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by William M. Morrow II and Lucien A. Schmit, Jr.

Prepared by
CASE WESTERN RESERVE UNIVERSITY
Cleveland, Ohio
for Langley Research Center

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STRUCTURAL SYNTHESIS OF A STIFFENED CYLINDER

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SUMMARY

The problem of structural synthesis of a cylindrical shell stiffened in the longitudinal and circumferential directions with rectangular cross-section stiffeners is treated. The design variables are the dimensions and spacings of the stiffeners and the thickness of the skin, seven in all. The synthesis method uses the penalty function method of Fiacco and McCormick to transform the basic inequality constrained minimization problem into a sequence of unconstrained minimization problems. The stability analysis of the cylinder is a linear analysis including the effects of stiffener eccentricity. Numerical results are presented which illustrate the effectiveness of the penalty function approach, the importance of multiple load conditions, the influence of internal versus external stiffening, the effect of minimum gage limitations on the optimum design, and the advantage that stiffening in both directions exhibits compared with stiffening in only one direction.

INTRODUCTION

Stiffened cylinders have proven to be more efficient than monocoque cylinders for carrying compressive loads. However, it is not obvious how the cylinder should be stiffened in order to carry a given set of loads with the lightest possible cylinder. Many different methods of stiffening a cylinder are available. In order to determine the relative merits of the different configurations, the dimensions must be determined which define the lightest possible design for each configuration for the given system of loads. Once the best possible design for each configuration is known a comparison can be made to determine the best configuration and design.

The minimum weight design of stiffened cylinders subject to a single load condition of axial compression has been treated by several authors, for example, Gerard and Pipirno[1], Burns[2], and Hedgepeth and Hall[3].

Kicher[4] has treated the design of stiffened cylindrical shells subject to multiple load conditions.

In the work presented here one configuration is selected and a means of synthesizing the minimum weight design is developed. In this configuration the stiffeners are rectangular in cross section and integral with the skin of the cylinder. There are two sets, one in the longitudinal direction and one in the circumferential direction (see Figures 1 and 2). Either set may be inside or outside the cylinder, but the same set may not be both inside and outside. The skin is a solid orthotropic material with the principal material axes in the longitudinal and circumferential directions. The cylinder is required to support several different combinations of axial compression or tension and external or internal lateral pressure; that is, it is subjected to multiple load conditions. It is also possible to have different material properties in the skin and stiffeners and for these to be different in each load condition. The synthesis scheme determines the skin thickness, the dimensions of the stiffeners and the spacing of the stiffeners.

In order to conduct the synthesis the design requirements must be well defined. This means that ways of defining all modes of failure, both overall and local, as well as a means of handling constraints on the design variables, such as minimum gage, must be determined before the design process begins. The most important failure modes in the problem are the instability modes, both for the entire cylinder and for the elemental parts of the cylinder, the skin and stiffeners. Material yield failure is also guarded against in the skin and stiffeners.

The cylinder design problem is treated here as a nonlinear mathematical programming problem. The constrained minimization problem is converted to a sequence of unconstrained minimization problems using the penalty function method of Fiacco and McCormick.[5,6] The unconstrained minimization problems are solved using the method of Fletcher and Powell.[7] The unconstrained minimization technique generates a sequence of noncritical designs with decreasing weight and permits the use of approximate analysis during a major portion of the synthesis.

SYMBOLS

Scalars

C.B.L.	Circumferential stiffener buckling for a contraction of the cylinder, $\varepsilon_{\phi p}/\varepsilon_{\phi cr}$.
C.B.U.	Circumferential stiffener buckling for an expansion of the cylinder, $\varepsilon_{\phi p}/\varepsilon_{\phi cr}$.
C.Y.C.	Circumferential stiffener yield in compression, $\sigma_{\phi sp}/\sigma_{\phi OC}$.
C.Y.T.	Circumferential stiffener yield in tension, $\sigma_{\phi sp}/\sigma_{\phi OT}$.
D_1, D_2, D_ν	Bending stiffnesses of skin.
d	Stiffener depth.
E	Modulus of elasticity, lbs/inch^2.
E_x, E_ϕ	Moduli of elasticity of skin.
$E_{xs}, E_{\phi s}$	Moduli of elasticity of stiffeners.
e_x, e_ϕ	Eccentricity of stiffeners (+ inside, - outside).
$F(\bar{v}, r)$	Unconstrained function.
f	Actual value of behavior variable.
f_{cr}	Critical value of behavior variable.
G_x, G_ϕ	Shear modulus of stiffeners.
G	Shear modulus of skin.
G.B.	Gross buckling, N/N_{cr}.
$g_i(\bar{v})$	Constraint functions.
H_{s1}, H_{s2}, H_ν	Extensional stiffnesses of skin.
H_x, H_ϕ	Extensional stiffnesses of stiffeners.
J_x, J_ϕ	Torsional constants of stiffeners.
K	Torsional stiffness of skin.
L	Length of Cylinder, inches.
L_i	Lower bound on design variables.

L.B.	Lower bound.
L.C.	Load condition.
L.S.B.	Longitudinal stiffener buckling, σ_{xsp}/σ_{cr}.
L.Y.C.	Longitudinal stiffener yield compression, $\sigma_{xsp}/\sigma_{xOC}$.
L.Y.T.	Longitudinal stiffener yield compression, $\sigma_{xsp}/\sigma_{xOT}$.
$\ell_{\phi u},\ell_{xu},\ell_{\phi \ell},\ell_{x\ell}$	Bounds on ℓ_ϕ and ℓ_x.
$M_x,M_\phi,M_{x\phi},M_{\phi x}$	Moment resultants.
N_x,N_ϕ,N_x	Force resultants.
m,n	Wave numbers.
n_ϕ,n_x	Number of stiffeners in each direction.
N	Applied axial force per unit length of circumference.
p	Radial pressure.
P.B.	Panel buckling, N/N_{cr}.
R	Radius of cylinder.
r	Penalty function multiplier.
r_o	Initial value of the multiplier, r.
S	Shear stiffness of skin.
S.B.	Skin buckling, σ_{xp}/σ_{xcr}.
S.Y.	Skin yield, σ_D/σ_{OD}.
T_x,T_ϕ	Torsional stiffnesses of stiffeners.
T_{min}	Minimum move distance.
t	Stiffener thickness.
$\overline{t}$	Equivalent thickness of monocoque cylinder of equal weight.
$t_s,t_x,t_\phi,d_x,d_\phi,\ell_x,\ell_\phi$	Design variables (see Figure 2).
U_i	Upper bounds on design variables.
U.B.	Upper bound.

Δv	Finite difference increments in each design variable.
W	Weight of cylinder.
α_{ϕ}^{3}	Bending eccentricity term for circumferential stiffeners.
γ	Weight density, lbs/in.^{3}
$\gamma_s, \gamma_{\phi}, \gamma_x$	Weight densities of the skin, circumferential stiffeners and longitudinal stiffeners.
$\delta_{x\phi}, \delta_{xw}, \delta_{\phi w}$	Stiffener combination indicators in the weight function.
$\varepsilon_x, \varepsilon_{\phi}, \gamma_{x\phi}$	Strains.
$\varepsilon_{\phi cr}$	Buckling strain of circumferential stiffener.
$\varepsilon_{xp}, \varepsilon_{\phi p}$	Prebuckle strains.
λ, η	Wave parameters.
μ_x, μ_{ϕ}	Poisson's ratios of skin.
ν	Poisson's ratio.
ρ_x, ρ_{ϕ}	Radii of gyration of stiffeners about the skin midsurface.
σ_c	Critical buckling stress in longitudinal stiffeners.
$\sigma_{xp}, \sigma_{\phi p}$	Prebuckle stresses in skin.
$\sigma_{xsp}, \sigma_{\phi sp}$	Prebuckle stresses in stiffeners.
$\sigma_x, \sigma_{\phi}, \tau_{x\phi}$	Stresses in skin.
$\sigma_{xs}, \sigma_{\phi s}$	Stresses in stiffeners.
σ_y	Yield stress, lbs/in.^{2}
$\sigma_{OD}, \sigma_{xOT}, \sigma_{xOC}, \sigma_{\phi OT}, \sigma_{\phi OC}$	Yield stresses in skin.
$\sigma_{xSOC}, \sigma_{xSOT}, \sigma_{\phi SOC}, \sigma_{\phi SOT}$	Yield stresses in stiffeners.
θ_x, θ_{ϕ}	Rotations of shell per unit length.
ζ	Ratio of stiffener depth to the radius of its unsupported edge.

Vectors

$\overline{s}_i$	Move direction.
$\overline{v}$	Design variables.
$\overline{y}_i$	Change in successive gradients.
$\overline{\sigma}_i$	Vector to minimum along $\overline{s}_i$.

Matrices

H_i	Metric Matrix.

ANALYSIS

The analysis of the cylinder, as presented in Appendix A, is performed by assuming that the loaded cylinder can fail independently in any one or several of eleven different failure modes. For a cylinder of given design and material properties, a critical failure value of a load, stress, or strain is determined for each failure mode and load condition. The actual value of the corresponding load, stress and strain is determined for each load condition, and checked against the failure values to determine whether or not the cylinder can sustain the applied loads.

Three of the failure modes involve determining buckling failure load values of the cylindrical shell. These are buckling of the entire cylinder (gross buckling),buckling of the cylinder between the circumferential stiffeners (panel buckling), and buckling of the cylindrical skin (skin buckling). In the gross buckling and panel buckling analyses the effects of the stiffeners are averaged over the stiffener spacing. Besides the bending stiffnesses of the stiffeners the torsional stiffnesses and the effects of eccentricity are also taken into account. The cylinder buckling analysis is a linear classical small displacement analysis, assuming simply supported boundaries, and a uniform prebuckled membrane force and displacement distribution. The same analysis is used to determine the critical loads for gross, panel, and skin buckling, by substituting the appropriate stiffness properties and displacement patterns.

Using an expression for the buckling load in terms of the mode shape (Eqn. A20 or A22) the critical buckling load is found by determining the buckling loads for a large number of mode shapes and selecting the lowest of these loads as the critical value.

The stresses and strains in the skin and stiffeners prior to buckling are determined from the membrane force distribution (Eqs. A12, A13 and A14). The skin is assumed to be in a biaxial stress state and the stiffeners are assumed to be in a uniaxial stress state. The strains in the stiffeners where they join the skin are assumed to be the same as the corresponding strains in the skin.

There are three failure modes for the stiffeners. The longitudinal stiffeners can buckle only when they are under axial compression; a critical buckling stress is calculated for this (Eq. A23). Outside circumferential stiffeners can buckle either when the cylinder expands or contracts under load, but inside circumferential stiffeners can buckle only when the cylinder contracts. Two critical circumferential strains are calculated when the circumferential stiffeners are on the outside and one when they are on the inside (see Appendix A Sec. A.8). The expression for the circumferential stiffener critical strain (Eq. A24) derived in Appendix A Sec. A.9 is verified for two limiting cases in Appendix B. In the stiffener buckling analysis simply supported boundaries are assumed at all edges where the stiffener connects with the shell or the other stiffeners. In addition to the buckling failure modes there are five yield failure modes (Eqs. A27 and A29). A distortion energy type criterion is used in the skin for the biaxial state of stress. In the stiffeners the uniaxial state of stress must have a value between the compression yield value and the tension yield value.

SYNTHESIS SCHEME

Conversion of the Design Problem to an Unconstrained Minimization Problem

The cylinder is synthesized in such a way that the weight is minimized, at least locally, and all constraints on behavior variables and design variables are satisfied. (The details of the synthesis scheme are described in Appendix C.) The actual dimensions of the cylinder (See Figures 1 and 2) are considered as the independent design variables. These variables are the skin thickness, t_s, the longitudinal stiffener thickness, t_x, the circumferential stiffener thickness, t_ϕ, the longitudinal stiffener depth, d_x, the circumferential stiffener depth, d_ϕ, the circumferential stiffener spacing, ℓ_x, and the longitudinal stiffener spacing, ℓ_ϕ. Any of these variables may be held fixed during the synthesis and the two stiffener depths may be forced to be equal.

Upper and lower bounds are imposed on all the design variables as well as compatibility bounds to prevent the stiffener thicknesses from becoming greater than their spacings. The behavior variables are the loads, stresses, and strains in the structure. Bounds are imposed on these to prevent the structure from buckling or yielding, as described in the analysis.

The weight minimization is realized by minimizing a composite function formed by adding a penalty function to the weight. This penalty function includes all of the design requirements. This function is formed as follows:[8]

$$F(\bar{v},r) = W(\bar{v}) + r \sum_i 1/g_i(\bar{v}))$$

In this expression, $\bar{v}$ represents the vector of design variables; $W(\bar{v})$ is the weight (see Appendix C); r is a constant multiplier; and the $g_i(\bar{v})$ are the constraint functions which express the design requirements. These requirements include the constraints on the design variables as well as the behavior requirements for all load conditions. The $g_i(\bar{v})$ are formed so that the constraints appear in the form:

$$g_i(\bar{v}) \geq 0$$

The constraints are of two types, the behavior constraints and the constraints on the design variables. In order to put the behavior constraints in the required form they are written as

$$g_i(\bar{v})_B = 1 - \left(\frac{f}{f_{cr}}\right)_i$$

where f is the actual value of the behavior variable (load, stress or strain) and f_{cr} is the critical value of this same variable.

The constraints on design variables are of three different types, upper bounds, lower bounds, and compatibility bounds. The upper bound constraints have the form:

$$g_i(\overline{v})_{SU} = \frac{U_i - v_i}{U_i - L_i}$$

where U_i is the upper bound, and L_i is the lower bound. The lower bound constraints have the form:

$$g_i(\overline{v})_{SL} = \frac{v_i - L_i}{U_i - L_i}$$

The compatibility constraints keep the stiffener thicknesses from becoming larger than the corresponding stiffener spacing. Thus, these are

$$g_1(\overline{v})_c = \frac{\ell_\phi - t_x}{\ell_{\phi u} - \ell_{\phi \ell}}$$

and

$$g_2(\overline{v})_c = \frac{\ell_x - t_\phi}{\ell_{xu} - \ell_{x\ell}}$$

where $\ell_{\phi u}$ and ℓ_{xu}, and $\ell_{\phi \ell}$ and $\ell_{x\ell}$ are the upper and lower bounds on ℓ_ϕ and ℓ_x.

By examining the expressions for the constraints, $g_i(\overline{v})$, it can be seen that as a limiting value of a behavior variable or design variable is approached the $g_i(v)$ approach zero from the positive side. Thus, the corresponding contribution to the penalty term $(1/g_i(\overline{v}))$ approaches positive infinity. For example, the closer the critical buckling load for the cylinder is to the applied load the larger the penalty term. If, however, a design is obtained which yields a buckling load slightly lower than the applied load and the penalty term is calculated, it will be a large negative number. Thus, in passing from a noncritical design, through a critical design, the value of F would approach positive infinity for nearly critical acceptable designs and then change to negative infinity for unacceptable designs. Because of this F is not calculated for unacceptable designs and the minimum of F is sought only for acceptable designs.

Since the penalty function goes to infinity as the design becomes critical in some way,the design which yields a minimum of the composite function F is a noncritical design. If, however, the true minimum weight design is a critical design this design can be approached as closely as desired by reducing the value of the multiplier r. The procedure for weight minimization is as follows (see block diagram Figure 3), start with some relatively large value of r (see Appendix C Sec C.8) and an initial design which satisfies all the requirements on the design variables and is capable of sustaining all loads (this must be a noncritical design with no design variables or behavior variables equal to their critical values); find a design which yields a minimum of F for this value of r; reduce the value of r; starting with this new value of r and the design which gave a minimum of F for the previous value of r, again find a design which gives a minimum F; continue this process until a convergence criterion is satisfied (see Appendix C Sec. C.7). It should be noted that since the penalty term includes the requirements for all load conditions the minimum weight design obtained in this manner does not presuppose a critical load condition or even that there is a critical load condition.

Because of the fact that the design which yields a minimum of F is a noncritical design it is possible to use an approximate cylinder buckling analysis during much of the minimization. This reduces the computational effort substantially. The approximate analysis uses only a selected set of mode shapes in the determination of the critical buckling loads for gross, panel, and skin buckling. (see Appendix C, Sec. C.3). An analysis using the large number of possible modes is done for the design yielding the minimum of F for each value of the multiplier r.

Minimization of the Unconstrained Function

Since the design which gives a minimum to the composite function F is unconstrained, noncritical, a method of unconstrained minimization can be used to find this design. The method used here is a variable metric method (see Appendix C, Sec. C.4). A design is altered by moving along a line in the space defined by the design variables. The direction of move at a point i in the design space is given by[9]

$$\bar{s}_i = - H_i (\nabla_V F)_i$$

where H_i is square matrix and $(\nabla_V F)_i$ is the gradient vector of F with respect to the independent variables $\bar{V}$ at the point i. The minimum of the function, F, is found along the direction $\bar{s}$ (see Appendix C Sec. C.6). Once the minimum is known the new design is

$$\bar{V}_{i+1} = \bar{V}_i + \bar{\sigma}_i$$

where $\bar{\sigma}_i$ is $\alpha_i \bar{s}_i$ and α_i is the distance along $\bar{s}_i$ to the minimum. At this point $\bar{V}_{i+1}$ the gradient is calculated and the H matrix is altered (see Appendix C Sec. C.5). Using the new gradient and the new H matrix a new direction $\bar{s}_{i+1}$ is generated and the process is repeated until a convergence criterion is satisfied (see Appendix C Sec. C.7).

The minimum along a line is found by first finding two acceptable designs which lie on opposite sides of the minimum and then using the function value and its slope at these two points to do a cubic interpolation to the minimum. The two acceptable designs bracketing the minimum are found using an incrementation scheme (see Appendix C Sec. C.6).

NUMERICAL RESULTS

Results for over thirty design cases have been obtained (see Appendix D). These results demonstrate the following:

1. the capability of the synthesis method,
2. the influence of inside versus outside stiffening,
3. the sensitivity of the optimum design with respect to change in the magnitude of the load conditions,
4. the influence of manufacturing bounds on the optimum design,
5. the importance of including multiple load conditions,
6. relative minima in the design space,

7. the advantage of stiffening in both directions.

These results were obtained with a computer program, which is given in Appendix E.

A set of twelve design problems was developed based on one basic design problem (see Appendix D, Cases 1-I through 3-I,0). This is an aluminum cylinder with a radius of 60 in. and a length of 165 in. This was first to be subjected to the following three load conditions: 1) axial compression of 700 lb/in. and no radial pressure, 2) axial compression of 940 lb/in. and internal radial pressure of 2 lb/in^2, 3) axial compression of 212 lb/in. and external radial pressure of 0.4 lb/in^2. Four design problems were developed for this load condition. Three of these had no manufacturing bounds, that is no minimum gages. These three problems were distinguished by the location of the stiffeners. One had all the stiffeners inside, another had all outside, and the third had longitudinal stiffeners outside and circumferential stiffeners inside. The fourth problem had all the stiffeners on the inside but the following minimum gage limits were imposed: 0.019 in. on the cylinder skin, and 0.05 in. on the stiffeners (Case 1-I'). The other eight cases were obtained by first doubling the loads and using the same four stiffening combinations and then tripling the loads, of the first four, and again running the same stiffening combinations.

The minimum weights obtained for nine of the twelve problems are exhibited in Figure 4. The lowest weight designs are those for all inside stiffening; the next lowest (not shown in the figure) are for longitudinal stiffening outside and circumferential stiffening inside; the highest weight designs, for zero lower bounds on the thicknesses, are the ones with all outside stiffening. The effect of including lower bounds on the thicknesses also can be seen in Figure 4. For the cylinder with the lightest load the weight penalty for the minimum gage is substantial and much more than the penalty which results from selecting an off optimum stiffener combination. In the more heavily loaded structure the penalty for manufacturing bounds is less but still more than the penalty for choosing an off optimum stiffener combination.

It should be noted that while these are three load condition cases they are single load condition dominant (load condition two dominates).

However the other gross buckling constraints are approached closely in the outside stiffened cases. Even assuming that the dominant load condition can be identified in advance it should be recognized that an optimum design obtained considering a single load condition may not be acceptable in the other two load conditions.

Also note that this dominant load condition is one which contains internal pressure. This should be contrasted with the results of problem 6 where the load is only axial compression. In problem 6 the inside outside case is the lighter weight design (see Cases 6-I', 6-0', and 6-I,0').

Two other cases (see Appendix D) of data are presented which are based on this same basic design problem (Case 1-I). One is simply a second starting point for this case. The final designs do differ somewhat but the weights are nearly the same. The other is Case 1-It and is the same as Case 1-I' but has a 25% degradation of the modulus in the first load condition. This causes a shift in the dominant load condition from the second to the first but the second is still active and there is only a slight increase in weight.

Another set of problems was developed using an aluminum cylinder of 200 in. radius and a length of 500 in. subjected to the following loads: 1) axial compression 2100 lb/in. with an external radial pressure of 1.0 lb/in^2, 2) an axial compression of 8000 lb/in. and an internal radial pressure of 20 lb/in^2, 3) an axial compression of 5000 lb/in. and no radial pressure. The yield limit of the aluminum was taken as 50,000 psi (Cases 4-I through 4-0').

This problem was run with all inside stiffening and all outside stiffening. The all inside stiffened case was run with the design variables effectively unbounded (see Table 1, Case 4-I). The final design in this case is limited by five behavior constraints, three in load condition two and two in load condition three. One of them in load condition two is skin yield. Note here the importance of considering the multiple load conditions.

In running the outside stiffening case difficulty was encountered when the design variables were permitted to be unbounded (this is done in the computer program by using large bounds). Two different starting designs were attempted in which very heavy final designs were obtained where no further optimization was possible (see Table 1, Cases 4-0). Note that for the first starting point the final design is of a larger weight than the starting weight. This is due to the nature of the penalty function. A weight increase can occur for the first minimization of the penalty function. Note also that the stiffeners are thick and shallow.

The difficulty encountered in this case was resolved by putting an upper bound on one inch on the stiffener thicknesses. This forces a deep stiffener thin skin design. The stiffener thicknesses are not, however, bounded by these constraints (see Table 1, Case 4-0').

These results indicate the possibility of relative minima in the design space. One with a thick stiffener design and the other with a thin stiffener design.

For this problem the outside thin stiffener design and the inside stiffener design are almost the same weight and the values of the final design variables are not greatly different. Note, however, that no constraints of load condition three are limiting in the outside stiffened case as they are in the inside stiffened case but the gross buckling constraint of load condition one has become active. This again points out the importance of considering multiple load conditions and the difficulty in picking a critical load condition.

In Case 5-I (see Appendix D) the circumferential stiffener depth is fixed at 10 inches. The results of this case are not single load condition dominant. Both load conditions 2 and 3 have constraints which are active.

A number of cases were run where the only load on the cylinder was axial compression. The first series of these was for an aluminum cylinder of length 38.0 in., radius 9.55 in., subject to an axial compression of 800 lb/in. The first result obtained for this problem was for a starting design with all inside stiffening and liberal bounds on the design variables. The result of this was a completely unstiffened design (see Table 2, Case 6-I). This case was then rerun putting lower bounds of .05 in. on the stiffener depths. The result of this was a stiffened design with a lower weight than the previously obtained unstiffened design. This was done in three different cases to obtain results with all inside stiffening (Case 6-I'), all outside stiffening (Case 6-0'), and with longitudinal stiffeners outside and circumferential stiffeners inside (Case 6-I,0'). The lowest weight obtained for these three (Cases 6-I', 6-0', and 6-I,0') was for the inside outside case (Case 6-I,0'). This is about a 12% advantage over the all outside case (Case 6-0').

This is another situation where relative minima in the design space is the most likely explanation. The results for the cases where lower bounds are imposed, the stiffened cases, are in the acceptable region of the design space for the case with less stringent side constraints where the unstiffened design was obtained.

Other single axial load cases were run which can be used to compare stiffening in one direction with stiffening in both directions (see Table 2, Cases 6-OS, 7-I, 7-I', 7-I"). When cases were run with overall dimensions and load the same as the above single load condition cases (Cases 6-OS and 6-IS minimum longitudinal stiffener depth 0.05 in.) but with axial stiffening only the results were designs of slightly higher weight than the unstiffened design (Case 6-I). A second series of single load condition problems was run for a cylinder of length 291 in. and radius 95.5 in. subject to an axial compression of 800 lb/in. Using liberal side constraints and starting with inside stiffening the result for Case 7-I is unstiffened. This case was then rerun putting a lower bound of 0.5 in. on the longitudinal stiffener depth and the resulting design (Case 7-I')is stiffened in both directions. When another case was run with circumferential stiffening only (Case 7-I", lower bound circumferential stiffener depth 0.5), a result was obtained which was higher in weight than the unstiffened design (Case 7-I). The unstiffened design is of greater weight than the design for stiffening in both directions (Case 7-I').

These results indicate that a design with stiffening in both directions is the better design. They also indicate that a monocoque design is superior to stiffening in one direction. However, because of imperfection sensitivity it may not be advisable to use a monocoque cylinder. Using an analysis similar to the one used here but without including eccentricity effects, Timoshenko[10] has shown that in designing against gross buckling there is no weight advantage to be gained over the monocoque design by putting the stiffeners in only one direction.

Case 8-I,0 (see Appendix D) is again an axial compression case. This case is thought to be representative of future launch vehicles.

CONCLUSIONS

An efficient method of synthesis has been developed for a stiffened cylinder. This efficiency is due to a combination of several factors.

The use of the penalty function of Fiacco and McCormick causes the successive designs obtained during the synthesis process to stay away from the constraints. This alleviates one of the difficulties encountered with methods such as the gradient projection method and the alternate step technique. In the later methods the designs become bounded by constraints early in the synthesis and it is then difficult to move away from the constraints.

Another advantage of staying away from the constraints is that an approximate analysis can be used during the synthesis with some confidence that the designs obtained remain in the acceptable region of the design space. The approximate analysis is the largest factor in speeding the synthesis.

Still another advantage to using the penalty function is that the problem is converted to a sequence of unconstrained minimizations and the quite efficient method of Fletcher and Powell can be used to perform this minimization.

The experience reported herein supports the contention that the generation of efficient structural synthesis capabilities, for design problems involving relatively complex analyses, requires that the structural analysis and synthesis procedure be tailored together exploiting available insight and understanding for the characteristics of the problem at hand.

Using the computer program a number of numerical results have been generated, and some general conclusions can be drawn from these. There is evidence of relative minima both for single and multiple load condition problems, and there is a means of forcing the designs into these relative minima pockets (see Numerical Results). There does not appear to be any advantage to stiffening a cylinder in only one direction. Manufacturing bounds have a strong influence on the optimum design. In some situations there is a moderate weight advantage to be gained in the placing of the stiffeners, inside or outside, and this depends on the load conditions. The largest weight saving due to change of stiffener location obtained in this work is 12% (see problem 6, Appendix D).

Table 1

Summary of Numerical Results - Problem 4

	Case 4-I (Inside) Start	Finish	Case 4-0 (Outside 1) Start	Finish	Case 4-0 (Outside 2) Start	Finish	Case 4-0' (Outside 2) Start	Finish
t_s	.15	.162	.15	.216	.5	.283	.5	.163
t_x	.201	.188	.200	.315	.5	.972	.5	.184
t_ϕ	.271	.192	.489	1.98	.5	3.07	.5	.200
d_ϕ	1.85	1.78	-1.84	-1.38	-1.5	-1.03	-1.5	-1.74
d_x	2.87	2.54	-1.89	-0.865	-1.5	- .652	-1.5	-2.13
ℓ_ϕ	36.6	31.6	35.	23.7	35.	26.3	35.	27.0
ℓ_x	4.55	6.30	4.58	8.57	4.	14.6	4.	6.08
Weight lbs)	15,900.	14,600.	16,200.	21,300.	44,900.	26,900.	44,900.	14,700.
t	.251	.230	.255	.336	.707	.423	.707	.231
Active Behavior Constraints* Final Design	GB2 = .980 LSB2 = .963 SY2 = .994 GB3 = .980 PB3 = .912 SB3 = .964		GB1 = .999 PB2 = .962		GB1 = .999 PB2 = .948 SB2 = .989 SB3 = .982		GB1 = .923 GB2 = .957 PB2 = .926 LSB2 = .954 SY2 = .990	
L = 500	R = 200		E = 10 x 10^6		σ_y = 50,000			

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* The numbers in the failure mode abbreviations refer to the load condition (i.e., GB2 is Gross Buckling in load condition 2).

Table 2

Summary of Numerical Results - Problems 6 and 7

	L = 38.0 R = 9.55 N = 800					L = 291 R = 95.5 N = 800		
Case	6-I	6-I'	6-O'	6-I,O'	6-OS	7-I	7-I'	7-I"
t_s	.0363	.0107	.00932	.00832	.0357	.111	.0292	.114
t_x	.249	.0132	.0150	.0139	.0125	.725	.0441	3.56
t_ϕ	.263	.00991	.0203	.00145	.05	.940	.0943	.400
d_x	0.	.121	-.142	-.126	-1.56	0.	.718	0.
d_ϕ	0.	.291	-1.83	.512	0.	0.	.810	1.64
ℓ_x	1.96	1.49	2.75	2.28	38.0	8.57	18.2	42.0
ℓ_ϕ	1.41	.283	.264	.228	1.45	3.84	1.42	11.6
Weight(lbs)	8.35	4.20	4.30	3.76	8.54	1960.	979.	2240.
$\bar{t}$	.0363	.0182	.0187	.0163	.0371	.111	.0555	.127
Active Behavior Constraints Final Design	GB = .968	GB = .990 PB = .725 SB = .921 LSB= .966 SY = .958 LYC= .965	GB = .871 PB = .938 SB = .994 LSB = .984 SY = .907 LYC = .912	GB = .992 PB = .967 SB = .972 LSB= .986 SY = .993 LYC= .995	GB = .976 PB = .976	GB = .982 PB = .961	GB = .941 PB = .920 SB = .983 LSB= .984	GB = .884 PB = .925 SB = .993

