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THE FDM DEMODULATING CHARACTERISTICS OF NON-LINEAR PHASE-LOCKED LOOPS

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Abstract

The projections of the state space onto the various hyper-planes are used to determine the minimum noise bandwidth and maximum information bandwidth for non-linear phase-locked loops used to demodulate FM multiplexed subcarriers.

Introduction

In many space-communications FM systems, where receivers must operate in low signal to noise environments, the phase-lock loop (PLL) is utilized because of its threshold extending capabilities. Since threshold is determine by the loop bandwidth (ω_n) for a given input signal to noise ratio, it is desirable to have a minimum loop bandwidth, yet process the modulation in an acceptable manner. The constraints on ω_n for a minimum of modulation distortion and for linear operation of the loop have been determined for single sine-wave demodulation [1,2]. These constraints require ω_n to be much larger than various signal parameters; however, how much larger is not indicated and a minimum ω_n cannot be determined from the constraints. Since in certain application it is desirable to exchange signal fidelity for a smaller ω_n and hence, a lower threshold, it is desirable to know the minimun ω_n. This paper is concerned with the problem of determining the minimum loop bandwidth before loop unlock occurs because of FM multiplexed subcarriers. In particular, the problem of demodulating a wideband video signal and several subcarriers is considered. The lower bound on ω_n is determined as a function of modulation indices, modulating frequencies and peak frequency deviations. This work considers the problem of establishing a boundary on the parameter region that insures locked operation of the PLL. Depending upon the signal to noise environment, the optimum ω_n will be in the region between a large ω_n needed for a minimum of signal distortion and the small ω_n below which loop unlock will occur.

State Equations

In order to determine the minimum ω_n before the loop breaks lock, the non-linear differential equation of the baseband PLL model was formulated in terms of state variables. A loop filter of the form $F(p) = 1 + a/p$ was used. The resulting state variable equations are [3].

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -2\zeta\cos X_1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ -\sin X_1 & \dot{\phi}_1\end{bmatrix} \qquad (1)$$

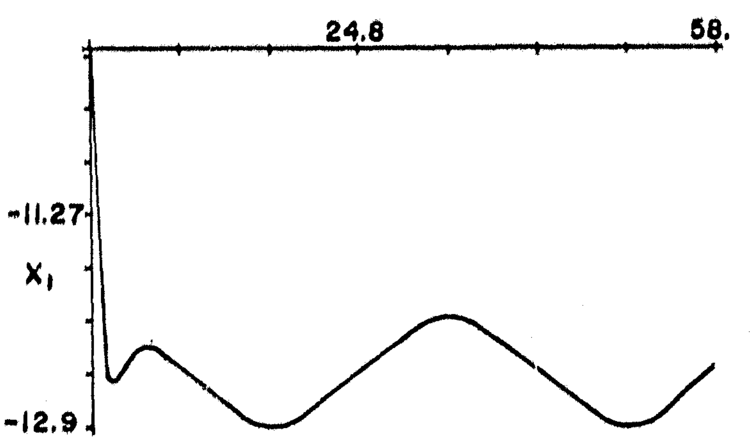

Figure 1. State space projection showing a properly locked PLL.

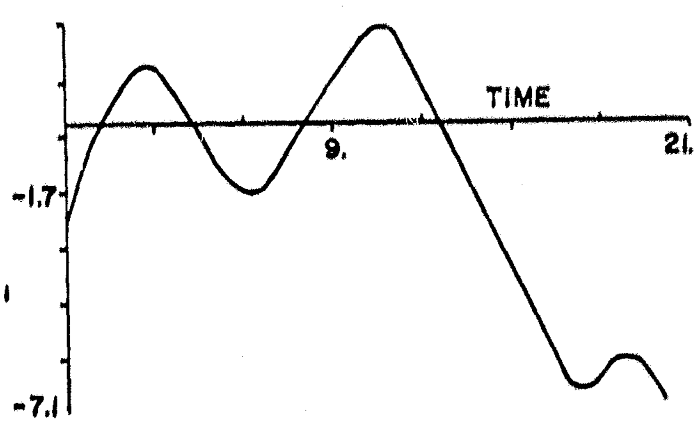

Figure 2. State space projection showing an unlocked PLL.

