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Department of Applied Mechanics

STANFORD UNIVERSITY

Technical Report No. 190

PITCH AND YAW MOTIONS OF A HUMAN BEING IN FREE FALL

by

M. P. Scher and T. R. Kane

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ABSTRACT

Two limb maneuvers, one producing pitch motion, the other yaw motion, are analysed. Numerical results are given for representative examples.

ACKNOWLEDGMENT

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INTRODUCTION

This report deals with limb motions that can be used by a man to alter the orientation of his body in space when he is in a state of free fall without initial rotation. (A man is said to be in free fall or "weightless" when the only forces acting on his body are gravitational forces.)

It is helpful to introduce at the outset three intersecting, mutually perpendicular lines and three planes which are fixed relative to the torso of the man. In accordance with familiar aeronautical usage, these lines are called pitch, roll, and yaw axes. The yaw axis has the same general orientation as the spine, and the pitch axis is perpendicular to the plane of symmetry of the torso. The location of the point of intersection of the axes is selected by the analyst in any convenient manner, and may differ from one analysis to the next. The plane determined by a pair of axes is designated by the name of the axis normal to it; e.g., the pitch plane is the plane normal to the pitch axis.

Two maneuvers are examined. The first, treated in Part I, produces rotation of the torso about the pitch axis; the second, discussed in Part II, is a yaw maneuver. (A previous report (see [1]) contains an analysis intended primarily for the study of roll motions.) Section 1 of each Part contains a description of the maneuver, Section 2 deals with the analysis, and numerical results are presented in Section 3.

I. PITCH MOTION

1. Description

The maneuver to be studied is discussed in [2] and is covered by the analyses in [1] and [3]. The present formulation is more concise than those in the aforementioned references, and the discussion of results is more extensive.

During the maneuver, the arms are held straight at the elbows and perform a rotary motion with respect to the torso, remaining symmetrically disposed with respect to the pitch plane at all times. The longitudinal axis of each arm travels on the surface of an imaginary torso-fixed cone whose vertex is at the shoulder. The arm motion can be described in terms of the cone semi-vertex angle and the orientation of the cone axis relative to the torso, any physically attainable orientation being permissible, as illustrated by the examples in Fig. 1.1. (Symmetry considerations show that these cone parameters need to be specified for only one of the arms.) The legs must be kept fixed relative to the torso and symmetrically located with respect to the pitch plane.

Consider the motion of the torso (and legs) during one cycle of the maneuver. The sense of pitching of the torso is opposite to that in which the arms travel on the surfaces of the cones. A reversal in the sense of motion of the arms produces a reversal in the sense of pitching.

Prior to starting the maneuver, the subject may have his limbs disposed in any way satisfying the requirements of symmetry with respect to the pitch plane, and the arms may then be brought to the starting

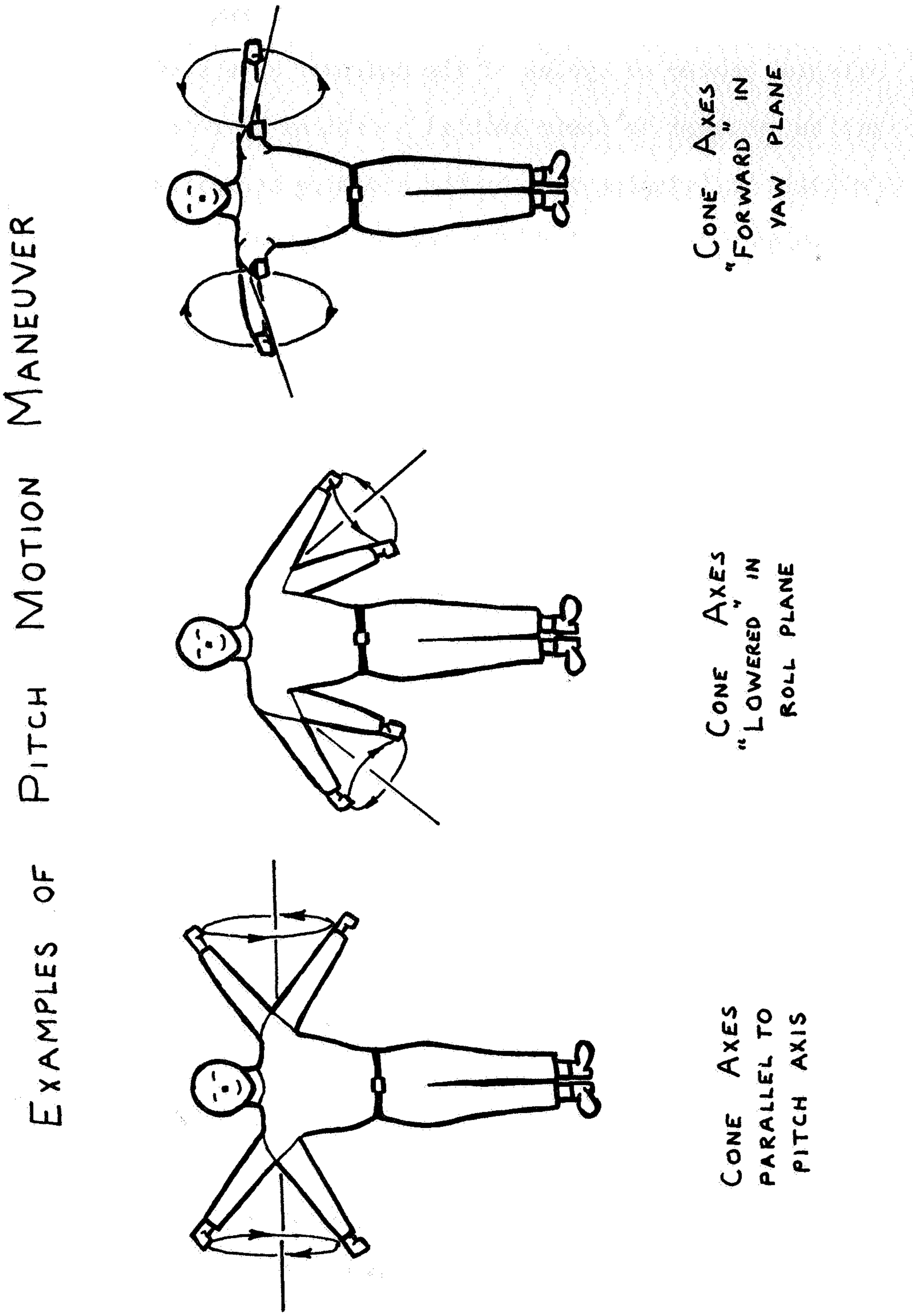
EXAMPLES OF PITCH MOTION MANEUVER
CONE AXES PARALLEL TO PITCH AXIS
CONE AXES "LOWERED" IN ROLL PLANE
CONE AXES "FORWARD" IN YAW PLANE

Fig. 1.1

position in any manner compatible with this requirement. After an integral number of cycles of the maneuver have been performed, the arms can be returned to their initial positions by retracing the paths followed in bringing them to the starting positions. Any pitching of the torso obtained while starting will be nullified during the return.

2. Analysis

For purposes of analysis, the human is modelled as a system S of three rigid bodies. One of these, designated A, represents the torso, head, and legs. The remaining two, B and B', each represent an arm. B and B' are connected to A at points O and O' (see Fig. 2.1), which represent shoulder joints.

Body A is presumed to be symmetric with respect to the pitch plane, so that its mass center A* lies in this plane. Points O and O' are symmetrically located at a distance a_2 to either side of the pitch plane, and the line P passing through them is designated the pitch axis. The yaw axis Y lies in the pitch plane and is fixed in body A, intersecting P at point C. As will be seen later, it is convenient to choose the orientation of Y in the pitch plane in such a way that Y passes through the mass center of a body comprised of only the torso and head, but not the legs. The position of A* relative to C is specified by a distance a_3, measured along Y, and a distance a_1, measured from the roll plane. The mass of A is m_A, and the moment of inertia of A about a line through A* parallel to P is I^A. Unit vectors $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ are fixed in A parallel to the roll, pitch, and yaw axes, respectively.

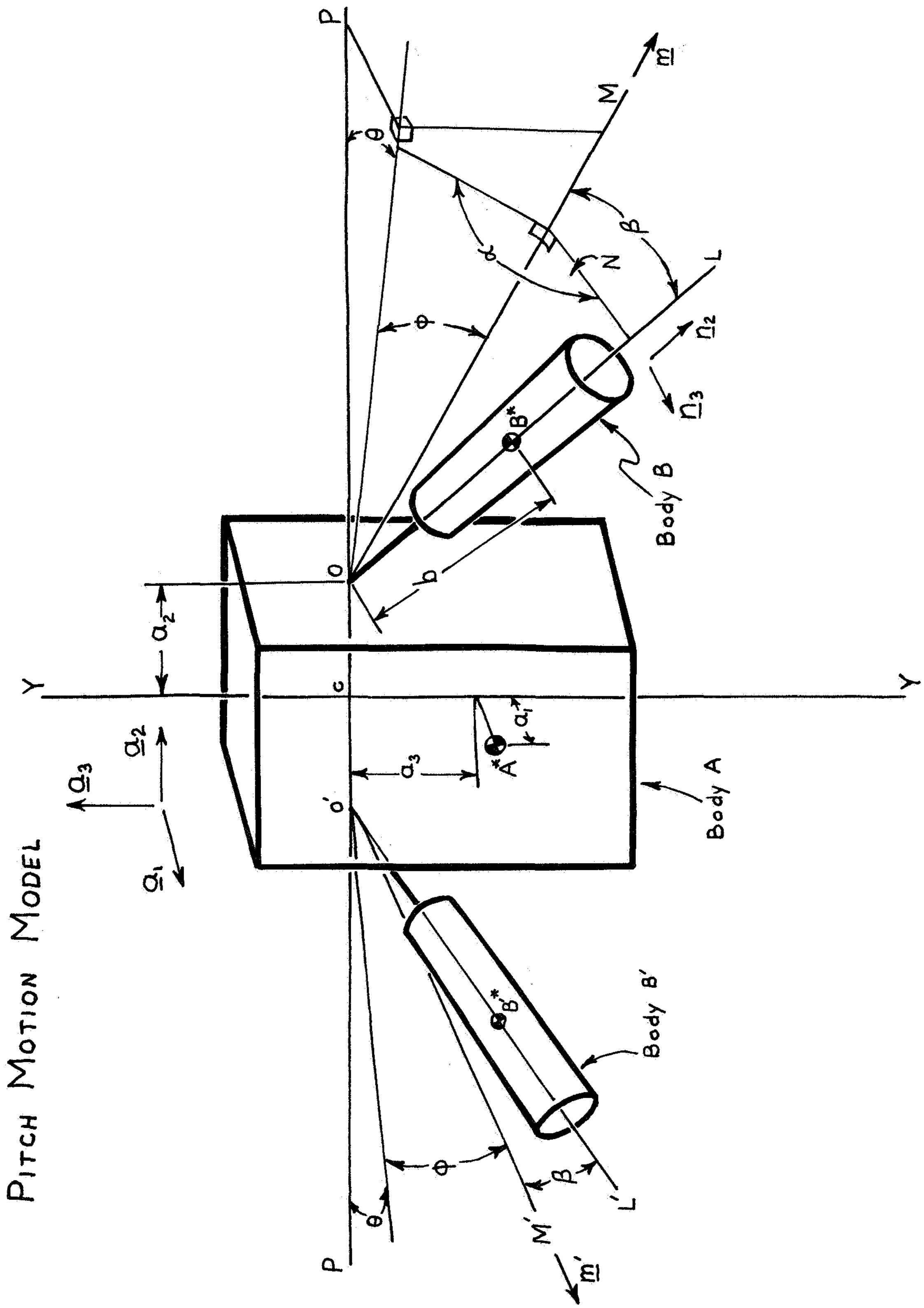

Fig. 2.1

Body B possesses an axis of symmetry, designated L. The mass center B^* of B is located on L at a distance b from point O. The mass of B is m_B, the moment of inertia of B about L is I_2^B, and the moment of inertia of B about any line passing through B^* and normal to L is I_1^B. The geometry and inertia properties of B' are identical to those of B.

The kinematical analysis in the sequel involves two lines, M and M', which are fixed with respect to A and are located symmetrically with respect to the pitch plane. Line M passes through O and its orientation is determined by angles θ and φ as shown in Fig. 2.1. (The lines forming θ lie in the yaw plane, and those forming φ lie in a plane normal to the yaw plane.) Symmetry considerations permit one to locate M'. Unit vectors $\underline{m}$ and $\underline{m}'$ are parallel to M and M', respectively.

