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COSMIC RAYS AND DYNAMICS OF THE SOLAR WIND

I

by

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SUMMARY

When investigating the propagation of cosmic rays in interplanetary space, solar wind is considered as given and independent of cosmic rays. It is shown that, considering rigorously the problem, one must take into account the inverse action of cosmic rays on the fluxes of solar plasma, which is materialized by way of magnetic fields. The integral-differential equation is derived, which describes the interaction of solar wind with cosmic rays, taking into account the intensity modulation and the deceleration of solar wind by cosmic rays.

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During the investigation of modulation of galactic cosmic rays and the propagation of solar cosmic rays in interplanetary space, solar wind is considered as given and independent of cosmic ray intensity. In particular, in the equation, describing the propagation of cosmic rays in interplanetary space and the modulation of their density n [1]

$$\frac{\partial n}{\partial t}+\sum_{\alpha=1}^{4}\frac{\partial j_\alpha}{\partial x_\alpha}=\Phi(r,\theta,\varphi,R,z,t) \quad , \tag{1}$$

the particle fluxes j_α with charge Ze (α = 1, 2, 3, 4 are the spatial coordinates r, θ, ϕ and the rigidity R; Φ is the function of the source) are determined by the given solar wind velocity u(r), while the pressure of galactic cosmic rays is comparable or even greater than the pressure of the galactic magnetic field [2]. This is why one must take into account that as the intensity of cosmic rays varies in time (with the variation of solar activity) and in space (with recession from the Sun), the action of cosmic rays on solar wind varies also, which leads in its turn to the variation of u(r). In the first part, we shall find the equation describing the interaction of solar wind with cosmic rays. In the second part [3], its approximate solution is found under specific assumptions on the properties of the low-energy part of cosmic radiation beyond the limits of the solar system.

Integral-Differential Equation describing the Solar Wind Interaction with Cosmic Rays. It was noted above that solar wind not only acts substantially on cosmic rays, but undergoes itself the influence of the latter, whereupon this influence is so substantial that it may be one of the basic causes of variation of solar wind velocity and limitation of its propagation in the interstellar space. This is why all the works where the modulation of solar wind by cosmic rays is investigated and the solar wind is considered as given (see, for example, [1, 4-6]), are not strict. It is necessary to take into account the inverse action of cosmic rays on solar wind, i. e. to resolve a self-consistent problem.

Let us consider the action of cosmic rays on the radial motion of an isolated plasmoid with frozen-in magnetic field, the transverse and longitudinal dimensions being respectively l and L. We shall consider that in the first approximation cosmic rays are distributed isotropically in interplanetary space. Then the pressure exerted by them will be isotropic, and at the distance r from the Sun the pressure will be determined by their energy density

$$\int_0^\infty n(r,R)E_K\,dR,$$

where E_k is the kinetic energy of particles, n(r, R) is the hard spectrum of cosmic ray concentration, The deceleration of the isolated plasmoid will be determined by the difference in pressure of cosmic rays on the forward and rear surfaces:

$$F_{dec} = l^2\int_0^\infty [n(r,R)-n(r+L,R)]E_K\,dR = -l^2L\int \frac{dn(r,R)}{dr}E_K\,dR. \tag{2}$$

The concentration of plasma, N(r), at the distance r from the Sun on account of continuity law, will be

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$$N(r) = N_{\odot} u_{\odot} r_{\odot}^{2} \; u^{-1} r^{-2}, \tag{4}$$

where $r_{\odot}$ = 1 a.u. is the distance Earth - Sun; $N_{\odot} \equiv N(r_{\odot})$; $u_{\odot} \equiv u(r_{\odot})$, where u is the radial velocity of plasma motion. Then the mass of the isolated area of plasma will be

$$M \doteq l^2 L N_{\odot} u_{\odot} r_{\odot}^{2} \; u^{-1} r^{-2} M_p, \tag{4}$$

where M_p is the mass of the hydrogen atom. On the basis of (2) and (4) we shall find the acceleration of deceleration du/dt = T_{dec}/M and the velocity gradient

$$\frac{du}{dr} = -\frac{r^2}{N_{\odot} u_{\odot} r^2_{\odot} M_p} \int_0^{\infty} \frac{dn(r, R)}{dr} E_k dR, \tag{5}$$

where it is taken into account that udt = dr.

