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PROJECT TECHNICAL REPORT

PROPULSION SYSTEMS DISPERSION ANALYSIS AND OPTIMUM PROPELLANT MANAGEMENT

NAS 9-8166 28 October 1968

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Prepared by
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TRW SYSTEMS GROUP

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ABSTRACT

The performance of a propulsion system during a given mission is not a fixed constant; it is a statistical variable. Statistical variation of the performance results from the fact that it is a function of quantities which exhibit varying degrees of uncertainty such as helium regulator outlet pressures, propellant temperatures and spacecraft weights. If the statistical variations of these variables are known, the variation in propulsion system performance can be determined. The report presents a procedure to develop dispersions for propulsion performance parameters. The approach is intended to be general so that it can be applied to all primary Apollo propulsion subsystems. By beginning with this general formulation, it is hoped that particular problems in determining the variations of certain independent propulsion parameters for individual subsystems (if they do not immediately fit the presented approach) may be solved. The report also presents a method of optimum propellant management. The discussions in this report are directed toward determining the dispersions in the independent variables. Although it will be stated that the independent variable dispersions will be input to a Monte-Carlo technique to determine the dependent variable dispersions, the results of the report are not restricted to the Monte-Carlo method. They are also equally applicable to other methods, such as "Root Sum Square", for determining the dependent variable dispersions.

CONTENTS

TABLES

ILLUSTRATIONS

SECTION I. PROPULSION SYSTEMS DISPERSION ANALYSIS

Introduction. When constructing Apollo propulsion preflight predictions, it is desirable to report propulsion system performance dispersions as well as nominal performance predictions. This is important so that the probability associated with the attainment of a particular mission objective can be determined. As an example, if a particular vehicle velocity requirement is established based on predicted propulsion system performance and payload requirements, the variability in propulsion performance may not indicate a sufficiently high probability that the predicted performance will be attained nor that the mission objectives can be achieved. The usual procedure is to allocate an additional amount of propellant (reserves) based on the probability that vehicle performance including that of the propulsion system will be less than predicted. If a vehicle performs as predicted, the reserves will not be consumed, and the additional propellant weight represents a decrease in payload capability. By decreasing the variability of performance, payload capability may be increased. A portion of a systems variability is inherent to the hardware of the system and cannot be improved without design changes; however, a substantial amount of the variability represents a lack of confidence in ones capability to predict a system's performance. Thus improved analysis techniques (both system modeling and dispersion analyses) may often afford greater payload potential by decreasing propellant reserve requirements. Another consideration of great importance is the possibility of defining variabilities that are too small; thus, insufficient propellant reserves may be allocated to meet the mission objectives with the actual variations of propulsion performance. The above reasons underline the necessity of defining systems variabilities as accurately as possible. Since the "Monte-Carlo" technique incorporated in the Propulsion Trajectory Analysis Program (PATS) will be used in determining the variability of the propulsion parameters such as engine thrust and specific impulse (i.e., the independent parameters), obtaining reliable results will depend on adequate descriptions of the variability in the propulsion system independent parameters, such as ullage pressures, temperatures, and system hydraulic resistances. Various approaches have been used in the industry, and in an effort to standardize the method of obtaining the variability of the independent parameters for Apollo propulsion systems, the approach presented in this report is recommended.

The principle advantages of using the Monte-Carlo technique are: 1) a large percentage of the burden of tedious mathematics is "lifted from the shoulders" of the user, and 2) once the types of (frequency) distributions of the input parameters are defined, the user need not be concerned with the mechanics of how these distributions must be combined to insure reliable results. At present, PATS is capable of handling two types of distributions (normal and uniform). However, if it is determined that other distributions need to be considered, the program could be modified. It should be noted that the distribution of all dependent parameters should be checked to determine their individual distributions.

Discussion. For each independent parameter, X, input into PATS (Reference 8) the parameters mean, $\overline{X}$, and standard deviation, σ, must be defined. Each parameter is thus defined by the following relationship:

$$X = \overline{X} \pm \sigma \qquad (1)$$

where: X = The random value of the independent parameter to be used in the simulation.

$\overline{X}$ = The mean value of the parameter, X.

σ = The standard deviation of X about the mean $\overline{X}$.

Although $\overline{X}$ and σ are the only parameters required for use in PATS, equation (1) is actually incomplete without a "confidence statement." A confidence statement reflects the degree of certainty one has that the values of $\overline{X}$ and σ are correct. The concept of confidence will be more clearly defined below.

The importance of the standard deviation, σ, is that it is an indicator of the variability of a given parameter, X, about its mean value, $\overline{X}$. If the frequency distribution of the parameter is known, then the standard deviation can be used to approximate the number of times a given value of X will be observed if sampled many times. If the parameter under consideration is "normally" distributed, then approximately 68.27% of the total observations (samples) will have a value within the band determined by $\overline{X} \pm \sigma$, 95.45% will have a value between $\overline{X} \pm 2\sigma$, and 99.73% will be bounded by $\overline{X} \pm 3\sigma$. The normal (Gaussian) distribution refers to a frequency distribution of X which is characterized by a symmetrical bell-shaped curve. Most propulsion input parameters that will be considered can, in general, be assumed to be normally distributed. The distribution of each parameter, however, should be checked to verify this assumption. If distributions are found to be other than normal or uniform, then an appropriate modification to PATS is indicated. Certain output parameters such as a vehicle velocity gain and propellant outage (i.e., the useable amount of one propellant remaining in the propulsion system after the other is totally consumed) are known to be non-normally distributed. Thus, the usual concept of a standard deviation (e.g., $\overline{X} \pm 1\sigma$ contains 68.3%) is not applicable, and the value for that function must be selected from the probability distribution function generated by the program.

It is important to realize the distinction between the sample standard deviation, S, calculated from a sample of measurements drawn from a population and the true standard deviation, σ, of the population. The sample standard deviation, S, can be calculated from a sample of measurements (X_j) by the following equation:

$$S = \left[\frac{\sum_{j=1}^{N}\left(X_j - \overline{X}\right)^2}{N - 1}\right]^{\frac{1}{2}} \tag{2a}$$

where

$$\overline{X} = \frac{\sum_{j=1}^{N} X_j}{N}, \tag{2b}$$

with the sample size represented by N. If N were equal to that of the total population, then S in equation (2a) would be numerically equal to σ. However, if N is less than that of the population, then S is only the "best estimate" of σ. In practice, one cannot sample all possible values of X. This is particularly true of parameters derived from rocket engine tests data where the sample size would be small. Equation (1) must then be rewritten as follows:

$$X = \overline{X} \pm S \tag{3}$$

Fortunately, statistical theory provides mathematical procedures to adjust the value of S to insure the inclusion of the true standard deviation, σ, (at a certain level of confidence) based on the sample size, N. These procedures have been incorporated in the following general equation and, thus, define the best estimate of the standard deviation:

$$S^2 = (kSrr)^2 + \left(\frac{t_c S_{INST}}{\sqrt{N}}\right)^2 + \left(S_{MODEL}\right)^2 + \left(kS_{EE}\right)^2; \qquad (4)$$

where: Srr = The true run-to-run sample standard deviation (instrumentation error removed) of the engine parameter.

k = The tolerance factor used to increase the uncertainty limits to achieve the desired probability and confidence.

t_c = The "Student t" factor used to compensate for uncertainties in the true mean, based on the sample size and desired population.