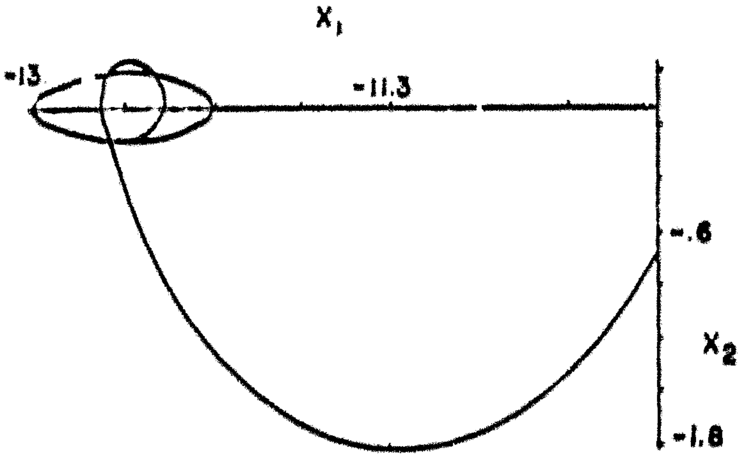

Figure 3. Phase plane showing a properly locked PLL.

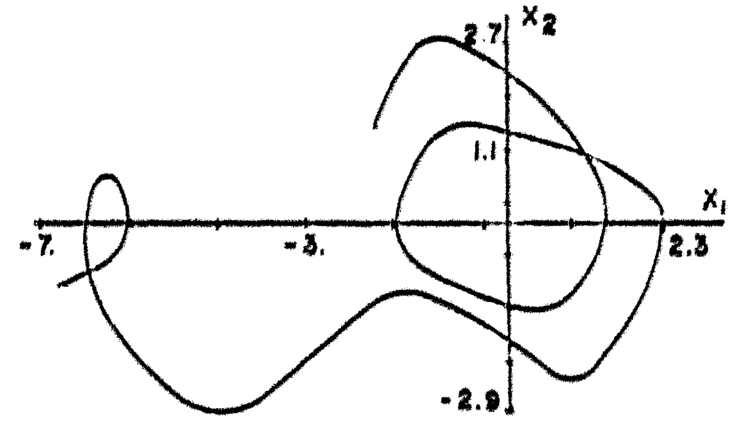

Figure 4. Phase plane showing an unlocked PLL.

where

X_1 = instantaneous phase error

$\dot{X}_1$ = instantaneous frequency error

$\phi_i(t)$ = input phase due to FM modulation

In equation (1), ω_n was normalized to unity. Digital computer algorithms were used to obtain solutions to equation (1) and to obtain projections of the state space onto the various hyperplanes. Analog computer simulations were also used to determine a minimum ω_n. Inspection of the state projections indicate when the loop is skipping cycles. For example figure 1 shows X_1, versus time, for a PLL properly locked onto the modulation. Figure 2 shows a PLL unlocked and skipping cycles because ω_n is too small. Figures 3 and 4 are the projections of the state space on the X_2 versus X_1 plane. These correspond to figures 1 and 2 respectively and may also be used to determine if the loop breaks lock or not. If the trajectories are contours around $\pm n\pi$(N = 0,2, 4,···) then the PLL is locked and tracking the modulation. Trajectories continuously increasing, such as figure 4, indicate cycle skipping.