The equation of motion for this system will be obtained by employing a consequence of Lagrange's equations for cases in which S is in free fall and possesses no initial angular momentum, namely

$$\frac{\partial K}{\partial \dot{\xi}} = 0 \tag{2.1}$$

where $\dot{\xi}$ is the time derivative of the angle ξ between Y and a line that is fixed in an inertial reference frame F and is normal to P; and K, the kinetic energy associated with motions of S relative to the system mass center, can be expressed as

$$K = K_\omega^A + K_\omega^B + K_\omega^{B'} + K_v^A + K_v^B + K_v^{B'} \tag{2.2}$$

The first three terms on the right-hand-side of this equation represent the rotational kinetic energies of A, B and B'. For example, if I_1^B, I_2^B, I_3^B are principal moments of inertia of B for B*, and ω_1^B, ω_2^B, ω_3^B are components of the angular velocity of B in F when this angular velocity is referred to principal axes of B for B*, then

$$K_\omega^B = \frac{1}{2}\left[I_1^B(\omega_1^B)^2 + I_2^B(\omega_2^B)^2 + I_3^B(\omega_3^B)^2\right] \tag{2.3}$$

The last three terms in Eq. 2.2 reflect the motions of the mass centers of A, B, and B'. For instance, if $\underline{v}_B$ is the velocity of B* relative to the system mass center S*, then

$$K_v^B = \frac{1}{2}\, m_B(\underline{v}_B^2) \tag{2.4}$$

In the following kinematical analysis, only pitch motions of A are considered. Consequently, the angular velocity of A in F can be expressed as

$$^F\underline{\omega}^A = \dot{\xi}\,\underline{a}_2 \tag{2.5}$$

Symmetry considerations demand that $\underline{a}_2$ be parallel to a principal axis of A for A*. Hence I^A is a principal moment of inertia and †

$$K_\omega^A \underset{(2.3,2.5)}{=} \frac{1}{2} I^A \dot{\xi}^2 \tag{2.6}$$

† Numbers beneath equal signs are intended to direct attention to corresponding equations.

The motions of B and B' are described in terms of the "coning" motion outlined in Sec. 1, lines M and M' being the axes of the cones. Line L intersects line M at O, thereby defining a plane N, and the angle β between L and M remains constant throughout the maneuver. Mutually perpendicular unit vectors $\underline{n}_1$, $\underline{n}_2$, $\underline{n}_3$ are fixed in N, with $\underline{n}_2$ parallel to L and $\underline{n}_1$ normal to N.

If the angle between N and a plane passing through M and normal to the yaw plane is designated α, then the angular velocity of N in A is given by

$$^{A}\underline{\omega}^{N} = \dot{\alpha}\,\underline{m} \tag{2.7}$$

When α increases from 0 to 2π, L travels once around the surface of a cone of semi-vertex angle β. During this motion, B must rotate in N in such a manner that no twisting of the arm relative to the torso occurs at the shoulder. This can be accomplished by specifying the angular velocity of B in N as follows:

$$^{N}\underline{\omega}^{B} = -\dot{\alpha}\,\underline{n}_2 \tag{2.8}$$

The angular velocity of B in F can be expressed as

$$^{F}\underline{\omega}^{B} = {}^{N}\underline{\omega}^{B} + {}^{A}\underline{\omega}^{N} + {}^{F}\underline{\omega}^{A} \tag{2.9}$$

In order to resolve this vector into components parallel to $\underline{n}_1$, $\underline{n}_2$, $\underline{n}_3$, we note that

$$\underline{n}_i = \sum_{j=1}^{3} c_{ij}\,\underline{a}_j \qquad i=1,2,3 \tag{2.10}$$

where the c_{ij} of interest are

$$c_{12} = - \sin\theta \cos\alpha - \cos\theta \sin\varphi \sin\alpha$$

$$c_{21} = \cos\theta \sin\beta \sin\alpha + \sin\theta \cos\varphi \cos\beta + \sin\theta \sin\varphi \sin\beta \cos\alpha$$

$$c_{22} = - \sin\theta \sin\beta \sin\alpha + \cos\theta \cos\varphi \cos\beta + \cos\theta \sin\varphi \sin\beta \cos\alpha$$

$$c_{23} = - \sin\varphi \cos\beta + \cos\varphi \sin\beta \cos\alpha$$

$$c_{32} = - \sin\theta \cos\beta \sin\alpha - \cos\theta \cos\varphi \sin\beta + \cos\theta \sin\varphi \cos\beta \cos\alpha$$

(2.11)

Furthermore,

$$\underline{m} = \cos\beta\, \underline{n}_2 - \sin\beta\, \underline{n}_3 \tag{2.12}$$

It now follows that (see Eqs. 2.9, 2.5, 2.10, 2.7, 2.12, 2.8)

$${}^F\underline{\omega}^B = [\dot{\xi} c_{12}]\underline{n}_1 + [\dot{\xi} c_{22} + \dot{\alpha}(\cos\beta - 1)]\underline{n}_2 + [\dot{\xi} c_{32} - \dot{\alpha} \sin\beta]\underline{n}_3 \tag{2.13}$$

and, since $\underline{n}_1$, $\underline{n}_2$, $\underline{n}_3$ are parallel to principal axes of B for B^*,

$$K_\omega^B \underset{(2.3, 2.13)}{=} \frac{1}{2}\Big\{ \dot{\xi}^2 [I_1^B (c_{12}^2 + c_{32}^2) + I_2^B c_{22}^2] + 2\dot{\xi}\dot{\alpha}[I_2^B c_{22}(\cos\beta - 1) - I_1^B c_{32} \sin\beta] + \dot{\alpha}^2 [I_1^B \sin^2\beta + I_2^B (\cos\beta - 1)^2] \Big\} \tag{2.14}$$

In a similar fashion, an identical expression is obtained for $K_\omega^{B'}$.

The velocity $\underline{v}_A$ of A^* relative to S^* is the time derivative in F of $\underline{r}^{A^*/S^*}$, the position vector of A^* relative to S^*; i.e.,

$$\underline{v}_A = \frac{{}^F d}{dt}\, \underline{r}^{A*/S*} \tag{2.15}$$

Symmetry requires that $S*$ lie in the pitch plane. This fact makes it possible to express $\underline{r}^{A*/S*}$ as

$$\underline{r}^{A*/S*} = -\frac{2m_B}{m_A+2m_B}\left[\underline{r}^{B*/A*}\cdot\underline{a}_1\,\underline{a}_1 + \underline{r}^{B*/A*}\cdot\underline{a}_3\,\underline{a}_3\right] \tag{2.16}$$

where $\underline{r}^{B*/A*}$ is the position vector of $B*$ relative to $A*$, which, by reference to Fig. 2.1, is seen to be given by

$$\underline{r}^{B*/A*} = -a_1\,\underline{a}_1 + a_2\,\underline{a}_2 + a_3\,\underline{a}_3 + b\,\underline{n}_2 \tag{2.17}$$

Consequently, (see Eqs. 2.15, 2.16, 2.17, 2.10, 2.11, 2.5)

$$\underline{v}_A = \frac{2m_B}{m_A+2m_B}\left\{-\left[b\frac{\partial c_{21}}{\partial\alpha}\dot{\alpha} + \dot{\xi}(bc_{23}+a_3)\right]\underline{a}_1 + \left[\dot{\xi}(bc_{21}-a_1) - b\frac{\partial c_{23}}{\partial\alpha}\dot{\alpha}\right]\underline{a}_3\right\} \tag{2.18}$$

and it follows that

$$\begin{aligned} K_v^A = \frac{1}{2} m_A\left(\frac{2m_B}{m_A+2m_B}\right)^2 \Bigg\{ & \dot{\xi}^2\left[b^2(c_{23}^2+c_{21}^2)+2b(a_3c_{23}-a_1c_{21}) + a_1^2 + a_3^2\right] \\ & + 2\dot{\xi}\dot{\alpha}\left[b^2\left(c_{23}\frac{\partial c_{21}}{\partial\alpha} - c_{21}\frac{\partial c_{23}}{\partial\alpha}\right) + b\left(\frac{\partial c_{21}}{\partial\alpha}a_3 + \frac{\partial c_{23}}{\partial\alpha}a_1\right)\right] \\ & + \dot{\alpha}^2 b^2\left[\left(\frac{\partial c_{21}}{\partial\alpha}\right)^2 + \left(\frac{\partial c_{23}}{\partial\alpha}\right)^2\right]\Bigg\} \end{aligned} \tag{2.19}$$

(2.4,2.18)

Next, $\underline{v}_B$ is given by

$$\underline{v}_B = \underline{v}^{B*/0} + \underline{v}^{0/A*} + \underline{v}_A \tag{2.20}$$

where

$$\underline{v}^{B*/O} = {}^{F}\underline{\omega}^{B} \times b\,\underline{n}_2 \tag{2.21}$$

and

$$\underline{v}^{O/A*} = {}^{F}\underline{\omega}^{A} \times (-a_1\,\underline{a}_1 + a_2\,\underline{a}_2 + a_3\,\underline{a}_3) \tag{2.22}$$

Thus (see Eqs. 2.20, 2.18, 2.21, 2.13, 2.10, 2.22, 2.5)

$$\begin{aligned}
\underline{v}_B = &\left\{\frac{m_A}{m_A+2m_B}\left[\dot{\xi}(bc_{23}+a_3)+\dot{\alpha}\,b\,\frac{\partial c_{21}}{\partial\alpha}\right]\right\}\underline{a}_1 \\
&+\{\dot{\alpha}b\,\sin\beta\;c_{12}\}\underline{a}_2 \\
&+\left\{\frac{m_A}{m_A+2m_B}\left[\dot{\xi}(a_1-bc_{21})+\dot{\alpha}b\,\frac{\partial c_{23}}{\partial\alpha}\right]\right\}\underline{a}_3
\end{aligned} \tag{2.23}$$

K_v^B is, therefore, given by

$$\begin{aligned}
K_v^B \underset{(2.4,2.23)}{=} \frac{1}{2}m_B\Bigg\{&\dot{\xi}^2\left(\frac{m_A}{m_A+2m_B}\right)^2\left[b^2(c_{21}^2+c_{23}^2)+2b(c_{23}a_3-c_{21}a_1)+(a_1^2+a_3^2)\right] \\
&+2\dot{\xi}\dot{\alpha}\left(\frac{m_A}{m_A+2m_B}\right)^2\left[b^2\left(c_{23}\,\frac{\partial c_{21}}{\partial\alpha}-c_{21}\frac{\partial c_{23}}{\partial\alpha}\right)+b\left(a_3\,\frac{\partial c_{21}}{\partial\alpha}+a_1\,\frac{\partial c_{23}}{\partial\alpha}\right)\right] \\
&+\dot{\alpha}^2b^2\left[\sin^2\beta\;c_{12}^2+\left(\frac{m_A}{m_A+2m_B}\right)^2\left(\left(\frac{\partial c_{21}}{\partial\alpha}\right)^2+\left(\frac{\partial c_{23}}{\partial\alpha}\right)^2\right)\right]\Bigg\}
\end{aligned} \tag{2.24}$$

and the expression for $K_v^{B'}$ is identical. The kinetic energy associated with motions of S relative to S^* can now be expressed as (see Eqs. 2.2, 2.6, 2.14, 2.19, 2.24)

$$
\begin{aligned}
K = \;& \dot{\xi}^2 \Big\{ \frac{I^A}{2} + I_1^B(c_{12}^2 + c_{32}^2) + I_2^B c_{22}^2 \\
& \quad + \frac{m_A m_B}{m_A + 2m_B} \left[b^2(c_{21}^2 + c_{23}^2) + 2b(a_3 c_{23} - a_1 c_{21}) + (a_1^2 + a_3^2) \right] \Big\} \\
& + 2\dot{\xi}\dot{\alpha} \Big\{ I_2^B c_{22}(\cos\beta - 1) - I_1^B c_{32} \sin\beta \\
& \quad + \frac{m_A m_B}{m_A + 2m_B} \left[b^2 \left(c_{23} \frac{\partial c_{21}}{\partial \alpha} - c_{21} \frac{\partial c_{23}}{\partial \alpha} \right) + b \left(\frac{\partial c_{21}}{\partial \alpha} a_3 + \frac{\partial c_{23}}{\partial \alpha} a_1 \right) \right] \Big\} \\
& + \dot{\alpha}^2 \Big\{ 2 I_2^B (1 - \cos\beta) + (I_1^B - I_2^B + m_B b^2) \sin^2\beta \\
& \quad + \frac{m_A m_B}{m_A + 2m_B} b^2 \left[\left(\frac{\partial c_{21}}{\partial \alpha} \right)^2 + \left(\frac{\partial c_{23}}{\partial \alpha} \right)^2 \right] \Big\}
\end{aligned} \tag{2.25}
$$

The equation of motion can thus be formulated as

$$
\begin{aligned}
& \dot{\xi} \Big\{ \frac{I^A}{2} + I_1^B(c_{12}^2 + c_{32}^2) + I_2^B c_{22}^2 \\
& \qquad + \frac{m_A m_B}{m_A + 2m_B} \left[b^2(c_{21}^2 + c_{23}^2) + 2b(a_3 c_{23} - a_1 c_{21}) + (a_1^2 + a_3^2) \right] \Big\} \\
& + \dot{\alpha} \Big\{ I_2^B c_{22}(\cos\beta - 1) - I_1^B c_{32} \sin\beta \\
& \qquad + \frac{m_A m_B}{m_A + 2m_B} \left[b^2 \left(c_{23} \frac{\partial c_{21}}{\partial \alpha} - c_{21} \frac{\partial c_{23}}{\partial \alpha} \right) + b \left(a_3 \frac{\partial c_{21}}{\partial \alpha} + a_1 \frac{\partial c_{23}}{\partial \alpha} \right) \right] \Big\} \\
& = 0
\end{aligned} \tag{2.26}
$$

