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FLIGHT TRAJECTORIES AROUND PLANETS WITH RETURN TO EARTH

by

V. R. E. Brauze
A. A. Da hkov
V. N. Kubasov

(USSR)

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A. A. Dashkov
& V. N. Kubasov

SUMMARY

The method of calculations is discussed of close planet fly-by trajectories, taking account of planets' perturbing action, with subsequent return of the spacecraft to Earth. The peculiarities are investigated of flight trajectories around Venus and Mars as a function of total flight time and fly-by distance near the planet.

It is shown that during flight to Venus in 1967, the fly-off velocities from Earth to ensure return, exceed insignificantly the values of optimum velocities of flight without return.

It is shown also that trajectories, similar to those considered in this paper, exist not only for the Mars take off in 1969 or to Venus in 1967, but also in other years, with intervals between optimum take-off dates to Mars of about 2.14 years and to Venus about 1.6 years.

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* *

The flight trajectories around planets with return to Earth but without delay near the planet offer great interest. Considered below are the methods and the results of the solution of such a problem, using as an example the flights from Earth to Mars and Venus.

When investigating the trajectories assuring return to Earth, it is indispensable to determine in the first place the dependences of the initial geocentric velocities on the take-off dates, flight time to the planet and the minimum distance to it during the passage. It is also important to determine the total flight time from takeoff to return to Earth as a function of fly-by conditions near the planet.

The calculations of unperturbed motion show that the flight to Mars and Venus and return may be performed in two years. However, the results of such calculations cannot be used as reference for the subsequent study.

The results of calculations of perturbed motion are then applied to allow us to bring to light the possibility of curtailing the total flight time and decreasing energy consumptions at the expense of variations of flight distances near the planet.

The exposure of these mechanisms provides us with the possibility of choosing the most convenient flight trajectories, taking into account the fundamental facts, and subsequently curtail the volume of operations when conducting the computation of flight parameters and also of trajectories for other optimum cycles and to other planets.

The mass computations of similar trajectories may be conducted by a simple and fast-acting method, based upon the breakdown of the entire trajectory in sectors and the determination of docking conditions of these trajectories.

It is conditionally assumed that the entire flight trajectory consists of the following portions:

First Portion: heliocentric flight trajectory from Earth to planet;
second portion: heliocentric flight trajectory from planet to Earth;
flight near the planet: planetocentric trajectory;
flight near the Earth: geocentric trajectory at fly-off and return.

The determination of the first portion (Earth-Planet) and of the second one (Planet-Earth) may be conducted in the assumption that the spheres of actions of planets are drawn together into points. The flight trajectories are determined with the aid of Lambert-Euler equation (see [1]).

For the determination of energy required to bringing the probe into the interplanetary orbit, one has to determine the difference of spacecraft's and planet's (Earth's) heliocentric velocities

$$\vec{v}_{rel} = \vec{v}_{sc} - \vec{v}_{pl}.$$

It is further estimated that over the planetocentric sector, the trajectory must be such that the equality $\vec{V}_{rel} = \vec{V}_\infty$ be assured; here $\vec{V}_\infty$ is the planetocentric velocity of the spacecraft at "infinity". The magnitude of this velocity is linked with the true semiaxis a of the orbit by hhe relation

$$V_\infty = \frac{\mu}{|a|},$$

where μ is the product of planet's mass by the gravitational constant.

From the condition brought out it is possible to determine the parameters of planetocentric trajectories.

Thus, the "docking" of interplanetary and planetocentric trajectories is performed by parameters at "infinity".

For the determination of the flight orbit parameters between two points, it is necessary to assign oneself the radius-vectors $\vec{r}_1$ and $\vec{r}_2$ of the latter and the flight time t_f. Fixing the specific position of the planet (t_M for Mars, t_V for Venus) and varying the flight times accrrding to the first (t_1) an-and second (t_2) sectors of the orbit in required ranges, one may determine the portions of the orbit corresponding to one another, that is having near-planet relative velocities equal in absolute value, for at close planet fly-by, the relative velocity at "infinity" at fly up to and fly off varies only in direction without changing in its absolute value. It may be shown that the selection of such a calculation scheme hardly alters the total flight time, inasmuch as the fly-up-to acceleration near the planet at rapprochement is approximatley equivalent to the flight deceleration at take-off from it.