S_{INST} = The sample standard deviation of the random instrumentation error associated with the data sample.

N = The number of samples or data points used to calculate the mean value, X.

S_{MODEL} = The standard deviation of the propulsion model error in terms of equations or numerical technique.

S_{EE} = The sample standard deviation of the parameter's true engine-to-engine dispersion.

In equation (4), the first term accounts for the true run-to-run variability of the engine parameter; the second term accounts for the parameter uncertainty due to the instrumentation used to measure the parameter; the third term is to account for uncertainties in the characterizations used to model the parameter while the fourth term accounts for parameter variability from engine-to-engine. The complete equation (4) describes the total uncertainty of an engine parameter given the observed performance from ground tests (sigma "log-to-launch"). If the S_{EE} term is zero, the uncertainty applies to a specific engine rather than the engine class. Whether all of the terms in equation (4) will be applicable to a given parameter will depend on the particular propulsion system (i.e., SPS, DPS, or APS) being analyzed and the type of study (i.e., for a particular engine system or a general case study).

To illustrate the development of mean and dispersion values, assume that a particular engine has had n acceptance test runs and that it is one of a family of M engines.

When a rocket engine is fired a number of times, the engine performance from test-to-test will contain some variation even though the test conditions may appear to be identical each time. This dispersion, commonly known as the engine "run-to-run" variation, contains the true run-to-run variability of the engine plus instrumentation non-repeatability. If more than one engine of a family is being considered, there will be an additional dispersion in performance due to small hardware differences. This dispersion is known as "engine-to-engine" variation.

The Sample Run-to-Run Mean, $\bar{X}$. During each engine test, the values of certain parameters (e.g., interface pressure, propellant flow rate) are measured many times. The parameter values are averaged over several two to three second spans (time slices) selected from the test. The effect of this averaging is to generally eliminate the effects of noise in the measured values. Assume that the total number of time slices for a given test is represented by m. Let the average values for each time slice be represented by X_j, where j=1 to m. Then the mean value of the parameter for the particular engine, k, under consideration, is represented by:

$$\bar{X}_k = \frac{\sum_{i}^{n}\left(\frac{\sum_{j}^{m} X_j}{m}\right)_i}{n} \qquad (5)$$

The averaged parameter, X_j, could represent a quantity directly measured during the test, such as propellant temperature, or it could represent a calculated parameter such as hydraulic line resistance.

If a general engine study is being conducted, then the average value of the parameter is defined as:

$$\bar{\bar{X}} = \frac{\sum_{k}^{M} \bar{X}_k}{M} \qquad (5a)$$

The Sample True Run-To-Run Standard Deviation, Srr. In order to obtain a representative value for Srr, a large number of samples or test runs is desirable. However, since each engine experiences only a few runs before a flight, the following approach was adopted. It is assumed that although each particular engine of a family may display different mean values for the particular parameter under consideration, the run-to-run variability of each engine is the same for all engines. The apparent run-to-run dispersion, developed from the measured test parameters, deviates from the true run-to-run dispersion which we actually seek since the test measurements inherently contain instrumentation error. The sample dispersion of the true run-to-run variability may thus be represented by the following relationship:

$$(kSrr)^2 = (k_1 S\overset{*}{r}r)^2 - \left(k_2 S_{INST}\right)^2$$

where: $S\overset{*}{r}r$ = The apparent run-to-run dispersion

S_{INST} = The random instrumentation uncertainty

k_i = Tolerance factor, insuring compatible confidence and population.

Although it is expected that the apparent run-to-run dispersion, $k_1 S\overset{*}{r}r$, will be greater than the random instrumentation error, $k_2 S_{INST}$ in actual practice the reverse situation may occur. If such a situation arises, it is common to define: Srr = kSrr = 0 (i.e., the engine performance is extremely repeatable).

For each engine of the family, a mean for a parameter may be determined from the n engine tests as shown above. The variability of the parameter about this mean can be determined as follows:

$$\Delta\overline{X}_p = \frac{\sum_i^n \left(\frac{\sum_j^m X_j}{m}\right)_i}{n} - \left(\frac{\sum_j^m X_j}{m}\right)_p$$

$$p = 1 \ldots n$$

or

$$\Delta\overline{X}_p = \overline{X}_p - \left(\frac{\sum_j^m X_j}{m}\right)_p \qquad (6)$$

then

$$(kSrr)^2 = k^2 \frac{\sum_p^N \left(\Delta\overline{X}_p\right)^2}{N - 1} - S^2_{INST}, \qquad (7)$$

$$N = \Sigma n(M)$$

where N represents the total number of engine runs performed on all engines of the family. The first term to the right of the equal sign is the square of the apparent run-to-run dispersion, $k_1 S\overset{*}{r}r$.

The Tolerance Factor, k. It is desirable to predict the behavior of a parameter in such a way that at least a given percentage of future samples can be expected to lie within a computed interval with a specified degree of certainty. The values that specify the computed interval are called tolerance limits and can be generally represented by $\overline{X} \pm ks$. The tolerance factor k accounts for the sampling errors in $\overline{X}$ and s as well as for the percent population in the interval, P, and the confidence desired. The degree of certainty (confidence) is given by the confidence coefficient, γ. Suppose that $\overline{X} \pm ks$ is selected to include 68.27% (one standard deviation) of the population at a confidence level (γ) of 95%. Then, if many sets of random samples are taken from the population, the interval $\overline{X} \pm ks$ will contain at least 68.27% of the population in 95% of the sets chosen. The factor k may be obtained by the following:

$$k = ru \tag{8}$$

where r and u are presented in Tables 1 and 2 respectively. The statistical background and justification for the k factor may be found in References 1 and 2. The derivation of the factors r and u are found in Reference 1. Tables 1 and 2 were reproduced from this reference. The factor r is a function of the sample size, N, used to determine $\overline{X}_k$, and the proportion of the population, P, desired. The factor u is a function of degrees of freedom, f, associated with Srr and the desired degree of confidence, γ. The number of degrees of freedom is defined as the number of independent observations in the sample (i.e., sample size) minus the number of statistical quantities (i.e., $\overline{X}$, standard deviation, etc.) which must be estimated from sample observations. Since only the sample standard deviation is being computed, f becomes the sample size used to calculate Srr minus one. The k factor as used above is only applicable for normal or nearly normal distributions. However, it has already been noted that all input parameters (to which the tolerance factor will be applied) are assumed to be normal. It is thus expected that this constraint will have little effect on the approach.