(2.1,2.25)

and, eliminating time and making the substitutions

$$
M = \frac{m_A m_B}{m_A + 2m_B}
$$

$$
J = I_1^B - I_2^B + Mb^2
$$

$$E_1 = \cos\theta \cos\varphi[J \sin^2\beta - I_2^B(\cos\beta - 1)]$$

$$E_2 = \sin\beta[\sin\theta(J \cos\beta + I_2^B - Mba_3 \sin\varphi) - \cos\varphi \, Mb\, a_1]$$

$$E_3 = \cos\theta \sin\beta[M b a_3 - \sin\varphi(J \cos\beta + I_2^B)]$$

$$F_1 = \frac{I^A}{2} + M(a_1^2 + a_3^2) + I_2^B + J[\cos^2\beta(\sin^2\theta - \cos^2\varphi \cos^2\theta) + \cos^2\theta]$$

$$- 2\, Mb \cos\beta\, (\sin\varphi\, a_3 + \cos\varphi \sin\theta\, a_1)$$

$$F_2 = J \sin^2\beta(\sin^2\theta - \sin^2\varphi \cos^2\theta)$$

$$F_3 = 2J \sin\theta \cos\theta \sin\varphi \sin^2\beta$$

$$F_4 = 2 \sin\beta \cos\theta\, (J \cos\beta \sin\theta \cos\varphi - Mb\, a_1)$$

$$F_5 = 2 \sin\beta\, [Mb(a_3 \cos\varphi - a_1 \sin\theta \sin\varphi) - J \cos^2\theta \sin\varphi \cos\varphi \cos\beta] \tag{2.27}$$

one finds that

$$\frac{d\xi}{d\alpha} = - \frac{E_1 + E_2 \sin\alpha + E_3 \cos\alpha}{F_1 + F_2 \cos^2\alpha + F_3 \sin\alpha \cos\alpha + F_4 \sin\alpha + F_5 \cos\alpha} \tag{2.28}$$

Integration of this equation for $0 \leq \alpha \leq 2\pi$ yields $\Delta\xi$, the pitch reorientation per cycle of the maneuver. An analytical solution is not readily available, but a computer program for numerical integration was written and is described in Sec. 4.

Before turning to the discussion of results, it is worth noting that in one special case, namely when the cone axes coincide with the pitch axis $(\theta = \varphi = 0)$, an analytical solution can be obtained easily. Specifically, after making the definitions

$$\ell = \sqrt{a_1^2 + a_3^2}$$

$$\delta = \text{arc cos } \frac{a_1}{\ell} = \text{arc sin } \frac{a_3}{\ell}$$

$$\eta = \alpha - \delta$$

$$E = J \sin^2\beta + I_2^B(1-\cos\beta)$$

$$F = I^A/2 + M\ell^2 + I_2^B + J \sin^2\beta$$

$$G = M b \ell \sin\beta \tag{2.29}$$

one can in this case bring Eq. 2.28 into the form

$$\frac{d\xi}{d\eta} = - \frac{E - G \sin\eta}{F - 2G \sin\eta} \tag{2.30}$$

and integration between the limits zero and 2π now leads directly to

$$\Delta\xi = \pi\left[1 + \frac{2E-F}{\sqrt{F^2-4G^2}}\right] \tag{2.31}$$

Equation 2.28 (or 2.31) cannot be used until suitable values have been selected for the inertia properties. Moreover, I^A, a_1, and a_3 depend on the position in which the legs are held, as well as on the inertia properties of the limbs. The computer program discussed in Sec. 4 accomodates a wide class of leg positions, and the following table provides values suitable for two leg positions. These values are based on the Hanavan model for the 50 percentile USAF man (see [4]).

Table 2.1

Symbol	Value	Units
Arm		
m_B	0.288	slugs
b	0.903	feet
I_1^B	0.1325	slug-ft.^{2}
I_2^B	0.002335	slug-ft.^{2}

Torso, Head, and Legs

	Legs in Position of "Attention"	Legs in Tucked Position (see Sec. 4 for details)	
m_A	4.458	4.458	slugs
a_1	0	0.309	feet
a_3	1.481	1.027	feet
I^A	8.150	3.895	slug-ft.^{2}

3. Results

It is of interest to observe how the pitch obtained per cycle, $\Delta\xi$, varies as a function of the cone semi-vertex angle, β. In Fig. 3.1, $\Delta\xi$ is plotted as a function of β for three cases:

(1) the maneuver performed with the legs straight as in the position of "attention",

(2) the maneuver performed with the legs tucked close to the body[‡], and

(3) the maneuver performed with the legs tucked and a five-pound weight (0.1556 slugs) held in each hand.

In all three cases, the cone axes are parallel to the pitch axis, Eq. 2.31 is used, and the requisite inertia properties are taken from Table 2.1.

It can be seen that pitch increases monotonically with β. Since the construction of the shoulder joints places an upper limit on β once a particular cone axis has been chosen, the maximum possible β being about 45 deg. when the cone axes are parallel to the pitch axis, a man with his legs straight can expect only about 11 deg. of pitch when performing a cycle of the maneuver in this fashion. Tucking the legs in markedly improves the effectiveness of the maneuver. For instance, with β equal to 45 deg., the value of $\Delta\xi$ is doubled to 20 deg. by tucking.

---

‡ A detailed explanation of this "tucked" position is presented in the third paragraph of Sec. 4.

PITCH VS. CONE SEMI-VERTEX ANGLE

(CONE AXES PARALLEL TO PITCH AXIS, $\Theta = \Phi = 0$)

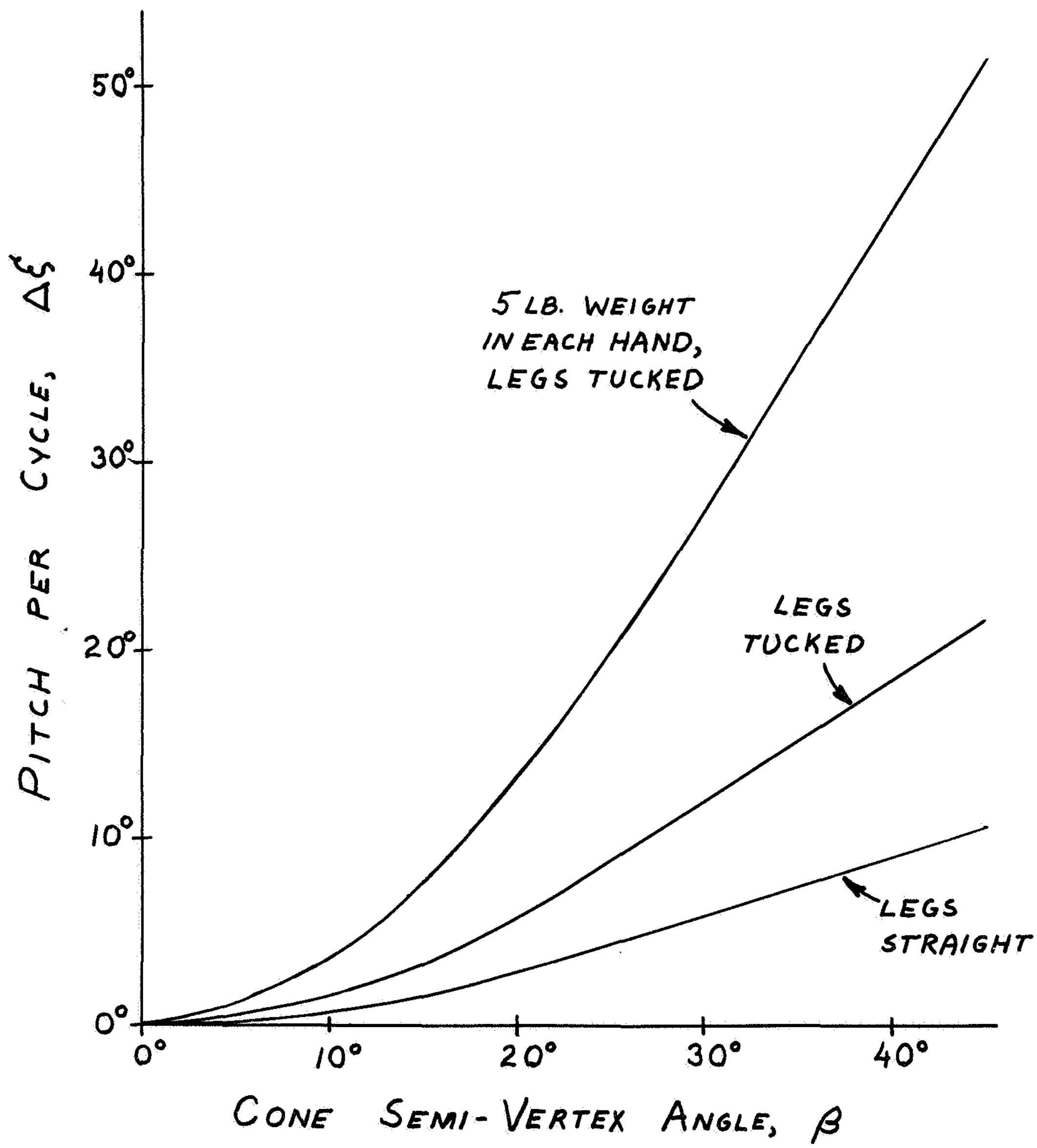

Fig. 3.1

The addition of five-pound weights to the hands more than doubles $\Delta\xi$; e.g., if the weights are used while the legs are tucked and β equals 45 deg., 52 degrees of pitch per cycle can be obtained.

The location of the cone axes relative to the torso has a significant effect on the amount of pitch obtained per cycle. In Fig. 3.2A, $\Delta\xi$ is plotted as a function of θ, with $\varphi = 0$ (see Fig. 2.1 for θ and φ), for two values of β, i.e., 20 and 45 deg., when the legs are tucked. (The computer program described in Sec. 4 was used to make the calculations.) $\Delta\xi$ is seen to decrease with increasing θ until, when θ becomes equal to 90 deg. (i.e., when the cone axes are parallel to the roll axis), no pitch is obtained. This suggests that it is advantageous to maintain the cone axes nearly parallel to the roll plane.

When the cone axes are "lowered" in the roll plane, $\Delta\xi$ may increase or decrease. This can be seen in Fig. 3.2B, which shows the pitch per cycle as a function of φ, with $\theta = 0$, the legs tucked, and β again equal to 20 deg. and 45 deg. Both curves possess a maximum when φ is about 15 deg. The relative flatness of the curves between 0 and 30 deg. is important since physiological constraints at the shoulder joint are such that the semi-vertex angles β that can be used become larger as φ is increased. For instance, while the upper bound on β is about 45 deg. with φ equal to zero, it is closer to 60 deg. when φ is 30 deg. Consequently, the greatest amount of pitch per cycle obtained without weights in the tucked position is about 32 degrees, and this is achieved by taking β equal to 60 deg., φ equal to 30 deg., and θ nearly equal to zero.