$E = 10.5 \times 10^6$ $\nu = .33$ $\sigma_y = 50,000$

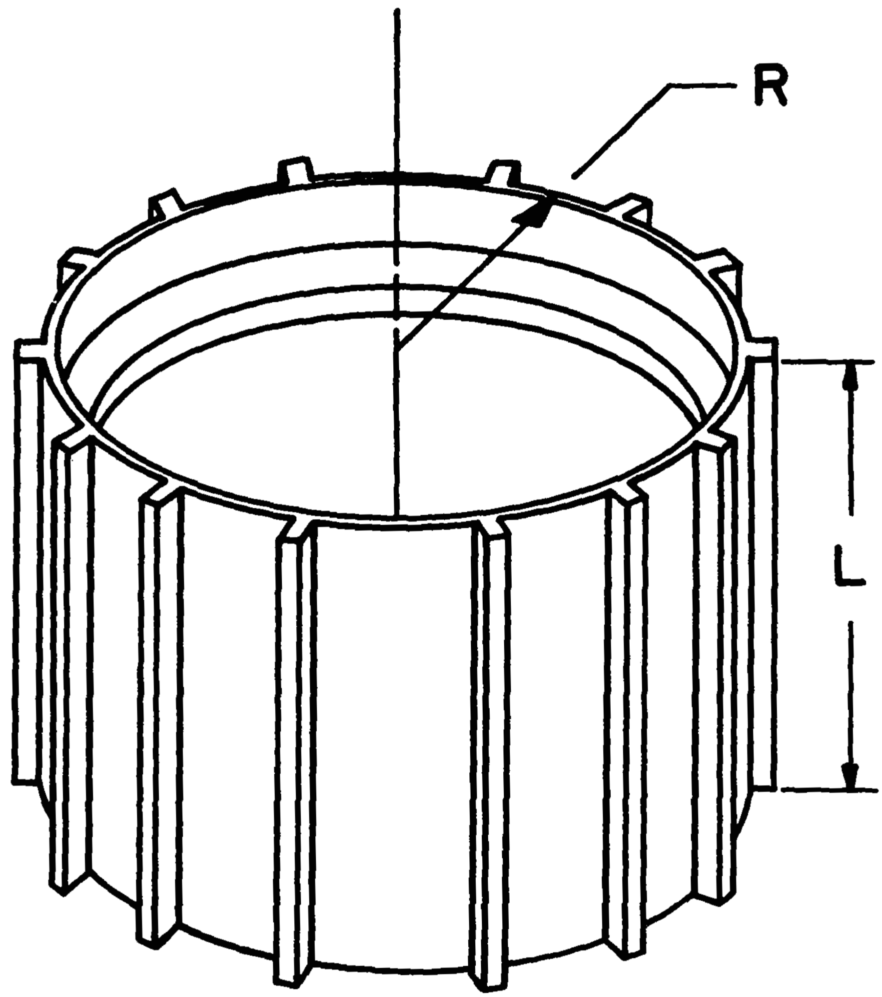

Figure 1 An Integrally Stiffened Cylinder

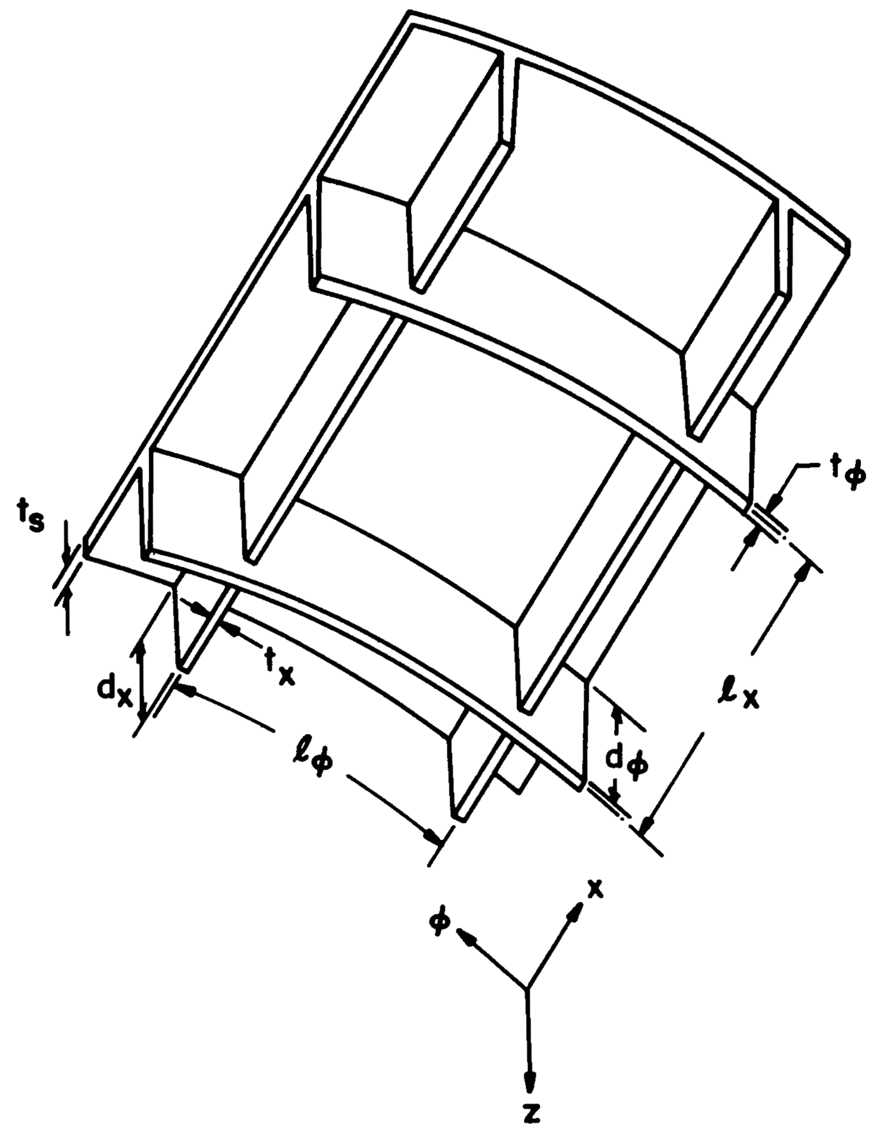

Figure 2 An Element of a Stiffened Cylinder

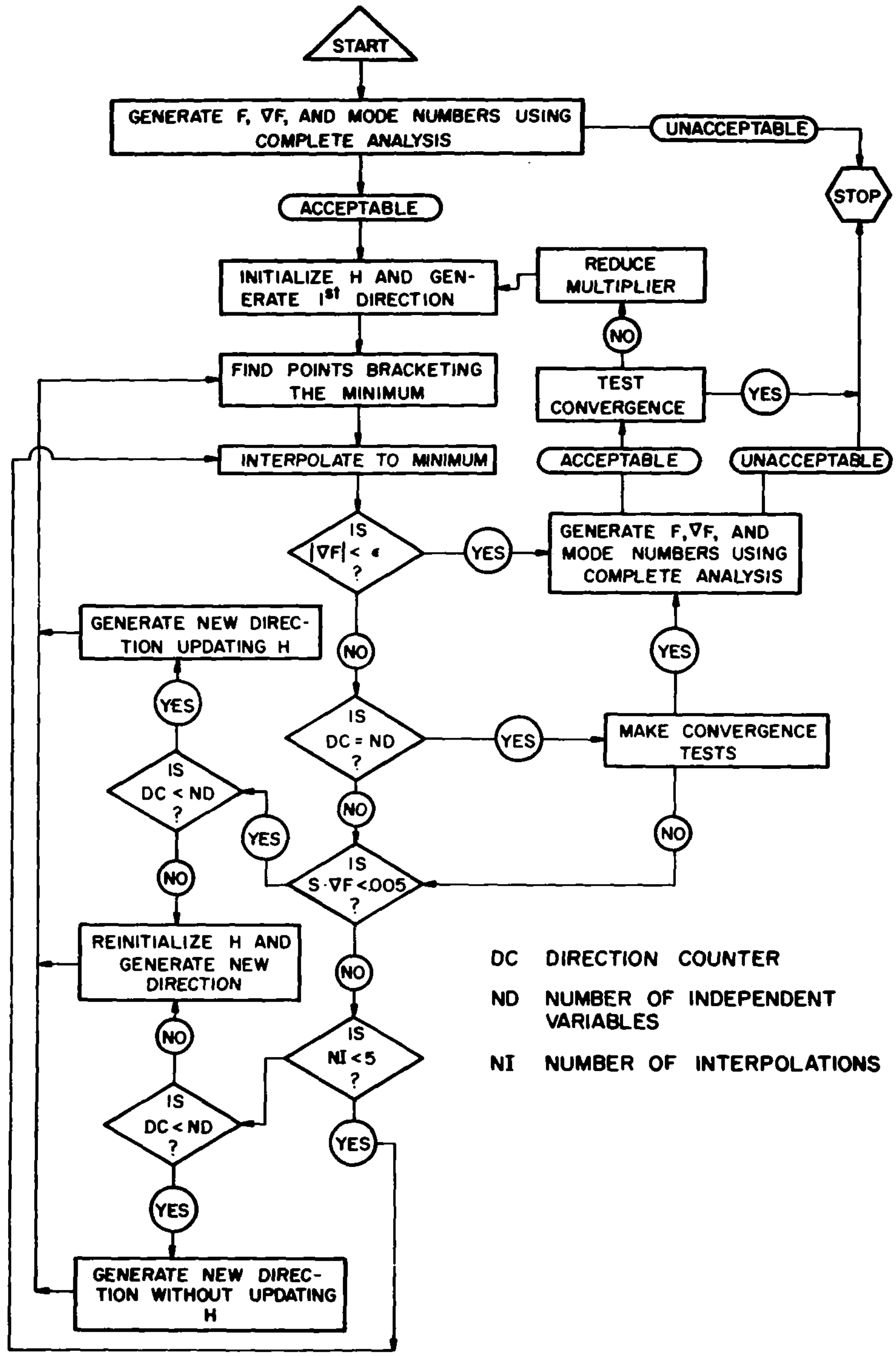

Figure 3 Synthesis Scheme

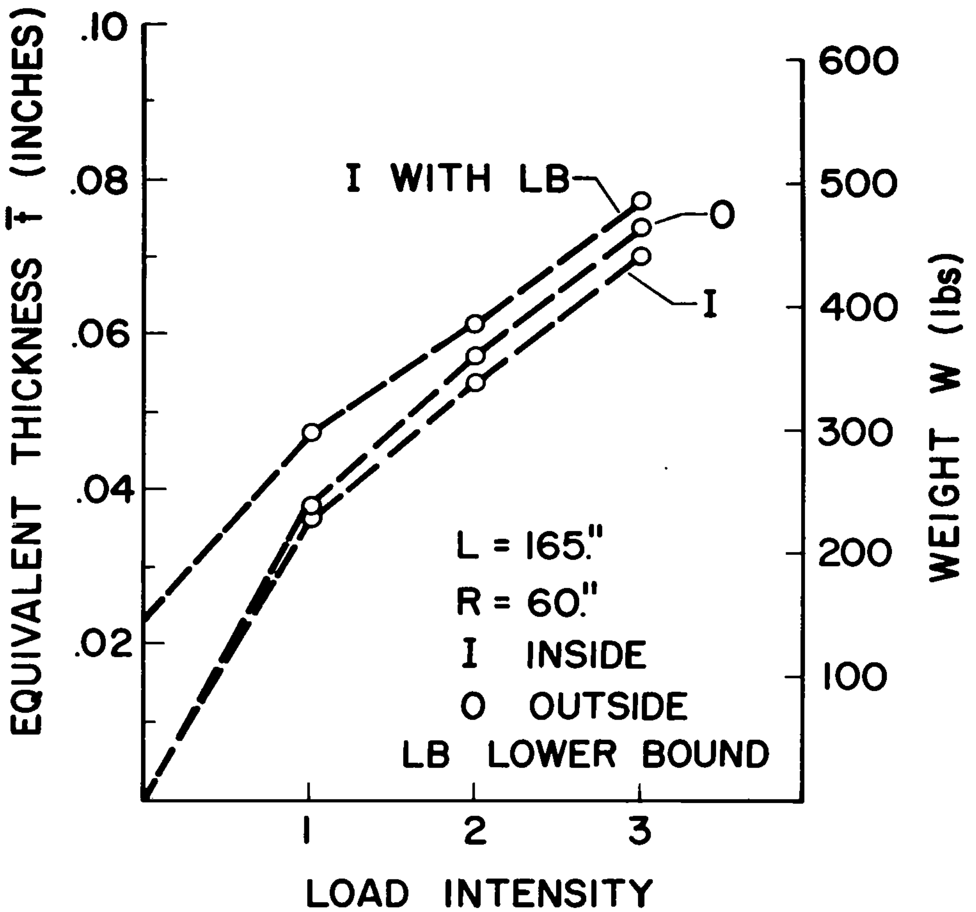

Figure 4 Weight Increase with Load Increase

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APPENDIX A

Development of the Analysis of the Stiffened Cylinder

A.1 Introduction

In this appendix all the equations needed to analyze the stiffened cylinder are presented. These include the overall buckling analysis of the cylinder as well as the buckling, stress and yield analyses of the skin and stiffeners.

It is well known that there is a large discrepancy between the buckling failure loads for monocoque cylinders which are predicted by classical buckling theory and the failure loads obtained in tests. However, it has been found recently that this is not necessarily the case for stiffened cylinders.[11] Linear theory is used here but it has been found that this may not apply in some cases.[12] The importance of including the effect of eccentricity of the stiffeners has been pointed out both experimentally[13] and analytically.[14,15,16] Earlier investigators have also treated this effect analytically.[17,18] In the analysis used here eccentricity effects are included. This analysis follows closely that of Flügge.[19]

A.2 Stress-Strain Relations

The skin of the cylinder is assumed to be in a biaxial state of stress. The axes of elastic symmetry are in the longitudinal and circumferential directions. The x axis is in the longitudinal direction and the ϕ axis is in the circumferential direction. With these assumptions the stress-strain relations in the sheet are

$$\sigma_x = \frac{E_x}{1-\mu_x\mu_\phi} (\epsilon_x + \mu_\phi \epsilon_\phi) \qquad \text{(A1)}$$

$$\sigma_\phi = \frac{E_\phi}{1-\mu_x\mu_\phi} (\mu_x \epsilon_x + \epsilon_\phi)$$

$$\tau_{x\phi} = G \gamma_{x\phi}$$

The stiffeners are assumed to be in a uniaxial state of stress so that the stress-strain relations are

$$\sigma_{xs} = E_{xs}\,\varepsilon_x \tag{A2}$$

$$\sigma_{\phi s} = E_{\phi s}\,\varepsilon_\phi$$

in the longitudinal and circumferential stiffeners respectively.

A.3 Strain-Displacement Relations

The reference surface of the shell is taken as the midsurface of the skin. With the z axis taken positive inward from the reference surface and u, v, and w being the displacements of the reference surface respectively in the positive x, ϕ, and z coordinate directions, the strain displacement relations are taken to be

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\,\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_\phi = \frac{1}{R}\frac{\partial v}{\partial \phi} - \frac{w}{R-z} - \frac{z}{R(R-z)}\,\frac{\partial^2 w}{\partial \phi^2} \tag{A3}$$

$$\gamma_{x\phi} = \frac{1}{R-z}\,\frac{\partial u}{\partial \phi} + \left(\frac{R-z}{R}\right)\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \phi}\left(\frac{z}{R} + \frac{z}{R-z}\right)$$

where ε_x, ε_ϕ, and $\gamma_{x\phi}$ are the strains at a point in the shell. ε_x and ε_ϕ are assumed to be continuous in the skin and x and ϕ stiffeners respectively. These relations may be derived in a geometric manner as done by Flügge[20] or by reducing the linear three dimensional strain displacement relations in cylindrical coordinates.[21] The later is done by assuming the displacements vary linearly with the depth of the shell[22] and by setting the transverse shear strains and the extensional strain in the z direction to zero.

The displacements of a point in the cylinder corresponding to these strain midsurface displacements are

$$\tilde{u} = u - z\frac{\partial w}{\partial x}$$

$$\tilde{v} = \frac{R-z}{R} v - \frac{z}{R}\frac{\partial w}{\partial \phi} \qquad \text{(A4)}$$

$$\tilde{w} = w$$

(see Figure A1).

The rotations of the normal used in the above displacements are

$$\omega_x = \frac{v}{R} + \frac{1}{R}\frac{\partial w}{\partial \phi}$$

$$\omega_\phi = \frac{\partial w}{\partial x}$$

The relative rotations per unit length are then

$$\theta_x = \frac{\partial \omega_x}{\partial x} = \frac{1}{R}\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \phi}\right)$$

$$\qquad \text{(A5)}$$

$$\theta_\phi = \frac{1}{R}\frac{\partial \omega_\phi}{\partial \phi} = \frac{1}{R}\frac{\partial^2 w}{\partial x \partial \phi}$$

A.4 Force Resultants

The force and moment resultants per unit length are obtained by performing the appropriate integrations of the stresses over the thickness of the skin and then adding to these the corresponding force and moment resultants per unit length in the stiffeners. The force and moment resultants per unit length in the stiffeners are obtained by dividing the resultant forces and moments by the stiffener spacings.

The extensional forces and bending moments in the stiffeners are obtained by performing the appropriate integrations of the extensional stresses in the stiffeners over the areas of the stiffeners. The stiffeners are assumed to carry no shear load; so they have no contribution to the shear force resultants, but they are assumed to have a twisting moment resultant. The angle of twist is assumed to be the same as that of the normal to the skin. The torsional stiffnesses of the stiffeners are obtained from an approximate curve for data given by Crandal and Dahl[23] for torsion of bars of rectangular cross section. Thus, the force resultants are obtained by substituting the strain displacement relations (A3) into the stress strain relations (A1) and (A2), then substituting the resulting stress displacement relations into the following formulas, and performing the integrations:

$$N_x = \int_{-t_s/2}^{+t_s/2} \left(\frac{R-z}{R}\right) \sigma_x \, dz + \frac{t_x}{\ell_\phi} \int_{t_s/2}^{d_x+t_s/2} \sigma_{xs} \, dz$$

$$N_\phi = \int_{-t_s/2}^{+t_s/2} \sigma_\phi \, dz + \frac{t_\phi}{\ell_x} \int_{t_s/2}^{d_\phi+t_s/2} \sigma_{\phi s} \, dz$$

$$N_{x\phi} = \int_{-t_s/2}^{+t_s/2} \tau_{x\phi} \left(\frac{R-z}{R}\right) dz \tag{A6}$$

$$N_{\phi x} = \int_{-t_s/2}^{+t_s/2} \tau_{x\phi} \, dz$$

$$M_x = \int_{-t_s/2}^{+t_s/2} \sigma_x \left(\frac{R-z}{R}\right) z\, dz + \frac{t_x}{\ell_\phi} \int_{t_s/2}^{d_x+t_s/2} \sigma_{xs}\, z\, dz$$

$$M_\phi = \int_{-t_s/2}^{+t_s/2} \sigma_\phi\, z\, dz + \frac{t_\phi}{\ell_x} \int_{t_s/2}^{d_\phi+t_s/2} \sigma_{\phi s}\, z\, dz$$

$$M_{x\phi} = \int_{-t_s/2}^{+t_s/2} \tau_{x\phi} \left(\frac{R-z}{R}\right) z\, dz - \frac{G_x J_x}{\ell_\phi} \theta_x$$

$$M_{\phi x} = \int_{-t_s/2}^{+t_s/2} \tau_{x\phi}\, z\, dz - \frac{G_\phi J_\phi}{\ell_x} \theta_\phi$$

The above expressions apply for stiffeners on the inside of the cylinder. For stiffeners on the outside of the cylinder the limits of integration on the stiffener integrals, the second terms, must be changed to go from $-(d_x + t_s/2)$ to $-t_s/2$ and $-(d_\phi + t_s/2)$ to $-t_s/2$ (see Figures 2 and A2). θ_x and θ_ϕ are the angles of twist of the normal to the skin, given in section A.3. J_x and J_ϕ are the section constants for a rectangular cross-section in torsion. These correspond to a polar moment of inertia and are approximate by the expression,

$$J = c\, ab^3 \qquad b \leq a$$

where c is given by

$$c = -0.285\, e^{-0.49(a/b)} + 0.316$$

and a and b are the cross sectional dimensions of the stiffener, t_x and d_x, and t_ϕ and d_ϕ; b is taken as the dimension of smaller magnitude. After making the substitutions described above, performing the integrations, and neglecting terms of the order of the thickness of the skin divided by the radius and square of the depth of the stiffeners divided by the square of the radius with respect to 1, the force and moment resultants can be written:

$$N_x = (H_{s1} + H_x) \frac{\partial u}{\partial x} + H_\nu \frac{1}{R} \frac{\partial v}{\partial \phi} - H_\nu \frac{w}{R} - (H_x e_x - \frac{D_1}{R}) \frac{\partial^2 w}{\partial x^2}$$

$$N_\phi = H_\nu \frac{\partial u}{\partial x} + (H_{s2} + H_\phi) \frac{1}{R} \frac{\partial v}{\partial \phi} - (H_{s2} + H_\phi (1 + \frac{e_\phi}{R})) \frac{w}{R}$$

$$- (\frac{D_2}{R} + H_\phi (e_\phi + \frac{\rho_\phi^2}{R})) \frac{1}{R^2} \frac{\partial^2 w}{\partial \phi^2}$$

$$N_{x\phi} = \frac{S}{R} \frac{\partial u}{\partial \phi} + S \frac{\partial v}{\partial x} + \frac{K}{R^2} \frac{\partial^2 w}{\partial x \partial \phi} \qquad \text{(A7)}$$

$$N_{\phi x} = \frac{S}{R} \frac{\partial u}{\partial \phi} + S \frac{\partial v}{\partial x} - \frac{K}{R^2} \frac{\partial^2 w}{\partial x \partial \phi}$$

$$M_x = (H_x e_x - \frac{D_1}{R}) \frac{\partial u}{\partial x} - \frac{D_\nu}{R^2} \frac{\partial v}{\partial \phi}$$

$$- (D_1 + H_x \rho_x^2) \frac{\partial^2 w}{\partial x^2} - \frac{D_\nu}{R^2} \frac{\partial^2 w}{\partial \phi^2}$$

$$M_\phi = H_\phi \frac{e_\phi}{R} \frac{\partial v}{\partial \phi} - (\frac{D_2}{R} + H_\phi (e_\phi + \frac{\rho_\phi^2}{R})) \frac{w}{R}$$

$$- D_\nu \frac{\partial^2 w}{\partial x^2} - (D_2 + H_\phi (\rho_\phi^2 + \frac{\alpha_\phi^3}{R})) \frac{1}{R^2} \frac{\partial^2 w}{\partial \phi^2} .$$

$$M_{x\phi} = -\left(\frac{2K}{R} + \frac{T_x}{R}\right)\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \phi}\right)$$

$$M_{\phi x} = \frac{K}{R^2}\frac{\partial u}{\partial \phi} - \frac{K}{R}\frac{\partial v}{\partial x} - \left(\frac{2K}{R} + \frac{T_\phi}{R}\right)\frac{\partial^2 w}{\partial x \partial \phi}$$

where the constants in the above expressions are given in terms of the material properties and the dimensions of the stiffened cylinder by

$$H_{s1} = \frac{E_x\, t_s}{1-\mu_x\mu_\phi} \qquad H_{s2} = \frac{E_\phi\, t_s}{1-\mu_x\mu_\phi}$$

$$H_x = \frac{E_{xs}\, t_x\, |d_x|}{\ell_\phi} \qquad H_\phi = \frac{E_{\phi s}\, t_\phi\, |d_\phi|}{\ell_x}$$

$$D_1 = \frac{E_x\, t_s^3}{12(1-\mu_x\mu_\phi)} \qquad D_2 = \frac{E_\phi\, t_s^3}{12(1-\mu_x\mu_\phi)}$$

$$D_\nu = \frac{E_x\, \mu_\phi\, t_s^3}{12(1-\mu_x\mu_\phi)} = \frac{E_\phi\, \mu_x\, t_s^3}{12(1-\mu_x\mu_\phi)}$$

$$H_\nu = \frac{E_x\, \mu_\phi\, t_s}{1-\mu_x\mu_\phi} = \frac{E_\phi\, \mu_x\, t_s}{1-\mu_x\mu_\phi}$$

$$S = Gt_s \qquad K = \frac{Gt_s^3}{12}$$

$$T_x = \frac{G_x\, J_x}{\ell_\phi} \qquad T_\phi = \frac{G_\phi\, J_\phi}{\ell_x} \tag{A8}$$

$$\rho_x^2 = \frac{4d_x^2 + 6\,|d_x|\, t_s + 3t_s^2}{12}$$

$$\rho_\phi^2 = \frac{4d_\phi^2 + 6\,|d_\phi|\, t_s + 3t_s^2}{12}$$

$$e_x = \pm \frac{|d_x| + t_s}{2} \qquad \begin{array}{l} + \text{ inside} \\ - \text{ outside} \end{array}$$

$$e_\phi = \pm \frac{|d_\phi| + t_s}{2}$$

$$\alpha_\phi^{\;3} = \pm \frac{2\,|d_\phi|^3 + 4t_s\, d_\phi^{\;2} + 3t_s^{\;2}\,|d_\phi| + t_s^{\;3}}{8}$$

The effects of the eccentricity of the stiffeners are seen in the terms e_x, e_ϕ, and $\alpha_\phi^{\;3}$, which have a positive sign when the stiffeners are on the inside and a negative sign when the stiffeners are on the outside.

A.5 Prebuckle Forces and Stresses

It is assumed that when the cylinder is loaded there is a uniform change in length and a uniform change in radius. This implies that u, v, and w are independent of ϕ; w and v are independent of x; and that u is a linear function of x. Applying these assumptions to the force displacement relations (A.7), the forces in the cylinder are

$$N_x = (H_{s1} + H_x)\frac{\partial u}{\partial x} - H_\nu \frac{w}{R}$$

$$N_\phi = H_\nu \frac{\partial u}{\partial x} - \left(H_{s2} + H_\phi\left(1 + \frac{e_\phi}{R}\right)\right)\frac{w}{R} \qquad \text{(A9)}$$

$$M_x = \left(H_x\, e_x - \frac{D_1}{R}\right)\frac{\partial u}{\partial x}$$

$$M_\phi = -\left(\frac{D_2}{R} + H_\phi\left(e_\phi + \frac{\rho_\phi^{\;2}}{R}\right)\right)\frac{w}{R}$$

$$N_{x\phi} = N_{\phi x} = M_{x\phi} = M_{\phi x} = 0$$

By substituting these into the equilibrium equations[24] the internal forces may be obtained in terms of the applied loads. The result is

$$N_x = -N$$
$$N_\phi = -pR \tag{A10}$$

where N is the applied axial compression load per unit length of circumference, and p is the applied external pressure per unit surface area.

With the assumptions about the prebuckled deformation, the midsurface prebuckle strains are obtained from the strain displacement relations as

$$\varepsilon_{xp} = \frac{\partial u}{\partial x}, \qquad \varepsilon_{\phi p} = -\frac{w}{R} \tag{A11}$$

By equating the expressions for the force resultants in terms of the displacements with the values of the force resultants in terms of the external forces and identifying the strains, the following expressions for the midsurface strains are obtained, after neglecting terms of the order of the depth of the stiffener divided by the radius with respect to one:

$$\varepsilon_{xp} = \frac{H_\nu \, pR - (H_{s2} + H_\phi)\, N}{(H_{s1} + H_x)(H_{s2} + H_\phi) - H_\nu^2}$$
$$\tag{A12}$$
$$\varepsilon_{\phi p} = \frac{H_\nu \, N - (H_{s1} + H_x)\, pR}{(H_{s1} + H_x)(H_{s2} + H_\phi) - H_\nu^2}$$

Substituting these into the stress-strain relations (A.1) for the skin, the expressions for the stresses in the skin are obtained:

$$\sigma_{xp} = -\frac{1}{t_s} \frac{\left[\left(1 + \frac{H_\phi}{H_{s2}}\right) - \mu_x \mu_\phi\right] N + \mu_x \frac{H_x}{H_{s1}} pR}{\left(1 + \frac{H_x}{H_{s1}}\right)\left(1 + \frac{H_\phi}{H_{s2}}\right) - \mu_x \mu_\phi} \tag{A13}$$

$$\sigma_{\phi p} = -\frac{1}{t_s} \frac{\mu_\phi \frac{H_\phi}{H_{s2}} N - [\mu_x \mu_\phi - (1 + \frac{H_x}{H_{s1}})] pR}{(1 + \frac{H_x}{H_{s1}})(1 + \frac{H_\phi}{H_{s2}}) - \mu_x \mu_\phi}$$

The expressions for the stresses in the ribs neglecting terms involving the depth of the stiffener divided by the radius with respect to one are obtained by multiplying the stiffener modulus by the corresponding value of the midsurface prebuckle strain. These are

$$\sigma_{xsp} = E_{xs} \varepsilon_{xp}$$

$$\sigma_{\phi sp} = E_{\phi s} \varepsilon_{\phi p} \tag{A14}$$

A.6 Buckling of the Cylinder and Skin

An expression for the critical buckling load of the cylinder is obtained in terms of two integer parameters representing the buckling mode shape. The lowest buckling load is then obtained by searching the buckling loads obtained from a large number of possible mode shapes.

The expression for the buckling load is obtained from the determinant of a set of homogeneous equations. These are obtained by substituting into the buckling equilibrium equations, in terms of displacements, an assumed solution which satisfies these equations and simple support boundary conditions. The displacement functions contain the two integer parameters representing the mode shape and arbitrary constants.

The buckling equilibrium equations are obtained in terms of displacements, by substituting the force resultants, in terms of the displacements, into the buckling equilibrium equations in terms of forces. The buckling equilibrium equations used are those given by Flügge[25] but contain only the buckling force terms recommended by Hedgepeth and Hall.[26] With the changes required because of the different coordinate system used here these equations are

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi} = 0$$

$$\frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} - \frac{1}{R^2}\frac{\partial M_\phi}{\partial \phi} - \frac{1}{R}\frac{\partial M_{x\phi}}{\partial x} - N\frac{\partial^2 v}{\partial x^2} = 0 \qquad \text{(A15)}$$

$$\frac{\partial^2 M_\phi}{\partial \phi^2} + R\frac{\partial^2 M_{x\phi}}{\partial x \partial \phi} + R\frac{\partial^2 M_{\phi x}}{\partial x\, \partial \phi} + R^2\frac{\partial^2 M_x}{\partial x^2} + R N_\phi$$

$$- NR^2\frac{\partial^2 w}{\partial x^2} - pR\left(\frac{\partial^2 w}{\partial \phi^2} + w\right) = 0$$

where N is the applied axial compressive force per unit length and p is the applied external radial pressure per unit area.

After substituting the force displacement relations (A7) into these equations, the buckling equilibrium equations in terms of displacement are obtained. These can be written in the form:

$$R\left(\frac{H_{s1}}{H_{s2}} + \frac{H_x}{H_{s2}}\right)\frac{\partial^2 u}{\partial x^2} + \frac{S}{H_{s2}}\frac{1}{R}\frac{\partial^2 u}{\partial \phi^2} + \left(\frac{H_\nu}{H_{s2}} + \frac{S}{H_{s2}}\right)\frac{\partial^2 v}{\partial x \partial \phi}$$

$$- \frac{H_\nu}{H_{s2}}\frac{\partial w}{\partial x} - R\left(\frac{H_x}{H_{s2}}e_x - \frac{D_1}{H_{s2}R}\right)\frac{\partial^3 w}{\partial x^3} - \frac{K}{H_{s2}}\frac{1}{R^2}\frac{\partial^3 w}{\partial x \partial \phi^2} = 0$$

$$\left(\frac{H_\nu}{H_{s2}} + \frac{S}{H_{s2}}\right)\frac{\partial^2 u}{\partial x \partial \phi} + \left(1 + \frac{H_\phi}{H_{s2}}\left(1 - \frac{e_\phi}{R}\right)\right)\frac{1}{R}\frac{\partial^2 v}{\partial \phi^2}$$

$$\text{(A16)}$$

$$+ \left(\frac{SR}{H_{s2}} + \frac{T_x}{H_{s2}R}\right)\frac{\partial^2 v}{\partial x^2} - \frac{NR}{H_{s2}}\frac{\partial^2 v}{\partial x^2} - \left(1 + \frac{H_\phi}{H_{s2}}\right)\frac{1}{R}\frac{\partial w}{\partial \phi}$$

$$- \left(\frac{H_\phi e_\phi}{H_{s2}}\right)\frac{1}{R^2}\frac{\partial^3 w}{\partial \phi^3} + \left(\frac{3K}{H_{s2}} + \frac{D_\nu}{H_{s2}} + \frac{T_x}{H_{s2}}\right)\frac{1}{R}\frac{\partial^3 w}{\partial x^2 \partial \phi} = 0$$

$$\frac{H_\nu}{H_{s2}} \frac{\partial u}{\partial x} + \left(\frac{H_x e_x}{H_{s2}} - \frac{D_2}{H_{s2}R}\right) R \frac{\partial^3 u}{\partial x^3} + \frac{K}{H_{s2}R^2} \frac{\partial^3 u}{\partial x \partial \phi^2}$$

$$(1 + \frac{H_\phi}{H_{s2}}) \frac{1}{R} \frac{\partial v}{\partial \phi} - \left(\frac{3K}{H} + \frac{D_\nu}{H_{s2}} + \frac{T_x}{H_{s2}}\right) \frac{1}{R} \frac{\partial^3 v}{\partial x^2 \partial \phi} + \frac{H_\phi e_\phi}{H_{s2}} \frac{1}{R^2} \frac{\partial^3 v}{\partial \phi^3}$$

$$- 2 \left(\frac{D_2}{H_{s2}R} + \frac{H_\phi}{H_{s2}} \left(e_\phi + \frac{\rho_\phi^2}{R}\right)\right) \frac{1}{R^3} \frac{\partial^2 w}{\partial \phi^2}$$

$$- \left(\frac{2D_\nu}{H_{s2}} + \frac{4K}{H_{s2}} + \frac{T_x}{H_{s2}} + \frac{T_\phi}{H_{s2}}\right) \frac{1}{R} \frac{\partial^4 w}{\partial x^2 \partial \phi^2}$$

$$- \left(\frac{D_2}{H_{s2}} + \frac{H_\phi}{H_{s2}} \left(\rho_\phi^2 + \frac{\alpha_\phi^3}{R}\right)\right) \frac{1}{R^2} \frac{\partial^4 w}{\partial \phi^4} - \left(\frac{D_1}{H_{s2}} + \frac{H_x \rho_x^2}{H_{s2}}\right) R \frac{\partial^4 w}{\partial x^4}$$

$$- \left(1 + \frac{H_\phi}{H_{s2}} \left(1 + \frac{e_\phi}{R}\right)\right) \frac{w}{R} - \frac{NR}{H_{s2}} \frac{\partial^2 w}{\partial x^2} - \frac{p}{H_{s2}} \left(\frac{\partial^2 w}{\partial \phi^2} + w\right) = 0$$

The assumed displacements which satisfy the above equations and the simple support boundary conditions are

$$u = A \sin \eta\phi \cos \lambda x$$

$$v = B \cos \eta\phi \sin \lambda x$$

$$w = C \sin \eta\phi \sin \lambda x$$

where for the complete cylinder

$$\lambda = \frac{m\pi}{L}, \qquad m = 1, 2, \ldots$$

$$\eta = n \qquad n = 0, 1, 2, \ldots$$

and L is the length of the cylinder.

For a cylindrical plate (the skin between stiffeners)

$$\lambda = \frac{m\pi}{\ell_x} \qquad m = 1, 2, \ldots$$

$$\eta = \frac{n\pi R}{\ell_\phi} \qquad n = 1, 2, \ldots$$

After the displacements are substituted into the displacement buckling equilibrium equations, these equations can be written in the form:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} + \frac{NR\lambda^2}{H_{s2}} & C_{23} \\ C_{31} & C_{23} & C_{33} + \frac{NR\lambda^2}{H_{s2}} + \frac{p}{H_{s2}}(\eta^2 - 1) \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(A18)}$$

where the C's are given by

$$C_{11} = - R \left(\frac{H_{s1}}{H_{s2}} + \frac{H_x}{H_{s2}}\right) \lambda^2 - \frac{S}{H_{s2}} \frac{\eta^2}{R}$$

$$C_{12} = - \left(\frac{H_\nu}{H_{s2}} + \frac{S}{H_{s2}}\right) \eta \lambda$$

$$C_{13} = - \frac{H_\nu}{H_{s2}} \lambda + R \left(\frac{H_x e_x}{H_{s2}} - \frac{D_1}{H_{s2} R}\right) \lambda^3 + \frac{K}{H_{s2}} \frac{\lambda \eta^2}{R^2}$$

$$C_{31} = - \frac{H_\nu}{H_{s2}} \lambda + R \left(\frac{H_x e_x}{H_{s2}} - \frac{D_2}{H_{s2} R}\right) \lambda^3 + \frac{K}{H_{s2}} \frac{\lambda \eta^2}{R^2} \qquad \text{(A19)}$$

$$C_{22} = - \left(1 + \frac{H_\phi}{H_{s2}} \left(1 - \frac{e_\phi}{R}\right)\right) \eta^2 - \left(\frac{SR}{H_{s2}} + \frac{T_x}{H_{s2} R}\right) \lambda^2$$

$$C_{23} = -\left(1 + \frac{H_\phi}{H_{s2}}\right)\frac{n}{R} + \left(\frac{H_\phi e_\phi}{H_{s2}}\right)\frac{n^3}{R^2} - \left(\frac{3K}{H_{s2}} + \frac{D_\nu}{H_{s2}} + \frac{T_x}{H_{s2}}\right)\frac{\lambda^2 n}{R}$$

$$C_{33} = 2\left(\frac{D_2}{H_{s2}R} + \frac{H_\phi}{H_{s2}}\left(e_\phi + \frac{\rho_\phi^2}{R}\right)\right)\frac{n^2}{R^2}$$

$$- \left(2\frac{D_\nu}{H_{s2}} + \frac{4K}{H_{s2}} + \frac{T_x}{H_{s2}} + \frac{T_\phi}{H_{s2}}\right)\lambda^2 \frac{n^2}{R}$$

$$-\left(\frac{D_2}{H_{s2}} + \frac{H_\phi}{H_{s2}}\left(\rho_\phi^2 + \frac{\alpha_\phi^3}{R}\right)\right)\frac{n^4}{R^3} - \left(\frac{D_1}{H_{s2}} + \frac{H_x \rho_x^2}{H_{s2}}\right) R\,\lambda^4$$

$$- \frac{1}{R}\left(1 + \frac{H_\phi}{H_{s2}}\left(1 + \frac{e_\phi}{R}\right)\right)$$

Since the values of the applied loads are known, a ratio between the axial load and pressure can be calculated. Letting

$$p = \alpha N$$

and setting to zero the determinant of the coefficients of A, B, and C in the last set of equations the expression for the critical axial load is obtained. This is

$$\left(\frac{N}{H_{s2}}\right)_{cr} = \frac{-\overline{B} \pm \sqrt{\overline{B}^2 - 4\,\overline{A}\,\overline{C}}}{2\overline{A}} \tag{A20}$$

where

$$\overline{A} = C_{11}\left(R\,\lambda^4 + \lambda^2 (n^2 - 1)\,\alpha\right) R$$

$$\overline{B} = [(C_{11}\,C_{22} - C_{12}^2) + (C_{11}\,C_{33} - C_{13}\,C_{31})]\,R\,\lambda^2$$

$$+ (C_{11}\,C_{22} - C_{12}^2)\,\alpha\,(n^2 - 1) \tag{A21}$$

$$\overline{C} = C_{11} C_{22} C_{33} + C_{12} C_{23} C_{31} + C_{13} C_{12} C_{23}$$
$$- C_{12}^2 C_{33} - C_{13} C_{31} C_{22} - C_{23}^2 C_{11}$$

For each combination of the parameters m and n there are two possible values of $(N/H_{s2})_{cr}$. The one which has to be used as critical is the one with smallest magnitude and having the same sign as the applied load N. The critical buckling load is obtained by finding $(N/H_{s2})_{cr}$ for a large number of values of both m and n and then selecting the lowest magnitude value out of all of these.

For the special case when N = 0 the critical pressure must be found. This is given by

$$\left(\frac{p}{H_{s2}}\right)_{cr} = \frac{-\overline{C}}{(C_{11} C_{22} - C_{12}^2)(\eta^2 - 1)} \tag{A22}$$

The above analysis is used for gross buckling, panel buckling, and sheet buckling. For gross buckling all the constants are calculate as given in the C's and the full length of the cylinder is used. For panel buckling, the terms which contain the properties of the circumferential stiffeners are set to zero and the length of the cylinder is taken as the circumferential ring spacing. For skin buckling, all terms containing stiffener properties are set to zero, η is changed to apply to a cylindrical plate with a width of the longitudinal stiffener spacing and the length of the cylinder is again taken as the length between circumferential stiffeners.

A.7 Longitudinal Stiffener Buckling

The critical buckling stress for the longitudinal stiffeners is obtained by applying a solution for the critical buckling stress of a flat rectangular plate to several different possible assumed modes of buckling of the stiffener. In all the possible assumed modes the longitudinal stiffener is assumed to be simply supported on three edges and free on the fourth. The critical buckling stress for such a flat plate is given by Bleich[27] as

$$\sigma_c = \frac{\pi^2 E_{xs}}{12(1-\nu^2)} \left(\frac{t}{d}\right)^2 \left[\left(\frac{d}{\ell}\right)^2 + 0.425\right] \qquad \text{(A23)}$$

The notation has been changed here; t is the thickness of tne plate (i.e. the width of the stiffener), d the width of the plate (i.e. the depth of portion of the stiffener under consideration), and ℓ the length of stiffener under consideration.

The first failure mode to which this expression is applied is in the situation where the circumferential stiffeners are either on the opposite side of the cylinder from the longitudinal ones or where they are non-existent. In this case d is taken as d_x, the full depth of the stiffener, and ℓ is taken as L, the full length of the cylinder.

The second mode is where the circumferential stiffeners are on the same side of the cylinder as the longitudinal ones and are the deepest. In this case the critical buckling stress of the longitudinals is taken as that of a plate with depth d_x, the full depth of the longitudinal stiffeners, and a length ℓ_x, the length between circumferential stiffeners.

The third mode is where the circumferential stiffeners are on the same side as the longitudinal ones but are not as deep. In this case one would expect the stiffener to buckle in a manner coupling the material between the circumferential stiffeners with the material above the circumferential stiffeners. To obtain an estimation of the critical buckling stress two cases are considered. One assumes that the portion of the material between the circumferential stiffeners does not buckle but the outstanding portion does. In this case the formula is applied to a plate of the dimensions of the depth of the outstanding portion and the length of the entire cylinder ($d_x - d_\phi$ by L). The other case assumes that the material between the circumferential stiffeners does buckle with the outstanding portion of the longitudinal stiffener but that the circumferential stiffeners force nodes in the buckling of the longitudinal and these notes occur at the location of the circumferential stiffeners. The buckling stress in this case is taken as that for a plate and d equal to the full depth, d_x, of the stiffener and ℓ equal to ℓ_x, the circumferential stiffener spacing. This is the same as the case where the circumferential stiffeners are deeper than the longitudinals.