In practice, the determination of the mutually corresponding portions of the trajectory is performed by the method of orderly excess (trial-and-error method ?) as follows. The relative take-off velocity from the planet (V_{M2} or V_{V2}) is determined for each of the trajectories in the required range of t_2; thus, the dependence is obtained: $V_2 = f(t_2)$. Then, from the assigned range, the relative flight velocity V_1 to the planet is determined*, and with the aid of the dependence $V_2 = f(t_2)$ we may find the velocity V_2 , equal to it and the corresponding t_2 (one or several), that is, the heliocentric flight trajectory Earth-planet-Earth is fully determined.

(*) for each flight time t_1

With such a statement of the problem, i. e., when the planet's sphere of action is assumed to be a point, the flight orbit at the point of planet location is artificially "broken", i. e., the planetocentric hyperbola must be such that the velocity "at infinity", characteristic for it, $V_{\infty hyp}$, be equal to the relative velocity V_1 at fly up to and V_2 at fly off, i. e., the following equalities must be fulfilled:

$$V_{\infty hyp} = V_1 = V_2,$$
$$V_{\infty 1} = V_{rel\,1},$$
$$V_{\infty 2} = V_{rel\,2},$$

where the index 1 is related to the first trajectory sector, and the index 2 to the second.

In this way, the "docking" of heliocentric and planetocentric sectors of the orbit is performed according to parameter values "at infinity".

The given method of computation offers the advantage of rapid calculations with satisfactory precision. We shall consider, as an example, the trajectories with take off to Mars in 1969 and to Venus in 1967.

In the case of flight to Mars we considered the cases when spacecraft performs one orbit around the Sun. In that case the encounter with Mars is possible on either the first (angular flight range $2f < 180°$), or the second ($180° < 2f < 360°$) half turns of orbit. Considered also was the return to Earth after two orbits. Then the encounter with Mars takes place on one of the four half turns of the orbit. The first case corresponds to orbits with total flight time of about two years, the second one - of about three years.

The dependences of take-off velocity from Earth and of total aggregate flight time till return to Earth on the date of start, $t_{\Sigma} = f(t_{st}, V_{fly\ off} = const, r_{\pi} = const)$ are plotted for each type of trajectories characterized by flight near Mars on a specific half turn of the orbit. Examples of these dependences are shown in Figures 2—5.

When studying the fly-around trajectories, we excluded the collision trajectories, i.e., we assumed for Mars $r_{\pi} \geqslant 3400$ km and for Venus $r_{\pi} \geqslant 6000$ km.

It is also appropriate to limit the take-off velocity from Earth, related to infinity, to the value $V_{fly\ off} \leqslant 6.5$ km/sec.

In the graph for $t_\Sigma = f(t_{st})$ (Fig.2) the line $t_\Sigma = 2$ years is drawn which corresponds to flight "at infinity" ($r_\pi = \infty$) and divides the family of trajectories in two halves, in each of which one may find orbits with common start date t_{st} and the same flight time t_1 to Mars, having a different total flight time t_Σ , i. e. differing by the second portions of trajectory as a function of the disposition of the flight hyperbola relative to Mars.

