of the coefficients β_1, β_2, and ϕ quite slowly ($\sim\delta/\nu$) approach their corresponding values for the semi-infinite partition. For example, when $\mu_1 = 0$ and $\nu/\delta > 2$,

$$\phi - \phi_\infty = \left(\frac{12 \ln 2}{\pi} + \frac{4\nu}{\delta}\right)^{-1} \qquad (3.3)$$

where ϕ_∞ is the integral conductivity of the duct with a semi-infinite partition.

A similar solution of the problem when $\mu_1 = 0$ where the magnetic field is absent beyond the electrodes is presented in the paper [ref. #11].

References

1. Sutton, G.W., Vista Astronautics, 3, 53, 1960.
2. Vatazhin, A.B., Nyemkova, N.G., PMTF, 2, 40, 1964.
3. Lavrent'yev, I.V., *Magnetic Hydrodynamics (Magnitnaya Gidrodinamika)*, 3, 153, 1967.
4. Vatazhin, A.B., Regiser, S.A., PMM, 3, 548, 1962.
5. Volkovyskii, L.I., Lunts, G.L., Aramanovich, I.G.,*A Collection of Problems on the Theory of Functions of Complex Variables (Sbornik zadach po teorii funktsii kompleksnogo peremennogo)*, Fizmatgiz (Translator's note: Publishing House for Material in Physics), 1960.
6. Sutton, G.W., Hurwitz, H. Jr., and Poritsky, H., Commun. and Electron., 58, 687, 1962.
7. Muskhelishvili, N.I., *Singular Integral Equations (Singulyarnyye integral'nyye uravneniya)* Fizmatgiz, 1962.
8. Lavrent'yev, I.V., *Magnetic Hydrodynamics*, 4, 89, 1967.
9. Vatazhin, A.B., PMTF, 5, 59, 1962.
10. Vatazhin, A.B., PMTF, 4, 122, 1964.
11. Lavrent'yev, I.V., *Magnetic Hydrodynamics*, 1, 1968.

Figures

End Effects in a MHD Duct with Nonconducting Partitions.

Fig.1. Schematic of a duct and conforming reflection.
Fig.2. Schematic of a duct and conforming reflection.
Fig.3. Dependence of the coefficient of β_1 on μ/H.
Fig.4. Dependence of the coefficient of β_2 on μ/H.
Fig.5. Dependence of the coefficient of β on the number of partitions.
Fig.6. Schematic of a duct with conforming reflection.