A.8 Circumferential Stiffener Buckling

Similar situations are encountered with the buckling of the circumferential stiffeners as with the buckling of the longitudinal stiffeners. Here, however, an additional mode of buckling is encountered (see Tables A1 and A2). The external stiffeners not only can buckle when they are compressed, but due to their curvature can also buckle when they are expanded. An expression for the critical circumferential strain in the skin of the cylinder, or at the edge of the stiffener, is obtained (in Section A.9) by doing an assumed mode solution of the buckling problem. This expression is

$$\varepsilon_{\phi cr} = -\left(\frac{t_\phi}{d}\right)^2 \left(\frac{1}{12(1-\nu^2)}\right) \left(\frac{2 + 2(1-\nu)\ (\zeta + \zeta^2/2)}{1 + 2\zeta + \zeta^2}\right) \times$$

$$\left[\frac{(1 + 2n^2(1-\nu))\ \zeta + (2n^2(2-\nu)-1)\ \frac{\zeta^2}{2} + (n^2 - 1)^2\ \zeta^3/3 + \dots}{1 + \left(\frac{2n^2 - 1}{3}\right) \zeta + \left(\frac{3-4n^2}{12}\right) \zeta^2 + \dots} \right] \tag{A24}$$

where d is the depth of the stiffener portion in question; and ζ is the ratio of the stiffener depth, d, to the radius of the unsupported edge of the stiffener; ζ is a positive number if the stiffener is inside and negative if the stiffener is outside; and n is the number of full waves in the circumferential direction.

With the circumferential stiffeners on the inside of the cylinder, ζ positive, the value of $\varepsilon_{\phi cr}$ is negative for all values of n and increases in magnitude as n increases. This means that inside circumferential stiffeners can buckle only when the cylinder contracts under load.

With the circumferential stiffeners inside the cylinder and the longitudinals outside or non-existent the critical buckling value for $\varepsilon_{\phi cr}$ is obtained with n = 0.

With both the circumferential and the longitudinal stiffeners inside the cylinder and with the longitudinal stiffeners deeper than the circumferential ones, the circumferential stiffeners are physically restrained from buckling into a smaller number of half waves than the number of spaces between longitudinal stiffeners. Since $\varepsilon_{\phi cr}$ increases in magnitude as n increases, the critical buckling value for this situation is obtained by using for n the number of spaces in half the circumference of the cylinder.

In the situation with the circumferential stiffeners deeper than the longitudinal ones two values of $\varepsilon_{\phi cr}$ are obtained. One is for the unsupported portion of the circumferential with $d = d_\phi - d_x$ and $n = 0$. The other is obtained as above, for the supported stiffeners, for the full depth of the stiffener assuming that nodes are forced at the locations of the longitudinal stiffeners. This is similar to the case of the longitudinal stiffeners.

With the circumferential stiffeners outside, ζ negative, $\varepsilon_{\phi cr}$ is positive for small values of n and increases in magnitude as n increases. When n becomes large enough $\varepsilon_{\phi cr}$ becomes negative and then as n increases the magnitude of $\varepsilon_{\phi cr}$ decreases while the value remains negative. The magnitude of $\varepsilon_{\phi cr}$ decreases until for some value of n a minimum is obtained. Thus the circumferential stiffener can buckle for small values of n when the cylinder expands, $\varepsilon_{\phi cr}$ positive, or can buckle for large values of n when the cylinder contracts, $\varepsilon_{\phi cr}$ negative.

For the case of external circumferential stiffeners with the longitudinal stiffeners inside or non-existent, the critical positive value of $\varepsilon_{\phi cr}$ is obtained with $n = 0$, and the critical negative value is found by searching for the lowest magnitude negative value of $\varepsilon_{\phi cr}$.

With the longitudinal stiffeners also on the outside and deeper than the circumferential the circumferential stiffeners are again physically restrained from buckling into a smaller number of half waves than the number of spaces between the longitudinal stiffeners. A value of $\varepsilon_{\phi cr}$ is calculated for n equal to the number of spaces in half the circumference $n = \pi R/\ell_\phi$. If this value is positive then this is the critical value for an expansion of the cylinder. If it is negative there is no critical value for an expansion of the cylinder. Several possibilities exist for the negative buckling value. If the above value of $\varepsilon_{\phi cr}$ is positive then the negative value is the one given by the minimum magnitude value found for the unsupported stiffener, since this value has a larger number of circumferential waves than spaces between stiffeners. If the value for $n = \pi R/\ell_\phi$ is negative then there is a choice between this value and the value minimum $|\varepsilon_{\phi cr}|$. The one which has the larger value of n is used. The reasons for this are as follows: if $n = \pi R/\ell_\phi$ is the largest then a smaller n is physically impossible; if the n for minimum $|\varepsilon_{\phi cr}|$ is larger then this gives smallest $|\varepsilon_{\phi cr}|$ for $\varepsilon_{\phi cr}$ negative and is physically possible. For the case of all external stiffeners with the circumferential ones having the greater depth the problem is again split into two parts, one an unsupported circumferential stiffener with depth $d_\phi - d_x$ and the other a stiffener with the full depth d_ϕ, assuming nodes at the location of the longitudinal stiffeners. Values are then obtained for each case in a manner similar to that described above for external stiffeners. Two results are then obtained and compared to find the critical value.

In this treatment n is considered as a continuous variable instead of integer as it actually is and no arguments about the compatibility of the mode shapes are made. Introducing these restrictions would increase the buckling values so that the treatment used is conservative.

This buckling solution does not apply where the circumferential stiffener is thick compared with its depth. This is because the assumed simply supported boundary condition does not apply. In situations where the outstanding portion of the stiffener has a depth to thickness ratio of less than ten the yield limit is substituted for the buckling limit.

A.9 Solution of the Circumferential Stiffener Critical Buckling Strain

An approximate solution is obtained for the buckling of a circular plate with a large hole in it. The plate is assumed to be simply supported at one edge and free at the other (see Figure A3). The simply supported edge is the edge which attaches to the cylinder and thus must have the same displacements as the cylinder. The critical buckling parameter is taken as the tangential strain on the simply supported edge. The solution is obtained using an assumed mode variational method.

The variational formulation of the problem of elastic stability is given by Novozhilov[28] as

$$\delta[\ A^{(2)}\ ] = \delta\ R_2^{(2)}$$

In the cases in which the initial stress state can be determined using classical theory, this is such a case, $A^{(2)}$ is given in cylindrical coordinates as

$$A^{(2)} = \frac{E}{2(1+\nu)} \iiint \{\frac{1}{1-\nu}\ (b_2')^2 - 2b_1'\}\ r\ dr\ d\theta\ dz$$

$$+ \frac{1}{2} \iiint \{\sigma_r^\circ\ (\omega_\theta'^2 + \omega_z'^2) + \sigma_\theta^\circ\ (\omega_r'^2 + \omega_z'^2) + \sigma_z^\circ\ (\omega_r'^2 + \omega_\theta'^2)$$

$$- 2[\tau_{r\theta}^\circ\ \omega_r'\omega_\theta' + \tau_{rz}^\circ\ \omega_r'\omega_z' + \tau_{\theta z}^\circ\ \omega_\theta'\omega_z']\ \}\ r\ dr\ d\theta\ dz$$

where

$$b_2' = \varepsilon_r' + \varepsilon_\theta' + \varepsilon_z'$$

$$b_1' = \varepsilon_r'\varepsilon_\theta' + \varepsilon_r'\varepsilon_z' + \varepsilon_\theta'\varepsilon_z' - \frac{1}{4}\ (\varepsilon_{r\theta}'^2 + \varepsilon_{rz}'^2 + \varepsilon_{\theta z}'^2)$$

and

$$\varepsilon_r' = \frac{\partial \tilde{u}'}{\partial r}, \qquad \varepsilon_\theta' = \frac{1}{r}\frac{\partial \tilde{v}'}{\partial \theta} + \frac{\tilde{u}'}{r}, \qquad \varepsilon_z = \frac{\partial \tilde{w}'}{\partial z}$$

$$\varepsilon_{r\theta}' = \frac{\partial \tilde{v}'}{\partial r} - \frac{\tilde{v}'}{r} + \frac{1}{r}\frac{\partial \tilde{u}'}{\partial \theta}, \quad \varepsilon_{rz}' = \frac{\partial \tilde{u}'}{\partial z} + \frac{\partial \tilde{w}'}{\partial r}, \quad \varepsilon_{\theta z}' = \frac{1}{r}\ \frac{\partial \tilde{w}'}{\partial \theta} + \frac{\partial \tilde{v}'}{\partial z}$$

$$2\omega_r' = \frac{1}{r}\frac{\partial \tilde{w}'}{\partial \theta} - \frac{\partial \tilde{v}'}{\partial z}, \quad 2\omega_\theta' = \frac{\partial \tilde{u}'}{\partial z} - \frac{\partial \tilde{w}'}{\partial r}, \quad 2\omega_z' = \frac{\partial \tilde{v}'}{\partial r} + \frac{\tilde{v}'}{r} - \frac{1}{r}\frac{\partial \tilde{u}'}{\partial \theta}$$

The primes denote the buckle state and zero the initial state. By the same type of procedure as used for the derivation of the strain displacement relations in the cylinder the above strain displacement relations can be reduced to strain midsurface displacement relations. Thus, by assuming the displacements vary linearly with depth, the transverse shear strains are zero, and the normal strain is zero, the strain displacement relations reduce to

$$\varepsilon_r' = \frac{\partial u'}{\partial r} - z \frac{\partial^2 w'}{\partial r^2}$$

$$\varepsilon_\theta' = \frac{1}{r}\frac{\partial v'}{\partial \theta} + \frac{u'}{r} - \frac{z}{r}\left(\frac{1}{r}\frac{\partial^2 w'}{\partial \theta^2} + \frac{\partial w'}{\partial r}\right)$$

$$\varepsilon_{r\theta}' = \frac{\partial v'}{\partial r} - \frac{v'}{r} + \frac{1}{r}\frac{\partial u'}{\partial \theta} - \frac{2z}{r}\left(\frac{\partial^2 w'}{\partial r\,\partial \theta} - \frac{1}{r}\frac{\partial w'}{\partial \theta}\right)$$

$$\omega_r' = \frac{1}{r}\frac{\partial w'}{\partial \theta}, \quad \omega_\theta' = -\frac{\partial w'}{\partial r}, \quad \omega_z' = \frac{\partial v'}{\partial r} + \frac{v'}{r} - \frac{1}{r}\frac{\partial u'}{\partial \theta}$$

Note that the displacements in these expressions are the midsurface displacements and not the displacements of a point as in the previous expressions, also u', v', and w' are the displacements in the r, θ, and z directions and are not the same as the u, v, and w for the cylinder.

The stresses in the plate prior to buckling are given by Timoshenko[29] in terms of a radially inward pressure at the outer edge as

$$\sigma_r^\circ = -P\frac{b^2}{r^2}\frac{(r^2 - a^2)}{(b^2 - a^2)}$$

$$\sigma_\theta^\circ = -P\frac{b^2}{r^2}\frac{(r^2 + a^2)}{(b^2 - a^2)}$$

$$\tau_{r\theta}^\circ = \tau_{rz} = \tau_{\theta z} = \sigma_z = 0$$

P is the radial pressure. These may be transformed to be in terms of the tangential strain at the edge $r = b$, $\bar{\varepsilon}_\theta$, by using the stress strain relations to solve for P in terms of $\bar{\varepsilon}_\theta$ and substituting the result into the above expressions for the stresses

$$\sigma_r{}^\circ = \frac{b^2 \, \bar{\varepsilon}_\theta \, (1 - \frac{a^2}{r^2}) \, E_{\phi s}}{a^2 + b^2 + \nu(a^2 - b^2)} \tag{A25}$$

$$\sigma_\theta{}^\circ = \frac{b^2 \, \bar{\varepsilon}_\theta \, (1 + \frac{a^2}{r^2}) \, E_{\phi s}}{a^2 + b^2 + \nu(a^2 - b^2)}$$

The following set of buckling displacements satisfy the displacement boundary conditions:

$$u' = v' = 0$$

$$w' = A\,(b-r)\sin n\theta$$

These displacements are then substituted into the strain-displacement relations and the resulting expressions along with the prebuckle stresses are then substituted into the expression for $A^{(2)}$. When the integration is carried out and $\delta[A^{(2)}]$ is set to zero ($\delta R_2^{(2)}$ is zero for the problem since the forces are constant on one edge and the displacements are constant on the other)[30] the following expression is obtained for the critical $\bar{\varepsilon}_\theta$:

$$\bar{\varepsilon}_\theta = - \frac{t^2}{12(1-\nu^2)} \left[\frac{a^2 + b^2 + \nu(a^2 - b^2)}{b^2}\right]$$

$$\left[\frac{(n^2-1)^2 \ln \frac{b}{a} + 2n^2 (1-n^2) \frac{b-a}{a} + n^2 \frac{(n^2 + 2(1-\nu))}{2} (\frac{b^2 - a^2}{a^2})}{\frac{1-2n^2}{2} (b^2 - a^2) + (b^2 n^2 + a^2 (n^2-1)) \ln \frac{b}{a}}\right] \tag{A26}$$

Now $(\frac{b-a}{a})$ is set equal to ζ and the expression is expanded in terms of this quantity. The result of doing this and setting the critical $\bar{\varepsilon}_\theta$ equal to the critical strain in the cylinder is the expression:

$$\varepsilon_{\phi cr} = -\left(\frac{t}{d}\right)^2 \left(\frac{1}{12(1-\nu^2)}\right) \frac{\left(2 + 2(1-\nu)(\zeta + \frac{\zeta^2}{2})\right)}{(1 + 2\zeta + \zeta^2)}$$

$$\left[\frac{(1+2n^2(1-\nu))\zeta + (2n^2(2-\nu)-1)\frac{\zeta^2}{2} + (n^2-1)^2\left(\frac{\zeta^3}{3} - \frac{\zeta^4}{4}\right) + \ldots}{1 + \left(\frac{2n^2-1}{3}\right)\zeta + \frac{3-4n^2}{12}\zeta^2 + \ldots}\right]$$

Specialization of this solution in two limiting cases, for which solutions are available in the literature, is given in Appendix B.

A.10 Yield Failure

The principal stresses in the skin are given by σ_{xp} and $\sigma_{\phi p}$ (A13). It is assumed that the yield criterion for the cylindrical shell skin material is of the following form:[31]

$$f_D^2 = \left(\frac{\sigma_{xp}}{\sigma_{x0\alpha}}\right)^2 - \kappa_{\alpha\beta}\frac{\sigma_{xp}}{|\sigma_{x0\alpha}|}\frac{\sigma_{\phi p}}{|\sigma_{\phi 0\beta}|} + \left(\frac{\sigma_{\phi}}{\sigma_{\phi 0\beta}}\right)^2 \leq 1 \qquad \text{(A27)}$$

where

$\sigma_{x0\alpha} = \sigma_{x0T}$ the longitudinal tension yield stress in the skin if $\sigma_{xp} > 0$

$\sigma_{x0\alpha} = \sigma_{x0C}$ the longitudinal compression yield stress in the skin if $\sigma_{xp} < 0$

$\sigma_{\phi 0\beta} = \sigma_{\phi 0T}$ the circumferential tension yield stress in the skin if $\sigma_{\phi p} > 0$

$\sigma_{\phi 0\beta} = \sigma_{\phi 0C}$ the circumferential compression yield stress in the skin if $\sigma_{\phi p} < 0$

$\kappa_{\alpha\beta} = \kappa_{TT}$ constant defining yield envelope in first quadrant $\sigma_{xp} > 0$ and $\alpha_{\phi p} > 0$

$\kappa_{\alpha\beta} = \kappa_{CT}$ constant defining yield envelope in second quadrant $\sigma_{xp} < 0$ and $\sigma_{\phi p} > 0$

$\kappa_{\alpha\beta} = \kappa_{CC}$ constant defining yield envelope in third quadrant $\sigma_{xp} < 0$ and $\sigma_{\phi p} < 0$

$\kappa_{\alpha\beta} = \kappa_{TC}$ constant defining yield envelope in fourth quadrant $\sigma_{xp} > 0$ and $\sigma_{\phi p} < 0$

For the case of an isotropic material that behaves identically in tension and compression with yield stress σ_{0D} Eqs. A27 when specialized by the following substitutions:

$$\sigma_{x0T} = \sigma_{x0C} = \sigma_{\phi 0T} = \sigma_{\phi 0C} = \sigma_{0D}$$

$$\kappa_{\alpha\beta} = 1$$

reduce to the distortion energy yield criterion

$$\sigma_{xp}^2 - \sigma_{xp}\sigma_{\phi p} + \sigma_{\phi p}^2 \leq \sigma_{0D}^2 \qquad \text{(A28)}$$

The stiffeners are in a uniaxial state of stress so the stresses (A14) must satisfy the yield conditions:

$$\begin{aligned} \sigma_{xS0C} &\leq \sigma_{xsp} \leq \sigma_{xS0T} \\ \sigma_{\phi S0C} &\leq \sigma_{\phi sp} \leq \sigma_{\phi S0T} \end{aligned} \qquad \text{(A29)}$$

where the subscript x refers to the longitudinal stiffener; ϕ refers to the circumferential stiffener; 0 refers to yield; C refers to compression; and T refers to tension.

Table A 1

Selection of Circumferential Stiffener Buckling Mode

$(\varepsilon_\phi)_{cr}$ - Contraction

d_ϕ	Opposite Sides	$\lvert d_\phi \rvert \leq \lvert d_x \rvert$	d	n	$(\varepsilon_\phi)_{cr}$
Inside +	Yes	N/A	d_ϕ	$n = 0$	$(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}\rvert_{n=0}$
Outside -	Yes	N/A	d_ϕ	$n = n^*$	$(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}\rvert_{n=n^*}$
Inside +	No	Yes	d_ϕ	$n = \frac{\pi R}{\ell_\phi}$	$(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}\rvert_{n=\pi R/\ell_\phi}$
Outside -	No	Yes	d_ϕ d_ϕ	$n = \frac{\pi R}{\ell_\phi}$ $n = n^*$	if $(\varepsilon_\phi)_{cr}\rvert_{n=\pi R/\ell_\phi} \geq 0$ then use $(\varepsilon_\phi)_{cr}\rvert_{n=n^*}$ otherwise use $(\varepsilon_\phi)_{cr}\rvert_{n=\max(n,n^*)}$
Inside +	No	No	$d_\phi - d_x$ d_ϕ	$n = 0$ $n = \frac{\pi R}{\ell_\phi}$	$\left.\begin{matrix}(\varepsilon_\phi)^{(1)}_{cr}\\(\varepsilon_\phi)^{(2)}_{cr}\end{matrix}\right\}$ if $\lvert(\varepsilon_\phi)^{(1)}_{cr}\rvert \geq \lvert(\varepsilon_\phi)^{(2)}_{cr}\rvert$ then $(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)^{(2)}_{cr}$ otherwise $(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)^{(1)}_{cr}$
Outside -	No	No	$\lvert d_\phi\rvert - \lvert d_x\rvert$	$n = n^*$	$(\varepsilon_\phi)^{(1)}_{cr}$
			d_ϕ d_ϕ	$n = \frac{\pi R}{\ell_\phi}$ $n = n^*$	if $(\varepsilon_\phi)_{cr}\ {}_{n=\pi R/\ell_\phi} \geq 0$ then $(\varepsilon_\phi)^{(2)}_{cr}\ \rvert_{n=n^*}$ otherwise $(\varepsilon_\phi)_{cr}\rvert_{n=\max(n,n^*)}$
				$\left.\begin{matrix}(\varepsilon_\phi)^{(1)}_{cr}\\(\varepsilon_\phi)^{(2)}_{cr}\end{matrix}\right\}$	if $(\varepsilon_\phi)^{(1)}_{cr}\rvert \geq \lvert(\varepsilon_\phi)^{(2)}_{cr}\rvert$ then $(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)^{(2)}_{cr}$ otherwise $(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)^{(1)}_{cr}$

$\zeta = \frac{d_\phi}{R - d_\phi - \frac{t_s}{2}}$; $d_\phi > 0$ inside

$\zeta = \frac{d_\phi}{R + \lvert d_\phi \rvert + \frac{t_s}{2}}$; $d_\phi < 0$ outside

n^* positive integer such that $(\varepsilon_\phi)_{cr} < 0$ and $\lvert(\varepsilon_\phi)_{cr}\rvert\rvert_{n=n^*}$ is a minimum.

Table A 2

Selection of Circumferential Stiffener Buckling Mode

$(\varepsilon_\phi)_{cr}$ - Expansion

d_ϕ	Opposite Side	$\|d_\phi\| \leq \|d_x\|$	d	n	$(\varepsilon_\phi)_{cr}$
Outside -	Yes	N/A	d_ϕ	0	$(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}\|_{n=0}$
Outside -	No	Yes	d_ϕ	$\frac{\pi R}{\ell_\phi}$	$(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}\|_{n=\frac{\pi R}{\ell_\phi}}$ $(\varepsilon_\phi)_{cr}\|_{n=\frac{\pi R}{\ell_\phi}} > 0$ otherwise no buckling in this case
Outside -	No	No	$\|d_\phi\|-\|d_x\|$ d_ϕ	0 $\frac{\pi R}{\ell_\phi}$	$(\varepsilon_\phi)_{cr}^{(1)}$, $(\varepsilon_\phi)_{cr}^{(2)}$: if $(\varepsilon_\phi)_{cr}^{(2)} \leq 0$ then $(\varepsilon_\phi)_{cr} = (\varepsilon_\phi)_{cr}^{(1)}$ if $(\varepsilon_\phi)_{cr}^{(2)} > 0$ then $(\varepsilon_\phi)_{cr} = \min((\varepsilon_\phi)_{cr}^{(1)}, (\varepsilon_\phi)_{cr}^{(2)})$

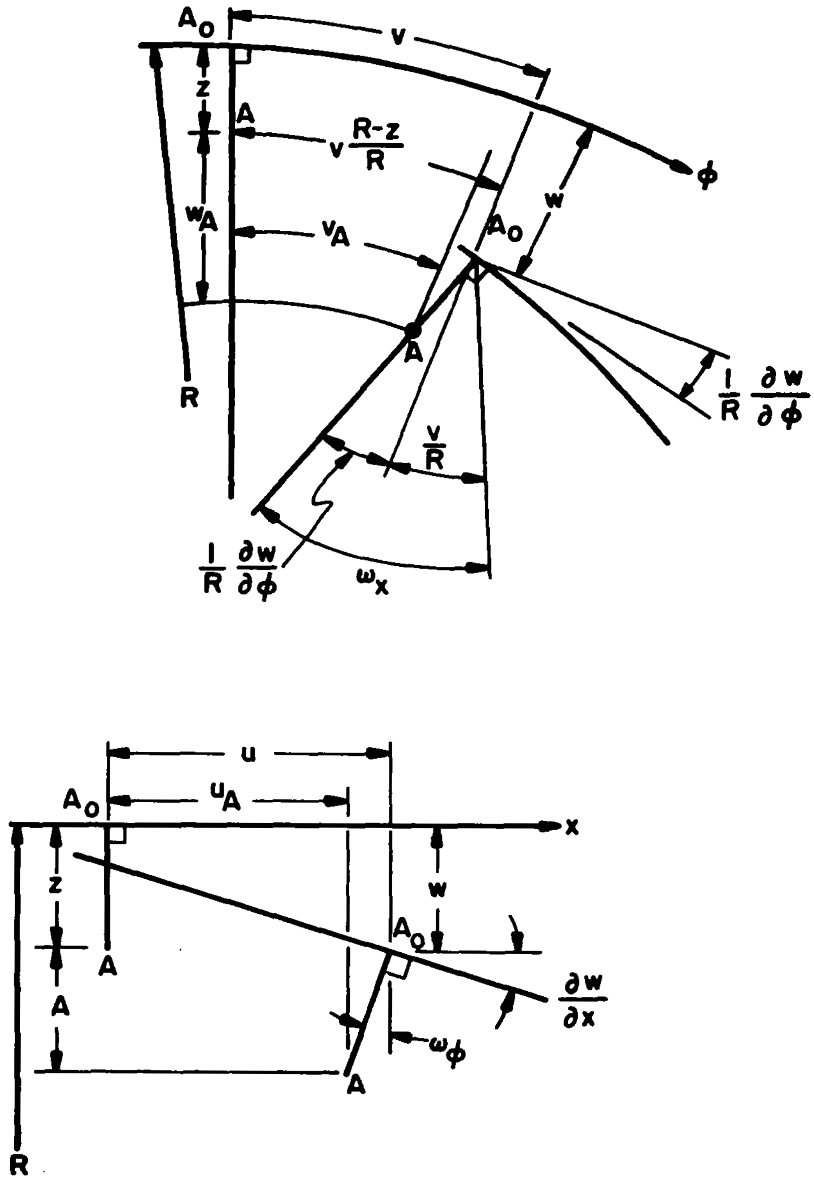

Figure A 1 Displacements and Rotations of a Shell Element

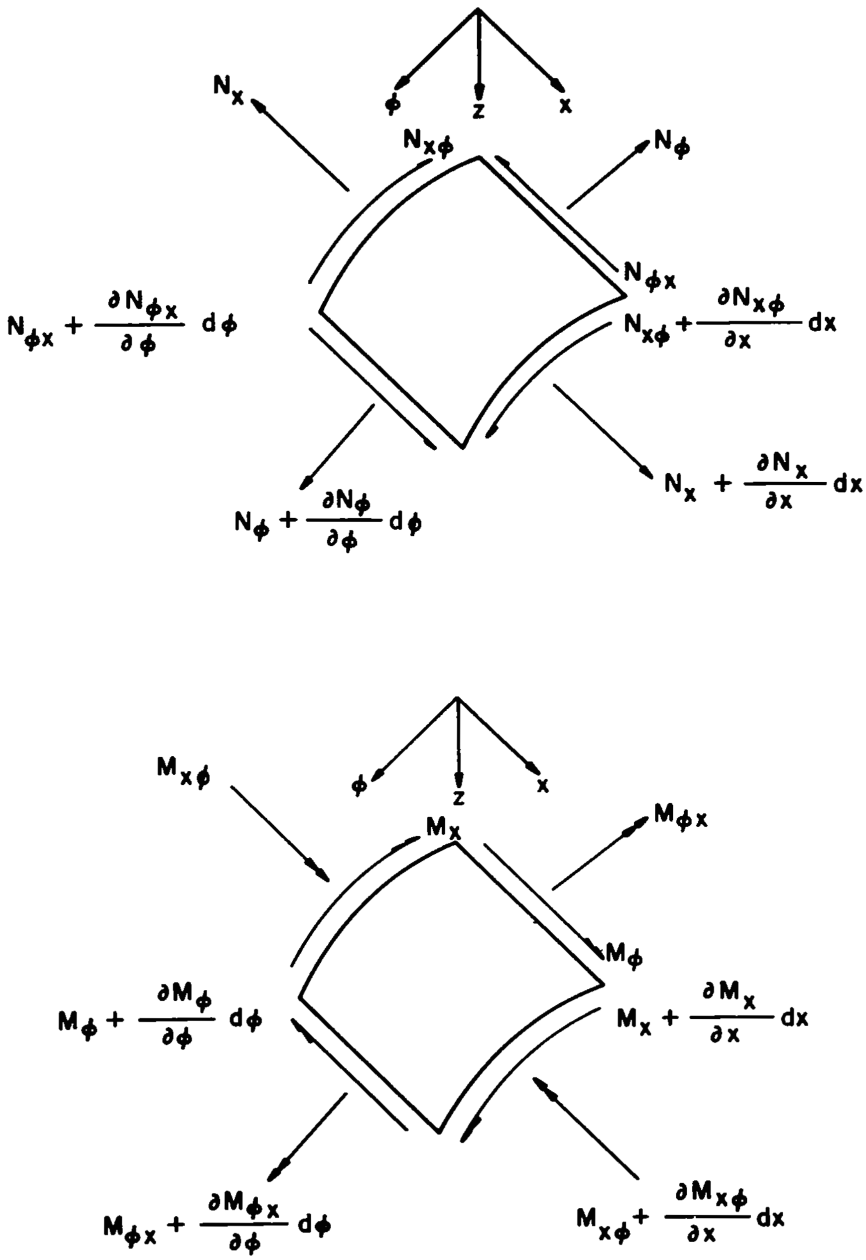

Figure A 2 Force Resultants

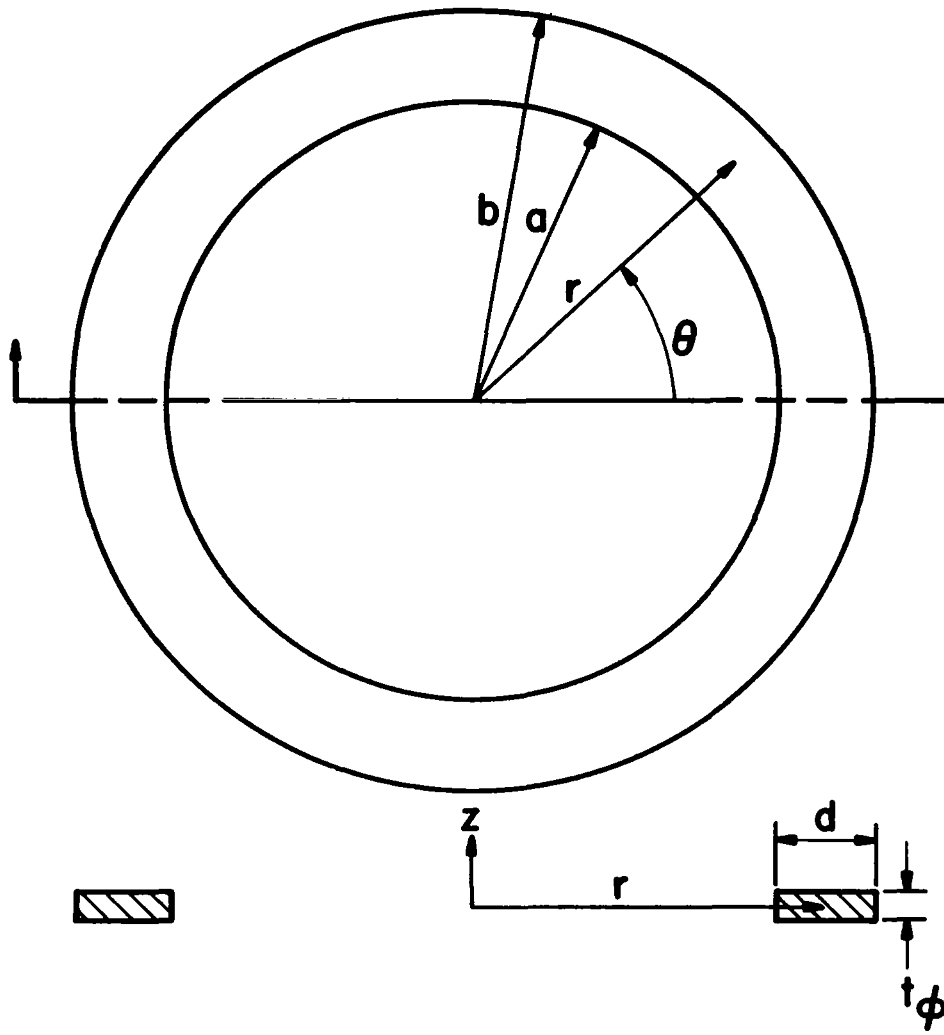

Figure A 3 Circumferential Stiffener

APPENDIX B

Verification of the Circumferential Stiffener Buckling Solution

Two limiting cases for $\varepsilon_{\phi cr}$ are obtained. One is for the washer mode of buckling with no circumferential waves. The other case is for a large number of waves. These cases are checked with existing solutions.

With n = 0 and neglecting terms involving the depth of the stiffener divided by the radius, ζ, with respect to one, the expression for the washer mode is

$$\varepsilon_{\phi cr} = -\left(\frac{t}{d}\right)^2 \frac{1}{12(1-\nu^2)} (2)\zeta \tag{B1}$$

With n large and again neglecting ζ with respect to one the expression for $\varepsilon_{\phi cr}$ is

$$\varepsilon_{\phi cr} = -\left(\frac{t}{d}\right)^2 \frac{2}{12\,(1-\nu^2)} \left(\frac{n^2 \zeta^2}{2} + 3(1-\nu)\right) \tag{B2}$$

Substituting the expression for the critical strain in the washer mode (B1) into the expression for the stress in the radial direction (A25) and obtaining the value at the supported boundary, r = b, the following expression is obtained:

$$\sigma_{rcr} = \frac{-E_{\phi s}}{12\,(1-\nu^2)} \left(\frac{t}{d}\right)^2 \zeta\,(2\zeta)$$

Writing ζ as d/R and making the definition $D = E_{\phi s} t^2/12(1-\nu^2)$ this expression is

$$\sigma_r = -\frac{2D}{R^2}$$

The negative sign signifies that the stress is compressive. The above value agrees closely with the exact solution for the axisymmetric case done by Meissner[32] who obtain a value for the coefficient 1.86 instead of 2.

By substituting the value of $\varepsilon_{\theta cr}$ for n large (B2) into the expression for the tangential stress, σ_θ, (A25) at the supported edge, r = b, the following expression is obtained:

$$\sigma_\theta = \frac{-E_{\phi s}}{12(1-\nu^2)} \left(\frac{t}{d}\right)^2 \left(3(1-\nu) + \frac{n^2 \zeta^2}{2}\right) (2)$$

n is expressed in terms of the half wavelength, ℓ, as $\pi R/\ell$. Substituting for ζ and n the following expression is obtained:

$$\sigma_{\theta cr} = \frac{-E_{\phi s} \pi^2}{12\,(1-\nu^2)} \left(\frac{t}{d}\right)^2 \left(\left(\frac{d}{\ell}\right)^2 + \frac{6(1-\nu)}{\pi^2}\right)$$

If the value ν = .3 is used this becomes

$$\sigma_{\theta cr} = \frac{-E_\phi \pi^2}{12(1-\nu^2)} \left(\frac{t}{d}\right)^2 \left(\left(\frac{d}{\ell}\right)^2 + 0.425\right)$$

which is the same as the expression for the critical buckling stress for a rectangular plate (see Section A.7).

APPENDIX C

Development of Synthesis Scheme

C.1 Introduction

In the section entitled synthesis scheme it is explained how the design problem is converted to an unconstrained minimization problem. Fiacco and McCormick[33] have shown that when this method is applied to a convex programming problem that as the multiplier, r, approaches zero the optimal solution to the unconstrained problem approaches the optimal solution to the constrained problem.

In this appendix the equations necessary to implement the scheme are given as well as more detailed discussion of how the scheme operates. A discussion of the computational experience is also given.

C.2 Weight of the Stiffened Cylinder

The expression for the weight of the cylinder in terms of the design variables and over all dimensions of the cylinder is

$$W = 2\pi RL\, t_s \gamma_s + \left|2R\, d_\phi - d_\phi^{\,2} - t_s |d_\phi|\right| \pi\, t_\phi \gamma_\phi n_\phi +$$

$$L\, |d_x|\, t_x \gamma_x n_x - \min(|d_x|, |d_\phi|)\, \delta_{x\phi}\, t_x t_\phi\, (\gamma_x \delta_{xw} + \gamma_\phi \delta_{\phi w})\, n_\phi n_x$$

where γ_s, γ_ϕ , γ_x are the weight densities respectively of the skin, circumferential stiffener, and longitudinal stiffener; n_ϕ and n_x are respectively the number of circumferential stiffeners and the number of longitudinal stiffeners. The last term in the expression accounts for the fact that the stiffeners may cross when they are on the same side of the cylinder and the weight of the material at this intersection must not be counted twice. The quantities $\delta_{x\phi}$, δ_{xw}, and $\delta_{\phi w}$ are introduced to account for the possible combinations. These are defined as follows:

$$\delta_{x\phi} = \begin{cases} 0 & \text{stiffener sets on the opposite sides of the skin} \\ 1 & \text{stiffener sets on the same side of the skin} \end{cases}$$

$$\delta_{xw} = \begin{cases} 0 & \text{longitudinal stiffeners continuous} \\ 1 & \text{circumferential stiffeners continuous} \end{cases}$$

$$\delta_{\phi w} = \begin{cases} 0 & \text{circumferential stiffeners continuous} \\ 1 & \text{longitudinal stiffeners continuous} \end{cases}$$

The quantity $\min(|d_x|, |d_\phi|)$ takes the value of the magnitude of stiffener depth which is the smallest in magnitude.