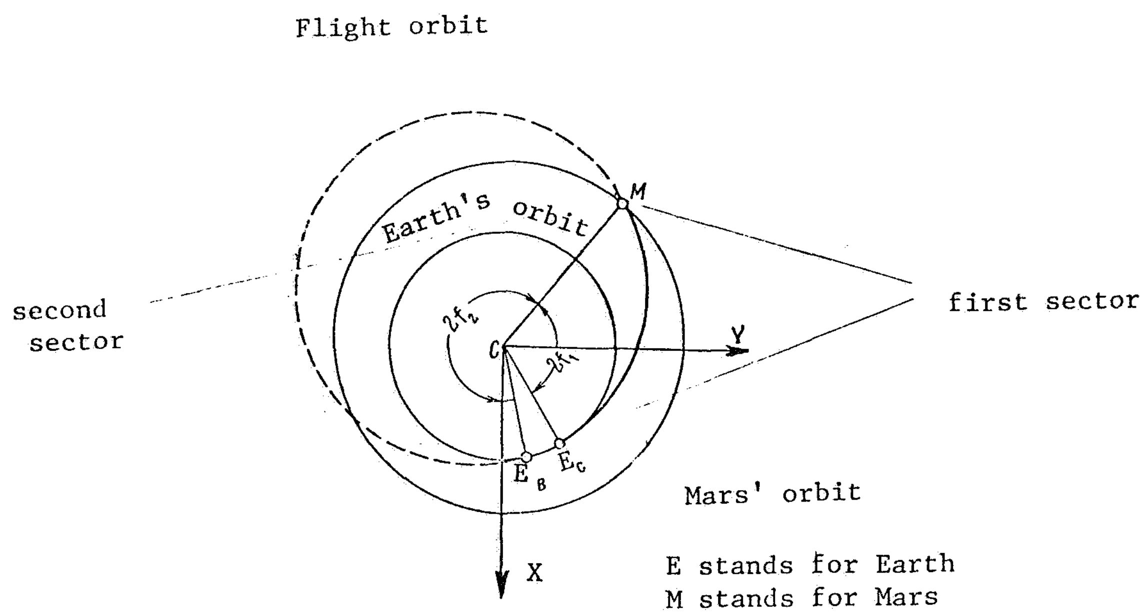

Fig.1

Sketch of the flight Earth-Mars-Earth

Isolines are drawn in the graph for constant flight distances near Mars r_π = const alongside with isolines of fly-off velocities $V_{fly\ off\infty}$ = const from Earth, with the help of which it is possible to determine the optimum date for start, as well as the minimum velocity for a given flight distance r_π near Mars.

It may be seen from the results of calculations that the optimum start date in 1969 for "two-year" orbits is 11 March. At the same time, the minimum fly-off velocity from Earth, determined "at infinity", constitutes 5.075 km/sec, the flight time to Mars is t_1 = 112 days, and the total flight time is 730 days.