Initial Conditions

If it is assumed that the PLL is initially locked onto the carrier and that the subcarriers are applied at t = 0, the initial conditions are given by

$$X_1(0^+) = -\sum_{i=1}^{N} \frac{\Delta\omega_i}{\omega_i} \cos\theta_i \qquad (2)$$

$$\dot{X}_1(0^+) = \sum_{i=1}^{N} \Delta\omega_i \sin(\theta_i) - 2\zeta \sin X(0^+). \qquad (3)$$

Equations (2) and (3) are the result of letting $\phi(t)$ of equation (1) represent the multiplexed subcarriers and expressing $\phi(t)$ as

$$\phi(t) = -\sum_{i=1}^{N} \frac{\Delta\omega_i}{\omega_i} \cos(\omega_i t+\theta_i) \qquad (4)$$

Varying θ_i of each subcarrier allows various combinations of sinusoids to be used to determine the minimum ω_n for the worst case. It might be expected that θ_i equal to either $\pi/2$ or to zero would give the worst case since the maximum rate of frequency change occurs periodically for $\theta = \pi/2$ and the peak phase deviation occurs periodically for $\theta = 0$. This was found to be true, and further, the minimum ω_n was approximately the same for $\theta_i = \pi/2$ or zero. The initial conditions do determine what singular point the stable contours will enclose. For example, if $\theta = 0$, equation (2) becomes $X(0^+) = -\sum \Delta\omega/\omega_i$ and if the modulation parameters are such that the loop locks quickly, then the singular point nearest the value of the sum of the modulation indices will be enclosed in the contour. Figure 3 shows this case for a single subcarrier.

Parameter Regions

The minimum ω_n was determined as a function of peak frequency deviations, modulation indices and the subcarrier frequencies, for one, two and three subcarriers. The minimum ω_n for a given set of signal or modulation parameters was determined using the criterion of cycle skipping. In general, the condition $X_1 \geq \pi/2$ does not result in the loop breaking lock and hence, cannot be used as an equivalent criterion. The condition $X_1 \geq \pi/2$ is a sufficient condition for unlock in a restricted parameter region, however. For example, in the single subcarrier case the condition $X_1 \geq \pi/2$ does cause the loop to unlock if the parameters are such that the inequality $\omega_n \leq .5\omega_m$ is satisfied. However, in the parameter region defined by $.6\omega_m \leq \omega_n \leq \omega_m$ instantaneous phase errors greatly in excess of $\pi/2$ can occur without the loop breaking lock. This is analogous to the response of a PLL to a frequency step with a magnitude close to the value of the separtix. With a frequency step of this size the trajectory in the phase plane will enclose the point $\pi/2$ by a considerable margin. For the multi-subcarrier case parameter regions also exist where the condition $X_1 \geq \pi/2$ is a sufficient condition for unlock; however, these regions are not as well defined.