While n_ϕ and n_x are in fact integers they are not taken as such here because this would cause the gradient to the weight to be discontinuous. Thus, their values are taken as

$$n_\phi = \frac{L}{\ell_x} - 1.0$$

$$n_x = \frac{2\pi R}{\ell_\phi}$$

C.3 Approximate Analysis

The approximate analysis is approximate in the sense that in the cylinder buckling analysis only a small number of possible buckling mode shapes are checked in the search for the minimum buckling load. These mode shapes are determined by doing a search of a large number of possible modes, at the start of each minimization cycle, and sorting out a predetermined number of modes to be kept for the partial analysis. These modes are selected by ordering them according to the value of the buckling load which they give. The ones which give the lowest buckling loads are kept. An approximate analysis then consists of calculating buckling loads for each proposed design just as in the complete analysis but using only the selected set of mode shape numbers. The rest of the analysis remains the same.

This is an unconservative method and the reasons that this can be used are as follows: at the outset of the minimization, when the value of the multiplier is large, the designs are far away from the constraints and an error in calculating the buckling load does not result in leaving the acceptable region of the design space; near the optimum only small changes are made in the design so the buckling modes do not change much, if at all, and the approximate analysis is very close to being as accurate as the complete analysis.

C.4 Altering the H Matrix

The H matrix for the direction $\bar{s}_{i+1}$ is calculated using the information at the minimum along the direction $\bar{s}_i$ in the following manner:

$$H_{i+1} = H_i + A_i + B_i$$

where

$$A_i = \frac{\bar{\sigma}_i \, \bar{\sigma}_i^T}{\bar{\sigma}_i^T \, \bar{y}_i}$$

$$B_i = \frac{-H_i \, \bar{y}_i \, \bar{y}_i^T \, H_i}{\bar{y}_i^T \, H_i \, \bar{y}_i}$$

and

$$\bar{y}_i = (\nabla_v F)_{i+1} - (\nabla_v F)_i$$

The first value of H is taken as the identity matrix. Fletcher and Powell[34] have shown that for a quadratic function the method will converge by using only the same number of directions as there are independent variables.

There are situations which arise where the minimum along a line cannot be found within the prescribed tolerance (see one-dimensional minimization). In such a case the H matrix is not updated but a new direction is generated using the new gradient at the new point, the one which estimates the minimum along the line, and the same H matrix.

The H matrix is reinitialized after the number of directions in which attempts are made to find a minimum exceeds the number of independent variables (see block diagram, Figure 3).

C.5 Gradient

The gradient to the function, F, is calculated partially explicitly and partially by finite difference. The gradient to the weight and the gradient to the part of penalty function which contains the side constraints are obtained explicitly. The gradient to the part of the penalty function which contains the behavior constraints is obtained by finite difference.

The unconstrained function, F, is rewritten as follows in order to calculate the separate parts of the components to the gradient:

$$F = W + r(\sum_u + \sum_\ell + \sum_C + \sum_B)$$

where $\sum_u$ is the sum of the $1/g_i(\overline{v})$ for all the upper bound design variable constraints; $\sum_\ell$ is the sum for the lower bound design variable constraints; $\sum_C$ is the sum for the compatibility constraints; and $\sum_B$ is the sum for all the behavior constraints.

The gradient to F is then

$$\nabla_v F(\overline{v},r) = \nabla_v W + r(\nabla_v \sum_u + \nabla_v \sum_\ell + \nabla_v \sum_C + \nabla_v \sum_B)$$

The components to the gradient to the weight are given by the following expressions:

$$\frac{\partial W}{\partial t_s} = 2\pi \; RL \; \gamma_s - \pi \; d_\phi \; t_\phi \; \gamma_\phi \; n_\phi$$

$$\frac{\partial W}{\partial t_x} = |d_x|\, L\gamma_x n_x - \min(|d_x|,|d_\phi|)\ \delta_{x\phi} t_\phi\ (\gamma_x \delta_{xw} + \gamma_\phi\ \delta_{\phi w})\ n_\phi\ n_x$$

$$\frac{\partial W}{\partial t_\phi} = \ |2R\ d_\phi - d_\phi^{\ 2} - t_s\ |d_\phi||\ \gamma_\phi\ \pi\ n_\phi$$

$$- \min\ (|d_x|,|d_\phi|)\ \delta_{x\phi}\ t_x\ (\gamma_x \delta_{xw}\ +\ \gamma_\phi\ \delta_{\phi w})\ n_\phi\ n_x$$

$$\frac{\partial W}{\partial d_x} = \ \varepsilon_{I0}\ [L\ t_x\ \gamma_x\ n_x - \delta_{dx}\delta_{x\phi}\ t_x t_\phi\ (\gamma_x \delta_{xw} + \gamma_\phi\ \delta_{\phi w})\ n_\phi\ n_x]$$

$$\frac{\partial W}{\partial d_\phi} = \varepsilon_{I1}\ [|2R - 2d_\phi - t_s\ \varepsilon_{I1}|\gamma_\phi\ \ t_\phi\ \pi\ \ n_\phi$$

$$- \delta_{d\phi}\ \delta_{x\phi}\ t_x\ t_\phi\ (\gamma_x\ \delta_{xw} + \gamma_\phi\ \delta_{\phi w})\ n_\phi\ n_x]$$

$$\frac{\partial W}{\partial \ell_x} = -\ [|2R\ d_\phi - d_\phi^{\ 2} - t_s\ |d_\phi||\ \gamma_\phi\ t_\phi\ \pi$$

$$- \min\ (|d_x|,|d_\phi|)\ \delta_{x\phi}\ t_x\ t_\phi\ (\gamma_x \delta_{xw} +\ \ \gamma_\phi\ \delta_{\phi w})\ n_x]\ \frac{L}{\ell_x^{\ 2}}$$

$$\frac{\partial W}{\partial \ell_\phi} = \ -\ [L\ t_x\ \gamma_x\ |d_x|$$

$$- \min(|d_x|,|d_\phi|)\delta_{x\phi} t_x\ t_\phi\ (\gamma_x\ \delta_{xw} + \gamma_\phi\ \delta_{\phi w})\ n_\phi]\ \frac{2\pi R}{\ell_\phi^{\ 2}}$$

where

$$\varepsilon_{I0} = \begin{cases} 1 & d_x > 0 \\ -1 & d_x < 0 \end{cases}$$

$$\varepsilon_{I1} = \begin{cases} 1 & d_\phi > 0 \\ -1 & d_\phi < 0 \end{cases}$$

$$\delta_{dx} = \begin{cases} 1 & |d_x| \leq |d_\phi| \\ 0 & |d_x| > |d_\phi| \end{cases}$$

$$\delta_{d\phi} = \begin{cases} 1 & |d_\phi| < |d_x| \\ 0 & |d_\phi| \geq |d_x| \end{cases}$$

The upper bound gradient components are

$$\frac{\partial \sum_u}{\partial v_i} = - \frac{U_i - L_i}{(U_i - v_i)^2}$$

The lower bound components are

$$\frac{\partial \sum_\ell}{\partial v_i} = - \frac{U_i - L_i}{(v_i - L_i)^2}$$

The compatibility bound components are

$$\frac{\partial \sum_c}{\partial t_x} = - \frac{\ell_{\phi\ell} - \ell_{\phi u}}{(\ell_\phi - t_x)^2}$$

$$\frac{\partial \sum_c}{\partial t_\phi} = - \frac{\ell_{xu} - \ell_{x\ell}}{(\ell_x - t_\phi)^2}$$

$$\frac{\partial \sum_C}{\partial \ell_\phi} = - \frac{\ell_{\phi u} - \ell_{\phi \ell}}{(\ell_\phi - t_x)^2}$$

$$\frac{\partial \sum_C}{\partial \ell_x} = - \frac{(\ell_{xu} - \ell_{x\ell})}{(\ell_x - t_\phi)^2}$$

The gradient to the behavior constraints part of the penalty function is calculated using a forward difference method. Thus these components are

$$\frac{\partial \sum_B}{\partial v_i} = \frac{1}{\Delta v_i} \left(\sum_B \Big|_{v_i + \Delta v_i} - \sum_B \Big|_{v_i} \right)$$

In calculating the sums involved here an analysis is performed by using the critical mode shapes of the design point $\bar{v}$. This eliminates the complete modal search.

C.6 One-Dimensional Minimization

In finding the minimum of F along a move direction, it is assumed that the function is unimodal in this direction. Using the last calculated value of the weight as an estimate for the minimum of F a linear interpolation is made along the line to the estimated minimum design. This gives an initial guess at an increment size. Then using an incrementation scheme and the slope along the line as a test two points are found which lie on opposite sides of the minimum. A cubic interpolation is then made to an estimated minimum design. The value of the function, F, and its gradient are calculated at the interpolated minimum. The inner product of the unit vector in the gradient direction is taken with the unit vector in the move direction. If this is less than a prescribed value, .005, the point is accepted as the minimum. If not the point is substituted for one of the points bracketing the minimum and another interpolation is made.

The following cubic interpolation formula is used:[35]

$$\frac{\alpha}{\lambda} = 1 - \frac{(\nabla_v F_b)^T \bar{s}_i + w - z}{(\nabla_v F_b)^T \bar{s}_i - (\nabla_v F_a)^T \bar{s}_i + 2w}$$

where a and b are the points which bracket the minimum, α is the distance to the minimum from a; and λ is the distance from a to b; α and λ are positive from a to b.

$$w = (z^2 - ((\nabla_v F_a)^T \bar{s}_i)((\nabla_v F_b)^T \bar{s}_i)))^{1/2}$$

$$z = \frac{3}{\lambda} (F_a - F_b) + (\nabla_v F_a)^T \bar{s}_i + (\nabla_v F_b)^T \bar{s}_i$$

F_a and F_b are the values of the function at points a and b.

If the convergence criterion is not satisfied after a prescribed number of interpolation attempts, five is used, the direction is abandoned and a new one generated without updating the H matrix. The last design point estimated as the minimum is used as a starting point for the new direction. Also if the distance between points a and b gets to be less than a minimum value the direction is abandoned and a new one generated, again without updating the H matrix. In this case the point a is used as the starting point for the new direction. Note here that while the H matrix is not changed the gradient is, because the design point is changed.

The minimum move distance is selected as follows: the component of $\bar{s}$, the move direction, which has the largest magnitude is selected; the minimum distance is then calculated as the finite difference increment in the variable corresponding to the selected component of $\bar{s}$ divided by the magnitude of this component of $\bar{s}$. The minimum distance is

$$T_{min} = \frac{\nabla v_i}{|s_i|_{max}}$$

C.7 Convergence Criteria

Three alternative tests of convergence are used to determine convergence of one of the unconstrained minimization problems. Convergence is assumed when any one of these is satisfied. One convergence test is contained in the computer program for convergence of the overall problem. The program may also be stoped after a predetermined number of cycles, unconstrained minimizations.

The first of the three convergence criteria for terminating a cycle is that the magnitude of the gradient be less than a prescribed value, a test for convergence using this criterion is made for every design.

The second is that the estimated value by which the value of F exceeds its minimum is less than prescribed percentage of the value of F. The amount by which F exceeds its minimum is given by

$$\frac{1}{2} \nabla_V F(\bar{V},r)^T A^{-1} \nabla_V F(\bar{V},r)$$

if F is quadratic. A is the Hessian matrix of F evaluated at the design point $\bar{V}$. Since the H matrix[36] in the direction finding method tends to A^{-1} as $\bar{V}$ approaches the minimum, H is used in this method to get an estimate of the amount by which F exceeds its minimum. Since H will only approximate A^{-1} after minimizations have been achieved along a number of directions equal to the number of independent variables, this convergence test is only applied after this number of directions is searched.

A third test is used to prevent an attempt to make moves which are within the finite difference star. This test proceeds as follows: after the above test fails a move is made in the gradient direction which is twice the minimum move distance (see one dimensional minimization); if the sign of the slope of the function at the new point along the old gradient direction is opposite from the sign of the slope at the old point or if the new point is in violation then convergence is assumed.

The final convergence criterion is based on the primal-dual nature of the method. Once a minimum is obtained for one value of the multiplier, r, bounds can be placed on the value of the minimum weight. The minimum weight value is bounded below by the value of the dual objective and above by the current value of the weight. This leads to the following convergence criterion:[37]

$$\frac{W - G}{G} \leq \varepsilon$$

where ε is some small percentage and G is the value of the dual objective. G is given by

$$G = W - r \sum_i \frac{1}{g_i}$$

It is convenient for computational purposes to write the convergence criterion in terms of F and W. This is

$$\frac{F - W}{2W - F} \leq \varepsilon$$

The convergence criterion applies only when $G = 2W - F > 0$.

C.8 Computational Experience

Even though a finite difference method is used to get the components of the gradient to behavior portion of the penalty function this causes only a small increase in computational effort, since the mode shape numbers are used from the analysis at the design point and no modal search is involved in performing the analyses at the points of the finite star. Only a forward difference is used but it works well and probably little advantage would be gained in using a more accurate method.

Convergence for three load condition cases on CWRU's Univac 1107 computer using the Fortran IV language takes fifteen to twenty minutes of computer time. The one load condition cases take from five to eight minutes. Approximately twelve cycles (minimization for a value of r) are required for convergence. A complete analysis for a three load condition case including sorting the most critical mode shapes takes about thirty-five seconds and a partial analysis takes about one half second.

For the cycle convergence criterion on the magnitude of the gradient the value used is 10% of the weight. This seems quite large but the fact is that it is difficult to satisfy.

The second criterion, the estimation of the percentage by which the function exceeds its minimum, works well. A value of 2% is used here. Using this criterion convergence is achieved the majority of the time after minimizing along the same number of directions as there are independent variables.

The third convergence criterion, the move size criterion becomes active once the function value, F, becomes close to the value of the weight. In this situation the move distances become small and it is difficult to satisfy the convergence criterion along a line so the H matrix is not updated on every move. The H matrix does not then approximate A^{-1} and in some cases remains the identity matrix. Thus, the second convergence criterion is not satisfied.

Fiacco and McCormick[38] have found that the computational effort involved in achieving the over all minimum is relatively insensitive to the rate at which the multiplier, r, is reduced. This is because a greater reduction in r requires a greater effort to find the minimum of the unconstrained problem.

In the work presented here there is a factor involved in selecting the rate of reduction of r which does not occur in their problems. This is the fact that the individual minimizations are being performed on an approximation to the function, F. The approximate analysis is used for the individual optimization. Too large a reduction in r will result in large changes in the design variables and the approximations will then be poor. This can and does result quite frequently in optima for the unconstrained problem which when analyzed completely are found not to lie in the acceptable region of the design space. The synthesis cannot be continued from such a point.

The reduction of r and the number of modes used in the partial analysis are thus intimately tied together. Their connection is a difficult one to establish since this is an optimization problem in itself. It was found, however, that once a value of the reduction of r was established which kept the designs in the acceptable region for most problems, using a relatively small number of saved modes, that the convergence time could be improved by increasing the number of saved modes. This occurred even though the individual analysis times go up when the number of modes saved is increased, both for the complete analysis and the approximate analysis. The function, F, seems to become better behaved when more modes are saved and less effort is involved, in terms of number of designs checked, in finding the minimum of F. Thus, even though the individual analyses take longer the convergence is faster because less analyses are performed.

The values which work well are as follows: reduction of r, factor of 1/2; number of modes saved in gross buckling, 40 (except when there is an external pressure load and then 10 can be used); number of modes saved in panel buckling, 20; number of modes saved in skin buckling, 10.

The problem of choosing the initial value of r is an important one here, because too small a choice leads to the same difficulty as reducing r too rapidly, and too large a value leads to longer convergence times. No automatic method is incorporated in the computer program. The values used here have been found by trial. Good starting values for the aluminum cylinders vary from 1/10 to 1/30 of the weight of the initial design. These values cause the value of the penalty term in F to be of the same order of magnitude as the weight.

APPENDIX D
Summary of Results

In this section summary charts are presented of each case. In these cases the designation I means that all the stiffening is inside; the design O means that all the stiffening is outside; and the designation I,O means that the circumferential stiffeners are inside and the longitudinal stiffeners outside. The prime indicates a change in the design parameter bounds from the case with the same number-letter designation. The t in case 1-It designates a temperature degradation effect. The cases with the same number designation are all the same problem; that is, the load conditions, the material, and the size of the cylinders are all the same.

Problems one through four are the same as those presented by Kicher.[39]

Case 1 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0221	.0298	.00917	.347	1.93	6.65	849	231.	.0368
Initial	.099	.06	.06	.5	.5	6.	3.	715.	.114
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	.0	.0	.0	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.769	.997	.240
P.B.	.793	.932	.345
S.B.	.840	.934	.309
L.S.B.	.690	.963	.202
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.402	.582	.128
L.Y.T.	----	----	----
L.Y.C.	.404	.564	.118
C.Y.T.	.122	.257	.182
C.Y.C.	----	----	----

L = 165.

Aluminum

$E = 10 \times 10^6$

R = 60.

$\gamma = .101$

$\nu = .333$

$\sigma_y = 50,000.$

$r_o = 10.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	15	1	8
	n	9	33	1
2	m	15	1	9
	n	9	25	1
3	m	14	1	6
	n	9	101	1

Loads

L.C.	N	p
1	700.	0.
2	940.	-2.
3	212.	.4

Case 1 - I Starting Point 2

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0200	.0312	.00907	.365	1.92	7.15	.792	230.	.0367
Initial	.04	.03	.03	.5	.5	.25	.25	1000.	.160
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.768	.989	.241
P.B.	.765	.899	.373
S.B.	.899	.971	.338
L.S.B.	.698	.974	.204
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.402	.582	.128
L.Y.T.	----	----	----
L.Y.C.	.404	.563	.118
C.Y.T.	.121	.265	.162
C.Y.C.	----	----	----

$L = 165.$

Aluminum

$E = 10 \times 10^6$

$R = 60.$

$\gamma = .101$

$\nu = .333$

$\sigma_y = 50,000.$

$r_o = 60.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	13	1	9
	n	9	30	1
2	m	14	1	10
	n	8	23	1
3	m	13	1	7
	n	9	116	1

Loads

L.C.	N	p
1	700.	0.
2	940.	-2.
3	212.	.4

Case 1 - I'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0229	.0513	.0523	.351	.906	8.13	.986	293.	.0467
Initial	.099	.06	.06	.5	.5	6.	3.	715.	.114
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	.019	.05	.05	-2.	-2.	.05	.05		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.797	.999	.259
P.B.	.855	.975	.381
S.B.	.915	.999	.342
L.S.B.	.198	.276	.058
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.332	.477	.968
L.Y.T.	----	----	----
L.Y.C.	.336	.466	.098
C.Y.T.	.912	.202	.011
C.Y.C.	----	----	----

$L = 165.$

Aluminum

$E = 10 \times 10^6$

$R = 60.$

$\gamma = .101$

$\nu = .333$

$\sigma_y = 50{,}000.$

$r_o = 10.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	12	1	8
	n	10	29	1
2	m	12	1	9
	n	10	22	1
3	m	1	1	6
	n	5	91	1

Loads

L.C.	N	p
1	700.	0
2	940.	-2.
3	212.	.4

Case 1 - 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0217	.0312	.0492	-.372	-.470	7.25	.862	240.	.0382
Initial	.099	.06	.06	-.5	-.5	6.	3.	715.	.114
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.850	.989	.969
P.B.	.655	.879	.342
S.B.	.892	.979	.328
L.S.B.	.705	.984	.207
C.B.U.	.116	.249	.166
C.B.L.	----	----	----
S.Y.	.392	.566	.113
L.Y.T.	----	----	----
L.Y.C.	.395	.550	.115
C.Y.T.	.116	.249	.116
C.Y.C.	----	----	----

$L = 165.$ $R = 60.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50{,}000.$

$r_o = 10.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	5	1	8
	n	11	0	1
2	m	14	1	10
	n	11	0	1
3	m	1	1	7
	n	6	111	1

Loads

L.C.	N	p
1	700.	0.
2	940.	-2.
3	212.	.4

Case 1 - I, 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\overline{t}$
Final	.0212	.0339	.00514	-.409	1.15	8.02	.894	235.	.0374
Initial	.099	.06	.06	-.5	.5	6.	3.	715.	.114
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.822	.993	.686
P.B.	.594	.798	.350
S.B.	.925	.985	.353
L.S.B.	.704	.986	.205
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.391	.592	.119
L.Y.T.	----	----	----
L.Y.C.	.380	.532	.111
C.Y.T.	.123	.269	.164
C.Y.C.	----	----	----

$L = 165.$

Aluminum

$E = 10 \times 10^6$

$R = 60.$

$\gamma = .101$

$\nu = .333$

$\sigma_y = 50,000.$

$r_o = 10.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	10	1	9
	n	12	5	1
2	m	13	1	11
	n	11	0	1
3	m	1	1	7
	n	6	114	1

Loads

L.C.	N	p
1	700.	0
2	940.	-2.
3	212.	.4

Case 1 - It

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0246	.0506	.0510	.341	.931	7.45	.977	303.	.0483
Initial	.099	.06	.06	.5	.5	6.	3.	715.	.114
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	.019	.05	.05	-2.	-2.	.05	.05		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.996	.953	.242
P.B.	.992	.867	.296
S.B.	.999	.832	.279
L.S.B.	.250	.260	.055
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.323	.463	.094
L.Y.T.	----	----	----
L.Y.C.	.326	.453	.096
C.Y.T.	.884	.193	.012
C.Y.C.	----	----	----

$L = 165.$

Aluminum

$E = 10 \times 10^6$

$R = 60.$

$\gamma = .101$

$\nu = .333$

Note: $E = 7.5 \times 10^6$ in L.C. 1

$\sigma_y = 50,000.$

$r_o = 10.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	12	1	7
	n	10	30	1
2	m	12	1	8
	n	10	23	1
3	m	12	1	6
	n	10	83	1

Loads

L.C.	N	p
1	700.	0.
2	940.	-2.
3	212.	.4

Case 2 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0344	.0450	.00684	.460	2.94	7.65	1.19	340.	.0542
Initial	.037	.0443	.00977	.495	2.17	8.29	1.22	370.	.0590
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	0.	0.	0.	-3.	-3.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.770	.996	.241
P.B.	.803	.943	.323
S.B.	.897	.993	.329
L.S.B.	.709	.990	.207
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.536	.777	.154
L.Y.T.	----	----	----
L.Y.C.	.538	.751	.157
C.Y.T.	.168	.350	.258
C.Y.C.	----	----	----

L = 165. R = 60.

Aluminum γ = .101

E = 10×10^6 ν = .333 σ_y = 50,000.

r_o = .07

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	11	1	7
	n	9	27	1
2	m	12	1	8
	n	9	20	1
3	m	10	1	5
	n	9	68	1

Loads

L.C.	N	p
1	1400.	0.
2	1880.	-4.
3	424.	.8

Case 2 - I'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0328	.0503	.0511	.525	1.30	9.76	1.16	389.	.0619
Initial	.037	.0513	.0525	.519	1.29	9.68	1.16	418.	.0664
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	.019	.05	.05	-2.	-2.	.05	.05		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.806	.998	.263
P.B.	.847	.964	.384
S.B.	.909	.996	.336
L.S.B.	.685	.953	.201
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.493	.710	.143
L.Y.T.	----	----	----
L.Y.C.	.497	.692	.145
C.Y.T.	.498	.305	.019
C.Y.C.	----	----	----

$L = 165.$ $R = 60.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50,000.$

$r_o = .07$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	10	1	8
	n	9	24	1
2	m	10	1	9
	n	9	19	1
3	m	9	1	6
	n	9	75	1

Loads

L.C.	N	p
1	1400.	0.
2	1880.	-4.
3	424.	.8

Case 2 - 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0288	.0487	.0436	-.506	-.593	5.60	1.01	363.	.0577
Initial	.1	.1	.1	-.5	-.5	3.	3.	836.	.133
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	0.	0.	0.	-3.	-3.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.926	.991	.947
P.B.	.302	.406	.141
S.B.	.915	.975	.349
L.S.B.	.702	.979	.206
C.B.U.	.152	.341	.186
C.B.L.	----	----	----
S.Y.	.520	.783	.051
L.Y.T.	----	----	----
L.Y.C.	.522	.727	.153
C.Y.T.	.152	.341	.186
C.Y.C.	----	----	----

$L = 165.$ $R = 60.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50{,}000.$

$r_o = 30.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	5
	n	5	0	1
2	m	11	1	6
	n	10	0	1
3	m	1	1	4
	n	6	150	1

Loads

L.C.	N	p
1	1400.	0.
2	1880.	-4.
3	424.	.8

Case 2 - I, 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\overline{\tau}$
Final	.0314	.0518	.00772	-.543	1.34	7.57	1.15	358.	.0571
Initial	.099	.06	.06	-.5	.5	3.	3.	746.	.119
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	0.	0.	0.	-3.	-3.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.827	.991	.622
P.B.	.443	.595	.213
S.B.	.916	.976	.354
L.S.B.	.696	.973	.203
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.499	.731	.143
L.Y.T.	----	----	----
L.Y.C.	.500	.699	.145
C.Y.T.	.160	.355	.206
C.Y.C.	----	----	----

$L = 165.$

Aluminum

$E = 10 \times 10^6$

$R = 60.$

$\gamma = .101$

$\nu = .333$

$\sigma_y = 50,000.$

$r_o = 15.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	9	1	7
	n	11	0	1
2	m	11	1	8
	n	10	0	1
3	m	1	1	5
	n	5	113	1

Loads

L.C.	N	p
1	1400.	0.
2	1880.	-4.
3	424.	.8

Case 3 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0411	.0611	.0115	.560	2.81	9.55	1.33	445.	.0707
Initial	.1	.1	.1	.5	.5	3.	3.	835.	.133
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	0.	0.	0.	-3.	-3.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.781	.993	.250
P.B.	.830	.961	.349
S.B.	.903	.980	.337
L.S.B.	.702	.980	.205
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.620	.905	.177
L.Y.T.	----	----	----
L.Y.C.	.617	.861	.180
C.Y.T.	.191	.412	.027
C.Y.C.	----	----	----

$L = 165.$ $\quad R = 60.$

Aluminum $\quad \gamma = .101$

$E = 10 \times 10^6$ $\quad \nu = .333$ $\quad \sigma_y = 50,000.$

$r_o = 30.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	10	1	7
	n	8	24	1
2	m	10	1	8
	n	7	18	1
3	m	10	1	6
	n	8	67	1

Loads

L.C.	N	p
1	2100.	0.
2	2820.	-6.
3	636.	1.2

Case 3 - I'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0455	.0606	.0520	.590	1.63	9.72	1.47	490.	.0780
Initial	.1	.1	.1	.5	.5	3.	3.	835.	.133
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	.019	.05	.05	-2.	-2.	.05	.05		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.790	.996	.254
P.B.	.850	.974	.343
S.B.	.893	.999	.324
L.S.B.	.710	.988	.208
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.592	.846	.170
L.Y.T.	----	----	----
L.Y.C.	.594	.826	.174
C.Y.T.	.169	.355	.255
C.Y.C.	----	----	----

L = 165. R = 60.

Aluminum γ = .101

$E = 10 \times 10^6$ ν = .333 σ_y = 50,000.

r_o = 15.

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	10	1	6
	n	8	24	1
2	m	10	1	7
	n	8	18	1
3	m	9	1	5
	n	8	55	1

Loads

L.C.	N	p
1	2100.	0
2	2820.	-6.
3	636.	1.2

Case 3 - 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0396	.0625	.0597	-.589	-.708	7.04	1.26	468.	.0745
Initial	.1	.1	.1	-.5	-.5	3.	3.	836.	.133
U.B.	.5	.5	.5	3.	3.	10.	10.		
L.B.	0.	0.	0.	-3.	-3.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.902	.969	.819
P.B.	.410	.550	.173
S.B.	.878	.951	.330
L.S.B.	.673	.937	.197
C.B.U.	.178	.390	.235
C.B.L.	----	----	----
S.Y.	.598	.849	.174
L.Y.T.	----	----	----
L.Y.C.	.605	.843	.177
C.Y.T.	.178	.390	.235
C.Y.C.	----	----	----

L = 165.

Aluminum

$E = 10 \times 10^6$

R = 60.

γ = .101

ν = .333

σ_y = 50,000.

r_o = 30.

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	5
	n	5	0	1
2	m	10	1	6
	n	9	0	1
3	m	1	1	4
	n	5	107	1

Loads

L.C.	N	p
1	2100.	0.
2	2820.	-6.
3	636.	1.2

Case 3 - I, 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{T}$
Final	.0409	.0663	.00737	-.642	1.51	8.81	1.39	457.	.0727
Initial	.1	.1	.1	-.5	.5	3.	3.	835.	.133
U.B.	.5	.5	.5	2.	2.	10.	10.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.878	.992	.765
P.B.	.486	.653	.225
S.B.	.920	.976	.353
L.S.B.	.699	.977	.203
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.585	.858	.167
L.Y.T.	----	----	----
L.Y.C.	.586	.819	.171
C.Y.T.	.190	.417	.025
C.Y.C.	----	----	----

L = 165.

Aluminum

$E = 10 \times 10^6$

R = 60.

γ = .101

ν = .333

σ_y = 50,000.

r_o = 15.

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	6	1	6
	n	10	0	1
2	m	9	1	8
	n	9	0	1
3	m	1	1	5
	n	6	90	1

Loads

L.C.	N	p
1	2100.	0.
2	2820.	1.2
3	636.	-6.

Case 4 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{T}$
Final	.162	.188	.192	1.78	2.54	31.6	6.30	14600.	.230
Initial	.15	.201	.271	1.85	2.87	36.6	4.55	15900.	.251
U.B.	1.	20.	40.	4.	4.	40.	20.		
L.B.	0.	0.	0.	-4.	-4.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.705	.980	.980
P.B.	.467	.897	.912
S.B.	.463	.882	.964
L.S.B.	.213	.963	.522
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.185	.994	.458
L.Y.T.	----	----	----
L.Y.C.	.188	.850	.461
C.Y.T.	.375	.665	.141
C.Y.C.	----	----	----

$L = 500.$ $R = 200.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50{,}000.$

$r_o = 50.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	4
	n	6	36	1
2	m	12	1	8
	n	10	13	1
3	m	8	1	5
	n	12	26	1

Loads

L.C.	N	p
1	2100.	1.
2	8000.	-20.
3	5000.	0.

Case 4 - 0 Starting Point 1

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.216	.315	1.98	-1.38	-0.865	23.7	8.57	21300.	.336
Initial	.15	.200	.489	-1.84	-1.89	35.	4.58	16200.	.255
U.B.	1.	20.	40.	4.	4.	40.	20.		
L.B.	0.	0.	0.	-4.	-4.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.999	.878	.780
P.B.	.285	.962	.622
S.B.	.400	.876	.839
L.S.B.	.0364	.160	.0887
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.148	.727	.362
L.Y.T.	----	----	----
L.Y.C.	.150	.662	.367
C.Y.T.	.0258	.424	.094
C.Y.C.	----	----	----

$L = 500.$ $R = 200.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50,000.$

$r_o = 600.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	2
	n	6	33	1
2	m	19	1	4
	n	0	0	1
3	m	1	1	3
	n	6	19	1

Loads

L.C.	N	p
1	2100.	1.
2	8000.	-20.
3	5000.	0.

Case 4 - 0 Starting Point 2

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.283	.972	3.07	-1.03	-.652	26.3	14.6	26,900.	.423
Initial	.5	.5	.5	-1.5	-1.5	35.	4.	44900.	.707
U.B.	1.	20.	40.	4.	4.	40.	20.		
L.B.	0.	0.	0.	-4.	-4.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.999	.829	.742
P.B.	.311	.948	.682
S.B.	.455	.989	.982
L.S.B.	.002	.007	.004
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.112	.563	.276
L.Y.T.	----	----	----
L.Y.C.	.114	.506	.279
C.Y.T.	.021	.339	.751
C.Y.C.	----	----	----

$L = 500.$ $R = 200.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50{,}000.$

$r_o = 1500.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	1
	n	7	25	1
2	m	21	1	3
	n	0	4	1
3	m	1	1	2
	n	6	20	1

Loads

L.C.	N	p
1	2100.	1.
2	8000.	-20.
3	5000.	0.

Case 4 - 0' Starting Point 2

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{\tau}$
Final	.163	.184	.200	-1.74	-2.13	27.0	6.08	14,700.	.231
Initial	.5	.5	.5	-1.5	-1.5	35.	4.	44,900.	.707
U.B.	1.	1.	1.	4.	4.	40.	20.		
L.B.	0.	0.	0.	-4.	-4.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.923	.957	.769
P.B.	.305	.926	.595
S.B.	.425	.815	.883
L.S.B.	.211	.954	.516
C.B.U.	.037	.661	.141
C.B.L.	----	----	----
S.Y.	.184	.990	.457
L.Y.T.	----	----	----
L.Y.C.	.187	.848	.460
C.Y.T.	.037	.661	.140
C.Y.C.	----	----	----

$L = 500.$ $R = 200.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 50,000.$

$r_o = 375.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	4
	n	6	51	1
2	m	16	1	7
	n	0	0	1
3	m	4	1	4
	n	10	19	1

Loads

L.C.	N	p
1	2100.	1.
2	8000.	-20.
3	5000.	0.

Case 5 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.112	.190	.0224	1.71	10.	31.5	4.14	50,000.	.197
Initial	.25	.25	.3	2.	10.	2.5	4.	124,000.	.490
U.B.	.5	20.	1.	5.	10.1	100.	10.		
L.B.	.1	.02	.01	-5.	-10.1	1.	1.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

L.C.	1	2	3
G.B.	.527	.997	.742
P.B.	.480	.920	.867
S.B.	.499	.736	.984
L.S.B.	.218	.992	.535
C.B.U.	----	----	----
C.B.L.	----	----	----
S.Y.	.145	.847	.362
L.Y.T.	----	----	----
L.Y.C.	.148	.673	.363
C.Y.T.	.025	.629	.115
C.Y.C.	----	----	----

$L = 2000.$ $R = 200.$

Aluminum $\gamma = .101$

$E = 10 \times 10^6$ $\nu = .333$ $\sigma_y = 72,000.$

$r_o = 1000.$

Wave Numbers

L.C.		Gross	Panel	Skin
1	m	1	1	6
	n	2	70	1
2	m	45	1	13
	n	7	12	1
3	m	39	1	7
	n	8	25	1

Loads

L.C.	N	p
1	2100.	1.
2	8000.	-20.
3	5000.	0.

Case 6 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.0363	.249	.263	0.	0.	1.96	1.41	8.35	.0363
Initial	.04	.04	.04	.25	.25	1.0	1.0	13.7	.0596
U.B.	1.	1.	1.	2.	2.	5.	2.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

	f/f_{cr}
G.B.	.968
P.B.	.908
S.B.	.657
L.S.B.	----
C.B.U.	----
C.B.L.	----
S.Y.	.443
L.Y.T.	----
L.Y.C.	.441
C.Y.T.	.146
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	1	1	1
n	5	15	1

$N = 800.$

$p = 0.$

$L = 38.0$

$R = 9.55$

Aluminum

$\gamma = .101$

$E = 10.5 \times 10^6$

$\nu = .33$

$\sigma_y = 50,000.$

$r_o = .5$

Case 6 - I'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{t}$
Final	.0107	.0132	.00991	.121	.291	1.49	.283	4.20	.0182
Initial	.028	.05	.05	.1	.1	1.5	.25	11.78	.0512
U.B.	1.	1.	1.	2.	2.	5.	2.		
L.B.	0.	0.	0.	.05	.05	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

G.B.	.990
P.B.	.725
S.B.	.921
L.S.B.	.966
C.B.U.	----
C.B.L.	----
S.Y.	.958
L.Y.T.	----
L.Y.C.	.965
C.Y.T.	.274
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	13	1	5
n	8	21	1

$N = 800.$

$p = 0.$

$L = 38.0$

Aluminum

$E = 10.5 \times 10^6$

$R = 9.55$

$\gamma = .101$

$\nu = .33$

$\sigma_y = 50,000.$

$r_o = .5$

Case 6 - 0'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	$\bar{r}$
Final	.00932	.0150	.0203	-.142	-.183	2.75	.264	4.30	.0187
Initial	.028	.05	.05	-.1	-.1	1.5	.25	11.8	.0512
U.B.	1.0	1.0	1.0	-.05	-.05	5.	2.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

G.B.	.871
P.B.	.938
S.B.	.994
L.S.B.	.984
C.B.U.	.267
C.B.L.	----
S.Y.	.907
L.Y.T.	----
L.Y.C.	.912
C.Y.T.	.267
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	1	1	10
n	4	18	1

N = 800.

p = 0.