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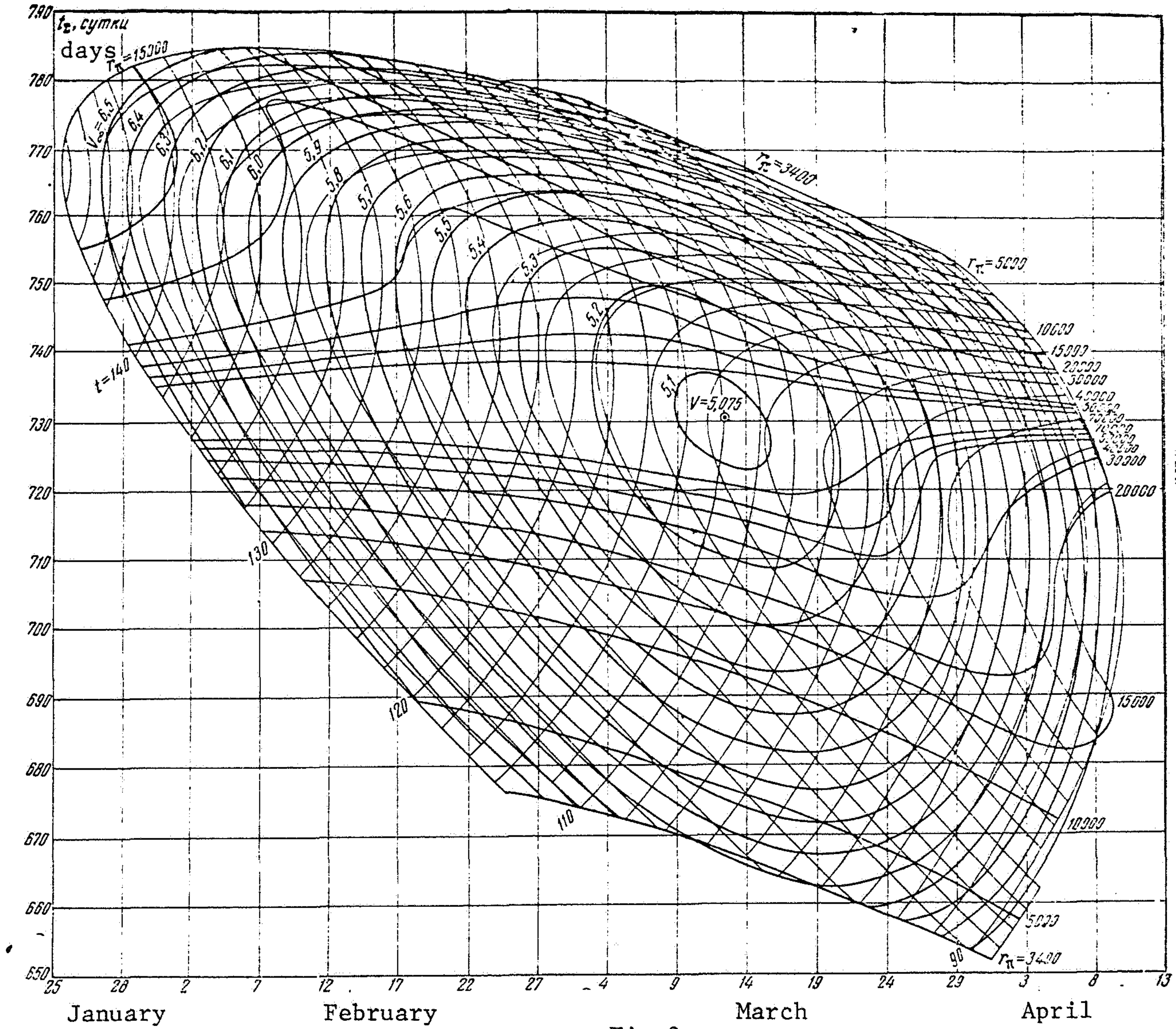

Fig.2

Dependence of total flight time to Mars with return to Earth when starting in 1969.