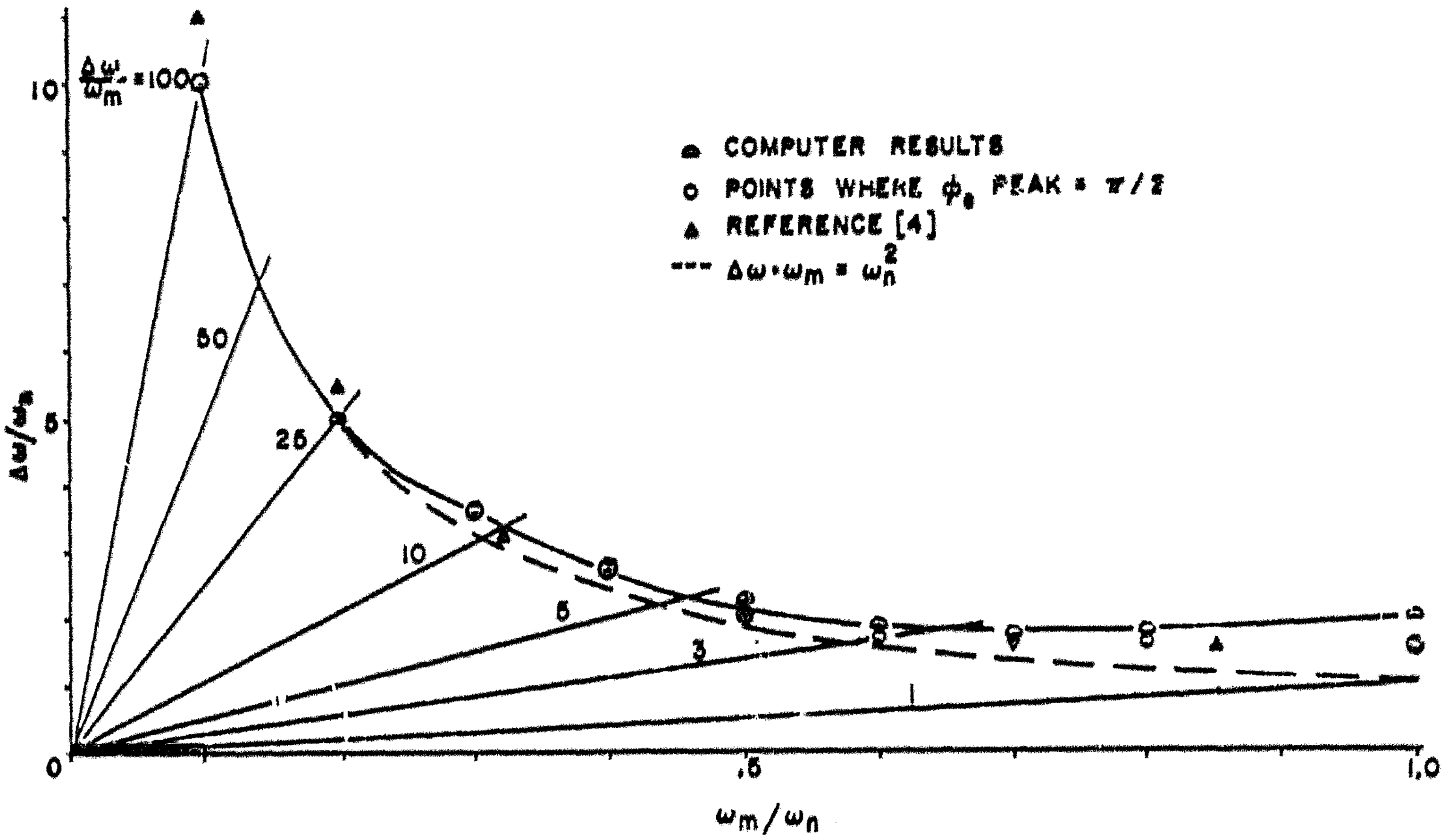

Figure 5. Partitioned parameter space for single subcarrier.

For the single subcarrier, figure 5 shows the minimum ω_n as a function of $\Delta\omega$, ω_m and D, the modulation index. That is, the parameter space is divided into two regions. For all values of $\Delta\omega$, ω_m in the region below the smooth continuous curve locked operation of the PLL will result. If the parameters are selected from the region below the curve indicated in figure 5, then phase errors in excess of $\pi/2$ will not occur. The region below the smooth curve of figure 5 agrees closely with region defined by the inequality $\Delta\omega \cdot \omega \leq \omega_n^2$ for $\omega_m < .5\omega_n$. This is a condition for locked operation developed in an earlier paper [3]. Locked operation can also occur in the region above the smooth curve and therefore, it appears that the smooth curve defines a narrow region of instability. Therefore, the signal and system parameters should be chosen such that this region is avoided. In particular, the parameters of a FM video signal and the parameters of the demodulating PLL should be selected from the region below the continuous curve.

Another recent paper [4], assuming $X_1 \geq \pi/2$ causes unlock, derived an expression for the minimum ω_n using describing function. After normalizing ω_n to unity the expression becomes.

$$\frac{\Delta\omega}{\omega_m}_{max} = \pi/2 \left| 1 - \frac{1}{\omega_m^2} - j\frac{1}{\omega_m} \right|$$

This equation agrees very closely with the points of figure 5 that indicate $X_1 = \pi/2$.

The conditions on the parameters for a minimum of distortion are [1,2],

$$\frac{\Delta\omega}{\omega_n} << 1 \qquad (5.a)$$

$$\frac{\omega_m}{\omega_n} << 1 \qquad (5.b)$$

These conditions are satisfied by the parameters in the cross hatched region of figure 5. This assumes the much less than requirement is satisfied by ratios less than .1. These inequalities require too large an ω_n for low signal to noise ratios. For operation of a PLL in a low signal to noise environment the optimum ω_n should be in the region bounded above by the smooth curve defining the absolute minimum ω_n and bounded below by the region defined by the inequalities of equation (5).