L = 38.0

R = 9.55

Aluminum

γ = .101

E = 10.5×10^6

ν = .33

σ_y = 50,000.

r_o = .3

Case 6 - I, 0'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.00832	.0139	.00145	-.126	.512	2.28	.228	3.76	.0163
Initial	.028	.05	.05	-.1	.1	1.5	.25	11.8	.0512
U.B.	1.	1.	1.	-.05	2.	5.	2.		
L.B.	0.	0.	0.	-2.	.05	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

G.B.	.992
P.B.	.967
S.B.	.972
L.S.B.	.986
C.B.U.	----
C.B.L.	----
S.Y.	.993
L.Y.T.	----
L.Y.C.	.995
C.Y.T.	.317
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	12	1	10
n	7	14	1

N = 800.

p = 0.

L = 38.0

R = 9.55

Aluminum

γ = .101

E = 10.5×10^6

ν = .33

σ_y = 50,000.

r_o = .3

Case 6 - OS

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.0357	.0125	.05	-.156	0.	38.0	1.45	8.54	.0371
Initial	.035	.05	.05	-.1	0.	38.0	.25	12.7	.0550
U.B.	1.	1.	1.	-.05	2.	5.	2.		
L.B.	0.	0.	0.	-2.	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

	f/f_{cr}
G.B.	.976
P.B.	.976
S.B.	.698
L.S.B.	.812
C.B.U.	----
C.B.L.	----
S.Y.	.431
L.Y.T.	----
L.Y.C.	.431
C.Y.T.	.142
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	1	1	20
n	5	5	1

$N = 800.$

$p = 0.$

$L = 38.$ $R = 9.55$

Aluminum $\gamma = .101$

$E = 10.5 \times 10^6$ $\nu = .33$ $\sigma_y = 50{,}000.$

$r_o = .3$

Case 6 - IS

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.0358	.0132	.05	.111	0.	38.0	1.95	8.40	.0365
Initial	.035	.05	.05	.1	0.	38.0	.25	12.7	.0550
U.B.	1.	1.	1.	2.	2.	5.0	2.0		
L.B.	0.	0.	0.	.05	-2.	0.	0.		
$\Delta v \times 10^5$	.1	1.	1.	10.	10.	100.	100.		

f/f_{cr}

G.B.	.993
P.B.	.993
S.B.	.810
L.S.B.	.913
C.B.U.	----
C.B.L.	----
S.Y.	.438
L.Y.T.	----
L.Y.C.	.438
C.Y.T.	.144
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	1	1	19
n	5	5	1

$N = 800.$

$p = 0.$

$L = 38.0$ $\quad R = 9.55$

Aluminum $\quad \gamma = .101$

$E = 10.5 \times 10^6$ $\quad \nu = .33$ $\quad \sigma_y = 50{,}000.$

$r_o = .3$

Case 7 - I

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.111	.725	.940	0.	0.	8.57	3.84	1,960.	.111
Initial	.05	.1	.05	1.	2.	8.	3.	1,680.	.0934
U.B.	10.	10.	10.	20.	20.	50.	20.		
L.B.	0.	0.	0.	-20.	-20.	0.	0.		
$\Delta v \times 10^5$	1.	10.	10.	100.	100.	1000.	1000.		

f/f_{cr}

G.B.	.982
P.B.	.961
S.B.	.215
L.S.B.	0.
C.B.U.	----
C.B.L.	----
S.Y.	.139
L.Y.T.	----
L.Y.C.	.144
C.Y.T.	.0474
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	1	1	2
n	7	25	1

N = 800.

p = 0.

L = 291.

R = 95.5

Aluminum

γ = .101

$E = 10.5 \times 10^6$

ν = .33

σ_y = 50,000.

r_o = 40.

Case 7 - I'

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.0292	.0441	.0943	.718	.810	18.2	1.42	979.	.0555
Initial	.05	.1	.05	1.	2.	8.	3.	1,680.	.0954
U.B.	10.	10.	10.	20.	20.	50.	20.		
L.B.	0.	0.	0.	0.5	0.	0.	0.		
$\Delta v \times 10^5$	1.	10.	10.	100.	100.	1000.	1000.		

	f/f_{cr}
G.B.	.941
P.B.	.920
S.B.	.983
L.S.B.	.984
C.B.U.	----
C.B.L.	----
S.Y.	.314
L.Y.T.	----
L.Y.C.	.307
C.Y.T.	.090
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	8	1	12
n	12	25	1

$N = 800.$

$p = 0.$

$L = 291.$

$R = 95.5$

Aluminum

$\gamma = .101$

$E = 10.5 \times 10^6$

$\nu = .33$

$\sigma_y = 50,000.$

$r_o = 5.$

Case 7 - I''

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.114	3.56	.400	0.	1.64	42.0	11.6	2240.	.127
Initial	.05	.1	.05	1.	2.	8.	3.	1680.	.0934
U.B.	10.	10.	10.	20.	20.	50.	20.		
L.B.	0.	0.	0.	-20.	.5	0.	0.		
$\Delta v \times 10^5$	I.	10.	10.	100.	100.	1000.	1000.		

f/f_{cr}

G.B.	.884
P.B.	.925
S.B.	.993
L.S.B.	0.
C.B.U.	----
C.B.L.	----
S.Y.	.138
L.Y.T.	----
L.Y.C.	.139
C.Y.T.	.0407
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	50	1	2
n	8	18	1

N = 800.

p = 0.

L = 291.

R = 95.5

Aluminum

γ = .101

E = 10.5×10^6

ν = .33

σ_y = 50,000.

r_o = 40.

Case 8 - I, 0

Design Variables

	t_s	t_x	t_ϕ	d_x	d_ϕ	ℓ_x	ℓ_ϕ	W	τ
Final	.226	.320	.206	-3.57	8.72	71.8	7.59	39,400.	.397
Initial	.231	.340	.268	-3.92	3.86	17.7	7.17	46,800.	.472
U.B.	1.	10.	10.	-.5	10.	80.	20.		
L.B.	.019	.05	.05	-10.	.5	.05	.05		
$\Delta v \times 10^5$	10.	10.	10.	100.	100.	1000.	1000.		

f/f_{cr}

G.B.	.993
P.B.	.913
S.B.	.973
L.S.B.	.978
C.B.U.	----
C.B.L.	----
S.Y.	.435
L.Y.T.	----
L.Y.C.	.437
C.Y.T.	.132
C.Y.C.	----

Wave Numbers

	Gross	Panel	Skin
m	3	1	9
n	10	18	1

$N = 12,150.$

$p = 0.$

$L = 361.$

$R = 433.$

Aluminum

$\gamma = .101$

$E = 10.4 \times 10^6$

$\nu = .33$

$\sigma_y = 73,000.$

$r_o = 157.$

APPENDIX E

COMPUTER PROGRAM

E.1 Introduction

The program is written in the Fortran IV language and consists of eleven separately compiled subroutines. It starts by reading the data and then runs automatically until it is terminated either by satisfying the convergence criterion or by exceeding the cycle limit.

The program is capable of handling a maximum of ten separate load conditions. The material of the skin and each set of stiffeners may be different and temperature degradation effects in material properties are allowed; therefore the material constants for the skin and each set of stiffeners must be read for each load condition. This is not necessarily ten, since the number of load conditions is also an input. Although the program is written to include the case of an orthotropic skin material the program has not been tested in other than an isotropic case.

E.2 Description of Input

The required input data is described below in the order it is read. The Fortran symbolic representation is given on the left and an explanation on the right, where order refers to reading from left to right on the data cards. Load conditions are always in increasing order. Except where noted a new data card is started for each Fortran symbolic given below.

Identifier	Explanation
I	Number of load conditions. 10 maximum. Integer.
NSM(,)	Number of modes saved for the approximate analysis. The values for the first load condition are read in the order gross, panel, skin, and then this is repeated for load condition two etc. Integers (I x 3). A separate card is required for each load condition.
DV( )	Initial values of design variables, the order is t_s, t_x, t_ϕ, d_x, d_ϕ, ℓ_x, ℓ_ϕ (inches).

E 1( )	Longitudinal modulus of skin for each load condition, E_x (lbs/in^2).
NUX( )	Poisson's ratio of skin for each load condition, μ_x.
E 2 ( )	Circumferential modulus of skin for each load condition, E_ϕ.
NUY( )	Poisson's ratio of skin for each load condition, μ_ϕ.
GSM( )	Shear modulus of skin for each load condition, G.
EY ( )	Modulus of circumferential stiffener for each load condition, $E_{\phi s}$.
NU 1( )	Poisson's ratio for the circumferential stiffeners for each load condition.
EX( )	Longitudinal stiffener modulus for each load condition, E_{xs}.
NU 2( )	Poisson's ratio for the longitudinal stiffeners for each load condition.
P 1( )	Applied axial compressive loads for each load condition, N (lbs/inch).
P 2( )	Applied external radial pressure for each load condition, p (lbs/inch^2).
GAM( )	Densities of the skin, circumferential stiffeners, and longitudinal stiffeners respectively (lbs/inch^3).
L	Length of cylinder (inches).
R	Radius of cylinder (inches) Note, L and R must be placed on the same data card.
DLT(2)	Indicator, zero when the longitudinal stiffeners are continuous, one otherwise, δ_{xw}.
DLT(3)	Indicator, zero when circumferential stiffeners are continuous, one otherwise, $\delta_{\phi w}$. Note, DLT(2) and DLT(3) must be on the same data card.
ML(,)	Limit on the number of half wave numbers searched in the longitudinal direction for each load condition for each cylinder failure mode. The order is load condition then failure modes. Integer. (3 x I).

NL(,)	Limits on the number of full wave numbers searched. Same order as above. Integer. (3 X I).
LRCU( )	Tensile yield stress for the longitudinal stiffeners for each load condition, σ_{xSOT} (lbs/in^2).
LRCL( )	Compressive yield stress for longitudinal stiffeners for each load condition, σ_{xSOC} (lbs/in^2).
CRCU( )	Tensile yield stress for circumferential stiffeners for each load condition, $\sigma_{\phi SOT}$ (lbs/in^2).
CRCL( )	Compressive yield stress for circumferential stiffeners for each load condition, $\sigma_{\phi SOC}$ (lbs/in^2).
DVL(,)	Upper and lower bounds on the design parameters. The order is upper t_s, t_x, t_ϕ, d_x, d_ϕ, ℓ_x, ℓ_ϕ, and then lower. After the data for the upper bounds a new data card is started for the lower bounds.
BDV( )	Logical variables determining active design variables-one for each variable plus one to tell when the two depth variables are to be kept equal. There are seven quantities in the same order as the design variables plus the additional eighth quantity. The letters T for true and F for false are placed in the even columns 2 through 16 on the data card. T means a variable is active F means it is not. To make the two depth variables equal the last entry is made T and the entry corresponding to d_ϕ is made F.
RD	Initial value of multiplier, r.
RDC	Factor by which r is divided at each cycle.
INCF( )	Finite difference increments for each design variable (inches).
TLIM	Maximum number of cycles.
KAPA(,)	Constants defining yield envelope, $\kappa_{\alpha\beta}$. κ_{TT} is read first for each load condition then starting a new data card κ_{CT} for each load condition. Similarly κ_{CC} and κ_{TC} are read.

SO(,) Skin yield stresses. First S_{x0T} is read for each load condition and then starting a new data card S_{x0C} is read for each load condition. Similarly $S_{\phi 0T}$ and $S_{\phi 0C}$ are read.

All the above variables are real except where designated, integer or logical. The format for preparing the data cards for the logical variables is explained above. The data card for the integer variables are prepared as follows: the integers are punched on the data cards with the rightmost digit in column 5, 10, 15, etc. to column 50, with the exception of NSM(,) which only goes to column 15. The real variables are punched on the data cards in the following manner: Up to five variables are placed on each data card. Fifteen columns are available for each variable. The first variable in an array is placed in columns 1 through 15, the second in 16 through 30, etc. For variable lists (arrays) containing more than five variables the list is continued on the next card starting in the columns 1 through 15. In the 15 columns available for a variable the values are punched as decimal numbers with or without an exponent. For example the value 126.2 may be placed anywhere in the fifteen columns available in any of the following forms: 126.2, 1.262 E02, or 1262.E-01 where E is the power of ten, that is E02 is equivalent to $X10^2$ and E-01 is equivalent to $X10^{-1}$.

Note that even when a design variable is not active its values must be read, in DV, and bounds must be placed on it, in DVL. Also no variable may be exactly equal to a bound or exactly equal to zero.

E.3 Description of Output

The data is written in the same order in which it is read and with the identifiers used in the program. In the output of the number of modes saved, successive load conditions are written on successive lines. For all other data the load conditions are in successive columns. The upper bounds on the m and n search ML and NL are in the order gross, panel, skin, on successive lines. The upper and lower bounds on the design parameters are designated DVLU and DVLL respectively and have the same order as the design variables. BDV and INCF also are written in the same order as the design variables.

The variables κ_{TT}, κ_{CT}, κ_{CC} and κ_{TC} (in the array KAPA(,)) are designated KTT, KCT, KCC, and KTC respectively in the output. The variables S_{x0T}, S_{x0C},

$S_{\phi OT}$, and $S_{\phi OC}$ (in the array SO(,)) are designated SXOT, SXOC, SYOT, AND SYOC in the output.

For each complete analysis the following output is obtained:

CLT	These are the critical buckling load values divided by H_{s2} for the modes saved for the skin for the last load condition.
SMS	The values of m saved, starting with all the values for gross buckling for all load conditions, followed by panel buckling for all load conditions, followed by skin buckling for all load conditions.
SNS	Same as SMS but for the values of n, which are saved.
CRITICAL LOADS	Each line contains the critical buckling load for gross, panel, and skin buckling for one load condition, successive load conditions are on successive lines.
MODE SHAPES	Same order as above giving the values of m and n.
LRS	Stress in the longitudinal rib for each load condition.
CRS	Stress in the circumferential rib for each load condition.
DES	Actual value of distortion energy stress squared for each load condition.
EBU	Critical strain value, circumferential rib, for an expansion of the cylinder, for each load condition.
EBL	Critical strain value, circumferential rib for a contraction of the cylinder.
LRCB	Critical buckling stress for the longitudinal rib, for each load condition.
BEU	Logical variables signifying the existence of a critical strain EBU, T for true, F for false.
BEL	Same as above for EBL.
BLR	Same as above for LRCB.
EPA	Actual value of circumferential strain for each load condition.

TS,TX,TY,DX,DY,LX,LY These correspond to t_s, t_x, t_ϕ, d_x, d_ϕ, ℓ_x, ℓ_ϕ.

AX,AY Areas of the longitudinal and circumferential stiffeners respectively.

The following eleven lines of output are the ratios of the actual values of the behavior variables to the critical values, in columns for each load condition.

G.B. Gross Buckling.

P.B. Panel Buckling.

S.B. Skin Buckling.

LRB Longitudinal Stiffener Buckling.

CRBU Circumferential stiffener buckling for an expansion of the cylinder.

CRBL Circumferential stiffener buckling for a contraction of the cylinder.

S.Y. Skin yield.

LRYU Longitudinal stiffener yield in tension.

LRYL Longitudinal stiffener yield in compression.

CRYU Circumferential stiffener yield in tension.

CRYL Circumferential stiffener yield in compression.

WT Weight of the cylinder in pounds.

TB Equivalent thickness of an unstiffened cylinder made of the skin material.

After each partial analysis the following are written:

WT Weight.

VDP Vector of variables being changed.

SIG The $\sum_i \frac{1}{g_i(\overline{v})}$ for the behavior variables for each point of the finite difference star. In the same order as the design variables with the first element of the vector being the value for the central point.

GS — Elements of gradient to F.

The following variables are printed at other times during the synthesis:

FEM — Value of F at the estimated minimum along a line.

TS — Magnitude of gradient to F.

TEST — Inner-product of unit vector in gradient direction with vector in move direction.

D — Negative of the move direction vector.

CONVERGENCE TEST — The number following this is the amount by which F is estimated to exceed the minimum.

H — The metric matrix.

TT — Twice the minimum move distance.

TP — Vector of design variables a distance TT along the negative gradient direction.

S — Gradient direction.

When a minimum for one value of RD has been obtained this value of RD is printed along with the value of FEM and WT, and the final design point is printed out as XEM. The above is followed by the output for a partial analysis and then by the output for the complete analysis. In some instances the value of WT which is printed on the line with RD and the value which is printed with the complete analysis will differ; the one printed with the complete analysis is the correct one.

Messages of various types will be printed out while the program is running. These are either self explanatory or are explained in the Operational Hints part of the program listing.