Flight near Mars over the first half turn of the 2-year orbit

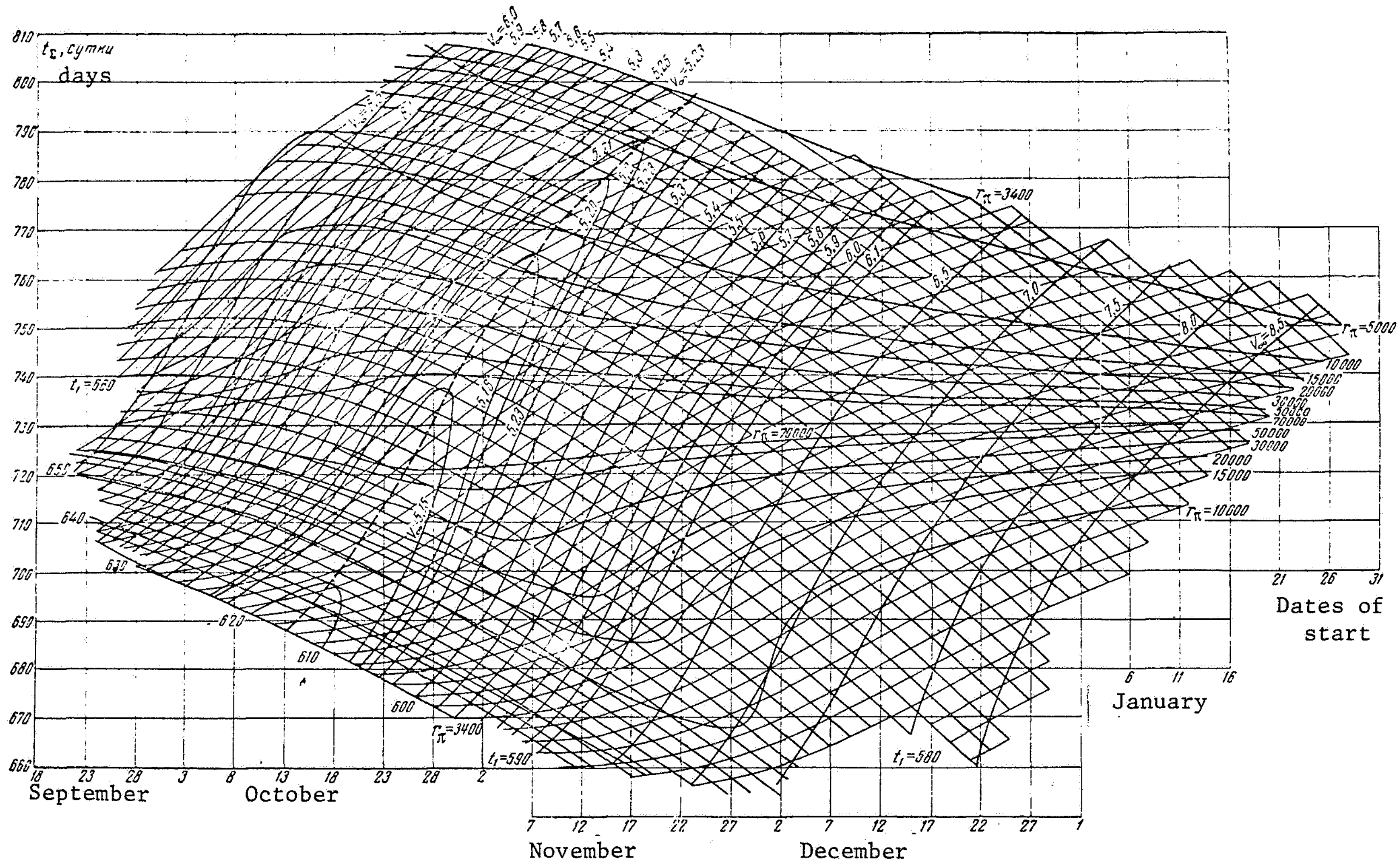

Fig.3

Dependence of total flight time to Mars with return to Earth for a start at the end 1969-begin.1970