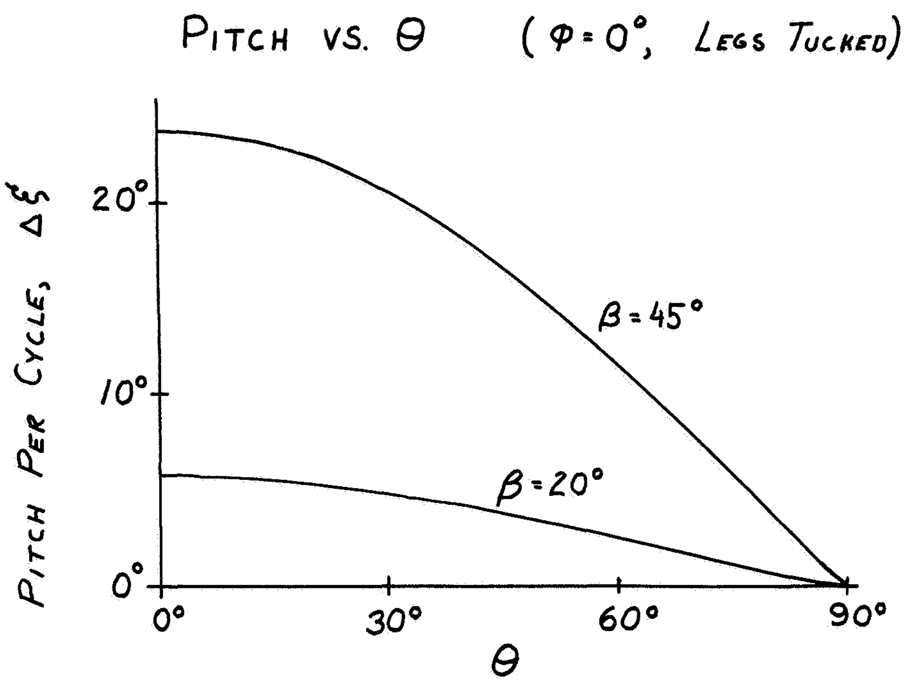

Fig. 3.2.A

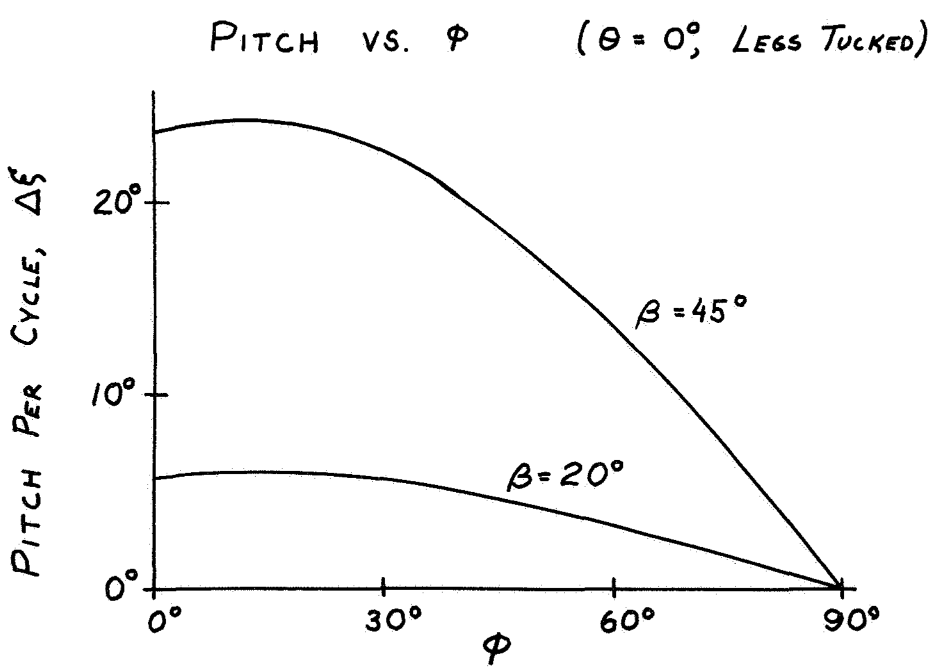

Fig. 3.2.B

The limb motions just described are reasonable ones for an unencumbered man in a "shirt sleeve" environment. For an astronaut in a pressure suit, the range of mobility may be sharply reduced, and the added mass of the suit may have a substantial effect on performance. In particular, $\Delta\xi$ seems to be most sensitive to changes in the distance from the shoulder to the mass center of the arm and to the distance from the line joining the shoulders to the mass center of the torso, head, and legs.

4. Computer Program

This section contains documentation for a FORTRAN IV, level H, computer program which performs the numerical integration necessary to obtain a solution to Eq. 2.28. The program has been used on the IBM 360/67 at Stanford University.

The program also serves a second function, namely computation of the quantities I^A, a_1, a_3, and m_A which are dependent on the inertia properties and positions of the legs and torso. Body A is assumed to be composed of five bodies, B_1, B_2, B_2', B_3, and B_3' where B_1 represents the torso, neck, and head, B_2 and B_2' represent the upper legs, and B_3 and B_3' represent the lower legs and feet (see Fig. 4.1). Two restrictions are placed on leg positions:

(1) the longitudinal axes of all leg segments must remain parallel to the pitch plane, and

(2) the legs must remain symmetrically located with respect to the pitch plane.

Consequently, all permissible leg positions can be described by

Model for Body A

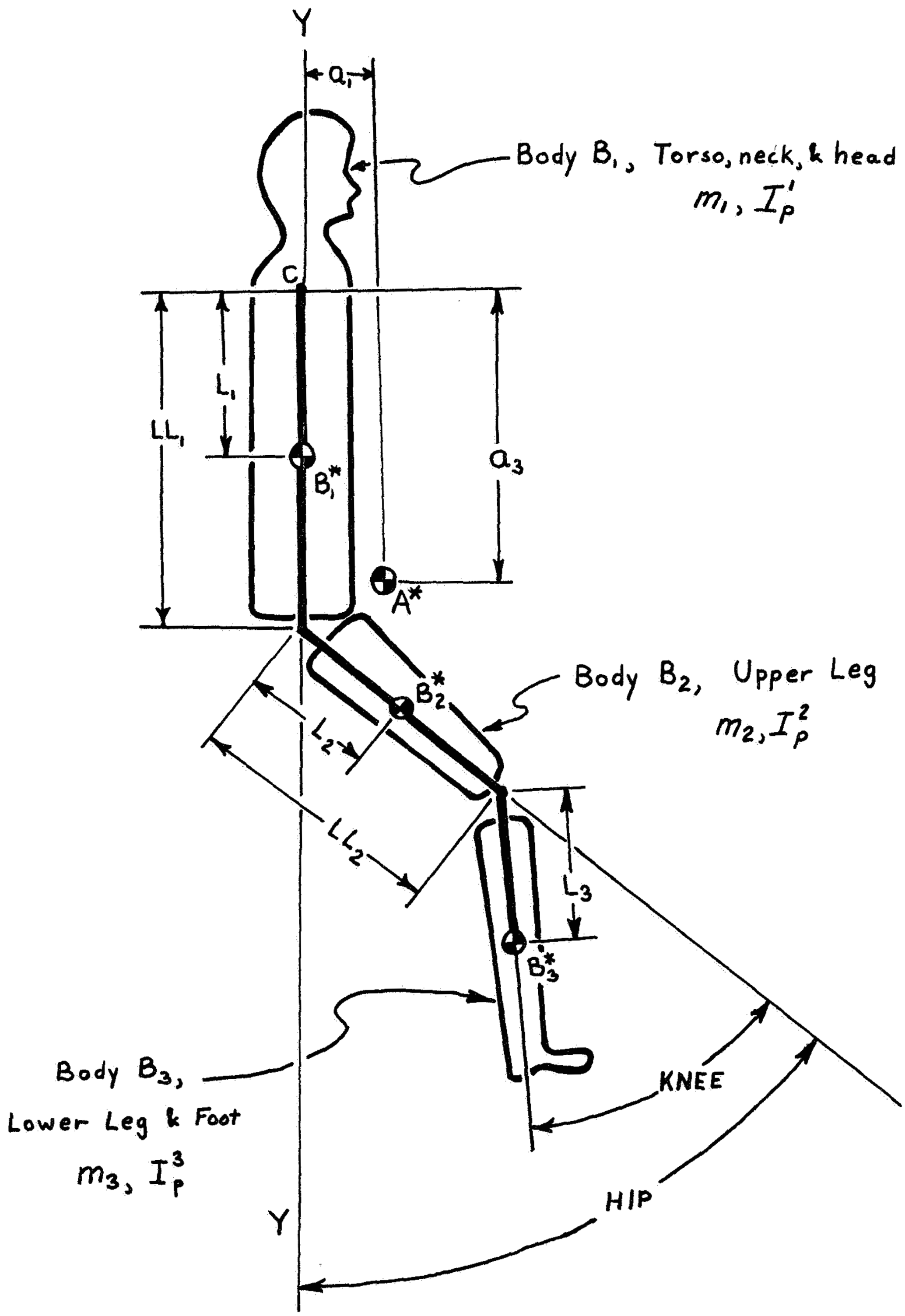

Fig. 4.1

specifying two angles, HIP and KNEE, where HIP is the angle between the roll plane and the longitudinal axis of an upper leg segment, and KNEE is the angle between the longitudinal axes of the upper and lower leg segments. HIP and KNEE are zero when the legs are straight. In the tucked position, HIP is 90 deg. and KNEE is 150 deg.

Input data are submitted to the computer on cards in the following manner. The first data card contains inertia properties of an arm and of the torso, neck, and head regarded as a single rigid body. The second data card contains inertia properties of the upper and lower leg segments and an integer, NUMBER, which designates the number of angle input data cards that follow. Each of these angle input data cards contains five angles in degrees, i.e., BETA, THETA, PHI, HIP, KNEE; and they may be followed by a new pair of cards listing inertia properties, the second of these containing a new NUMBER and being followed by angle input data cards. The procedure can thus be repeated any number of times. Table 4.1 lists the quantities appearing in the data cards in detail. The inertia properties may be introduced in any consistent system of mass and length units. Appropriate British Engineering System units are indicated in Table 4.1.

TABLE P.4.1

Data Card # 1 FORMAT (8F10.3)

Symbol In Program	Symbol in Analysis or in Fig. 4.1	Units	Definition
MB	m_B	slugs	mass of entire arm and hand
B	b	feet	distance from shoulder to mass center of arm
IB1	I_1^B	slug-ft.^{2}	moment of inertia of B for B* about any line normal to L
IB2	I_2^B	slug-ft.^{2}	moment of inertia of B about L
M1	m_1	slugs	mass of torso, neck, and head
L1	L_1	feet	distance from line connecting shoulders to mass center of torso, head, and neck
LL1	LL_1	feet	distance from line connecting shoulders to line connecting hips
I1P	I_P^1	slug-ft.^{2}	moment of inertia of torso, head, and neck about a line parallel to P and passing through their combined mass center

TABLE P.4.1 cont.

Data Card # 2 FORMAT (7F10.3,I10)

Symbol in Program	Symbol in Analysis or in Fig. 4.1	Units	Definition
M2	m_2	slugs	mass of upper leg segment
L2	L_2	feet	distance from hip to mass center of upper leg
LL2	LL_2	feet	distance from hip to knee
I2P	I^2_P	slug-ft.^{2}	moment of inertia of upper leg about a line parallel to P and passing through its mass center
M3	m_3	slugs	mass of lower leg and foot
L3	L_3	feet	distance from knee to mass center of lower leg
I3P	I^3_P	slug-ft.^{2}	moment of inertia of lower leg about a line parallel to P and passing through its mass center
NUMBER		(integer)	the number of angle input data cards that follow

Angle Input Data Cards FORMAT (5F10.2)

BETA	β	degrees	cone semi-vertex angle
THETA	θ	degrees	angle between P and the projection of the cone axis on the yaw plane
PHI	φ	degrees	angle between cone axis and yaw plane
HIP	HIP	degrees	angle of bending at hip
KNEE	KNEE	degrees	angle of bending at knee

The program is composed of three parts, a main program and two REAL FUNCTION subprograms. The main program reads input data, computes the requisite constants (such as those in Eq. 2.27), and writes the output. The subprogram FUNCT provides the single, first-order differential equation (Eq. 2.28) which is integrated for $0 \leq \alpha \leq 2\pi$ by subprogram QUADS3. QUADS3 is a Stanford Computation Center library subprogram which numerically integrates a single integrand of one variable between upper and lower limits, with specified accuracy. A listing of the entire program is contained in Appendix 1. A sample page of output is listed in Appendix 2. This contains the values of inertia properties appropriate for the Hanavan model of the USAF 50 percentile man (see [4]).

II. YAW MOTION

1. Description

The limb movements to be discussed were suggested by James Jones of the NASA Ames Research Center. The description and illustrations that follow, deal with a maneuver performed with the legs. However, the description applies also to a maneuver performed with the arms (but not with the arms and legs together.)

In all cases, the limbs remain straight at the knees and elbows, and the pair of limbs that is not used must remain fixed relative to the torso in such a way that the yaw axis is a principal axis of inertia; e.g., if the legs are used, the arms may be kept at the sides, as in a position of "attention".

The maneuver is performed in two phases. For definiteness, suppose that a rotation of the torso to the left is desired. Then phase 1 begins with the right leg extended forward from the torso, and the left leg extended rearward, through equal angles β_o (see Fig. 1.1A): The right leg is swept to the right and then to the rear (relative to the torso) while the left leg is swept leftward and forward (see Figs. 1.1B to 1.1E); that is, the longitudinal axis of each leg moves on the surface of an imaginary, torso-fixed cone whose vertex is at the hip and whose axis is parallel to the yaw axis. In the course of this "coning" motion, no twisting of the leg occurs. Thus, the toes always point nearly forward. At the conclusion of phase 1, the right leg is extended rearward and the left leg forward relative to the torso (see Fig. 1.1E).