Instrumentation Error, S_{INST}. In general, instrumentation error may be considered as random in nature. The characteristic of random error is that if enough run-to-run data samples from a measurement are taken, then the effects of random error on the mean of all samples will approach zero (i.e., random error has a zero mean). The existence of random error results in an uncertainty in the mean value ($\overline{X}$) of a measured parameter that was derived from a small number of data samples. In equation (4) this uncertainty is taken into account by dividing the random instrumentation error standard deviation by the square root of the number of observations. Thus, as the number of observations or samples increases, the effect of random instrumentation uncertainty on the knowledge of the true mean decreases. If the standard deviation of the random instrumentation error, S_{INST}, is a sample value rather than a population (or true) standard deviation, then it is necessary to include the "Student t" factor, t_c, to estimate, with the desired confidence, the interval which contains the true parameter mean. The statistical background for the "Student t" factor may be found in References 2 to 4. Table 3 from Reference 3 presents values of t_c as a function of degrees of freedom and confidence coefficients where the subscript c is given by $\frac{1}{2} + \frac{P}{2}$, where P is the fractional population desired. The degrees of freedom are determined by the number of samples (used to determine the random instrumentation standard deviation term) minus one. If a good estimate of the standard deviation of the random instrumentation error is available, then t_c will approach a value which simply reflects the desired population in terms of the number of standard deviations.

The tolerance factor, k, when used with the term for instrumentation error is determined in the same manner as is the tolerance factor for the apparent run-to-run dispersion, S^*rr. The components of k (r and u) are related to the number of samples and degrees of freedom used in obtaining S_{INST} while the population desired and confidence level are defined by the user. Under normal circumstances, it is anticipated that the standard deviation of the instrumentation error, as supplied by the engine manufacturer, will be of a sufficient confidence level and population so that no adjustment using the tolerance factor will be necessary.

Modeling Error, S_{MODEL}. When the engine is operating at conditions removed from the nominal or outside of the acceptance test conditions, it may be desirable to assign a value to S_{MODEL}. In general, the value of S_{MODEL} may be arbitrarily assigned by the user. This is necessary since one is actually extrapolating along the model equation into a region that may not have been substantiated by test data. To illustrate a possible error source, suppose that an equation for some given parameter is developed from many test firings. Suppose that all tests were conducted with propellant temperatures of approximately 70°F. In using regression techniques, the equation representing this parameter would not contain propellant temperature as a variable. If it is known from other engines of a similar type that the parameter is effected by propellant temperature, then one would be justified in establishing an appropriate value for S_{MODEL}. The above discussion of modeling error is related only to the effects of independent parameters upon dependent parameters. The need may also arise to apply an uncertainty to the dependent (output) parameters if errors due to computer convergence criterion or numerical integration techniques are to be accounted for.

Engine-to-Engine Sample Standard Deviation, S_{EE}. When a particular engine is to be used in a study, then $S_{EE} = 0$ in equation (4). However, if it is necessary to make a general propulsion study (i.e., what is the uncertainty associated with the performance if any engine is selected at random), then a term such as S_{EE} is needed. In this case:

$$k^2 S_{EE}^2 = \frac{k_1^2 \sum_i^N \left(\overline{X}_i - \overline{\overline{X}}\right)^2}{N - 1} - \left(k_2 S_{INST}\right)^2 \qquad (9)$$

where: $\overline{X}_i$ = The parameter mean for each engine.

$\overline{\overline{X}}$ = The average of the parameter means for all engines.

N = The number of engines considered.

S_{INST} = The random instrumentation error.

k_i = The tolerance factor as described above.

Equation (4) will then estimate the dispersion, S, required to insure that $\overline{X} \pm S$ will contain 68.27% of the population of all engines at some prescribed level of confidence. This equation should be applied to each parameter needed as input for a PATS simulation.

Example Problem. The total run-to-run and engine-to-engine variability of the oxidizer system resistance for the DPS engine will be determined (i.e., one standard deviation at 95% confidence will be calculated). This will be the resistance that corresponds to the pressure drop as measured from engine interface to injector face while the engine is operating at the fixed throttle position. The resistance is defined by:

$$R = \frac{\Delta P \cdot \rho \cdot 144}{\dot{w}^2} \qquad (10)$$

This particular parameter is not measured directly, but is calculated from the measured pressure drop (ΔP), oxidizer density (ρ) and oxidizer flowrate ($\dot{w}$); thus, its accuracy is a function of the variability of the measured parameters. Data from DPS engines 1026, 1037, and 1030, each with two acceptance tests, each test with several time slices, were analyzed. These calculated resistances are tabulated in Table 4 and are referred to as Cases A, B, and C, respectively. As previously mentioned, it is assumed that run-to-run variations for each engine are the same; thus the data from several engines may contribute to the run-to-run statistic. Mean values for each engine are:

$$\overline{X}_A = \frac{3913.2 + 3916.9}{2} = 3915.05$$

$$\overline{X}_B = \frac{3802.2 + 3799.0}{2} = 3800.6$$

$$\overline{X}_C = \frac{3918.6 + 3926.9}{2} = 3922.8$$

Using equation (6) one obtains

$$\Delta\overline{X}_1 = 3915.05 - 3913.2 = 1.85$$

$$\Delta\overline{X}_2 = 3915.05 - 3916.9 = -1.85$$

$$\Delta\overline{X}_3 = 3800.6 - 3802.2 = -1.6$$

$$\Delta\overline{X}_4 = 3800.6 - 3799.0 = 1.6$$

$$\Delta\overline{X}_5 = 3922.8 - 3918.6 = 4.2$$

$$\Delta\overline{X}_6 = 3922.8 - 3926.9 = -4.1$$

Before equation (7) can be used to determine Srr the instrumentation error (S_{INST}) must be defined. Since resistance is not measured directly, there is no direct instrumentation error. However, there is an uncertainty in the observed variability of R due to the instrumentation error in measuring ΔP, ρ, and $\dot{w}$. This variability is defined by the following equation (Reference 5)

$$S^2_{INST} = \left(\frac{\partial R}{\partial \rho} \cdot S_\rho\right)^2 + \left(\frac{\partial R}{\partial \dot{w}} \cdot S_{\dot{w}}\right)^2 + \left(\frac{\partial R}{\partial \Delta P} \cdot S_{\Delta P}\right)^2 \qquad (11)$$

where S_ρ, $S_{\dot{w}}$, and $S_{\Delta P}$ are standard deviations of the variabilities of the subscripted parameters and are obtained from engine manufacturer instrumentation error analysis. From Reference 6, assuming 95% confidence for this example:

$$S_\rho = 0.00067\ \rho$$

$$S_{\dot{w}} = 0.00083\ \dot{w}$$

$$S_P = 0.00173\ P$$

From equation (10), the following quantities may be determined:

$$\frac{\partial R}{\partial \rho} = \frac{\Delta P}{\dot{w}^2}(144) = \frac{R}{\rho}$$

$$\frac{\partial R}{\partial \Delta P} = \frac{\rho}{\dot{w}^2}(144) = \frac{R}{\Delta P}$$

$$\frac{\partial R}{\partial \dot{w}} = \frac{-2\rho \Delta P}{\dot{w}^3}(144) = \frac{-2R}{\dot{w}}$$