Flight near Mars on the second half-turn of a 2-year orbit

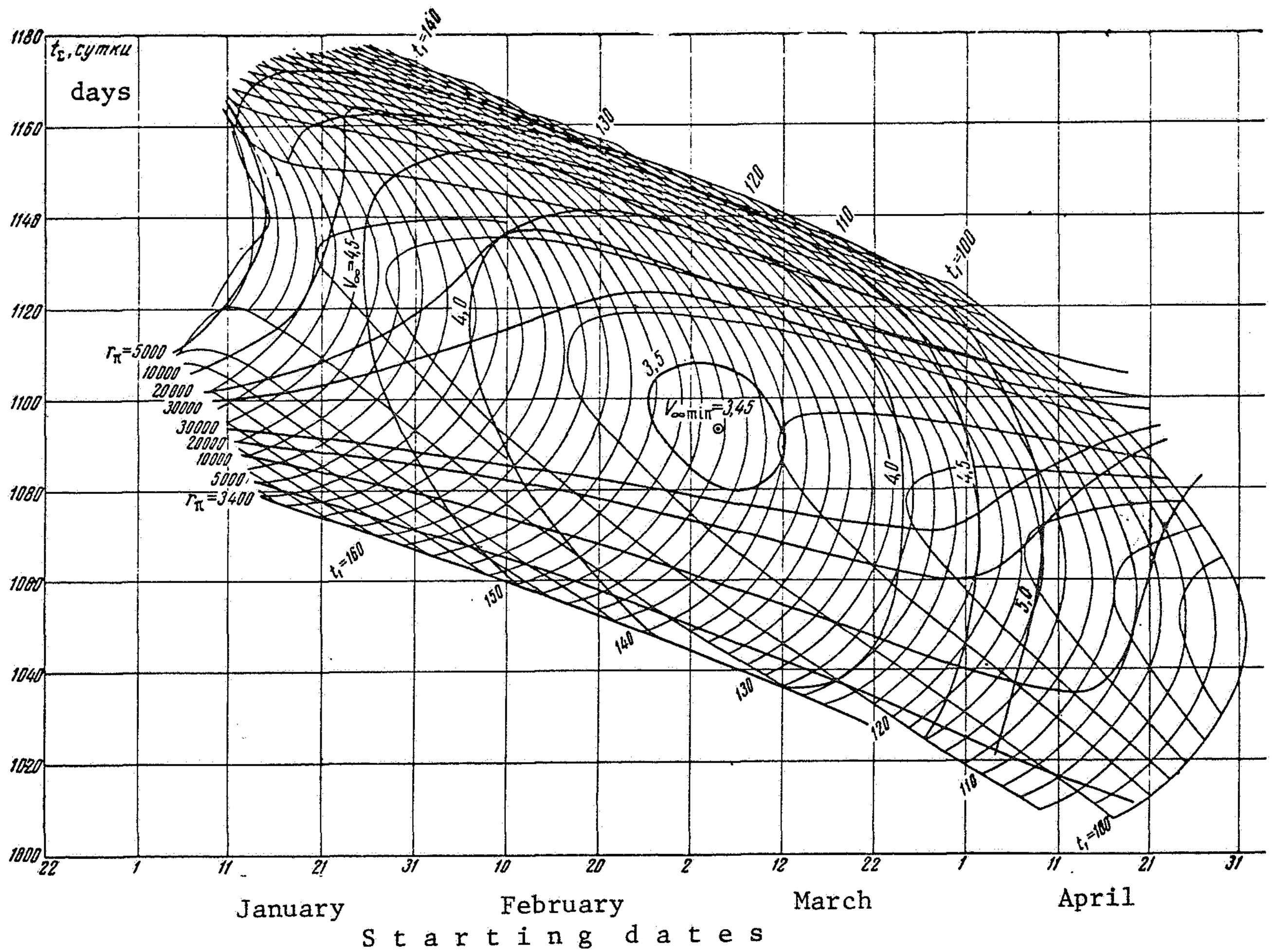

Fig.4

Dependence of the total flight time to Mars with return to Earth after 3 years

Start in 1969. Flight near Mars over the first orbit half turn

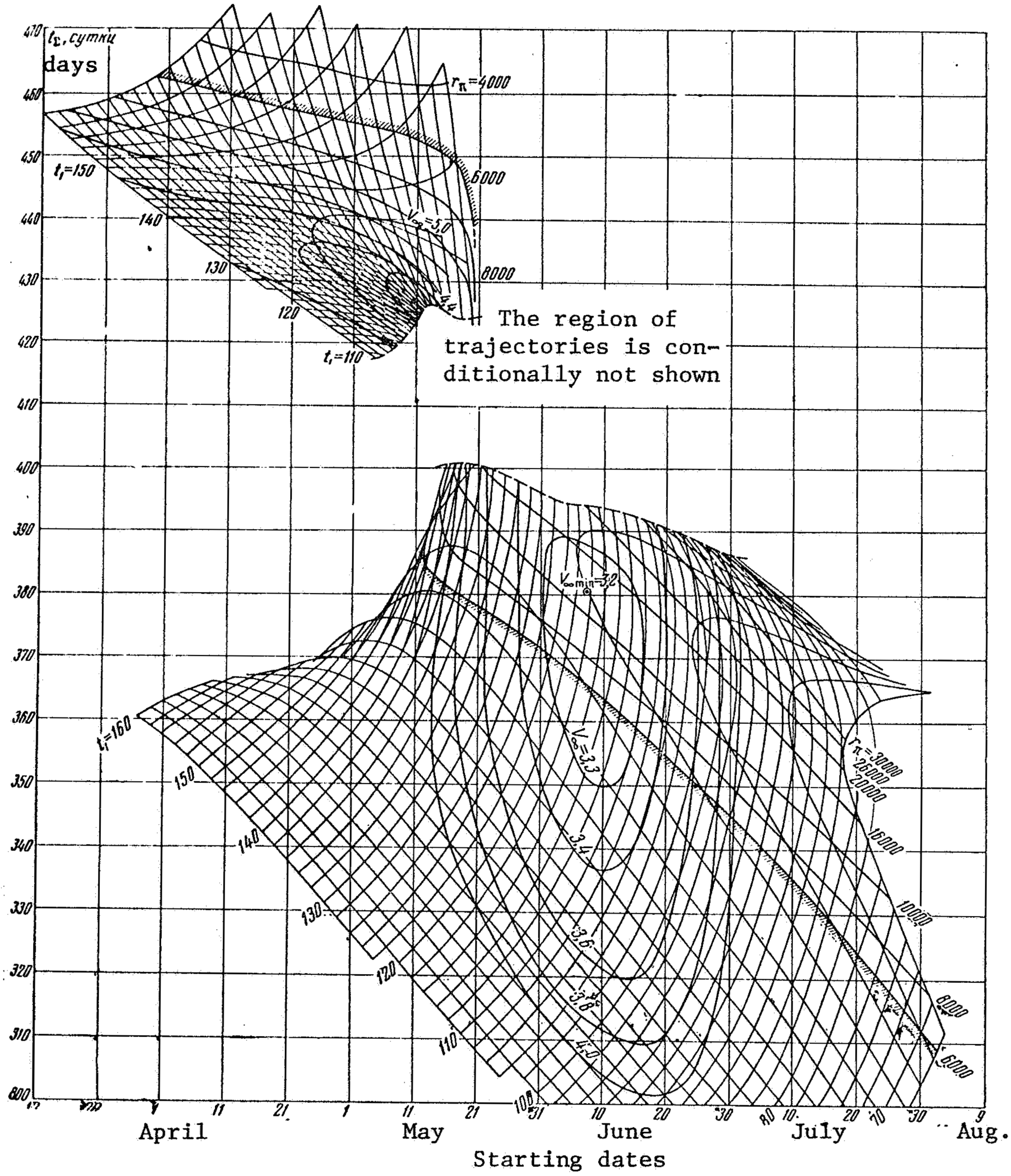

Fig.5

Dependence of total flight time to Venus with return to Earth at start in 1967

As the fly-off velocity increases to 6.5 km/sec, it becomes possible to start toward Mars beginning from 25 January and ending on 10 April.

At encounter on the first half turn, the flight time range to Mars constitutes t_f = 100 - 130 days. The close flight by Mars allows us to economize the flight time by about 60 days (by comparison with the 2-year orbit), but requires a higher fly off velocity than the minimum velocity at flight "at infinity".

The region of trajectories encountering Mars over the second half turn (see Fig.3), has no sharply expressed minimum; the velocity minimum at take-off varies from 5.070 km/sec at start on 14 October to 5.210 km/sec at start on 25 November, forming a peculiar kind of "line of minima". Contrary to trajectories encountering Mars over the first half turn, all curves r_{π} = const intersect this line of minima, i. e. a close flight near Mars' surface may be materialized with minimum take-off velocity from Earth. At encounter with Mars over the second half turn of the orbit, the flight time to the planet is 590 to 670 days, the starting period being between 25 September and 25 December 1969.