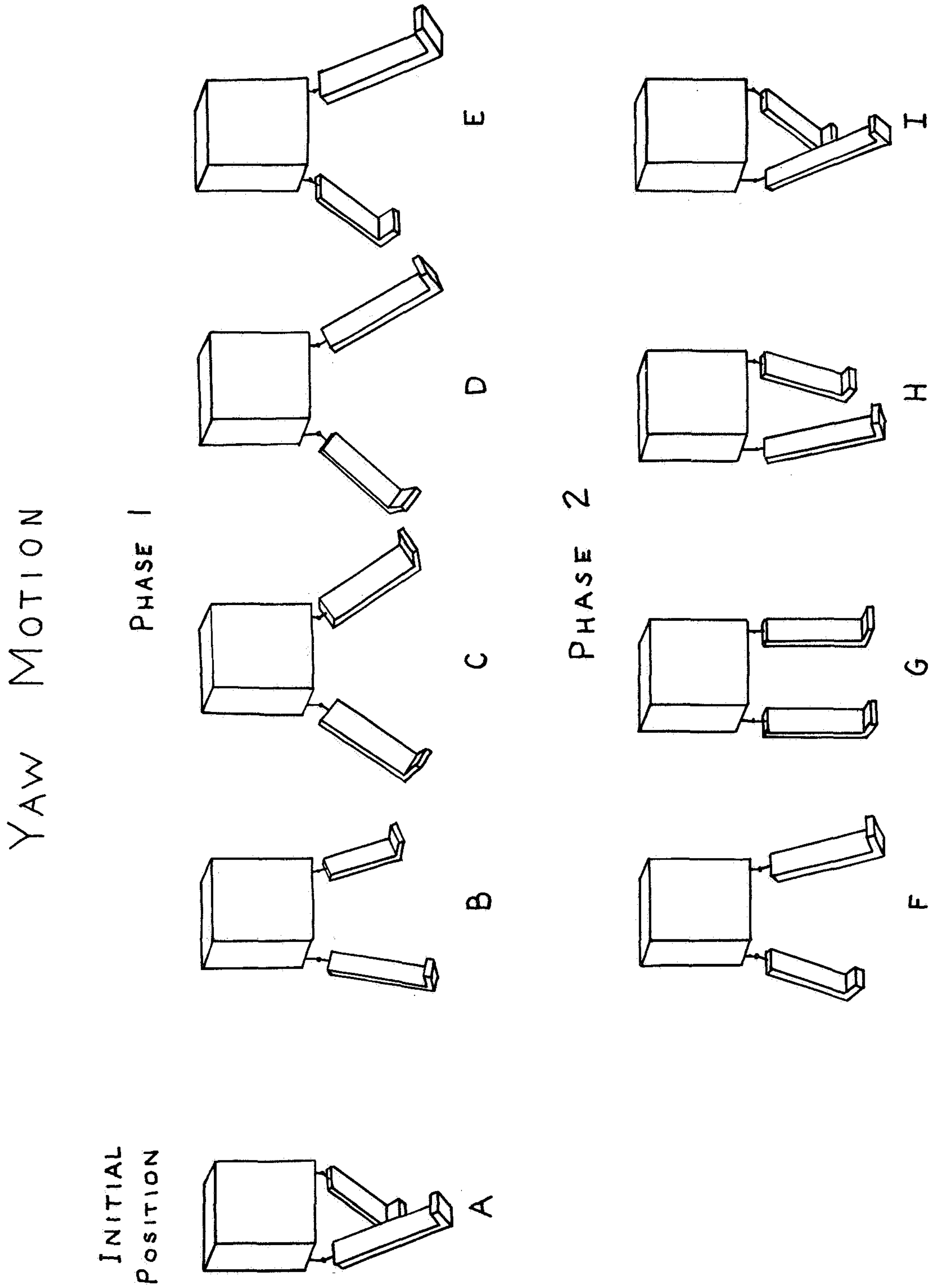

Fig. 1.1

In phase 2, the legs travel simultaneously in planes parallel to the pitch plane until each leg has returned to the position it occupied (with respect to the torso) at the beginning of phase 1 (see Figs. 1.1F through 1.1I). The entire cycle may then be repeated.

Consider the behavior of the torso during this maneuver. As phase 1 progresses, the torso rotates in an inertial reference frame to its left about the yaw axis, as desired, while the orientation of the yaw axis remains fixed. During phase 2, the torso turns back to the right, but this regression is not sufficient to nullify the rotation obtained in phase 1. A net rotation to the left is thus obtained from each complete cycle of the maneuver. The direction of rotation can be reversed by starting phase 1 with the left leg forward, rather than the right one.

An astronaut may be in a position of "attention" prior to starting the maneuver. The starting position for phase 1 may then be attained by moving the legs in planes parallel to the pitch plane. This will cause a regressive rotation of the torso about the yaw axis. After an integral number of cycles of the maneuver have been completed (a cycle consists of one performance of phase 1 and phase 2), the legs may be returned to a position of "attention", and the resulting yaw motion then nullifies the regressive rotation obtained while starting.

2. Analysis

For purposes of analysis, the human is modelled as a system S of three rigid bodies. One of these, designated A, represents the torso, head, and arms. The remaining two, B and B', each represent a leg. B and B' are connected to A at points O and O', which represent the hips (see Fig. 2.1).

YAW MOTION MODEL

PHASE I

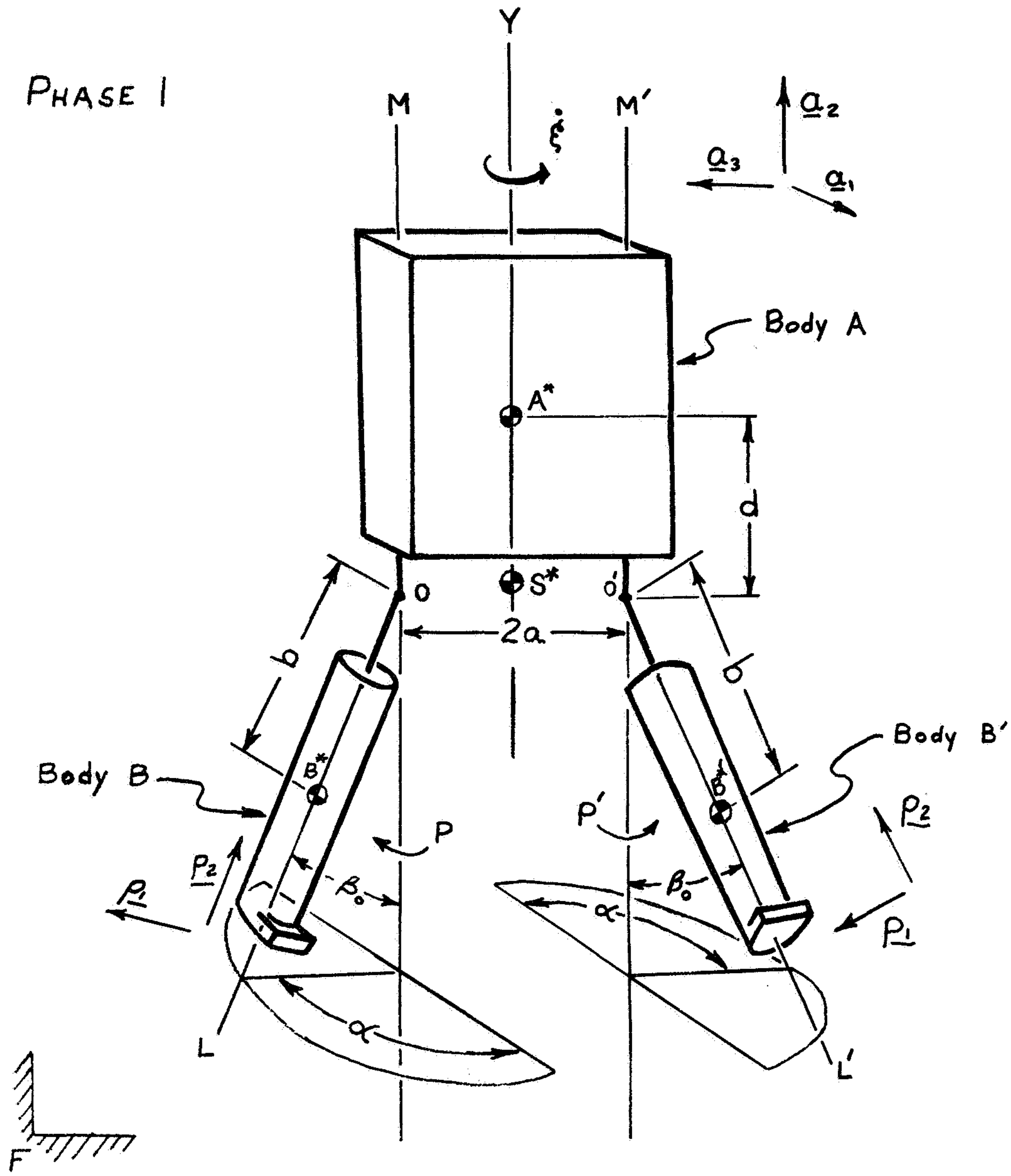

Fig. 2.1

The yaw axis Y is presumed to coincide with a principal axis of A for its mass center A^*. The pitch axis is chosen so that points O and O' lie thereon, each at a distance a from Y. The analysis in the sequel involves two additional lines fixed in A, namely M and M', which are parallel to Y and pass through O and O', respectively. Body A has a mass m_A and a moment of inertia I^A about Y. Mutually perpendicular unit vectors $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ are fixed in A parallel to the roll, yaw, and pitch axes respectively.

It is assumed that bodies B and B' each possess an axis of symmetry.† The axis of minimum moment of inertia of B for its mass center is designated L, and the associated principal moment of inertia has the value I_2^B. The moment of inertia of B about any line perpendicular to L and passing through B^* is I_1^B, and the mass of B is m_B. Line L passes through O, and the distance from B^* to O is b. The inertia properties of leg B' are identical to those of B.

As different variables are used for the mathematical description of the two phases of the maneuver, two analyses are required. In both cases, confining our attention solely to yaw motions of A, we exploit a consequence of Lagrange's equations of motion, namely the fact that

$$\frac{\partial K}{\partial \dot{\xi}} = 0 \qquad (2.1)$$

---

† The maximum and intermediate principal moments of inertia of a leg for its mass center differ from each other by less than 1% in the Hanavan model (see [4]).

where $\dot{\xi}$ is the time derivative of the angle ξ between the roll axis and a line fixed in F normal to the yaw axis, and K is the kinetic energy associated with motion of S relative to the system mass center, which can be expressed as

$$K = K_{\omega}^{A} + K_{\omega}^{B} + K_{\omega}^{B'} + K_{v}^{A} + K_{v}^{B} + K_{v}^{B'} \tag{2.2}$$

The first three terms on the right-hand-side of this equation represent the rotational kinetic energies of bodies A, B, and B' respectively, whereas the last three reflect the motions of the mass centers of A, B, and B'. For example, if ${}^{F}\underline{\omega}^{B}$, the angular velocity of B in F, is expressed as

$${}^{F}\underline{\omega}^{B} = \omega_1^B \, \underline{p}_1 + \omega_2^B \, \underline{p}_2 + \omega_3^B \, \underline{p}_3 \tag{2.3}$$

where $\underline{p}_1$, $\underline{p}_2$, $\underline{p}_3$ are unit vectors, each parallel to a principal axis of B for B*, and I_1^B, I_2^B, I_3^B are the corresponding principal moments of inertia, then

$$K_{\omega}^{B} = \frac{1}{2}\left[I_1^B(\omega_1^B)^2 + I_2^B(\omega_2^B)^2 + I_3^B(\omega_3^B)^2\right] \tag{2.4}$$

and, if $\underline{v}_B$ denotes the velocity of B* relative to the mass center S* of S, then

$$K_{v}^{B} = \frac{1}{2}\, m_B(\underline{v}_B^2) \tag{2.5}$$

Turning to the kinematical analysis for phase 1, we hypothesize that A has a simple angular velocity in F, i.e.,

$${}^{F}\underline{\omega}^{A} = \dot{\xi}\, \underline{a}_2 \tag{2.6}$$

Since Y is a principal axis of A, and ${}^{F}\underline{\omega}^{A}$ is parallel to Y,

$$K_{\omega}^{A} = \frac{1}{2} I^{A} \dot{\xi}^{2} \tag{2.7}$$

(2.4,2.6)

The motions of B and B' are described in terms of the "coning" motion outlined in Sec. 1. We regard line M as the axis of a cone on whose surface L moves. The angle β_o between L and M remains fixed, and if P is the plane determined by L and M, and mutually perpendicular unit vectors $\underline{p}_1$, $\underline{p}_2$, $\underline{p}_3$ are fixed in P, with $\underline{p}_2$ parallel to L and $\underline{p}_3$ normal to P, then $\underline{p}_1$, $\underline{p}_2$, $\underline{p}_3$ are related to $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ as indicated in Table 2.1, where α is the angle between P and the pitch plane.

Table 2.1

	$\underline{a}_1$	$\underline{a}_2$	$\underline{a}_3$
$\underline{p}_1$	$\cos\beta_o\cos\alpha$	$\sin\beta_o$	$\cos\beta_o\sin\alpha$
$\underline{p}_2$	$-\sin\beta_o\cos\alpha$	$\cos\beta_o$	$-\sin\beta_o\sin\alpha$
$\underline{p}_3$	$-\sin\alpha$	0	$\cos\alpha$

The angular velocity of P in A is given by

$${}^{A}\underline{\omega}^{P} = -\dot{\alpha}\,\underline{a}_2 \tag{2.8}$$

Leg B must rotate in P about L in order to avoid twisting in a physically impossible manner, as mentioned in Sec. 1. A suitable rotation of B in P results if the angular velocity of B in P is taken to be

$$ {}^{P}\underline{\omega}^{B} = \dot{\alpha}\, \underline{p}_2 \tag{2.9} $$

The angular velocity of B in F is then given by

$$ {}^{F}\underline{\omega}^{B} = {}^{P}\underline{\omega}^{B} + {}^{A}\underline{\omega}^{P} + {}^{F}\underline{\omega}^{A} $$

$$ = \dot{\alpha}\, \underline{p}_2 - \dot{\alpha}\, \underline{a}_2 + \dot{\xi}\, \underline{a}_2 \tag{2.10} $$

(2.9, 2.8, 2.6)

or, in view of Table 2.1, by

$$ {}^{F}\underline{\omega}^{B} = \omega_1^B\, \underline{p}_1 + \omega_2^B\, \underline{p}_2 \tag{2.11} $$

(2.10)

where

$$ \omega_1^B = (\dot{\xi} - \dot{\alpha})\sin \beta_o \tag{2.12} $$

and

$$ \omega_2^B = \dot{\alpha} + (\dot{\xi} - \dot{\alpha})\cos \beta_o \tag{2.13} $$

Consequently

$$K_\omega^B = \frac{1}{2}\left[I_1^B(\omega_1^B)^2 + I_2^B(\omega_2^B)^2\right]$$

(2.4)

$$= \frac{1}{2}\{\dot{\xi}^2[I_1^B\sin^2\beta_o + I_2^B\cos^2\beta_o]$$

(2.12,2.13)

$$-2\dot{\xi}\dot{\alpha}[I_1^B\sin^2\beta_o + I_2^B\cos^2\beta_o - I_2^B\cos\beta_o]$$

$$+\dot{\alpha}^2[I_1^B\sin^2\beta_o + I_2^B\cos^2\beta_o + I_2(1-2\cos\beta_o)]\} \quad (2.14)$$

In the same manner, an identical expression is obtained for $K_\omega^{B'}$.