The boundaries for the parameter region insuring lock for two and three subcarriers are shown in figure 6,7, 8 and 9 respectively. The modulation indices for the family of curves are plotted inversely along the ω_1 and ω_2 axis. Parameters selected from the space below the smooth curves will give locked PLL operation. For a carrier tracking PLL the parameters should be selected from the region far above the smooth curve; however, this region was not investigated in this paper. The region in the vicinity of the smooth curve should be avoided. In particular, if one of the ω's represents the highest fre-

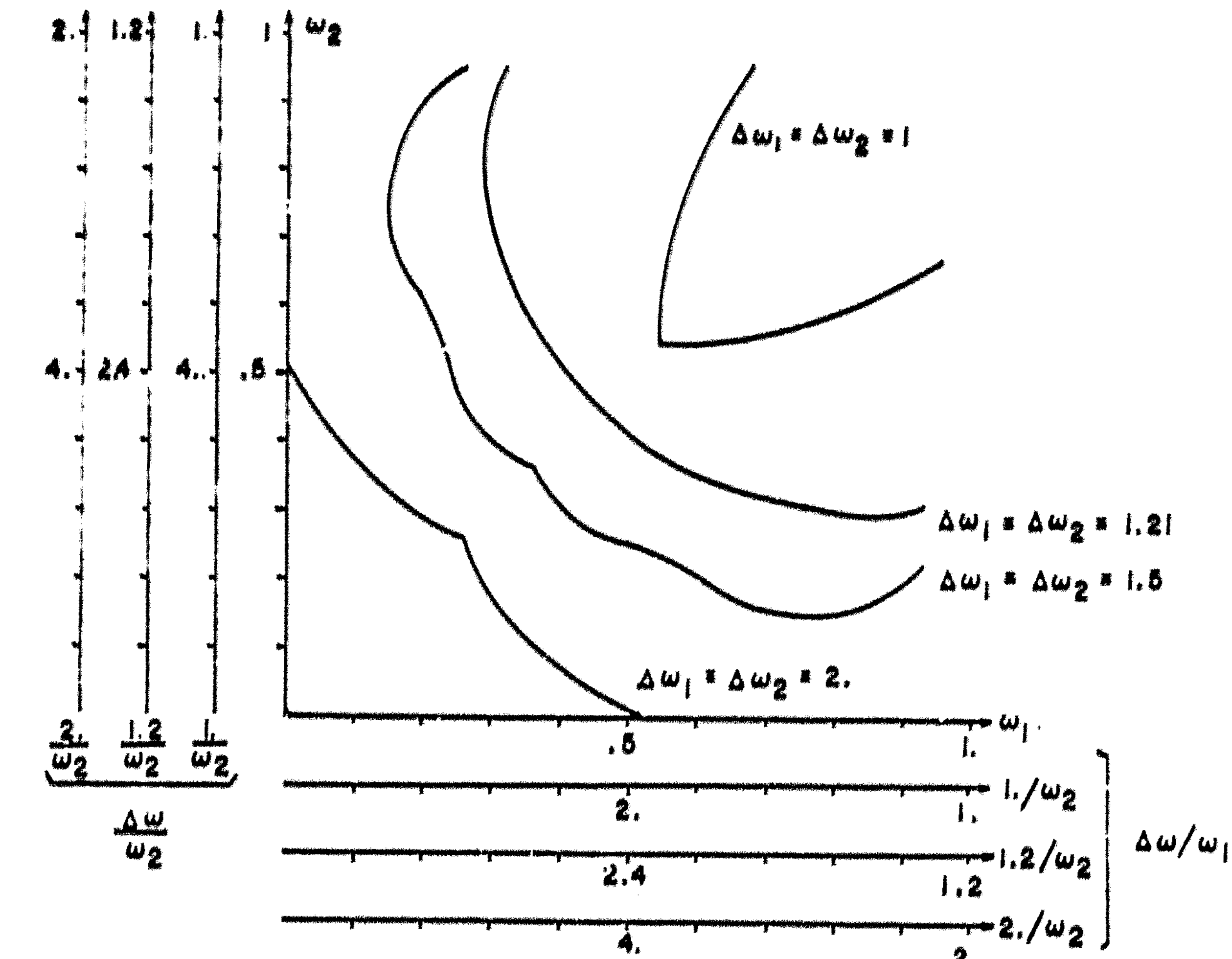

Figure 6. Partitioned parameter space for two subcarriers.