We considered above the trajectories of flight around Mars and return to Earth within more or less 1 years. In this case the probe effects more or less one revolution around the Sun. However, possible also are trajectories at which the probe performs more or less two revolutions around the Sun and returns to Earth within more or less three years. We shall designate such orbits as three-year ones.

At encounter with Mars over the first half turn (see Fig.4) for the three-year orbits the qualitative pattern is analogous to that of the two-year orbit, with the only difference that the period of start is more spread, from 1 January to 20 April 1969. At start on 7 March 1969, the velocity minimum is 3.45 km/sec, the duration of the journey to Mars is 190 days and the maximum possible economy of time by comparison with the three-year orbit constitutes 60 to 70 days.

The encounter with Mars on the fourth half turn of the three-year orbit takes place from 900 to 1000 days after take-off. In the region of minimum fly-off velocities, equal to 3.3 - 3.4 km/sec, the total flight time economy constitutes 30 - 40 days, while at increase of $V_{fly-off\infty}$ to 6.5 km/sec, it may be decreased by 90 to 100 days. For the fourth half turn, the starting period lasts from September 1968 through January 1969 (the range of possible dates of start corresponding to the limitation of $V_{fly-off\infty}$ to 6.5 km/sec).

When flying to Venus, the regions of return trajectories are determined for the start in 1967, under the condition for the probe to perform one orbit around the Sun, whereupon this total flight time till the return to Earth is of about one year.

At flight near Venus on the first half turn of the orbit (see Fig.5), the optimum take-off date is on 7 June 1967. At the same time, the fly-off velocity, determined at infinity, is minimum and equal to 3.2 km/sec, the fli ht time to Venus t_1 = 119 days, the total flight time till returning to Earth is t_Σ = 384 days, the flight by Venus taking place at a distance of about 10,000 km.

If we limit the fly-off velocity to 6.5 km/sec, it is possible to start between 24 March and 24 July. If for the start we consider a date range of about 300 days in the optimum region, the flight by Venus at distances cf 1000 to ~ 15,000 kilometers and more from the surface takes place along trajectories for which the fly-off velocity from Earth lies in the region of the minimum and constitutes 3.2 - 3.5 km/sec, the flight time from Earth to Venus constitutes t_1 = 100 - 130 days, the total flight time is t_Σ = 365 - 385 days, with start from 20 May to 20 June. Start at flight by Venus over the second orbit half turn and limitation of fly-off velocity to 6.5 km/sec is possible between 9 January and 13 November 1967. The optimum date for start is 18 June, whereupon the flight time to Venus t_1 = 174 days and the minimum fly-off velocity is 3.42 km/sec.

In a range of starting dates of ±15 days from the optimum (from 3 June to 3 July) the fly-off velocity $V_{fly-off\infty}$ = 3.42 - 3.7 km/sec, the flight tine to Venus is t_1 = 160 - 186 days, the total flight time t_Σ = 480 - 520 days.

Considering the basic energy characteristic of flight trajectories around Mars and Venus, that is, the required fly-off velocity at infinite range from the Earth, $V_{fly-off}\ \infty$, one may derive the following basic conclusions:

— at flight to Mars with return to Earth, the velocities $V_{fly-off\infty}$ exceed substantially the optimum velocities for reaching Mars for the case, when no return to Earth takes place;

— at flight to Venus with return to Earth, the velocities $V_{fly-off\infty}$ are insignificantly higher than the optimum velocities for the case when return to Earth does not take place;

— the flight around Venus may be accomplished in about one year. The distance of minimum rapprochement with Venus may constitute from several hundred to several tens of thousand kilometers for an insignificant energy variation.

Trajectories, analogous to those considered in the present work, exist not only for the start to Mars in 1969 and the start to Venus in 1967, but also for other years, with time interval between optimum starting dates of about 2.14 years to Mars, and of about 1.6 years to Venus.

**** T H E E N D ****

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