The multiplier dn(r, R)/dr, entering into (5), is determined by plasma velocity u(r, t) and by transport free path for particle scattering Λ(r, R, t) by means of a nonstationary equation of anisotropic diffusion [1], or, to be more precise, by the kinetic equation [7]. However, it is then impossible to obtain a simple analytical expression and difficult to investigate Eq.(5). This is why we shall make use of the following two cases, substantially alleviating our problem. First, as is shown in [1], when investigating long-period variations of cosmic rays, one may use with great precision the stationary equation, considering t as a parameter. Secondly, according to [1], a rigorous analytical solution of the modulation problem in two simplest cases, when Λ = const and when $\Lambda \sim r$, may be represented with a sufficient precision (to 10%) in the form

$$n(r, R, t) = n_0(R) \exp\left[-\int_r^{r_0} \frac{3u(r, t)\,dr}{v\Lambda(r, R, t)}\right], \tag{6}$$

where n_0(R) is spectrum of cosmic ray concentration beyond the limits of solar wind; r_0 are the dimensions of solar wind assigned in [8] on the basis of hysteretic phenomena in cosmic rays; v is the velocity of particles. However, in our case one should spread the integration over r in the right-hand part of (6) from r to ∞, inasmuch as the solar wind limitation is materialized by the quantity u(r,t) with the aid of Eq.(5). As already noted, we may consider in accord with [1, 9], t in (6) and (5) as a paramete- and in the following we shall drop it. Differentiating (6) with respect to r and substituting into (5), we shall obtain for the determination of u(r) the integral-differential equation

$$u' = \frac{du}{dr} = -\frac{3ur^2}{N_\odot u_\odot r_\odot M_p}\int_0^\infty \frac{n_0(R)E_K}{v(R)\Lambda(r,R)}\exp\left[-\int_r^\infty \frac{3udr}{v\Lambda}\right]dR. \qquad (7)$$

The substitution of velocity u(r), determined with the aid of (7), into (6) (as $r \to \infty$) will allow us to find also u(r,R).

Transformation of the Integral-Differential Equation into a Nonlinear Differential Equation of Second Order. Selection of $n_0(R)$. According to [2], for the density spectrum of nucleons beyond the limits of the modulating volume, one should choose the expression

$$n_0(R) = \begin{cases} aR^{-(\gamma+1)} & \text{if} \quad R \geqslant R_{\min}, \\ 0 & \text{if} \quad R < R_{\min}. \end{cases} \qquad (8)$$

Here $\gamma \approx 1{,}5$, $n_0(R)$ per nucleon $cm^{-3}\cdot Bv^{-1}$. As to the quantity R_{min}, it may be asserted on the basis of experimental data (see [2]) that it is knowingly smaller than 0.5 Bv (the value $R_{min} \approx 0.5$ Bv corresponds to energy density of cosmic rays in the interstellar medium ~ 1 ev/cm^3). Subsequently, calculations will be conducted for R_{min} = 0.5, 0.2, 0.1 and 0.05 Bv. Taking into account that in the interstellar medium cosmic rays are distributed nearly isotropically and that the number of nucleons in nuclei and protons is about identical, we find

$$a \approx \frac{4\pi}{c}\cdot 1{,}4 \approx 6\cdot 10^{-10} \text{ nucleon cm}^{-3}\cdot\text{Bv}^{1.5} \qquad (9)$$

Approximate Expression for E_k. For protons the kinetic energy is

$$E_k = b(\sqrt{R^2 + 0.88} - 0.94) \text{ ergs;}$$

R is given in Bv, $b = 1.6\cdot 10^{-3}$ is the conversion factor to the CGS system. For nucleons in nuclei $E_k = b(\sqrt{R^2 + 3{,}52} - 1.88)$. Inasmuch as the number of nucleons in nuclei and protons is about identical, the mean value of E_k will be:

$$E_k = 0,5b[\sqrt{R^2+0,88}+\sqrt{R^2+3,52}-2,82] \approx \frac{0,44bR^2}{1+0,4R} \tag{10}$$

where the last expression is valid with a relative error of no more than 10%:

R, Bv	0	0.5	1	2	4	8	∞
$\frac{E_{k\ ex}}{E_{k\ appr}}$	0.90	0.98	1.07	1.09	1.06	1.00	0.90

Approximate Expression for v(R). Inasmuch as for protons $v(R) = c(R^2 + 0.88)^{-1/2}$, and for nucleons in nuclei $v(R) = cR(R^2 + 3.52)^{-1/2}$ the average is

$$v(R) = 0,5\ cR[(R^2+0,88)^{-1/2} + (R^2+3,52)^{-1/2}] = cR(R^2+1,57)^{-1/2}, \tag{11}$$

where the last expression is correct with a relative error not exceeding 6%:

R, Bv	0	0.5	1	2	4	8	∞
$\frac{E_{k\ ex}}{E_{k\ appr}}$	1.06	1.05	1.04	1.02	1.01	1.00	1.00