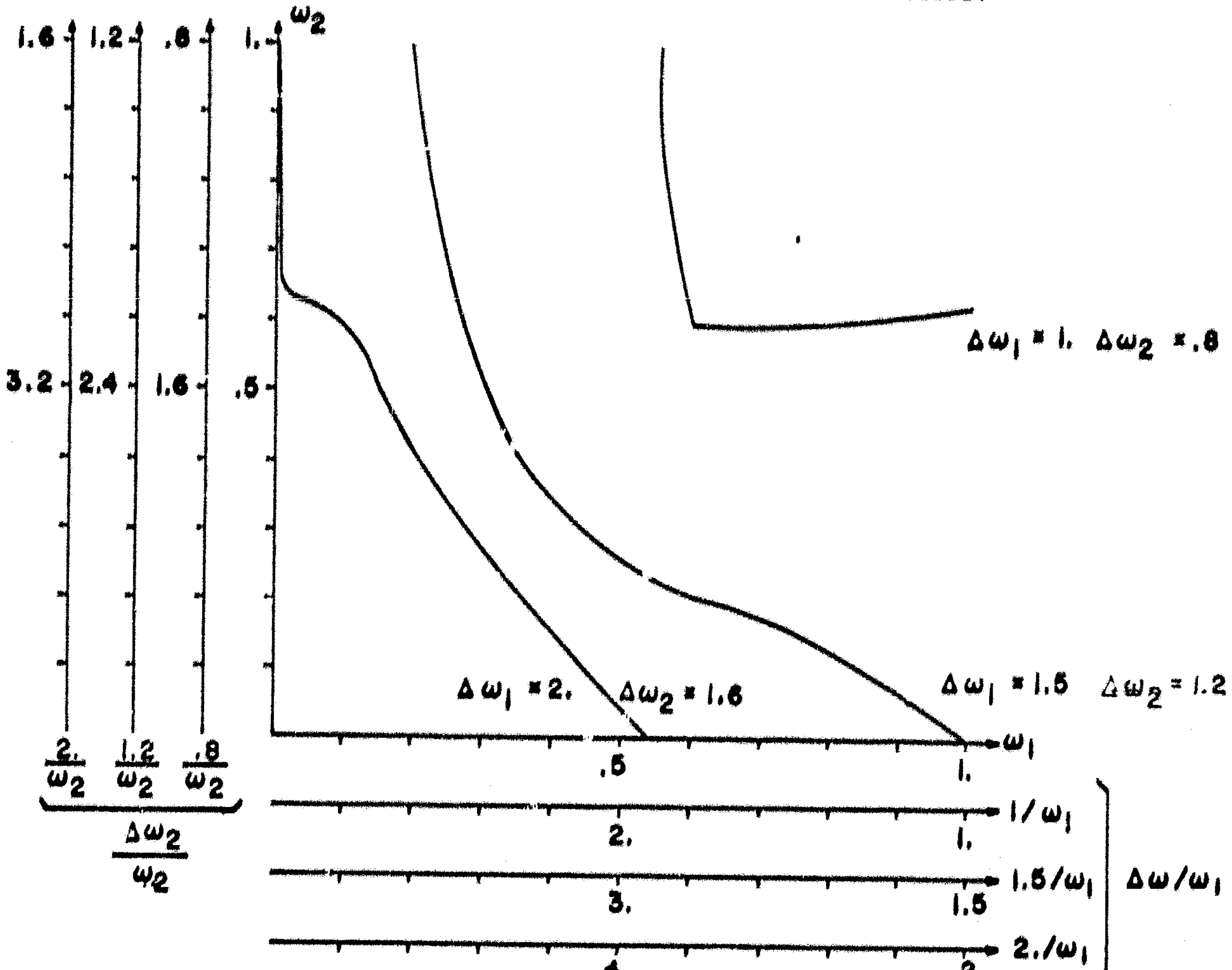

Figure 7. Partitioned parameter space for two subcarriers.