E.4 Computer Listings

Following is a list of the computer program, a sample set of output, and a sample set of data.

```
      PROGRAM CONTROL  (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     CONTROL SEGMENT
C  THIS IS THE MAIN PROGRAM.  THE DATA IS READ AND PRINTED.  SOME VARIABLES
C          ARE INITIALIZED.  THE SYNTHESIS SCHEME (SUBROUTINE SYN) IS CALLED.
      REAL LX,LY,L,NU2,NU1,NU,LRCU,LRCL,INCF,LRCB,NUX,NUY
      REAL KAPA
      REAL INCF1
      LOGICAL BDV
      COMMON/RIBP2/GX(10),GY(10)
      COMMON/DIMEN/TS,TX,TY,DX,DY,LX,LY
     1/RIBPRO/EX(10),EY(10),NU2(10),NU1(10)
     2/NLC/I
     4/CPI/PI
     5/SHEPRO/E1(10),E2(10),NUX(10),NUY(10),GSM(10)
     6/LOADS/P1(10),P2(10)
     7/YIELD/LRCU(10),LRCL(10),CRCU(10),CRCL(10),KAPA(4,10),SO(4,10)
     8/MODLIM/ML(3,10),NL(3,10),LM(3,10),LN(3,10)
     9/DEN/GAM(3),DLT(3)
      COMMON/SYNCON/DVL(2,7),INCF(7)
     1/DESVAR/DV(7)
     2/CRTRIB/EBU(10),EBL(10),LRCB(10)
     3/BOODV/BDV(8)
     4/CLR/L,R
     5/CNSM/NSM(10,3)
     6/CRD/RD,RDC
      COMMON/CTIM/TLIM
     2/CINCF1/INCF1(7)
      DIMENSION VDP(7)
   26 FORMAT(5E15.8)
   27 FORMAT(8L2)
  450 FORMAT(3I5)
  451 FORMAT(10I5)
    1 FORMAT(4H0TS=E12.5,4H TX=E12.5,4H TY=E12.5,4H DX=E12.5,4H DY=E12.5
     1,4H LX=E12.5,4H LY=E12.5)
    2 FORMAT(5H0 E1=8E15.5,3X)
    3 FORMAT(6H  NUX=8E15.8)
  402 FORMAT(6H  NUY=8E15.8)
  403 FORMAT(6H   E2=8E15.8)
  404 FORMAT(6H  GSM=8E15.8)
    4 FORMAT(5H  EY=8E15.5,3X)
    5 FORMAT(6H  NU1=8E15.5,2X)
    6 FORMAT(5H  EX=8E15.5,3X)
```

```
   28 FORMAT(6H   NU2=8E15.5,2X)
    7 FORMAT(5H   P1=8E15.5,3X)
    8 FORMAT(5H   P2=8E15.5,3X)
    9 FORMAT(6H   GAM=8E15.5,3X)
   10 FORMAT(3H0L=E12.5,3H R=E12.5,8H DLT(2)=E12.5,8H DLT(3)=E12.5)
   11 FORMAT(3H ML,10I5)
   12 FORMAT(3H NL,10I5)
   13 FORMAT(7H0 LRCU=8E15.5,1X)
   14 FORMAT(7H   LRCL=8E15.5,1X)
   15 FORMAT(7H   CRCU=8E15.5,1X)
   16 FORMAT(7H   CRCL=8E15.5,1X)
   18 FORMAT(7H   DVLU=7E15.5)
   19 FORMAT(7H   DVLL=7E15.5)
   20 FORMAT(5H0BDV=8L5)
   21 FORMAT(4H RD=E15.8,5H RDC=E15.8)
   22 FORMAT(6H INCF=7E15.5)
   29 FORMAT(1H1,7X,16HNO. LOAD COND. =,I3,5X)
   30 FORMAT(1H0,7X,39HNO. MODES SAVED   GROSS   PANEL   SHEET)
   34 FORMAT(1H0,27X,I3,5X,I3,5X,I3)
  400 FORMAT(1H0,9X,3HFM=,E15.8)
  401 FORMAT(1H0,4X,3HG= ,7E15.8)
  405 FORMAT(6H TLIM=E15.8//)
  407 FORMAT(* KTT    *,8E15.5)
  408 FORMAT(* KCT    *,8E15.5)
  409 FORMAT(* KCC    *,8E15.5)
  410 FORMAT(* KTC    *,8E15.5)
  411 FORMAT(* SX0T   *,8E15.5)
  412 FORMAT(* SX0C   *,8E15.5)
  413 FORMAT(* SY0T   *,8E15.5)
  414 FORMAT(* SY0C   *,8E15.5)
 2468 FORMAT(1H )
      READ(5,450)I
      READ(5,450)((NSM(J,M),M=1,3),J=1,I)
      READ(5,26)DV
      READ(5,26)(E1(J),J=1,I)
      READ(5,26)(NUX(J),J=1,I)
      READ(5,26)(E2(J),J=1,I)
      READ(5,26)(NUY(J),J=1,I)
      READ(5,26)(GSM(J),J=1,I)
      READ(5,26)(EY(J),J=1,I)
      READ(5,26)(NU1(J),J=1,I)
      READ(5,26)(EX(J),J=1,I)
      READ(5,26)(NU2(J),J=1,I)
```

```
      READ(5,26)(P1(J),J=1,I)
      READ(5,26)(P2(J),J=1,I)
      READ(5,26)GAM
      READ(5,26)L,R
      READ(5,26)DLT(2),DLT(3)
       READ(5,451)(ML(1,J),J=1,I)
       READ(5,451)(ML(2,J),J=1,I)
       READ(5,451)(ML(3,J),J=1,I)
       READ(5,451)(NL(1,J),J=1,I)
       READ(5,451)(NL(2,J),J=1,I)
       READ(5,451)(NL(3,J),J=1,I)
      READ(5,26)(LRCU(J),J=1,I)
      READ(5,26)(LRCL(J),J=1,I)
      READ(5,26)(CRCU(J),J=1,I)
      READ(5,26)(CRCL(J),J=1,I)
      READ(5,26)(DVL(1,J),J=1,7)
      READ(5,26)(DVL(2,J),J=1,7)
      READ(5,27)BDV
      READ(5,26)RD,RDC
      READ(5,26)(INCF(J),J=1,7)
      READ(5,26)TLIM
      READ(5,26)  (KAPA(1,J),J=1,I)
      READ(5,26)  (KAPA(2,J),J=1,I)
      READ(5,26)  (KAPA(3,J),J=1,I)
      READ(5,26)  (KAPA(4,J),J=1,I)
      READ(5,26)(SO(1,J),J=1,I)
      READ(5,26)(SO(2,J),J=1,I)
      READ(5,26)(SO(3,J),J=1,I)
      READ(5,26)(SO(4,J),J=1,I)
      WRITE(6,29)I
      WRITE(6,2468)
      WRITE(6,30)
      DO 50  J=1,I
      WRITE(6,2468)
   50 WRITE(6,34)(NSM(J,M),M=1,3)
      WRITE(6,2468)
      WRITE(6,1)DV
      WRITE(6,2468)
      WRITE(6,2)(E1(J),J=1,I)
      WRITE(6,3)(NUX(J),J=1,I)
      WRITE(6,403)(E2(J),J=1,I)
      WRITE(6,402)(NUY(J),J=1,I)
      WRITE(6,404)(GSM(J),J=1,I)
```

```
      WRITE(6,4)(EY(J),J=1,I)
      WRITE(6,5)(NU1(J),J=1,I)
      WRITE(6,6)(EX(J),J=1,I)
      WRITE(6,28)(NU2(J),J=1,I)
      WRITE(6,7)(P1(J),J=1,I)
      WRITE(6,8)(P2(J),J=1,I)
      WRITE(6,9)GAM
      WRITE(6,2468)
      WRITE(6,10)L,R,DLT(2),DLT(3)
      WRITE(6,11)(ML(1,J),J=1,I)
      WRITE(6,11)(ML(2,J),J=1,I)
      WRITE(6,11)(ML(3,J),J=1,I)
      WRITE(6,12)(NL(1,J),J=1,I)
      WRITE(6,12)(NL(2,J),J=1,I)
      WRITE(6,12)(NL(3,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,13)(LRCU(J),J=1,I)
      WRITE(6,14)(LRCL(J),J=1,I)
      WRITE(6,15)(CRCU(J),J=1,I)
      WRITE(6,16)(CRCL(J),J=1,I)
      WRITE(6,18)(DVL(1,J),J=1,7)
      WRITE(6,19)(DVL(2,J),J=1,7)
      WRITE(6,2468)
      WRITE(6,20)BDV
      WRITE(6,21)RD,RDC
      WRITE(6,22)INCF
      WRITE(6,405)TLIM
      WRITE(6,407)  (KAPA(1,J),J=1,I)
      WRITE(6,408)  (KAPA(2,J),J=1,I)
      WRITE(6,409)  (KAPA(3,J),J=1,I)
      WRITE(6,410)  (KAPA(4,J),J=1,I)
      WRITE(6,411)  (SO(1,J),J=1,I)
      WRITE(6,412)  (SO(2,J),J=1,I)
      WRITE(6,413)  (SO(3,J),J=1,I)
      WRITE(6,414)  (SO(4,J),J=1,I)
      PI=3.141592653589793
C   INITIALIZATION OF LOWER LIMIT ARRAYS FOR M AND N SEARCH FOR CYLINDER AND
C           SKIN BUCKLING FAILURE MODES
      DO 32 N=1,2
      DO 32 M=1,I
      LM(N,M)=1
   32 LN(N,M)=0
      DO 33 M=1,I
```

```
      LM(3,M)=1
   33 LN(3,M)=1
C  DESIGN VARIABLE AND DESIGN VARIABLE INCREMENT SHIFT FOR ACTIVE VARIABLES
      N=0
      DO 31 M=1,7
      IF (.NOT.BDV(M)) GO TO 31
      N=N+1
      VDP(N)=DV(M)
      INCF1(N)=INCF(M)
   31 CONTINUE
      DO 35 J=1,I
      GX(J)=EX(J)/(2.0*(1.0+NU2(J)))
   35 GY(J)=EY(J)/(2.0*(1.0+NU1(J)))
      CALL SYN (N,VDP)
      END
```

```
      SUBROUTINE ANAL(DV,ML,NL,LM,LN,BPA)
C  THIS SUBROUTINE ANALYZES THE CYLINDER AND CALCULATES THE RATIOS OF
C          ACTUAL VALUES OF THE BEHAVIOR VARIABLES TO THE CRITICAL VALUES.
C          THE CYLINDER BUCKLING LOADS FOR GROSS, PANEL, AND SKIN BUCKLING
C          ARE OBTAINED BY CALLING THE SUBROUTINE CYBL.  THE CRITICAL
C          VALUES FOR THE STIFFENERS ARE OBTAINED BY CALLING THE
C          SUBROUTINE RIBBA.  DV IS THE VECTOR OF DESIGN VARIABLES (COMPLETE).
C          ML AND NL ARE THE ARRAYS OF LOWER LIMITS ON THE M AND N SEARCH.
C          LM AND LN ARE THE UPPER BOUNDS ON THE M AND N SEARCH.
       LOGICAL BPA,BEU,BEL,BLR,BDV
      LOGICAL BML
      REAL LX,LY,NU,NU1,NU2,LRCB,LRCU,LRCL,L,K1,K2,LRS,JNK1,JNK2,NUX,NUY
      REAL KAP,KAPA
      COMMON/DIMEN/TS,TX,TY,DX,DY,LX,LY
     1/RIBPRO/EX(10),EY(10),NU2(10),NU1(10)
     2/CRTRIB/EBU(10),EBL(10),LRCB(10)
     3/CBVA/BEU(10),BEL(10),BLR(10)
     4/NLC/I
     5/CPI/PI
     6/SHEPRO/E1(10),E2(10),NUX(10),NUY(10),GSM(10)
     7/LOADS/P1(10),P2(10)
     8/YIELD/LRCU(10),LRCL(10),CRCU(10),CRCL(10),KAPA(4,10),SO(4,10)
     9/DESVAR/DV(7)
      COMMON/BOODV/BDV(8)
     1/CLR/L,R/CBF/BF(11,10)
     2 /CJA/SR1,SR2,SR3,SR4,SR5,SR6,SR7,SR8,SR9,RG1,RG2,RG3,RG4,RG5,RG6,
     3RG7,
     3HS1,HS2,H2,H3,K1,K2
     4/CNSM/NSM(10,3)
     5/CMSNS/MS(3,10),NS(3,10)
      DIMENSION SP1(10),SP2(10),LRS(10),CRS(10),DES(10),EPA(10),CL1(10),
     1CL2(10),CL3(10),                  ML(3,10),LM(3,10),NL(3,10),
     2LN(3,10)
  401 FORMAT(1H09X,14HCRITICAL LOADS,5X,5HGROSS,10X,5HPANEL,10X,5HSHEET)
  402 FORMAT(1H023X,3E15.8)
  403 FORMAT(1H09X,11HMODE SHAPES,8X,5HM   N,10X,5HM   N,10X,5HM   N)
  404 FORMAT(1H0,18X,3(I11,I4))
  405 FORMAT(8H0  LRS  ,8E15.8)
  406 FORMAT(8H   CRS  ,8E15.8)
  407 FORMAT(8H   DES  ,8E15.8)
  408 FORMAT(8H   EBU  ,8E15.8)
  409 FORMAT(8H   EBL  ,8E15.8)
```

```
 410 FORMAT(8H    LRCB ,8E15.8)
 411 FORMAT(8H    BEU  ,10L3)
 412 FORMAT(8H    BEL  ,10L3)
 413 FORMAT(8H    BLR  ,10L3)
 414 FORMAT(8H    EPA  ,8E15.8)
 415 FORMAT(10H ENTER ANA)
 416 FORMAT(* WARNING, A MODE SHAPE NUMBER IS APPROACHING ITS UPPER LIM
    1IT*)
2468 FORMAT(1H )
     BML=.FALSE.
     TS=DV(1)
     TX=DV(2)
     TY=DV(3)
     DX=DV(4)
     DY=DV(5)
     LX=DV(6)
     LY=DV(7)
     IF (BDV(8)) DY=DX
  20 RG1=((DX*DX)/3.0)+(ABS(DX)*TS/2.0)+((TS*TS)/4.0)
     RG2=((DY*DY)/3.0)+(ABS(DY)*TS/2.0)+((TS*TS)/4.0)
     RG3=(ABS(DX)+TS)/2.0
     IF (DX.LT.0.0) RG3=-RG3
     RG4=(ABS(DY)+TS)/2.0
     IF (DY.LT.0.0) RG4=-RG4
     IF ( ABS(DX) .LT. TX) GO TO 38
     DR1=TX*TX
     DR2=ABS(DX)
     DR3=DR2/TX
     GO TO 39
  38 DR1=DX*DX
     DR2=TX
     DR3=TX/ABS(DX)
  39 DR3=0.49*DR3
     DR4=1000.0
     IF ( DR3 .LT. 15.0) DR4=EXP(DR3)
     DR5=0.316-(0.285/DR4)
     RG5=DR1*DR5
     IF ( ABS(DY) .LT. TY) GO TO 40
     DR1=TY*TY
     DR2=ABS(DY)
     DR3=DR2/TY
     GO TO 41
  40 DR1=DY*DY
```

```
      DR2=TY
      DR3=TY/ABS(DY)
   41 DR3=0.49*DR3
      DR4=1000.0
      IF ( DR3 .LT. 15.0) DR4=EXP(DR3)
      DR5=0.316-(0.285/DR4)
      RG6=DR1*DR5
      RG7=((2.0*DY*DY*ABS(DY))+(4.0*TS*DY*DY)+(3.0*TS*TS*ABS(DY))
     1+(TS*TS*TS))/8.0
      IF (DY.LT.0.0) RG7=-RG7
       K1=PI/L
      K2=1.0
       SR1=(TS*TS)/12.0
      DO 23 J=1,I
      SR7=NUX(J)*NUY(J)
      SR9=1.0-SR7
      HS1=(E1(J)*TS)/SR9
      HS2=(E2(J)*TS)/SR9
      H2=(EX(J)*ABS(DX)*TX)/LY
       H3=(EY(J)*ABS(DY)*TY)/LX
      SR2=H2/HS1
      SR3=H3/HS2
       SR4=1.0+SR2
       SR5=1.0+SR3
       SR6=SR5*SR4
       SR8=SR6-SR7
C   SKIN LOADS
      SP1(J)=(((SR5-SR7)*P1(J))+(NUX(J)*SR2*R*P2(J)))/SR8
      SP2(J)=(((SR4-SR7)*P2(J))+(NUY(J)*SR3*(P1(J)/R)))/SR8
C   STIFFENER STRESSES
      LRS(J)=-((EX(J)/HS1)*((SR5*P1(J))-(NUX(J)*P2(J)*R)))/SR8
      CRS(J)=+((EY(J)/HS2)*((NUY(J)*P1(J))-(SR4*P2(J)*R)))/SR8
C   SKIN STRESS X
      JNK1=-SP1(J)/TS
C   SKIN STRESS PHI
      JNK2=-SP2(J)*R/TS
      IF(JNK1.LT.0.0)   GO TO 42
      SXA=SO(1,J)
      IF(JNK2.LT.0.0)   GO TO 43
      SFB=SO(3,J)
      KAP=KAPA(1,J)
      GO TO 44
   43 SFB=SO(4,J)
```

```
      KAP=KAPA(4,J)
      GO TO 44
   42 SXA=SO(2,J)
      IF(JNK2.LT.0.0)  GO TO 45
      SFB=SO(3,J)
      KAP=KAPA(2,J)
      GO TO 44
   45 SFB=SO(4,J)
      KAP=KAPA(3,J)
C     DISTORTION ENERGY CRITERION
   44 DES(J)=JNK1**2/SXA**2-JNK1*JNK2/(SXA*SFB)+JNK2**2/SFB**2
C     CIRCUMFERENTIAL RIB STRAIN
      EPA(J)=CRS(J)/EY(J)
C     CRITICAL LOAD FOR GROSS BUCKLING, CL1(J)
      CALL CYBL(P1(J),P2(J),R,L,CL1(J),MS(1,J),NS(1,J),ML(1,J),NL(1,J),
     1LM(1,J),LN(1,J),BPA,1,J)
   23 CL1(J)=CL1(J)*HS2
      K1=PI/LX
      H3=0.0
      RG2=0.0
      RG4=0.0
      RG6=0.0
      RG7=0.0
      SR3=0.0
      SR5=1.0
      DO 9 J=1,I
      IF (LX.NE.L) GO TO 24
      CL2(J)=CL1(J)
       GO TO 9
   24 SR7=NUX(J)*NUY(J)
      SR9=1.0-SR7
      HS1=(E1(J)*TS)/SR9
      HS2=(E2(J)*TS)/SR9
       H2=(EX(J)*ABS(DX)*TX)/LY
      SR2=H2/HS1
      SR4=1.0+SR2
C     CRITICAL LOAD FOR PANEL BUCKLING, CL2(J)
      CALL CYBL(P1(J),P2(J),R,LX,CL2(J),MS(2,J),NS(2,J),ML(2,J),
     1NL(2,J),LM(2,J),LN(2,J),BPA,2,J)
      CL2(J)=CL2(J)*HS2
    9 CONTINUE
      K1=PI/LX
      K2=PI*R/LY
```

```
      RG1=0.0
      RG3=0.0
      RG5=0.0
      SR2=0.0
      SR4=1.0
      SR6=1.0
      H2=0.0
      DO 25 J=1,I
      SR7=NUX(J)*NUY(J)
      SR9=1.0-SR7
      HS1=(E1(J)*TS)/SR9
      HS2=(E2(J)*TS)/SR9
C   CRITICAL LOAD FOR SKIN BUCKLING, CL3(J)
      CALL CYBL(SP1(J),SP2(J),R,LX,CL3(J),MS(3,J),NS(3,J),ML(3,J),
     1NL(3,J),LM(3,J),LN(3,J),BPA,3,J)
   25 CL3(J)=CL3(J)*HS2
C   STIFFENER BUCKLING ANALYSIS
      CALL RIBBA
C   PRINTOUT FOR END OF CYCLE
      IF ((.NOT.BPA).OR.(ML(2,1).EQ.LM(2,1))) GO TO 37
      WRITE(6,2468)
      WRITE(6,401)
      DO 100 J=1,I
      WRITE(6,2468)
  100 WRITE(6,402)CL1(J),CL2(J),CL3(J)
      WRITE(6,2468)
      WRITE(6,403)
      DO 101 J=1,I
      DO 102 M=1,3
      IF((MS(M,J).GT.(ML(M,J)-5)).OR.(NS(M,J).GT.(NL(M,J)-5)))BML=.TRUE.
  102 CONTINUE
      WRITE(6,2468)
  101 WRITE(6,404)( MS(M,J),NS(M,J) ,M=1,3)
      IF(BML)  WRITE(6,416)
      WRITE(6,2468)
      WRITE(6,405)(LRS(J),J=1,I)
      WRITE(6,406)(CRS(J),J=1,I)
      WRITE(6,407)(DES(J),J=1,I)
      WRITE(6,408)(EBU(J),J=1,I)
      WRITE(6,409)(EBL(J),J=1,I)
      WRITE(6,410)(LRCB(J),J=1,I)
      WRITE(6,411)(BEU(J),J=1,I)
      WRITE(6,412)(BEL(J),J=1,I)
```

```
      WRITE(6,413)(BLR(J),J=1,I)
      WRITE(6,414)(EPA(J),J=1,I)
C   CALCULATION OF RATIOS OF ACTUAL VALUES OF BEHAVIOR VARIABLES TO CRITICAL
C           VALUES
   37 DO 26 J=1,I
      IF (P1(J).NE.0.0) GO TO 27
       BF(1,J)=P2(J)/CL1(J)
      IF(CL1(J).EQ.0.)  BF(1,J)=0.
       BF(2,J)=P2(J)/CL2(J)
      IF(CL2(J).EQ.0.)  BF(2,J)=0.
       GO TO 28
   27 BF(1,J)=P1(J)/CL1(J)
      IF(CL1(J).EQ.0.)  BF(1,J)=0.
      BF(2,J)=P1(J)/CL2(J)
      IF(CL2(J).EQ.0.)  BF(2,J)=0.
   28 IF (SP1(J).EQ.0.0) GO TO 29
      BF(3,J)=SP1(J)/CL3(J)
      IF(CL3(J).EQ.0.)  BF(3,J)=0.
      GO TO 30
   29 BF(3,J)=SP2(J)/CL3(J)
      IF(CL3(J).EQ.0.)  BF(3,J)=0.
   30 IF (BLR(J)) GO TO 31
      BF(4,J)=0.0
      GO TO 32
   31 BF(4,J)=LRS(J)/LRCB(J)
   32 IF (BEU(J)) GO TO 33
      BF(5,J)=0.0
      GO TO 34
   33 BF(5,J)=EPA(J)/EBU(J)
   34 IF (BEL(J)) GO TO 35
      BF(6,J)=0.0
       GO TO 36
   35 BF(6,J)=EPA(J)/EBL(J)
   36 BF(7,J)=SQRT(DES(J))
      BF(8,J)=LRS(J)/LRCU(J)
       BF(9,J)=LRS(J)/LRCL(J)
       BF(10,J)=CRS(J)/CRCU(J)
   26 BF(11,J)=CRS(J)/CRCL(J)
      RETURN
      END
```

```
      SUBROUTINE CYBL(LD1,LD2,R,L,CL1,MS,NS,MLT,NLT,LMT,LNT,BPA,FMI,J)
C  THIS SUBROUTINE CALCULATES THE NON-DIMENSIONAL BUCKLING LOAD FOR THE
C          STIFFENED CYLINDER AND STORES A SELECTED SET OF MODE NUMBERS
C          TO BE USED IN THE APPROXIMATE ANALYSIS.
C  INPUT
C          LD1    AXIAL COMPRESSION LOAD
C          LD2    EXTERNAL LATERAL PRESSURE
C          R      RADIUS
C          L      LENGTH
C          MLT,NLT   UPPER LIMITS ON M AND N SEARCH
C          LMT,LNT   LOWER LIMITS ON M AND N SEARCH
C          BPA       IF TRUE, COMPLETE ANALYSIS.  IF FALSE, APPROXIMATE ANALYSIS
C          FMI       FAILURE MODE INDEX
C                    1 - GROSS,  2 - PANEL,  3 - SKIN
C          J         LOAD CONDITION INDEX
C  OUTPUT
C          CL1       NONDIMENSIONAL CRITICAL LOAD
C          MS,NS     VALUES OF M AND N FOR THE CRITICAL LOAD
      LOGICAL BPA
      REAL LD1,LD2,L,LMD,JNK1,JNK2,JNK3,JNK4,JNK5,K1,K2,LX,LY,NU,NU1,NU2
      REAL LMDS,JNKA,JNKB,JNKC,JNKD,JNKE,JNKF,JNKG,JNKH,JNKI,NUX,NUY
      INTEGER FMI,SMS,SNS,CTS,RI,TNSM
      COMMON/DIMEN/TS,TX,TY,DX,DY,LX,LY
     1/CJA/SR1,SR2,SR3,SR4,SR5,SR6,SR7,SR8,SR9,RG1,RG2,RG3,RG4,RG5,RG6,
     2RG7,
     2HS1,HS2,H2,H3,K1,K2
     3/SHEPRO/E1(10),E2(10),NUX(10),NUY(10),GSM(10)
     4/CNSM/NSM(10,3)
     5/NLC/I
     6/RIBPRO/EX(10),EY(10),NU2(10),NU1(10)
      COMMON/RIBP2/GX(10),GY(10)
      DIMENSION SMS(100,3,10),SNS(100,3,10),CLT(100)
  400 FORMAT(*1*,5X*CLT*)
  401 FORMAT(8H     SMS)
  402 FORMAT(8H     SNS)
  403 FORMAT(8H        ,8E15.5)
  404 FORMAT(3H   25I5)
  405 FORMAT(8H  JNK1  5E15.5)
  406 FORMAT(8H     C= 7E15.5)
  407 FORMAT(11H ENTER CYBL)
  408 FORMAT(1H ,4X,9HAB BB CB ,3E15.8)
  409 FORMAT(1H ,4X,4HM  N,2I4)
```

```
  410 FORMAT(1H ,2X,10E12.5)
      ML=MLT
      NL=NLT
      LM=LMT
      LN=LNT
      NST=NSM(J,FMI)
      DO 30 M=1,NST
   30 CLT(M)=0.0
C   TEST FOR ALL TENSILE LOADS
      IF ((LD1.GT.0.0).OR.(LD2.GT.0.0)) GO TO 1
      CL1=0.0
      GO TO 31
C   TEST FOR AXIAL LOAD
    1 IF (LD1.NE.0.0) GO TO 17
      ALF=1.0
      DP=0.0
      GO TO 18
   17 ALF=LD2/LD1
      DP=1.0
   18 RI=0
C   CALCULATION OF TERMS INDEPENDENT OF LAMBDA AND ETA
      F1=(GSM(J)*SR9)/E1(J)
      F2=(GSM(J)*SR9)/E2(J)
      SR10=SR2*RG3
      SR11=SR3*RG4
      F3=H2/HS2
      SR12=F3*RG1
      F4=F3*RG3
      SR13=SR3*RG2
      SR14=(GX(J)*TX*ABS(DX)*RG5)/(LY*HS2)
      SR15=(GY(J)*TY*ABS(DY)*RG6)/(LX*HS2)
      C11B=F1/R
      C11A1=R*(HS1+H2)/HS2
      C12A=NUY(J)+F1
      C13A=-NUX(J)
      C13B1=H2*RG3*R/HS2
      C13B=C13B1-E1(J)*SR1/E2(J)
      C31B=C13B1-SR1
      RSQ=R*R
      C13C=F2*SR1/RSQ
      C22B=(SR5-(SR11/R))/R
      C23A=SR5/R
      C23B=SR11/RSQ
```

```
      JNKA=(F2*R)+(SR14/R)
      JNKC=((F2*SR1*3.0)+(NUX(J)*SR1)+SR14)/R
      C33A=C23A+(SR11/RSQ)
      C33B=2.0*((SR1/R)+SR11+(SR13/R))/RSQ
      JNKF=((NUX(J)*2.0*SR1)+(F2*SR1*4.0)+SR14+SR15)/R
      C33D=(SR1+SR13+((SR3*RG7)/R))/(R*RSQ)
      JNKG=((SR1*E1(J)/E2(J))+SR12)*R
      IF (BPA) GO TO 19
      TNSM=NSM(J,FMI)
      GO TO 20
   19 TNSM=1
C  MODE SELECTION FOR APPROXIMATE ANALYSIS
   20 DO 10 CTS=1,TNSM
      IF (BPA) GO TO 22
      LM=SMS(CTS,FMI,J)
      ML=LM
      LN=SNS(CTS,FMI,J)
      NL=LN
C  CALCULATION OF TERMS DEPENDING ON LAMBDA
   22 DO 5 M=LM,ML
      LMD=K1*FLOAT(M)
      LMDS=LMD*LMD
      C11A=LMDS*C11A1
      C12B=LMD*C12A
      C13D=(C13A*LMD)+(C13B*LMD*LMDS)
      C31D=(C13A*LMD)+(C31B*LMD*LMDS)
      C13E=C13C*LMD
      C22A=LMDS*JNKA
      C23C=JNKC*LMDS
      C33C=JNKF*LMDS
      C33E=JNKG*LMDS*LMDS
C  CALCULATION OF TERMS DEPENDING ON ETA
      DO 5 N=LN,NL
      ETA=K2*FLOAT(N)
      ETAS=ETA*ETA
      C11=-C11A-(C11B*ETAS)
      C12=-ETA*C12B
      C13G=C13E*ETAS
      C13=C13D+C13G
      C31=C31D+C13G
      C22=-C22A-(ETAS*C22B)
      C23=-(C23A*ETA)+(C23B*ETA*ETAS)-(C23C*ETA)
      C33=-C33A+(C33B*ETAS)-(C33C*ETAS)-(C33D*ETAS*ETAS)-C33E
```

```
      AB=(C11*LMDS*((R*LMDS)+((ETAS-1.0)*ALF)))*DP*R
      JNK1=C11*C33-C13*C31
      JNK2=(C11*C22)-(C12*C12)
      BB=((JNK2+JNK1)*R*(LMD*LMD)*DP)+(JNK2*((ETA*ETA)-1.0)*ALF)
      CB=(C11*C22*C33)-(C12*C12*C33)+(C12*C23*C31)+
     1(C13*C12*C23)-(C13*C31*C22)-(C11*C23*C23)
C   TEST FOR AXIAL LOAD
      IF (DP.NE.0.0) GO TO 24
      PC=-CB/BB
      IF ( PC .LE. 0.0 ) GO TO 5
      GO TO 15
   24 JNK4=(4.0*AB)*(CB/BB)
      JNK1=BB*(BB-JNK4)
      IF (JNK1.LT.0.0) GO TO 2
      JNK1=SQRT(JNK1)
      JNK2=(-BB-JNK1)/(2.0*AB)
      JNK3=(-BB+JNK1)/(2.0*AB)
      JNK4=JNK4/BB
      IF (ABS(JNK4).GT.0.1) GO TO 25
      JNK5=(JNK4/2.0)+((JNK4*JNK4)/8.0)+((JNK4**3)/16.0)+
     1((5.0*(JNK4**4))/128.0)
      JNK2=(BB*(JNK5-2.0))/(2.0*AB)
      JNK3=(-BB*JNK5)/(2.0*AB)
   25 IF (LD1.GT.0.0) GO TO 3
      IF (JNK2.LT.0.0) GO TO 4
      IF (JNK3.GT.0.0) GO TO 5
      GO TO 6
    4 IF (-BB.LT.0.0) GO TO 7
    9 PC=JNK2
      GO TO 8
    7 IF (JNK3.GT.0.0) GO TO 9
    6 PC=JNK3
    8 IF (RI.EQ.0) GO TO 11
      IF (PC.GT.CL1) GO TO 11
      GO TO 16
    3 IF (JNK2.GT.0.0) GO TO 12
      IF (JNK3.GT.0.0) GO TO 13
   34 CONTINUE
      WRITE(6,405)JNK1,JNK2,JNK3,JNK4,JNK5
      WRITE(6,406)C11,C12,C13,C22,C23,C33
      WRITE(6,408)AB,BB,CB
      WRITE(6,409)M,N
      WRITE(6,410)LMD,ETA,K1,K2,SR4,R
```

```
      GO TO 5
   12 IF (-BB.GT.0.0) GO TO 14
      IF (JNK3.LT.0.0) GO TO 14
   13 PC=JNK3
      GO TO 15
   14 PC=JNK2
C  SELECTION OF MODES FOR APPROXIMATE ANALYSIS
   15 IF (RI.EQ.0) GO TO 11
      IF (PC.GT.CL1) GO TO 16
   11 CL1=PC
      MS=M
      NS=N
      RI=1
   16 IF (ML.EQ.LM) GO TO 27
      P=PC
      MT=M
      NT=N
      DO 28 ISC=1,NST
      IF (CLT(ISC).NE.0.0) GO TO 26
      CLT(ISC)=P
      SMS(ISC,FMI,J)=MT
      SNS(ISC,FMI,J)=NT
      GO TO 27
   26 CONTINUE
      IF (ABS(P).GT.ABS(CLT(ISC))) GO TO 28
      PT=CLT(ISC)
      MTT=SMS(ISC,FMI,J)
      NTT=SNS(ISC,FMI,J)
      CLT(ISC)=P
      SMS(ISC,FMI,J)=MT
      SNS(ISC,FMI,J)=NT
      P=PT
      MT=MTT
      NT=NTT
   28 CONTINUE
   27 CONTINUE
C  END OF SELECTION OF MODES FOR APPROXIMATE ANALYSIS
      GO TO 5
C  PRINTOUT FOR IMAGINARY ROOTS
    2 WRITE(6,406)C11,C12,C13,C22,C23,C33
      WRITE(6,408)AB,BB,CB
      WRITE(6,409)M,N
      WRITE(6,405)JNK1
```

```
    5 CONTINUE
   10 CONTINUE
   31 IF (((ML.EQ.LM).OR.(FMI.NE.3)).OR.(J.NE.I)) GO TO 29
C     PRINTOUT FOR END OF CYCLE
      WRITE(6,400)
      WRITE(6,403)(CLT(JD),JD=1,NST)
      WRITE(6,401)
      DO 32 ID=1,3
      DO 32 LD=1,I
      NST=NSM(LD,ID)
   32 WRITE(6,404)(SMS(JD,ID,LD),JD=1,NST)
      WRITE(6,402)
      DO 33 ID=1,3
      DO 33 LD=1,I
      NST=NSM(LD,ID)
   33 WRITE(6,404)(SNS(JD,ID,LD),JD=1,NST)
   29 CONTINUE
      RETURN
      END
```

```
      FUNCTION EPB(N,NU,D,TY,DY)
C  CRITICAL EDGE STRAIN
C  THIS FUNCTION CALCULATES THE CRITICAL BUCKLING STRAIN FOR A CIRCUMFERENTIAL
C           STIFFENER SIMPLY SUPPORTED ON ONE EDGE AND FREE ON THE OTHER.
C           N      NUMBER OF WAVES IN CIRCUMFERENTIAL DIRECTION
C           NU     POISSON≠S RATIO
C           D      RATIO OF STIFFENER DEPTH TO RADIUS OF SUPPORTED EDGE
C           TY     THICKNESS OF STIFFENER
C           DY     DEPTH OF STIFFENER PORTION IN QUESTION
C           EPB    CRITICAL CIRCUMFERENTIAL STRAIN
      REAL NU,D,TY,DY,A,B,C,N
  400 FORMAT(12H NU D TY DY ,4E15.5)
      A=(((2.0*(N**2)*(1.0-NU))+1.0)*D)
     2+((((2.0*N*N*(2.0-NU))-1.0)/2.0)*D*D)
     3+((((N*N)-1.0)*((N*N)-1.0))*
     4(((D**3)/3.0)-((D**4)/4.0)))
      B=1.0+((((2.0*(N**2))-1.0)/3.0)*D)
     2 +((0.25-((N**2)/3.0))*(D**2))
      C=-(1.0/(6.0*(1.0-(NU**2))))*((TY/DY)**2)
     2*((1.0+((1.0*(1.0-NU))*(D+((D**2)/2.0))))/(1.0+(2.0*D)+(D**2)))
      EPB=(C*A)/B
      RETURN
      END
```

```
      SUBROUTINE FP(VDP,RD,FM,G,B1)
C  THIS SUBROUTINE USES THE VALUES OF THE ACTIVE DESIGN VARIABLES, VDP,
C          AND THE VALUE OF RD TO CALCULATE THE UNCONSTRAINED FUNCTION, FM,
C          AND ITS GRADIENT, G.  THE DESIGN VARIABLE BOUNDS ARE CHECKED.
C          USING THE SUBROUTINE ANAL THE BEHAVIOR FUNCTIONS ARE GENERATED.
C          THE BEHAVIOR IS CHECKED.  IF B1 IS TRUE A COMPLETE ANALYSIS
C          IS USED.  IF B1 IS FALSE AN APPROXIMATE ANALYSIS IS USED.
      LOGICAL B1,BDV
      LOGICAL BDVV1,BDVV2
      REAL INC,LBS,NR,NX,LX,LY,L,MND,INCF
      INTEGER CC
      COMMON/DIMEN/TS,TX,TY,DX,DY,LX,LY
     1/CBF/BF(11,10)
     1/DEN/GAM(3),DLT(3)
     3/NLC/I
     4/SYNCON/DVL(2,7),INCF(7)
     5/BOODV/BDV(8)
     6/CLR/L,R
     7/CPI/PI
     8/CWT/WT
     9/DESVAR/DV(7)
      COMMON/MODLIM/ML(3,10),NL(3,10),LM(3,10),LN(3,10)
     1/CMSNS/MS(3,10),NS(3,10)
     2/CBDVV/BDVV1,BDVV2
      DIMENSION VDP(7),G(7),                  WG(7),DVI(7),INC(7),SIG(8)
     1,GSC(7),GS(7)
  400 FORMAT(8H0  WT   ,E15.8)
  401 FORMAT(8H     WG  ,7E16.8)
  402 FORMAT(8H     GSC ,7E16.8)
  403 FORMAT(4H SIG,8E15.8)
  404 FORMAT(8H     GS  ,7E16.8)
  415 FORMAT(4H0TS=E15.8,4H TX=E15.8,4H TY=E15.8,4H DX=E15.8,4H DY=E15.8
     1,4H LX=E15.8,4H LY=E14.7)
  416 FORMAT(8H0  G.B. ,8E15.8)
  417 FORMAT(8H0  P.B. ,8E15.8)
  418 FORMAT(8H0  S.B. ,8E15.8)
  419 FORMAT(8H0  LRB. ,8E15.8)
  420 FORMAT(8H0  CRBU ,8E15.8)
  421 FORMAT(8H0  CRBL ,8E15.8)
  422 FORMAT(8H0  S.Y. ,8E15.8)
  423 FORMAT(8H0  LRYU ,8E15.8)
  424 FORMAT(8H0  LRYL ,8E15.8)
```

```
  425 FORMAT(8H0  CRYU ,8E15.8)
  426 FORMAT(8H0  CRYL ,8E15.8)
  427 FORMAT(9H ENTER FP)
  428 FORMAT(1H ,4X,4HVDP=,7E15.8)
  429 FORMAT(4H0AX=E15.8,4H AY=E15.8)
  430 FORMAT(8H0  TB   ,E15.8//)
  431 FORMAT(* WARNING, DX LESS THAN 5XTS,  SKIN BUCKLING FAILURE MODE M
     1AY NOT BE MEANINGFUL.*)
  432 FORMAT(* WARNING.  DY LESS THAN 5XTS.  SKIN BUCKLING AND PANEL BUC
     1KLING FAILURE MODES MAY NOT BE MEANINGFUL.*)
 2468 FORMAT(1H )
      BDVV1=.FALSE.
      BDVV2=.FALSE.
      N=0
       DO 31 M=1,7
       IF (.NOT.BDV(M)) GO TO 31
      N=N+1
      DV(M)=VDP(N)
   31 CONTINUE
      IF (BDV(8)) DV(5)=DV(4)
C  BEGINNING OF DESIGN VARIABLE CHECK
      DO 34 M=1,7
      IF ((DV(M).LT.DVL(1,M)).AND.(DV(M).GT.DVL(2,M))) GO TO 34
      FM=1.0E30
      BDVV1=.TRUE.
      GO TO 7
   34 CONTINUE
C  COMPATIBILITY BOUND CHECK
      IF ((DV(2).LT.DV(7)).AND.(DV(3).LT.DV(6)))  GO TO 35
      FM=1.0E30
      BDVV2=.TRUE.
      GO TO 7
C  END OF DESIGN VARIABLE CHECK
   35 CC=0
C  ANALYSIS OF INPUT DESIGN---COMPLETE IF B1 IS TRUE, APPROXIMATE IF B1 IS FALSE
      CALL ANAL(DV,ML,NL,LM,LN,B1)
      IF (.NOT.B1) GO TO 54
C  PRINTOUT FOR END OF CYCLE
      WRITE(6,2468)
      WRITE(6,415)TS,TX,TY,DX,DY,LX,LY
      IF(ABS(DV(4)).LT.(5.*DV(1)))  WRITE(6,431)
      IF(ABS(DV(5)).LT.(5.*DV(1)))  WRITE(6,432)
      AX=TX*DX
```

```
      AY=TY*DY
      WRITE(6,2468)
      WRITE(6,429)AX,AY
      WRITE(6,2468)
      WRITE(6,416)(BF(1,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,417)(BF(2,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,418)(BF(3,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,419)(BF(4,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,420)(BF(5,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,421)(BF(6,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,422)(BF(7,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,423)(BF(8,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,424)(BF(9,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,425)(BF(10,J),J=1,I)
      WRITE(6,2468)
      WRITE(6,426)(BF(11,J),J=1,I)
   54 CONTINUE
C  BEGINNING OF BEHAVIOR VARIABLE CHECK
      DO 36 M=1,11
      DO 36 J=1,I
      IF (BF(M,J).LT.1.0) GO TO 36
      FM=1.0E30
      GO TO 7
C  END OF BEHAVIOR VARIABLE CHECK
   36 CONTINUE
C  BEGINNING OF CALCULATION OF WEIGHT AND GRADIENT TO WEIGHT
      IF (((DX.GT.0.0).AND.(DY.GT.0.0)).OR.((DX.LT.0.0).AND.(DY.LT.0.0))
     1) GO TO 38
      DLT(1)=0.0
      GO TO 39
   38 DLT(1)=1.0
   39 GD=(GAM(2)*DLT(2))+(GAM(3)*DLT(3))
      MND=AMIN1(ABS(DX),ABS(DY))
      NR=(L/LX)-1.0
```

```
      NX=(2.0*PI*R)/LY
      SW=ABS((2.0*R*DY)-(DY*DY)-(TS*ABS(DY)))
      WT=(2.0*PI*R*TS*L*GAM(1))+(SW*GAM(2)*TY*PI*NR)+
     1(L*TX*GAM(3)*ABS(DX)*NX)-(MND*DLT(1)*TX*TY*GD)
C   END OF CALCULATION OF WEIGHT
      WRITE(6,2468)
      WRITE(6,400)WT
      IF (.NOT.B1) GO TO 58
      TB=WT/(2.0*PI*R*L*GAM(1))
      WRITE(6,2468)
      WRITE(6,430)TB
   58 CONTINUE
      WRITE(6,428) VDP
      IF (ABS(DY).GE.ABS(DX)) GO TO 40
      DDX=0.0
      DDY=1.0
      GO TO 41
   40 DDX=1.0
      DDY=0.0
   41 EI0=-1.0
      IF (DX.GT.0.0) EI0=1.0
      EI1=-1.0
      IF (DY.GT.0.0) EI1=1.0
      IF (DX.EQ.0.0) EI0=0.0
      IF (DY.EQ.0.0) EI1=0.0
C   WEIGHT GRADIENT COMPONENTS
      WG(1)=PI*((2.0*R*L*GAM(1))-(DY*GAM(2)*TY*NR))
      WG(2)=(ABS(DX)*L*GAM(3)*NX)-(MND*DLT(1)*TY*GD*NR*NX)
      WG(3)=(SW*GAM(2)*PI*NR)-(MND*DLT(1)*TY*GD*NR*NX)
      WG(4)=EI0*((L*TX*GAM(3)*NX)-(DDX*DLT(1)*TX*TY*GD*NR*NX))
      WG(5)=EI1*((ABS((2.0*R)-(2.0*DY)-(TS*EI1))*GAM(2)*TY*PI*NR)-
     1(DDY*DLT(1)*TX*TY*GD*NR*NX))
      IF (BDV(8)) WG(4)=WG(4)+WG(5)
      WG(6)=((SW*GAM(2)*TY*PI)-(MND*DLT(1)*TX*TY*GD*NX))*(-L/(LX*LX))
      WG(7)=((L*TX*GAM(3)*ABS(DX))-(MND*DLT(1)*TX*TY*GD*NR))
     1*((-2.0*PI*R)/(LY*LY))
C   END OF CALCULATION OF GRADIENT TO WEIGHT
C   DESIGN VARIABLES PUT IN NEW ARRAY (COMPLETE SET)
      DVI(1)=TS
      DVI(2)=TX
      DVI(3)=TY
      DVI(4)=DX
      DVI(5)=DY
```

```
      DVI(6)=LX
      DVI(7)=LY
C   BEGINNING OF CALCULATION OF  DESIGN VARIABLE CONSTRAINT PENALTY TERMS
C           AND GRADIENT COMPONENTS
      SIGS=0.0
      DO 42 M=1,7
C   UPPER AND LOWER BOUND PENALTY TERM GRADIENT COMPONENTS
      IF (.NOT.BDV(M)) GO TO 42
      UBS=(DVL(1,M)-DVI(M))**2
      LBS=(DVI(M)-DVL(2,M))**2
      DBS=DVL(1,M)-DVL(2,M)
      GSC(M)=DBS/UBS
      GSC(M)=GSC(M)-(DBS/LBS)
   42 CONTINUE
C   COMPATIBILITY BOUND PENALTY TERMS AND GRADIENT COMPONENTS
      UBS=(LY-TX)**2
      LBS=(LX-TY)**2
      DBS=DVL(2,7)-DVL(1,7)
      DBC=DVL(2,6)-DVL(1,6)
      IF (.NOT.(BDV(2).AND.BDV(7))) GO TO 44
C           GRADIENT COMPONENTS
      GSC(2)=GSC(2)-(DBS/UBS)
      GSC(7)=GSC(7)+(DBS/UBS)
C           PENALTY TERM
      SIGS=SIGS-(DBS/(LY-TX))
   44 IF (.NOT.(BDV(3).AND.BDV(6))) GO TO 45
C           GRADIENT COMPONENTS
      GSC(3)=GSC(3)-(DBC/LBS)
      GSC(6)=GSC(6)+(DBC/LBS)
C           PENALTY TERM
      SIGS=SIGS-(DBC/(LX-TY))
   45 CONTINUE
C   UPPER AND LOWER BOUND PENALTY TERMS
      DO 46 J=1,7
      IF (.NOT.BDV(J)) GO TO 46
      UBS=DVL(1,J)-DVI(J)
      LBS=DVI(J)-DVL(2,J)
      DBS=DVL(1,J)-DVL(2,J)
C           PENALTY TERMS
      SIGS=SIGS+(DBS/UBS)
      SIGS=SIGS+(DBS/LBS)
C   END OF CALCULATION OF DESIGN VARIABLE CONSTRAINT PENALTY TERMS AND
C           GRADIENT COMPONENTS
```

```
   46 CONTINUE
C  BEGINNING OF BEHAVIOR CONSTRAINT PENALTY TERMS AND GRADIENT COMPONENTS
      DO 47 J=1,7
   47 DV(J)=DVI(J)
C  CALCULATION OF BEHAVIOR CONSTRAINT PENALTY TERMS  --  FINITE DIFFERENCE
    4 SIG(CC+1)=0.0
      DO 48 M=1,11
      DO 48 J=1,I
   48 SIG(CC+1)=SIG(CC+1)+(1.0/(1.0-BF(M,J)))
      IF (CC.GT.0) DV(CC)=DVI(CC)
      IF((CC.EQ.4).AND.BDV(8))  DV(5)=DV(4)
    2 CC=CC+1
C  TEST TO SEE IF ALL COMPONENTS HAVE BEEN CALCULATED
      IF (CC.EQ.8) GO TO 1
C  TEST TO SEE IF A DESIGN VARIABLE IS ACTIVE
      IF (BDV(CC)) GO TO 55
      SIG(CC+1)=0.0
      GO TO 2
   55 INC(CC)=INCF(CC)
C  DESIGN VARIABLE ALTERED BY FINITE DIFFERENCE INCREMENT
    3 DV(CC)=DVI(CC)+INC(CC)
      IF((CC.EQ.4).AND.BDV(8))  DV(5)=DV(4)
C  DESIGN VARIABLE UPPER AND LOWER BOUND CHECK
      IF ((DV(CC).GT.DVL(1,CC)).OR.(DV(CC).LT.DVL(2,CC))) GO TO 5
C  COMPATIBILITY BOUND VIOLATION CHECK
      IF (.NOT.((CC.EQ.2).OR.(CC.EQ.7))) GO TO 49
      IF (DV(2).GT.DV(7)) GO TO 5
   49 IF (.NOT.((CC.EQ.3).OR.(CC.EQ.6))) GO TO 6
      IF (DV(3).GT.DV(6)) GO TO 5
      GO TO 6
C  END OF COMPATIBILITY BOUND VIOLATION CHECK
    5 INC(CC)=INC(CC)/2.0
      IF(INC(CC).LT.(INCF(J)/2.))  GO TO 4
      GO TO 3
C  ANALYSIS FOR FINITE DIFFERENCE MOVE, ONE BUCKLING WAVE PATTERN FOR EACH
C          FAILURE MODE FOR EACH LOAD CONDITION
    6 CALL ANAL(DV,MS,NS,MS,NS,.TRUE.)
      DO 50 M=1,11
      DO 50 J=1,I
C  BEHAVIOR VIOLATION CHECK
   50 IF (BF(M,J).GT.1.0) GO TO 5
      GO TO 4
C  CALCULATION OF UNCONSTRAINED FUNCTION VALVE
```

```
    1 FM=WT+(RD*(SIG(1)+SIGS))
      WRITE(6,403) SIG
      DO 51 J=1,7
      IF (.NOT.BDV(J)) GO TO 52
      IF (INC(J).LT.(INCF(J)/2.0)) GO TO 56
C   BEHAVIOR CONSTRAINT PENALTY TERM GRADIENT COMPONENTS
      GS(J)=(SIG(J+1)-SIG(1))/INC(J)
      GO TO 57
   56 GS(J)=1.0E6
C   UNCONSTRAINED FUNCTION GRADIENT TERMS
   57 GS(J)=WG(J)+(RD*(GS(J)+GSC(J)))
      GO TO 51
   52 GS(J)=0.0
   51 CONTINUE
    7 N=0
      DO 53 M=1,7
      IF (.NOT.BDV(M)) GO TO 53
      N=N+1
      G(N)=GS(M)
   53 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INTER3(N,X,S,T,F1,G1,A,B,FA,FB,GA,GB,GAS,GBS,R,B4,XX)
C  GIVEN THE NUMBER OF ACTIVE DESIGN VARIABLES N, THE VALUE OF THE MULTIPLIER R,
C          THE VECTOR OF DESIGN VARIABLE X, THE DIRECTION OF MOVE S,
C          THE INITIAL DISTANCE OF MOVE T, AND THE FUNCTION VALUE AND ITS
C          GRADIENT AT THE INITIAL POINT, T=0, THIS SUBROUTINE FINDS TWO
C          POINTS A AND B, B GREATER THAN A, WITH FUNCTION VALUES FA AND FB,
C          GRADIENTS GA AND GB, AND SLOPES GAS AND GBS, WHICH LIE ON OPPOSITE
C          SIDES OF THE MINIMUM.  IF THE MOVE DISTANCE BECOMES TOO SMALL,
C          B4 IS SET TO FALSE AND THE DESIGN IS SET TO XX.
      LOGICAL B1,B2,B3,B4
      REAL INCF
      REAL INCF1
      COMMON/SYNCON/DVL(2,7),INCF(7)
     1/CINCF1/INCF1(7)
      DIMENSION TGA(7),XX(7),X(7),S(7),G1(7),GA(7),GB(7)
  400 FORMAT(* MOVE DISTANCE TOO SMALL, MINIMUM NOT FOUND IN THIS DIRECT
     1ION*)
      B1=.TRUE.
      B2=.FALSE.
      B3=.FALSE.
      B4=.TRUE.
C  CALCULATION OF MINIMUM MOVE DISTANCE  TM
      TM=INCF1(1)/ABS(S(1))
      DO 14 J=2,N
      SM=INCF1(J)/ABS(S(J))
      IF(SM.LT.TM)  TM=SM
   14 CONTINUE
    1 H=T
C  T MINIMUM TEST
    2 IF (ABS(H).GT.TM) GO TO 17
      FA=F1
      DO 18 I=1,N
      XX(I)=X(I)
   18 GA(I)=G1(I)
      T=H
      B4=.FALSE.
      WRITE(6,400)
      GO TO 11
   17 CONTINUE
      DO 3 I=1,N
    3 XX(I)=X(I)-(H*S(I))
      B=H
```

```
C   GENERATION OF FUNCTION VALUE AND GRADIENT USING APPROXIMATE ANALYSIS
       CALL FP(XX,R,FB,GB,.FALSE.)
       IF (FB.LT.1.0E28) GO TO 4
       T=T/2.0
       B2=.TRUE.
       IF (.NOT.B3) GO TO 5
       H=H-T
       GO TO 2
     5 GO TO 1
     4 TS=0.0
       DO 6 I=1,N
     6 TS=TS-(GB(I)*S(I))
       GBS=TS
       IF (ABS(GBS).LT.1.0E-4) GO TO 7
       IF (GBS.LT.0.0) GO TO 8
       IF (.NOT.B1) GO TO 9
       A=0.0
       TS=0.0
       DO 10 I=1,N
       TS=TS-(G1(I)*S(I))
    10 GA(I)=G1(I)
       GAS=TS
       FA=F1
       GO TO 11
     9 T=H
       A=TA
       FA=TFA
       DO 12 I=1,N
    12 GA(I)=TGA(I)
       GAS=TGAS
       GO TO 11
     8 TA=H
       TFA=FB
       B1=.FALSE.
       DO 13 I=1,N
    13 TGA(I)=GB(I)
       TGAS=GBS
     7 B3=.TRUE.
       IF (B2) T=T/2.0
C   T MINIMUM TEST
        IF (ABS(T).GT.TM) GO TO 15
       B4=.FALSE.
       WRITE(6,400)

       GO TO 9
    15 H=H+T
       GO TO 2
    11 CONTINUE
       RETURN
       END
```

```
      SUBROUTINE LOC3(N,X,S,A,B,FA,FB,GAS,GBS,TE,XTE,FXTE,GXTE,R)
C   THIS SUBROUTINE MAKES A CUBIC INTERPOLATION TO FIND AN ESTIMATED
C           MINIMUM DESIGN.
C   INPUT
C           N       NUMBER OF INDEPENDENT DESIGN VARIABLES
C           X       THE VECTOR OF INDEPENDENT DESIGN VARIABLES
C           S       THE DIRECTION OF MOVE
C           A,B     DISTANCES ALONG S FROM X TO POINTS BRACKETING THE MINIMUM
C           FA,FB   FUNCTION VALUES AT POINTS A AND B
C           GAS,GBS SLOPES AT POINTS A AND B
C           R       PENALTY FUNCTION MULTIPLIER
C   OUTPUT
C           TE      DISTANCE TO THE ESTIMATED MINIMUM FROM X
C           XTE     DESIGN AT THE ESTIMATED MINIMUM
C           FXTE    FUNCTION VALUE (NOT ESTIMATED) AT THE ESTIMATED MINIMUM
C                        DESIGN
C           GXTE    GRADIENT OF THE FUNCTION AT THE ESTIMATED MINIMUM DESIGN
      DIMENSION X(7),S(7),XTE(7),GXTE(7)
      Z=(3.0*(FA-FB))/(B-A)+GAS+GBS
      W=SQRT(Z*Z-(GAS*GBS))
      TE=B-(((GBS+W-Z)/(GBS-GAS+2.0*W))*(B-A))
      DO 1 I=1,N
    1 XTE(I)=X(I)-(TE*S(I))
      CALL FP(XTE,R,FXTE,GXTE,.FALSE.)
      RETURN
      END
```

```
      SUBROUTINE RIBBA
C     RIB BUCKLING ANALYSIS
C  THIS SUBROUTINE OBTAINS A CRITICAL BUCKLING STRAIN FOR THE CIRCUMFERENTIAL
C           STIFFENERS IN EACH LOAD CONDITION, FOR AN EXPANSION OF THE CYLINDER,
C           EBU(), FOR CONTRACTION OF THE CYLINDER, EBL(), AND A BUCKLING STRESS
C           FOR THE LONGITUDINAL STIFFENERS, LRCB().  LOGICAL VARIABLES ARE
C           ALSO SET TO INDICATE THE EXISTENCE OF THESE CRITICAL VALUES.
C           IN THE SAME ORDER AS ABOVE THESE ARE BEU(), BEL(), BLR().
      LOGICAL BEU,BEL,BLR,BYCU
      REAL NU2,NU1,L,LRCB,N2,LRB,NU,LX,LY
      COMMON/DIMEN/TS,TX,TY,DX,DY,LX,LY
     1/RIBPRO/EX(10),EY(10),NU2(10),NU1(10)
     2/CRTRIB/EBU(10),EBL(10),LRCB(10)
     3/CBVA/BEU(10),BEL(10),BLR(10)
     4/NLC/I
     5/CPI/PI
     6/CLR/L,R
     7/YIELD/LRCU(10),LRCL(10),CRCU(10),CRCL(10)
      DIMENSION LRB(2),EB(3)
  400 FORMAT(12H ENTER RIBBA)
  401 FORMAT(8E15.8)
C  FUNCTION FOR LONGITUDINAL STIFFENER CRITICAL BUCKLING STRESS
      SIGMAX(E,T,B,L,NU)=-((PI**2)*E*((T/B)**2)*(((B/L)**2)+0.425))/
     1(12.0*(1.0-(NU**2)))
      DO 1 J=1,I
      BEU(J)=.TRUE.
      BEL(J)=.TRUE.
      BLR(J)=.TRUE.
      BYCU=.FALSE.
      IF (ABS(DY).LT.(10.*TY)) BYCU=.TRUE.
      EYU=CRCU(J)/EY(J)
      EYL=CRCL(J)/EY(J)
      IF ( DY.NE. 0.0 ) GO TO 14
      BEU(J)=.FALSE.
      BEL(J)=.FALSE.
      IF ( DX .NE. 0.0 ) GO TO 15
      BLR(J)=.FALSE.
      GO TO 1
C  LONGITUDINAL STIFFENER BUCKLING STRESS FOR THE CASE OF NO CIRCUMFERENTIAL
C           STIFFENERS
   15 LRCB(J)=SIGMAX(EX(J),TX,DX,L,NU2(J))
      GO TO 1
```

```
   14 CONTINUE
      IF ( DY .GT. 0.0 ) GO TO 2
C  EXTERNAL CIRCUMFERENTIAL STIFFENERS
      D=DY/(R-DY+(TS/2.0))
      IF (.NOT.BYCU) GO TO 22
      EB(3)=EYL
      N2=0.0
      GO TO 23
   22 CONTINUE
C          BUCKLING STRAIN FOR EXTERNAL CIRCUMFERENTIAL STIFFENERS,
C                CONTRACTION (FULL DEPTH)
      CALL SEPBM(NU1(J),D,TY,DY,N2,EB(3))
   23 IF ( DX .LT. 0.0 ) GO TO 3
C          BUCKLING STRAIN FOR EXTERNAL CIRCUMFERENTIAL STIFFENERS,
C                EXPANSION (FULL DEPTH)
      EBU(J)=EPB(0,NU1(J),D,TY,DY)
      IF (BYCU) EBU(J)=EYU
      EBL(J)=EB(3)
      IF ( DX .NE. 0.0 ) GO TO 16
      BLR(J)=.FALSE.
      GO TO 24
C          BUCKLING STRESS FOR LONGITUDINAL STIFFENERS ON OPPOSITE SIDE
C                (INSIDE) OF CYLINDER FROM CIRCUMFERENTIALS
   16 LRCB(J)=SIGMAX(EX(J),TX,DX,L,NU2(J))
      GO TO 1
C  ALL STIFFENERS ON OUTSIDE (DX AND DY NEGATIVE)
C          LONGITUDINAL STIFFENER BUCKLING STRESS BETWEEN CIRCUMFERENTIALS
C                (FULL DEPTH)
    3 LRB(2)=SIGMAX(EX(J),TX,DX,LX,NU2(J))
   24 G=(PI*R)/LY
C          CIRCUMFERENTIAL STIFFENER BUCKLING STRAIN WITH NUMBER OF HALF
C                WAVES EQUAL TO THE NUMBER OF STIFFENER SPACES
      EB(2)=EPB(G,NU1(J),D,TY,DY)
      IF ( G .GT. N2 ) GO TO 4
      IF ( EB(2) .GT. 0.0 ) GO TO 5
      EBL(J)=EB(3)
      GO TO 6
    4 EBL(J)=EB(2)
    6 BEU(J)=.FALSE.
      GO TO 7
    5 IF (BYCU) EB(2)=EYU
      EBU(J)=EB(2)
      EBL(J)=EB(3)
```

```
    7 IF ( DX .LT. DY ) GO TO 8
      IF ( DX .EQ. DY ) GO TO 9
      D=(DY-DX)/(R+(TS/2.0)-DY)
C             WASHER MODE FOR OUTSTANDING PORTION OF CIRCUMFERENTIAL STIFFENER
      EB(1)=EPB(0,NU1(J),D,TY,DY-DX)
C  YIELD SUBSTITUTION
      IF (ABS(DY-DX).LT.(10.*TY)) EB(1)=EYU
      IF (BEU(J)) GO TO 18
      BEU(J)=.TRUE.
      GO TO 17
   18 IF ( EB(1) .LT. EBU(J) ) GO TO 17
      GO TO 9
   17 EBU(J)=EB(1)
    9 LRCB(J)=LRB(2)
      GO TO 1
C             BUCKLING STRESS FOR OUTSTANDING PORTION OF LONGITUDINAL STIFFENER
    8 LRB(1)=SIGMAX(EX(J),TX,DY-DX,L,NU2(J))
      IF ( LRB(2) .LT. LRB(1) ) GO TO 19
      GO TO 9
   19 LRCB(J)=LRB(1)
      GO TO 1
C  END EXTERNAL CIRCUMFERENTIAL STIFFENERS
C  INTERNAL CIRCUMFERENTIAL STIFFENERS
    2 D=DY/(R-(TS/2.0)-DY)
      IF ( DX .GT. 0.0 ) GO TO 10
C  WASHER MODE FOR INTERNAL CIRCUMFERENTIAL STIFFENERS
      EBL(J)=EPB(0,NU1(J),D,TY,DY)
      IF (BYCU) EBL(J)=EYL
      IF ( DX .NE. 0.0 ) GO TO 20
      BLR(J)=.FALSE.
      GO TO 13
C             LONGITUDINAL STIFFENER BUCKLING STRESS FOR FULL DEPTH STIFFENERS
C                  FULL LENGTH
   20 LRCB(J)=SIGMAX(EX(J),TX,DX,L,NU2(J))
      GO TO 13
   10 G=(PI*R)/LY
C             CRITICAL BUCKLING STRAIN FOR FULL DEPTH CIRCUMFERENTIAL STIFFENERS
C                  WITH THE NUMBER OF HALF WAVES EQUAL TO THE NUMBER OF
C                  LONGITUDINAL STIFFENER SPACES
      EB(2)=EPB(G,NU1(J),D,TY,DY)
      IF (BYCU) EB(2)=EYL
C  LONGITUDINAL STIFFENER BUCKLING STRESS FOR FULL DEPTH STIFFENER BETWEEN
C             CIRCUMFERENTIALS
```

```
      LRB(2)=SIGMAX(EX(J),TX,DX,LX,NU2(J))
      IF ( DX .GT. DY ) GO TO 11
      LRCB(J)=SIGMAX(EX(J),TX,DX,LX,NU2(J))
      IF ( DX .EQ. DY ) GO TO 12
      D=(DY-DX)/(R-(TS/2.0)-DY)
C   WASHER MODE FOR OUTSTANDING PORTION OF STIFFENER
      EB(1)=EPB(0,NU1(J),D,TY,DY-DX)
C   YIELD SUBSTITUTION
      IF ((DY-DX) .LT. (10.*TY)) EB(1)=EYL
      IF ( EB(1) . LT. EB(2) ) GO TO 12
      EBL(J)=EB(1)
      GO TO 13
C   BUCKLING STRESS FOR OUTSTANDING PORTION OF STIFFENER FOR FULL LENGTH
   11 LRB(1)=SIGMAX(EX(J),TX,DX-DY,L,NU2(J))
      IF ( LRB(1) .LE. LRB(2)) GO TO 21
      LRCB(J)=LRB(1)
      GO TO 12
   21 LRCB(J)=LRB(2)
   12 EBL(J)=EB(2)
   13 BEU(J)=.FALSE.
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SEPBM(NU,D,TY,DY,N,EPBM)
C     MINIMUM CRITICAL EDGE STRAIN
C  THIS SUBROUTINE OBTAINS THE NEGATIVE BUCKLING STRAIN CLOSEST TO ZERO FOR A
C          CIRCUMFERENTIAL STIFFENER OR PORTION OF A CIRCUMFERENTIAL STIFFENER
C          WHICH IS NOT SUPPORTED BY LONGITUDINAL STIFFENERS AND THE NUMBER
C          OF CIRCUMFERENTIAL WAVES.
C  INPUT
C          NU    POISSON≠S RATIO
C          D     RATIO OF STIFFENER DEPTH TO RADIUS OF SUPPORTED EDGE
C          TY    THICKNESS OF STIFFENER
C          DY    DEPTH OF STIFFENER
C  OUTPUT
C          N     NUMBER OF CIRCUMFERENTIAL WAVES
C          EPBM  CRITICAL VALUE OF THE STRAIN
      REAL NU,D,TY,DY,N,ET,TE,EPBM
  400 FORMAT(6H0   N=,E15.8)
      N=0.0
      TE=0.0
    1 N=N+1.0
      ET=EPB(N,NU,D,TY,DY)
      IF (ET .GE. 0.0) GO TO 1
      IF (TE .GE.ET) GO TO 2
      TE=ET
      GO TO 1
    2 IF (TE .NE. 0.0) GO TO 3
      TE=ET
      GO TO 1
    3 EPBM=TE
      N=N-1.0
      RETURN
      END
```

```
      SUBROUTINE SGEN2(N,S,T,GX,GXEM,B1,B2)
C  FLETCHER-POWELL DIRECTION GENERATOR
C  THIS SUBROUTINE GENERATES CONJUGATE DIRECTIONS BY MULTIPLYING THE CURRENT
C          GRADIENT BY A MATRIX OBTAINED FROM INFORMATION ACQUIRED FROM
C          THE PREVIOUS DIRECTIONS.
C  INPUT
C          N      NUMBER OF INDEPENDENT DESIGN VARIABLES
C          S      DIRECTION VECTOR
C          T      DISTANCE ALONG DIRECTION TO MINIMUM
C          GX     GRADIENT OF FUNCTION AT PT FROM WHICH PREVIOUS SEARCH STARTED
C          GXEM   GRADIENT OF FUNCTION AT MINIMUM OF PREVIOUS SEARCH
C          B1     .TRUE.  THE H MATRIX WILL NOT BE UPDATED
C          B2     .TRUE.  THE H MATRIX IS REPLACED BY THE IDENTITY MATRIX
C  OUTPUT
C          S      NEW DIRECTION (NORMALIZED)
      LOGICAL B1,B2
      REAL LV
      COMMON/COH/H(7,7)
      DIMENSION S(7),GX(7),GXEM(7),HY(7),Y(7),DELTA(7)
  400 FORMAT(1H ,22X,7E15.8)
      IF (.NOT.B2) GO TO 20
C  H REPLACED BY IDENTITY MATRIX
      DO 21 I=1,N
      DO 21 J=1,I
      H(J,I)=((J/I)*(I/J))
   21 H(I,J)=H(J,I)
      GO TO 1
   20 IF (.NOT.B1) GO TO 22
      B1=.FALSE.
      GO TO 1
C  UPDATING H
   22 LV=0.0
      TS=0.0
      DO 23 I=1,N
      DELTA(I)=-T*S(I)
      Y(I)=GXEM(I)-GX(I)
      TS=TS+DELTA(I)*Y(I)
   23 CONTINUE
      DO 24 I=1,N
      DV=0.0
      DO 25 J=1,N
   25 DV=DV+H(I,J)*Y(J)
```

```
      HY(I)=DV
   24 LV=Y(I)*HY(I)+LV
      DELTAY=TS
      YHY=LV
      DO 26 I=1,N
      DO 26 J=1,I
      IF(DELTAY.EQ.0..OR.YHY.EQ.0.) GO TO 261
      H(I,J)=H(I,J)+((DELTA(I)*DELTA(J))/DELTAY)-((HY(I)*HY(J))/YHY)
  261 CONTINUE
   26 H(J,I)=H(I,J)
C  END UPDATING H
    1 DO 27 I=1,N
      DV=0.0
      DO 28 J=1,N
   28 DV=DV+H(I,J)*GXEM(J)
   27 S(I)=DV
C  NORMALIZING S
      TS=0.0
      DO 29 J=1,N
   29 TS=TS+S(J)*S(J)
      DV=SQRT(TS)
      IF (DV.EQ.0.0) GO TO 2
      DO 30 J=1,N
   30 S(J)=S(J)/DV
    2 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SYN(N,VDP)
C  THIS IS THE MASTER SUBROUTINE FOR THE SYNTHESIS ROUTINE.  N IS THE NUMBER
C          OF DESIGN VARIABLES AND VDP IS THE VECTOR OF ACTIVE DESIGN VARIABLES.
C  THIS SUBROUTINE USES THE FOLLOWING SUBROUTINES
C          FP      GENERATES FUNCTION VALUE AND ITS GRADIENT
C          SGEN2   GENERATES MOVE DIRECTION
C          INTER3  FINDS TWO POINTS ALONG A DIRECTION WHICH LIE ON OPPOSITE
C                              SIDES OF THE MINIMUM
C          LOC3    DOES A CUBIC INTERPOLATION TO THE MINIMUM DESIGN ALONG A
C                              DIRECTION
      INTEGER RELOCS,RCOUNT,COUNT
      LOGICAL B1,BI,B4
      LOGICAL BID
      LOGICAL BDVV1,BDVV2
      REAL MG,INCF,NG
      REAL INCF1
      DIMENSION G(7),GXEM(7),XEM(7),GA(7),GB(7),D(7),X(7),S(7),NG(7),
     1TP(7),GP(7),VDP(7),EXMI(7),XX(7)
      COMMON/CWT/WT/CRD/RD,RDC
     1/CBDVV/BDVV1,BDVV2
     2/CINCF1/INCF1(7)
     1/SYNCON/DVL(2,7),INCF(7)
     2/COH/H(7,7)
      COMMON/CTIM/TLIM
      COMMON/BOODV/BDV(8)
  400 FORMAT(1H ,4X,27HCLEARED H-MATRIX DUE TO -GS)
  401 FORMAT(1H ,4X,4HFEM=,E15.8)
  402 FORMAT(1H ,3X,4HTEST,E15.7)
  403 FORMAT(1H ,4X,19HUNACCEPTABLE DESIGN)
  404 FORMAT(1H0,4X,7HFOR RD=,E13.5,5H FEM=,E13.5,4H WT=,E13.5)
  405 FORMAT(1H ,4X,3HXEM,7E15.7//)
  406 FORMAT(1H+,25X,3HTS=,E15.8)
  407 FORMAT(1H ,4X,16HCONVERGENCE TEST)
  408 FORMAT(1H ,4X,3HTT=,E15.8)
  409 FORMAT(1H ,4X,3HTP=,7E15.8)
  410 FORMAT(1H ,4X,3HS= ,7E15.8)
  411 FORMAT(1H ,4X,2HD=,7E15.8)
  412 FORMAT(1H ,8E15.8)
  413 FORMAT(1H ,10I5)
  414 FORMAT(2H0H)
  415 FORMAT(* INITIAL DESIGN UNACCEPTABLE*)
  416 FORMAT(* MOVE SIZE CONVERGENCE TEST SATISFIED*)
```

```
  417 FORMAT(* QUADRATIC CONVERGENCE TEST SATISFIED*)
  418 FORMAT(* GRADIENT CONVERGENCE TEST SATISFIED*)
  419 FORMAT(* SUGGEST INCREASING INITIAL RD AND/OR NO. MODES SAVED*)
  420 FORMAT(* RD INCREASED, CYCLE RESTARTED*)
  421 FORMAT(* DESIGN VARIABLE(S) EXCEEDING UPPER BOUND(S) AND/OR LESS T
     1HAN LOWER BOUNDS(S)*)
  422 FORMAT(* STIFFENER THICKNESS(ES) GREATER THAN THE CORRESPONDING SP
     1ACING(S)*)
  423 FORMAT(* BEHAVIOR IN VIOLATION*)
  424 FORMAT(* 5 INTERPOLATIONS ATTEMPTED, MINIMUM NOT FOUND IN THIS DIR
     1ECTION*)
  425 FORMAT(* TS INCREASED BY 20 PERCENT*)
 2468 FORMAT(1H )
      RCOUNT=0
      BID=.TRUE.
C  ANALYSIS OF INITIAL DESIGN
   44 CONTINUE
      CALL FP(VDP,RD,F1,G,.TRUE.)
      IF(F1.LT.1.E28)  GO TO 6
      IF(BID)  GO TO 43
      WRITE(6,403)
      GO TO 14
   43 CONTINUE
      WRITE(6,415)
      IF(BDVV1)  WRITE(6,421)
      IF(BDVV2)  WRITE(6,422)
      IF(BDVV1.OR.BDVV2)  GO TO 14
      WRITE(6,423)
      IF(.NOT.BDV(1))  GO TO 14
      VDP(1)=1.2*VDP(1)
      WRITE(6,425)
      BID=.FALSE.
      GO TO 44
    6 DO 15 I=1,N
      GP(I)=G(I)
   15 X(I)=VDP(I)
   10 COUNT=0
      FEST=WT
C  INITIALIZE H MATRIX
      CALL SGEN2(N,D,T,G,G,.FALSE.,.TRUE.)
    2 TS=0.0
      WRITE(6,411) D
      DO 30 I=1,N
```

```
   30 TS=G(I)*D(I)+TS
      GG=TS
      IF (GG.GT.0.0) GO TO 4
C  REINITIALIZE H MATRIX WHEN DIRECTION IS BAD
      CALL SGEN2(N,D,DDT,G,G,.FALSE.,.TRUE.)
      WRITE(6,400)
      GO TO 2
    4 RELOCS=0
C  INITIAL ESTIMATE OF DISTANCE TO MINIMUM ALONG D
      TS=F1-FEST
      IF (TS.GT.0.0) GO TO 16
      FEST=FEST+2.0*TS
      GO TO 4
   16 T=(2.0*TS)/GG
C  LIMITS ON THE INITIAL ESTIMATE OF MOVE DISTANCE
      IF (T.LT.0.25) T=0.5
      IF (T.GT.1.0) T=0.5
C  FINDING TWO POINTS WHICH LIE ON OPPOSITE SIDES OF MINIMUM ALONG THE
C          DIRECTION, D
   12 CALL INTER3(N,X,D,T,F1,G,A,B,FA,FB,GA,GB,GAS,GBS,RD,B4,XX)
      IF (B4) GO TO 13
C  REPLACEMENT FOR TOO SMALL A MOVE SIZE
      FEM=FA
      RELOCS=4
      DO 35 I=1,N
      GXEM(I)=GA(I)
   35 XEM(I)=XX(I)
      GO TO 36
C  MAKING CUBIC INTERPOLATION TO MINIMUM ALONG DIRECTION, D
   13 CALL LOC3(N,X,D,A,B,FA,FB,GAS,GBS,T,XEM,FEM,GXEM,RD)
   36 CONTINUE

      WRITE(6,401) FEM
      IF (FEM.LT.1.0E28) GO TO 17
C  UNACCEPTABLE DESIGN  ---  MOVE DISTANCE CUT
      T=T/2.0
      GO TO 12
   17 TS=0.0
      DO 18 I=1,N
   18 TS=TS+GXEM(I)*GXEM(I)
      TS=SQRT(TS)
      WRITE(6,2468)
      WRITE(6,406) TS
```

```
C   GRADIENT CONVERGENCE TEST
       IF(TS.GT.0.1*WT)  GO TO 40
       WRITE(6,418)
       GO TO 3
C   COUNT IS THE NUMBER OF DIRECTIONS SEARCHED
    40 IF(COUNT.LT.N)  GO TO 11
C   THIS PERMITS A WEIGHT INCREASE IF THE FUNCTION IS NOT UNIMODEL
       IF((F1.LT.FEM).AND.B4)  GO TO 11
       WRITE(6,407)
       DO 32 J=1,N
       EXMI(J)=0.0
       DO 32 I=1,N
    32 EXMI(J)=(H(J,I)*GXEM(I))+EXMI(J)
       EXM=0.0
       DO 33 J=1,N
    33 EXM=(GXEM(J)*EXMI(J))+EXM
       WRITE(6,412) EXM
       EXM=EXM/(2.0*FEM)
       WRITE(6,2468)
       WRITE(6,414)
       DO 34 I=1,N
    34 WRITE(6,412) (H(I,J),J=1,N)
C   QUADRATIC CONVERGENCE TEST
       IF(EXM.GT.0.02)  GO TO 39
       WRITE(6,417)
       GO TO 3
    39 DO 19 J=1,N
    19 S(J)=GXEM(J)/TS
C   MINIMUM MOVE DISTANCE
       TT=INCF1(1)/ABS(S(1))
       DO 20 J=2,N
       MG=INCF1(J)/ABS(S(J))
       IF(MG.LT.TT)  TT=MG
    20 CONTINUE
       WRITE(6,412) INCF(I),MG
       WRITE(6,413) I
       TT=2.*TT
       DO 21 J=1,N
    21 TP(J)=XEM(J)-(TT*S(J))
       WRITE(6,408) TT
       WRITE(6,409) TP
       WRITE(6,410) S
C   GENERATING FUNCTION VALUE AND GRADIENT USING APPROXIMATE ANALYSIS
```

```
      CALL FP(TP,RD,DF,NG,.FALSE.)
C     MOVE SIZE CONVERGENCE TEST
      IF(DF.LT.1.E28)   GO TO 38
      WRITE(6,416)
      GO TO 3
   38 PIP=0.
      DO 31  J=1,N
   31 PIP=(S(J)*NG(J))+PIP
      IF(PIP.GT.0.0)   GO TO 11
      WRITE(6,416)
      GO TO 3
C     END MOVE SIZE CONVERGENCE TEST
   11 GS=0.0
      AGXEM=0.0
      AG=0.0
      DO 22  I=1,N
      GS=GS-GXEM(I)*D(I)
      AGXEM=AGXEM+GXEM(I)*GXEM(I)
   22 AG=AG+D(I)*D(I)
      TEST=GS/(SQRT(AGXEM)*SQRT(AG))
      WRITE(6,402) TEST
      IF (FEM.GT.F1) TEST=1.0
C     TEST OF CONVERGENCE ALONG DIRECTION, D
      IF (ABS(TEST).LT.5.0E-2) GO TO 5
      RELOCS=RELOCS+1
      IF(RELOCS.LT.5)   GO TO 42
      WRITE(6,424)
      GO TO 5
   42 IF(GS.GT.0.0)   GO TO 23
      FA=FEM
      A=T
      GAS=GS
      DO 24 I=1,N
   24 GA(I)=GXEM(I)
      GO TO 13
   23 FB=FEM
      B=T
      GBS=GS
      DO 25 I=1,N
   25 GB(I)=GXEM(I)
      GO TO 13
    5 COUNT=COUNT+1
      BI=.FALSE.
```

```
      IF (COUNT.NE.N+1) GO TO 26
      BI=.TRUE.
      COUNT=0
   26 B1=.FALSE.
      IF (RELOCS.EQ.5) B1=.TRUE.
C  GENERATING A DIRECTION  ---  FLETCHER-POWELL DIRECTION GENERATOR
      CALL SGEN2(N,D,T,G,GXEM,B1,BI)
      DO 27 I=1,N
      G(I)=GXEM(I)
   27 X(I)=XEM(I)
      F1=FEM
      GO TO 2
C  THE FOLLOWING IS THE END OF A MINIMIZATION CYCLE
      WRITE(6,2468)
    3 WRITE(6,404)RD,FEM,WT
      WRITE(6,405) XEM
      RCOUNT=RCOUNT+1
      CALL FP(XEM,RD,FEM,GXEM,.FALSE.)
      CTEST=(FEM-WT)/((2.0*WT)-FEM)
      IF ( CTEST .LT. 0.0 ) CTEST=10.0
      RD=RD/RDC
      CALL FP(XEM,RD,F1,G,.TRUE.)
      TID=RCOUNT
      IF(CTEST.LT.0.03)  TID=TLIM
      IF ( TID.GE. TLIM) GO TO 14
      IF(F1.LT.1.E28)  GO TO 37
      WRITE(6,403)
      IF(RCOUNT.GT.1)  GO TO 41
      WRITE(6,419)
      GO TO 14
C  INCREASE OF MULTIPLIER RD IN THE CASE OF AN UNACCEPTABLE DESIGN AT THE
C          END OF A MINIMIZATION CYCLE
   41 RD=(RD*RDC+RDO)/2.
      WRITE(6,420)
      CALL FP(VDP,RD,F1,G,.TRUE.)
      GO TO 6
   37 RDO=RD*RDC
      DO 28 I=1,N
   28 VDP(I)=XEM(I)
      GO TO 6
   14 CONTINUE

C                            OPERATIONAL HINTS
```

```
C  INITIAL VALUES OF RD AND RDC
C          THE VALUES OF RD SHOULD BE PICKED SO THAT THE MAGNITUDE OF THE
C          UNCONSTRAINED FUNCTION IS ABOUT TWICE THE MAGNITUDE OF THE WEIGHT.
C          THUS, IN THE FIRST MINIMIZATION CYCLE THE VARIABLE FEM SHOULD BE
C          ABOUT TWICE THE MAGNITUDE OF THE VARIABLE WT.  A VALUE FOR RDC THAT
C          WORKS WELL IS 2.0.

C  MESSAGE  --  MINIMUM NOT FOUND IN THIS DIRECTION
C          A REPEATED OCCURRENCE OF THIS MESSAGE IS AN INDICATION THAT THE
C          DESIGN CHANGES ARE OF THE ORDER OF MAGNITUDE OF THE FINITE DIFFERENCE
C          INCREMENTS.  SEE BELOW.

C  MESSAGE  --  MOVE SIZE CONVERGENCE TEST SATISFIED
C          THIS IS AN INDICATION THAT THE DESIGN CHANGES ARE OF THE ORDER
C          OF MAGNITUDE OF THE FINITE DIFFERENCE INCREMENTS.  SEE BELOW.

C  MESSAGES  --  QUADRATIC CONVERGENCE TEST SATISFIED
C            --  GRADIENT CONVERGENCE TEST SATISFIED
C          THESE MEAN THE MINIMIZATION IS PROCEEDING NORMALLY.

C  MESSAGE  --  DY LESS THAN 5 TIMES TS,  ETC
C          THIS INDICATES THAT THE CIRCUMFERENTIAL STIFFENERS MAY NOT BE STIFF
C          ENOUGH TO FORCE NODES IN THE BUCKLING PATTERN.  IT MAY BE AN
C          INDICATION THAT THESE STIFFENERS ARE NOT NEEDED AND THE PROBLEM
C          SHOULD BE RERUN WITHOUT THEM.

C  MESSAGE  --  DX LESS THAN 5 TIMES TS, ETC
C          THIS INDICATES THAT THE LONGITUDINAL STIFFENERS MAY NOT BE STIFF
C          ENOUGH TO FORCE NODES IN THE BUCKLING PATTERN.  IT MAY BE AN
C          INDICATION THAT THESE STIFFENERS ARE NOT NEEDED AND THE PROBLEM
C          SHOULD BE RERUN WITHOUT THEM.

C  MESSAGE  --  WARNING, A MODE SHAPE NUMBER IS APPROACHING ITS UPPER LIMIT.
C          THE UPPER LIMIT ON THE MODAL SEARCH SHOULD PROBABLY BE INCREASED.

C  DESIGN CHANGES THE ORDER OF THE FINITE DIFFERENCE INCREMENT
C          EITHER THE FINITE DIFFERENCE INCREMENTS SHOULD BE REDUCED (NOT
C          NECESSARILY) OR THE PROBLEM SHOULD BE CONSIDERED CONVERGED.

C  DO NOT SET DESIGN VARIABLES EXACTLY EQUAL TO ZERO OR EQUAL TO A BOUND.

      RETURN
```

```
NO. LOAD COND. =  3

NO. MODES SAVED   GROSS   PANEL   SHEET

                    40      20      10

                    40      20      10

                    20      15      10

TS= 9.90000E-02 TX= 6.00000E-02 TY= 6.00000E-02 DX= 5.00000E-01 DY= 5.00000E-01 LX= 6.00000E+00 LY= 3.00000E+00

 E1=     1.00000E+07    1.00000E+07    1.00000E+07
 NUX= 3.33000000E-01 3.33000000E-01 3.33000000E-01
  E2= 1.00000000E+07 1.00000000E+07 1.00000000E+07
 NUY= 3.33000000E-01 3.33000000E-01 3.33000000E-01
 GSM= 3.75093770E+06 3.75093770E+06 3.75093770E+06
 EY=     1.00000E+07    1.00000E+07    1.00000E+07
 NU1=     3.33000E-01    3.33000E-01    3.33000E-01
 EX=     1.00000E+07    1.00000E+07    1.00000E+07
 NU2=     3.33000E-01    3.33000E-01    3.33000E-01
 P1=     7.00000E+02    9.40000E+02    2.12000E+02
 P2=     0.            -2.00000E+00    4.00000E-01
 GAM=     1.01000E-01    1.01000E-01    1.01000E-01

L= 1.65000E+02 R= 6.00000E+01 DLT(2)= 0.          DLT(3)= 1.00000E+00
ML   50   50   30
ML   20   20   20
ML   20   20   20
NL   30   30   30
NL  100  100  150
NL   15   15   15

 LRCU=    5.00000E+04    5.00000E+04    5.00000E+04
 LRCL=   -5.00000E+04   -5.00000E+04   -5.00000E+04
 CRCU=    5.00000E+04    5.00000E+04    5.00000E+04
 CRCL=   -5.00000E+04   -5.00000E+04   -5.00000E+04
 DVLU=    5.00000E-01    5.00000E-01    5.00000E-01    2.00000E+00    2.00000E+00    1.00000E+01    1.00000E+01
 DVLL=    1.90000E-02    5.00000E-02    5.00000E-02   -2.00000E+00   -2.00000E+00    5.00000E-02    5.00000E-02

BDV=    T    T    T    T    T    T    T    F
RD= 1.00000000E+01 RDC= 2.00000000E+00
INCF=     1.00000E-06    1.00000E-05    1.00000E-05    1.00000E-04    1.00000E-04    1.00000E-03    1.00000E-03
TLIM= 2.00000000E+00


KTT         1.00000E+00    1.00000E+00    1.00000E+00
KCT         1.00000E+00    1.00000E+00    1.00000E+00
KCC         1.00000E+00    1.00000E+00    1.00000E+00
KTC         1.00000E+00    1.00000E+00    1.00000E+00
SXOT        5.00000E+04    5.00000E+04    5.00000E+04
SXOC        5.00000E+04    5.00000E+04    5.00000E+04
SYOT        5.00000E+04    5.00000E+04    5.00000E+04
SYOC        5.00000E+04    5.00000E+04    5.00000E+04
```

```
 CLT
      3.20727E-03     3.66439E-03     4.01252E-03     5.45054E-03     7.40003E-03     9.82525E-03     1.25640E-02     1.26465E-02
      1.27125E-02     1.38769E-02
SMS
  9    11    10    10    12    11     8     8    11    12     9     9     7    13    12    13    10    10    12    13    14     7     6    13    11
 14     8     7    11    14     8     9     6    12    15    14    15     9    15    13
 13    12    13    12    14    11    14    14    13    15    15    12    11    11    13    15    12    10    14    16    16    10    14    15    16
 15    13    16    17    17    12    10    11    11     9    13    16    17    17    14
  1     1     1     1     2     2     1     2     3     2     3     3     1     4     4     2     5     4     3     5
  1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
  1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
  1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
  2     3     1     4     5     6     7     4     5     3
  2     3     4     5     6     1     7     4     5     8
  2     1     3     4     5     6     3     4     7     5
SNS
 14    15    15    14    15    14    14    13    16    16    15    13    13    16    14    15    16    13    17    17    16    12    12    14    13
 15    15    14    17    17    12    16    13    13    16    14    17    12    15    18
 15    14    14    15    15    14    14    16    16    15    16    13    15    13    13    14    16    14    13    15    16    13    17    17    14
 13    17    17    16    15    12    15    16    12    13    12    13    14    17    12
  6     7     8     5     8     9     9    10    10     7     9    11    10    11    12    11    12    10    12    13
 23    24    22    25    21    26    20    27    19    28    18    29    17    30    16    15    31    14    32    13
 19    20    18    21    17    22    16    23    15    24    14    13    25    12    26    11    10    27     9    28
 27    28    26    29    25    30    24    31    23    32    22    33    21    34    20
  1     1     1     1     1     1     1     2     2     2
  1     1     1     1     1     1     1     2     2     1
  1     1     1     1     1     1     2     2     1     2

     CRITICAL LOADS     GROSS           PANEL           SHEET

                    2.02985181E+03 3.74486614E+03 3.98637038E+03

                    2.40752048E+03 3.97541935E+03 4.59316698E+03

                    6.80931608E+02 3.49259135E+03 3.57120810E+03

     MODE SHAPES          M    N           M    N           M    N

                          9   14           1   23           2    1

                         13   15           1   19           2    1

                          1    6           1   27           2    1

LRS  -6.39090707E+03-8.93122622E+03-1.86570165E+03
CRS   2.03671423E+03 3.87768285E+03 3.88300130E+02
DES   1.62529415E-02 3.34407249E-02 1.34838745E-03
EBU            IIIII          IIIII          IIIII
EBL  -5.00000000E-03-5.00000000E-03-5.00000000E-03
LRCB -5.75377534E+04-5.75377534E+04-5.75377534E+04
BEU    F   F   F
BEL    T   T   T
BLR    T   T   T
EPA   2.03671423E-04 3.87768285E-04 3.88300130E-05
```

```
TS= 9.90000000E-02 TX= 6.00000000E-02 TY= 6.00000000E-02 DX= 5.00000000E-01 DY= 5.00000000E-01 LX= 6.00000000E+00 LY= 3.0000000E+00

AX= 3.00000000E-02 AY= 3.00000000E-02

  G.B.   3.44852760E-01 3.90443199E-01 3.11338169E-01

  P.B.   1.86922569E-01 2.36453042E-01 6.06999156E-02

  S.B.   1.59566440E-01 1.85207231E-01 5.41393775E-02

  LRB.   1.11073281E-01 1.55223756E-01 3.24256952E-02

  CRBU   0.             0.             0.

  CRBL  -4.07342845E-02-7.75536571E-02-7.76600259E-03

  S.Y.   1.27487025E-01 1.82868053E-01 3.67203956E-02

  LRYU  -1.27818141E-01-1.78624524E-01-3.73140331E-02

  LRYL   1.27818141E-01 1.78624524E-01 3.73140331E-02

  CRYU   4.07342845E-02 7.75536571E-02 7.76600259E-03

  CRYL  -4.07342845E-02-7.75536571E-02-7.76600259E-03

  WT     7.14917932E+02

  TB     1.13794102E-01


    VDP= 9.90000000E-02 6.00000000E-02 6.00000000E-02 5.00000000E-01 5.00000000E-01 6.00000000E+00 3.00000000E+00
SIG 3.64464920E+01 3.64464368E+01 3.64461968E+01 3.64463794E+01 3.64461280E+01 3.64461112E+01 3.64467905E+01 3.64472015E+01
    D= 7.95109528E-02-7.01914796E-01-7.07806864E-01 1.57805768E-03 5.28323407E-04-2.43017322E-05-5.48525906E-04

  WT     1.00952553E+03
    VDP= 5.92445236E-02 4.10957398E-01 4.13903432E-01 4.99210971E-01 4.99735838E-01 6.00001215E+00 3.00027426E+00
SIG 3.59419947E+01 3.59418560E+01 3.59419344E+01 3.59419970E+01 3.59413498E+01 3.59419027E+01 3.59420392E+01 3.59446207E+01

  WT     8.17383051E+02
    VDP= 8.51929827E-02 1.81886977E-01 1.82910130E-01 4.99725971E-01 4.99908257E-01 6.00000422E+00 3.00009525E+00
SIG 3.52870187E+01 3.52869741E+01 3.52869671E+01 3.52870013E+01 3.52866551E+01 3.52868139E+01 3.52871540E+01 3.52878438E+01
    FEM= 1.57811033E+03

                         TS= 4.84852044E+03
   TEST  6.8104294E-02
```

```
  WT     7.54371883E+02
    VDP= 9.36861589E-02 1.06910062E-01 1.07303838E-01 4.99894536E-01 4.99964691E-01 6.00000162E+00 3.00003666E+00
SIG 3.56234880E+01 3.56234443E+01 3.56233945E+01 3.56234426E+01 3.56231333E+01 3.56232017E+01 3.56236903E+01 3.56241831E+01
    FEM= 1.59350499E+03

                            TS= 5.11302687E+03
   TEST -2.5928862E-01

  WT     7.84670455E+02
    VDP= 8.96032670E-02 1.42953426E-01 1.43649760E-01 4.99813503E-01 4.99937562E-01 6.00000287E+00 3.00006483E+00
SIG 3.53975860E+01 3.53975430E+01 3.53975218E+01 3.53975614E+01 3.53972317E+01 3.53973691E+01 3.53977421E+01 3.53983300E+01
    FEM= 1.56687415E+03

                            TS= 4.94864050E+03
   TEST -1.3362270E-02
    D= 9.95454259E-01 5.20498096E-02-5.03766980E-02 5.41783228E-02 2.68933117E-02-1.97721430E-03-1.27032753E-02

  WT     6.48737514E+02
    VDP= 6.81356659E-02 1.41830938E-01 1.447361655E-01 4.98645113E-01 4.99357591E-01 6.00004551E+00 3.00033878E+00
SIG 3.71160804E+01 3.71159267E+01 3.71159419E+01 3.71160526E+01 3.71154093E+01 3.71157684E+01 3.71162483E+01 3.71186050E+01

  WT     5.80772795E+02
    VDP= 5.74018653E-02 1.41269695E-01 1.45279368E-01 4.98060918E-01 4.99067605E-01 6.00006683E+00 3.00047576E+00
SIG 4.08103034E+01 4.08094911E+01 4.08098192E+01 4.08103363E+01 4.08085546E+01 4.08100892E+01 4.08101254E+01 4.08230382E+01

  WT     6.06357839E+02
    VDP= 6.14425829E-02 1.41480974E-01 1.45074880E-01 4.98280837E-01 4.99176769E-01 6.00005881E+00 3.00042419E+00
SIG 3.86627594E+01 3.86624102E+01 3.86625144E+01 3.86627453E+01 3.86617311E+01 3.86624428E+01 3.86628498E+01 3.86683407E+01
    FEM= 1.46567342E+03

                            TS= 3.95408956E+02
   TEST -4.3731620E-01

  WT     6.04397598E+02
    VDP= 6.11329974E-02 1.41464786E-01 1.45090547E-01 4.98263987E-01 4.99168406E-01 6.00005942E+00 3.00042814E+00
SIG 3.87742744E+01 3.877390069E+01 3.87740198E+01 3.87742620E+01 3.87732160E+01 3.87739602E+01 3.87743558E+01 3.87801394E+01
    FEM= 1.46565370E+03

                            TS= 3.67048518E+02
   TEST  1.3358397E-01

  WT     6.04817434E+02
    VDP= 6.11993032E-02 1.41468253E-01 1.45087192E-01 4.98267596E-01 4.99170197E-01 6.00005929E+00 3.00042730E+00
SIG 3.87499086E+01 3.874954452E+01 3.87496561E+01 3.87498959E+01 3.87488568E+01 3.87495938E+01 3.87499920E+01 3.87557109E+01
    FEM= 1.46565207E+03

                            TS= 3.61928314E+02
   TEST  3.4154170E-04
    D=-3.20311055E-02 4.47585913E-01-4.52426795E-01 6.47978810E-01 4.13913760E-01-3.52392150E-02-3.88318541E-02
```

```
  WT    5.74816526E+02
    VDP= 6.52031914E-02 8.55200143E-02 2.01640541E-01 4.17270245E-01 4.47430977E-01 6.00446419E+00 3.00528128E+00
SIG 4.01323717E+01 4.01319718E+01 4.01319326E+01 4.01323534E+01 4.01306612E+01 4.01319180E+01 4.01328177E+01 4.01381954E+01

  WT    5.90147359E+02
    VDP= 6.29389041E-02 1.17159980E-01 1.69658372E-01 4.63076037E-01 4.76690649E-01 6.00197312E+00 3.00253624E+00
SIG 3.91851907E+01 3.91848099E+01 3.91849077E+01 3.91851757E+01 3.91840803E+01 3.91848105E+01 3.91853799E+01 3.91909833E+01
    FEM= 1.45820568E+03

                         TS= 3.93261569E+02
   TEST  1.5124891E-01

  WT    5.91962225E+02
    VDP= 6.27064212E-02 1.20408575E-01 1.66374642E-01 4.67779090E-01 4.79694850E-01 6.00171735E+00 3.00225440E+00
SIG 3.91223940E+01 3.91220151E+01 3.91221162E+01 3.91223793E+01 3.91212947E+01 3.91220233E+01 3.91225661E+01 3.91281939E+01
    FEM= 1.45802159E+03

                         TS= 3.16327578E+02
   TEST  2.2864357E-03
    D=-1.05779025E-02-5.10185407E-02 4.67680127E-02 7.89226087E-01 6.07229960E-01-4.98676642E-02-3.17040082E-02

  WT    5.36624609E+02
    VDP= 6.53508968E-02 1.33163210E-01 1.54682639E-01 2.70472569E-01 3.27887360E-01 6.01418427E+00 3.01018040E+00
SIG 4.51083585E+01 4.51077230E+01 4.51076348E+01 4.51081478E+01 4.50993549E+01 4.51054448E+01 4.51103259E+01 4.51155355E+01

  WT    5.56745685E+02
    VDP= 6.44256223E-02 1.28700496E-01 1.58773549E-01 3.39508067E-01 3.81003221E-01 6.00982223E+00 3.00740718E+00
SIG 4.11946621E+01 4.11942474E+01 4.11943018E+01 4.11945918E+01 4.11916428E+01 4.11936425E+01 4.11953238E+01 4.12004219E+01
    FEM= 1.43324687E+03

                         TS= 3.99621112E+02
   TEST -7.8205202E-02

  WT    5.53121161E+02
    VDP= 6.45951008E-02 1.29517911E-01 1.58024236E-01 3.26863142E-01 3.71274225E-01 6.01062120E+00 3.00791514E+00
SIG 4.16202909E+01 4.16198602E+01 4.16199021E+01 4.16202068E+01 4.16167843E+01 4.16190940E+01 4.16210729E+01 4.16261454E+01
    FEM= 1.43301581E+03

                         TS= 5.14467281E+02
   TEST  3.0038162E-02
    D= 1.32767384E-03 5.30805511E-02-5.38894564E-02-3.80297300E-01 9.20618323E-01-4.28736383E-02-1.65630957E-02

  WT    5.43168407E+02
    VDP= 6.44291415E-02 1.22882842E-01 1.64760418E-01 3.74400304E-01 2.56196934E-01 6.01598041E+00 3.00998552E+00
SIG 4.36657753E+01 4.36652454E+01 4.36653807E+01 4.36653679E+01 4.36629845E+01 4.36585409E+01 4.36671781E+01 4.36711965E+01

  WT    5.49369222E+02
    VDP= 6.45267122E-02 1.26783730E-01 1.60800083E-01 3.46452268E-01 3.23853146E-01 6.01282962E+00 3.00876830E+00
SIG 4.18593304E+01 4.18588963E+01 4.18589472E+01 4.18592003E+01 4.18561862E+01 4.18573621E+01 4.18601047E+01 4.18649758E+01
    FEM= 1.43254862E+03
```

```
                         TS= 5.54124676E+02
  TEST  5.5208843E-02

 WT    5.50981491E+02
   VDP= 6.45550171E-02 1.27915363E-01 1.59651205E-01 3.38344647E-01 3.43479960E-01 6.01191559E+00 3.00841519E+00
SIG 4.17090259E+01 4.17085973E+01 4.17086318E+01 4.17089175E+01 4.17056385E+01 4.17074396E+01 4.17097904E+01 4.17147933E+01
   FEM= 1.43225606E+03

                         TS= 5.17577137E+02
  TEST -3.7068864E-02
   D=-6.66965604E-03-1.01345876E-02 9.61173693E-03-6.31067965E-02-1.65021300E-04-6.59330167E-01-7.49040491E-01

 WT    5.68377991E+02
   VDP= 6.78898452E-02 1.32982657E-01 1.54845336E-01 3.69898045E-01 3.43562471E-01 6.34158068E+00 3.38293544E+00
SIG 4.15909002E+01 4.15904344E+01 4.15905522E+01 4.15908042E+01 4.15883245E+01 4.15895080E+01 4.15916159E+01 4.15967319E+01

 WT    5.56282783E+02
   VDP= 6.55898012E-02 1.29487725E-01 1.58159963E-01 3.48135542E-01 3.43505563E-01 6.11420938E+00 3.12462736E+00
SIG 4.16567562E+01 4.16563171E+01 4.16563943E+01 4.16566410E+01 4.16538309E+01 4.16550892E+01 4.16575250E+01 4.16624901E+01
   FEM= 1.43193059E+03

                         TS= 4.80190433E+02
  TEST  2.9079948E-03
   D= 4.24968824E-03 2.86949637E-03-3.06107740E-03-2.16378854E-02-3.03766787E-02-8.66214423E-01 4.98242882E-01

 WT    5.51745679E+02
   VDP= 6.34649571E-02 1.28052976E-01 1.59690501E-01 3.58954485E-01 3.58693903E-01 6.54731659E+00 2.87550592E+00
SIG 4.09619052E+01 4.09615540E+01 4.09615798E+01 4.09617954E+01 4.09592840E+01 4.09604188E+01 4.09628237E+01 4.09662216E+01

 WT    5.53628820E+02
   VDP= 6.44474006E-02 1.28716347E-01 1.58982841E-01 3.53952236E-01 3.51671418E-01 6.34706503E+00 2.99068977E+00
SIG 4.12512909E+01 4.12509073E+01 4.12509349E+01 4.12511771E+01 4.12483977E+01 4.12496772E+01 4.12519115E+01 4.12567488E+01
   FEM= 1.43128135E+03

                         TS= 3.68992114E+02
  TEST -3.5904454E-03
   D= 1.02433117E-02-3.04620128E-02 2.78955893E-02-3.13108387E-03-9.27271942E-02-7.73503993E-01 6.25517717E-01

 WT    5.44243030E+02
   VDP= 5.93257447E-02 1.43947353E-01 1.45035047E-01 3.55517778E-01 3.98035015E-01 6.73381702E+00 2.67793091E+00
SIG 4.08461428E+01 4.08457733E+01 4.08458253E+01 4.08460228E+01 4.08433396E+01 4.08447496E+01 4.08468943E+01 4.08514696E+01

 WT    5.51773778E+02
   VDP= 6.35959160E-02 1.31248530E-01 1.56663995E-01 3.54212510E-01 3.59379450E-01 6.41136325E+00 2.93869304E+00
SIG 4.11464240E+01 4.11460452E+01 4.11460958E+01 4.11463199E+01 4.11437560E+01 4.11449642E+01 4.11469723E+01 4.11518473E+01
   FEM= 1.43088421E+03

                         TS= 2.83421243E+02
   CONVERGENCE TEST
 1.25162925E+00
```

```
H
 2.37888993E-05 2.45640122E-05-2.38585450E-05-7.25756142E-05-5.28914298E-05-2.19713470E-03 1.42147758E-03
 2.45640122E-05 7.71141110E-05-7.54200627E-05-1.36825326E-04-1.73481197E-05-2.95465504E-03 1.52186567E-03
-2.38585450E-05-7.54200627E-05 7.75437689E-05 1.33075800E-04 1.26217621E-05 2.96482848E-03-1.48664139E-03
-7.25756142E-05-1.36825326E-04 1.33075800E-04 6.16315585E-04 3.24183402E-04 1.01428440E-02-4.13733256E-03
-5.28914298E-05-1.73481197E-05 1.26217621E-05 3.24183402E-04 1.16526420E-03 1.75283230E-02-3.07563959E-03
-2.19713470E-03-2.95465504E-03 2.96482848E-03 1.01428440E-02 1.75283230E-02 6.09110131E-01-1.33966625E-01
 1.42147758E-03 1.52186567E-03-1.48664139E-03-4.13733256E-03-3.07563959E-03-1.33966625E-01 9.38554199E-02
QUADRATIC CONVERGENCE TEST SATISFIED
    FOR RD=  1.00000E+01 FEM=  1.43088E+03 WT=  5.51774E+02
    XEM  6.3595916E-02  1.3124853E-01  1.5666399E-01  3.5421251E-01  3.5937945E-01  6.4113632E+00  2.9386930E+00


  WT    5.51773778E+02
    VDP= 6.35959160E-02 1.31248530E-01 1.56663995E-01 3.54212510E-01 3.59379450E-01 6.41136325E+00 2.93869304E+00
SIG 4.11464240E+01 4.11460452E+01 4.11460958E+01 4.11463199E+01 4.11437560E+01 4.11449642E+01 4.11469723E+01 4.11518473E+01
```