Then equation (11) becomes:

$$S^2_{INST} = \left(R\frac{S_\rho}{\rho}\right)^2 + \left(-2R\frac{S_{\dot{w}}}{\dot{w}}\right)^2 + \left(R\frac{S_{\Delta P}}{\Delta P}\right)^2 \qquad (12)$$

From Reference 6, the following equation may be written:

$$S^2_{\Delta P} = \left(\frac{\partial \Delta P}{\partial P_1}S_{P_1}\right)^2 + \left(\frac{\partial \Delta P}{\partial P_2}S_{P_2}\right)^2$$

or

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = \left(\frac{\partial \Delta P}{\partial P_1}\ \frac{S_{P_1}}{\Delta P}\right)^2 + \left(\frac{\partial \Delta P}{\partial P_2}\ \frac{S_{P_2}}{\Delta P}\right)^2 \qquad (13)$$

where

$$\Delta P = P_1 - P_2 \qquad (14)$$

where: P_1 = oxidizer interface pressure, psia

P_2 = chamber pressure, psia.

Then,

$$\frac{\partial \Delta P}{\partial P_1} = 1 \qquad\qquad \frac{\partial \Delta P}{\partial P_2} = -1 .$$

and

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = \left(\frac{S_{P_1}}{\Delta P}\right)^2 + \left(\frac{S_{P_2}}{\Delta P}\right)^2 \qquad (15)$$

It was given above that:

$$S_{P_1} = .00173\ P_1$$

and

$$S_{P_2} = .00173\ P_2$$

then,

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = \left(\frac{.00173P_1}{\Delta P}\right)^2 + \left(\frac{.00173P_2}{\Delta P}\right)^2$$

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = (.00173)^2\left[\left(\frac{P_1}{\Delta P}\right)^2 + \left(\frac{P_1 - \Delta P}{\Delta P}\right)^2\right]$$

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = (.00173)^2\left[2\left(\frac{P_1}{\Delta P}\right)^2 - 2\frac{P_1}{\Delta P} + 1\right]$$

At the FTP throttle position, $P_1 \approx 220$ and $P_2 \approx 110$, $\Delta P \approx 110$ and $\frac{P_1}{\Delta P} = 2$.

Therefore,

$$\left(\frac{S_{\Delta P}}{\Delta P}\right)^2 = (.00173)^2(5)$$

and substituting in equation (12)

$$S^2_{INST} = R^2(.00067)^2 + 4R^2(.00083)^2 + 5R^2(.00173)^2$$

$$S^2_{INST} = \left(18.11 \cdot 10^{-6}\right)R^2$$

$$S_{INST} = 4.26 \cdot 10^{-3}R$$

For an average resistance R of 3880,

$$S^2_{INST} = 16.51\frac{\sec^2}{ft^5}$$

The apparent run-to-run dispersion has been defined as:

$$(S^*_{rr})^2 = \frac{\sum_{p}^{N}\left(\Delta\bar{X}_p\right)^2}{N - 1}$$

For this example, then,

$$(S^{*}rr)^2 = \frac{(1.85)^2 + (-1.85)^2 + (1.6)^2 + (-1.6)^2 + (4.2)^2 + (-4.2)^2}{6 - 1}$$

$$= 9.45 \frac{sec^2}{ft^5}$$

It is now necessary to determine the tolerance factor, k. For

$N = 2$ (number of points used to establish mean value, $\overline{X}_k$)

$f = 5$ (degrees of freedom associated with $S^{*}rr$)

$\gamma = .95$ (confidence level)

$P = 68.27\%$ (proportion of population considered)

from Tables 1 and 2, the values of u and r of 2.0893 and 1.2778 respectively are obtained. Then,

$$k = (2.0893)(1.2778) = 2.6697$$

Equation (7) may now be applied:

$$(kSrr)^2 = (2.6697)^2(9.45) - (16.51)^2 = 67.4 - 272.5$$

As indicated as being possible, the variability due to random instrumentation uncertainties is greater than the apparent run-to-run variability. Inherent, however, in each value of R is the inability to determine the resistance more accurately than any one of the independent parameters (P, ρ, or $\dot{w}$) can be determined. Thus, we must set the true run-to-run variability to zero.

The final quantity needed is S_{MODEL} which should be given by those responsible for the engine model. Assume that

$$S_{MODEL} = .097\%$$

Equation (4) may now be used to determine the value of the standard deviation to be used in conjunction with a specific engine's Monte-Carlo analysis. For this case, Srr and S_{EE} are equal to zero.

For

$N = 2$ (number of points used to determine mean, $\overline{X}_k$)

$P = 68.27\%$ (proportion of population considered)

Assuming that S_{INST} is a good estimate of the true standard deviation, the "Student t" factor (t_c) is equal to a value of 1.0.

Using equation (4), one then obtains

$$S = \left[0.0 + \left(\frac{1.0(16.51)}{\sqrt{2}}\right)^2 + (00097 \cdot 3880)^2\right]^{\frac{1}{2}} = 12.27 \frac{sec^2}{ft^5}$$

If a Monte-Carlo analysis is to be performed for a particular engine, engine 1026, as an example, the mean value of resistance ($\bar{X}_A$) and the associated standard deviation (S) to be used are 3915.05 and 12.27, respectively If a study is to be completed for the class of three engines, the mean value of resistance is

$$\bar{\bar{X}} = \frac{\bar{X}_A + \bar{X}_B + \bar{X}_C}{N} = \frac{3915.05 + 3800.6 + 3922.8}{3} = 3879.4833$$

The parameter kS_{EE} must then be calculated for use in equation (9). For

γ = .95 (confidence level)

n = 3 (sample size)

P = 68.27% (proportion of population considered

f = n - 1 = 3 - 1 = 2 (degrees of freedom)

Tables 1 and 2 yield values of r and u of 1.3412 and 4.4154 respectively; thus, by equation (8) one gets

k = 1.3412 · 4.4154 = 5.922

As with run-to-run dispersions, the true engine-to-engine dispersion must be estimated from the observed engine-to-engine dispersion and random instrumentation error. Thus, using equation (9):

$$(kS_{EE})^2 = \left[5.922\left(\frac{(3915.05 - 3879.48)^2 + (3800.6 - 3879.48)^2 + (3922.8 - 3879.48)^2}{3 - 1}\right)^{\frac{1}{2}}\right]^2$$

$$- (16.51)^2$$

$$= [(5.92)(68.42)]^2 - (16.51)^2$$

$$= (404.71)^2$$

The magnitude of kS_{EE} reflects the very small sample size used in this example.