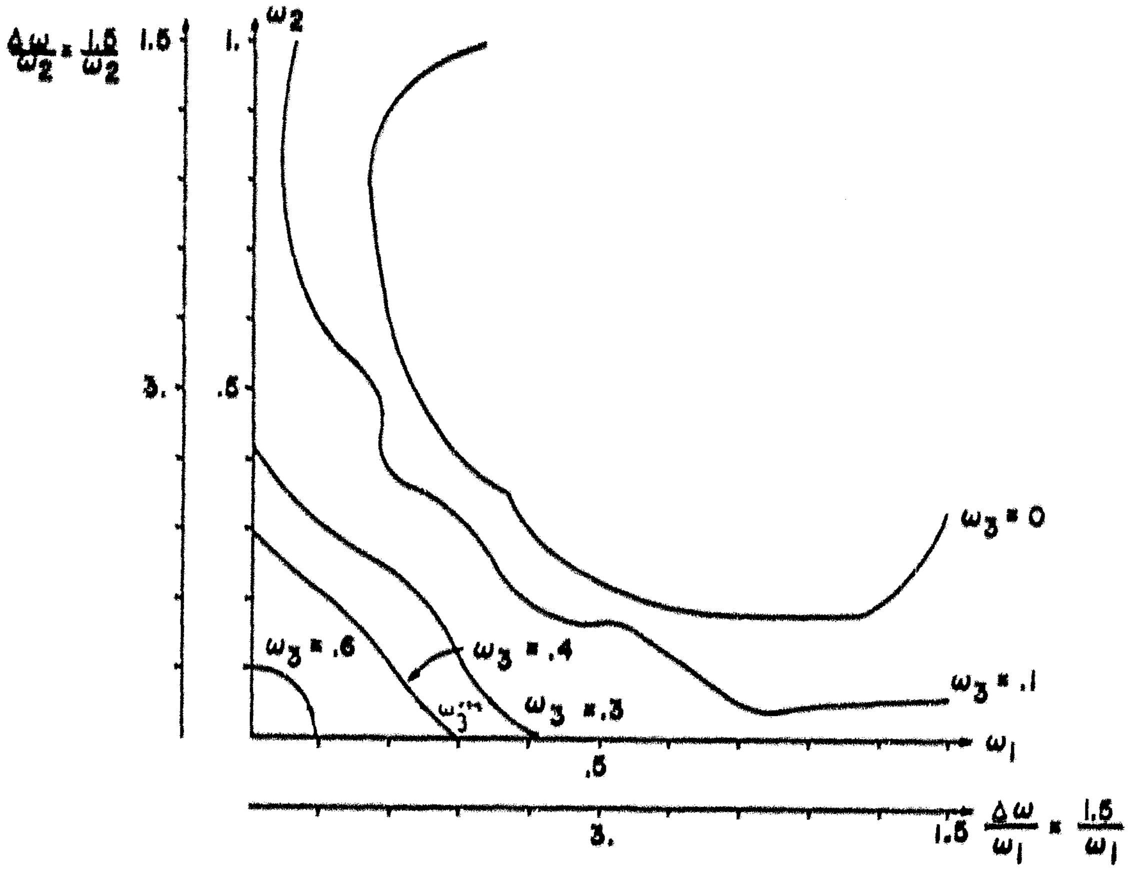

Figure 8. Partitioned parameter space for three subcarriers.