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 CLT
        1.34425E-03     1.52741E-03     1.61174E-03     2.11494E-03     2.80834E-03     3.67730E-03     4.71557E-03     5.21019E-03
        5.26247E-03     5.59503E-03
SMS
 10   11   11   12    9   10   12    9   13   12   10   13    8   11   11   13   14   14    8    9   12   13    7   10    8
 14   14   12    7   15    9   15   13   10   15   11    6   15   11   16
 12   13   13   14   14   12   11   12   15   11   14   13   13   15   15   14   11   10   16   16   10   12   16 . 15   15
 14   12   11   13   16   17   17   10   13   16   14    9   17   17   10
  1    1    1    1    2    2    1    2    3    2    3    3    4    5    4    1    4    6    5    6
  1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
  1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
  1   .1    1    1    1    1    1    1    1    1    1    1    1    1    1
  2    3    4    1    5    6    7    4    8    5
  2    3    4    5    6    7    1    8    5    4
  2    1    3    4    5.   6    7    4    3    5
SNS
 14   15   14   15   14   15   14   13   15   16   13   16   13   16   13   14   15   16   14   15   13   17   13   16   12
 14   17   17   12   16   12   15   13   12   17   17   12   14   12   16
 14   14   15   15   14   15   14   13   15   13   16   13   16   14   16   13   15   13   15   16   14   16   14   13   17
 17   12   12   12   17   15   16   12   17   13   12   13   14   17   15
  6    7    8    5    8    9    9   10   10    7    9   11   11   12   10   10   12   13   11   12
 27   28   26   29   25   30   24   31   23   32   33   22   34   21   35   20   36   19   37   18
 23   22   24   21   25   20   26   19   27   18   28   17   29   16   30   15   31   14   32   13
 33   32   34   31   35   30   36   29   37   28   38   39   27   40   26
  1    1    1    1    1    1    1    2    1    2
  1    1    1    1    1    1    1    1    2    2
  1    1    1    1    1    1    1    2    2    2