Substituting into equation (4) the yields:

$$S = \left[(12.27)^2 + (404.71)^2\right]^{\frac{1}{2}} = 404.90$$

Thus for a class study, the mean value of resistance ($\bar{\bar{X}}$) and the standard deviation (S) are 3879.48 and 404.90 respectively.

SECTION II. OPTIMUM PROPELLANT MANAGEMENT--BIASING AND RESERVES

Introduction. Section I was concerned with the probable variability of propulsion performance; Section II will define the means of minimizing the impact of such variability on the vehicle velocity (ΔV) capability.

Discussion. The ideal velocity equation (1) approximates the relationship between the propulsion system's performance and the attainable velocity of the Apollo vehicles to an accuracy sufficient to illustrate the value of optimum propellant biasing and flight performance reserves:

$$V_f = \bar{I}_{sp} g_c \ln(MR) - \bar{g} t_b + V_i \quad (1)$$

where

V_f = maximum attainable cutoff velocity

V_i = velocity at ignition

$\bar{I}_{sp}$ = average specific impulse over the duty cycle

MR = vehicle mass ratio = $\frac{\text{Initial Vehicle Mass}}{\text{Final Vehicle Mass}} = \frac{M_i}{M_f}$

g_c = weight to mass conversion factor

$\bar{g}$ = average gravitational attraction during the duty cycle

t_b = engine burn time during the duty cycle

Temporarily ignoring the acceleration term ($\bar{g}t_b$) in equation (1), rearranging and substituting, one gets

$$\Delta V = V_f - V_i = \bar{I}_{sp} g_c \ln\left(\frac{M_i}{M_f}\right) \quad (2)$$

Since there is some lack of confidence associated with predicted performance, it becomes necessary to carry reserve propellants in order to guarantee the attainment of a particular ΔV. Of course, the carrying of such reserves represents a vehicle performance loss since they are traded off with payload. Since the reserves are proportional to the variability of predicted propulsion performance, it becomes obvious that appreciable payload gains may be made by reducing systems variabilities and improving confidence in flight prediction capabilities. Confidence (tolerance bands) in predicted values can be improved only by improved analysis, techniques, testing, and instrumentation; however, the impact of the system variability may be reduced by optimum propellant biasing and reserves.

Ideally, the vehicle cutoff velocity (V_f) would be attained at the same time both usable propellants are simulatneously consumed. Since most vehicles and environmental influences cannot be predicted with 100% confidence, the probability of attaining that terminal velocity (with the nominal propellant loading) will be considerably less than 1. However, from the proper statistical analyses of all performance affecting influences, the probability of occurrence of potential terminal velocities can be determined; that is the cutoff velocity will be in the band $V_f - \delta v_f$ to $V_f + \zeta v_f$ with V_f usually having the highest probability of occurrence. The upper and lower bands were established with different probability terms (δ and ζ) since the distribution is not usually Gaussian. The better than predicted performance is of trivial importance for the Apollo spacecrafts since cutoff is initiated at V_f by the guidance system and not by propellant depletion. Since the lower velocity value is possible, it is necessary to carry reserve propellants to guarantee the attainment of V_f. Typical influences on terminal velocity are uncertainties associated with the following items:

1. Final mass (due to propellant outage)
2. Average specific impulse
3. Thrust
4. Guidance system
5. Trajectory
6. Stage weight (structure, miscellaneous items, payload)
7. Initial propellant weight and MR

The individual variabilities of terminal velocity due to items 2 through 7 are usually nearly normally distributed. However, because mixture ratio (by weight) is other than unity, normally distributed variations in it result in non-normal distributions in propellant outage. Furthermore, any propellant outage results in a velocity loss as compared to the no outage case. The outage affect is by far the predominate influence on the non-normal distribution of V_f, although the log term of equation (2) shows that the effect of any weight term is non-normal. Likewise, one would not expect thrust variations, because of the coupled gravity affect, to yield normal variations on V_f.

Uncorrelative Independent Parameters. For optimal propellant management calculations, the typical procedure is to assume that the listed influences act independently of one another; then their equivalent velocity variations are statistically combined to obtain δv_f and ζv_f. Since outage is a very strong influence on V_f its minimization can appreciably increase the final velocity at the desired probability and confidence. Before developing the optimum fuel bias equation to minimize outage, the following definitions are made:

μ = average vehicle mixture ratio over the duty cycle

σ_μ = one standard deviation in mixture ratio variability (includes) propulsion system, propellant loading, and unusable propellant variability)

k = tolerance factor for desired probability and confidence

WC = propellant nominally consumed during the duty cycle

WOT = propellant outage; that value of usable propellant that would be consumed if an apportioned amount of the other propellant were available for combustion.

o = subscript oxidizer

f = subscript fuel

p = subscript sum of oxidizer and fuel

If the nominal propellant loads are such that simultaneous depletion takes place at the same time V_f is attained, the maximum oxidizer and fuel outage that will exist for a Gaussian variability in μ and an open loop propellant utilization system is determined by equations (3) and (4) (Reference 9).

$$WOT_o = \frac{k\sigma_\mu \, WC_p}{(\mu + 1)} \tag{3}$$

$$WOT_f = \frac{k\sigma_\mu \, WC_p}{(\mu + k\sigma_\mu)(\mu + 1)} \tag{4}$$

If outage is plotted versus mixture ratio variability, it is found to consist of two legs that are approximately linear but which have different slopes, as illustrated in Figure 1-a by the solid line. Since the system mixture ratio is greater than one, negative variations in μ cause maximum (oxidizer) outages. Such variations would result in an outage frequency distribution similar to the solid line of Figure 1-b. From Figures 1-a and 1-b, it is apparent that outage is not normally distributed even if μ is. If the fuel bias (WB) is defined as that amount of fuel that causes the maximum oxidizer and fuel outage to be equal, one easily obtains equations (5) and (6) for oxidizer and fuel outage respectively, and equation (7) for the optimum fuel bias.

$$WOT_o = \frac{k\sigma_\mu \, WC_p}{(\mu + 1)} - \left(\mu_c - k\sigma_\mu\right) WB \quad (5)$$

$$WOT_f = \frac{k\sigma \, WC_p}{\left(\mu + k\sigma_\mu\right)\left(\mu + 1\right)} + WB \quad (6)$$

$$WB = \frac{k\sigma_\mu \, WC_p \left(\mu - 1 + k\sigma_\mu\right)}{\left(\mu_c + 1 - k\sigma_\mu\right)\left(\mu + k\sigma_\mu\right)\left(\mu + 1\right)} \quad (7)$$

where μ_c is the mixture ratio at cutoff.