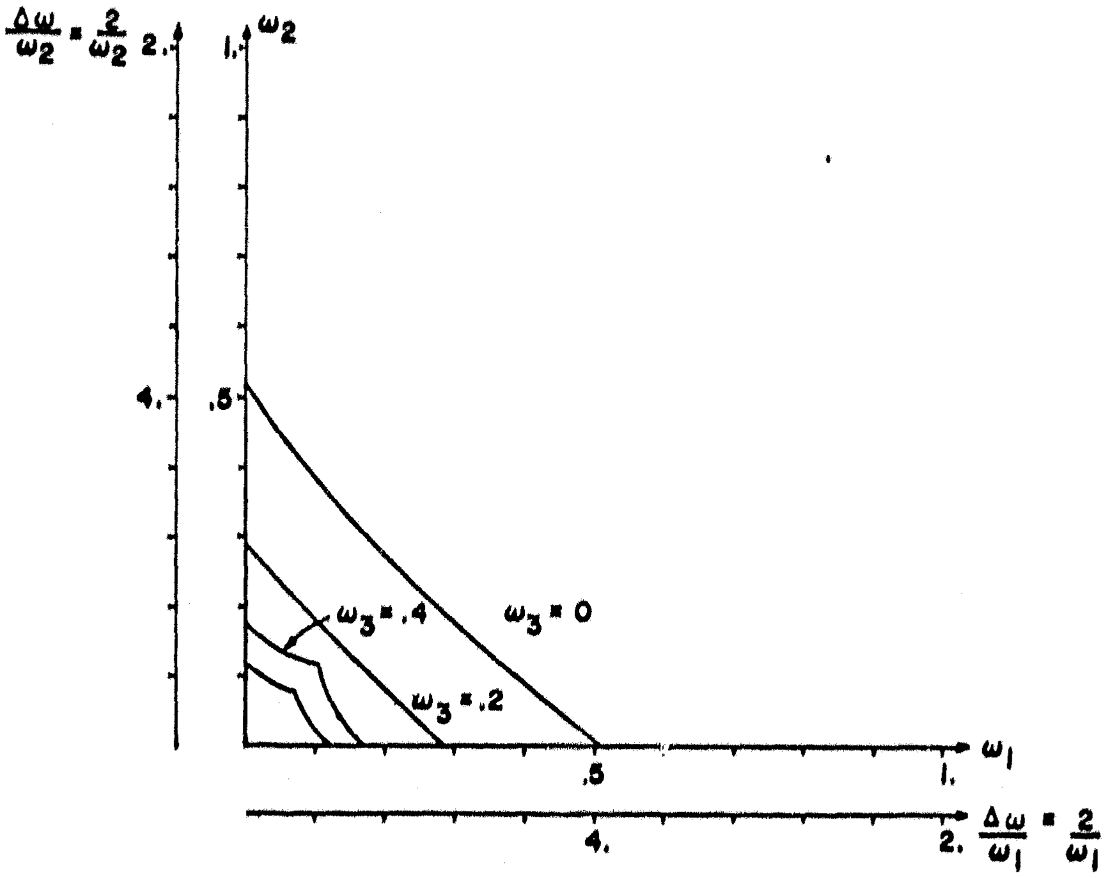

Figure 9. Partitioned parameter space for three subcarriers.

quency present in a video signal that has been frequency modulated onto a carrier concurrently with several subcarriers then all parameters should be selected from the region below the smooth curve. Further, the optimum ω_n should be in the region above the cross hatched region of minimum distortion shown in figure 5 for the single subcarrier or for the highest frequency in a video signal.

CONCLUSION

The boundary of the parameter region for locked operation was determined and a region for a minimum of distortion was shown that required a large ω_n. For low signal signal to noise environments the optimum ω_n should be selected below the minimum ω_n boundary and above the low distortion region.

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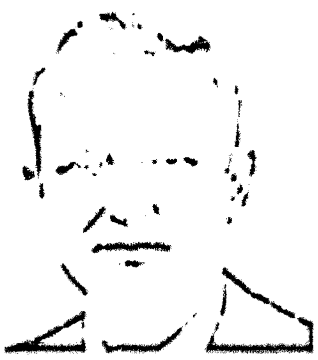
This work was supported by NASA grant #NGR-32-003-037



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