Approximate Expression for the Dependence of Λ on R. According to [10], in the region (R $\lesssim$ 1 Bv) is almost independent of R. For $3 \gtrsim R \gtrsim 1$ Bv, $\Lambda \sim \sqrt{R}$ [11]. In the region R $\gtrsim$ 2 – 3 Bv, Λ rises by ∿ R through R ∿ 15 Bv [9]. This is why it is possible to approximately represent

$$\Lambda = 0,63\ \Lambda_0\sqrt{R^2+1,57}, \tag{12}$$

where Λ_0 is the transport free path for scattering at R $\approx$ 1 Bv. The character of Λ dependence on R is as follows:

R, Bv	0	0.5	1	2	4	8	16
Λ/Λ_0	0.79	0.85	1.00	1.48	2.63	5.1	10.1

Note that the expression (12) satisfactorily describes the dependence of Λ on R only for R $\lesssim$ 15 Bv (according to [9], for R $\gtrsim$ 15 Bv, we apparently have Λ ∿ R ; however, this energy region is no longer essential for our problem).

Density of Cosmic Rays in Interstellar Space. Strictly speaking, the form of the function $n_0(R)$ in the region of small rigidities is unknown. In accord with [2], we choose $n_0(R) \sim R^{-2.5}$. The energy density of cosmic rays W_{cr} in the interstellar space is then very feebly dependent on the quantity R_{min}.

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$$W_{cr} = \int_{R_{min}}^{\infty} n_0(R) E_k(R)\, dR \approx 0{,}84 \left(\frac{\pi}{2} - \operatorname{arctg} \sqrt{0{,}4 R_{min}}\right) \quad \text{ev} \cdot \text{cm}^{-3}, \tag{13}$$

where the last expression was obtained by utilizing for $E_k(R)$ formula (10), the values of W_{cr} as a function of R_{min} are as follows:

R_{min}, Bv	10	2	1	0.5	0.2	0.1	0.05	0
W_{cr}, ev/cm^3	0.39	0.71	0.84	0.96	1.09	1.15	1.20	1.32

Transformations of Eq.(7). Substituting (8), (10) - (12) into (7), we obtain

$$-\frac{u'\Lambda_0}{kur^2} = \int_{R_{min}}^{\infty} \frac{R^{-\gamma}}{1+0{,}4R} e^{-\beta(r,u)/R}\, dR \tag{14}$$

where

$$k \equiv \frac{2{,}1ab}{N_{\odot} u_{\odot} c r_{\odot}^2 M_p}, \quad \beta(r,u) \equiv \int_r^{\infty} \frac{4{,}8u\,dr}{c\Lambda_0}. \tag{15}$$

By variable substitution we transform Eq.(14) to the form

$$-\frac{u'\Lambda_0}{kur^2} = \beta^{-\gamma+1} \int_0^{\beta/R_{min}} \frac{x^{\gamma-1} e^{-x}}{x+0{,}4\beta}\, dx \equiv Y(\beta, R_{min}). \tag{16}$$

The results of numerical calculations of $Y(\beta, R_{min})$ at $\gamma = 1.5$, are compiled in Table 1.

TABLE 1

R_{min}, Bv \ β	0	0,1	0,2	0,3	0,5	0,7	1,0	1,5	2	4	10
0,5	1,34	1,30	1,15	1,053	0,900	0,739	0,590	0,439	0,333	0,166	0,048
0,2	2,84	2,33	1,90	1,595	1,183	0,895	0,657	0,459	0,339	0,166	0,048
0,1	4,60	3,19	2,32	1,805	1,235	0,912	0,659	0,460	0,340	0,166	0,048
0,05	7,15	3,80	2,47	1,850	1,240	0,913	0,660	0,460	0,340	0,166	0,048

The asymptotic expressions for $Y(\beta, R_{min})$ at $\beta \ll 1$ and $\beta \gg 1$ are the following

$$Y(\beta, R_{min}) = 2\left[R_{min}^{-1/2} - \sqrt{0{,}4}\operatorname{arctg}(0{,}4R_{min})^{-1/2}\right] - 2\beta\left[R_{min}^{-3/2}/3 - 0{,}4R_{min}^{-1/2} + (0{,}4)^{3/2}\operatorname{arctg}(0{,}4R_{min})^{-1/2}\right] \quad \text{for} \quad \beta \ll 1, \tag{17}$$

$$Y(\beta, R_{min}) \approx 1{,}5 \cdot \beta^{-3/2} \quad \text{for} \quad \beta \gg 1. \tag{18}$$