The dotted line of Figure 1-a illustrates the typical variation of outage (with fuel bias included) for anticipated variations in μ while the dotted line of Figure 1-b typifies the resulting change for the propellant outage frequency distribution. Although the fuel bias causes a v_f decrease for the nominal case and increases the probability of fuel outage, the total outage probability will be minimized; particularly the occurrence of extreme values of outage that affect ΔV the greatest. By minimizing propellant outage, the bias appreciably reduces the propellant reserve requirements. In fact, it is easily shown that at the desired probability and confidence of accomplishing a mission,

Baised Propellant Reserve + Bias < Unbiased Propellant Reserve;

thus, propellant reserve requirements can be appreciably reduced, with the significance of the reduction proportional to the average vehicle mixture ratio and total propellant load.

Correlative Independent Parameters. In the preceding section the parameters that affect vehicle maximum velocity gain variability were considered to act independently of one another, when in fact, certain of those parameters may be highly correlated. Thus, the method of combining their influences (root sum square, for example) may result in either pessimistic or optimistic answers, depending on the actual correlation. As an example, the correlation of mixture ratio (μ), specific impulse (I_{sp}), and outage (WOT) will be considered. As noted in the preceding section, anticipated velocity variations due to outage are proportional to the uncertainty associated with predicted mixture ratio; likewise, there are velocity uncertainties due to the uncertainty associated with predicted specific impulse. Portions of these respective uncertainties are truly independent of one another: that due to instrumentation error, run-to-run variability, etc. However, portions of the total uncertainties may be highly correlated; e.g. I_{sp} which is strongly influenced by μ. Possible variations in mixture ratio due to the uncertainties in propellant temperatures, pressures, etc., result in corresponding variations in specific impulse. The effects of these variations are fixed and predictable. Figure 2 (a and b) are plots of correlative outage and specific impulse as functions of mixture ratio, for a hypothetical engine. Figure 2c represents the corresponding maximum velocity variations due to both influences, while the solid line of Figure 2d shows the resultant maximum velocity variations. This composite maximum velocity distribution may be considered independently of all other velocity distributions (if there are no other correlative parameters) and combined (RSS) with them, and with the distributions due to the random variations in mixture ratio and specific impulse. For the engine considered, ignoring the correlation of μ and I_{sp} would result in a propellant reserve greater than actually required; however, if the slope of the I_{sp}-μ relationship were positive, the reserve would be inadequate. Figure 2 also shows that the total variability of terminal velocity may be minimized by carrying an oxidizer bias which, by reducing the possible fuel outage, would result in the distribution illustrated in Figure 2d by the dotted line. Thus, the reserve requirement can be further reduced. Ideally, both the independent and correlative variations would be utilized to determine the optimum combination of bias and reserves. Prime parameters that are usually partially correlative are mixture ratio, specific impulse, thrust, the guidance system effects, and the trajectory. Since optimum biasing and reserve determination for correlative variables do not lend themselves to simple closed form solutions, iterative techniques are usually employed in conjunction with complete system models.

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7. TRW I.O.C. 4721.3.68-114, "LM-3 Engine Serial No. 1030 Descent Engine Characteristic Equations," J. T. Latimore and B. A. Raine, May 1968.
8. TRW Report 05952-6106-R000, "User's Manual for Propulsion Analysis Trajectory Simulation Program (PATS)," G. E. Hull and A. T. Baumgartner.
9. NASA-MSFC Report, Design Criteria for Propellant Management Program, January 15, 1965.

TABLE 1*

Values of "r" for Tolerance Factors with Population Fraction, P, and Sample Size, N.

N \ P	.50	.75	.90	.95	.99	.999
1	1.0505	1.6859	2.2844	2.6463	3.3266	4.0903
2	0.8557	1.4333	2.0078	2.3624	3.0368	3.7983
3	0.7929	1.3412	1.8979	2.2457	2.9128	3.6708
4	0.7622	1.2940	1.8388	2.1815	2.8422	3.5965
5	0.7442	1.2654	1.8019	2.1408	2.7963	3.5472
6	0.7322	1.2463	1.7768	2.1127	2.7640	3.5119
7	0.7237	1.2326	1.7587	2.0922	2.7399	3.4853
8	0.7175	1.2224	1.7448	2.0765	2.7211	2.4644
9	0.7127	1.2144	1.7340	2.0641	2.7066	3.4476
10	0.7088	1.2080	1.7253	2.0541	2.6945	3.4338
11	0.7056	1.2027	1.7182	2.0459	2.6845	3.4223
12	0.7030	1.1984	1.7122	2.0390	2.6760	3.4125
13	0.7008	1.1947	1.7071	2.0331	2.6688	3.4040
14	0.6989	1.1915	1.7027	2.0280	2.6625	3.3967
15	0.6973	1.1887	1.6990	2.0236	2.6571	3.3902
16	0.6958	1.1863	1.6956	2.0197	2.6523	3.3845
17	0.6945	1.1842	1.6926	2.0163	2.6480	3.3794
18	0.6934	1.1823	1.6901	2.0132	2.6441	3.3748
19	0.6924	1.1807	1.6877	2.0105	2.6407	3.3707
20	0.6915	1.1792	1.6855	2.0080	2.6376	3.3670
21	0.6907	1.1778	1.6837	2.0058	2.6348	3.3636
22	0.6900	1.1765	1.6819	2.0037	2.6322	3.3605
23	0.6893	1.1754	1.6803	2.0018	2.6298	3.3576
24	0.6887	1.1743	1.6788	2.0001	2.6276	3.3550
25	0.6881	1.1734	1.6775	1.9985	3.6256	3.3526
26	0.6875	1.1725	1.6762	1.9971	2.6238	3.3503
27	0.6870	1.1717	1.6750	1.9957	2.6221	3.3482
28	0.6866	1.1709	1.6740	1.9945	2.6205	3.3462
29	0.6862	1.1702	1.6730	1.9933	2.6190	3.3444
30	0.6858	1.1695	1.6721	1.9922	2.6176	3.3427
40	0.6830	1.1647	1.6653	1.9842	2.6074	3.3301
50	0.6813	1.1618	1.6612	1.9794	2.6012	3.3225
60	0.6801	1.1600	1.6585	1.9762	2.5970	3.3173
70	0.6793	1.1586	1.6566	1.9739	2.5940	3.3135
80	0.6787	1.1575	1.6551	1.9722	2.5917	3.3107
90	0.6782	1.1568	1.6540	1.9708	2.5900	3.3085
100	0.6779	1.1561	1.6531	1.9697	2.5886	3.3067
∞	0.6745	1.1504	1.6449	1.9600	2.5758	3.2905

*A. Weissburg and G. L. Beatty, "Tables of Tolerance-Limit Factors for Normal Distributions," Technometrics, Vol. 2, No. 4, November 1960.