Evaluation of K_v^A, K_v^B, and $K_v^{B'}$ necessitates determination of $\underline{v}_A$, $\underline{v}_B$, and $\underline{v}_{B'}$, the velocities relative to S^* of the mass centers of A, B, and B'. In phase 1, the position of S^* remains fixed in A, so that $\underline{v}_A$ vanishes. $\underline{v}_B$ can be expressed as

$$\underline{v}_B = \underline{v}^{B^*/O} + \underline{v}^{O/S^*} \quad (2.15)$$

where

$$\underline{v}^{B^*/O} = {}^F\underline{\omega}^B \times (-b\,\underline{p}_2) \quad (2.16)$$

and, since S^* is fixed in body A and lies on line Y,

$$\underline{v}^{O/S^*} = {}^F\underline{\omega}^A \times (a\,\underline{a}_3) \quad (2.17)$$

Referring to Table 2.1, one can thus obtain (see Eqs. 2.15, 2.16, 2.10, 2.17, 2.6)

$$\underline{v}_B = [\dot{\xi}a+(\dot{\xi}-\dot{\alpha})b \sin\beta_o \sin\alpha]\underline{a}_1$$
$$-[(\dot{\xi}-\dot{\alpha})b \sin\beta_o \cos\alpha]\underline{a}_3 \qquad (2.18)$$

and it follows that

$$K_v^B = \frac{1}{2} m_B\{\dot{\xi}^2[a^2+b^2\sin^2\beta_o+2ab \sin\beta_o \sin\alpha]$$

(2.5,2.18)

$$-2\dot{\xi}\dot{\alpha}[b^2\sin^2\beta_o+ab \sin\beta_o \sin\alpha]$$
$$+ \alpha^2[b^2\sin^2\beta_o]\} \qquad (2.19)$$

The expression for $K_v^{B'}$ is identical to that for K_v^B. Hence

$$K = \dot{\xi}^2\left[\frac{I^A}{2} + I_2^B+m_Ba^2+(I_1^B-I_2^B+m_Bb^2)\sin^2\beta_o+2m_Bab\sin\beta_o \sin\alpha\right]$$

(2.2,2.7,2.14,2.19)

$$- 2\dot{\xi}\dot{\alpha}\left[I_2(1-\cos\beta_o) + (I_1^B-I_2^Bm_Bb^2)\sin^2\beta_o+m_Bab\sin\beta_o\sin\alpha\right]$$
$$+ \dot{\alpha}^2\left[2I_2^B(1-\cos\beta_o) + (I_1^B-I_2^B+m_Bb^2)\sin^2\beta_o\right] \qquad (2.20)$$

and the dynamical equation of motion is

$$\dot{\xi}\left[\frac{I^A}{2} + I_2^B+m_Ba^2+(I_1^B-I_2^B+m_Bb^2)\sin^2\beta_o+2m_Bab\sin\beta_o\sin\alpha\right]$$
$$- \dot{\alpha}\left[I_2^B(1-\cos\beta_o) + (I_1^B-I_2^B+m_Bb^2)\sin^2\beta_o+m_Bab \sin\beta_o \sin\alpha\right]$$
$$= 0 \qquad (2.21)$$

(2.1,2.20)

Dependence on time can be eliminated and, making the substitutions

$$p_1 = \frac{\left(\frac{I^A}{2} + I_2^B + m_B a^2\right) + \left(I_1^B - I_2^B + m_B b^2\right)\sin^2\beta_o}{2m_B a\, b \sin\beta_o}$$

$$p_2 = \frac{I_2^B(1-\cos\beta_o)+(I_1^B - I_2^B + m_B b^2)\sin^2\beta_o}{2m_B a\, b \sin\beta_o} \tag{2.22}$$

one can relate the yaw angle ξ to the cone "sweep" angle α by means of the differential equation

$$\frac{d\xi}{d\alpha} = \frac{p_1 + \frac{1}{2}\sin\alpha}{p_2 + \sin\alpha} \tag{2.23}$$

(2.21,2.22)

This equation can be integrated in closed form for $0 \leq \alpha \leq \pi$ to yield an expression for $\Delta\xi_1$, the change in yaw during phase 1, as a function of the inertia properties and the semi-angle of limb spread:

$$\Delta\xi_1 = \frac{\pi}{2} + \frac{2p_1 - p_2}{\sqrt{p_2^2-1}} \left\{\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{p_2^2-1}}\right)\right\} \tag{2.24}$$

(2.23)

In phase 2, the legs move parallel to the pitch plane. A convenient variable is β (see Fig. 2.2), the angle between L and M (or L' and M'). β ranges from $-\beta_o$ to β_o.

In the kinematical analysis of phase 2, K_ω^A is again furnished by Eq. 2.7. The angular velocity of B in F may be written as

Yaw Motion Model

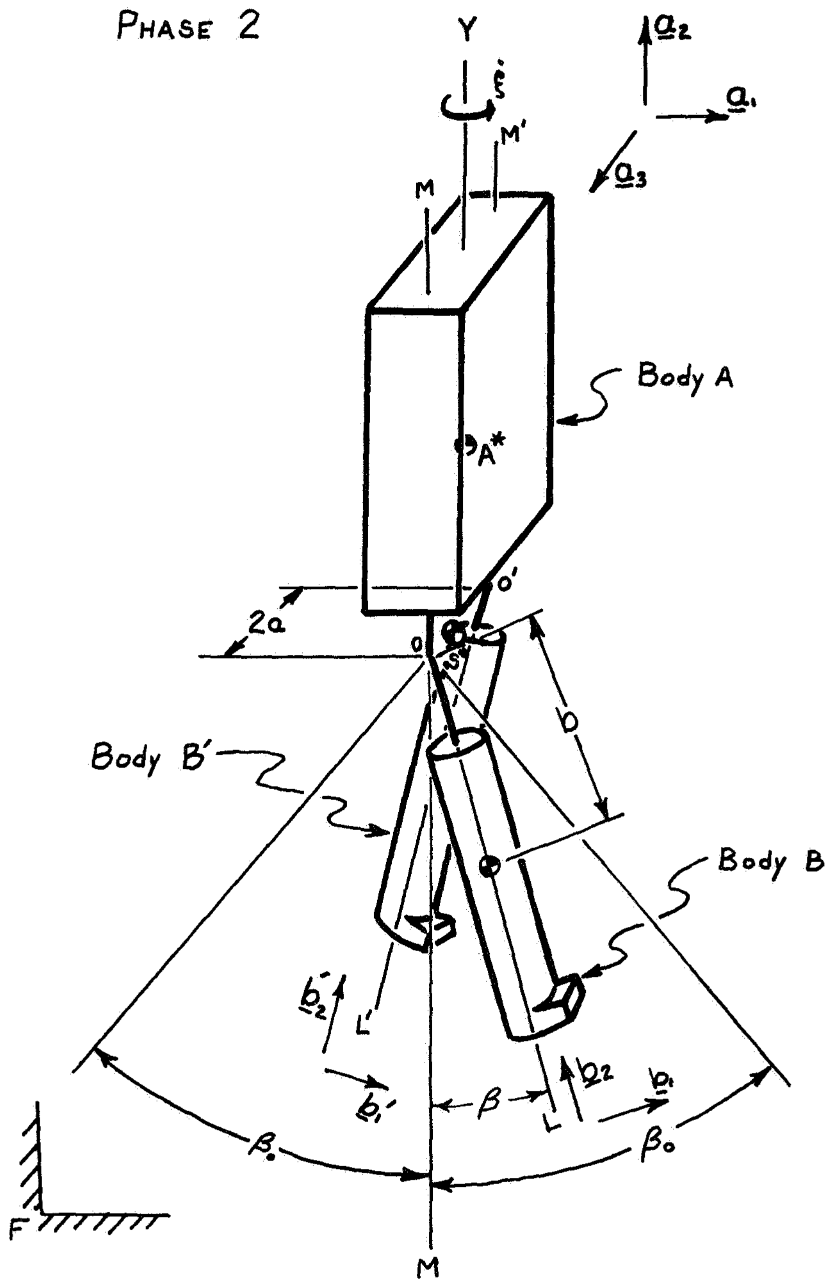

Fig. 2.2

$$ {}^{F}\underline{\omega}^{B} = {}^{A}\underline{\omega}^{B} + {}^{F}\underline{\omega}^{A} \tag{2.25} $$

where ${}^{F}\underline{\omega}^{A}$ is given by Eq. 2.6 while

$$ {}^{A}\underline{\omega}^{B} = \dot{\beta}\, \underline{a}_3 \tag{2.26} $$

If unit vectors $\underline{b}_1$, $\underline{b}_2$, $\underline{b}_3$ are fixed in body B, with $\underline{b}_2$ parallel to L and $\underline{b}_3$ parallel to the pitch axis, so that they are related to $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ as shown in Table 2.2, then

$$ {}^{F}\underline{\omega}^{B} = \dot{\xi} \sin\beta\, \underline{b}_1 + \dot{\xi} \cos\beta\, \underline{b}_2 + \dot{\beta}\, \underline{b}_3 \tag{2.27} $$

(2.25,2.6,2.26)

and, since $\underline{b}_1$, $\underline{b}_2$, $\underline{b}_3$ are parallel to principal axes of B for B*,

$$ K_{\omega}^{B} = \frac{1}{2}\left[\dot{\xi}^2 (I_1^B \sin^2\beta + I_2^B \cos^2\beta) + I_1^B \dot{\beta}^2\right] \tag{2.28} $$

(2.4,2.27)

An identical expression is obtained for $K_{\omega}^{B'}$.

Table 2.2

	$\underline{a}_1$	$\underline{a}_2$	$\underline{a}_3$
$\underline{b}_1$	$\cos\beta$	$\sin\beta$	0
$\underline{b}_2$	$-\sin\beta$	$\cos\beta$	0
$\underline{b}_3$	0	0	1

In this phase, K_v^A does not vanish. It is, however, independent of $\dot{\xi}$ and therefore does not contribute to the equation of motion. $\underline{v}_B$ is again given by Eq. 2.15, and $\underline{v}^{B*/0}$ by Eq. 2.16, but Eq. 2.17 must be replaced with a relationship that reflects the motion of S^* in A. Thus

$$\underline{v}_B = (\dot{\xi}a+b\dot{\beta}\cos\beta)\,\underline{a}_1 + \frac{m_A}{m_A+2m_B}\,b\sin\beta\,\dot{\beta}\,\underline{a}_2 - \dot{\xi}\,b\sin\beta\,\underline{a}_3 \tag{2.29}$$

Consequently,

$$\underset{(2.5,2.29)}{K_v^B} = \frac{1}{2}m_B\Big\{\dot{\xi}^2[a^2+b^2\sin^2\beta] + 2\dot{\xi}\dot{\beta}[ab\cos\beta] + \dot{\beta}^2\Big[b^2\cos^2\beta + \Big(\frac{m_A}{m_A+2m_B}\Big)^2 b^2\sin^2\beta\Big]\Big\} \tag{2.30}$$

and the expression for $K_v^{B'}$ is again identical. Hence

$$\underset{(2.2,2.7,2.28,2.30)}{K} = \Big\{\dot{\xi}^2\Big[\frac{I^A}{2} + m_Ba^2+I_2^B+(I_1^B-I_2^B+m_Bb^2)\sin^2\beta\Big] + 2\dot{\xi}\dot{\beta}[m_Bab\cos\beta] + \dot{\beta}^2\Big[I_1^B+m_Bb^2\cos^2\beta + \Big(\frac{m_Am_B}{m_A+2m_B}\Big)b^2\sin^2\beta\Big]\Big\} \tag{2.31}$$

The appropriate dynamical equation is again Eq. 2.1, and, when time is eliminated and two non-dimensional constants q_1 and q_2 are defined as

$$q_1 = \frac{m_B ab}{I_1^B - I_2^B + m_B b^2}$$

$$q_2 = \frac{\frac{I^A}{2} + I_2^B + m_B a^2}{I_1^B - I_2^B + m_B b^2} \qquad (2.32)$$

the relation between yaw and limb position is provided by the differential equation

$$\frac{d\xi}{d\beta} = -\, q_1 \frac{\cos\beta}{q_2 + \sin^2\beta} \qquad (2.33)$$

(2.1,2.31,2.32)

The change in yaw during phase 2, $\Delta\xi_2$, is obtained by integrating Eq. 2.33 for $-\beta_o \leq \beta \leq \beta_o$:

$$\Delta\xi_2 = \frac{-2q_1}{\sqrt{q_2}} \tan^{-1}\left(\frac{\sin\beta_o}{\sqrt{q_2}}\right) \qquad (2.34)$$

(2.33)

The total yaw rotation per cycle of the maneuver, $\Delta\xi$, is the sum of $\Delta\xi_1$ and $\Delta\xi_2$. With p_1, p_2, q_1, q_2 as defined in Eqs. 2.22 and 2.32, the yaw per cycle (in radians) is thus given by

$$\Delta\xi = \frac{\pi}{2} + \frac{2p_1 - p_2}{\sqrt{p_2^2 - 1}} \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{p_2^2 - 1}}\right)\right] - \frac{2q_1}{\sqrt{q_2}} \tan^{-1}\left(\frac{\sin\beta_o}{\sqrt{q_2}}\right) \qquad (2.35)$$

(2.24,2.34)

In Sec. 1, it was mentioned that the maneuver could be performed either with the arms or with the legs. Different values for the inertia properties must be used, depending upon which limbs are employed. Table 2.3 shows representative values for the two cases. These values are based on the Hanavan model for the U.S. Air Force 50 percentile man (see [4]).