     CRITICAL LOADS     GROSS          PANEL          SHEET

                    1.29979800E+03 2.07340219E+03 1.09559259E+03

                    1.52339488E+03 2.28246449E+03 1.30432639E+03

                    5.10607910E+02 1.85660148E+03 9.61512205E+02

     MODE SHAPES         M   N          M   N          M   N

                        10  14          1  27          2   1

                        12  14          1  23          2   1

                         1   6          1  33          2   1

LRS  -8.71914195E+03-1.21518742E+04-2.55199198E+03
CRS   2.58598776E+03 5.09831963E+03 4.58043366E+02
DES   3.00330914E-02 6.14230737E-02 2.51524641E-03
EBU   5.00000000E-03 5.00000000E-03 5.00000000E-03
EBL  -5.00000000E-03-5.00000000E-03-5.00000000E-03
LRCB -5.43651325E+05-5.43651325E+05-5.43651325E+05
```

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  BEU    F  F  F
  BEL    T  T  T
  BLR    T  T  T
  EPA    2.58598776E-04 5.09831963E-04 4.58043366E-05

TS= 6.35959160E-02 TX= 1.31248530E-01 TY= 1.56663995E-01 DX= 3.54212510E-01 DY= 3.59379450E-01 LX= 6.41136325E+00 LY= 2.9386930E+00

AX= 4.64898711E-02 AY= 5.63018204E-02

  G.B.   5.38545220E-01 6.17042903E-01 4.15191374E-01

  P.B.   3.37609367E-01 4.11835542E-01 1.14187133E-01

  S.B.   5.13022753E-01 5.73290857E-01 1.78497689E-01

  LRB.   1.60381140E-02 2.23523307E-02 4.69417044E-03

  CRBU   0.             0.             0.

  CRBL  -5.17197552E-02-1.01966393E-01-9.16086732E-03

  S.Y.   1.73300581E-01 2.47836788E-01 5.01522324E-02

  LRYU  -1.74382839E-01-2.43037485E-01-5.10398396E-02

  LRYL   1.74382839E-01 2.43037485E-01 5.10398396E-02

  CRYU   5.17197552E-02 1.01966393E-01 9.16086732E-03

  CRYL  -5.17197552E-02-1.01966393E-01-9.16086732E-03

  WT     5.51773778E+02

  TB     8.78263069E-02


   VDP= 6.35959160E-02 1.31248530E-01 1.56663995E-01 3.54212510E-01 3.59379450E-01 6.41136325E+00 2.93869304E+00
SIG 4.11464240E+01 4.11460452E+01 4.11460958E+01 4.11463199E+01 4.11437560E+01 4.11449642E+01 4.11469723E+01 4.11518473E+01
   D= 9.95053042E-01 8.02544695E-02 2.80871778E-02 4.50167101E-02 2.41439349E-02-1.27602253E-03-5.35843806E-03

  WT     4.97488920E+02
   VDP= 5.50727326E-02 1.30561105E-01 1.56423413E-01 3.53826917E-01 3.59172644E-01 6.41137418E+00 2.93873894E+00
SIG 4.94168341E+01 4.94139781E+01 4.94154284E+01 4.94168859E+01 4.94093617E+01 4.94154100E+01 4.94147321E+01 4.94618947E+01

  WT     5.18707481E+02
   VDP= 5.84042497E-02 1.30829804E-01 1.56517451E-01 3.53977637E-01 3.59253480E-01 6.41136991E+00 2.93872100E+00
SIG 4.41563705E+01 4.41554705E+01 4.41557927E+01 4.41562719E+01 4.41523531E+01 4.41546074E+01 4.41565385E+01 4.41699217E+01
   FEM= 9.80511215E+02

                         TS= 3.09271974E+02
   TEST -8.4787555E-01
```

```
 WT     5.15905819E+02
    VDP= 5.79643637E-02 1.30794325E-01 1.56505034E-01 3.53957736E-01 3.59242806E-01 6.41137047E+00 2.93872337E+00
SIG 4.45774806E+01 4.45764779E+01 4.45768575E+01 4.45773871E+01 4.45732498E+01 4.45757039E+01 4.45775491E+01 4.45926612E+01
    FEM= 9.80512207E+02

                                TS= 3.47953869E+02
  TEST   8.2262282E-01

 WT     5.17293176E+02
    VDP= 5.81821913E-02 1.30811894E-01 1.56511183E-01 3.53967591E-01 3.59248092E-01 6.41137019E+00 2.93872220E+00
SIG 4.43632800E+01 4.43623306E+01 4.43626804E+01 4.43631838E+01 4.43591588E+01 4.43615094E+01 4.43634007E+01 4.43776148E+01
    FEM= 9.80481426E+02

                                TS= 1.80259266E+02
  TEST   1.3633900E-02
    D=-3.66762148E-02 6.56112728E-01 5.19028276E-01 3.95350900E-01 3.43777538E-01-3.45502068E-02 1.51991925E-01

 WT     4.82677384E+02
    VDP= 6.04744547E-02 8.98048484E-02 1.24071915E-01 3.29258159E-01 3.37761996E-01 6.41352958E+00 2.92922270E+00
SIG 4.84178568E+01 4.84167699E+01 4.84164051E+01 4.84174702E+01 4.84077745E+01 4.84134660E+01 4.84196650E+01 4.84316770E+01

 WT     5.04874237E+02
    VDP= 5.89642373E-02 1.16821620E-01 1.45443957E-01 3.45537534E-01 3.51917733E-01 6.41210690E+00 2.93548128E+00
SIG 4.51892481E+01 4.51883100E+01 4.51885230E+01 4.51890804E+01 4.51841027E+01 4.51867870E+01 4.51897424E+01 4.52029640E+01
    FEM= 9.78729007E+02

                                TS= 1.59911620E+02
  TEST   4.7118819E-01

 WT     5.07159129E+02
    VDP= 5.88173276E-02 1.19449734E-01 1.47522968E-01 3.47121145E-01 3.53294762E-01 6.41196851E+00 2.93609009E+00
SIG 4.50008077E+01 4.49998717E+01 4.50001054E+01 4.50006524E+01 4.49958259E+01 4.49984603E+01 4.50012284E+01 4.50146459E+01
    FEM= 9.78616210E+02

                                TS= 1.27180061E+02
  TEST   1.8856205E-01

 WT     5.07801679E+02
    VDP= 5.87762699E-02 1.20184230E-01 1.48104002E-01 3.47563727E-01 3.53679609E-01 6.41192983E+00 2.93626024E+00
SIG 4.49503956E+01 4.49494602E+01 4.49497002E+01 4.49502437E+01 4.49454632E+01 4.49480789E+01 4.49507973E+01 4.49642671E+01
    FEM= 9.78621281E+02

                                TS= 1.21396526E+02
  TEST   8.2257111E-02

 WT     5.08056926E+02
    VDP= 5.87599907E-02 1.20475452E-01 1.48334378E-01 3.47739207E-01 3.53832199E-01 6.41191450E+00 2.93632770E+00
SIG 4.49306748E+01 4.49297394E+01 4.49299816E+01 4.49305241E+01 4.49257590E+01 4.49283701E+01 4.49310687E+01 4.49445606E+01
    FEM= 9.78627571E+02
```

```
                    TS= 1.19246064E+02
 TEST  3.9306593E-02
  D= 1.60914422E-02-4.09511492E-01 5.66377969E-01 2.68297720E-01 4.88906312E-01-7.06639537E-02 4.41872106E-01

 WT    4.95423900E+02
  VDP= 5.67485605E-02 1.71664389E-01 7.75371323E-02 3.14201992E-01 2.92718910E-01 6.42074749E+00 2.88109369E+00
SIG 8.33552061E+01 8.33394923E+01 8.33496774E+01 8.32650009E+01 8.33254675E+01 8.27097797E+01 8.34657013E+01 8.33917966E+01

 WT    4.99818049E+02
  VDP= 5.74813392E-02 1.53015885E-01 1.03329087E-01 3.26419845E-01 3.14982928E-01 6.41752957E+00 2.90121585E+00
SIG 4.83340366E+01 4.83329238E+01 4.83333156E+01 4.83330162E+01 4.83268169E+01 4.83244349E+01 4.83367975E+01 4.83450657E+01
  FEM= 9.98804585E+02

                    TS= 1.40421187E+03
 TEST  5.9496942E-01

 WT    5.06134285E+02
  VDP= 5.84720367E-02 1.27803603E-01 1.38199125E-01 3.42938058E-01 3.45083290E-01 6.41317902E+00 2.92842047E+00
SIG 4.52170629E+01 4.52161362E+01 4.52163785E+01 4.52168755E+01 4.52117385E+01 4.52144673E+01 4.52174932E+01 4.52306027E+01
  FEM= 9.78208727E+02

                    TS= 1.35983088E+02
 TEST  2.0079047E-01

 WT    5.06601951E+02
  VDP= 5.85426394E-02 1.26006831E-01 1.40684163E-01 3.44115240E-01 3.47228414E-01 6.41286897E+00 2.93035922E+00
SIG 4.51385344E+01 4.51376066E+01 4.51378488E+01 4.51383568E+01 4.51333187E+01 4.51360144E+01 4.51389548E+01 4.51521508E+01
  FEM= 9.78155898E+02

                    TS= 1.26164799E+02
 TEST  3.1401953E-02
  D= 5.86765027E-03-7.98516774E-02-2.68165676E-01 2.64140685E-01 4.71246576E-01-1.13791765E-01 7.85419791E-01

 WT    4.96416789E+02
  VDP= 5.70757269E-02 1.45969750E-01 2.07725582E-01 2.78080069E-01 2.29416770E-01 6.44131691E+00 2.73400428E+00
SIG 5.73109675E+01 5.73085963E+01 5.73091088E+01 5.73095625E+01 5.72863031E+01 5.72772889E+01 5.73172404E+01 5.73270519E+01

 WT    5.04855585E+02
  VDP= 5.79311488E-02 1.34328484E-01 1.68630748E-01 3.16588115E-01 2.98117980E-01 6.42472765E+00 2.84850758E+00
SIG 4.69641127E+01 4.69630989E+01 4.69633524E+01 4.69637969E+01 4.69567234E+01 4.69588500E+01 4.69663190E+01 4.69751096E+01
  FEM= 9.79186668E+02

                    TS= 3.86236906E+02
 TEST  1.4577831E-01

 WT    5.06333853E+02
  VDP= 5.83013716E-02 1.29290196E-01 1.51710679E-01 3.33254225E-01 3.27851557E-01 6.41754790E+00 2.89806410E+00
SIG 4.56564722E+01 4.56555369E+01 4.56557772E+01 4.56562531E+01 4.56505799E+01 4.56533401E+01 4.56583016E+01 4.56671783E+01
  FEM= 9.77487700E+02

                    TS= 7.11883560E+01
 TEST  9.0558927E-03
  D= 1.57957959E-02 1.01641626E-01 1.47047131E-01-2.74862544E-01-2.13918043E-01 6.78500386E-02 9.17541628E-01
```

```
  WT    4.43823192E+02
    VDP= 5.04034737E-02 7.84693831E-02 7.81871136E-02 4.70685497E-01 4.34810578E-01 6.38362288E+00 2.43929328E+00
SIG 4.56384203E+01 4.56369724E+01 4.56371843E+01 4.56380869E+01 4.56353527E+01 4.56362868E+01 4.56396019E+01 4.56588653E+01

  WT    4.86178482E+02
    VDP= 5.50390991E-02 1.08298365E-01 1.21341345E-01 3.90021008E-01 3.72031600E-01 6.40353497E+00 2.70856616E+00
SIG 4.47922537E+01 4.47912261E+01 4.47915051E+01 4.47920420E+01 4.47883332E+01 4.47899152E+01 4.47935013E+01 4.48057416E+01
    FEM= 9.79855899E+02

                                TS= 7.36135827E+02
   TEST  6.9585748E-02

  WT    4.99839689E+02
    VDP= 5.71151326E-02 1.21657072E-01 1.40667674E-01 3.53895963E-01 3.43916461E-01 6.41245247E+00 2.82915819E+00
SIG 4.51610795E+01 4.51601330E+01 4.51603662E+01 4.51608700E+01 4.51559192E+01 4.51583450E+01 4.51626617E+01 4.51726732E+01
    FEM= 9.76686482E+02

                                TS= 1.09586608E+02
   TEST  6.0730897E-03
    D= 7.16599492E-03-2.43759390E-02 9.01243093E-02 2.96090666E-01-5.22501995E-01 2.64085533E-01 7.48874412E-01

  WT    4.85723409E+02
    VDP= 5.53236338E-02 1.27751057E-01 1.18136597E-01 2.79873297E-01 4.74541959E-01 6.34643108E+00 2.64193958E+00
SIG 4.72751140E+01 4.72739704E+01 4.72739803E+01 4.72749473E+01 4.72625931E+01 4.72735272E+01 4.72780602E+01 4.72903484E+01

  WT    4.97144379E+02
    VDP= 5.66678828E-02 1.23178443E-01 1.35042764E-01 3.35416116E-01 3.76527273E-01 6.39597015E+00 2.78241884E+00
SIG 4.51793647E+01 4.51784013E+01 4.51786463E+01 4.51791930E+01 4.51737384E+01 4.51772707E+01 4.51809901E+01 4.51913994E+01
    FEM= 9.76365647E+02

                                TS= 2.23425636E+02
   TEST  3.8051396E-02
    D= 3.58451076E-03 5.71959336E-03-5.13823401E-03 1.96934359E-02 1.80739901E-02 9.31740375E-01 3.62040618E-01

  WT    4.90831348E+02
    VDP= 5.48756274E-02 1.20318646E-01 1.37611881E-01 3.25569398E-01 3.67490278E-01 5.93009996E+00 2.60139853E+00
SIG 4.49203180E+01 4.49194311E+01 4.49196308E+01 4.49201168E+01 4.49146935E+01 4.49178418E+01 4.49219542E+01 4.49311590E+01

  WT    4.85427269E+02
    VDP= 5.30833720E-02 1.17458849E-01 1.40180998E-01 3.15722680E-01 3.58453283E-01 5.46422977E+00 2.42037822E+00
SIG 4.47733085E+01 4.47724722E+01 4.47726184E+01 4.47730580E+01 4.47673311E+01 4.47701939E+01 4.47750133E+01 4.47831789E+01

  WT    4.89786348E+02
    VDP= 5.45532828E-02 1.19804300E-01 1.38073947E-01 3.23798424E-01 3.65864937E-01 5.84631127E+00 2.56884127E+00
SIG 4.48843784E+01 4.48835030E+01 4.48836925E+01 4.48841693E+01 4.48787089E+01 4.48818022E+01 4.48860233E+01 4.48950300E+01
    FEM= 9.74433352E+02

                                TS= 2.93569834E+01
GRADIENT CONVERGENCE TEST SATISFIED
    FOR RD=  5.00000E+00 FEM=  9.74433E+02 WT=  4.89786E+02
    XEM  5.4553283E-02  1.1980430E-01  1.3807395E-01  3.2379842E-01  3.6586494E-01  5.8463113E+00  2.5688413E+00


  WT    4.89786348E+02
    VDP= 5.45532828E-02 1.19804300E-01 1.38073947E-01 3.23798424E-01 3.65864937E-01 5.84631127E+00 2.56884127E+00
SIG 4.48843784E+01 4.48835030E+01 4.48836925E+01 4.48841693E+01 4.48787089E+01 4.48818022E+01 4.48860233E+01 4.48950300E+01
```

```
 CLT
       1.27096E-03     1.47050E-03     1.49282E-03     1.93246E-03     2.54342E-03     3.31094E-03     4.22886E-03     4.96002E-03
       5.06675E-03     5.26123E-03
SMS
 11   12   12   13   11   10   13   13   14   10   12   14   11    9   10    9   14   12   15   15   11    8   13   14   13
 15   15   16   16    9   14    8    9   12   10    8   16   11   10    7
 13   14   14   13   12   15   15   13   12   15   14   12   16   14   11   11   16   16   13   15   17   17   15   17   16
 16   10   12   11   13   14   14   11   17   12   10   18   18   15   17
  1    1    1    1    2    2    1    2    3    3    2    4    3    4    5    9    7    6    5    8
  1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
  1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
  1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
  2    3    4    1    5    6    7    8    4    5
  3    2    4    5    6    7    8    1    5    9
  2    1    3    4    5    6    7    4    3    5
SNS
 14   15   14   15   15   14   14   16   15   13   16   16   13   13   15   14   14   13   15   16   16   13   13   17   17
 14   17   16   15   12   13   12   15   17   12   14   17   12   16   12
 14   15   14   15   14   15   14   13   13   16   16   15   15   13   14   13   16   14   16   13   15   16   17   14   17
 13   13   12   15   12   17   12   12   17   16   14   15   16   12   13
  6    7    5    8    8    9    9    7   10    9   10   11   11   10   12   14   13   12   11   14
 29   30   28   31   27   32   26   33   25   34   24   35   23   36   37   22   38   21   39   20
 24   25   23   26   22   27   21   20   28   19   29   18   30   17   31   16   32   15   33   14
 36   35   37   38   34   39   33   40   32   41   31   42   30   43   29
  1    1    1    1    1    1    1    1    2    2
  1    1    1    1    1    1    1    1    2    1
  1    1    1    1    1    1    1    2    2    2

    CRITICAL LOADS     GROSS           PANEL           SHEET

                   1.10897772E+03 1.86107316E+03 8.97320247E+02

                   1.27240508E+03 2.03602721E+03 1.06375342E+03

                   4.72776811E+02 1.67078506E+03 7.79821813E+02

    MODE SHAPES         M    N            M    N            M    N

                       11   14            1   29            2    1

                       13   14            1   24            3    1

                        1    6            1   36            2    1

LRS  -9.92987925E+03-1.38312894E+04-2.90795883E+03
CRS   2.89846753E+03 5.75160242E+03 5.05946674E+02
DES   3.88975373E-02 7.94198727E-02 3.26459977E-03
EBU   5.00000000E-03 5.00000000E-03 5.00000000E-03
EBL  -5.00000000E-03-5.00000000E-03-5.00000000E-03
LRCB -5.42088416E+05-5.42088416E+05-5.42088416E+05
```

```
BEU     F   F   F
BEL     T   T   T
BLR     T   T   T
EPA     2.89846753E-04 5.75160242E-04 5.05946674E-05

TS= 5.45532828E-02 TX= 1.19804300E-01 TY= 1.38073947E-01 DX= 3.23798424E-01 DY= 3.65864937E-01 LX= 5.84631127E+00 LY= 2.5688413E+00

AX= 3.87924435E-02 AY= 5.05164160E-02

G.B.    6.31211962E-01 7.38758446E-01 4.48414548E-01

P.B.    3.76127072E-01 4.61683417E-01 1.26886459E-01

S.B.    6.12989019E-01 6.87313136E-01 2.15544744E-01

LRB.    1.83178222E-02 2.55148220E-02 5.36436261E-03

CRBU    0.             0.             0.

CRBL   -5.79693506E-02-1.15032048E-01-1.01189335E-02

S.Y.    1.97224586E-01 2.81815317E-01 5.71366762E-02

LRYU   -1.98597585E-01-2.76625789E-01-5.81591766E-02

LRYL    1.98597585E-01 2.76625789E-01 5.81591766E-02

CRYU    5.79693506E-02 1.15032048E-01 1.01189335E-02

CRYL   -5.79693506E-02-1.15032048E-01-1.01189335E-02

WT      4.89786348E+02

TB      7.79597144E-02

  VDP= 5.45532828E-02 1.19804300E-01 1.38073947E-01 3.23798424E-01 3.65864937E-01 5.84631127E+00 2.56884127E+00
SIG 4.48843784E+01 4.48835030E+01 4.48836925E+01 4.48841693E+01 4.48787089E+01 4.48818022E+01 4.48860233E+01 4.48950300E+01
```

INPUT DATA FOR SAMPLE PROBLEM

```
  3
 40   20    10
 40   20    10
 20   15    10
            .099          .06          .06          .5          .5
              6.           3.
            .1E8          .1E8         .1E8
            .333          .333         .333
            .1E8          .1E8         .1E8
            .333          .333         .333
       3750937.7     3750937.7    3750937.7
            .1E8          .1E8         .1E8
            .333          .333         .333
            .1E8          .1E8         .1E8
            .333          .333         .333
            700.          940.         212.
              0.           -2.           .4
            .101          .101         .101
            165.           60.
              0.            1.
 50   50    30
 20   20    20
 20   20    20
 30   30    30
100  100   150
 15   15    15
           .5E5          .5E5         .5E5
          -.5E5         -.5E5        -.5E5
           .5E5          .5E5         .5E5
          -.5E5         -.5E5        -.5E5
             .5            .5           .5          2.          2.
            10.           10.
           .019           .05          .05         -2.         -2.
            .05           .05
T  T  T  T  T  T  T  F
            10.            2.
          .1E-5         .1E-4        .1E-4       .1E-3       .1E-3
          .1E-2         .1E-2
             2.
             1.            1.           1.
             1.            1.           1.
             1.            1.           1.
             1.            1.           1.
           .5E5          .5E5         .5E5
           .5E5          .5E5         .5E5
           .5E5          .5E5         .5E5
           .5E5          .5E5         .5E5
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