N = Number of measurements used to obtain $\overline{X}_k$ (the sample estimate of the population mean).

P = Proportion of population included between limits.

TABLE 2*

Values of "u" for Tolerance Factors With Confidence γ and f degrees of freedom

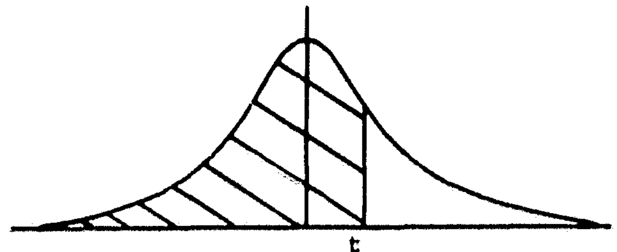
f \ γ	.50	.90	.95	.99
1	1.482	7.9579	15.9472	79.7863
2	1.201	3.0808	4.4154	9.9749
3	1.126	2.2658	2.9200	5.1113
4	1.092	1.9393	2.3724	3.6692
5	1.072	1.7621	2.0893	3.0034
6	1.059	1.6499	1.9154	2.6230
7	1.050	1.5719	1.7972	2.3769
8	1.044	1.5141	1.7110	2.2043
9	1.039	1.4694	1.6452	2.0762
10	1.035	1.4337	1.5931	1.9771
11	1.031	1.4043	1.5506	1.8980
12	1.029	1.3797	1.5153	1.8332
13	1.026	1.3587	1.4854	1.7792
14	1.024	1.3406	1.4597	1.7332
15	1.023	1.3248	1.4373	1.6936
16	1.021	1.3108	1.4176	1.6592
17	1.020	1.2983	1.4001	1.6288
18	1.019	1.2871	1.3845	1.6019
19	1.018	1.2770	1.3704	1.5778
20	1.017	1.2678	1.3576	1.5560
21	1.016	1.2594	1.3460	1.5363
22	1.015	1.2517	1.3353	1.5184
23	1.015	1.2446	1.3255	1.5020
24	1.014	1.2380	1.3165	1.4868
25	1.014	1.2319	1.3081	1.4729
26	1.013	1.2262	1.3002	1.4600
27	1.013	1.2209	1.2929	1.4479
28	1.012	1.2159	1.2861	1.4867
29	1.012	1.2112	1.2797	1.4263
30	1.011	1.2068	1.2737	1.4164
40	1.008	1.1734	1.2284	1.3434
50	1.007	1.1518	1.1993	1.2973
60	1.006	1.1364	1.1787	1.2651
70	1.005	1.1248	1.1631	1.2411
80	1.004	1.1156	1.1510	1.2224
90	1.004	1.1082	1.1410	1.2072
100	1.003	1.1019	1.1328	1.1947
∞	1.000	1.0000	1.0000	1.0000

*A. Weissburg and G. L. Beatty, "Tables of Tolerance - Limit Factors for Normal Distributions," Technometrics, Vol. 2, No. 4, November 1960.

f = Number of degrees of freedom associated with s (sample estimate of population standard deviation).

γ = Confidence coefficient associated with limits.

TABLE 3*



PERCENTILE VALUES (t_p)
for
STUDENTS t DISTRIBUTION
with f degrees of freedom
(shaded area = p)

f	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.80}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	6.85	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

*Murray R. Speigel, Theory and Problems of Statistics, Schaum Publishing Company, New York, 1961.

TABLE 4

DPS ENGINE, OXIDIZER RESISTANCE AT FTP

DPS Engine Serial Number	Acceptance Test Number	Time Slice	Calculated Oxidizer Line Resistance, R_0, $\frac{sec^2}{ft^5}$ (Engine Interface To Thrust Chamber)	Average Line Resistance, R, For Test
1026[1]	HATS 176	20	3916.89	
				3916.9
	HATS 177	7	3913.22	
				3913.2
1037[2]	HATS 190	6	3755.84	
		8	3819.42	
		12	3810.89	
		16	3812.91	
		18	3825.62	
		21	3788.52	
				3802.2
	HATS 191	7	3799.05	
				3799.0
1030[3]	HATS 219	7	3912.54	
		9	3921.97	
		11	3915.25	
		13	3918.50	
		15	3921.67	
		17	3919.27	
		21	3920.79	
				3918.6
	HATS 221	7	3916.53	
		9	3921.59	
		11	3926.66	
		13	3932.03	
		15	3928.03	
		17	3928.85	
		19	3934.89	
		21	3927.50	
				3926.9

[1]TRW Report No. 01827-6076-T000, "TRW LM Descent Engine S/N 1026 Acceptance Test Performance Report," dated 22 May 1967.

[2]TRW Report No. 01827-6098-T000, "TRW LM Descent Engine S/N 1037 Acceptance Test Performance Report," dated 29 July 1967.

[3]TRW Report No. 01827-6122-T000, "TRW LM Descent Engine S/N 1030 Acceptance Test Performance Report," dated 11 December 1967.