Table 2.3

	Leg Maneuver (Arms at sides)	Arm Maneuver (Legs parallel to yaw axis)	Units
I^A	.504	.3895	slg.ft.^{2}
a	.252	.665	ft.
m_B	.837	.288	slg.
b	1.353	.903	ft.
I_1^B	.5625	.1325	slg.-ft.^{2}
I_2^B	.0193	.002335	slg.-ft.^{2}

3. Results

It is of interest to know how the yaw obtained per cycle, $\Delta\xi$, varies as a function of the semi-angle of leg spread, β_o. In Fig. 3.1, $\Delta\xi$ is plotted as a function of β_o for three cases:

(1) the maneuver performed with the legs while the arms are held at the sides,

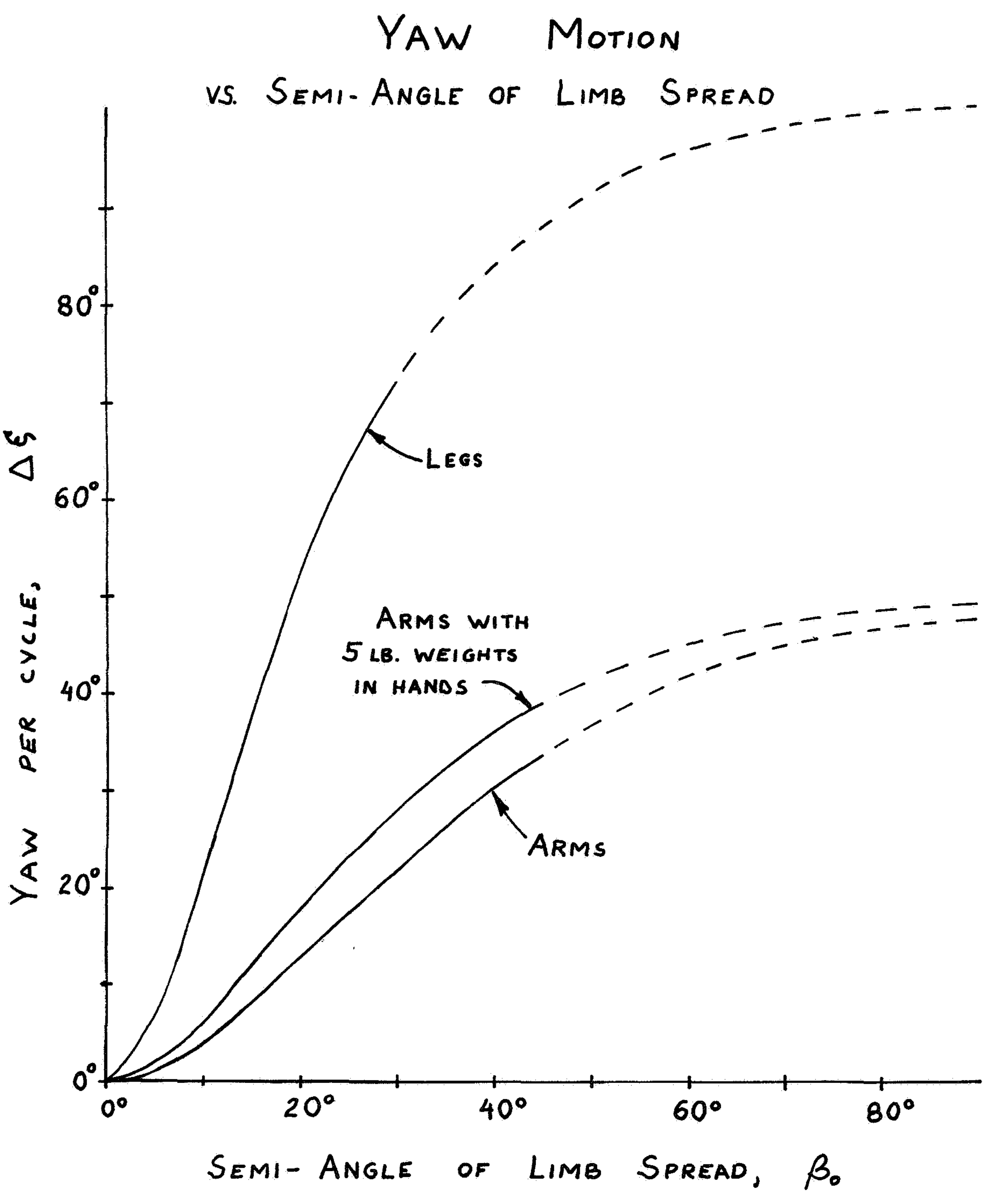
YAW MOTION
VS. SEMI-ANGLE OF LIMB SPREAD
YAW PER CYCLE, $\Delta\xi$
80°
60°
40°
20°
0°
LEGS
ARMS WITH
5 LB. WEIGHTS
IN HANDS
ARMS
0°
20°
40°
60°
80°
SEMI-ANGLE OF LIMB SPREAD, β_o

Fig. 3.1

(2) the maneuver performed with the arms while the legs are parallel to the yaw axis, and

(3) the maneuver performed with the arms when a five-pound weight (.1556 slg.) is held in each hand.

(The inertia properties are those for the Hanavan model of the 50 percentile USAF man, as listed at the end of the last section.)

It can be seen that yaw increases monotonically with β_o. Of course, the construction of the human hip and shoulder joints places an upper bound on β_o. For the legs, a maximum of 30 deg. is reasonable, whereas for the arms β_o may be as large as 45 deg. Consequently, realistic values for the maximum yaw obtainable per cycle are 72 deg. with the legs and 33 deg. with the arms. Holding a five-pound weight in each hand improves the performance, but not spectacularly, a reasonable upper limit for the yaw per cycle obtainable with five-poind weights being 39 deg.

Use of the legs can be seen to be more effective than that of the arms. However, occasions may arise in which the arms are the more convenient limbs. For example, constraints imposed by an astronaut's pressure suit might be such as to render the arms more mobile than the legs.

In conclusion, it is worth noting that it is possible to perform a maneuver in which the arms and the legs are used simultaneously. However, this case is not covered by the analysis of Sec. 2, and the reader should be aware that the results plotted in Fig. 3.1 are not additive. In fact, rough calculations indicate that such a maneuver would be less effective than the one involving the legs alone.

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Appendix 1

```
C      PITCH MOTION FROM SYMMETRIC ROTARY MOTION OF THE ARMS
C
C      MARK SCHER - STANFORD UNIVERSITY   AUGUST 1968
C
C      THIS PROGRAM COMPUTES THE PITCH OBTAINED BY PERFORMING ONE CYCLE
C      OF SYMMETRIC ROTARY MOTION OF THE ARMS.
C
C      DECLARATIONS:
       REAL PITCH, BETA,THETA,PHI,HIP,KNEE, RAD/1.745329E-02/,
     1      MB,B,IB1,IB2, M1,L1,LL1,I1P, M2,L2,LL2,I2P, M3,L3,I3P,
     2      SBE,CBE,STH,CTH,SPH,CPH,SHI,CHI, SBE2,CBE2,STH2,CTH2,CPH2,
     3      MA,A1,A3,IA, J,MM, E1,E2,E3, F1,F2,F3,F4,F5,
     4      Y3,R71,R73,R91,R93,RCM3
       INTEGER NLVL, NUMBER
       COMMON E1,E2,E3, F1,F2,F3,F4,F5
       EXTERNAL FUNCT
C
C      WRITE HEADING
       WRITE (6,110)
       WRITE (6,120)
  110  FORMAT ('1',      'PITCH MOTION PRODUCED BY SYMMETRIC ROTARY ',
     1          'MOTION OF THE ARMS'/ '0'/
     2          'ODEFINITIONS:'/
     3          'OINERTIA PROPERTIES:'
     4/'        MB  - MASS OF ENTIRE ARM & HAND'
     5/'        B   - DISTANCE FROM SHOULDER TO CM. OF ARM'
     6/'        IB1 - TRANSVERSE MOMENT OF INERTIA OF ARM FOR ITS CM.'
     7/'        IB2 - LONGITUDINAL MOMENT OF INERTIA OF ARM FOR ITS CM.'
     8/'        M1  - MASS OF TORSO, NECK & HEAD'
     9/'        L1  - DISTANCE FROM LINE JOINING SHOULDERS TO TORSO CM.'
     A/'        LL1 - DISTANCE FROM LINE JOINING SHOULDERS TO LINE '
     B,               'JOINING HIPS'
     C/'        I1P - MOMENT OF INERTIA OF TORSO ABOUT PITCH AXIS THRU '
     D,               'TORSO CM.'
     E/'        M2  - MASS OF UPPER LEG'
     F/'        L2  - DISTANCE FROM HIP TO CM. OF UPPER LEG'
     G/'        LL2 - DISTANCE FROM HIP TO KNEE'
     H/'        I2P - MOMENT OF INERTIA OF UPPER LEG ABOUT PITCH AXIS '
     I,               'THRU ITS CM.'
  120  FORMAT (
     1 '        M3  - MASS OF LOWER LEG (& FOOT)'
     2/'        L3  - DISTANCE FROM KNEE TO CM. OF LOWER LEG'
     3/'        I3P - MOMENT OF INERTIA OF LOWER LEG ABOUT PITCH AXIS '
     4,               'THRU ITS CM.'
     5/         'OANGLES (IN DEGREES):'
     6/'        PITCH - AMOUNT OF PITCH PER CYCLE OF MANEUVER'
```