Assume that Eq,(16) is resolved relative to β, i.e. that $\beta = \beta(Y, R_{min})$ has been found. Then, inasmuch as

$$\frac{d\beta}{dr} = \frac{d\beta}{dY}\frac{dY}{dr} \tag{19}$$

and, according to (16) and (17),

$$\frac{d\beta}{dr} = -\frac{4{,}8u}{c\Lambda_0}, \quad \frac{dY}{dr} = -\frac{(u''\Lambda_0 + u'\Lambda_0')ur^2 - (2ru + u'r^2)u'\Lambda_0}{k(ur^2)^2}, \tag{20}$$

we obtain

$$\psi'' = -\frac{\varkappa\rho^2}{\lambda_0^2\varphi(Y)} + \frac{2\psi'}{\rho} + \frac{(\psi')^2}{\psi} - \frac{\psi'\lambda_0'}{\lambda_0}; \tag{21}$$

Eq.(21) is written in dimensionless variables

$$\psi = u / u_{\odot}, \qquad \rho = r / r_{\delta}, \qquad \lambda = \Lambda_0 / r_{\delta} \tag{22}$$

and the denotation $\varphi(Y) \equiv d\beta / dY$. The dimensionless parameter

$$\varkappa = \frac{4{,}8ku_{\delta}r_{\delta}^2}{c} = \frac{10ab}{N_{\delta}c^2M_p} = 6{,}4\ 10^{-9} / N_{\delta} \tag{23}$$

is determined only by solar wind concentration near the Earth's orbit and by the density of cosmic rays in the interstellar space (coefficient a).

Determination of Function $\phi(Y)$. In dimensionless variables (22), and according to (16) and (23), parameter Y will be

$$Y = -2{,}5\cdot 10^{-2}u_{\delta}N_{\delta}\lambda_0\rho^{-2}\psi'\psi^{-1}. \tag{24}$$

Functions $\phi(Y) = d\beta / dY$, which are found by graphical differentiation with respect to data of Table 1, are compiled in Table 2(when finding $d\beta / dY$ at small and great β, asymptotic expressions (17) and (18) were used). Note that Y varies within the limits $0 \leqslant Y \leqslant Y_{max}$, where Y_{max} = 1.34, 2.84, 4.60 and 7.15, respectively at R_{min} = 0.5, 0.2, 0.1 and 0.05 Bv.

TABLE 2

$R_{min} = 0{,}5$ Bv	Y	1,34	1,30	1,15	1,053	0,900	0,739	0,590	0,439	0,33	0,166	0,048
	$-\varphi(Y)$	0,595	1,28	1,31	1,62	1,58	1,63	2,30	3,99	6,23	21,3	158,3
$R_{min} = 0{,}2$ Bv	Y	2,84	2,33	1,90	1,595	1,183	0,895	0,657	0,459	0,339	0,166	0,048
	$-\varphi(Y)$	0,149	0,166	0,287	0,397	0,686	0,975	1,71	3,29	5,60	21,3	158.3
$R_{min} = 0{,}1$ Bv	Y	4,60	3,19	2,32	1,805	1,235	0,912	0,657	0,459	0,339	0,166	0,048
	$-\varphi(Y)$	0,051	0,099	0,166	0,264	0,532	0,955	1,71	3,29	5,60	21,3	158.3
$R_{min} = 0{,}05$ Bv	Y	7,15	3,80	2,47	1,85	1,240	0,912	0,657	0,459	0,339	0,166	0,048
	$-\varphi(Y)$	0,0175	0,046	0,112	0,216	0,528	0.955	1.71	3,29	5,60	21,3	158.3

It follows from Table 2 that in the region $0.05 \leqslant Y \leqslant Y_{max}$ function $\phi(Y)$ may be approximately (with relative precision ±10%) represented in the form

$$\varphi(Y) = -0{,}8\, Y^{-2}. \tag{25}$$

As $Y \to 0$, we have in accord with (18), $\phi(Y) = -0.87\, Y^{-5/3}$. This differs little from (25). Besides, the region of very small Y (correspondnng to the regions of very great β) has no essential value for our problem and this is why we shall spread the variation of Y over the entire region.

<u>Differential Equation for the Determination of ψ.</u> Substituting (25) into (21) and taking into account (24) and (25), we shall obtain a nonlinear differential equation of second order for the determination of ψ:

$$\psi'' = \psi'[5\cdot 10^{-12} u_{\delta}{}^{2} N_{\delta}\, \rho^{-2} \psi' \psi^{-2} + 2\rho^{-1} + \psi'\psi^{-1} - \lambda_0'\lambda_0^{-1}]. \tag{26}$$

As to boundary conditions, we actually have only one condition: by definition $\psi(1) = 1$. The second condition for $\psi'(1)$ must be obtained with the aid of the differential equation (7) or relations equivalent to it.

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