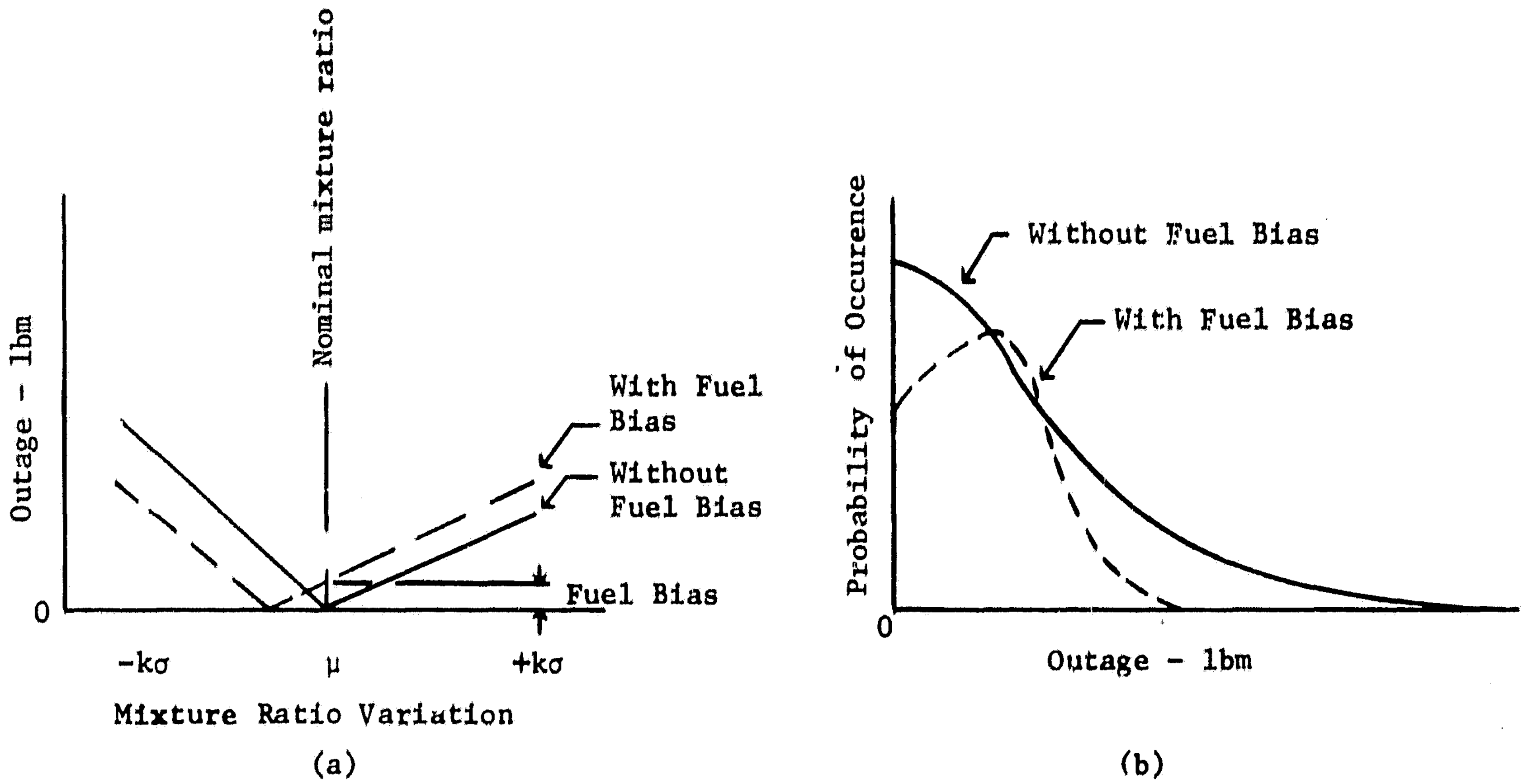


Figure 1

Typical Propellant Outage Distributions

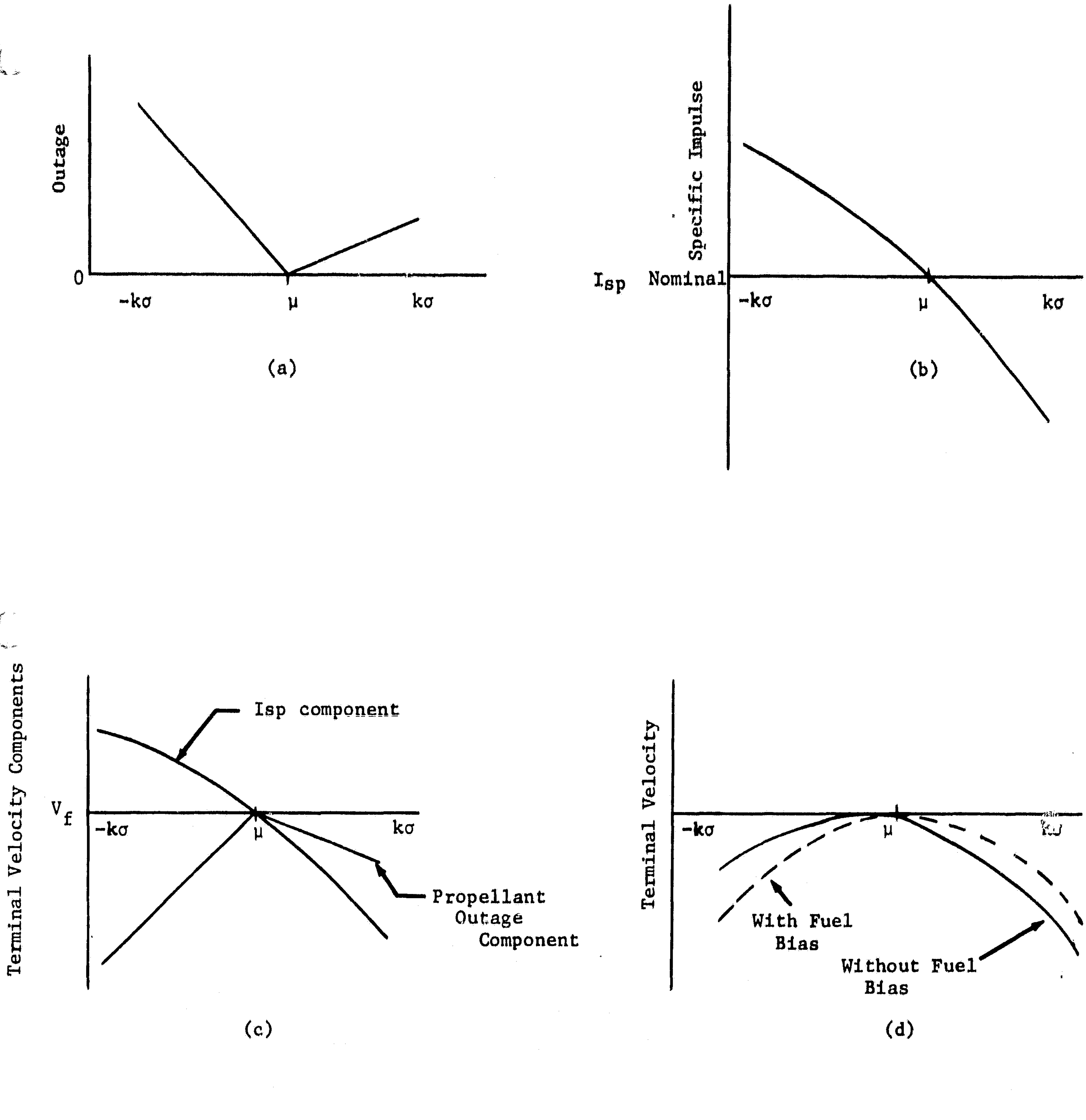


Figure 2

Correlated Specific Impulse and Mixture Ratio Relationship