Pitch Motion Program Listing

```
     7/'        BETA  - SEMI-VERTEX ANGLE OF CONES SWEPT OUT BY ARMS'
     8/'        THETA - ANGLE BETWEEN PITCH AXIS AND PROJECTION OF CONE '
     9,                'AXIS ON YAW PLANE'
     A/'        PHI   - ANGLE BETWEEN CONE AXIS AND YAW PLANE'
     B/'        HIP   - ANGLE BETWEEN THIGH AND TORSO (POSITIVE FORWARD)'
     C/'        KNEE  - ANGLE BETWEEN SHANK & THIGH (POSITIVE REARWARD)' )
C
C
C     READ INERTIA PROPERTIES
    1 READ (5,210,END=90,ERR=90) MB,B,IB1,IB2, M1,L1,LL1,I1P
  210 FORMAT (8F10.3)
      READ (5,220,END=90,ERR=90) M2,L2,LL2,I2P, M3,L3,I3P, NUMBER
  220 FORMAT (7F10.3,I10)
C
C     WRITE OUT INERTIA PROPERTIES AND LABEL COLUMNS
      WRITE(6,130) MB,B,IB1,IB2, M1,L1,LL1,I1P, M2,L2,LL2,I2P, M3,L3,I3P
  130 FORMAT ('0'/      'OINERTIA PROPERTIES:'/
     1          '       ARM:               MB =',F7.4, '      B =',F7.4,
     2          '     IB1 =',F7.4, '     IB2 =',F9.6/
     3          '       TORSO:             M1 =',F7.4, '     L1 =',F7.4,
     4          '     LL1 =',F7.4, '     I1P =',F8.5/
     5          '       UPPER LEG:         M2 =',F7.4, '     L2 =',F7.4,
     6          '     LL2 =',F7.4, '     I2P =',F8.5/
     7          '       LOWER LEG:         M3 =',F7.4, '     L3 =',F7.4,
     8          20X,'I3P =',F8.5/
     9          '0'/'         BETA      THETA       PHI        HIP        KNEE',
     A          '              PITCH')
C
      DO 20  I=1,NUMBER
      READ (5,230,END=90,ERR=90) BETA, THETA, PHI, HIP, KNEE
  230 FORMAT (5F10.2)
C
C     COMPUTE TRIG FUNCTIONS FOR INPUT ANGLES
      SBE  = SIN(BETA*RAD)
      CBE  = COS(BETA*RAD)
      STH  = SIN(THETA*RAD)
      CTH  = COS(THETA*RAD)
      SPH  = SIN(PHI*RAD)
      CPH  = COS(PHI*RAD)
      SHI  = SIN(HIP*RAD)
      CHI  = COS(HIP*RAD)
      SBE2  = SBE*SBE
      CBE2  = CBE*CBE
      CPH2  = CPH*CPH
      STH2  = STH*STH
      CTH2  = CTH*CTH
C
C     COMPUTE INERTIA PROPERTIES OF SEGMENT A
      MA  = M1 + 2.0*(M2 + M3)
      Y3  = LL1 - L1
      R73  = Y3 + L2*CHI
      R71  = L2*SHI
      R93  = Y3 + LL2*CHI + L3*COS((HIP-KNEE)*RAD)
      R91  = LL2*SHI + L3*SIN((HIP-KNEE)*RAD)
```

```
      RCM3 = 2.0*(M2*R73 + M3*R93)/MA
C
      A1 = 2.0*(M2*R71 + M3*R91)/MA
      A3 = RCM3 + L1
      IA = I1P + 2.0*(I2P + I3P) + M1*(RCM3*RCM3 + A1*A1)
     1     + 2.0*M2*( (R71-A1)*(R71-A1) + (R73-RCM3)*(R73-RCM3) )
     2     + 2.0*M3*( (R91-A1)*(R91-A1) + (R93-RCM3)*(R93-RCM3) )
C
C     DEFINE INTEGRAND CONSTANTS  (SEE EQS. 2.27)
      MM = MA*MB/(MA + 2.0*MB)
      J = IB1 - IB2 + MM*E*B
C
      E1 = CPH*CTH*( J*SBE2 + IB2*(1.0-CBE) )
      E2 = SBE*( STH*( J*CBE + IB2 - MM*B*A3*SPH) - MM*B*A1*CPH)
      E3 = SBE*CTH*( MM*B*A3 - SPH*(J*CBE + IB2) )
C
      F1 = IA/2.0 + IB2 + MM*(A1*A1+A3*A3) + J*(CBE2*(STH2-CPH2*CTH2)
     1     + CTH2) - 2*MM*B*CBE*(SPH*A3 + CPH*STH*A1)
      F2 = J*SBE2*(STH2 - CTH2*SPH*SPH)
      F3 = 2.0*J*STH*CTH*SPH*SBE2
      F4 = 2.0*SBE*CTH*( J*CBE*STH*CPH - MM*B*A1 )
      F5 = 2.0*SBE*( MM*B*(A3*CPH - A1*SPH*STH) - J*CBE*SPH*CPH*CTH2 )
C
C     PERFORM INTEGRATION
      PITCH = -QUADS3(FUNCT, 0.0, 6.283185, 1.0E-4, NLVL)/RAD
C
      WRITE (6,140) BETA, THETA,PHI,HIP, KNEE, PITCH
  140 FORMAT (' ',5F10.2,F15.3)
      IF (NLVL .NE. 0)  WRITE (6,150) NLVL
  150 FORMAT ('+',70X,'MAXLEVEL OF QUADS3 EXCEEDED',I4,' TIMES')
   20 CONTINUE
C
      GO TO 1
C
C     END OF DATA, SKIP PAGE
   90 WRITE (6,160)
  160 FORMAT ('1')
      RETURN
C
C     END OF MAIN PITCH PROGRAM
      END
```

```
      REAL FUNCTION FUNCT(ALPHA)
C
C     THIS SUBPROGRAM PROVIDES THE DIFFERENTIAL EQUATION RELATING PITCH
C     TO 'SWEEP' OF THE ARMS ON IDENTICAL SYMMETRIC CONES
C
      REAL E1,E2,E3, F1,F2,F3,F4,F5, SA,CA, ALPHA
      COMMON E1,E2,E3, F1,F2,F3,F4,F5
C
C     DEFINE TERMS
      SA = SIN(ALPHA)
      CA = COS(ALPHA)
C
C     FORM EQ. 2.28
      FUNCT = -( E1+E2*SA+E3*CA ) / ( F1+F2*CA*CA+F3*CA*SA+F4*SA+F5*CA )
      RETURN
C
C     END OF REAL FUNCTION FUNCT
      END
```

```
      REAL FUNCTION QUADS3(FUNCT, LOWER, UPPER, EPSLON, NLVL)
C
C     SINGLE INTEGRATION (ADAPTIVE, NONRECURSIVE)
         REAL FUNCT, LOWER, UPPER, EPSLON
         INTEGER NLVL
C
C        APPROXIMATES THE INTEGRAL OF THE FUNCTION FUNCT(X) BETWEEN
C        THE LIMITS OF LOWER AND UPPER BY APPLYING SIMPSON'S RULE AND
C        ROMBERG CORRECTION TO VARIOUS LENGTH SUBINTERVALS AS DICTATED
C        BY THE INTEGRAND AND THE TOLERANCE EPSLON.  NLVL IS SET NON-
C        ZERO IF CERTAIN CONDITIONS ARE NOT MET.
C
C        CALLING SEQUENCE...
C
C           Z=QUADS3(FUNCT,LOWER,UPPER,EPSLON,NLVL)
C
C        PARAMETERS...
C
C           FUNCT  REAL FUNCTION        THE NAME OF THE SUBPROGRAM THAT
C                  SUBPROGRAM           COMPUTES THE VALUES OF THE
C                                       INTEGRAND
C
C           LOWER, REAL VARIABLE        THE LOWER AND UPPER LIMITS OF
C           UPPER  SHORT PRECISION      THE INTEGRATION
C
C           EPS    REAL VARIABLE        ERROR TOLERANCE
C                  SHORT PRECISION
```

```
C
C            NLVL    FULL WORD INTEGER    SET TO THE NUMBER OF TIMES THE
C                                         MAXIMUM SUBDIVISION LEVEL WAS
C                                         REACHED
C
C         LIBRARY PROGRAM NUMBER  C014
C         JOHN H. WELSCH (SLAC)
C         JANUARY 26,1967
C
      INTEGER LEVEL, MINLVL/3/, MAXLVL/24/, RETRN(50), I
      REAL VALINT*8(50,2), MX(50), RX(50), FMX(50), FRX(50),
     1   FMRX(50), ESTRX(50), EPSX(50)
C *** TO CHANGE MAXIMUM DESCENT LEVEL, SET ALL 50'S ABOVE TO NEW LEVEL
      REAL L, R, FL, FML, FM, FMR, FR, EST, ESTL, ESTR, ESTINT,
     1   AREA, ABAREA*8, M, COEF, ROMBRG*8, EPS
C *** SET UP PARAMETERS FOR INITIAL CALL OF SUBINTEGRAL
         LEVEL = 0
         NLVL = 0
         ABAREA = 0.0
         L = LOWER
         R = UPPER
         FL = FUNCT(L)
         FM = FUNCT(0.5*(L+R))
         FR = FUNCT(R)
         EST = 0.0
         EPS = EPSLON
C *** PROCEDURE SUBINTEGRAL(L, R, FL, FM, FR, EST, EPS)
C ***    COMPARE ESTIMATE WITH SUM OF TWO SUBESTIMATES (ESTL & ESTR)
  100 LEVEL = LEVEL+1
      M = 0.5*(L+R)
      COEF = R-L
C *** CHECK FOR INTERVAL COLLAPSE
      IF(COEF.NE.0) GO TO 150
         ROMBRG = EST
         GO TO 300
  150 FML = FUNCT(0.5*(L+M))
      FMR = FUNCT(0.5*(M+R))
      ESTL = (FL+4.0*FML+FM)*COEF
      ESTR = (FM+4.0*FMR+FR)*COEF
      ESTINT = ESTL+ESTR
      AREA = ABS(ESTL)+ABS(ESTR)
      ABAREA = AREA+ABAREA-ABS(EST)
C *** CHECK FOR MAXIMUM LEVEL
      IF(LEVEL.NE.MAXLVL) GO TO 200
         NLVL = NLVL+1
         ROMBRG = ESTINT
         GO TO 300
C *** CHECK TOLERANCE AND MINIMUM LEVEL
  200 IF((ABS(EST-ESTINT).GT.(EPS*ABAREA)) .OR.
     1          (LEVEL.LT.MINLVL))  GO TO 400
C *** ACCEPT ROMBERG CORRECTION AS VALUE OF SUBINTEGRAL
         ROMBRG = (1.6D1*ESTINT-EST)/15.0
C *** ASCEND ONE LEVEL
  300    LEVEL = LEVEL-1
```

```
      I = RETRN(LEVEL)
      VALINT(LEVEL, I) = ROMBRG
      GO TO (500, 600), I
C *** DESCEND ONE LEVEL AND EVALUATE LEFT HALF SUBINTEGRAL
C ***   SAVE ITEMS NEEDED TO EVALUATE RIGHT HALF SUBINTEGRAL
  400 RETRN(LEVEL) = 1
      MX(LEVEL) = M
      RX(LEVEL) = R
      FMX(LEVEL) = FM
      FMRX(LEVEL) = FMR
      FRX(LEVEL) = FR
      ESTRX(LEVEL) = ESTR
      EPSX(LEVEL) = EPS
      EPS = EPS/1.4
C *** CALL SUBINTEGRAL
      R = M
      FR = FM
      FM = FML
      EST = ESTL
      GO TO 100
C *** CALL SUBINTEGRAL TO EVALUATE RIGHT HALF SUBINTEGRAL
  500 RETRN(LEVEL) = 2
      L = MX(LEVEL)
      R = RX(LEVEL)
      FL = FMX(LEVEL)
      FM = FMRX(LEVEL)
      FR = FRX(LEVEL)
      EST = ESTRX(LEVEL)
      EPS = EPSX(LEVEL)
      GO TO 100
C *** ADD BOTH HALF SUBINTEGRALS AND ASCEND ONE LEVEL
  600 ROMBRG = VALINT(LEVEL,1)+VALINT(LEVEL,2)
      IF(LEVEL.GT.1) GO TO 300
C *** FINALLY AT LEVEL ONE WITH THE ANSWER
      QUADS3 = ROMBRG /12.0
      RETURN
C *** LAST OF SUBROUTINE QUADS3
      END
```

Appendix 2

PITCH MOTION PRODUCED BY SYMMETRIC ROTARY MOTION OF THE ARMS

DEFINITIONS:

INERTIA PROPERTIES:

- MB - MASS OF ENTIRE ARM & HAND
- B - DISTANCE FROM SHOULDER TO CM. OF ARM
- IB1 - TRANSVERSE MOMENT OF INERTIA OF ARM FOR ITS CM.
- IB2 - LONGITUDINAL MOMENT OF INERTIA OF ARM FOR ITS CM.
- M1 - MASS OF TORSO, NECK & HEAD
- L1 - DISTANCE FROM LINE JOINING SHOULDERS TO TORSO CM.
- LL1 - DISTANCE FROM LINE JOINING SHOULDERS TO LINE JOINING HIPS
- I1P - MOMENT OF INERTIA OF TORSO ABOUT PITCH AXIS THRU TORSO CM.
- M2 - MASS OF UPPER LEG
- L2 - DISTANCE FROM HIP TO CM. OF UPPER LEG
- LL2 - DISTANCE FROM HIP TO KNEE
- I2P - MOMENT OF INERTIA OF UPPER LEG ABOUT PITCH AXIS THRU ITS CM.
- M3 - MASS OF LOWER LEG (& FOOT)
- L3 - DISTANCE FROM KNEE TO CM. OF LOWER LEG
- I3P - MOMENT OF INERTIA OF LOWER LEG ABOUT PITCH AXIS THRU ITS CM.

ANGLES (IN DEGREES):

- PITCH - AMOUNT OF PITCH PER CYCLE OF MANEUVER
- BETA - SEMI-VERTEX ANGLE OF CONES SWEPT OUT BY ARMS
- THETA - ANGLE BETWEEN PITCH AXIS AND PROJECTION OF CONE AXIS ON YAW PLANE
- PHI - ANGLE BETWEEN CONE AXIS AND YAW PLANE
- HIP - ANGLE BETWEEN THIGH AND TORSO (POSITIVE FORWARD)
- KNEE - ANGLE BETWEEN SHANK & THIGH (POSITIVE REARWARD)

INERTIA PROPERTIES:

ARM:	MB = 0.2880	B = 0.9030	IB1 = 0.1325	IB2 = 0.002335
TORSO:	M1 = 2.7840	L1 = 0.6320	LL1 = 1.5420	I1P = 1.67400
UPPER LEG:	M2 = 0.5110	L2 = 0.7900	LL2 = 1.5050	I2P = 0.07200
LOWER LEG:	M3 = 0.3260	L3 = 0.7270		I3P = 0.07860

BETA	THETA	PHI	HIP	KNEE	PITCH
45.00	0.00	15.00	90.00	150.00	24.330
60.00	0.00	30.00	90.00	150.00	32.562
60.00	15.00	30.00	90.00	150.00	31.806

Pitch Motion Program Output